Impact of Stair-Step Incentives and Dealer Structures on a Manufacturer's Sales Variance

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September 21, 2005

Abstract

In this paper we analyze the impact of stair-step incentive schemes, commonly used in the automotive industry, on both expected sales and sales variability. We model the effect of stair-step incentives in two specific scenarios: an exclusive dealership selling cars for only one manufacturer and a non-exclusive dealership selling cars for multiple manufacturers. For an exclusive dealer we show that appropriate stair-step incentives, with a positive bonus on crossing the threshold, not only increase the expected sales, but more importantly, decrease the coefficient of variation of sales. We show that if the manufacturer associates a positive cost with sales variance, a stair-step incentive, with a positive bonus, is superior to the scheme without a bonus. We then show that manufacturers continuing to offer stair-step incentives to non-exclusive dealers experience an increase in variance and a decrease in profits. This implies that when manufacturers must compete for dealer effort, stair-step incentives can hurt manufacturers.

1. Introduction

In this paper we study the impact of two phenomenon, that are observed in the automotive industry, on the variance of sales. We focus on the variance because most recent literature has assumed the firm to be risk neutral and thus ignored variance. In this paper we take the position that manufacturing and distribution costs increase with sales variability. Thus, both the expected value and the variance of sales affect the manufacturer's profits.

We consider the following stair-step incentive that is commonly used in the automotive industry (such incentives have been used by Daimler Chrysler and Nissan among others. This incentive structure is also used in several other industries where a principal sells through an agent.): The dealer is paid an additional amount per unit when sales exceed a threshold value; additionally, a fixed bonus may also be offered if sales exceed the threshold.

The goal of this paper is to understand how stair-step incentives and dealer structure (*exclusive* or *non-exclusive*) affect the effort exerted by dealers and the resulting sales and variance of sales. We do not focus on what the optimal incentive should be, but on the impact of an incentive structure that is commonly observed in the automotive industry. This is in the spirit of the study of "turn-and-earn" in the automotive industry by Cachon and Lariviere (1999).

The Chrysler experience (St. Louis Post-Dispatch, 2001) motivated our study because a change in incentives was followed by a fluctuation in sales that exceeded the average fluctutation for the industry as a whole. Under the stair-step incentive plan, Chrysler gave dealers cash based on the percent of a monthly vehicle sales target met. A dealer got no additional cash for sales below 75% of the sales target, \$150 per vehicle for sales between 75.1% and 99.9% of the sales target, \$250 per vehicle for sales between 100% and 109.9%, and \$500 per vehicle for reaching 110% of the sales target. The downturn in the automobile sector had an undesired effect. Chrysler's sales fell 20% when the industry average fall was between 8 to 12 %. Clearly, Chrysler observed a higher variability in sales than the industry. This paper offers an explanation for this increase in variability observed that is linked to stair-step incentives and the fact that many dealers have become non-exclusive and sell cars for multiple manufacturers. Similar to Chen (2000), Taylor (2002), Krishnan et al (2004), and Cachon and Lariviere (2005) we consider final sales to be affected by a market signal and dealer effort. In our paper we assume that the market signal is not common knowledge but is only visible to the dealer. The dealer then makes his effort decision after observing the market signal based on the incentive offered. We investigate how stair-step incentives and dealer structure (exclusive or non-exclusive) affect the effort decision by the dealer. This allows us to understand how stair-step incentives and dealer structure affect the mean and variance of manufacturer's sales.

The contributions of this paper are twofold. First, for an exclusive dealer we show that appropriate stair-step incentives, with a fixed bonus on crossing the threshold, can decrease the variance as well as the coefficient of variation of sales. Next, we prove that if the manufacturer associates a positive cost with sales variance, a stair-step incentive, with a bonus payment, is superior to the scheme without a positive bonus. For a non-exclusive dealer, however, we show that stair-step incentives reduce the variability of sales for the dealer but increase the same for each manufacturer under reasonable conditions. Specifically, we show that for a given market signal for a manufacturer, a non-exclusive may exert different efforts depending on the signal for the second manufacturer. Thus, the presence of stair-step incentives and non-exclusive dealers helps to partially explain the higher variability in sales observed by Chrysler.

The rest of the paper is organized as follows. Section 2 provides a brief literature review. Section 3 presents the basic models and related assumptions for the exclusive and non-exclusive dealer scenarios. In Section 4 we identify the optimal effort exerted by an exclusive dealer and characterize the expected sales and variance of sales functions with and without bonus payments. The main result for the manufacturer's problem is highlighted in Section 4.2, where we show that the providing a positive bonus reduces the coefficient of sales variation and reduces the cost associated with sales variance. We proceed to discuss the non-exclusive dealer's model in Section 5 and in Section 5.1 we show how dealers benefit from non-exclusivity. In Section 5.2 we analyze the effect of incentive parameters on the sales variance and coefficient of variation for the dealer and manufacturer. We compare the optimal threshold for a manufacturer under both scenarios, exclusive dealer and non-exclusive dealer, and show that under reasonable conditions the optimal threshold is lower in the non-exclusive dealer scenario. This implies that the manufacturer's profits decrease when dealers become non-exclusive; especially when a manufacturer has a high cost of variation. We provide a numerical example to validate our findings in Section 6. Finally, we conclude the paper in Section 7. Proofs for some of the important propositions and claims are provided in Appendix A.

2. Literature Review

Related research can be broadly classified into three areas: economics, marketing, and marketing–operations interface. In the economics domain, seminal work by Harris and Raviv (1979) and Holmström (1979) addresses the issue of information asymmetry between the principal and the agent. In particular, Holmström shows that any additional information about the agent's action (effort) can be used to design better contracts for both entities.

In the marketing literature, Farley (1964) laid the analytic foundation with deterministic demand functions. Weinberg (1975) shows that when salespeople are paid a commission based on gross margin and are allowed to control prices, they set prices to maximize their own income and the company profits simultaneously. Other references, which assume deterministic sales response functions, include Weinberg (1978) and Srinivasan (1981). Chowdhury (1993) empirically tests the motivational function of quotas. The results indicate that as quota levels increase the effort expended increases only up to a certain point, beyond which any increase in the quota level decreases the effort expended. Basu et al. (1985) were the first to apply the agency theory framework to characterize optimal compensation. They model compensation contracts as a Stackelberg game where both the firm (principal) and the agent (salesperson) are symmetrically informed about the sales response function. The risk-neutral firm declares a compensation plan and the agent decides on the effort level which influences the final sales level. Based on the response of the salesperson, to a given compensation contract, the firm chooses a compensation plan which maximizes its profits. The moral hazard problem arises because the relationship between effort and sales is stochastic. The salesperson does not influence costs and has no authority to set prices. Lal and Staelin (1986) extend this by presenting an analysis that relaxes the symmetric information assumption. Rao (1990) provides an alternate approach to the problem by analyzing the issue using a self-selection framework with a heterogeneous salesforce, wherein the salesperson picks a commission level and a quota by maximizing a utility function. Holmström and Milgrom (1987) show that, under certain assumptions, linear compensation schemes developed earlier can indeed be optimal. Lal and Srinivasan (1993) use this framework to model salesforce compensation and gain some interesting insights into single-product and multi-product salesforce compensation. They apply the Holmström Milgrom model and show that the commission income goes up in effectiveness of effort functions. All these papers have primarily focussed on the agency theory and a few studies, such as Coughlan and Narasimhan (1992) and John and Weitz (1989), have found empirical evidence to support this theory. Coughlan and Sen (1989) and Coughlan (1993) provide a comprehensive review on studies in marketing literature. Bruce et al (2004) study a two period model, with trade promotions (incentives) for durable goods, where an active secondary market (e.g. used cars) is present. They study an exclusive dealer setting.

Several recent papers in Operations Management have used agency models to study the marketing-operations interface. The influential paper by Porteus and Whang (1991) studied coordination problems between one manufacturing manager (MM) and several product managers (PM) where the PMs make sales efforts while the MM makes efforts for capacity realization and decides inventory levels for different products. They develop incentive plans that induce the managers to act in such a way that owner of the firm can attain maximum possible returns. Plambeck and Zenios (2000) develop a dynamic principal-agent model and identify an incentive-payment scheme that aligns the objectives of the owner and manager. Chen (2000) (2005) considers the problem of salesforce compensation by considering the impact of salesforce incentives on a firm's production inventory costs. Taylor (2002) considers the problems of coordinating a supply chain when the dealer exerts a sales effort to affect total sales. He assumes that the dealer's effort decision is made before market demand is realized. Krishnan et al. (2004) discuss the issue of contract-induced moral hazard arising when a manufacturer offers a contract to coordinate the supply chain and the dealer exerts a promotional effort to increase sales. Their paper assumes that the dealer's effort decision is made after observing initial sales. Our paper makes a similar assumption. Cachon and Lariviere (2005) also discuss the situation when revenue sharing contracts do not coordinate a supply chain if a dealer exerts effort to increase sales. They develop a variation on revenue sharing for this setting. Overall, this line of literature mainly focuses on maximizing the manufacturer's profits when they are assumed to be risk neutral. Our focus, however, is on understanding the impact on the variance of sales.

3. Model Basics and Assumptions

We consider sales to be the sum of a stochastic market signal and a function of the dealer effort. The manufacturer's total sales, s, are determined by the dealer's selling effort (b) and the market signal (x) by the following additive form: $s = x + g(b)$. The market signal is observed by the dealer but not the manufacturer. The manufacturer only observes the total sales s. The dealer bases the effort decision on the observed signal and the incentive offered by the manufacturer. We assume that the dealer observes the market signal x before he makes his effort decision b. As commonly assumed in the literature (see Chen, 2000), the growth of sales $g(b)$ with respect to the dealer effort b is concave and the cost of the effort $c(b)$ is convex and increasing. The input market signal, x, follows a continuous and twice differential cumulative distribution function, F, with a bounded probability density function

f. We also assume that probability distributions are log-concave, $f(y) = 0$ for all $y < 0$, $f(y) > 0$ for $y > 0$, and $F(0) = 0$.

The stair-step incentive is organized as follows: The dealer makes a standard margin (excluding cost of effort) p for every unit sold up to the threshold K. For every additional unit sold above, K, the manufacturer pays an additional Δ to the dealer. Thus, the dealer's margin (excluding the cost of effort) increases to $p+\Delta$ for every unit sold above the threshold K. In addition, the manufacturer offers a fixed bonus of $D \geq 0$, if sales reach the threshold of K. We assume that the distribution of the market signal is independent of the incentive parameters K, D and Δ. We analyze the impact of incentives on sales variability under two specific scenarios: an exclusive dealership scenario where a dealer sells product for a single manufacturer and a non-exclusive dealership scenario where the dealer sells products for two manufacturers.

In Section 4, we study the sale of a manufacturer's product through an exclusive dealer. We study the dealer's optimal response to a given incentive. This allows us to characterize how the expected value and variance of sales changes with the threshold K and bonus D. We show that, for certain values of K, the introduction of a positive bonus, D, increases the manufacturer's expected sales and decreases the variance. Our analysis shows that a manufacturer, whose costs increase with sales variability, can improve profits by offering a positive bonus D to an exclusive dealer.

In Section 5, we study the case when two manufacturers sell products through a non-exclusive dealer. Each manufacturer offers a stair-step incentive to the dealer. The dealer observes market signals x_i, $i = 1, 2$, and then decides on the effort levels b_i, $i = 1, 2$, across the two manufacturers. Our analysis assumes the market signals across the two manufacturers to be independent. The sales for each manufacturer are $s_i = x_i + g(b_i)$. The cost of the effort is assumed to be $2\,c\left(\frac{b_1+b_2}{2}\right)$. It is reasonable to assume that the dealer's cost is a function of the total effort because common resources are used by the dealer to spur sales across all the products (cars) they sell. Our results indicate that manufacturers offering stair-step incentives observe higher sales variability with non-exclusive dealers compared to exclusive dealers. Numerical simulations indicate that in case of non-exclusive dealers, a positive bonus may not be as helpful in reducing the manufacturer's sales variance.

4. The Exclusive Dealer

Our first objective is to identify an exclusive dealer's optimal response when facing a stair step incentive. Assume that an exclusive dealer exerts effort b given an input market signal x. Given a stair-step incentive, the dealer makes one of the following three profit levels depending on the sales $x + g(b)$.

$$\Pi_1(x,\ b) = p\ (x + g(b)) - c(b) \quad \text{if } x + g(b) < K,$$
$$\Pi_2(x,\ b) = (p + \Delta)\ (x + g(b)) + D - \Delta K - c(b) \text{ if } x + g(b) \geq K, \text{ and}$$
$$\Pi_K(x,\ b) = p\ K + D - c(b) \ \text{ where } \ x + g(b) = K. \tag{1}$$

Π_1 is the profit realized when the total sale is less that K, Π_2 is the profit when the sale exceeds K, and Π_K is the dealer's profit when the sale equals K. Π_1, Π_2, and Π_K are concave functions with respect to b because g is concave and c is convex. Let b_1^* and b_2^* be the optimal efforts that maximize Π_1 and Π_2 respectively. Let B^K represent the set of effort levels defined by $B^K \equiv \{b^K : b^K = g^{-1}(K - x)\}$. First order KKT optimality conditions imply that b_1^* and b_2^* must satisfy the following conditions: $g'(b_1^*) = \frac{c'(b_1^*)}{p}$, $g'(b_2^*) = \frac{c'(b_2^*)}{p+\Delta}$. The dealer compares the profits on exerting effort b_1^*, b_2^*, and b^K and exerts the effort that results in the highest profit. With a slight abuse of notation we can represent the optimal effort chosen by the exclusive dealer as

$$b^*(x) = \arg \max_{\{b_1^*,\ b_2^*,\ b^K\}} \left\{\Pi_1(x,\ b_1^*),\ \Pi_2(x,\ b_2^*),\ \Pi_K\left(x,\ b^K\right)\right\} \tag{2}$$

Notice that $\Pi_1(x,\ b_1^*)$ and $\Pi_2(x,\ b_2^*)$ are linear in x and $\Pi_K\left(x,\ b^K\right)$ is a concave function of x. The slope of $\Pi_2(x,\ b_2^*)$ is greater than that of $\Pi_1(x,\ b_1^*)$. The plot to the left, in Figure 1, shows the optimal-effort profits, as a function of x, when $K > g(b_2^*)$ and D is such that $\Pi_1(0,\ b_1^*) > \Pi_2(0,\ b_2^*)$. For low market signals, the dealer exerts effort b_1^* and resulting sales are below K. At some point it is optimal for the dealer to exert enough effort b^K to raise the sales to K. For higher market signals, the dealer exerts effort b_2^* and the resulting sales exceed K. The two cutoff points, δ_{x1} and δ_{x2}, represent the level of the market signals at which the dealer switches optimal effort levels. For market signals below δ_{x1}, the dealer exerts effort that keeps sales below K. Between δ_{x1} and δ_{x2}, the dealer exerts effort such that sales are exactly K. For a market signal above δ_{x2}, the dealer exerts effort such that sales exceed K.

