Dynamic competitive pricing strategies *

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Pricing decisions have long been of interest to researchers in the competitive strategy area. Static game theory models of competitive pricing commonly assume that rivals act with full information and full rationality. Such models imply that rivals immediately reach a Nash equilibrium. This implication is consistent neither with experimental research results nor with casual observation of real markets. We therefore relax the assumption of full information to investigate how competitors learn and update their marketing strategies over time, as well as to identify the ultimate equilibrium they reach. This dual focus on the learning process as well as its outcome is important to understanding how rivals compete over time. We find that the ultimate equilibrium takes some time to reach, and is neither the standard Nash solution nor a collusive one. These differences from the usual static game solutions suggest the value of models that better capture the incomplete information facing a firm in a competitive marketplace.

1. Introduction

Static game theory models of competitive strategy are now familiar to marketers, having been used to explain competitive pricing (Nagle, 1984), dealing behavior (Narasimhan, 1988; Rao, 1991), and distribution channel structure (Coughlan, 1985; Coughlan and Wernerfelt, 1989; Jeuland and Shugan, 1983; and McGuire and Staelin, 1983), among other phenomena [Eliashberg and Chatterjee (1985) and Moorthy (1985) provide reviews of some of these issues as well]. Modelers of competitive pricing strategy seek to understand both how firms react to the pricing changes of their rivals and what is the nature of the pricing equilibrium. Two common assumptions in these models about the behavior of rivals are first, that they possess full information, and second, that they act with full rationality. The first assumption refers to the availability of information to all firms about their rivals' optimization problems (including demand and cost parameters), as well as about their likely thought processes. The second assumption refers to the ability of all rivals to process and use all available information. Full information and full rationality imply that rivals reach a Nash equilibrium instantaneously and with certainty.

In many competitive situations in the real world one sees apparent contradictions to these strategic modeling predictions. For instance, the airline industry did not seem to instantly adjust pricing to a new Nash equilibrium following deregulation, nor does it seem that that individual airlines react optimally to all the changing prices they see in the market every day. 1
Even in the more controlled environment of the laboratory, researchers find results that are inconsistent with standard predictions of static game theory models; see, for example, Alger (1986), Camerer and Weigelt (1988), Friedman and Hoggatt (1980), Holt (1985), and Murphy (1966); Plott (1982) surveys the literature. One is that equilibrium may never be reached by a set of subjects playing a competitive game, or if it is reached at all, it is not achieved for many periods. Further, the equilibrium need not be the Nash outcome: typically, equilibria may range from the perfectly competitive to the collusive. Finally, even when several different sets of subjects, all endowed with the same initial information, play the same competitive game, the experimenters typically find a wide range of outcomes.

This controversy may appear to be of only passing interest to the marketing field. But our increasing use of competitive models, grounded in economic analysis, requires us to question and modify their assumptions to apply to our specific concerns. Marketing decisions must be constantly updated as firms gather new information about the market, and it is therefore important to understand how updating can be done based on observing competitive actions, rather than fully knowing competitors' strategies. Further not just one, but all, competitors engage in such learning and updating behavior, suggesting that we should allow for successive learning episodes by rivals. Finally, real-world equilibria are difficult to obtain outside of financial markets. Random events, new products, and technological change constantly make old marketing strategies obsolete. Therefore, understanding how equilibria are achieved may be as important as identifying the equilibria themselves.

As a first step, in this paper we model a duopoly whose firms possess limited information about their rivals' motivations and behaviors. While we can expect rivals to "do the best they can with what they've got" (i.e., be rational), limitation on "what they've got" (i.e., information incompleteness) can be reasonably suspected to alter their behavior in the competitive context. We therefore assume in our model that each firm can observe its rival's product price, but does not know the exact form of its rival's optimization problem.

Our model is similar in spirit to that of Shugan (1985), who models a single manufacturer–retailer distribution channel. He assumes that each channel member has limited knowledge of the other's incentives, and investigates the conditions under which "implicit understandings" can arise as profit-enhancing mechanisms. Shugan's model provides for experimentation in the quest to better understand the other channel member's goals. Our model also provides for firms to learn from observations of the other firm's prices and thereby improve their information.

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1 Jouzaitis (1988) notes that "On an average day, airlines change more than 1 million fares." While the airlines employ analysts whose sole job is pricing, it is impossible to instantaneously foresee and respond to these changes in competitive pricing policies in a jointly optimal way. Analysts say that they may study a major fare restructuring for a day or two before responding at all, suggesting a discrete response strategy rather than an instantaneously adjusting one, a la Nash.

2 Alger (1986), for instance, discovers that it is quite common for a game to proceed for over 100 periods before an equilibrium is reached. Other researchers find that equilibrium is achieved in a considerably shorter time, but almost never is it achieved immediately. More common is for rivals to interact for ten to twenty periods before settling at an equilibrium.

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3 An emerging interest in this path of inquiry in economics research (Williamson, 1975; Akerlof and Yellen, 1985 and 1987) suggests that this may be a useful direction to follow. Further, there is support for the notion of limitations on processing ability in the behavioral literature in psychology and marketing. See, for example, Petty and Cacioppo (1981, Ch. 3).
about the rival’s incentives and strategies.\textsuperscript{4} However, our industry is a competitive duopoly, and our demand structure and learning mechanism also differ from Shugan’s.

Work in progress by Jeck and Staelin (1989) also looks at competitive pricing practices in a channel context when the information sets of manufacturers and retailers in a channel setting are very restricted. Like Jeck and Staelin, we seek to evaluate the robustness of commonly investigated pricing equilibria, given firms’ ability to gain information over time about competitive responsiveness. But we assume that a firm can observe its rival’s prevailing market price, while they model manufacturers who lack even that information. Also, in contrast to their simulation approach, we are able to provide an analytical, closed-form solution to the problem of dynamic competitive pricing strategies in a duopoly. Further, our steady-state equilibrium pricing strategies are not the Nash equilibrium strategy, while Jeck and Staelin’s rivals eventually converge to the Nash price.

Finally, our model goes beyond the existing literature to provide a set of internally consistent informational conditions leading to a “consistent conjectures” equilibrium.\textsuperscript{5} Our story of learning and the resultant updating of competitive pricing strategies, unlike standard discussions of conjectural variations, does not require rivals to know each other’s optimization problems; this suggests that the consistent conjectures equilibrium may be a useful tool for predicting the ultimate pricing equilibrium in a price-setting oligopoly.

The specific questions that our model addresses are as follows:

— Does learning result in convergent expectations about competitive pricing strategy? That is, are a firm’s expectations of rival pricing reaction always wrong, or do they become “less” wrong as learning progresses?

