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The Effect of Lead Time Uncertainty on Safety Stocks

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ABSTRACT

The pressure to reduce inventory investments in supply chains has increased as competition expands and product variety grows. Managers are looking for areas they can improve to reduce inventories without hurting the level of service provided. Two areas that managers focus on are the reduction of the replenishment lead time from suppliers and the variability of this lead time. The normal approximation of lead time demand distribution indicates that both actions reduce inventories for cycle service levels above 50%. The normal approximation also indicates that reducing lead time variability tends to have a greater impact than reducing lead times, especially when lead time variability is large. We build on the work of Eppen and Martin (1988) to show that the conclusions from the normal approximation are flawed, especially in the range of service levels where most companies operate. We show the existence of a service-level threshold greater than 50% below which reorder points increase with a decrease in lead time variability. Thus, for a firm operating just below this threshold, reducing lead times decreases reorder points, whereas reducing lead time variability increases reorder points. For firms operating at these service levels, decreasing lead time is the right lever if they want to cut inventories, not reducing lead time variability.

Subject Areas: Inventory Management, Mathematical Programming/ Optimization, Probability Models and Supply Chain Management.

INTRODUCTION AND FRAMEWORK

Managers have been under increasing pressure to decrease inventories as supply chains attempt to become leaner. The goal, however, is to reduce inventories without hurting the level of service provided to customers. Safety stock is a function of the cycle service level, the demand uncertainty, the replenishment lead time, and

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the lead time uncertainty. For a fixed-cycle service level, a manager thus has three levers that affect the safety stock—demand uncertainty, replenishment lead time, and lead time uncertainty. In this paper we focus on the relationship between lead time uncertainty and safety stock and the resulting implications for management.

Traditionally, a normal approximation has been used to estimate the relationship between safety stock and demand uncertainty, replenishment lead time, and lead time uncertainty. According to Eppen and Martin (1988), this approximation is often justified by using an argument based on the central limit theorem, but in reality, they say, “the normality assumption is unwarranted in general and this procedure can produce a probability of stocking out that is egregiously in error.” Silver and Peterson (1985), however, argue that trying to correct this effect with a more accurate representation of demand during lead time may be ineffectual because the gain in precision may only induce minimal improvement in the cost. Tyworth and O’Neill (1997) also address this issue in a detailed empirical study for *fast-moving* finished goods (demand per unit time have coefficients of variation [c.v.] below 40%) in seven major industries. Their investigations reveal that “the normal approximation method can lead to large errors in contingency stock—say, greater than 25%. Such errors have relatively little influence on the optimal solutions, however, because contingency stock holding cost comprises a small portion of the total logistics system cost” (p. 183). They further conjecture that reducing the *fill rate*, the proportion of orders filled from stock, “makes total system costs less sensitive to normal theory misspecifications” (p. 178) since this will in turn reduce the required safety stock level and thus make the total holding costs a smaller percentage of total system cost.

In this paper our focus is not on the size of the error resulting from using the normal approximation (that has been captured very well by Eppen and Martin [1988]), but on the flaws in the managerial prescriptions implied by the normal approximation. In particular, we focus on two prescriptions of the normal approximation:

1. For cycle service levels above 50%, reducing lead time variability reduces the reorder point and safety stock.
2. For cycle service levels above 50%, reducing lead time variability is more effective than reducing lead times because it decreases the safety stock by a larger amount.

In this paper we show that for cycle service levels that are commonly used in industry, both prescriptions are false if we consider the exact demand during the lead time. Using the exact demand during the lead time instead of the normal approximation we infer the following:

1. For cycle service levels above 50% but below a threshold, reducing lead time variability **increases** the reorder point and safety stock.
2. For cycle service levels above 50% but below a threshold, reducing the lead time variability **increases** the reorder point and safety stock, whereas reducing the lead time **decreases** the reorder point and safety stock.

Table 1: Safety stocks for gamma lead times and different service levels.

Row	Lead Time Process	CSL (α)	Safety Stock (Normal Approximation)	Safety Stock (Exact value)
1	Gamma, $L = 10$, $s_L = 5$	.6	28	20
2	Gamma, $L = 10$, $s_L = 4$	.6	23	22
3	Gamma, $L = 8$, $s_L = 5$	.6	27	15
4	Gamma, $L = 10$, $s_L = 5$	.95	182	218
5	Gamma, $L = 10$, $s_L = 4$	.95	153	181
6	Gamma, $L = 8$, $s_L = 5$	.95	179	218

CSL = cycle service level.

Both effects are more pronounced when the coefficient of variation of demand is high and less pronounced when the coefficient of variation of demand is low. This is consistent with the conclusion of Tyworth and O'Neill (1997) that the normal approximation is quite effective for low c.v. Our inference also support the results in Song (1994), who assumes a periodic demand that follows a compound Poisson process and derives a threshold value underneath which base stocks increase with a reduction in lead-time uncertainty. It is easy to see that under the normal approximation, this threshold equals .5. For the distributions we analyze, assuming a normal period demand, we show that this threshold lies in a range where most firms operate (between .5 and .7). The comparison of the prescriptions is illustrated using Table 1.

Consider rows 1–3 of Table 1. For a cycle service level of .6, the normal approximation predicts that reducing the standard deviation of the lead time from 5 to 4 should decrease the safety stock from 28 to 23. The exact calculation, however, shows that reducing the standard deviation of lead time **increases** the required safety stock from 20 to 22. The normal approximation predicts that reducing the standard deviation of lead time by 20% (5 to 4) is much more effective at reducing the safety stock than reducing the lead time by 20% (10 to 8). The exact calculation, however, shows that for a cycle service level of .6, decreasing lead time is more effective (safety stock decreases from 20 to 15) than reducing the standard deviation of lead time (safety stock increases from 20 to 22).

For a cycle service level of 95%, however, the prescriptions of the normal approximation are correct (see rows 4–6). At this cycle service level both the normal approximation and the exact calculation show that reducing the standard deviation of lead time decreases the safety stock and is more effective than decreasing the lead time itself.

We next argue that many firms in practice operate at cycle service levels in the 50–70% range rather than the 95–99% that is often assumed. In practice, managers often focus on the *fill rate* as a service quality measure (Aiginger, 1987; Lee & Billington, 1992; Byrne & Markham, 1991), rather than the cycle service level (CSL or α). The fill rate measures the proportion of demand that is met from stock, whereas the cycle service level measures the proportion of replenishment cycles where a stockout does not occur. Table 2 considers the cycle service level and fill rate for different reorder points for a product that has a weekly demand of 2,500, standard deviation of weekly demand of 500, lead time of two weeks, and

Table 2: Cycle service level and fill rate as a function of safety stock.

Reorder Point	Safety Stock	Cycle Service Level	Fill Rate
5000	0	.500	.9718
5040	40	.523	.9738
5080	80	.545	.9756
5120	120	.567	.9774
5160	160	.590	.9791
5200	200	.611	.9807
5240	240	.633	.9822
5280	280	.654	.9836
5320	320	.675	.9850
5360	360	.695	.9862
5400	400	.714	.9874

a reorder quantity of 10,000. Calculations for fill rate are detailed in chapter 11 of Chopra and Meindl (2003).

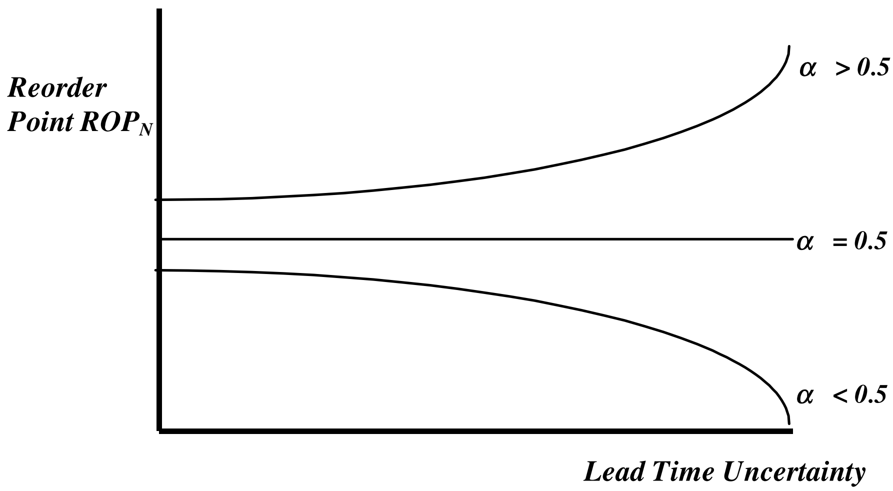
In this example, Table 2 illustrates that fill rates of between 97 and 99% are achieved for cycle service levels between 50 and 70%. Most firms aim for fill rates of between 97 and 99% (and not cycle service levels). This implies cycle service levels of between 50 and 70%. As we show in this paper, it is for cycle service levels between 50 and 70% that the prescriptions of the normal approximation are most distorted and lead to managers pushing the wrong levers to reduce inventories. Our main point is that for cycle service levels where most firms operate, the normal approximation erroneously encourages managers to focus on reducing the variability of lead times when they would be better off reducing the lead time itself.