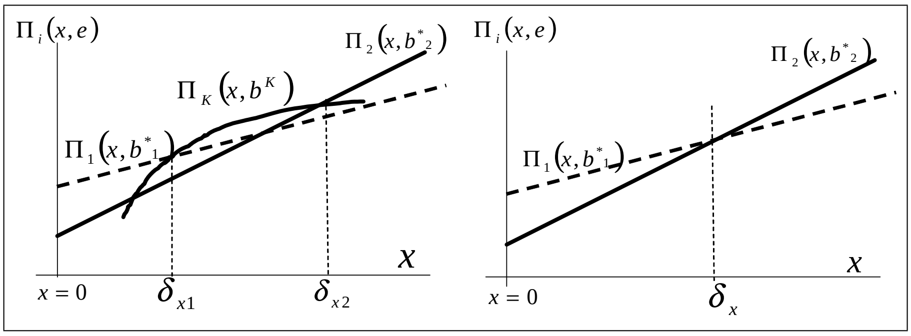

Figure 1: Exclusive dealer optimal profit functions when $D > 0$ and $D = 0$.

When the bonus $D = 0$, Π_K is now equivalent to Π_2 and can be eliminated. As shown in the second plot in Figure (1), the only transition point for the dealer's effort is denoted by δ_x. The exclusive dealer's problem can be expressed as an equivalent non-linear optimization problem (EDP).

$$EDP: \quad \min\ \Theta$$

$$g_1: \quad \Pi_1(x,\, b_1) - \Theta \leq 0 \quad (u_1) \tag{3}$$

$$g_2: \quad \Pi_2(x,\, b_2) - \Theta \leq 0 \quad (u_2) \tag{4}$$

$$g_3: \quad x + g(x,\, b^K) - K = 0 \quad (u_3) \tag{5}$$

$$g_4: \quad \Pi_3(x,\, b^K) - \Theta \leq 0 \quad (u_4) \tag{6}$$

$$b_1,\ b_2,\ b^K,\ \Theta \in \mathbb{R}_+ \tag{7}$$

For a given x and b, $\Pi_1(x,b) > \Pi_2(x,b)$ when $x + g(b) < K$ and $\Pi_2(x,b) > \Pi_1(x,b)$ when $x + g(b) > K$. Further, if $D > 0$ and $x + g(b) = K$, then $\Pi_3(x,b) = \Pi_2(x,b) > \Pi_1(x,b)$. Constraint (5) models the fact that the dealer may put a different effort, b^K, if the bonus D is strictly positive, so that the threshold sales is just achieved. The values $u_1,\ u_2,\ u_3$,and u_4, shown in brackets next to the constraints (3), (4), (5), and (6), are the lagrangian multipliers. Claim 1 characterizes the optimal cutoff points, where the dealer changes effort levels, when the manufacturer offers a positive bonus $D > 0$. Recall that the effort levels b_1^* and b_2^* are as follows:

$$b_1^* : g'(b_1^*) = \frac{c'(b_1^*)}{p} \text{ and } b_2^* : g'(b_2^*) = \frac{c'(b_2^*)}{p + \Delta} \tag{8}$$

Claim 1 *When $D > 0$ the exclusive dealer's optimal effort levels are*

$$b^* = \begin{cases} b_1^* & : 0 \leq x < \delta_{x1} \\ b^K & : \delta_{x1} \leq x < \delta_{x2} \text{ and } b^K = g^{-1}(K - x) \\ b_2^* & : \delta_{x2} \leq x \end{cases}$$

where the cutoff $\delta_{x2} = K - g(b_2^*)$, *and the cutoff* $\delta_{x1} = K - \gamma_x$. γ_x *is defined by* $p\,\gamma_x - c[g^{-1}(\gamma_x)] + D = p\,g(b_1^*) - c(b_1^*)$.

Proof. The proof uses the first order optimality conditions. The cutoffs are can be calculated using the dominance of one profit function over the other. See Appendix A for the proof. ■

Claim 2 characterizes the optimal cutoff point when the bonus $D = 0$.

Claim 2 *When $D = 0$, the exclusive dealer's optimal efforts are b_1^*, when $0 \leq x < \delta_x$, and b_2^* when $x \geq \delta_x$. The cutoff* $\delta_x = K - g(b_2^*) + \frac{1}{\Delta}([p\,g(b_1^*) - c(b_1^*)] - [p\,g(b_2^*) - c(b_2^*)])$.

Proof. The proof uses the first order optimality conditions. See Appendix A for a detailed proof. ■

Define

$$\varepsilon_x \equiv \frac{1}{\Delta}\left([p\,g(b_1^*) - c(b_1^*)] - [p\,g(b_2^*) - c(b_2^*)]\right) \tag{9}$$

Using the fact that $\frac{c'}{g'}$ is an increasing function it is easy to show that $b_1^* < b_2^*$. Further, if $D = 0$ then using simple convexity arguments it can be shown that $g'(b_2^*)(b_2^* - b_1^*) \geq \varepsilon_x \geq 0$.

Claims 1 and 2 imply that if total sales are below K, it is always more profitable to exert the lower effort b_1^* rather than the higher effort level b_2^*. Claims 1 and 2 are illustrated in Figure 2. The plot to the left in Figure 2, shows the optimal effort levels when $D = 0$ for different input market signals. If the market signal x is below δ_x, the dealer exerts an effort, b_1^*, with resulting sales below the threshold limit, K. If the market signal is at least δ_x, the dealer exerts an higher effort, b_2^*, and the resulting sales exceed K. The plot to the right in Figure 2 shows the optimal effort levels when $D > 0$. For market signals below δ_{x1}, the dealer spends an effort, b_1^*, and the resulting sales are less than K. When the market signal x is such that $\delta_{x1} \leq x < \delta_{x2}$, the dealer exerts an effort, $b^K = g^{-1}(K - x)$, to push sales to the bonus limit K and capture the bonus payment D. For all market signals x that are at least δ_{x2}, the dealer exerts an effort, b_2^* with resulting sales above K. The important fact to note is that the introduction of a bonus $D > 0$ induces the dealer to exert additional effort to reach the threshold K when market signals are between δ_{x1} and δ_{x2}.

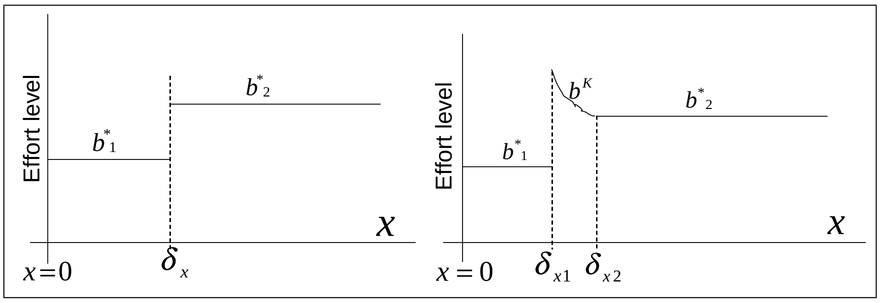

Figure 2: Optimal effort levels for an exclusive dealer when $D = 0$ and $D > 0$.

Remark 3 *When the bonus is small, i.e.* $D < \Delta\,\varepsilon_x$, *the dealer behaves as if* $D = 0$ *(has only two optimal effort levels,* b_1^* *and* b_2^**).*

This remark follows from the values of δ_{x1} and δ_{x2} in Claim 1. When the bonus D is not large enough, $\delta_{x1} > \delta_{x2}$ in Claim 1 and there is no region where the dealer spends effort b^K to just reach the threshold K.

Having characterized the exclusive dealer's optimal response, we next compute the expected sales and variance in Section 4.1.

4.1 Impact of D and K on the Dealer's Sales

Our goal in this section is to understand how the mean and variance of sales is affected by the threshold K and bonus D in the stair-step incentive. We first compute the expected sales and variance when $D > 0$. The expected sales, $E(s)$, can be expressed as

$$\begin{aligned} E(s) &= \int_0^{\delta_{x1}} (x + g(b_1^*))\ f(x)\ dx + \int_{\delta_{x1}}^{\delta_{x2}} K\ f(x)\ dx + \int_{\delta_{x2}}^{\infty} (x + g(b_2^*))\ f(x)\ dx \\ &= E(x) + g(b_2^*) + F(\delta_{x1})\ \left[g(b_1^*) - \gamma_x\right] + (\delta_{x2} - \delta_{x1}) \\ &+ \int_{\delta_{x2}}^{\infty} (x - \delta_{x2})\ f(x)\ dx - \int_{\delta_{x1}}^{\infty} (x - \delta_{x1})\ f(x)\ dx \end{aligned} \tag{10}$$

The variance of sales is given by $V(s) = E(s^2) - [E(s)]^2$. Before we show how $E(s)$ and $V(s)$ vary with K for log-concave distributions in Claim 4 we define the function $t(K) \equiv \frac{F\left(K - g\left(b_2^*\right)\right) - F(K - \gamma_x)}{f(K - \gamma_x)}$.

Claim 4 *If* $K \geq \gamma_x$, f *is log-concave and bounded, and* $\alpha = (\gamma_x - g(b_1^*)) > 1$ *then*

1. $\exists$ *a* $K_1^* \geq \gamma_x$ *such that and* $\frac{\partial E(s)}{\partial K} \geq 0 \ \ \forall\ \gamma_x \leq \ K \leq K_1^*$ *and* $\frac{\partial V(s)}{\partial K} \leq 0 \ \ \forall \ \ K \geq K_1^*$. *If* $\alpha < t(\gamma_x)$ *then* $t(K_1^*) = \alpha$*; Otherwise,* $K_1^* = \gamma_x$, *and*

2. *$V(s)$ is an inverted-U shaped function; i.e. $\exists$ a K_2^* such that* $\frac{\partial V(s)}{\partial K} \geq 0 \ \forall \gamma_x \leq \ K \leq K_2^*$ *and* $\frac{\partial V(s)}{\partial K} \leq 0 \ \ \forall \ \ K \geq K_2^*$.

Proof. See Appendix A. ■

The condition on α is not very restrictive. It is satisfied for reasonable values of incentive parameters for uniform and normal distributions.

In Proposition 5 we characterize the expected sales and variance of sales relative to $E(x)$ and $V(x)$ for different values of K when $D > 0$. We show the existence of threshold values for which the variance of sales is less than the variance of the market signal.

Proposition 5 *If $D > 0$, f is log-concave and bounded, and $\alpha = (\gamma_x - g(b_1^*)) > 1$ then*

1. $$E(s) - E(x) = \begin{cases} g(b_2^*) & \text{if } 0 \leq K \leq g(b_2^*) \\ \text{monotonically increases with } K & \text{if } g(b_2^*) \leq K \leq K_1^* \\ \text{monotonically decreases with } K & \text{if } K_1^* < K \end{cases}$$

2. $$V(s) - V(x) = \begin{cases} 0 & \text{if } 0 \leq K \leq g(b_2^*) \\ \text{monotonically decreases with } K & \text{if } g(b_2^*) \leq K \leq \gamma_x \\ \text{inverted-U shaped with maximum above } V(x) & \text{if } \gamma_x < K \end{cases}$$

3. *$E(s)$ is maximized and $V(s)$ is minimized at $K = \gamma_x$*

Proof. See Appendix A. ■

From Proposition 5 observe that if K is between $g(b_2^*)$ and γ_x, the variance of sales is less than the variance of the market signal. In other words, the manufacturer can select a threshold K and a bonus $D > 0$, such that the exclusive dealer finds it optimal to absorb some of the market variance by adjusting effort, thus lowering sales variance for the manufacturer.

In Proposition 6, we characterize the expected sales, $E(s)$, and variance of sales, $V(s)$, for different values of K when $D = 0$. We show that in the absence of a positive bonus D, the sales variance is never below the variance of the market signal.

Proposition 6 *If $D = 0$, f is log-concave and bounded, and $\alpha = (\gamma_x - g(b_1^*)) > 1$ then*

1. $$E(s) - E(x) = \begin{cases} g(b_2^*) & \text{if } 0 \leq K \leq g(b_2^*) - \varepsilon_x \\ \text{monotonically decreases with } K & \text{if } g(b_2^*) - \varepsilon_x \leq K \end{cases}$$

2. $$V(s) - V(x) = \begin{cases} 0 & \text{if } 0 \leq K \leq g(b_2^*) - \varepsilon_x \\ \text{inverted-U shaped with maximum above } V(x) & \text{if } g(b_2^*) - \varepsilon_x \leq K \end{cases}$$

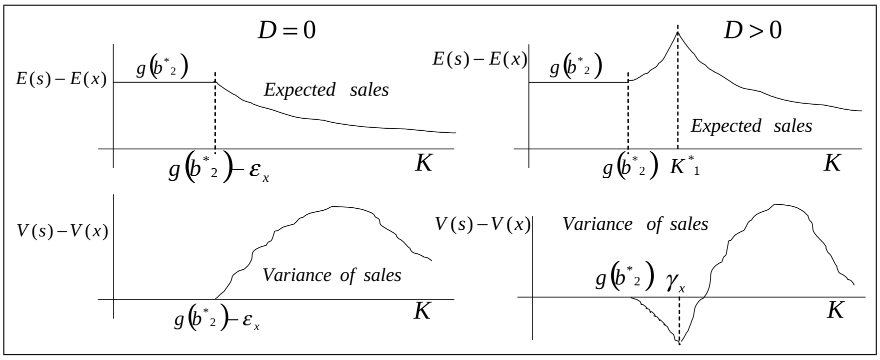

Figure 3: Expected profit and variance functions for an exclusive dealer when $D = 0$ and $D > 0$.

3. The expected sales is maximized and variance is minimized for $K \leq g(b_2^*) - \varepsilon_x$

Proof. See Appendix A. ■

Figure 3 summarizes the results of Propositions (5) and (6). As shown in the plot to the left in Figure 3, when $D = 0$, expected sale is constant until $K \leq g(b_2^*) - \varepsilon_x$ and starts decreasing (though never falls below the expected market signal $E(x)$) when $K \geq g(b_2^*) - \varepsilon_x$. For $D = 0$ the variance of sales curve is never below the variance of the market signal and is higher than of the input market signal for values of K beyond $g(b_2^*) - \varepsilon_x$. The plot to the right shows that when $D > 0$ the expected sales are maximized at $K = K_1^*$and variance minimized at $K = \gamma_x$. The variance of sales is below that of the market signal at this point. This implies that, with exclusive dealers, the manufacturer can offer a positive bonus, $D > 0$, and choose an appropriate threshold, K, such that the dealer exerts effort levels that increase expected sales and reduce variance below the market signal. The reduction in variance is driven by the fact that the presence of a positive bonus $D > 0$ leads the dealer to exert effort such that sales are raised exactly to K over a range of market signals. In other words, with a positive bonus, an exclusive dealer reduces the variance of sales for the manufacturer by varying his effort to absorb some of the market signal variance. For a manufacturer with a high cost of operational sales variance, this fact is significant while designing an appropriate stair-step incentive plan. We discuss how the manufacturer can select an appropriate incentive structure in Section 4.2.

4.2 How Should a Manufacturer Structure Stair-Step Incentives for an Exclusive Dealer

In Section 4.1 we described the impact of incentive parameters on the expected sales and variance of sales. In this section we consider the manufacturer's problem of designing the stair-step incentive and describe the structure of such incentives that maximizes manufacturer profits when the manufacturer's costs increase with sales variance. Our main result shows that if the manufacturer has a high operational cost associated with sales variance, it is preferable for the manufacturer to offer a positive bonus that encourages the exclusive dealer exert effort in a way that reduces sales variance.