— How do competitive pricing strategies evolve over time? Does pricing converge to an equilibrium as learning progresses?

— If so, what does the competitive pricing equilibrium look like? In particular, where does it fall along a continuum ranging from perfectly competitive, through Nash, to collusive pricing strategies?

— How long does it take to reach equilibrium?

Below, we first develop the model. We then describe rational learning patterns by the rivals, given their limited information. Next, we characterize the steady-state competitive pricing strategy of the market. We follow with a discussion of the pricing dynamics of the system, with a view to answering the questions posed above. We close with conclusions and future research directions.

\textbf{2. The model}

We consider a duopoly where each manufacturer makes one product. The manufacturers take turns setting prices in alternate periods, so that if manufacturer $i$ sets price $p_i$ in time period $t$, manufacturer $j$ sets price $p_j$ in period $(t+1)$, while manufacturer $i$
next sets $p_i$ in period $(t + 2)$. This structure of alternating price-setting moves is a natural representation of true pricing behavior, where one firm would change its price after observing the pricing actions of another firm in the market.

The demand functions for the two products at time $t$, where $p_i$ is to be set in $t$ and $p_j$ in $(t - 1)$, is assumed to be:

$$q_{i,t} = 1 - p_{i,t} + \theta(p_{j,t-1} - p_{i,t}), \quad j \neq i, \quad (1a)$$

$$q_{j,t} = 1 - p_{j,t-1} + \theta(p_{i,t} - p_{j,t-1}), \quad j \neq i. \quad (1b)$$

Similarly, the demand functions for the two products at time $(t + 1)$ are symmetric functions of $p_{j,t+1}$ and $p_{i,t}$:

$$q_{i,t+1} = 1 - p_{i,t} + \theta(p_{j,t+1} - p_{i,t}), \quad j \neq i, \quad (2a)$$

$$q_{j,t+1} = 1 - p_{j,t+1} + \theta(p_{i,t} - p_{j,t+1}), \quad j \neq i. \quad (2b)$$

This function posits that demand is a negative function of own price, and a positive function of a disparity between the competitive and own prices. The two products are substitutes in demand if $\theta$ is positive, and complements in demand if $\theta$ is negative. Finally, the marginal cost of production and marketing is assumed to be $k$, a constant per unit, for both products.

It is next important to specify the information set available to each competitor. We assume that firm $i$ knows its own profit function parameters: the price it charges, its marginal cost, and the form of its demand function. Further, firm $i$ can observe the price that firm $j$ set in the last period in which $j$ in fact set a price, but firm $i$ does not know the form of firm $j$'s demand function, nor does it know what price firm $j$ will set in the next period. The assumption reflects a market where firms lack the extensive market intelligence necessary to infer a rival's internal decision processes, and also provides a crisp contrast to the common assumption of full information (nevertheless, there are undoubtedly markets where some information about rival demand and costs is available, but modeling such a market is beyond the scope of this work).

Despite a lack of hard market intelligence, each firm can form an expectation regarding the rival's price in the next period and takes this expected price into account in setting its own price in the present period. Specifically, suppose we are in period $t$ and it is firm $i$'s turn to set its price, $p_{i,t}$. Firm $i$ expects firm $j$'s price in period $(t + 1)$ will be some $\tilde{p}_{j,t+1}$. We assume that firm $i$ has a two-period horizon and therefore chooses the $p_{i,t}$ that solves the problem:

$$\max_{p_{i,t}} (\Pi_{i,t} + \Pi_{i,t+1})$$

$$p_{i,t} = (p_{i,t} - k) \times \left[2 - 2(1 + \theta)\tilde{p}_{i,t} + \theta(p_{j,t-1}^* + \tilde{p}_{j,t+1})\right], \quad (3)$$

We show later that real-valued solutions to our model are insured for $\theta$ in the interval $[-0.5857, +\infty)$. This is plausible because the firm has no reason to change its price again until after observing a rival price change. An alternative formulation with a longer decision horizon would reflect a firm with greater foresight, but would also be less tractable.
where $\Pi_{i,t}$ are profits in period $t$, given by:

$$
\Pi_{i,t} = (p_{i,t} - k)\left[1 - p_{i,t} + \theta(p_{j,t-1}^* - p_{i,t})\right],
$$

(4)

and $p_{j,t-1}^*$ is firm $j$'s price chosen in period $(t-1)$. Let $p_{j,t}^*$ denote the solution to (3). Then in period $(t+1)$, having observed $p_{i,t}^*$, firm $j$ selects the price $p_{j,t+1}$ that solves:

$$
\max_{p_{j,t+1}} (\Pi_{j,t+1} + \Pi_{j,t+2})
$$

$$
= (p_{j,t+1} - k) 
\times \left[2 - 2(1 + \theta)p_{j,t+1} + \theta(p_{i,t}^* + \tilde{p}_{j,t+2})\right],
$$

(5)

where $\tilde{p}_{j,t+2}$ is the price firm $j$ expects firm $i$ to charge in period $(t+2)$.

A firm may or may not expect the rival's price in the next period to depend on its own price chosen in the present period. A passive competitive pricing strategy, for example, would imply that each firm thinks its rival will not respond to its price changes at all. This is the standard assumption in Nash competitive models. However, rather than one treating the other as a passive onlooker, we assume that each firm does expect its rival to react actively to changes in its price. Further, as firms compete over time, we can expect them to learn more and more about each other's behavior, and to incorporate this learning into their expectations about each other's future behavior. Then the question of interest is how competitive prices move over time—the topic of the next section.

### 3. Competitive price-setting over time

Let us suppose (without loss of generality) that at the beginning of play in period 1, firm $i$ is the first entrant in the market, and is therefore a monopolist. Firm $i$ remains a monopolist until period 2, when firm $j$ enters. However, firm $i$ has no reason to suspect when firm $j$ will enter or what price it would charge if it did enter. Because $i$ is a monopolist, its period-one demand function accounts for total market demand. At one extreme, this demand could be as large as the sum of $q_{i,t}$ and $q_{j,t}$ in equations (1a) and (1b) (for $t = 1$ and $i = j$): that is, total market size could be invariant to the number of competitors and entry by firm $j$ could display firm $i$'s sales at a one-to-one rate. Another possibility is that the size of the total market grows with the number of competitors. Our model's solution (that is, its steady-state prices and the time it takes to reach steady-state) is invariant to any such assumption we wish to make; it is only necessary that firm $i$'s experience before and after firm $j$'s entry not give it enough information to fully infer $j$'s demand function or optimization rules. We assume here that total demand is indeed independent of the number of rivals, so that firm $i$'s period-one demand is given by:

$$
a_{i,1} = 2 - 2p_{i,1}.
$$

(6)