Our general results range from specific theoretical outcomes when the lead time follows a uniform distribution (see section "Effect of Lead-time Uncertainty: The Exact Distribution Effect") to numerical observations when the lead time follows a uniform, gamma, or normal distribution (see section "Numerical Results and Analysis"). In the next section, we formalize our model and reexamine the response to reducing uncertainty when the normal approximation is used rather than an exact characterization. We conclude with the scope and managerial implications of our findings in the final section.

EFFECT OF LEAD-TIME UNCERTAINTY: THE NORMAL APPROXIMATION

For a given cycle service level, determining the required safety stock levels is predicated on characterizing the distribution of demand during the lead time. We assume that there is an indivisible period of analysis; for example, a day. Demand during day i, X_i, are independent and identically distributed random variables drawn from a normal distribution with mean μ_X and standard deviation σ_X. For a generic lead time distribution of mean L and standard deviation s_L, the demand during lead time under the normal approximation has mean $M = L\mu_X$ and standard deviation $S = \sqrt{L\sigma_X^2 + \mu_X^2 s_L^2}$ (Silver & Peterson, 1985).

Figure 1: Reorder point as lead time uncertainty increases for service levels above, below, and equal to .5.



Let $F(\bullet)$ represent the cumulative distribution function (CDF) of the standard normal distribution with mean 0 and standard deviation 1. Define z as the solution to $F(z) = \alpha$ and ROP_N as the reorder point for a cycle service level of α. Under the normal approximation, we have $ROP_N = M + zS$ with zS as the safety stock. Observe that under the normal approximation S increases (decreases) as s_L increases (decreases). Thus, whether the safety stock S and the reorder point ROP_N, rise or fall as s_L increases depends only on the sign of z. For a given mean lead-time L, Figure 1 depicts the relationship between the lead-time uncertainty (represented by s_L) and the reorder points predicted by the normal approximation for three α's. As can be seen from Figure 1, the safety stock S and reorder point ROP_N rise with an increase in s_L for a CSL above .5 (since $z > 0$) and drop with an increase in s_L for CSL below .5 (since $z < 0$). For a CSL of .5, the reorder point remains at the level of the deterministic case ($s_L = 0$) and does not change with an increase in s_L (since $z = 0$, $ROP_N = M$). These observations lead to the following conclusion:

Theorem 1

Suppose that the lead-time uncertainty represented by s_L increases. Then for a given CSL $= \alpha$, the following is true:

1. If $\alpha < .5$, then ROP_N falls;
2. if $\alpha = .5$, then ROP_N is invariant; and
3. if $\alpha > .5$, then ROP_N rises.

Theorem 1 indicates that, as management works on the reduction of lead time uncertainty (reduction of s_L), the reorder point drops for CSLs above .5. Unfortunately, as demonstrated in Eppen and Martin (1988), this neat prescription is a consequence of the normal approximation. In the next section, we show the existence of a threshold $\bar{\alpha} > 0.5$ such that for CSLs in $[.5, \bar{\alpha}]$, the reorder point and safety stock actually increase as s_L decreases.

EFFECT OF LEAD-TIME UNCERTAINTY: THE EXACT DISTRIBUTION

In this section, we show how the prescriptions of the normal approximation in Theorem 1 are flawed for the case when periodic demand follows the normal distribution and the lead time has a discrete uniform distribution with a mean of Y and a range of $Y \pm y$. Denote the reorder point by R and let $G_y(R)$ be the (unconditional) probability that demand during the lead time is less than or equal to R when the lead time is uniformly distributed between $Y \pm y$. If μ_x is the expected demand per period and σ_x is the standard deviation of demand per period, we define

$$z_Y(R) = (R - Y\mu_x)/(\sigma_x\sqrt{Y}). \tag{1}$$

It is clear from the definition of $z_Y(R)$ that it represents the number of standard deviations R is away from the expected value of demand given that the lead time is Y. Let $F(z_Y(R))$ represent the probability that the standard normal is less than or equal to $z_Y(R)$. As in Eppen and Martin (1988) it then follows that

$$G_y(R) = \left(\frac{1}{2y+1}\right) \sum_{W=Y-y}^{Y+y} F(z_W(R)). \tag{2}$$

From (1) and (2), it thus follows that

$$G_y(R_1) > G_y(R_2) \text{ if and only if } R_1 > R_2. \tag{3}$$

Observe that the case when $y = 0$ corresponds to the case of deterministic lead time. We are interested in examining how $G_y(R)$ behaves as the lead time uncertainty represented by y changes. We begin by examining the effect of increasing uncertainty by increasing y by one period. Then, simple algebra yields:

$$G_{y+1}(R) - G_y(R) = \left(\frac{1}{2y+3}\right)[F(z_{Y+y+1}(R)) + F(z_{Y-y-1}(R)) - 2G_y(R)], \tag{4}$$

and

$$G_{y+1}(R) = \left(\frac{2y+1}{2y+3}\right)G_y(R) + \left(\frac{1}{2y+3}\right)[F(z_{Y+y+1}(R)) + F(z_{Y-y-1}(R))]. \tag{5}$$

Since $Y + y + 1 > Y - y - 1 \geq 0$, it readily follows that $z_{Y-y-1}(R) \geq z_{Y+y+1}(R)$, so that

$$1 > F(z_{Y-y-1}(R)) > F(z_{Y+y+1}(R)) > 0. \tag{6}$$

Our objective for the rest of the section is to try to come up with an analogue to Figure 1 for the case when we use the exact distribution of demand during the lead time—shown in (2). To proceed we need the following lemma.

Lemma 1

Let R_y and R_{y+1} be such that $G_y(R_y) = G_{y+1}(R_{y+1}) = \alpha$. We have $R_{y+1} > (<)\ R_y$ if and only if $F(z_{Y+y+1}(R_y)) + F(z_{Y-y-1}(R_y)) < (>)\ 2G_y(R_y)$.

Proof

Observe that if $F(z_{Y+y+1}(R_y)) + F(z_{Y-y-1}(R_y)) < (>)\, 2G_y(R_y)$, we have $G_{y+1}(R_y) < (>)\ G_y(R_y) = \alpha$ by (3.4). Since $G_{y+1}(R_{y+1}) = \alpha$, using (3.3) we thus have $R_{y+1} > (<)\ R_y$.

On the other hand, if $R_{y+1} > (<)\ R_y$, (3) implies that $\alpha = G_{y+1}(R_{y+1}) > (<)\ G_{y+1}(R_y)$. Since $\alpha = G_y(R_y)$, we have $G_y(R_y)\ > (<)\ G_{y+1}(R_y)$. From (4) we thus have $F(z_{Y+y+1}(R_y)) + F(z_{Y-y-1}(R_y)) < (>)\ 2G_y(R_y)$. The result thus follows. Another result needed is presented below. The proof follows from the definition of the standard normal distribution.

Lemma 2

Let $F(\cdot)$ be the standard normal cumulative distribution function. If $z_1 < 0 < z_2$, then $1 < F(z_1) + F(z_2)$ if and only *if* $-z_1 < z_2$.