Assume that the manufacturer's margin per unit is p_m when the dealer's margin is p per unit, and the manufacturer's cost of variance is v. Given a sales variance of V and market signal x the manufacturer's profit is evaluated as

$$\Pi = \begin{cases} p_m\left(x + g\left(b\right)\right) - v\,V & \text{if } x + g\left(b\right) < K \\ p_m K - D + \left(p_m - \Delta\right)\left(x + g\left(b\right) - K\right) - v\,V & \text{if } x + g\left(b\right) \geq K \end{cases} \tag{11}$$

Using Propositions 5 and 6 we can thus structure the manufacturer's optimal incentive for an exclusive dealer.

Proposition 7 *When designing a stair-step incentive with $D = 0$, the manufacturer maximizes his profits either by setting $K = g\left(b_2^*\right) - \varepsilon_x$, where b_2^* is defined by equation 8 and ε_x is defined by equation 9, or by setting K to be extremely large such that $F\left(\delta_x\right) \to 1$; i.e. the market signal is guaranteed to be below the cutoff δ_x.*

Proof. The proof uses Proposition 6. See Appendix A for a detailed proof. ■

Next, we characterize the optimal stair-step incentive for the case when $D > 0$.

Proposition 8 *When designing a stair-step incentive with $D > 0$, the manufacturer maximizes profits by either setting $K \geq \gamma_x$, where γ_x is defined in Claim 1, or by setting a large enough $K = g\left(b_2^*\right)$.*

Proof. The proof uses Proposition 5. See Appendix A for a detailed proof. ■

We now show that for a high enough operational cost of variance, the manufacturer is better off by offering a stair-step incentive with $D > 0$ compared to the case when $D = 0$.

Proposition 9 *For a high enough value of v the manufacturer can increase profits by setting a positive bonus payment $D > 0$.*

Proof. See Appendix A. ■

Proposition 9 shows that a manufacturer with a high cost of sales variance is better off offering a stair-step incentive with a positive bonus. The positive bonus encourages the dealer to exert effort in a way that reduces sales variance. In the next section we show that as dealers become non-exclusive, manufacturers face a greater sales variance than when dealers are exclusive.

5. The Non-Exclusive Dealer

In this section, we study the effect of stair-step incentives when the dealer is no longer exclusive and sells products for multiple manufacturers. In the automotive industry in the United States, most dealers today are non-exclusive. Auto malls, for example, sell cars from multiple manufacturers from the same lot. In our model, dealers that sell cars for different manufacturers from different lots are also non-exclusive as long as they can shift effort across manufacturers. This often occurs in practice because a dealer selling for two manufacturers is likely to shift advertising effort and cost across the manufacturers depending upon market conditions. Our goal is to understand how the loss of exclusivity affects sales variance for manufacturers offering stair-step incentives. Consider two manufacturers (1 and 2) selling their products (also denoted by index 1 and 2) through a single non-exclusive dealer. We assume that both manufacturers offer similar stair-step incentives, in terms of p, Δ, D, and K. While maintaining symmetry simplifies the analysis and exposition, most of the results can be extended to the asymmetric case.

The sequence of events is as follows. The dealer observes market signals x_i, $i = 1, 2$, then decides on the efforts, b_i, resulting in sales $x_i + g(b_i)$ for $i = 1, 2$. Our analysis assumes the market signals to be independent. The dealer's cost of effort is based on total effort and is given by $2\ c(\frac{b_1+b_2}{2})$ which is convex and increasing. Observe that when $b_1 = b_2$ the cost of effort for the non-exclusive dealer is equal to the sum of the cost of efforts for the two exclusive dealers. Next, we study the dealer's optimal response function when $D = 0$.

5.1 The Non-Exclusive Dealer's Problem for $D = 0$

For $D = 0$ there are four possible profit outcomes for the dealer, Π_i $(i = 1, \ldots, 4)$ with four distinct effort levels b_{ki} $(k = 1, \ \ldots, 4)$ for each manufacturer $i = 1, 2$ (see Table 1). Each profit function, Π_i in Table 1, is concave in the effort levels b_{i1} and b_{i2}.

	Profit	
Π_1	$p(x_1+g(b_{11}))+p(x_2+g(b_{12}))-2\ c(\frac{b_{11}+b_{12}}{2})$	$: x_1+g\,(b_{11})<K,$ and $x_2+g\,(b_{12})<K$
Π_2	$p(x_1+g(b_{21}))+(p+\Delta)(x_2+g(b_{22}))$ $-\Delta\,K-2\ c(\frac{b_{21}+b_{22}}{2})$	$: x_1+g\,(b_{21})<K,$ and $x_2+g\,(b_{22})\geq K$
Π_3	$(p+\Delta)(x_1+g(b_{31}))+p(x_2+g(b_{32}))$ $-\Delta\,K-2\ c(\frac{b_{31}+b_{32}}{2})$	$: x_1+g\,(b_{31})\geq K,$ and $x_2+g\,(b_{32})<K$
Π_4	$(p+\Delta)(x_1+g(b_{41}))+(p+\Delta)(x_2+g(b_{42}))$ $-2\ \Delta\,K-2\ c(\frac{b_{41}+b_{42}}{2})$	$: x_1+g\,(b_{41})\geq K,$ and $x_2+g\,(b_{42})\geq K$

Table 1: The four possible profit outcomes for a non-exclusive dealer.

Let b^*_{i1} and b^*_{i2} denote the optimal effort levels that maximize the dealer's profit functions Π_i $(i=1,\ldots,4)$. Observe that the profit functions $\Pi_i(x_1,\ x_2,\ b^*_{i1},\ b^*_{i2})$, $i=1,\ldots,4$, are a linear in x_1 and x_2. The non-exclusive dealer chooses optimal efforts such that

$$(b^*_{i1},\ b^*_{i2})=\arg\max_{i=1,\ldots,4}\{\Pi_i\} \qquad (12)$$

In order to characterize the optimal effort levels and compute the cutoff ranges for x_1 and x_2 we express the non-exclusive dealer's problem as an equivalent non-linear profit maximization model. The values in the brackets to the right of the constraints are the corresponding lagrangian multipliers.

$$NEDP: \min\ \Theta \qquad (13)$$

$$g1: \Pi_1(x_1,x_2,b_{11},b_{12})-\Theta\leq 0 \quad (u_1)$$

$$g2: \Pi_2(x_1,x_2,b_{21},b_{22})-\Theta\leq 0 \quad (u_2)$$

$$g3: \Pi_3(x_1,x_2,b_{31},b_{32})-\Theta\leq 0 \quad (u_3)$$

$$g4: \Pi_4(x_1,x_2,b_{41},b_{42})-\Theta\leq 0 \quad (u_4)$$

We define ε_1, ε_2, ε_3, and ε_4 as follows:

$$\varepsilon_1\equiv\frac{p\,[g(b^*_{11})-g(b^*_{21})]+p\,[g(b^*_{12})-g(b^*_{22})]+2\ c(\frac{b^*_{21}+b^*_{22}}{2})-2\ c(\frac{b^*_{11}+b^*_{12}}{2})}{\Delta} \qquad (14)$$

$$\varepsilon_2\equiv\frac{p\,[g(b^*_{11})-g(b^*_{31})]+p\,[g(b^*_{12})-g(b^*_{32})]+2\ c(\frac{b^*_{31}+b^*_{32}}{2})-2\ c(\frac{b^*_{11}+b^*_{12}}{2})}{\Delta} \qquad (15)$$

$$\varepsilon_3\equiv\frac{(p+\Delta)\,[g(b^*_{31})-g(b^*_{41})]+p\,[g(b^*_{32})-g(b^*_{42})]+2\ c(\frac{b^*_{41}+b^*_{42}}{2})-2\ c(\frac{b^*_{31}+b^*_{32}}{2})}{\Delta} \qquad (16)$$

$$\varepsilon_4\equiv\frac{p\,[g(b^*_{21})-g(b^*_{41})]+(p+\Delta)\,[g(b^*_{22})-g(b^*_{42})]+2\ c(\frac{b^*_{41}+b^*_{42}}{2})-2\ c(\frac{b^*_{21}+b^*_{22}}{2})}{\Delta} \qquad (17)$$

Claim 10 summarizes the optimal efforts and cutoff values for a non-exclusive dealer.

Claim 10 *The non-exclusive dealer exerts the following effort levels to maximize profits*

$$b^* = \begin{cases} (b_{11}^*, b_{12}^*) & : 0 < x_1 < \delta_2 \;\; \textit{and} \;\; 0 < x_2 < \delta_1 \\ (b_{21}^*, b_{22}^*) & : 0 < x_1 < \delta_2 \;\; \textit{and} \;\; \delta_1 \le x_2 \\ (b_{31}^*, b_{32}^*) & : \delta_2 \le x_1 < \delta_4 \;\; \textit{and} \;\; x_2 < x_1 \\ (b_{21}^*, b_{22}^*) & : \delta_2 \le x_1 < \delta_4 \;\; \textit{and} \;\; x_2 > x_1 \\ (b_{31}^*, b_{32}^*) & : \delta_4 \le x_1 \;\; \textit{and} \;\; 0 < x_2 < \delta_3 \\ (b_{41}^*, b_{42}^*) & : \delta_4 \le x_1 \;\; \textit{and} \;\; \delta_3 \le x_2 \end{cases}$$

where the effort levels and cutoffs satisfy the following conditions:

1. $g'(b_{11}^*) = g'(b_{12}^*) = \frac{c'\left(\frac{b_{11}^*+b_{12}^*}{2}\right)}{p}$;

2. $g'(b_{21}^*) = \frac{c'\left(\frac{b_{21}^*+b_{22}^*}{2}\right)}{p}$, $g'(b_{22}^*) = \frac{c'\left(\frac{b_{21}^*+b_{22}^*}{2}\right)}{p+\Delta}$;

3. $g'(b_{31}^*) = \frac{c'\left(\frac{b_{31}^*+b_{32}^*}{2}\right)}{p+\Delta}$, $g'(b_{32}^*) = \frac{c'\left(\frac{b_{31}^*+b_{32}^*}{2}\right)}{p}$;

4. $g'(b_{41}^*) = g'(b_{42}^*) = \frac{c'\left(\frac{b_{41}^*+b_{42}^*}{2}\right)}{p+\Delta}$;

5. $\delta_1 = K - g(b_{22}^*) + \varepsilon_1$, $\delta_2 = K - g(b_{31}^*) + \varepsilon_2$, $\delta_3 = K - g(b_{42}^*) + \varepsilon_3$, $\delta_4 = K - g(b_{41}^*) + \varepsilon_4$.

Proof. These results are proved using first order optimality conditions. The cutoffs are computed by finding the points of intersection of the various profit functions. See Appendix A for a sketch of the proof. ■

Corollary 11 *The following equalities hold:* $b_{11}^* = b_{12}^*$, $b_{41}^* = b_{42}^*$, *and* $\frac{g'(b_{21}^*)}{g'(b_{22}^*)} = \frac{g'(b_{32}^*)}{g'(b_{31}^*)} = \frac{p+\Delta}{p}$.

Proof. These relationships follow immediately from Claim 10. ■

Notice that for our problem the four levels of optimal effort exerted by the non-exclusive dealer are symmetric across the two manufacturers. That is to say $b_{31}^* = b_{22}^*$, $b_{32}^* = b_{21}^*$, $b_{11}^* = b_{12}^*$ and $b_{41}^* = b_{42}^*$.

Rearranging the terms in equations (14), (15), (16), and (17) we get $\delta_4 - \delta_2 = \delta_3 - \delta_1$. Figure 4 shows the cutoffs and optimal dealer efforts as the input market signals vary. For a value of x_1 between δ_2 and δ_4, the optimal effort exerted by the dealer for manufactuer 1 fluctuates from b_{31}^* to b_{21}^*depending upon the value of x_2. For a market signal $x_2 > x_1$, the dealer exerts a lower effort b_{21}^* for manufacturer 1. For a market signal $x_2 < x_1$, the dealer exerts a higher effort b_{31}^* for manufacturer 1. Unlike the case of the exclusive dealer, where each market signal resulted in a specific effort by the dealer, the non-exclusive dealer may exert different effort levels for the same market signal for a manufacturer. This result may partially explain the Chrysler experience mentioned at the beginning of the paper. With non-exclusive dealers and manufacturers that offer stair step incentives, Chrysler may have seen a large drop in sales because their market signal was lower than that of other manufacturers and dealers shifted their effort away from Chrysler to other manufacturers.

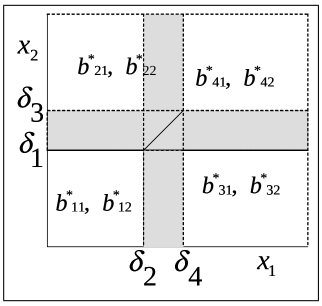

Figure 4: Optimal effort levels and cutoff values for a non-exclusive dealer.

Before comparing the optimal effort levels for the exclusive dealer and non-exclusive dealer, we compute the bounds on ε_i $(i = 1, \ldots, 4)$. These bounds can be easily proved using convexity arguments for the effort and cost functions, and the conditions on the optimal efforts from Claim 10. We summarize these results in Claim 12.

Claim 12 *The following inequalities hold for a non-exclusive dealer:* $0 \leq \varepsilon_1 \leq g'(b^*_{22})\,(b^*_{22} - b^*_{12})$*,* $0 \leq \varepsilon_2 \leq g'(b^*_{31})(b^*_{31} - b^*_{11})$*,* $0 \leq \varepsilon_3 \leq g'(b^*_{42})(b^*_{42} - b^*_{32})$*, and* $0 \leq \varepsilon_4 \leq g'(b^*_{41})(b^*_{41} - b^*_{21})$*.*

Proof. These results are proved using simple convexity arguments. See Appendix A for technical details. ■

Corollary 13 *The effort levels are nested as follows:* $b^*_{22} \geq b^*_{12}$*,* $b^*_{31} \geq b^*_{11}$*,* $b^*_{42} \geq b^*_{32}$*, and* $b^*_{41} \geq b^*_{21}$*.*

Proof. Follows from Claim 12. ■

These relationships help us compare the effort levels exerted by an exclusive dealer and a non-exclusive dealer. Suppose an exclusive dealer exerts optimal effort levels b^*_1 and b^*_2 for manufacturer 1. If the same dealer becomes non-exclusive to manufacturer 1, the expected effort levels change to b^*_{i1} $(i = 1, \ldots, 4)$ depending on the market signals ($x_1$and x_2). To understand how this will affect the expected sales and variance of sales, for both the dealer and the manufacturers, we need to understand the relationship between these effort levels. Claim 14 summarizes the nesting relationship between these effort levels.

Claim 14 *If $\Delta > 0$ then the effort levels for an exclusive and non-exclusive dealer are nested as follows:* $b_{31}^* > b_{41}^* = b_2^* > \frac{b_{31}^* + b_{21}^*}{2} > b_{11}^* = b_1^* > b_{21}^*$

Proof. The nested relationship is proved using the fact that $\frac{c'}{g'}$ is an increasing function and other convexity arguments. See Appendix A for technical details of the proof. ■

Claim 15 *For any given K the following relationship holds: $\delta_4 > \delta_x > \delta_2$.*

Proof. These results are proved using simple convexity arguments. See Appendix A for technical details. ■

Using Claim 15 we show that the non-exclusive dealer's expected profit is strictly greater than the sum of expected profits of two exclusive dealers.