The optimal first-period price for firm $i$, denoted $p_{i,1}^*$, is:

$$
p_{i,1}^* = \frac{1 + k}{2}.
$$

(7)

Firm $j$ enters in period 2 and notes firm $i$'s price given by (7). Note that $j$ observes the dollar and cents price itself, not the functional form of (7). Firm $j$ can reasonably assume that firm $i$ is a passive player at this stage, since firm $j$ has not observed any pricing reaction so far by firm $i$ to $j$'s own pricing action. Therefore, expecting that firm $i$'s price will stay at $p_{i,1}^*$, firm $j$'s optimal price is given by solving (5) for $t = 1$:

$$
p_{j,2}^* = \frac{1 + (1 + \theta)k + \theta p_{i,1}^*}{2(1 + \theta)}.
$$

(8)

Then in period 3, firm $i$ has noted the entry of firm $j$, and its dollar price, in period
2. Suppose that in order to learn something about the other's responsiveness, firm \( i \) perturbs its price slightly, say, by a small amount \( \epsilon \). Therefore, firm \( i \) sets:

\[
p_{i,3}^e = p_{i,1}^* + \epsilon,
\]

where \( p_{i,3}^e \) denotes "experimental price." Firm \( j \) maintains its expectation of zero competitive reaction from firm \( i \), and sets its period-four price as:

\[
p_{j,4}^* = \frac{1 + (1 + \theta)k + \theta p_{i,5}^e}{2(1 + \theta)}.
\]

Moving to period 5, firm \( i \) now has observed firm \( j \)'s prices twice, in periods 2 and 4. Firm \( i \) only sees the actual dollar prices, not the formulæ in (8) and (10). While firm \( i \) is naive about \( j \)'s algorithm for price-setting, \( i \) may now suspect that \( j \)'s pricing behavior has some element of competitive reaction to it. Without actually knowing \( j \)'s demand function, the simplest assumption that \( i \) can make is that \( j \) reacts linearly to \( i \)'s prices:

\[
p_{j,5} = \alpha_{i,5} + \beta_{i,5} p_{i,t},
\]

where \( \alpha_{i,5} \) denotes the intercept and \( \beta_{i,5} \) denotes the slope of firm \( j \)'s reaction conjectured by firm \( i \) in period 5. Firm \( i \) can go on making such experimental changes in its price and observing \( j \)'s reaction for as many more periods as it likes. However, two price-setting episodes by \( j \) (periods 2 and 4) are the minimum necessary to make an inference about the values of \( \alpha_{i,5} \) and \( \beta_{i,5} \) (two observations are sufficient to determine the coefficients of any linear function). Further experimentation will simply solidify firm \( i \)'s inferences. Thus, for expositional ease, we will describe competitive interactions as if each firm uses only two rival price-setting episodes to calibrate its beliefs about rival decision rules, but we keep in mind that this is shorthand for an experimentation phase that lasts at least this long.

Upon completing its experimentation, \( i \) is able to infer values of \( \alpha_{i,5} \) and \( \beta_{i,5} \) as follows:

\[
\hat{\alpha}_{i,5} = \frac{1 + (1 + \theta)k}{2(1 + \theta)}, \quad \hat{\beta}_{i,5} = \frac{\theta}{2(1 + \theta)}.
\]

[We as analysts can derive these expressions directly from \( j \)'s price-setting rule (8) or (10)]. Once again, note that firm \( i \) has estimated values of \( \alpha_{i,5} \) and \( \beta_{i,5} \) and does not know the mathematical expressions in (12) and (13).

Once firm \( i \) discovers the above values, it can create a forecast of \( j \)'s upcoming behavior in period 6:

\[
\bar{p}_{i,6} = \hat{\alpha}_{i,5} + \hat{\beta}_{i,5} p_{i,5}.
\]

Firm \( i \) now takes \( \bar{p}_{i,6} \) into account in setting \( p_{i,6} \). Specifically, \( i \)'s problem and the resulting solution are:

\[
\max_{p_{i,5}} (\Pi_{i,5} + \Pi_{i,6})
\]

\[
= (p_{i,5} - k)\left[2 + \theta p_{i,5}^* + \theta \hat{\alpha}_{i,5} - p_{i,5}\right]
\]

\[
\times \left[2(1 + \theta) - \theta \hat{\beta}_{i,5}\right],
\]

\[
p_{i,5}^* = \frac{2 + \theta \hat{\alpha}_{i,5} + k\left[2(1 + \theta) - \theta \hat{\beta}_{i,5}\right]}{4(1 + \theta) - 2 \theta \hat{\beta}_{i,5}}
\]

\[
\times \left(1 - \frac{\theta}{4(1 + \theta) - 2 \theta \hat{\beta}_{i,5}}\right) p_{j,4}^*. \quad \text{(16)}
\]

Now we move to period 6. After observing \( p_{i,5}^* \), firm \( j \) realizes that \( i \) is no longer a
passive player. However, \( j \) has too little information to infer \( i \)'s price behavior rule as yet—\( j \) has only observed \( i \) set two virtually identical prices \( (p_{i,1}^* \text{ and } p_{i,3}^*) \) and one quite different price \( (p_{i,5}^*) \). Firm \( j \) has an incentive to gain more information about \( i \)'s pricing rule before making any drastic (and likely, ill-informed) pricing changes of its own. One option would be for firm \( j \) to somehow "average in" its prior observations on firm \( i \)'s prices with its most recent observation. But if \( j \) is truly convinced that \( i \)'s decision rule has changed, it is likely to want to gather more new information before revising its own competitive behavior. This, it is plausible that \( j \) will keep its decision rule from period 4 [see equation (10) above] and wait for more information about \( i \)'s new behavior before revising this decision rule.