We start by considering the reorder point as y increases for the case where the CSL is .5. For a given value of lead time uncertainty y, let $R_y(0.5)$ be the reorder point such that $G_y(R_y(0.5)) = .5$. For the case $y = 0$, observe that $R_y(0.5)$ is the expected demand, $M = Y\mu_x$, during the lead time Y. We have $z_{Y+1}(R_0(0.5)) < 0 < -z_{Y+1}(R_0(0.5)) < z_{Y-1}(R_0(0.5))$ from (1). From Lemma 2 it thus follows that $F(z_{Y+1}(R_0(0.5))) + F(z_{Y-1}(R_0(0.5))) > 2G_0(R_0(0.5)) = 1$. Using Lemma 1 it thus follows that $R_1(0.5) < R_0(0.5)$.

In other words, the reorder point decreases as lead time uncertainty increases from $y = 0$ to $y = 1$ for a cycle service level of .5. In the next result we prove that this pattern continues to hold as lead time uncertainty (y) increases, that is, the reorder point continues to drop as lead time uncertainty (y) increases for a cycle service level of .5.

Theorem 2

For a cycle service level $\alpha = .5$, the reorder point $R_y(0.5)$ declines with an increase in lead time uncertainty y, that is, $R_{y+1}(0.5) < R_y(0.5)$.

Theorem 2 (proved in the appendix) is equivalent to stating that the median of the distribution of demand during the lead time declines as lead time uncertainty represented by y increases. In contrast, the median is invariant when the normal approximation is used. For the specific case of the median, Theorem 2 provides a complete characterization of the behavior of the reorder point as y increases. In general, the reorder point is the solution to

$$G_y(R) = \left(\frac{1}{2y+1}\right) \sum_{W=Y-y}^{Y+y} F(z_W(R)) = \alpha. \tag{7}$$

Let $R_y(\alpha)$ represent the unique solution to (7). To examine the effect of increasing the cycle service level α it is sufficient to specialize (4) to:

$$G_{y+1}(R_y(\alpha)) - G_y(R_y(\alpha)) = \left(\frac{1}{2y+3}\right)[F(z_{Y+y}(R_y(\alpha))) + F(z_{Y-y}(R_y(\alpha))) - 2\alpha] \quad (8)$$

Observe that Theorem 2 implicitly analyzes (8) for the special case $\alpha = .5$. By Lemma 1, for arbitrary α, determining whether the reorder point increases or decreases depends on the sign of the term $2\alpha - [F(z_{Y+y}(R_y(\alpha))) + F(z_{Y-y}(R_y(\alpha)))]$.

Theorem 3

1. If $0 < \alpha < .5$, there exists $y > 0$ such that $R_y(\alpha) < R_0(\alpha)$;
2. There exists $.5 < \alpha < 1$ and $y > 0$ such that $R_y\ (\alpha) < R_0\ (\alpha)$.

Part 1 of this theorem states that the optimal reorder point falls with increasing uncertainty if the CSL is less than .5. Part 2 states that there are service levels $\alpha > .5$ for which the reorder point *initially* falls with an increase in lead time uncertainty, which contradicts the prediction for the normal approximation. We prove both parts in the Appendix.

In the next section we numerically study the effect of decreasing lead time uncertainty on safety stocks for various lead time distributions.

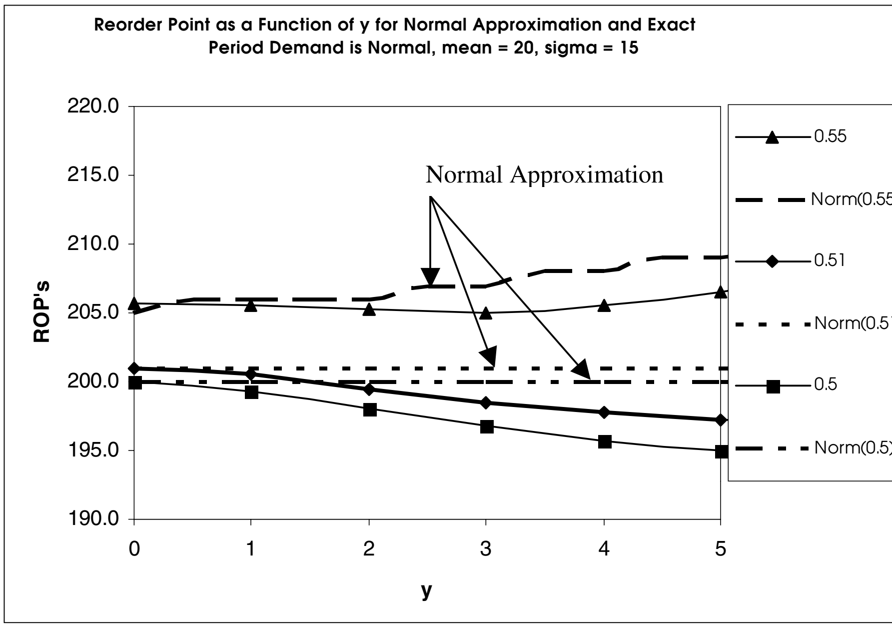
NUMERICAL RESULTS AND ANALYSIS

Theorems 2 and 3 show that there is a range of cycle service levels above 50% where decreasing the lead time uncertainty increases the reorder point and safety stock when the lead time is uniformly distributed. In this section we present computational evidence to show that these claims are valid when lead times follow the gamma, the uniform, or the normal distribution. For the gamma lead time distribution we show that for cycle service levels around 60%, decreasing lead time variability increases the reorder point. For the uniform lead time this effect is observed for cycle service levels close to, but above, 50%. As we have discussed earlier, most firms operate at cycle service levels in this range because they imply fill rates of around 98%. Using the computational results we also show that in this range, a manager is better off decreasing lead time rather than lead time variability if reducing inventories is the goal.

We first consider the effect of reducing lead time variability on reorder points and safety stock when the lead time follows the uniform or gamma distribution. In both cases we keep the mean lead time fixed and vary the standard deviation. Demand per period is assumed to be normal with a mean $\mu = 20$ and standard deviation $\sigma = 15$ or 5. This allows us to analyze the effect for both a high and low coefficient of variation of demand.

Figure 2 shows the effect of reducing lead time variability when periodic demand has a high coefficient of variation (15/20) and lead time is uniformly distributed with a mean of 10 and a range of $10 \pm y$, where y ranges from

Figure 2: ROP as a function of y for uniform lead time.



0 to 10. We plot the change in reorder point as y changes using both the normal approximation and the exact calculations for cycle service levels of .5, .51, and .55. The safety stock is calculated as $ss = \text{ROP} - 200$ because 200 is the mean demand during the lead time. The curves marked Norm($\cdot$) represent the results of the normal approximation, whereas the others represent the exact calculation of reorder points.

From Figure 2 we conclude that for a high coefficient of variation of periodic demand, if lead times are uniformly distributed, there is a range of cycle service levels above .5 (but close to .5), where reducing lead time uncertainty increases safety stocks, whereas the normal approximation predicts the opposite.

Figure 3 shows that the effect is even more pronounced when the lead time follows a gamma distribution. Once again we consider periodic demand to be normally distributed with a high coefficient of variation (15/20). Lead time is assumed to follow the gamma distribution with a mean of 10 and standard deviation varied from 6 to 1 and the reorder point calculated for cycle service levels of .50, .55, and .6. The plot compares the reorder points obtained using the normal approximation and the exact calculation.

Figure 3 shows that the normal approximation is even more erroneous when lead time follows the gamma distribution. Even for a cycle service level of .6, decreasing the lead time uncertainty increases safety stocks, whereas the normal approximation predicts just the opposite. When lead times are gamma distributed we thus conclude that there is a range of cycle service levels even beyond .6 when decreasing lead time variability increases the required safety stock.

Figure 3: ROP as a function of lead time standard deviation for gamma lead time.