Proposition 16 *When $D = 0$, and the stair-step incentives are the same, a non-exclusive dealer's expected profits are larger than the sum of the expected profits of the two exclusive dealers.*

Proof. The proof uses Claim 15 and the fact that for market signals such that $\delta_x > x_1 > x_2 > \delta_2$ the non-exclusive dealer makes more profits by adjusting effort between the manufacturers. See Appendix A for the detailed proof. ■

Proposition 16 shows that dealers benefit from becoming non-exclusive. Next, we analyze the situation for the manufacturers selling through a non-exclusive dealer in Section 5.2 and show that dealer non-exclusivity hurts the manufacturer's.

5.2 Impact of a Non-Exclusive Dealer on a Manufacturer's Sales when $D = 0$

In this section we show that the expected sales of a manufacturer decreases and the variance increases with an increase in K. We identify values of K when the manufacturer has lower expected sales and a higher variance of sales with a non-exclusive dealer when compared to having an exclusive dealer. After identifying the optimal threshold for a non-exclusive dealer we also characterize the effect of Δ on the manufacturer's sales variability. To proceed with our analysis we define $h(b) \equiv \frac{c''(b)}{c'(b)}$ and $m(b) \equiv \frac{g''(b)}{g'(b)}$. Observe that $h(b) > 0$, $m(b) < 0$ and are continuous. We also impose the following conditions on h, m, and g.

C1: h is non-decreasing or h non-increasing such that $h(z) > \frac{1}{2}h(y)\ \forall\ \frac{z}{2} < y < z$

C2: m is non-decreasing

C3: g and Δ are such that $\frac{b_{31}^*-b_{11}^*}{b_{11}^*-b_{21}^*} \geq 2$ and $\frac{\Delta}{p} < 1$.

These conditions, imposed on the cost function c and impact of effort function g, are not very restrictive. Condition C1 is satisfied by cost functions of the form $c(z) = e^{\beta z}$, with $\beta > 0$, or by polynomial cost functions of the form $c(z) = z^{\beta}$, with $\beta > 1$. For such polynomials $h(z) = \frac{\beta-1}{z}$ and $\frac{h(z)}{h(y)} = \frac{y}{z} > \frac{1}{2}$ for all $(z,\ y)$ such that $\frac{z}{2} < y < z$. Impact of effort functions of the form $g(z) = z^{\alpha}$ with $0 < \alpha < 1$ satisfy conditions C2. For these polynomials $m(z) = \frac{\alpha-1}{z}$. Notice that $m(z)$ increases with z since it is negative. Condition C3 implies $\frac{\Delta}{p}+1 < \frac{b_{31}^*-b_{11}^*}{b_{11}^*-b_{21}^*}$ which further implies $\frac{g'\left(b_{21}^*\right)}{g'\left(b_{31}^*\right)} < \frac{b_{31}^*-b_{11}^*}{b_{11}^*-b_{21}^*}$. Hence $g'(b_{31}^*)\ (b_{31}^* - b_{11}^*) \geq g'(b_{21}^*)\ (b_{11}^* - b_{21}^*)$.

In Claim 10 we defined the effort levels exerted by the non-exclusive dealer for various input market signals. Since the problem is symmetric the probability that $x_2 > x_1$, when $\delta_4 > x_1 > \delta_2$ and $\delta_4 > x_2 > \delta_2$, is the same as the probability when $x_1 > x_2$ in the same region. The expected sales function, $E(s)$, for the non-exclusive dealer can be written as:

$$\begin{aligned} E(s) = {} & E(x_1) + F(\delta_2)^2 g(b_{11}^*) + [1 - F(\delta_2)] F(\delta_2)\ g(b_{21}^*) \\ & + [F(\delta_4) - F(\delta_2)] F(\delta_2)\ g(b_{31}^*) + \frac{1}{2}[F(\delta_4) - F(\delta_2)]^2 [g(b_{31}^*) + g(b_{21}^*)] \\ & + [F(\delta_4) - F(\delta_2)][1 - F(\delta_4)]\ g(b_{21}^*) + [1 - F(\delta_4)] F(\delta_4) g(b_{31}^*) \\ & + [1 - F(\delta_4)]^2 g(b_{41}^*) \end{aligned} \tag{18}$$

Notice the first term on the right hand side is simply the expected level of the input signal. The next two terms are the expected effort exerted when x_1 is below δ_2. The next three terms correspond to the expected effort when $\delta_4 > x_1 > \delta_2$. Observe that when both signals are between δ_4 and δ_2 we assume that x_1 and x_2 dominate each other with the same probability. Finally, the last two terms correspond to the expected effort when $x_1 > \delta_4$.

Proposition 17 identifies the optimal threshold with a non-exclusive dealer.

Proposition 17 *If condition C3 holds, then the optimal K that maximizes a manufacturer's profit with a non-exclusive dealer is $g(b_{41}^*) - \varepsilon_4$.*

Proof. See Appendix A. ■

Denote the optimal threshold for a manufacturer selling through a non-exclusive dealer as K_{NED}^*. Similarly, let K_{ED}^* denote the optimal threshold when the dealer is exclusive (Proposition 7). Proposition 18 compares the expected sales and variance of sales for the manufacturer for the cases with exclusive and non-exclusive dealers. The proposition identifies values of K for which the manufacturer has lower expected sales and a higher variance of sales with non-exclusive dealers.

Proposition 18 *For a manufacturer selling through a non-exclusive dealer, if condition C3 holds, then*

1. *The optimal threshold for the non-exclusive dealer is less than the optimal threshold for the exclusive dealer, i.e.* $K_{NED}^* < K_{ED}^*$,

2. $\forall$ K *such that* $K \leq K_{NED}^*$ *the manufacturer's expected sales, and variance of sales, are the same as selling through an exclusive dealer,*

3. $\forall$ K *such that* $K_{NED}^* < K \leq K_{ED}^*$ *the manufacturer's expected sales is lesser than when the dealer is non-exclusive.*

Proof. See Appendix A. ■

Now consider any K such that $K_{NED}^* < K \leq K_{ED}^*$. The non-exclusive dealer exerts 3 effort levels $g(b_{21}^*)$, $g(b_{31}^*)$ and $g(b_{41}^*)$ depending on the other manufacturer's market signal level. Consider two market signals x_1^L and x_1^H for manufacturer 1 such that $x_1^L < x_1^H$. For any K within the specified region, the expected effort exerted by the non-exclusive dealer when the market signal for manufacturer 1 is y is

$$E\{b \mid x_1 = y\} = F(\delta_4)\begin{bmatrix} \Pr\{x_2 > x_1 \mid x_1 = y\}\ b_{21}^* \\ +\Pr\{x_2 \leq x_1 \mid x_1 = y\}\ b_{31}^* \end{bmatrix} + (1 - F(\delta_4))\begin{bmatrix} \Pr\{x_2 > \delta_4 \mid x_1 = y\}\ b_{41}^* \\ +\Pr\{x_2 \leq \delta_4 \mid x_1 = y\}\ b_{31}^* \end{bmatrix} \tag{19}$$

So the difference, $E\left\{b \mid x_1 = x_1^H\right\} - E\left\{b \mid x_1 = x_1^L\right\}$

$$\begin{aligned}
&= F(\delta_4)\ b_{21}^*\left[\Pr\left\{x_2 > x_1^H \mid x_1 = x_1^H\right\} - \Pr\left\{x_2 > x_1^L \mid x_1 = x_1^L\right\}\right] \\
&+ F(\delta_4)\ b_{31}^*\left[\Pr\left\{x_2 \leq x_1^H \mid x_1 = x_1^H\right\} - \Pr\left\{x_2 \leq x_1^L \mid x_1 = x_1^L\right\}\right] \\
&> F(\delta_4)\ b_{21}^*\begin{bmatrix} \Pr\left\{x_2 > x_1^H \mid x_1 = x_1^H\right\} + \Pr\left\{x_2 \leq x_1^H \mid x_1 = x_1^H\right\} \\ -\Pr\left\{x_2 > x_1^L \mid x_1 = x_1^L\right\} - \Pr\left\{x_2 \leq x_1^L \mid x_1 = x_1^L\right\} \end{bmatrix} \\
&= F(\delta_4)\ b_{21}^*\begin{bmatrix} 1 - \Pr\left\{x_2 \leq x_1^H \mid x_1 = x_1^H\right\} + \Pr\left\{x_2 \leq x_1^H \mid x_1 = x_1^H\right\} \\ -1 + \Pr\left\{x_2 \leq x_1^L \mid x_1 = x_1^L\right\} - \Pr\left\{x_2 \leq x_1^L \mid x_1 = x_1^L\right\} \end{bmatrix} = 0
\end{aligned}$$

The expected effort exerted by the non-exclusive dealer increases as the manufacturer's market signal increases. This implies that expected effort exerted is positively correlated to the corresponding market signal. Given this fact, we conjecture that the variance of sales for the manufacturer is larger than $V(x)$ for a non-exclusive dealer. For an exclusive dealer's case the variance of sales is simply $V(x)$ in this region (Proposition 6). Hence, it is likely that the manufacturer's sales variance increases with a non-exclusive dealer. Computational experiments in Section 6 validate our conjecture. Computational experiments in Section 6 also show that the manufacturer's sales variance may be higher with a non-exclusive dealer even when $K \geq K^*_{ED}$. This fact, together with Proposition 18, imply that, under certain conditions, a manufacturer's profits will be lower with a non-exclusive dealer if there is high cost associated with sales variance.

Next, in Claim 19, we characterize the effect of Δ on the sales variability of a manufacturer selling through a non-exclusive dealer.

Claim 19 *If conditions C1 and C2, hold, and the effort function, g, is such that* $g(b^*_{31}) - g(b^*_{41})$ *increases with* Δ*, then increasing* Δ *expands the range over which the dealer changes effort level based on the market signals of the other manufacturer. Furthermore, if the two manufacturers offer different* Δ*s, the one offering a higher* Δ *(say* Δ_2*) has a smaller range over which the dealer changes his effort level based on the other manufacturer's market signal.*

Proof. See Appendix A for the proof. ■

Claim 19 shows that increasing Δ for both manufacturer's may increase their sales variance resulting in greater reduction in a manufacturer's profits. In contrast, if one of the manufacturer maintains the same Δ, the other manufacturer can decrease his sales variance by increasing Δ to Δ_2.

5.3 Impact of $D > 0$ with Non-Exclusive Dealers

As shown in Table 2 there are 9 possible profit functions when $D > 0$.

In Claim 20 we show that the last five profit functions in Table 2 are dominated by the first four profit functions.

Claim 20 $\Pi_{LG} \geq \Pi_{LE}$, $\Pi_{GL} \geq \Pi_{EL}$, $\Pi_{GG} \geq \Pi_{EG}$, $\Pi_{GG} \geq \Pi_{GE}$, *and* $\Pi_{GG} \geq \Pi_{EE}$. *Further, if* $D \geq \Delta\,\varepsilon_2$ *then* $\Pi_{LG} - \Pi_{LL} \geq 0$ *for* $x_2 \geq K - g(b^*_{22})$

	Profit	
Π_{LL}	$p(x_1 + g(b_{11}^*)) + p(x_2 + g(b_{12}^*)) - 2\ c(\frac{b_{11}^*+b_{12}^*}{2})$	$: x_1 + g\,(b_{11}) < K,$ and $x_2 + g\,(b_{12}) < K$
Π_{LG}	$p\,(x_1 + g(b_{21}^*)) + (p+\Delta)\,(x_2 + g(b_{22}^*)) - \Delta K + D - 2\,c\left(\frac{b_{21}^*+b_{22}^*}{2}\right)$	$: x_1 + g\,(b_{21}) < K,$ and $x_2 + g\,(b_{22}) \geq K$
Π_{GL}	$(p+\Delta)\,(x_1 + g(b_{31}^*)) + p\,(x_2 + g(b_{32}^*)) - \Delta K + D - 2\,c\left(\frac{b_{31}^*+b_{32}^*}{2}\right)$	$: x_1 + g\,(b_{31}) \geq K,$ and $x_2 + g\,(b_{32}) < K$
Π_{GG}	$(p+\Delta)\,(x_1 + g(b_{41}^*)) + (p+\Delta)\,(x_2 + g(b_{42}^*)) + 2D - 2\Delta K - 2\,c\left(\frac{b_{41}^*+b_{42}^*}{2}\right)$	$: x_1 + g\,(b_{41}) \geq K,$ and $x_2 + g\,(b_{42}) \geq K$
Π_{LE}	$p\,(x_1 + g(b_1^{le})) + p\,(x_2 + g(b_2^{le})) + D - 2\,c\left(\frac{b_1^{le}+b_2^{le}}{2}\right)$	$: x_1 + g\left(b_1^{le}\right) < K,$ and $x_1 + g\left(b_2^{le}\right) = K$
Π_{EL}	$p\,(x_1 + g(b_1^{el})) + p\,(x_2 + g(b_2^{el})) + D - 2\,c\left(\frac{b_1^{el}+b_2^{el}}{2}\right)$	$: x_1 + g\left(b_1^{el}\right) = K,$ and $x_1 + g\left(b_2^{el}\right) < K$
Π_{EE}	$p\,(x_1 + g(b_1^{ee})) + p\,(x_2 + g(b_2^{ee})) + 2D - 2\,c\left(\frac{b_1^{ee}+b_2^{ee}}{2}\right)$	$: x_1 + g\left(b_1^{ee}\right) = K,$ and $x_1 + g\left(b_2^{ee}\right) = K$
Π_{GE}	$(p+\Delta)\,(x_1 + g(b_1^{ge})) + p\,(x_2 + g(b_2^{ge})) - \Delta K + 2D - 2\,c\left(\frac{b_1^{ge}+b_2^{ge}}{2}\right)$	$: x_1 + g\left(b_1^{ge}\right) \geq K,$ and $x_1 + g\left(b_2^{ge}\right) = K$
Π_{EG}	$p\,(x_1 + g(b_1^{eg})) + (p+\Delta)\,(x_2 + g(b_2^{eg})) - \Delta K + 2D - 2\,c\left(\frac{b_1^{eg}+b_2^{eg}}{2}\right)$	$: x_1 + g\left(b_1^{ge}\right) = K,$ and $x_1 + g\left(b_2^{ge}\right) \geq K$

Table 2: The nine possible profit outcomes for a non-exclusive dealer when D>0.

Proof. These results are proved using simple convexity arguments. See Appendix A for technical details of the proof. ■

Claim 20 implies that offering a bonus does not change the structure of the effort levels of the non-exclusive dealer. The only effect a positive bonus has is to lower cut off δ_2; i.e. the non-exclusive dealer exerts a higher effort level earlier. Thus, unlike the case of an exclusive dealer, the non-exclusive dealer makes no effort to absorb portions of the market signal variance and keep total sales constant even when offered a positive bonus D. We demonstrate these results using numerical experiments in Section 6.

6. Numerical Experiments

The two scenarios studied are denoted as ED (exclusive dealer) and NED (non-exclusive dealer). Table 3 shows the incentive parameters (p, Δ, D) for both scenarios and the manufacturers profit p_m. The exclusive dealer's effort function is defined as $g\,(b) \equiv \sqrt{b}$ and cost function, associated with the sales effort, is defined as $c\,(b) \equiv b^2$. The input market signal x is assumed to uniformly distributed between 0 and 150. For NED, similar effort functions and market signal distribution parameters are assumed for the for each product ($i = 1,\ 2$). However, the non-exclusive dealer's cost function depends on the effort exerted across both products and hence is assumed to be square of the sum of exerted efforts. The market input signals for the individual products are assumed to be uniformly distributed between 0 and 150. The manufacturer's profit (p_m) is considered only for the scenario ED to demonstrate the effect of offering a non-zero bonus.

	p	p_m	Δ	D	$g()$	$c()$
ED ~U(0, 150)	1500	1000	250	10000	$\sqrt{b}$	$0.001\ b^2$
NED ~U(0, 150)	1500	–	250	10000	$\sqrt{b_i}$ $i{=}1,\ 2$	$0.002\left(\frac{b_1+b_2}{2}\right)^2$

Table 3: Experiment setting for ED and NED.