Therefore, \( j \)'s price in period 6 is given by:

\[
p_{j,6}^* = \frac{1 + (1 + \theta)k}{2(1 + \theta)} + \frac{\theta}{2(1 + \theta)} p_{i,5}^*.
\]  

In period 7, firm \( i \) observes the dollar price \( p_{j,6}^* \) and confirms that \( \hat{\alpha}_{i,5} \) and \( \hat{\beta}_{i,5} \) in equations (12) and (13) still describe \( j \)'s pricing behavior. Therefore, \( i \)'s period-five price-setting rule is still optimal in period 7:

\[
p_{i,7}^* = \frac{2 + \theta \hat{\alpha}_{i,5} + k \left[ 2(1 + \theta) - \theta \hat{\beta}_{i,5} \right]}{4(1 + \theta) - 2 \theta \hat{\beta}_{i,5}} + \frac{\theta}{4(1 + \theta) - 2 \theta \hat{\beta}_{i,5}} p_{j,6}^*.
\]  

In period 8, firm \( j \) has observed the dollar values of \( p_{i,5}^* \) and \( p_{i,7}^* \). Firm \( j \)'s simple linear expectation can be expressed as:

\[
p_{j,8} = \alpha_{j,8} + \beta_{j,8} p_{j,7} - 1.
\]

Firm \( i \)'s pricing decisions in periods 5 and 7 are the minimum information necessary for firm \( j \) to infer the parameters of (19). Of course, \( j \) may use more than two observations on \( i \)'s behavior, but these will not change \( j \)'s beliefs. It infers \( \alpha_{j,8} \) and \( \beta_{j,8} \) to be:

\[
\hat{\alpha}_{j,8} = \frac{2 + \theta \alpha_{j,8} + k \left[ 2(1 + \theta) - \theta \beta_{j,5} \right]}{4(1 + \theta) - 2 \theta \beta_{j,5}}, \quad (20)
\]

\[
\hat{\beta}_{j,8} = \frac{\theta}{4(1 + \theta) - 2 \theta \beta_{j,5}}. \quad (21)
\]

Firm \( j \) thus forms a forecast of firm \( i \)'s upcoming pricing behavior in period 9 of:

\[
\hat{p}_{j,9} = \hat{\alpha}_{i,8} + \hat{\beta}_{i,8} p_{j,8}, \quad (22)
\]

and uses this forecast in its period-eight pricing decision:

\[
\max \left( \Pi_{j,8} + \Pi_{j,9} \right) \quad \text{with } p_{j,8} = (p_{j,8} - k) \times \left[ 2 - 2(1 + \theta)p_{j,8} + \theta (p_{i,5}^* + \hat{p}_{j,9}) \right]. \quad (23)
\]

In period 9, firm \( i \) has observed \( p_{j,8}^* \) and can immediately infer a change in \( j \)'s reactive pricing rule. Just as firm \( j \) did from periods 6 to 8, firm \( i \) can now update its estimates during periods 9 to 11 (or later, if it deems more experimental periods to be necessary) from \( \hat{\alpha}_{i,5} \) and \( \hat{\beta}_{i,5} \) to \( \hat{\alpha}_{i,11} \) and \( \hat{\beta}_{i,11} \). While we do not expect the competitors to discern an actual mathematical rule governing the updating of expectations, we as modelers can infer one fairly simply from the above discussion. Where \( t \) is a non-negative integer \((t = 0, 1, 2, 3, \ldots)\), we have:

\[
\hat{\alpha}_{j,8+6t} = \frac{2 + \theta \hat{\alpha}_{i,8+6t} + k \left[ 2(1 + \theta) - \theta \hat{\beta}_{i,8+6t} \right]}{4(1 + \theta) - 2 \theta \hat{\beta}_{i,8+6t}}, \quad (24)
\]

\[
\hat{\beta}_{j,8+6t} = \frac{\theta}{4(1 + \theta) - 2 \theta \hat{\beta}_{i,8+6t}}. \quad (25)
\]
and further,
\[
\hat{\alpha}_{i,11+6t} = \frac{2 + \theta \hat{\alpha}_{j,8+6t} + k \left[ 2(1 + \theta) - \theta \hat{\beta}_{j,8+6t} \right]}{4(1 + \theta) - 2\theta \hat{\beta}_{j,8+6t}}.
\]
(26)

\[
\hat{\beta}_{j,11+6t} = \frac{\theta}{4(1 + \theta) - 2\theta \hat{\beta}_{j,8+6t}}.
\]
(27)

Again, it is important to emphasize that it is we as analysts, and not the competing firms themselves, who can infer the generic rules (24)-(27) for the updating of expectations of rival pricing reactions. This updating is accomplished without the inference of the rival's actual optimization problem or first-order conditions. All that is observed is the rival's actual prices, not the calculations behind the setting of those prices.

We can determine some properties of the \(\hat{\beta}\) parameters analytically (Table 1 provides numerical examples to illustrate these properties):

- **a.** The \(\hat{\beta}\)s for firms \(i\) and \(j\) are positive or negative as the products are substitutes or complements, respectively. Intuitively, each firm expects its rival to raise price in response to own price increases for substitutes, but to lower price in response to own price increases for complements.

- **b.** \(\partial \hat{\beta}_i/\partial \theta\) and \(\partial \hat{\beta}_j/\partial \theta\) are positive: the more substitutable products are, the more positive is the \(\hat{\beta}\) parameter at any given level of learning, and the more complementary products are, the more negative the \(\hat{\beta}\) parameter is at any given level of learning. Intuitively, responsive price changes are more intense, the more intensely the demand for the two products interacts.

- **c.** Concerning the progression of \(\hat{\beta}\)s: (i) when the products are substitutes, the \(\hat{\beta}\)s monotonically decrease over time; (ii) when the products are complements, the absolute values of the \(\hat{\beta}\)s monotonically decrease over time. In other words, the absolute values of the \(\hat{\beta}\)s are decreasing with \(t\), for either substitutes or complements. Intuitively, as learning progresses, each firm expects its rival's propensity to react to own price changes to diminish in intensity. Properties of the sequence of \(\hat{\alpha}\)s are not obvious analytically. We therefore numerically analyzed the system, and show illustrative results in Table 1. Inspection of this table shows that:

  - **a.** The \(\hat{\alpha}\)s are always positive. Intuitively, each rival conjectures that the other will charge a positive price, even if he himself charges a zero price.

  - **b.** \(\partial \hat{\alpha}_i/\partial \theta\) and \(\partial \hat{\alpha}_j/\partial \theta\) are negative: the less complementary, or more substitutable, products are, the lower is the \(\hat{\alpha}\) parameter at any given level of learning. Intuitively, rivals expect each other's prices to be higher for complements and lower for substitutes, and the more competitive the market, the lower the expected price.