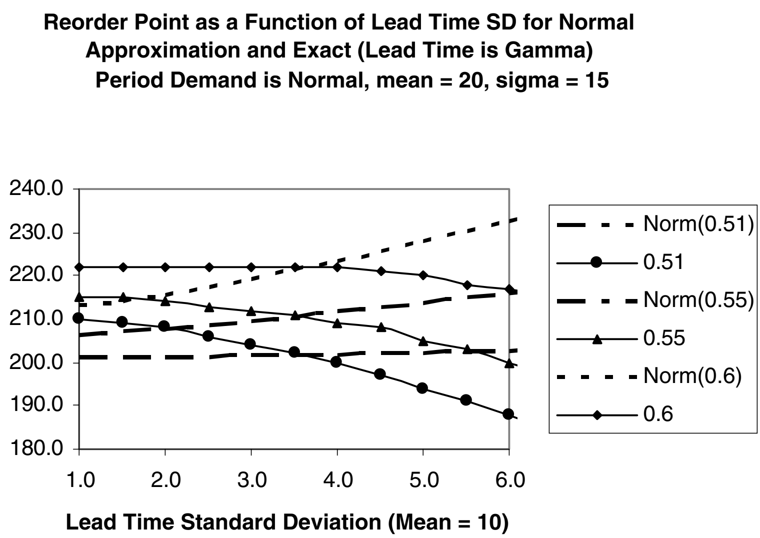


Figure 4: Reorder point as a function of lead time uncertainty for a low coefficient of variation.

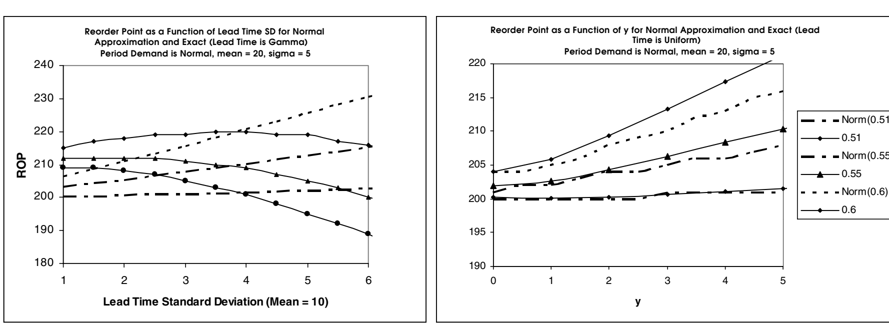


Figure 4 repeats the results of Figures 2 and 3 but for a low coefficient of variation (5/20) of periodic demand. Figure 4 shows that even with a low coefficient of variation of periodic demand, for cycle service levels between 50% and a threshold, the exact calculation shows that decreasing lead time variability increases the required safety stock, whereas the normal approximation predicts the opposite. The computational results show that the threshold value decreases as the coefficient of variation of periodic demand decreases. Thus, for a very low coefficient of variation of period demand, the error of the normal approximation is less pronounced.

In Figures 5 and 6, we compare the impact of reducing lead time variability and lead time on safety stocks. In both cases we consider periodic demand to have a high coefficient of variation (15/20). In Figure 5 we consider lead time to be uniformly distributed with a mean of 10 and a range of $10 \pm y$. The chart on the left

Figure 5: Safety stock as a function of lead time uncertainty (left) and lead time mean (right) for uniform lead times.

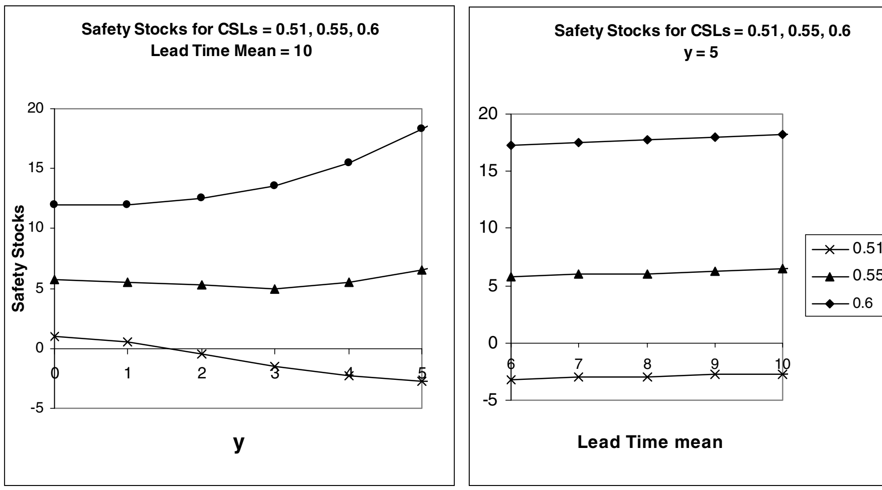
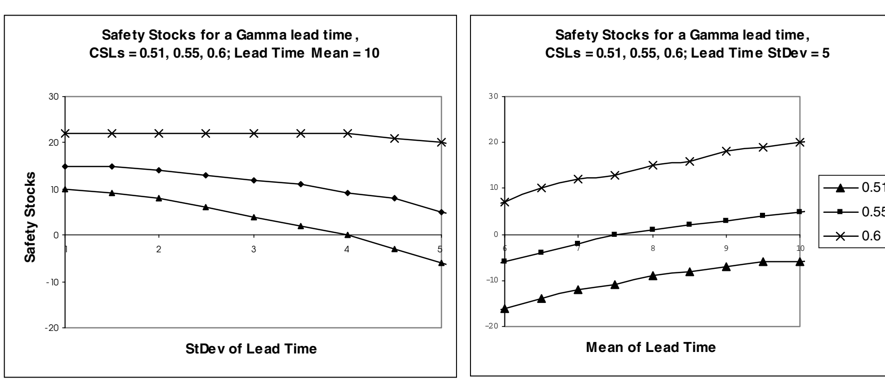


Figure 6: Safety stock as a function of lead time uncertainty (left) and lead time mean (right) for gamma lead times.

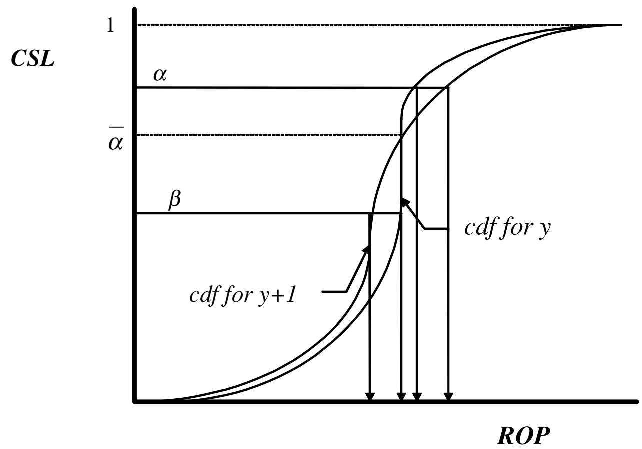


shows how the ROP changes as y is decreased from 5 to 0. The chart on the right shows how for $y = 5$, the ROP changes as the lead time decreases from 10 to 5. The results are shown for cycle service levels of .51, .55, and .6. In both cases the results show that the error of the normal approximation is less pronounced for a low coefficient of variation of periodic demand.

FINDING THE THRESHOLDS

We now show how to obtain the thresholds below which the conclusions of the normal approximation are flawed. The CDF of demand during lead time shows how the ROP changes as a function of the cycle service level. Recall that the safety stock

Figure 7: CDF for demand during lead time.



ss = ROP − mean demand during lead time. In Figure 7 we represent two CDFs corresponding to the cases where the lead time is uniformly distributed between $Y \pm y$ (represented by y) and $Y \pm (y + 1)$ (represented by $y + 1$). The crossover $\bar{\alpha}$ represents the CSL for which both distributions require the same safety stock.

For a cycle service level α, larger than the crossover point $\bar{\alpha}$, decreasing lead time range from $Y \pm (y + 1)$ to $Y \pm y$ results in a decrease in the safety stock. However, when the CSL is below $\bar{\alpha}$, say β, decreasing lead time range from $Y \pm (y + 1)$ to $Y \pm y$ results in an *increase* in the *ROP*. Thus, the *ROP* increases with a decrease in lead time uncertainty for cycle service levels below the crossover $\bar{\alpha}$. The crossover point $\bar{\alpha}$ is the threshold below which decreasing the lead time variability increases the required safety stock. For cycle service levels between 50% and the crossover point, decreasing the lead time uncertainty results in an increase of safety stock.

Next, we numerically describe the cumulative distribution functions for the case where the lead time distribution is uniform or gamma and obtain crossover points to explain the results in the section "Numerical Results and Analysis." Figure 8 shows the CDF for the demand during the lead time when lead time is uniformly distributed between $10 \pm y$ as y changes from 9 to 1. Observe that the crossover point between the CDF for $y = 3$ and $y = 1$ is at .564. This implies that for any cycle service level between 50 and 56.4%, decreasing y from 3 to 1 will increase the safety stock, whereas the normal approximation predicts otherwise. The crossover point thus establishes the threshold below which the normal approximation is directionally wrong when lead time is uniformly distributed.