First, we analyze scenario ED. The optimal efforts are shown in Table 4 under two situations, i.e. when $D = 0$ and $D = 10000$. When the threshold is fixed at 90, notice that $b^K > b_2^* > b_1^*$ and $\delta_{x2} > \delta_{x1}$ when $D = 10000$.

	b_1^*	b_2^*	δ_{x1} at $K = 90$	δ_{x2} at $K = 90$	$\gamma_x = g\left(b^K\right)$	ε_x
$D = 0$	5200.21	5763.045	15.95	–	–	1.87
$D = 10,000$	5200.21	5763.045	1.28	14.09	88.71	–

Table 4: Dealer's optimal efforts, in scenario ED, when $D = 0$ and $D > 0$.

Figure 5 shows the plot of expected total sales and sales variance , for an exclusive dealer, as K varies.

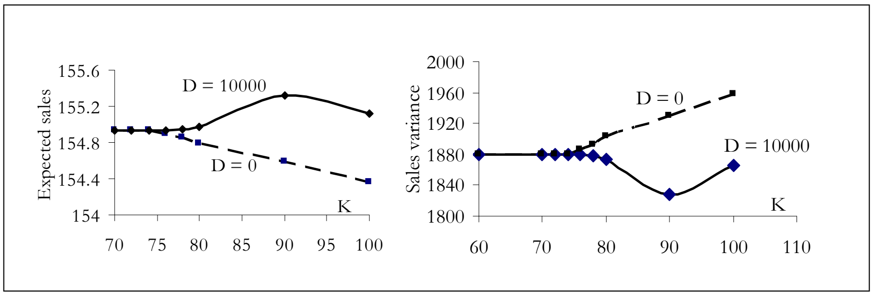

Figure 5: Expected sales and variance, for an exclusive dealer, when $D = 0$ and $D = 10,000$.

The plot to the left shows that when $D = 10,000$, the expected sales increases when $g\left(b_2^*\right) \leq K \leq \gamma_x$, i.e. when $75.91 \leq K \leq 88.71$. The plot to the right shows that the variance of sales dips in the same range when $D = 10,000$.

Figure 6 compares the coefficient of sales variation with and without a bonus offering for scenario ED. The coefficient of sales variation dips when $D = 10,000$ and $75.91 \leq K \leq 88.71$.

Figure 7 compares a manufacturer's objective function when a penalty for variance of sales, v, is included. In this particular case $v = 0.0018$. As can be seen in the plot to the left, if no penalty is included, the manufacturer makes lesser profit by offering a bonus. The

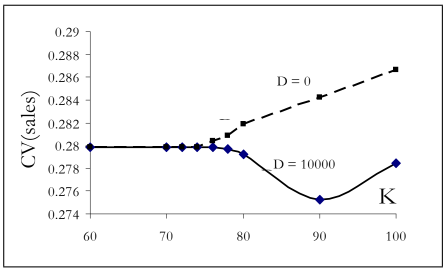

Figure 6: Comparing coefficient of sales variation when $D = 0$ and $D = 10,000$.

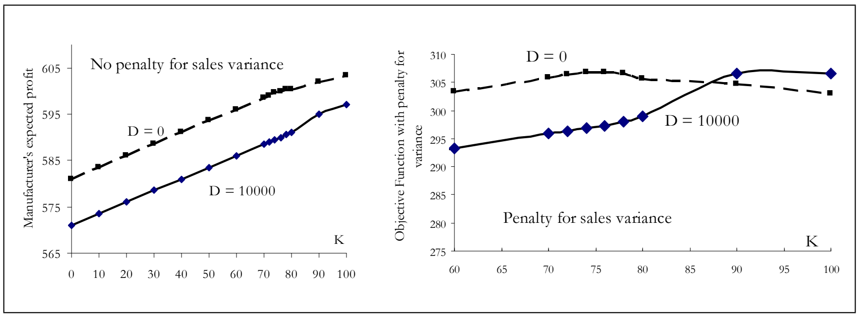

Figure 7: Comparison of manufacturer's profit functions when a penalty for sales variance is included in the objective.

plot to the right shows that, if such a penalty is included, offering a bonus of $D = 10000$, increases the operational profit for $75.91 \leq K \leq 88.71$.

Next we study the effect of varying the bonus payment on the expected sales and variance of sales for an exclusive dealer. We consider 4 values of D: 468, 10000, 15000, and 30000.

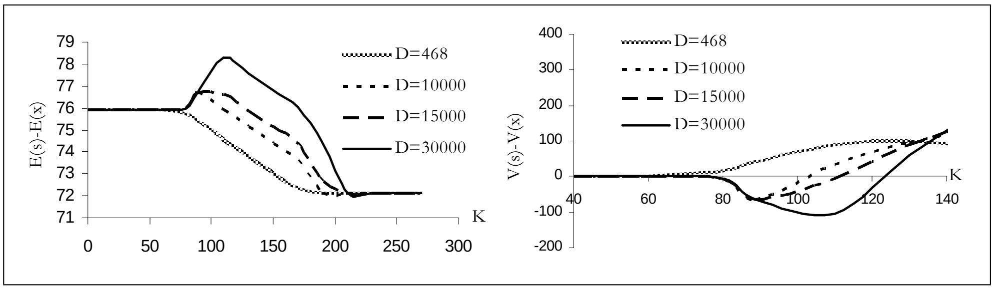

Figure 8: Effect of varying D on $E(s)$ and $V(s)$ for an exclusive dealer.

The plot to the left in Figure 8 shows that the maximal value of the expected sales, $E(s)$, increases as D increases. Furthermore, the value of K for which $E(s)$ is maximal, i.e. K_1^*, increases as D increases. The plot to the right depicts the effect of on variance of sales increasing K for different values of D. As D increases the minimum value of the variance of sales decreases. Also, the value of K at which variance is minimal, i.e. γ_1, increases. This implies the range for which $V(s)$ is below $V(x)$ also increases.

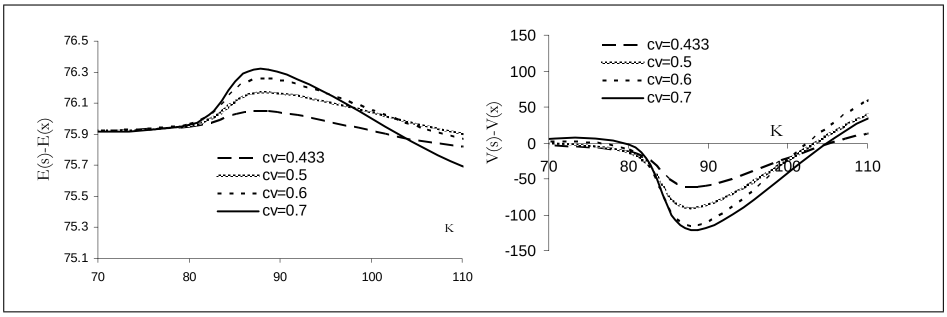

Figure 9: The effect of input signal coeffecient of variance on sales through an exclusive dealer when $D = 10000$.

In Figure 9 we study the effect of the coeffecient of variation of the input signal on the expected sales and variance of sales when $D = 10,000$. For these experiments we use a normally distributed input signal such that $x \geq 0$ and the mean is 150. We study 4 cases with the input signal coeffecient of variation set to 0.433, 0.5, 0.6 and 0.7. The plot to the left, in Figure 9, shows that the point at which expected sales $E(s)$ peaks, i.e. K_1^*, is constant irrespective of the coeffecient of variation of the input signal (denoted by cv in the plots). Furthermore, as cv increases the maximum expected sales also increases and rate of decreases beyond K_1^* is sharper for higher values of cv. The plot to the right shows the effect on the variance of sales $V(s)$. The minimum value of $V(s)$ is lower of higher values of cv and always happens at $K = \gamma_1 = 88.71$.

Next, we compare scenario NED with scenario ED. For this experimental setting $b_{11}^* = 5200.2$, $b_{21}^* = 4655.4$, $b_{31}^* = 6336.6$, and $b_{41}^* = 5763.04$.

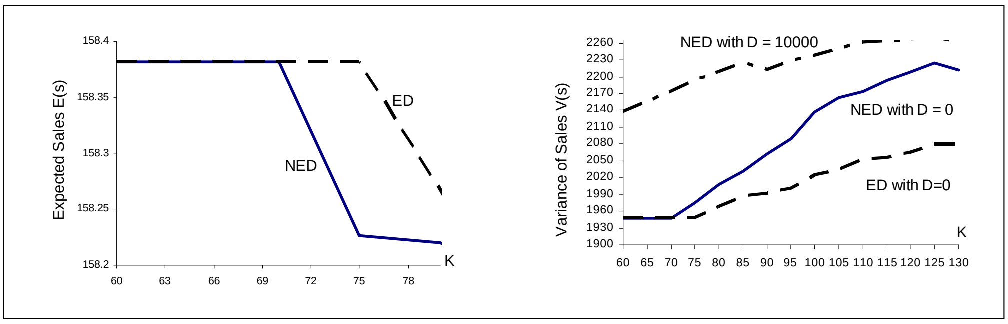

Figure 10: Comparing manufacturer's expected sales and variance with exclusive and non-exclusive dealers.

Figure 10 compares the expected sales and sales variation between the scenarios NED and ED for values of K between 60 and 130. The plot to the left shows that the manufacturer's expected sales is higher when dealing with an exclusive dealer (ED) and is lower with an non-exclusive dealer (NED). These results support our conjecture in Section 5.2. Figure 10 also shows that the variance of sales increase with the threshold K. As conceded by Daimler-Chrysler executives, high target values also contributed to the increased sales variability observed by them. As seen in the plot to the left, the optimal K^*_{NED} is lesser than K^*_{ED}. The plot to the right shows that a manufacturer's sales variation is much lower with an exclusive dealer (ED) than with a non-exclusive dealer. We also compare the variance of a manufacturer's sales when a positive bonus ($D = 10000$) is offered to the non-exclusive dealer. The plot to the right shows that a manufacturer's sales variability is the highest when $D = 10000$ and the dealer is non-exclusive. The variance reduces with a non-exclusive dealer when $D = 0$ though it is still higher than the sales variance when the dealer is exclusive. These plots also imply that the manufacturer's coefficient of variation for sales is higher with a non-exclusive dealer as compared to an exclusive dealer.

7. Conclusions

We analyze the impact of stair-step incentives on sales variability under two specific scenarios: an exclusive dealership scenario and a non-exclusive dealership scenario. In the case of an exclusive dealership we show that, if the manufacturer associates a positive cost with sales variance, a stair-step incentive with a bonus payment may be superior to the scheme without a fixed bonus. The presence of a positive bonus encourages the exclusive dealer to change his effort level with the market signal in a way that the manufacturer's sales has a lower variance than the variance of the market signal. In other words, a positive bonus leads the exclusive dealer to absorb some of the market signal variance by varying his effort.

Our study of stair step incentives for non-exclusive dealers shows two main results. The first is that non-exclusivity of dealers increases the sales variance observed by the manufacturer's. Even though the dealer observes a lower sales variance in terms of aggregate sales than an exclusive dealer, the variance of sales observed by each manufacturer goes up. Our second result is that in the case of non-exclusive dealers, a positive bonus does not lead to the dealer absorbing any market signal variance. In other words, a positive bonus is not helpful in reducing variance when dealers are non-exclusive.

The experience of Daimler Chrysler described at the beginning of the paper can partially be explained by the presence of non-exclusive dealers and stair step incentives. Overall our results indicate that manufacturers should rethink offering stair step incentives as dealers become non-exclusive, especially if they have a high cost associated with sales variance.

Acknowledgements

The second author's research is supported by NSF Grant number DMI-0457503. The first author wishes to thank the Kellogg School of Management for their support during this research effort. We thank Martin A. Lariviere for valuable comments on the paper.

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8. Appendix A: Proofs

Proof. The constraints (5) and (6), in model EDP, imply that a separate profit is made on just reaching K. First order KKT optimality conditions imply $u_1 + u_2 + u_4 = 1$, $u_1 \left[p\, g'(b_1^*) - c'(b_1^*)\right] = 0$, $u_2 \left[(p+\Delta)g'(b_2^*) - c'(b_2^*)\right] + u_3\, g'\left(b^K\right) = 0$, $u_4 \left[p\, g'(b^K) - c'(b^K)\right] = 0$, $u_1\, g_1 = 0$, $u_2\, g_2 = 0$, $u_3\, g_3 = 0$, $u_4\, g_4 = 0$. To calculate the cutoff between Π_1 and Π_2, we check when the condition when $u_1 > 0$, $u_4 > 0$, and $u_2 = 0$. This implies that the profit functions are equal (i.e. the corresponding constraints are tight). Thus, we have $p(K-x) - c[g^{-1}(K-x)] = p\, g(b_1^*) - c(b_1^*) - D$. The value of x satisfying this equation gives the first cutoff, δ_{x1}. To obtain the second cutoff, δ_{x2}, between Π_2 and Π_3, we check when $u_1 = 0$, $u_2 > 0$, and $u_4 > 0$. Observe that $u_4 > 0 \Longrightarrow (p+\Delta)\ K + D - c(b^K) = \Theta$ and $u_4 > 0 \Longrightarrow u_2 \left[p\ g'(b^K) - c'(b^K)\right] + u_3 g'(b^K) = 0$. Thus equating the two we get $x = K - g(b_2^*) + \frac{c(b_2^*) - c(b^K)}{p+\Delta}$. Further, $u_2 > 0 \Longrightarrow (p+\Delta)(x + g(b_2^*)) - \Delta K + D - c(b_2^*) = \Theta$. This also implies $u_3 = u_2 \left[p + \Delta - \frac{c'(b^K)}{g'(b^K)}\right]$. Now as $u_4 \to 0$, i.e. when constraint 6 is no longer binding, we must have $u_2 \to 1$. In this case $u_3 \to p + \Delta - \frac{c'(b^K)}{g'(b^K)}$ which implies $b^K \to b_2^*$. Hence, δ_{x2} reduces to $K - g(b_2^*)$. ■

Claim 2. In this particular case constraints (6) and (5) are absent in EDP. The optimal effort vectors are obtained by applying the first order KKT conditions. Now if $u_1 = 1$, $u_2 = 0$ (i.e. constraint $g1$ is tight), the optimal effort is b_1^* such that $p = \frac{c'(b_1^*)}{g'(b_1^*)}$. If $u_1 = 0, u_2 = 1$, i.e. constraint $g1$ is tight, the optimal effort decision is b_2^* such that $(p+\Delta) = \frac{c'(b_2^*)}{g'(b_2^*)}$. Suppose

$u_1 > 0$ and $u_2 > 0$, i.e. g_1 and g_2 are both tight, then we have $\Pi_1(b_1^*) = \Pi_2(b_2^*)$. Thus, we get the cutoff for x, when Π_2 exceeds Π_1 is $\delta_x = K - g(b_2^*) + \frac{p\left[g(b_1^*)-g(b_2^*)\right]+\left[c(b_2^*)-c(b_1^*)\right]}{\Delta}$. ■

Claim 4. The partial derivative of the expected sales with respect to K is given by $\frac{\partial E(s)}{\partial K}$ $= f(K-\gamma_x)\left[g(b_1^*)-\gamma_x\right] + \int_{\delta_{x1}}^{\delta_{x2}} f(x)\ dx$. So setting $\alpha = \gamma_x - g(b_1^*)$ and simplifying we get $\frac{\partial E(s)}{\partial K} = f(K-\gamma_x)(t(K)-\alpha)$. Observe that,

$$t'(K) = \frac{1}{f(K-\gamma_x)^2}\left[\begin{array}{c} f(K-\gamma_x)\ \ (f(K-g(b_2^*)) - f(K-\gamma_x)) \\ -f'(K-\gamma_x)\ \ (F(K-g(b_2^*)) - F(K-\gamma_x)) \end{array}\right]$$

Now $t'(K) \leq 0 \iff \frac{f\left(K-g\left(b_2^*\right)\right)-f(K-\gamma_x)}{F\left(K-g\left(b_2^*\right)\right)-F(K-\gamma_x)} \leq \frac{f'(K-\gamma_x)}{f(K-\gamma_x)}$. Further, notice that $f = F'$ and $f' = F''$. Define $\beta \equiv \gamma_x - g(b_2^*)$ and $y = K - \gamma_x$. Therefore $K - g(b_2^*) = y + \beta$.