The above general expectation rules imply general price-setting rules for firms \(i\) and \(j\) as well. Let \(N\) take on the values \(\{4, 10, 16, 22, \ldots\}\), and \(n\) take on the values 1, 3, or 5. Then the two firms' pricing rules are:

\[
p_{i,N+n}^* = \frac{2 + \theta \hat{\alpha}_{i,N+1} + k \left[ 2(1 + \theta) - \theta \hat{\beta}_{i,N+1} \right]}{4(1 + \theta) - 2\theta \hat{\beta}_{i,N+1}} + \frac{\theta}{4(1 + \theta) - 2\theta \hat{\beta}_{i,N+1}} p_{j,N+n-1}^*.
\]
(28)

13 The derivation of these properties is reported in Appendix A1.
Table 1
Sequences of conjectured reaction function parameters, for various $\theta$ and $k = 0.2$

<table>
<thead>
<tr>
<th>Time</th>
<th>$\theta = -0.5857$</th>
<th>$\theta = -0.3$</th>
<th>$\theta = -0.1$</th>
<th>$\theta = +0.1$</th>
<th>$\theta = +1.0$</th>
<th>$\theta = 10.0$</th>
<th>$\theta = +10,000.0$</th>
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<td>$b$</td>
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<td>$b$</td>
<td>$a$</td>
</tr>
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<td>1.306855</td>
<td>-0.70685</td>
<td>0.814286</td>
<td>-0.21429</td>
<td>0.655556</td>
<td>-0.05556</td>
<td>0.554545</td>
</tr>
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<td>0.757219</td>
<td>0.11230</td>
<td>0.639009</td>
<td>0.02786</td>
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<td>-0.10972</td>
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<td>-0.02782</td>
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<td>0.567937</td>
</tr>
</tbody>
</table>

In periods 5, 11, 17, 23, 29, and 35, the entries for $a$ and $b$ are the parameters inferred by firm $i$; in periods 8, 14, 20, 26, 32, and 38, the entries are those inferred by firm $j$.

For $\theta = -0.5857$, $a = a^* = 1.451582$ from period 659 on; $b = b^* = -0.68848$ from period 524 on.
\[ P_{j,N+n+3}^* = \frac{2 + \theta \hat{\alpha}_{j,N+4} + k \left[ 2(1 + \theta) - \theta \hat{\beta}_{j,N+4} \right]}{4(1 + \theta) - 2 \theta \hat{\beta}_{j,N+4}} \]
\[ + \frac{\theta}{4(1 + \theta) - 2 \theta \hat{\beta}_{j,N+4}} - P_{i,N+n+2}^*. \] (29)

4. Steady-state pricing strategies

In the steady state, \( \hat{\alpha}_{i,11} + b_i = \hat{\alpha}_{j,11} + b_j + 1 \) and \( \hat{\beta}_{i,11} + b_i = \hat{\beta}_{j,11} + b_j + 1 \) for firm \( i \), and \( \hat{\alpha}_{j,8} + b = \hat{\alpha}_{j,8} + b_2 + 1 \) and \( \hat{\beta}_{j,8} + b = \hat{\beta}_{j,8} + b_2 + 1 \) for firm \( j \). This implies that no further updating of expectations is necessary for either firm. Note that due to symmetry, the steady-state values of the \( \alpha \)s and \( \beta \)s are the same for both firms. Let \( \alpha^* \) and \( \beta^* \) denote these common steady-state values. We can use equations (24) and (26) together to solve for \( \alpha^* \) and equations (25) and (27) together to solve for \( \beta^* \). After some tedious algebra, these values are found to be:

\[ \beta^* = \frac{2(1 + \theta) \pm \sqrt{4(1 + \theta)^2 - 4\theta^2}}{2 \theta}, \] (30)

\[ \alpha^* = \frac{2 + k \left[ 2(1 + \theta) - \theta \beta^* \right]}{4 + 3\theta - 2 \theta \beta^*}. \] (31)

Several properties of \( \beta^* \) and \( \alpha^* \) deserve mention. First, for \( \theta \) equal to zero, \( \beta^* \) and \( \alpha^* \) are undefined; intuitively, this is because there is no reason to learn at all about a product which is totally unrelated to one's own demand. In this context, "competitive" pricing strategy is irrelevant. Next, note that \( \theta \) must be greater than or equal to \( (\sqrt{2} - 2) \), or approximately \(-0.5857\), to insure that \( \beta^* \) has real-valued roots. \(^14\) No maximum value of \( \theta \) is implied, however; therefore, the feasible range of \( \theta \) is \([-0.5857, +\infty)\). For \( \theta \) in this range, the sequence of \( \beta \)s converges to its negative, not positive, root. \(^15\) The corresponding value of \( \alpha^* \) then follows according to (31).

Finally, we can infer from the convergence of the sequence of \( \beta \)s and \( \alpha \)s that learning is in fact a convergent process. That is, each firm moves closer and closer to the "right guess" about its rival's competitive pricing strategy as learning progresses, even when it cannot infer the exact form of the rival's maximization problem, but only observes the prices charged by the competitor.

Further characterization of the steady-state equilibrium requires that prices (as well as expectations and pricing strategies) no longer change over time. The steady-state competitive prices are:

\[ \bar{p}_{i,L} = \bar{p}_{j,L} = \frac{2 + \theta \alpha^* + \left[ 2(1 + \theta) - \theta \beta^* \right] k}{4 + 3\theta - 2 \theta \beta^*}. \] (32)

Note that given the feasible ranges for \( \theta, \beta^* \), and \( \alpha^* \), \( \bar{p}_{i,L} \) can range from a maximum of \((0.82843 + 0.17157k)\) (for \( \theta \) equal to its minimum value, \( (\sqrt{2} - 2) \)) to a minimum of \( k \) (for \( \theta \) equal to its maximum value, \(-1\)). \(^17\)

The minimum value of \( \bar{p}_{i,L} \) as \( \theta \) approaches infinity reflects a perfectly competitive, zero-profit steady state. These feasible ranges (as well as those for a naive Nash or a collusive regime) are summarized in Table 2. A set of initial conditions is the last element necessary to completely characterize the optimal dynamic competitive pricing path for the two products. These are given by equations (7) and (8).

For comparison, Table 2 also reports the naive Nash pricing equilibrium and the collusive equilibrium. In the naive pricing equilib-

\(^{10}\) See Appendix A1.

\(^{14}\) For a proof of this point.