Figure 9 shows how the reorder point ROP varies with the cycle service level when lead time follows a gamma distribution with a mean of 10 and a standard deviation that varies from 9 to 1. Once again the crossover point helps explain the range of cycle service levels over which the conclusions of the normal approximation are directionally flawed. For example, the crossover point for the

Figure 8: CDF of lead time demand for different y's when $\mu = 20$, $\sigma = 15$, and L = 10.

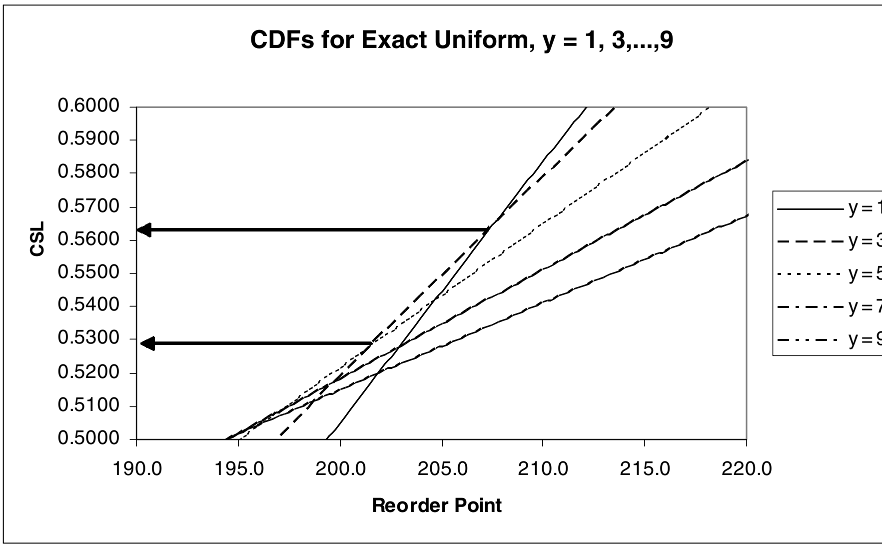


Figure 9: CDF of lead time demand for gamma lead time, $\mu = 20$, $\sigma = 15$, and L = 10.

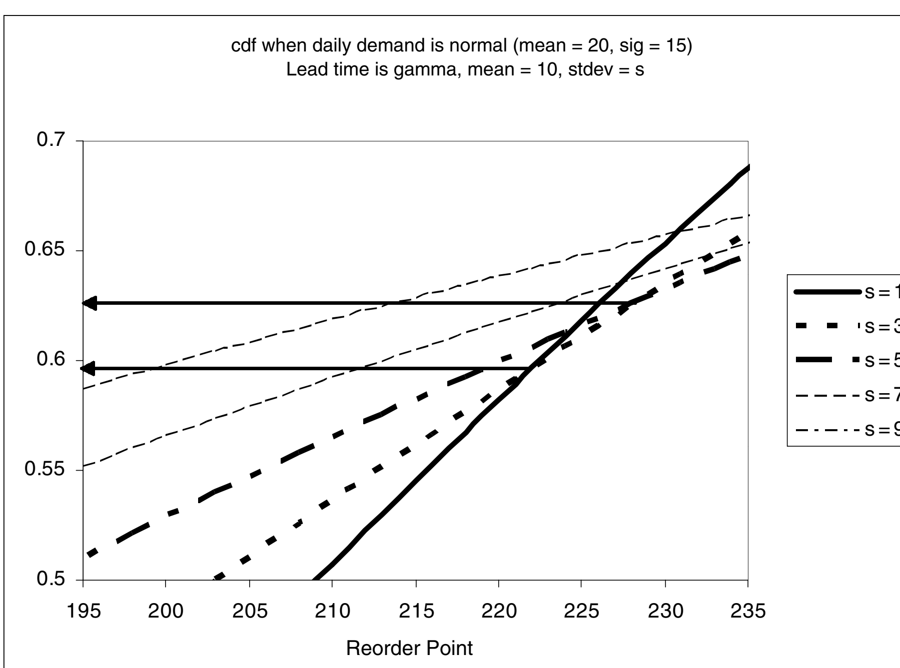
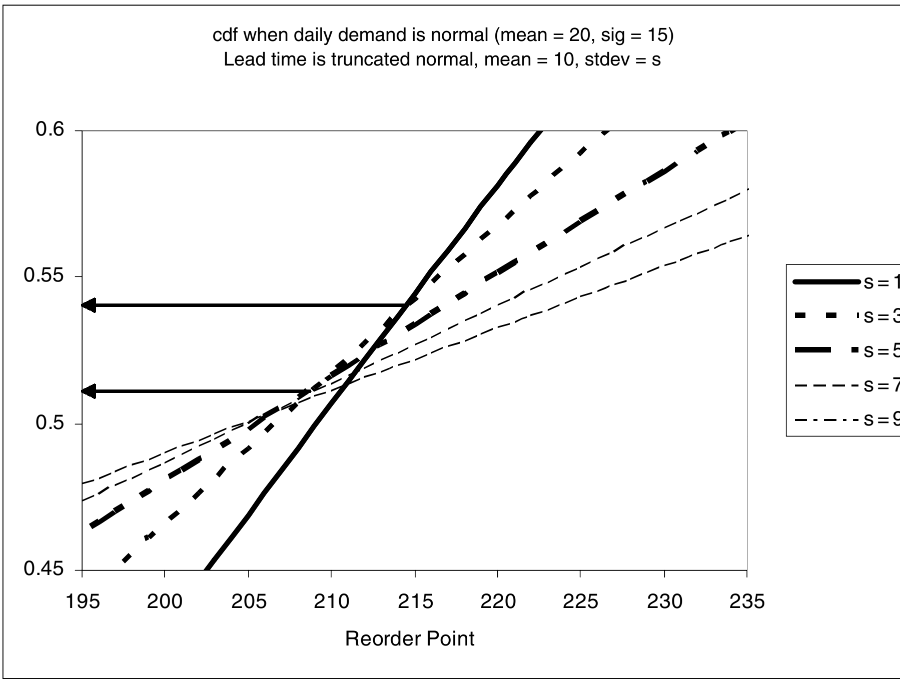


Figure 10: CDF of lead time demand for normal lead time, $\mu = 20$, $\sigma = 15$, and L = 10.



CDF for a standard deviation of 5 and 3 is at .628. Thus, for cycle service levels between 50 and 63%, decreasing the standard deviation of lead time from 5 to 3 increases the safety stock required, whereas the normal approximation predicts just the opposite.

Figure 10 shows how the reorder point ROP varies with the cycle service level when lead time follows a truncated normal distribution (with mean of 10 and a discrete support from 0 to 20 days). Here we note that the crossover points are closer to .5 (.54 when s = 1 and s = 3 intersect, and .51 when s = 3 and s = 5 intersect) illustrating that the width of the interval of service levels in which the reorder point decrease as variability increases is dependent on the shape of the lead time distribution.

The procedure detailed above for uniform, gamma, and normal lead times can be used for any lead time distribution to estimate the threshold below which decreasing lead time variability increases the safety stock required. To identify whether decreasing the lead time variability from σ_h to σ_l will increase or decrease the safety stock required, we plot the CDF of demand during the lead time for each lead time variability and identify the crossover point X. Cycle service levels between 50% and the crossover point represent the range over which the exact distribution predicts an increase in safety stock if lead time variability is decreased, whereas the normal approximation predicts the opposite.

CONCLUSION

For the most part, management's understanding of the effect on safety stocks of uncertainty in lead time is based on an approximate characterization of demand during lead time using the normal distribution. For cycle service levels above 50% the normal approximation predicts that a manager can reduce safety stocks by decreasing lead time uncertainty. Our analytical results and numerical experiments, however, indicate that for cycle service levels between 50% and a threshold, the prescriptions of the normal approximation are flawed, and decreasing the lead time uncertainty, in fact, *increases* the required safety stock. In this range of cycle service levels, a manager who wants to decrease inventories should focus on decreasing lead times rather than lead time variability. This contradicts the conclusion drawn using the normal approximation.