Cauchy's Mean Value Theorem states the following. Suppose functions ϕ_1 and ϕ_2 are differentiable on the open interval $(a,\, b)$ and continuous on the closed interval $[a, b]$. If $\phi_2'(y) \neq 0$ for any y in (a, b), then there is at least one point c in (a, b) such that $(\phi_1'(c))/(\phi_2'(c))$ $= (\phi_1(b) - \phi_1(a))/(\phi_2(b) - \phi_2(a))$. So, we must have a $\theta_y \in (y,\, y+\beta)$ such that $\frac{f(y+\beta)-f(y)}{F(y+\beta)-F(y)} = \frac{f'(\theta_y)}{f(\theta_y)}$. Since f is assumed to be log-concave $\frac{f'}{f}$ is monotonically decreasing. Hence, we must have $\frac{f(y+\beta)-f(y)}{F(y+\beta)-F(y)} = \frac{f'(\theta_y)}{f(\theta_y)} \leq \frac{f'(y)}{f(y)}$. This implies $t'(K) \leq 0$ which further implies that $t(K)$ is a decreasing function. So, $t(K) \leq t(\gamma_x) = \frac{F\left(\gamma_x - g\left(b_2^*\right)\right)}{f(0)}$. If $\alpha \geq t(\gamma_x)$, we must have $\frac{\partial E(s)}{\partial K} \leq 0$ for all $K \geq \gamma_x$. Now suppose that $t(\gamma_x) > \alpha$ .Observe that, for very large values of K, $t(K)$ approaches 0 since $f(K-\gamma_x) > 0$. So there exists a $K_1^* > \gamma_x$ such that $t(K_1^*)$ $= \alpha$. This proves the first part of the claim which also implies that $E(s)$ is a decreasing function for all $K \geq K_1^*$.

For any continuous cumulative distribution the following condition must hold: $0 \leq F(K-g(b_2^*)) - F(K-\gamma_x) \leq 1 < \alpha$. Defining $\varepsilon_v = \alpha - (F(K-g(b_2^*)) - F(K-\gamma_x))$ we rewrite the first derivative of the variance of sales as $\frac{\partial V(s)}{\partial K} = \frac{\partial E(s^2)}{\partial K} - 2E(s)\frac{\partial E(s)}{\partial K}$. Simplifying we get $\frac{\partial V(s)}{\partial K} = f(K-\gamma_x)\ \alpha^2 + 2\ \varepsilon_v\ \ (E(s) - K)$. From equation 10 it is clear that $E(s) > K$ when $K = \gamma_x$. This implies that $\frac{\partial V(s)}{\partial K} \geq 0$ at $K = \gamma_x$. From the first part of the claim we know that $E(s)$ continuously increases until $K \leq K_1^*$ beyond which it decreases as K increases further. So , $\frac{\partial V(s)}{\partial K} \geq 0$ for all $\gamma_x \leq K \leq K_1^*$. For $K \geq K_1^*$ we know that $(E(s) - K)$ is a monotonically decreasing function K. Now, we consider two cases. When $E(s) \geq K > K_1^*$ notice that ε_v is always positive for any value of K, since $\alpha \geq 1$. Furthermore, the first term is also positive for the entire region of K. Thus, $\frac{\partial V(s)}{\partial K} \geq 0$. When $E(s) < K$, $E(s) - K$ is always negative for any value of K. As K increases, $(E(s) - K)$

increases in the negative direction. We know that $\frac{\partial V(s)}{\partial K} < 0 \iff \frac{\alpha^2 f(K-\gamma_x)}{2\ \varepsilon_v} + E(s) \leq K$. Hence, if f is a bounded function we can always increase K to a large value such that the inequality holds. Thus, there exists $K_2^* > K_1^* \geq \gamma_x$ such that $\frac{\partial V(s)}{\partial K} < 0$. Since the derivative flips sign, $V(s)$ is inverted-U shaped when f is bounded. ■

Proposition 5. $K \leq g(b_2^*)$ implies $\delta_{x1} \leq 0$ and $\delta_{x2} \leq 0$. From equation (10) the expected sales, $E(s)$, reduces to $E(x) + g(b_2^*)$. Therefore, for $K \leq g(b_2^*)$ we have $E(s) = E(x) + g(b_2^*)$. For values of K satisfying $g(b_2^*) \leq K \leq \gamma_x$ we must have $\delta_{x1} = 0$ and $\delta_{x2} > 0$. Hence, $E(s) = K + \int_{\delta_{x2}}^{\infty} (x - \delta_{x2})\ f(x)\ dx = K + \int_{K-g(b_2^*)}^{\infty} [1 - F(x)]\ dx$. Notice that $\frac{\partial E(s)}{\partial K} = F(K - g(b_2^*)) > 0$. This implies that $E(s)$ monotonically increases beyond $K = g(b_2^*)$, because $\delta_{x2} = K - g(b_2^*) > 0$. Further, observe that $E(s) > K$ because $\delta_{x2} > 0$. In Claim 4 we have already shown that $E(s)$ continues to increase until $K \leq K_1^*$ and starts decreasing beyond for all values of $K \geq K_1^*$. This proves the first part of the proposition. When $K \leq g(b_2^*)$ we know that $E(s) = E(x) + g(b_2^*)$. variance of sales $V(s) = V(x)$ for these values of K. Before we evaluate the expression for the derivative of $V(s)$ relative to K, for $\gamma_x \geq K \geq g(b_2^*)$ we note that $E(s^2) = K^2 F(\delta_{x2}) + \int_{\delta_{x2}}^{\infty} x^2 f(x)\ dx + g(b_2^*)^2\ [1 - F(\delta_{x2})] + 2\, g(b_2^*) \int_{\delta_{x2}}^{\infty} x\ f(x)\ dx$. The derivative of the variance of sales, $V(s)$, on simplification reduces to $\frac{\partial V(s)}{\partial K} = 2\ F(\delta_{x2})\ \ (K - E(s))$. Since $K < E(s)$, this implies that $\frac{\partial V(s)}{\partial K} < 0$ in this region. Hence the variance of sales, $V(s)$, decreases below the signal variance $V(x)$ in this region. In other words, the dealer absorbs some of the signal variance by changing the effort level, thus reducing the sales variance observed by the manufacturer.

Now consider the case when $K \geq \gamma_x$. This implies $\delta_{x1} > 0$, $\delta_{x2} > 0$, and $\dot{K} \geq \delta_{x2}$. From Claim 4 we know that when $K \geq \gamma_x$, $V(s)$ is an inverted-U shaped function and $\frac{\partial V(s)}{\partial K} \geq 0$ at $K = \gamma_x$. This proves part 2 of the proposition.

Now as K becomes very large, the dealer may find it cost prohibitive to reach K by putting additional effort. So for very large values of K, the value of $E(s)$ approaches $E(x) + g(b_1^*)$. Thus, the expected sales are maximized when $K = \gamma_x$. Further observe that once the partial derivative of the variance of sales (when $K \geq \gamma_x$) falls below zero it always remains non-positive. For large values of K the dealer exerts only a constant effort $g(b_1^*)$ because it is impossible to reach K. Hence $\frac{\partial V(s)}{\partial K}$ approaches 0 for large values of K implying that $V(s) = V(x)$ for large values of K. The fact that the derivative of $V(s)$ remains non-positive, after switching signs, and approaches 0 for large values of K implies that $V(s)$ never falls below $V(x)$ after reaching its maximum value. This implies maximum value of the variance of sales $V(s) \geq V(x)$ (part 2 of the Proposition). This further implies that the minimum sales variance occurs at $K = \gamma_x$ (when the $V(s)$ is below $V(x)$ and the derivative of $V(s)$ becomes positive). This proves part 3. ■

Proposition 6. When $D = 0$ we have only one cutoff $\delta_x = K - g(b_2^*) + \varepsilon_x$, where ε_x is a positive constant given by equation 9. Note that δ_x is a linear function of K with $\frac{\partial \delta_x}{\partial K} = 1$. The expected values for sales and its square reduce to

$$E(s) = E(x) + g(b_2^*) + F(\delta_x)\left[g(b_1^*) - g(b_2^*)\right] \tag{20}$$

$$E(s^2) = \int_0^{\delta_x} x^2 f(x)dx + g(b_1^*)^2 F(\delta_x) + 2g(b_1^*)\int_0^{\delta_x} x\ f(x)dx + \int_{\delta_x}^{\infty} x^2 f(x)dx + g(b_2^*)^2\left[1 - F(\delta_x)\right] + 2g(b_2^*)\int_{\delta_x}^{\infty} x\ f(x)dx$$

We now evaluate the two cases, $K \leq g(b_2^*) - \varepsilon_x$ and $K > g(b_2^*) - \varepsilon_x$. If $K \leq g(b_2^*) - \varepsilon_x$ then $\delta_x \leq 0$. So we have only one effort strategy, b_2^* for the dealer. The expected value reduces to $E(s) = E(x) + g(b_2^*)$. Further, $E(s^2) = E(x^2) + g(b_2^*)^2 + 2g(b_2^*)\ E(x)$ implying $V(s) = V(x)$. If $K > g(b_2^*) - \varepsilon_x$ then $\delta_x > 0$. When the cutoff is strictly positive there are the two dealer effort levels, b_1^* and b_2^* (by Claim 2), and $E(s)$ is given by equation (20). Observe that $\frac{\partial E(s)}{\partial K} = f(\delta_x)\frac{dc^x}{dK}\ \left[g(b_1^*) - g(b_2^*)\right] \leq\ 0$. This implies that $E(s)$ is a monotonically decreasing function of K. Further,

$$\begin{aligned} E(s^2) &= \int_0^{\delta_x} (x + g(b_1^*)^2 f(x)\, dx + \int_{\delta_x}^{\infty} (x + g(b_2^*)^2 f(x)\, dx \\ &= E(x^2) + F(\delta_x)\left[g(b_1^*)^2 - g(b_2^*)^2\right] \\ &+ g(b_2^*)^2 + 2\left[g(b_1^*) - g(b_2^*)\right]\int_0^{\delta_x} x\, f(x)dx + 2\, g(b_2^*)E(x) \end{aligned}$$

and variance of sales

$$\begin{aligned} V(s) &= E(x^2) + F(\delta_x)\left[g(b_1^*)^2 - g(b_2^*)^2\right] \\ &+ g(b_2^*)^2 + 2\left[g(b_1^*) - g(b_2^*)\right]\int_{\delta_x}^{\infty} x\, f(x)\, dx + 2g(b_2^*)\, E(x) - (E(s))^2 \\ &= V(x) + F(\delta_x)\left[1 - F(\delta_x)\right]\left(g(b_1^*) - g(b_2^*)\right)^2 \\ &+ 2(g(b_1^*) - g(b_2^*))\int_0^{\delta_x} (x - E(x))\ f(x)\, dx. \end{aligned} \tag{21}$$

The partial derivative of the variance of sales with respect to K can be expressed as

$$\begin{aligned} \frac{\partial V(s)}{\partial K} &= f(\delta_x)\left[g(b_2^*) - g(b_1^*)\right]^2 - 2F(\delta_x)\, f(\delta_x)\left[g(b_2^*) - g(b_1^*)\right]^2 \\ &- 2\left(g(b_2^*) - g(b_1^*)\right)\ f(\delta_x)\left[\delta_x - E(x)\right] \\ &= f(\delta_x)\left[g(b_2^*) - g(b_1^*)\right]^2\left[1 - 2\left(F(\delta_x) + \frac{\delta_x - E(x)}{g(b_2^*) - g(b_1^*)}\right)\right]. \end{aligned}$$

Define $l(\delta_x) \equiv F(\delta_x) + \frac{\delta_x - E(x)}{g(b_2^*) - g(b_1^*)}$. Observe that $l(\delta_x)$ is a monotonic increasing function of δ_x and hence K. At $\delta_x = 0$, we have $F(\delta_x) = 0$, and hence $l(\delta_x) = \frac{-E(x)}{g(b_2^*) - g(b_1^*)} < 0$. Further, $1 - 2l(\delta_x) > 0$ which implies $\frac{\partial V(s)}{\partial K} \geq 0$. Since $l(\delta_x)$ monotonically increases with δ_x, there exists a δ^*, such that for $\delta_x < \delta^*$, we have $F(\delta_x) + \frac{\delta_x - E(x)}{g(b_2^*) - g(b_1^*)} < 1/2$ and $\frac{\partial V(s)}{\partial K} > 0$, and for $\delta_x > \delta^*$, $\frac{\partial V(s)}{\partial K} < 0$. At $\delta_x = \delta^*$ we have $t(\delta^*) = 0$. For symmetric distributions, such as the normal or uniform distribution, $\delta^* = E(x)$. Thus, $V(s) - V(x)$ is inverted-U in shape for all $K > g(b_2^*) - \varepsilon_x$ and 0 for all $K \leq g(b_2^*) - \varepsilon_x$. This proves parts 1 and 2 of the Proposition.