\(^{15}\) In the feasible range for \( \bar{p}_{i,L}, (0.82843 + 0.17157k) \) is always greater than \( k \), since \( k \) is restricted to be less than one by the form of the demand function. Note that total market demand in the steady state is \( \bar{d}_{i,L} = \bar{d}_{j,L} \). For quantity demanded to be positive, we require that \( \bar{p}_{i,L} < 1 \). But for positive profit margins, \( k \) must be less than \( \bar{p}_{i,L} \). Thus, \( k \) is less than one.
Table 2
Feasible ranges of parameters and prices in the learning, naive Nash, and collusive cases

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\theta = -2 + \sqrt{2}$</th>
<th>$\theta = +\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha^*$</td>
<td>$[2 + k(\sqrt{2} - 1)]/\sqrt{2}$</td>
<td>$k(2 + \sqrt{2})/(2 + 2\sqrt{2}) = 0.70711k$</td>
</tr>
<tr>
<td>$\beta^*$</td>
<td>$(\sqrt{2} - 1)/(\sqrt{2} - 2) = -0.70711$</td>
<td>$(2 - \sqrt{2})/2 = 0.29289$</td>
</tr>
<tr>
<td>$\bar{p}_{i,N}$ [learning—see (32)]</td>
<td>$(\sqrt{2} - 1X + k(\sqrt{2} - 1)) = 0.828843 + 0.17157k$</td>
<td>$k$</td>
</tr>
<tr>
<td>$\bar{p}_{i,N}$ [naive Nash—see (33)]</td>
<td>$[2 + 2k(\sqrt{2} - 1)]/(3(\sqrt{2} - 2))$</td>
<td>$2k/3 = 0.89181 + 0.36940k$</td>
</tr>
<tr>
<td>$\bar{p}_{i,C}$ [collusive—see (34)]</td>
<td>$1 + k/2$</td>
<td>$(1 + k)/2$</td>
</tr>
</tbody>
</table>

* L'Hôpital's Rule is used to derive entries in this column.
* $k$ is the (constant) marginal cost.
* Note that this value for $\bar{p}_{i,N}$ implies a negative profit margin in the naive Nash case. Rivals in such a duopoly will not sell, so this is a theoretical, rather than a real, pricing limit.

rium, each firm expects a zero reaction from its rival, implying that $p_{i,t} = p_{i,t+1} = \bar{p}_{i,N}$ and $p_{j,t-1} = p_{j,t+1} = \bar{p}_{j,N}$. Making these substitutions and solving the maximizations in (3) and (5) produces the ultimate equilibrium in a naive duopoly:

$$\bar{p}_{i,N} = \bar{p}_{j,N} = \frac{2[1 + (1 + \theta)k]}{4 + 3\theta}. \quad (33)$$

This price is the same as the one-period static Nash equilibrium price in such a duopoly. Note that it is possible for naive duopoly profits to be negative, given our feasible range for $\theta$ in the learning model of $[-0.5857, +\infty)$. Specifically, given any particular value of $k$, any $\theta$ greater than $[2(1 - k)/k]$ causes $(\bar{p}_{i,N} - k)$ to be negative. Since firms would choose not to sell rather than make negative profit margins, we should realistically restrict our attention to situation where $\theta$ is less than $[2(1 - k)/k]$ when comparing across cases. Thus, for example, in Table 2, $k$ equal to 0.2 implies that the range of $\theta$ in which comparisons across cases are

Table 3
Steady-state expectations and prices in the learning, naive Nash, and collusive cases, for $k = 0.2$

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>Updates of $\alpha$ and $\beta$</th>
<th>$\alpha^*$</th>
<th>$\beta^*$</th>
<th>$\bar{p}_{i,N}$</th>
<th>$\bar{p}_{i,C}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.5857</td>
<td>&gt; 200, &gt; 170</td>
<td>1.451582</td>
<td>-0.68848</td>
<td>0.859697</td>
<td>0.965589</td>
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<tr>
<td>-0.3</td>
<td>7, 4</td>
<td>0.749272</td>
<td>-0.10972</td>
<td>0.675189</td>
<td>0.735484</td>
</tr>
<tr>
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<td>4, 3</td>
<td>0.638648</td>
<td>-0.02782</td>
<td>0.621361</td>
<td>0.637838</td>
</tr>
<tr>
<td>+0.1</td>
<td>4, 3</td>
<td>0.567937</td>
<td>0.02275</td>
<td>0.581159</td>
<td>0.567442</td>
</tr>
<tr>
<td>+1.0</td>
<td>7, 5</td>
<td>0.411496</td>
<td>0.129171</td>
<td>0.472534</td>
<td>0.6</td>
</tr>
<tr>
<td>+8.0</td>
<td>9, 8</td>
<td>0.21667</td>
<td>0.25</td>
<td>0.288889</td>
<td>0.2</td>
</tr>
<tr>
<td>+10.0</td>
<td>9, 8</td>
<td>0.203978</td>
<td>0.257385</td>
<td>0.274675</td>
<td>0.188235</td>
</tr>
<tr>
<td>+10,000.0</td>
<td>8, 9</td>
<td>0.141496</td>
<td>0.292852</td>
<td>0.200094</td>
<td>0.133396</td>
</tr>
</tbody>
</table>

* Number of updates of $\alpha$ and $\beta$, respectively, before reaching steady state
* Numbers in parentheses are percentage deviations from the naive Nash price.
sensible is \([-0.5857, +8.0]\). Values of \(\theta\) above +8.0 imply zero profits and no sales for naive Nash duopolists.

Similarly, in a collusive duopoly, the two products’ prices should be set to maximize joint profits in any period. This produces equilibrium prices of:

\[
\bar{p}_{i,C} = \bar{p}_{j,C} = \frac{1 + k}{2}.
\]  

(34)

5. Dynamics of the system

We are now in a position to analyze the progression of the competitive pricing system over time. Several general insights about the optimal price path can be seen in Tables 3, 4, and 5. Table 3 details the steady-state values of the expectations parameter \(\beta^*\) and \(\alpha^*\), steady-state prices, and the number of times \(\beta\) and \(\alpha\) are updated before reaching the steady state, as well as the naive Nash and collusive pricing equilibria, for \(k\) equal to 0.2 and for several feasible values of \(\theta\). Tables 4 and 5 show the actual period-by-period dynamic pricing and expectations-updating processes for \(\theta\) equal to \(-0.3\) and +10.0, respectively.

In Tables 4 and 5, the sequence of expectations parameter is also convergent to the “ultimate” level of learning, \(\beta^*\) and \(\alpha^*\). Hence, learning is a successful process over time in this framework; firms get closer and closer to the correct expectations, until in the ultimate equilibrium, their conjectures are

<table>
<thead>
<tr>
<th>Time</th>
<th>(\hat{\alpha}_{i,t})</th>
<th>(\hat{\beta}_{i,t})</th>
<th>(\hat{\alpha}_{j,t})</th>
<th>(\hat{\beta}_{j,t})</th>
<th>(p_{i,t})</th>
<th>(p_{j,t})</th>
<th>(I_{i,t})</th>
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<td></td>
</tr>
</tbody>
</table>

Blank table entries indicate no change from the previous period. System is fully in steady state by period 26.
consistent about each other's competitive pricing strategies.