Our conclusion is more pronounced when demand has a high coefficient of variation. When the lead time follows a gamma distribution, the prescriptions of the normal approximation are flawed over a wide range of cycle service levels. This range is narrower when lead times are uniformly or normally distributed. Thus, using the normal approximation makes sense if lead times are normally distributed, but it would not make sense if lead times follow a distribution closer to the gamma. [Received: February 2002. Accepted: October 2003.]

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APPENDIX A: PROOF OF THEOREMS 2 AND 3

To lighten notation throughout, we let $c = \sigma_x/\mu_x$.

Theorem 2

For $\alpha = .5$, the reorder point $R_y(0.5)$ declines with an increase in lead time uncertainty y, that is, $R_{y+1}(0.5) < R_y(0.5)$.

Proof

The result is proved using induction. We first consider the case for $y = 0$, that is, the lead time is fixed at Y. For a fixed lead time Y, the reorder point for a cycle service level of .5 is given by $R_o(0.5) = Y\mu_x$.

Now consider the lead time to be uniformly distributed with equal support on $\{Y + 1, Y, Y - 1\}$, that is, $y = 1$. If the reorder point is kept at $R_0(.5) = Y\mu_x$, the cycle service level is given by

$$\frac{1}{3}\left\{F\left(\frac{(Y-(Y-1))}{c\sqrt{Y-1}}\right)+F\left(\frac{(Y-(Y+1))}{c\sqrt{Y+1}}\right)+F\left(\frac{(Y-Y)}{c\sqrt{Y}}\right)\right\}$$

We claim that $F(\frac{1}{c\sqrt{Y-1}}) + F(\frac{-1}{c\sqrt{Y+1}}) > 1$. By Lemma 2, this follows because $\frac{-1}{c\sqrt{Y+1}} < 0 < \frac{1}{c\sqrt{Y-1}}$ and $\frac{1}{c\sqrt{Y+1}} < \frac{1}{c\sqrt{Y-1}}$. This implies that if $y = 1$, the cycle service level for a reorder point of $R_0(.5)$ is strictly greater than .5. Thus, $R_1(.5) < R_0(.5)$. Define $\Delta_1 = (R_0(.5) - R_1(.5))/\mu_x$. The service level at $R_1(.5)$ is .5 and is given by

$$\frac{1}{3}\left\{F\left(\frac{(Y-(Y-1))-\Delta_1}{c\sqrt{Y-1}}\right)+F\left(\frac{(Y-(Y+1))-\Delta_1}{c\sqrt{Y+1}}\right)\right.$$
$$\left.+F\left(\frac{(Y-Y)-\Delta_1}{\sqrt{Y}}\right)\right\}=0.5. \tag{A1}$$

Since $\Delta_1 > 0$, we have $F(\frac{-\Delta_1}{c\sqrt{Y}}) < 0.5$.Thus, it must be the case that

$$F\left(\frac{1-\Delta_1}{c\sqrt{Y-1}}\right)+F\left(\frac{-1-\Delta_1}{c\sqrt{Y+1}}\right) > 1 \text{ or by Lemma 2}$$

$$\frac{1-\Delta_1}{c\sqrt{Y-1}} > \frac{1+\Delta_1}{c\sqrt{Y+1}}. \tag{A2}$$

Now consider raising the lead time uncertainty by assuming lead time to be uniformly distributed over $\{Y - 2, Y - 1, Y, Y + 1, Y + 2\}$. If the reorder point is kept at $R_1(.5) = Y\mu_x - \Delta_1\mu_x$, the cycle service level is given by

$$\frac{1}{5}\left\{F\left(\frac{(Y-(Y-2))-\Delta_1}{c\sqrt{Y-2}}\right)+F\left(\frac{(Y-(Y+2))-\Delta_1}{c\sqrt{Y+2}}\right)\right.$$
$$+F\left(\frac{(Y-(Y-1))-\Delta_1}{c\sqrt{Y-1}}\right)+F\left(\frac{(Y-(Y+1))-\Delta_1}{c\sqrt{Y+1}}\right)$$
$$\left.+F\left(\frac{(Y-Y)-\Delta_1}{\sqrt{Y}}\right)\right\}$$

We now claim that $F(\frac{2-\Delta_1}{c\sqrt{Y-2}}) + F(\frac{-2-\Delta_1}{c\sqrt{Y+2}}) > 1$.
By Lemma 2, this is equivalent to showing that

$$\frac{2-\Delta_1}{c\sqrt{Y-2}}=\frac{1}{c\sqrt{Y-2}}+\frac{1-\Delta_1}{c\sqrt{Y-2}}>\frac{2+\Delta_1}{c\sqrt{Y+2}}=\frac{1}{c\sqrt{Y+2}}+\frac{1+\Delta_1}{c\sqrt{Y+2}}. \tag{A3}$$

From (A2), we have

$$\frac{1-\Delta_1}{c\sqrt{Y-2}} > \frac{1-\Delta_1}{c\sqrt{Y-1}} > \frac{1+\Delta_1}{c\sqrt{Y+1}} > \frac{1+\Delta_1}{c\sqrt{Y+2}}.$$

Given the fact that $\frac{1}{c\sqrt{Y-2}} > \frac{1}{c\sqrt{Y+2}}$, (A3) thus follows. This implies that if $y = 2$, the cycle service level for a reorder point of $R_1(.5)$ is strictly greater than .5. Thus, $R_2(.5) < R_1(.5)$. Define $\Delta_2 = (R_1(.5) - R_2(.5))/\mu_x$.

We now use induction to complete the proof. Define $\Delta_y = (R_{y-1}(.5) - R_y(.5))/\mu_x$. To show that $R_{y+1}(.5) < R_y(.5)$ we start with the induction assumption that $\frac{y-\Delta_y}{c\sqrt{Y-y}} > \frac{y+\Delta_y}{c\sqrt{Y+y}}$. We now need to prove that $F(\frac{y+1-\Delta_y}{c\sqrt{Y-(y+1)}}) + F(\frac{-(y+1)-\Delta_y}{c\sqrt{Y+(y+1)}}) > 1$.
Observe that

$$\begin{aligned}
\frac{(y+1)-\Delta_y}{c\sqrt{Y-(y+1)}} &= \frac{y}{c\sqrt{Y-(y+1)}} + \frac{1-\Delta_y}{c\sqrt{Y-(y+1)}} \\
&> \frac{y}{c\sqrt{Y-(y+1)}} + \frac{1-\Delta_y}{c\sqrt{Y-y}} \\
&> \frac{y}{c\sqrt{Y+(y+1)}} + \frac{1+\Delta_y}{c\sqrt{Y+y}} \\
&> \{\text{This follows from the induction hypothesis}\}
\end{aligned}$$

$$\frac{y}{c\sqrt{Y+(y+1)}} + \frac{1+\Delta_y}{c\sqrt{Y+(y+1)}} = \frac{(y+1)+\Delta_y}{c\sqrt{Y+(y+1)}}.$$

The result thus follows using Lemma 2. This implies that $R_{y+1}(.5) < R_y(.5)$.

Theorem 3, Part 1

If $0 < \alpha \leq .5$, there exists a $y > 0$ such that $R_y(\alpha) < R_0(\alpha)$.