It is easy to show that $V(s) = V(x)$ and $E(s) = E(x) + g(b_1^*)$ for large values of K. Observe that when $K = g(b_2^*) - \varepsilon_x$ we must have $\delta_x = 0$. Therefore, $\frac{\partial V(s)}{\partial K} \geq 0$ at $K = g(b_2^*) - \varepsilon_x$. Hence, $V(s)$ must have a maximum strictly greater than $V(x)$. Part 3 can be proved using arguments similar to those in Proposition 5. ■

Proposition 7. The proof follows from Proposition 6. First we observe that the manufacturer should not set $K < g(b_2^*) - \varepsilon_x$ because it results in lower expected profit than setting $K = g(b_2^*) - \varepsilon_x$. Expected sales monotonically decrease as K increases beyond $g(b_2^*) - \varepsilon_x$. The variance of sales for $K > g(b_2^*) - \varepsilon_x$ is at least as large as the variance of sales when $K = g(b_2^*) - \varepsilon_x$. Thus, if the manufacturer were to set a reasonable K it is best to set $K = g(b_2^*) - \varepsilon_x$. Finally, the manufacturer should should compare the profit earned at this value of K to the profit earned at a threshold level that is never reached by the dealer. This follows from the manufacturer's profit function 11. If $p_m(x + g(b)) > p_m K - D + (p_m - \Delta)(x - K)$ then the manufacturer should set a threshold that is very large. Otherwise, $K = g(b_2^*) - \varepsilon_x$ is the best threshold level. ■

Proposition 8. From Proposition 5 observe that when $K = K_1^* \geq \gamma_x$ the manufacturer's expected sales, $E(s)$, is maximal. The variance of sales, $V(s)$, is minimal at $K = \gamma_x$. Thus the manufacturer's profit is maximized when $K \geq \gamma_x$ among all values of K. Finally, the manufacturer should should compare the profit earned at this value of K, say $\tilde{K}$, to the profit earned at a threshold level that is never reached by the dealer. This follows from the manufacturer's profit function 11. Let $V_{\min}$ be the variance of sales when $K = \tilde{K}$ and $S_{\max}$ be the corresponding expected sales. If $p_m(E(x) + g(b_2^*)) - v\,V(x) > (p_m - \Delta)\,S_{\max} - D - v\,V_{\min}$ then the manufacturer should set $K = g(b_2^*)$. Otherwise, $K = \tilde{K} \geq \gamma_x$. ■

Proposition 9. Let $\Pi_{D=0}$ and $\Pi_{D>0}$ be the manufacturer's profits respectively in the cases when $D = 0$ and $D > 0$. Let $V_{\min}$ be the variance of sales when $K = \gamma_x$ and

$S_{\max}$ be the corresponding expected sales. Recall that $V_{\min} < V(x)$ and $S_{\max} > E(x) + g(b_2^*)$. Thus, for a high enough v, it must be the case that $(p_m - \Delta)\, S_{\max} - D - v\, V_{\min} > p_m\,(E(x) + g(b_2^*)) - v\, V(x)$. In such a situation it is optimal for the manufacturer to set $D > 0$ and $K \geq \gamma_x$. ■

Claim 10. Any optimal solution Θ for $NEDP$ is the maximum of the four profit functions and the effort levels correspond to the values that maximize the particular profit function. At any stationary solution, the lagrangian multipliers $u_1, \ldots, u_4$ (shown in brackets next to the constraints) must satisfy the convexity constraint $\sum_{i=1}^{4} u_i = 1$. When u_i and u_j are non-zero, the profit functions must be equal since at optimality since $u_i.g_i(b_{i1}^*, b_{i2}^*) = 0$. The first order KKT optimality conditions also imply $u_i\, \frac{\partial \Pi_i}{\partial b_{i1}^*} = 0$ and $u_i\, \frac{\partial \Pi_i}{\partial b_{i2}^*} = 0$. The cutoff for x_1 and x_2 can be found when $u_i > 0$ and $u_j > 0$. Thus, using the first order optimality conditions for $NEDP$ we have $\frac{\partial \Pi_1}{\partial b_{11}} = p\, g'(b_{11}^*) - c'(\frac{b_{11}^* + b_{12}^*}{2}) = 0$ and $\frac{\partial \Pi_1}{\partial b_{12}} = p\; g'(b_{12}^*) - c'(\frac{b_{11}^* + b_{12}^*}{2}) = 0$. Simplifying, we have $\frac{g'(b_{12}^*)}{g'(b_{11}^*)} = 1$. This proves the first part. Similarly, using the first order KKT conditions and solving for other constraints, we can easily show parts 2, 3 and 4. The cutoffs for the profit function are calculated as follows. Cutoffs are defined when $\Pi_i = \Pi_j$ $\forall\, i \neq j$. We start with the base case Π_1 and compute the cutoffs represented by the following diagram:

$$\begin{array}{ccccc} & & \Pi_1 & & \\ & \swarrow \delta_1 & & \searrow \delta_2 & \\ \Pi_2 & & \longleftrightarrow & & \Pi_3 \\ & \searrow \delta_4 & & \swarrow \delta_3 & \\ & & \Pi_4 & & \end{array}$$

Cutoff δ_1 represents the value of x_2 when profit function $\Pi_2(x_1,\, x_2,\, b_{21}^*,\, b_{22}^*) = \Pi_1(x_1,\, x_2,\, b_{11}^*,\, b_{12}^*)$. Observe that for both Π_1 and Π_2 the non-exclusive dealer does not reach K for manufacturer 1. Cutoff δ_2 is the value of x_1 for which $\Pi_3(x_1,\, x_2,\, b_{31}^*,\, b_{32}^*) = \Pi_1(x_1,\, x_2,\, b_{11}^*,\, b_{12}^*)$. Notice that in this case the non-exclusive dealer exceeds K for manufacturer 1 only. Cutoff δ_3 represents the value of x_2 such that $\Pi_3(x_1,\, x_2,\, b_{31}^*,\, b_{32}^*) = \Pi_4(x_1,\, x_2,\, b_{41}^*,\, b_{42}^*)$ and cutoff δ_4 corresponds to the value of x_1 such that $\Pi_3(x_1,\, x_2,\, b_{21}^*,\, b_{22}^*) = \Pi_4(x_1,\, x_2,\, b_{41}^*,\, b_{42}^*)$.

■

Claim 12. We illustrate the proof for ε_2. The bounds for the rest are computed using similar steps. We know that

$$\varepsilon_2 = \frac{p\,[g(b_{11}^*) - g(b_{31}^*)] + p\,[g(b_{12}^*) - g(b_{32}^*)] + 2\, c(\frac{b_{31}^* + b_{32}^*}{2}) - 2\, c(\frac{b_{11}^* + b_{12}^*}{2})}{\Delta}$$

Thus, $\varepsilon_2 \geq \frac{1}{\Delta}\, [p\; g'(b_{11}^*)\,(b_{11}^* - b_{31}^*) + p\; g'(b_{12}^*)\,(b_{12}^* - b_{32}^*)] + \frac{2}{\Delta}\, [c'(\frac{b_{11}^* + b_{12}^*}{2})\; (\frac{b_{31}^* + b_{32}^* - b_{11}^* - b_{12}^*}{2})]$. The right hand side of this inequality is always non-negative (Corollary 11 and Claim 10).

The upper bound is computed as follows. We know that $\varepsilon_2 \leq \frac{1}{\Delta}\ [p\ g'(b^*_{31})(b^*_{11} - b^*_{31}) + p\ g'(b^*_{32})(b^*_{12} - b^*_{32})] + \frac{2}{\Delta}\ [c'(\frac{b^*_{31}+b^*_{32}}{2})(\frac{b^*_{31}+b^*_{32}-b^*_{11}-b^*_{12}}{2})]$. The right hand side can be simplified to $g'(b^*_{31})(b^*_{31} - b^*_{11})$. ■

Claim 14. Notice that $\frac{c'}{g'}$ is a monotonically increasing function. Since $b^*_{11} = b^*_{12}$ we have $g'\left(b^*_{11}\right) = g'\left(b^*_{12}\right) = \frac{c'\left(b^*_{11}\right)}{p}$. Similarly since $b^*_{41} = b^*_{42}$ we have $g'\left(b^*_{41}\right) = g'\left(b^*_{42}\right) = \frac{c'\left(b^*_{41}\right)}{p+\Delta}$. So $\frac{c'\left(b^*_{41}\right)}{g'\left(b^*_{41}\right)} = \frac{c'\left(b^*_{11}\right)}{g'\left(b^*_{11}\right)} + \Delta$ which implies that $b^*_{41} > b^*_{11}$. Further $\frac{c'\left(b^*_{2}\right)}{g'\left(b^*_{2}\right)} = p + \Delta = \frac{c'\left(b^*_{41}\right)}{g'\left(b^*_{41}\right)}$ implies $b^*_2 = b^*_{41}$. Since $\frac{g'\left(b^*_{21}\right)}{g'\left(b^*_{22}\right)} = \frac{p+\Delta}{p} > 1$ we must have $b^*_{21} < b^*_{22}$ which also implies $b^*_{21} < \frac{b^*_{21}+b^*_{22}}{2}$. Now suppose $b^*_{11} < b^*_{21}$. Then we would have $\frac{c'\left(b^*_{11}\right)}{g'\left(b^*_{11}\right)} < \frac{c'\left(b^*_{21}\right)}{g'\left(b^*_{21}\right)} < \frac{c'\left(\frac{b^*_{21}+b^*_{22}}{2}\right)}{g'\left(b^*_{21}\right)}$. This leads to a contradiction since $\frac{c'\left(\frac{b^*_{21}+b^*_{22}}{2}\right)}{g'\left(b^*_{21}\right)} = \frac{c'\left(b^*_{11}\right)}{g'\left(b^*_{11}\right)} = p$. Clearly $b^*_{11} \neq b^*_{21}$ and hence $b^*_{11} > b^*_{21}$. This also implies that $g'\left(b^*_{21}\right) > g'\left(b^*_{11}\right)$. Hence $c'\left(\frac{b^*_{21}+b^*_{22}}{2}\right) > c'\left(b^*_{11}\right)$ which then implies $\frac{b^*_{21}+b^*_{22}}{2} = \frac{b^*_{31}+b^*_{21}}{2} > b^*_{11}$. We know that $\frac{g'\left(b^*_{32}\right)}{g'\left(b^*_{31}\right)} = \frac{p+\Delta}{p}$. So $g'\left(b^*_{31}\right) = \frac{p}{p+\Delta} g'\left(b^*_{32}\right)$. This implies that $b^*_{31} > b^*_{32}$. We know that $p + \Delta = \frac{c'\left(b^*_{41}\right)}{g'\left(b^*_{41}\right)} = \frac{c'\left(\frac{b^*_{31}+b^*_{32}}{2}\right)}{g'\left(b^*_{31}\right)}$. Now suppose $b^*_{41} > b^*_{31}$. Then we must have $\frac{c'\left(b^*_{41}\right)}{g'\left(b^*_{41}\right)} > \frac{c'\left(b^*_{31}\right)}{g'\left(b^*_{31}\right)} > \frac{c'\left(\frac{b^*_{31}+b^*_{32}}{2}\right)}{g'\left(b^*_{31}\right)}$ leading to a contradiction. It is clear that $b^*_{31} \neq b^*_{41}$. Hence $b^*_{31} > b^*_{41}$. Thus $g'\left(b^*_{41}\right) > g'\left(b^*_{31}\right)$ which then implies $c'\left(\frac{b^*_{31}+b^*_{32}}{2}\right) < c'\left(b^*_{41}\right)$. This further implies that $\frac{b^*_{31}+b^*_{32}}{2} = \frac{b^*_{31}+b^*_{21}}{2} < b^*_{41}$. To prove that $b^*_1 = b^*_{11}$ we use the fact that $p = \frac{c'\left(b^*_{1}\right)}{g'\left(b^*_{1}\right)} = \frac{c'\left(b^*_{11}\right)}{g'\left(b^*_{11}\right)}$. Since $\frac{c'}{g'}$ is a monotonically increasing function we must have $b^*_1 = b^*_{11}$.
■

Claim 15. From Claim 2, and using the facts that $b^*_{41} = b^*_2$ and $b^*_{11} = b^*_1$, we have

$$\begin{aligned}
\delta_4 - \delta_x &= \frac{1}{\Delta}\left[\begin{array}{c} p\ \left(g\left(b^*_{21}\right) - g\left(b^*_{11}\right)\right) + \left(p+\Delta\right)\ \left(g\left(b^*_{31}\right) - g\left(b^*_{41}\right)\right) \\ + c\left(b^*_{41}\right) + c\left(b^*_{11}\right) - 2\,c\left(\frac{b^*_{31}+b^*_{21}}{2}\right) \end{array}\right] \\
&> \frac{1}{\Delta}\left[\begin{array}{c} p\,g'\left(b^*_{21}\right)\left(b^*_{21} - b^*_{11}\right) + \left(p+\Delta\right)\ g'\left(b^*_{31}\right)\left(b^*_{31} - b^*_{41}\right) \\ + c'\left(\frac{b^*_{31}+b^*_{21}}{2}\right)\left(b^*_{41} - \frac{b^*_{31}+b^*_{21}}{2}\right) + c'\left(\frac{b^*_{31}+b^*_{21}}{2}\right)\left(b^*_{11} - \frac{b^*_{31}+b^*_{21}}{2}\right) \end{array}\right]
\end{aligned}$$

On simplication the right hand side reduces to 0. Similarly, using the fact that $b^*_{11} < \frac{b^*_{31}+b^*_{21}}{2}$, we have

$$\begin{aligned}
\delta_x - \delta_2 = g\left(b^*_{31}\right) - g\left(b^*_{41}\right) + \frac{1}{\Delta}&\left[\begin{array}{c} p\ \left(g\left(b^*_{31}\right) - g\left(b^*_{41}\right)\right) + p\ \left(g\left(b^*_{11}\right) - g\left(b^*_{21}\right)\right) \\ + c\left(b^*_{41}\right) + c\left(b^*_{11}\right) - 2\,c\left(\frac{b^*_{31}+b^*_{21}}{2}\right) \end{array}\right] \\
\geq \frac{1}{\Delta}&\left[\begin{array}{c} \left(p+\Delta\right)\ g'\left(b^*_{31}\right)\left(b^*_{31} - b^*_{41}\right) + p\,g'\left(b^*_{11}\right)\left(b^*_{11} - b^*_{21}\right) \\ + c'\left(\frac{b^*_{31}+b^*_{21}}{2}\right)\left(b^*_{41} - \frac{b^*_{31}+b^*_{21}}{2}\right) \\ + c'\left(\frac{b^*_{31}+b^*_{21}}{2}\right)\left(b^*_{11} - \frac{b^*_{31}+b^*_{21}}{2}\right) \end{array}\right]
\end{aligned}$$

On simplification the right hand side reduces to $\frac{1}{\Delta}\,[p\,g'\left(b^*_{11}\right)\left(b^*_{11}-b^*_{21}\right)+\ c'(\frac{b^*_{31}+b^*_{21}}{2})\,(\frac{b^*_{31}-b^*_{21}}{2})$ $+\ c'(\frac{b^*_{31}+b^*_{21}}{2})\,(b^*_{11}\ -\ \frac{b^*_{31}+b^*_{21}}{2})]$ which is greater than $\frac{2\,c'\left(b^*_{11}\right)\left(b^*_{11}-b^*_{21}\right)}{\Delta}>0$. Hence we have $\delta_4>\delta_x>\delta_2$.