The sequences of optimal prices over time for the two products are also convergent, for all values of $\theta$. Consistent with equation (32), the equilibrium prices for the two products converge to the same value. $^{18}$

When products are complements (Table 4 is just one example of the general phenomenon), the first entrant's price first rises, then falls to reach the steady state, while the second entrant's price first falls, then rises in its progression to the steady state. When $\theta = 0$, no learning is really necessary about a "rival" firm, because its pricing decisions have no impact on one's own product's demand. The price paths for substitutes (see Table 5) exhibit a non-monotonic approach to the ultimate equilibrium, both briefly rising and then falling (once learning has begun) to the steady state. $^{19}$ In general, as

---

### Table 5

Expectations, prices, and profits over time: $\theta = +10.0$, $k = 0.2$

<table>
<thead>
<tr>
<th>Time</th>
<th>$\hat{a}_{i,t}$</th>
<th>$\hat{b}_{i,t}$</th>
<th>$\hat{a}_{j,t}$</th>
<th>$\hat{b}_{j,t}$</th>
<th>$p_{i,t}$</th>
<th>$p_{j,t}$</th>
<th>$\Pi_{i,t}$</th>
<th>$\Pi_{j,t}$</th>
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Blank table entries indicate no change from the previous period. System is fully in steady state by period 32.
Table 3 shows, the approach to the ultimate equilibrium occurs faster for products less related in demand because the true pricing reaction of the rival diverges less from the initial naive guess of zero reaction than when products are more highly related in demand.

Note that equilibrium is not expected to be achieved immediately, in contrast to the implications from full-information models of competitive strategy. Not only will several revisions of expectations (and hence competitive strategies) be expected, but each round of expectation revision will take at least three periods to complete.

In Table 3 we can also see that steady-state prices in our learning model are decreasing in $\theta$. This is consistent with the classic Bertrand (pricing) model results that are characterized by lower equilibrium prices with stiffer and stiffer competition. Clearly, if firms could choose $\theta$, they would seek maximum differentiation (i.e., $\theta$ close to zero) in this world.

Finally, we can ask whether the steady state that we have defined here is advantageous from a profit point of view, as compared to "naive" competitive pricing decisions. The benchmark for such a comparison is most naturally taken as the collusive outcome. If the learning equilibrium price lies between the collusive price and the naive price, then we can conclude that learning is a profit-enhancing activity. Comparing the prices in (32), (33), and (34), we see that:

$$\bar{p}_{i,C} > \bar{p}_{i,L} > \bar{p}_{i,N}, \text{ for } \theta > 0; \quad (35)$$

$$\bar{p}_{i,C} < \bar{p}_{i,L} < \bar{p}_{i,N} \text{ for } \theta < 0. \quad (36)$$

Such a pattern is common in the results shown graphically in Alger’s (1986) experimental work, suggesting that our model may posit behavior consistent with the empirical findings. Shugan (1985) also finds that the analytical approach to steady-state equilibrium is non-monotonic (specifically, oscillatory) in his monopoly distribution channel model.

That is, learning behavior between rival firms always enhances profitability relative to the naive model. This is so because the observation and learning process improves information about rival behavior patterns, which makes for less naive (and hence more profitable) pricing decisions in the steady state.

If firms are quantity-setters instead of price-setters, however, learning reduces profits. See Coughlan and Wernerfelt (1989) for a different model illustrating this concept, and Moorthy (1988) for a relevant discussion of strategic complements and substitutes.
Figures 1 and 2 show what proportion of the gap between Nash and collusive prices is bridged by the learning-case steady-state price. The effects are different for complements and substitutes. As products become more complementary ($\theta$ becomes more negative in Fig. 1), the learning equilibrium first closes an increasing, and then a decreasing, percentage of the gap between the static Nash and collusive prices. However, Fig. 2 shows that as substitutability increases, not only does the absolute gap between Nash and collusive pricing behavior (and thus profitability) widen, but also the ability of a learning process to bridge that pricing and profitability gap diminishes.

We can summarize these observations in a series of empirically testable hypotheses:

**Hypothesis 1.** Learning is a convergent process: prices converge to an equilibrium over time, and a firm's beliefs about its rival's competitive reaction converges as well.

**Hypothesis 2.** The price paths for product complements are non-monotonic, rising and then falling for the first entrant, and falling and then rising for the second entrant.

**Hypothesis 3.** The price paths for product substitutes are also non-monotonic, both firms' prices rising briefly until learning begins, and then falling to the steady state.

**Hypothesis 4.** The progression to a steady state takes longer, the more interrelated in demand are the products (whether substitutes or complements).

**Hypothesis 5.** Given price-setting firms, learning produces higher profits in the steady state than does a naive competitive process, but the percentage profit improvement diminishes with increasing product substitutability. The percentage improvement first increases, then decreases, with increasing product complementarity.

6. Conclusions and future research directions

One survey article on pricing strategy research in Marketing (Rao, 1984, p. S55) poses several topics for future research, among them: (a) When would a competitor react to a rival's price change? (b) How large a price change would that competitor put into effect? (c) What are the dynamics of competitive pricing behavior? In this paper, we seek some answers to questions like these in the context of a model where rivals lack full information.

We find that learning about reaction parameters is a time-consuming process, but does converge over time to the consistent-conjectures equilibrium. Thus, the model shows an institutional set of circumstances under which consistent conjectures “make sense.” Second, consistent conjectures are accompanied by steady-state pricing behavior as well. Further, the learning equilibrium is always more profitable than the naïve Nash equilibrium. This suggests that full-information models' predictions are not robust to informational limitations.

Finally, the more interrelated in demand products are (whether substitutes or complements), the longer it takes to reach equilibrium. The only situation where we expect competitors to reach equilibrium immediately is when their products are unrelated in demand. This is a trivial case, however, since then the two firms are essentially monopolists, with no learning necessary about rival “reactions” to one's own pricing decisions.

Do our results mean we should reject full-information, static competitive strategy mod-

21 Bresnahan (1981) refers to the possibility of experimentation as a means of updating expectations obliquely in his paper: “If firms have inconsistent conjectures and it is possible for them to learn how their industry reacts to exogenous shocks (a ‘natural experiment’) they will learn that their conjectures are wrong” (p. 943). However, he never explicitly posits the mechanism for this type of learning and updating.
els? Not entirely. For instance, consistent with classic Bertrand models, we find that equilibrium prices do decrease with the level of substitutability in demand of the products. Our equilibrium, like that in the static models, is not the same as a collusive one; so learning is not a mechanism for complete collusion. Further, the pricing predictions of the static, full-information type of model are likely to be more representative of true behavior in the presence of limited information when products are not too interrelated in demand, both in the speed of attainment of equilibrium and in the competitive pricing levels at equilibrium. Finally, we may not be so easily able to find analytical solutions to models of more complex competitive system if we impose these informational constraints on all players.