Proof

If the lead time is fixed at Y, the cycle service level for a reorder point of $R\mu_x$ ($R \in (0, Y)$) is given by $F(\frac{R-Y}{c\sqrt{Y}})$. Consider now lead time to be uniformly distributed on $\{Y-y, Y, Y+y\}$ where y is some small positive value. We show that the cycle service level under this setting (with the reorder point fixed at $R\mu_x$) increases, or equivalently

$$F\left(\frac{R-(Y+y)}{c\sqrt{Y+y}}\right) + F\left(\frac{R-(Y-y)}{c\sqrt{Y-y}}\right) > 2F\left(\frac{R-Y}{c\sqrt{Y}}\right) \tag{A4}$$

for some $y > 0$. By Lemma 1, this is equivalent to proving

$$\frac{R-(Y+y)}{c\sqrt{Y+y}}+\frac{R-(Y-y)}{c\sqrt{Y-y}}>2\frac{R-Y}{c\sqrt{Y}}$$

or

$$\frac{R-Y}{c\sqrt{Y}}-\frac{R-(Y+y)}{c\sqrt{Y+y}}<\frac{R-(Y-y)}{c\sqrt{Y-y}}-\frac{R-Y}{c\sqrt{Y}}.$$

This inequality is equivalent to

$$\begin{aligned}&(R-Y)\left(2\sqrt{Y+y}\sqrt{Y-y}-\sqrt{Y}\sqrt{Y-y}-\sqrt{Y}\sqrt{Y+y}\right)\\&<\ y\left(\sqrt{Y}\sqrt{Y+y}-\sqrt{Y}\sqrt{Y-y}\right).\end{aligned}$$

Observe that the right-hand side is clearly positive and $(R-Y)$ is nonpositive by assumption and thus maximized at $R=0$. Therefore, it remains to show that

$$\begin{aligned}&(-Y)\left(2\sqrt{Y+y}\sqrt{Y-y}-\sqrt{Y}\sqrt{Y-y}-\sqrt{Y}\sqrt{Y+y}\right)\\&<\ y\left(\sqrt{Y}\sqrt{Y+y}-\sqrt{Y}\sqrt{Y-y}\right).\end{aligned}$$

Without loss of generality, assume that $y=\beta Y$ for some $\beta\in[0,1]$.

Basic algebraic manipulations yield

$$\sqrt{1-\beta}\sqrt{1+\beta}\left(\sqrt{1+\beta}-1\right)<\sqrt{1-\beta}\sqrt{1+\beta}\left(1-\sqrt{1-\beta}\right)$$

or

$$\sqrt{1+\beta}+\sqrt{1-\beta}<2 \text{ which is true for } \beta\in[0,1].$$

Theorem 3, Part 2

There exist $\alpha>0.5$ and $y>0$ such that $R_y(\alpha)<R_0(\alpha)$.

Proof

Observe that (A4) is tight when $y=0$. Differentiating its left-hand side with respect to y yields

$$\begin{aligned}A=&\ f\left(\frac{R-Y+y}{c\sqrt{Y-y}}\right)\frac{c\sqrt{Y-y}-(R-Y+y)\frac{-c}{2\sqrt{Y-y}}}{c^2(Y-y)}\\&+f\left(\frac{R-Y-y}{c\sqrt{Y+y}}\right)\frac{-c\sqrt{Y+y}-(R-Y-y)\frac{c}{2\sqrt{Y+y}}}{c^2(Y+y)}\end{aligned}$$

which simplifies to

$$f\left(\frac{R-Y+y}{c\sqrt{Y-y}}\right)\frac{Y-y+R}{2c(Y-y)^{3/2}} - f\left(\frac{R-Y-y}{c\sqrt{Y+y}}\right)\frac{Y+y+R}{2c(Y+y)^{3/2}}$$

with $f(\cdot)$ being the density function of the standardized normal distribution. We show that $A > 0$ for a small positive y. Without loss of generality, assume that $R = (1+\gamma)Y$ and $y = \beta Y$ for $\gamma > 0$ and $0 < \beta < \gamma$. Substituting yields

$$\begin{aligned} A &= f\left(\frac{(\gamma+\beta)Y}{c\sqrt{Y(1-\beta)}}\right)\frac{(2+\gamma-\beta)Y}{2cY^{3/2}(1-\beta)^{3/2}} - f\left(\frac{(\gamma-\beta)Y}{c\sqrt{Y(1+\beta)}}\right)\frac{(2+\gamma+\beta)Y}{2cY^{3/2}(1+\beta)^{3/2}} \\ &= \frac{1}{2c\sqrt{2\pi Y}}\left[\exp\left(-k\frac{(\gamma+\beta)^2}{1-\beta}\right)\frac{2+\gamma-\beta}{(1-\beta)^{3/2}} - \exp\left(-k\frac{(\gamma-\beta)^2}{1+\beta}\right)\frac{2+\gamma+\beta}{(1+\beta)^{3/2}}\right] \end{aligned}$$

where

$$k = \frac{Y}{2c^2}.$$

We rewrite A as

$$\begin{aligned} A = \frac{\exp\left(-k\frac{(\gamma-\beta)^2}{1+\beta}\right)}{2c\sqrt{2\pi Y}}&\left[\exp\left(k\left\{\frac{(\gamma-\beta)^2}{1+\beta} - \frac{(\gamma+\beta)^2}{1-\beta}\right\}\right)\frac{2+\gamma-\beta}{(1-\beta)^{3/2}}\right. \\ &\left. - \frac{2+\gamma+\beta}{(1+\beta)^{3/2}}\right]. \end{aligned}$$

To show that the above is nonnegative for a suitable k, we need to show that

$$\exp\left(k\left\{\frac{(\gamma-\beta)^2}{1+\beta} - \frac{(\gamma+\beta)^2}{1-\beta}\right\}\right)\frac{2+\gamma-\beta}{(1-\beta)^{3/2}} - \frac{2+\gamma+\beta}{(1+\beta)^{3/2}} \geq 0$$

for a small enough β. Thus $A > 0$ reduces to

$$\exp\left(-k\left\{\frac{4\gamma\beta+2\gamma^2\beta+2\beta^3}{1-\beta^2}\right\}\right)\frac{2+\gamma-\beta}{(1+\beta)^{3/2}} - \frac{2+\gamma+\beta}{(1+\beta)^{3/2}} \geq 0. \qquad \text{(A5)}$$

For $k \leq \frac{1}{4\gamma+4\gamma^2}$, we have

$$-k\left\{\frac{4\gamma\beta+2\gamma^2\beta+2\beta^3}{1-\beta^2}\right\} \geq -\frac{\beta}{1-\beta^2}$$

or

$$\exp\left(-k\left\{\frac{\beta(4\gamma+2\gamma^2+2\beta^2)}{1-\beta^2}\right\}\right) \geq \exp\left(-\frac{\beta}{1-\beta^2}\right).$$

(A5) is satisfied if for a small $\beta > 0$ we can show that

$$H(\gamma,\beta) = \exp\left(-\frac{\beta}{1-\beta^2}\right)\frac{2+\gamma-\beta}{(1-\beta)^{3/2}} - \frac{2+\gamma+\beta}{(1+\beta)^{3/2}} > 0. \qquad \text{(A6)}$$

Since $H(\gamma, 0) = 0$ and $\frac{2+\gamma+\beta}{(1+\beta)^{3/2}}$ is decreasing in β, it remains to show that $H_1(\gamma, \beta)$ is nondecreasing initially for a small positive β, where

$$H_1(\gamma, \beta) = \exp\left(-\frac{\beta}{1-\beta^2}\right)\frac{2+\gamma-\beta}{(1-\beta)^{3/2}}.$$

We compute

$$\frac{\partial H_1(\gamma, \beta)}{\partial \beta} = \exp\left(-\frac{\beta}{1-\beta^2}\right)\frac{-(1-\beta)^{3/2} + \frac{3}{2}(2+\gamma-\beta)\sqrt{1-\beta}}{(1-\beta)^3}$$

$$+ \exp\left(-\frac{\beta}{1-\beta^2}\right)\left(\frac{2+\gamma-\beta}{(1-\beta)^{3/2}}\right)\left(\frac{-(1-\beta)^2-2\beta^2}{(1-\beta^2)^2}\right)$$

$$\text{At } \beta = 0, \left.\frac{\partial H_1(\gamma, \beta)}{\partial \beta}\right|_{\beta=0} = -1 + \frac{3}{2}(2+\gamma) - (2+\gamma) > 0$$

and by continuity we conclude that $\frac{\partial H_1(\gamma,\beta)}{\partial \beta} \geq 0$ for a small positive β and therefore $H_1(\gamma, \beta)$ is increasing initially. Thus, if $k = \frac{Y}{2c^2} \leq \frac{1}{4\gamma + 4\gamma^2}$ the cycle service level initially increases with an increase in lead time uncertainty and the associated reorder point decreases.

APPENDIX B: TECHNICAL METHODOLOGY

The Normal Approximation for the Exact Uniform Distribution

$ROP_N = \mathrm{F}^{-1}\{\alpha,\ \mu_y\mu_x, \sqrt{\mu_y\sigma_X^2 + y(y+1)\mu_X^2/3}\}$ where F^{-1} $\{\bullet,\ \bullet,\ \bullet\}$ is the inverse of the normal distribution (NORMINV) of given mean and standard deviation.