Our model is a highly stylized representation of competitive pricing behavior. By pointing out its limitations, we underscore the need for future research that increases our understanding of competitive behavior under limited information. Specifically:

— We refrain in our analysis from introducing any notions of reward or punishment in the repeated-games sense; further, we do not allow for pricing strategies that "send a signal" to the competitor about one's own desired pricing stance.

— Our learning and updating mechanism is only one possible way in which rivals can change their competitive strategies over time. One other mechanism would be a Bayesian updating process; see Coughlan and Mantrala (1990) for an example of such a process.

— Our informational assumptions are only one possibility. A model incorporating knowledge of market demand functions could, for example, use experimentation to uncover information about a rival's cost structure.

— We analyze only a linear demand function. We point out above why a Cobb-Douglas formulation is inappropriate for a competitive context. However, if demand is nonlinear but firms use a linear rule to update expectations of rival behavior, they will update the "wrong" parameters. See Kirman (1975) for a model of this sort that nevertheless reaches an equilibrium, albeit not the consistent-conjectures one.

— Our competitors are symmetric. An analysis of asymmetric competition could help us understand competitive interaction in leader-follower industries or given asymmetric cross-price demand effects.

A desirable next step in this research, of course, is to directly test this model's predictions in a laboratory setting. On the theoretical front, we see an opportunity for other modeling work, like that of Jeck and Staelin (1989), that posits different information sets for rivals to examine further the robustness of pricing equilibria in competitive markets.

Appendix: Derivation of points in the text

A1. Derivation of properties of the conjectured reaction function parameters

Two important points must be noted before examining the properties of the reaction function parameters: first, the second-order conditions require that \(4(1 + \theta) - 2\theta \hat{\beta}_i \) and \(4(1 + \theta) - 2\theta \hat{\beta}_j \) be positive for a profit maximum, and second, only values of \(\theta\) greater than \(-0.586\) are feasible in our model. This is established by recalling that \(\hat{\beta}_{i,5} = \theta/(2(1 + \theta))\). Therefore, in period 5, the second-order condition to be satisfied is: \(4(1 + \theta) - \theta^2/(1 + \theta)) > 0\). This may be rewritten as: \((3\theta^2 + 8\theta + 4) > 0\), which holds only for \(\theta > -2/3\). Further, to insure real values of \(\beta^*\) in equation (30) in the text, we require that \([4(1 + \theta)^2 - 2\theta^2]\) be greater than zero. This implies that \(\theta \geq (\sqrt{2} - 2)\), or \(-0.5857\) (which
of course satisfies the second-order condition).

Proofs of properties (a)–(c) of $\hat{\beta}_i$ or $\hat{\beta}_j$

Property (a) holds, as the signs of (25) and (27) are the sign of $\theta$, given the above-mentioned second-order conditions for a profit maximum. To see that property (b) holds, evaluate the derivatives of equations (25) and (27) with respect to $\theta$. To see that property (c(i)) holds, consider the progression of $\beta$s derived from equations (25) and (27):

$$\hat{\beta}_{i,5} = \frac{\theta}{2(1 + \theta)}$$
$$\hat{\beta}_{j,8} = \frac{\theta(1 + \theta)}{4(1 + \theta)^2 - \theta^2}$$
$$\hat{\beta}_{i,11} = \frac{\theta[4(1 + \theta)^2 - \theta^2]}{2(1 + \theta)[8(1 + \theta)^2 - 3\theta^2]}$$

Now, inspect the ratio of $\hat{\beta}_{i,11}$ to $\hat{\beta}_{i,5}$:

$$\frac{\hat{\beta}_{i,11}}{\hat{\beta}_{i,5}} = \frac{4 + 8\theta + 3\theta^2}{8 + 16\theta + 5\theta^2}$$

This ratio is always less than one for $\theta$ greater than zero. Now consider the ratio of $\hat{\beta}_{j,14}$ to $\hat{\beta}_{j,8}$:

$$\frac{\hat{\beta}_{j,14}}{\hat{\beta}_{j,8}} = \frac{4(1 + \theta) - 2\theta\hat{\beta}_{i,5}}{4(1 + \theta) - 2\theta\hat{\beta}_{i,11}}$$

This ratio is less than one if and only if $\hat{\beta}_{i,11}$ is less than $\hat{\beta}_{i,5}$—which is true by the above logic. By successive application of this logic, and using equations (25) and (27) to write down successive values of $\beta$s, it is straightforwardly seen that the $\hat{\beta}$s monotonically decrease over time.

To see that property (c(ii)) holds, note that $\hat{\beta}_{i,11}/\hat{\beta}_{i,5}$ is less than one for $\theta \in [-0.5857, 0)$. For $\theta$ in this range, the $\beta$s have negative sign and therefore, $|\hat{\beta}_{i,11}| < |\hat{\beta}_{i,5}|$.

A2. To show that the sequence of $\beta$s converges to the negative root of (30)

Note that $\hat{\beta}_{i,5} = \theta/[2(1 + \theta)]$. This has an absolute value less than one for all feasible $\theta$. Now, when $\theta$ is positive, the $\beta$s monotonically decrease over time (by property c(i)). Therefore, $\beta^*$ must be less than one as well. This can only occur if the sequence of $\hat{\beta}$s converges to the negative root of (30). Similarly, for $\theta \in [-0.5857, 0)$, the absolute values of the $\hat{\beta}$s decrease over time. The absolute value of $\hat{\beta}_{i,5}$ is less than one. Therefore, the absolute value of $\beta^*$ must also be less than one. It can be verified that this can only occur if the sequence of $\beta$s converges to the negative root of (30).

A3. Derivation of equation (32)

Equations (28) and (29) are respectively firm i's and firm j's price-setting rules at any given opportunity solely as a function of the parameters of the problem and the history of past prices. Thus, they characterize the price path over time for the two products. To find the steady-state prices (denoted by $\bar{\beta}_{i,L}$ and $\bar{\beta}_{j,L}$), we simply set $p_{i,N+n} = p_{i,N+n+2} = \bar{\beta}_{i,L}$ and $p_{i,N+n-1} = p_{i,N+n+3} = \bar{\beta}_{j,L}$ in equations (28) and (29). Further, since the steady state implies equilibrium expectations according to equations (30) and (31) above, we can set $\hat{\beta}_{i,N+1} = \hat{\beta}_{j,N+4} = \beta^*$ and $\hat{\alpha}_{i,N+1} = \hat{\alpha}_{j,N+4} = \alpha^*$ in equations (28) and (29). This creates a system of two equations in two unknowns, yielding the steady-state prices in (32) after some algebraic manipulation.

References

A. T. Coughlan, M.K. Mantrala / Dynamic competitive pricing strategies