The Normal Approximation for the Gamma Distribution

$ROP_N = \mathrm{F}^{-1}\{\alpha,\ L\mu_x, \sqrt{L\sigma_X^2 + s_L^2\mu_X^2}\}$ where F^{-1} $\{\bullet, \bullet, \bullet\}$ is the inverse of the normal distribution of given mean and standard deviation and L and s_L are the mean and standard deviation of the gamma distribution.

The Exact Uniform

Sheet 1. Generate Table indexed by (row) ROP and (column) y with CSL in the body of the Table.

Sheet 2. Use VLOOKUP function to extract ROP index that corresponds to given y and CSL.

The Discrete Gamma Distribution

We seek the inverse (G^{-1}) of the cumulative distribution function of the demand during lead time. The lead time distribution has mean L and standard deviation s_L. We know

$$ROP = G^{-1}\{P(D < X) = \sum_{l=1,\ldots,30} w_l * NORMDIST(X, l * \mu_X, \sigma_X * sqrt(l), 1)\}.$$

Sheet 1. Generate weights (w_l, l = 1, . . . , 30) Table indexed by (row) support (between 0 and 30) and (column) standard deviation with, in row j and column s the value

$$GAMMADIST\left(j, \left(\frac{L}{s}\right)^2, \frac{s^2}{L}, 1\right) - \sum_{i=0}^{j-1} GAMMADIST\left(i, \left(\frac{L}{s}\right)^2, \frac{s^2}{L}, 1\right)$$

for j = 1, . . . , 29 with 0 in row j = 0 and, in row j = 30,

$$GAMMADIST\left(30, \left(\frac{L}{s}\right)^2, \frac{s^2}{L}, 1\right) - \sum_{i=0}^{29} GAMMADIST\left(i, \left(\frac{L}{s}\right)^2, \frac{s^2}{L}, 1\right) + 1 - \sum_{i=0}^{30} GAMMADIST\left(i, \left(\frac{L}{s}\right)^2, \frac{s^2}{L}, 1\right).$$

That is, we add to j = 30 the mass of the tail to the right of 30. In Sheet 1, we also generate a table of NORMDIST values as per the ROP formula above.

Sheet 2. Matrix multiply the Sheet 1's Normdist table to Sheet 1's weights table.

Sheet 3. Use a VLOOKUP(CSL) on Sheet 2 to find the ROP that yields the given CSL.

The resulting discrete gamma distributions for lead time are illustrated in Figure A1.

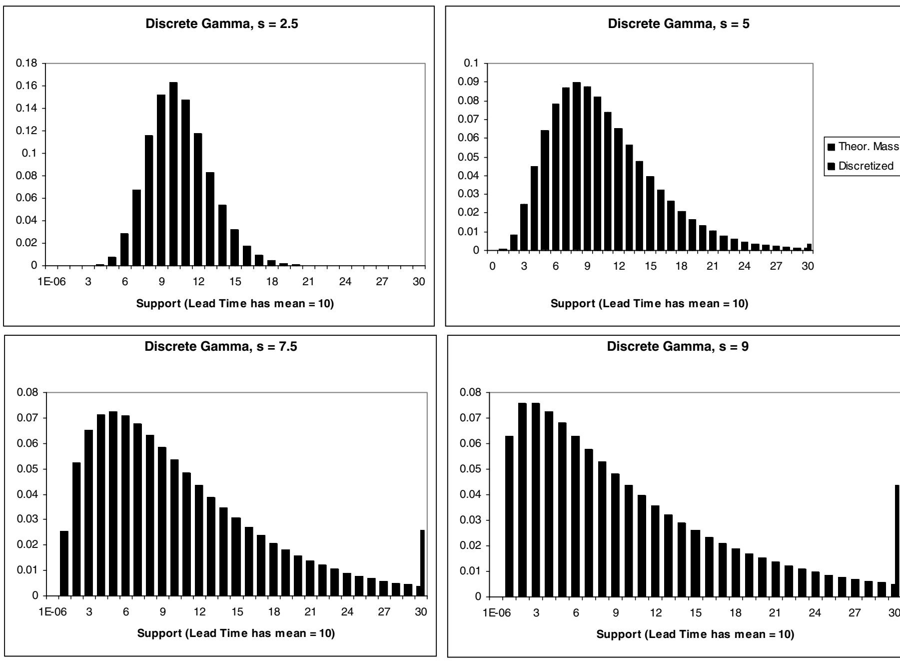
The Truncated Normal Distribution

We seek the inverse (G^{-1}) of the cumulative distribution function of the demand during lead time. The lead time distribution has mean L and standard deviation s_L. We know

$$ROP = G^{-1}\{P(D < X) = \sum_{l=1,\ldots,L} w_l * NORMDIST(X, l * \mu_X, \sigma_X * sqrt(l), 1)\}.$$

Sheet 1. Generate weights Table indexed by (row) support (between 0 and 30) and (column) standard deviation with, in row j and column s the value

Figure A1: Gamma lead time distributions for standard deviations of 2.5, 5, 7.5, and 9.



$$NORMDIST(j, L, s, 1) - \sum_{i=0}^{j-1} NORMDIST(i, L, s, 1)$$

for j $=1, \ldots, 29$ with $NORMDIST(0, L, s, 1)$ 0 in row j $=0$ and, in row j $=30$,

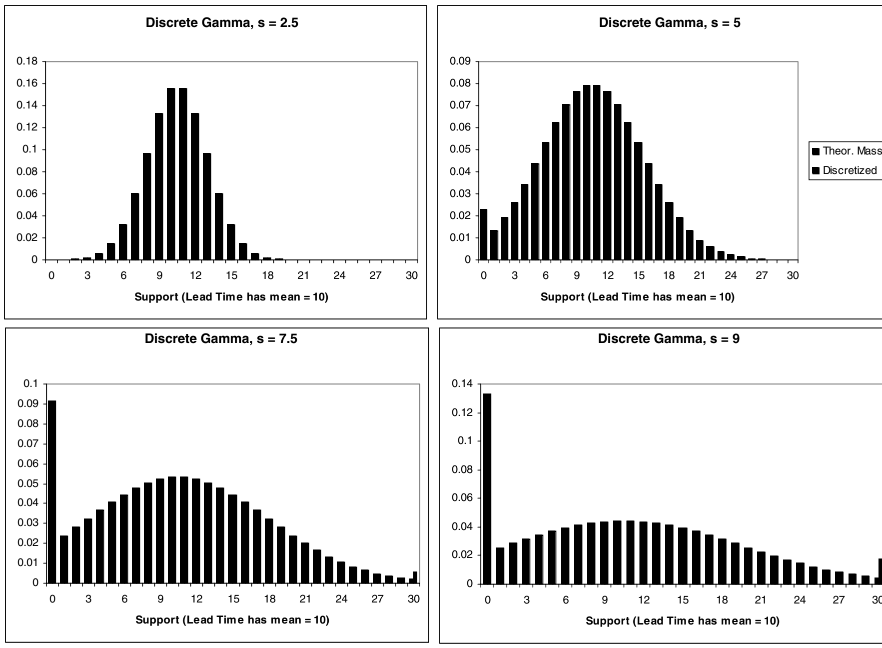
$$NORMDIST(30, L, s, 1) - \sum_{i=0}^{29} NORMDIST\left(i, \left(\frac{L}{s}\right)^2, \frac{s^2}{L}, 1\right)$$
$$+1 - \sum_{i=0}^{30} NORMDIST\left(i, \left(\frac{L}{s}\right)^2, \frac{s^2}{L}, 1\right).$$

That is, we add to j $=0$ and j $=30$, respectively, the mass of the tail to the left of 0 and to the right of 30. In Sheet 1, we also generate a table of NORMDIST values as per the ROP formula above.

Sheet 2. Matrix multiply the Sheet 1's Normdist table to Sheet 1's weights table.

Sheet 3. Use a VLOOKUP(CSL) on Sheet 2 to find the ROP that yields the given CSL.

Figure A2: Normal lead time distributions for standard deviations of 2.5, 5, 7.5, and 9.



The resulting truncated normal distributions for lead time are illustrated in Figure A2.

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