■

Proposition 16. For the market signals the non-exclusive dealer exerts one of the two effort levels b^*_1 and b^*_2 the resulting profit is at least as large as the sum of corresponding exclusive dealer profits. Thus, the impact of the effort is the same in both cases (i.e. $g\left(b^*_1\right)$ and $g\left(b^*_2\right)$ are the same) and the total cost incurred by the non-exclusive dealer, $2\,c\left(\frac{b^*_1+b^*_2}{2}\right)$, is no greater than the sum of the costs, $c\left(b^*_1\right)+c\left(b^*_2\right)$, incurred by the exclusive dealers. To show that the profits are strictly larger for a non-exclusive dealer we show that there exist market signals for individual manufacturers when the exclusive dealer only exerts effort b^*_1 while the non-exclusive dealer exerts different effort levels to make higher profits. Consider two market signals, x_1 and x_2, such that $\delta_2<x_2<x_1<\delta_x$. Claim 15 guarantees the existence of such market signals. From Claim 2 we know that the exclusive dealers exert efforts b^*_1 for both the manufacturers. From Claim 10 we know that, by exerting effort b^*_{31} for manufacturer 1 and $b^*_{32}=b^*_{21}$ for manufacturer 2, the non-exclusive dealer makes a higher combined profit than exerting $b^*_{11}=b^*_1$ effort for both manufacturers. Thus, the non-exclusive dealer makes strictly greater profit all such market signals resulting in a higher expected profit. ■

Proposition 17. Observe that the cutoffs δ_2 and δ_4 vary linearly with K. We now compute the derivative the expected sales (equation 18), $\frac{\partial E(s)}{\partial K}$

$$\begin{aligned}\frac{\partial E\left(s\right)}{\partial K}&=2\,f\left(\delta_2\right)\,F\left(\delta_2\right)\left[g\left(b^*_{11}\right)-g\left(b^*_{21}\right)\right]+f\left(\delta_2\right)\,g\left(b^*_{21}\right)\\&+\left[\left(F\left(\delta_4\right)-F\left(\delta_2\right)\right)\,f\left(\delta_2\right)+\left(f\left(\delta_4\right)-f\left(\delta_2\right)\right)\,F\left(\delta_2\right)\right]\,g\left(b^*_{31}\right)\\&+\left[\left(F\left(\delta_4\right)-F\left(\delta_2\right)\right)\,\left(f\left(\delta_4\right)-f\left(\delta_2\right)\right)\right]\,\left[g\left(b^*_{31}\right)+g\left(b^*_{21}\right)\right]\\&+\left[\left(f\left(\delta_4\right)\,-f\left(\delta_2\right)\right)\,\left(1-F\left(\delta_4\right)\right)-f\left(\delta_4\right)\,\left(F\left(\delta_4\right)-F\left(\delta_2\right)\right)\right]\,g\left(b^*_{21}\right)\\&+\left[f\left(\delta_4\right)\,\left(1-F\left(\delta_4\right)\right)-f\left(\delta_4\right)\,F\left(\delta_4\right)\right]\,g\left(b^*_{31}\right)-2\left(1-F\left(\delta_4\right)\right)\,f\left(\delta_4\right)\,g\left(b^*_{41}\right)\end{aligned}$$

Simplifying, we get $\frac{\partial E(s)}{\partial K}=f\left(\delta_2\right)\,F\left(\delta_2\right)\,\left[2\,g\left(b^*_{11}\right)-g\left(b^*_{31}\right)-g\left(b^*_{21}\right)\right]-f\left(\delta_4\right)\,\left(1-F\left(\delta_4\right)\right)$ $\left[2\,g\left(b^*_{41}\right)-g\left(b^*_{31}\right)-g\left(b^*_{21}\right)\right]$. Since $b^*_{41}>\frac{b^*_{31}+b^*_{21}}{2}$ (Claim 14) we have $g\left(b^*_{41}\right)>g\left(\frac{b^*_{31}+b^*_{21}}{2}\right)>\frac{g\left(b^*_{31}\right)+g\left(b^*_{21}\right)}{2}$. This implies that the second term of the derivative is always negative. Observe that $g\left(b^*_{11}\right)-g\left(b^*_{21}\right)\leq g'\left(b^*_{21}\right)\,\left(b^*_{11}-b^*_{21}\right)$ and $g\left(b^*_{31}\right)-g\left(b^*_{11}\right)\geq g'\left(b^*_{31}\right)\,\left(b^*_{31}-b^*_{11}\right)$. We now evaluate the following three cases. When $K<g\left(e^*_{41}\right)-\varepsilon_4$ we have $\delta_4\leq 0$. Since $\delta_4>\delta_2$

this also implies $\delta_2 \leq 0$. In this case the non-exclusive dealer only exerts one effort level, i.e. b^*_{41}, and the $E(s)$ reduces to $E(x_1) + g(b^*_{41})$. So, $\frac{\partial E(s)}{\partial K} = 0$. When $g(e^*_{41}) - \varepsilon_4 \leq K < g(e^*_{31}) - \varepsilon_2$ then only $\delta_2 \leq 0$. Hence effort level b^*_{11} is never exerted. The derivative reduces to $-f(\delta_4)\,(1 - F(\delta_4))\,[2\,g(b^*_{41}) - g(b^*_{31}) - g(b^*_{21})]$ which is non-positive. The last case is when $K \geq g(e^*_{31}) - \varepsilon_2$. In this situation the derivative is always negative if condition C3 holds implying that $E(s)$ is a decreasing function. This proves the Proposition. ■

Proposition 18. For any K we must have $\varepsilon_4 > \varepsilon_x$, where ε_x is defined by equation 9, because $\delta_4 > \delta_x$ by Claim 15. Part 1 follows immediately from Claim 2 and Proposition 17. For $K \leq K^*_{NED}$ the manufacturer's expected sales with either a non-exclusive dealer or an exclusive dealer is $E(x) + g(b^*_2)$ (since $b^*_{41} = b^*_2$). The variance of sales is equal to $V(x)$ in this case. So the manufacturer's profits are the same under both scenarios. This proves part 2 of the proposition. For the exclusive dealer we have already shown that when $K \leq K^*_{ED}$ the expected sales is $E(x) + g(b^*_2)$. Since the derivative of the expected sales is negative in this region the expected sales for the manufacturer is lower than $E(x) + g(b^*_{41}) = E(x) + g(b^*_2)$. This proves part 3 of the proposition. ■

Claim 19. First we show that b^*_{32} decreases, b^*_{31} increases, and b^*_{41} increases as Δ increases. We know that $\frac{c'\left(\frac{b^*_{31}+b^*_{32}}{2}\right)}{g'\left(b^*_{31}\right)} = p + \Delta$ and $\frac{c'\left(\frac{b^*_{31}+b^*_{32}}{2}\right)}{g'\left(b^*_{32}\right)} = p$. Now suppose we increase Δ to $\bar{\Delta}$. Further, suppose b^*_{32} changes to b_{32} such that $b^*_{32} < b_{32}$. This implies $g'(b_{32}) < g'(b^*_{32})$. If b^*_{31} changes to b_{31} then we have the following possibilities. Suppose $b_{31} \geq b^*_{31}$. Then $\frac{b_{31}+b_{32}}{2}$ increases and $\frac{c'\left(\frac{b_{31}+b_{32}}{2}\right)}{g'\left(b_{32}\right)} > p$ leading to a contradiction. Now suppose $b_{31} < b^*_{31}$ which implies $g'(b_{31}) > g'(b^*_{31})$. If $\frac{b_{31}+b_{32}}{2}$ also increases or remains the same then we have $\frac{c'\left(\frac{b_{31}+b_{32}}{2}\right)}{g'\left(b_{32}\right)} > p$ leading to a contradiction. On the other hand if $\frac{b_{31}+b_{32}}{2} < \frac{b^*_{31}+b^*_{32}}{2}$ then $\frac{c'\left(\frac{b_{31}+b_{32}}{2}\right)}{g'(b_{31})} < p + \bar{\Delta}$ leading to a contradiction. So $b_{32} \not> b^*_{32}$. Suppose $b^*_{32} = b_{32}$. Suppose $b_{31} \leq b^*_{31}$ implying $\frac{c'\left(\frac{b_{31}+b_{32}}{2}\right)}{g'(b_{31})} < p + \bar{\Delta}$ which leads to a contradiction. Hence, $b_{31} > b^*_{31}$. But this in turn implies that $\frac{c'\left(\frac{b_{31}+b_{32}}{2}\right)}{g'(b_{32})} > p$ which leads to a contradiction. Hence $b^*_{32} \neq b_{32}$. The only possibility is $b_{32} < b^*_{32}$. Since b^*_{32} decreases it must be the case that b^*_{31} increases with Δ. Since $\frac{c'\left(b^*_{41}\right)}{g'\left(b^*_{41}\right)} = p + \Delta$ and $\frac{c'}{g'}$ is an increasing function b^*_{41} must also increase with Δ. Given that conditions C1 and C2 hold, consider an effort function g such that $g(y) \equiv y^{\alpha}$, $0 < \alpha < 1$. We define the gap $q(\Delta) \equiv c_4 - c_2 = g(b^*_{31}) - g(b^*_{41}) + \varepsilon_4 - \varepsilon_2$. We now show the following: 1) b^*_{31} increases at a faster rate than b^*_{41}, 2) There exists a $\omega > 0$ such that, for all $\alpha \in [1 - \log_2(1 + \omega)\,,\ 1)$, $g(b^*_{31}) - g(b^*_{41})$ increases with Δ, and 3) If $g(b^*_{31}) - g(b^*_{41})$ increases with Δ then $q(\Delta)$ increases with Δ until $\delta_2 > 0$. Further, δ_2 decreases with Δ. Notice that $\frac{c'\left(\frac{b^*_{31}+b^*_{32}}{2}\right)}{g'\left(b^*_{31}\right)} = p + \Delta = \frac{c'\left(b^*_{41}\right)}{g'\left(b^*_{41}\right)}$. Equating the derivative on both sides we get $\frac{1}{g'\left(b^*_{41}\right)^2}$ $[g'\left(b^*_{41}\right) c''\left(b^*_{41}\right) - g''\left(b^*_{41}\right) c'\left(b^*_{41}\right) \frac{db^*_{41}}{d\Delta}] = \frac{1}{g'\left(b^*_{31}\right)^2} [\frac{g'\left(b^*_{31}\right)}{2} c''(\frac{b^*_{31}+b^*_{32}}{2}) - g''\left(b^*_{31}\right) c'(\frac{b^*_{31}+b^*_{32}}{2})] \frac{db^*_{31}}{d\Delta} +$ $\frac{1}{2\ g'\left(b^*_{31}\right)} c''(\frac{b^*_{31}+b^*_{32}}{2}) \frac{db^*_{32}}{d\Delta}$. Since b^*_{32} decreases as Δ increases, i.e. $\frac{db^*_{32}}{d\Delta} < 0$, we must have $\frac{1}{g'\left(b^*_{41}\right)^2}$ $[g'\left(b^*_{41}\right) c''\left(b^*_{41}\right) - g''\left(b^*_{41}\right)\ \ c'\left(b^*_{41}\right)] \frac{db^*_{41}}{d\Delta} < \frac{1}{g'\left(b^*_{31}\right)^2} [\frac{g'\left(b^*_{31}\right)}{2} c''(\frac{b^*_{31}+b^*_{32}}{2}) - g''\left(b^*_{31}\right) c'(\frac{b^*_{31}+b^*_{32}}{2})] \frac{db^*_{31}}{d\Delta}$. This implies $[\frac{c''\left(b^*_{41}\right)}{g'\left(b^*_{41}\right)} - \frac{g''\left(b^*_{41}\right)c'\left(b^*_{41}\right)}{g'\left(b^*_{41}\right)^2}] \frac{db^*_{41}}{d\Delta} < [\frac{c''(\frac{b^*_{31}+b^*_{32}}{2})}{2\ g'\left(b^*_{31}\right)} - \frac{g''\left(b^*_{31}\right)c'(\frac{b^*_{31}+b^*_{32}}{2})}{g'\left(b^*_{31}\right)^2}] \frac{db^*_{31}}{d\Delta}$. Notice that the terms in brackets, on both sides of the inequality, are strictly positive. On further simplification we get

$$\frac{\frac{db^*_{31}}{d\Delta}}{\frac{db^*_{41}}{d\Delta}} > \frac{\frac{c''\left(b^*_{41}\right)}{g'\left(b^*_{41}\right)} - \frac{g''\left(b^*_{41}\right)c'\left(b^*_{41}\right)}{g'\left(b^*_{41}\right)^2}}{\frac{c''\left(\frac{b^*_{31}+b^*_{32}}{2}\right)}{2\ g'\left(b^*_{31}\right)} - \frac{g''\left(b^*_{31}\right)\ c'\left(\frac{b^*_{31}+b^*_{32}}{2}\right)}{g'\left(b^*_{31}\right)^2}} = \frac{\frac{c'\left(b^*_{41}\right)}{g'\left(b^*_{41}\right)}\left[\frac{c''\left(b^*_{41}\right)}{c'\left(b^*_{41}\right)} - \frac{g''\left(b^*_{41}\right)}{g'\left(b^*_{41}\right)}\right]}{\frac{c'\left(\frac{b^*_{31}+b^*_{32}}{2}\right)}{g'\left(b^*_{31}\right)}\left[\frac{c''\left(\frac{b^*_{31}+b^*_{32}}{2}\right)}{2\ c'\left(\frac{b^*_{31}+b^*_{32}}{2}\right)} - \frac{g''\left(b^*_{31}\right)}{g'\left(b^*_{31}\right)}\right]}$$

$$= \frac{h\left(b^*_{41}\right) - m\left(b^*_{41}\right)}{\frac{1}{2}h\left(\frac{b^*_{31}+b^*_{32}}{2}\right) - m\left(b^*_{31}\right)}$$

Since $\frac{b^*_{41}}{2} < \frac{b^*_{31}+b^*_{32}}{2} < b^*_{41} < b^*_{31} + b^*_{32}$ we have $h\left(b^*_{41}\right) > \frac{1}{2}h\left(\frac{b^*_{31}+b^*_{32}}{2}\right)$ (by C1). Furthermore $m\left(b^*_{41}\right) < m\left(b^*_{31}\right) < 0$. Thus, $h\left(b^*_{41}\right) - m\left(b^*_{41}\right) > \frac{1}{2}h\left(\frac{b^*_{31}+b^*_{32}}{2}\right) - m\left(b^*_{31}\right) > 0$. So $\frac{db^*_{31}}{d\Delta} > \frac{db^*_{41}}{d\Delta}$. This proves the first part. Observe that as long as $b^*_{32} = b^*_{21} > 0$ the effort levels $b^*_{31} > b^*_{41} > \frac{b^*_{31}}{2}$. We know that $\frac{\frac{db^*_{31}}{d\Delta}}{\frac{db^*_{41}}{d\Delta}} > 1$. So let $\frac{\frac{db^*_{31}}{d\Delta}}{\frac{db^*_{41}}{d\Delta}} = 1 + \omega$ where $\omega > 0$. Since $g\left(x\right) = x^\alpha$, $g'\left(x\right) = \alpha\, x^{(\alpha-1)}$. Therefore, for $\frac{x}{2} < y < x$, we must have $\frac{g'(y)}{g'(x)} = \frac{y^{\alpha-1}}{x^{\alpha-1}} = \left(\frac{x}{y}\right)^{1-\alpha} < 2^{1-\alpha}$. Thus, $\frac{g'\left(b^*_{41}\right)}{g'\left(b^*_{31}\right)} < 2^{1-\alpha}$. If $\alpha = 1 - \log_2\left(1+\omega\right) + \delta$, then for some $\log_2\left(1+\omega\right) > \delta > 0$ we must have $2^{\log_2(1+\omega)-\delta} = \frac{1+\omega}{2^\delta} < 1+\omega$. This implies, for $\alpha \in [1-\log_2\left(1+\omega\right),\ 1)$, $\frac{\frac{db^*_{31}}{d\Delta}}{\frac{db^*_{41}}{d\Delta}} > \frac{g'\left(b^*_{41}\right)}{g'\left(b^*_{31}\right)}$ which further implies that $\frac{db^*_{31}}{d\Delta}g'\left(b^*_{31}\right) - \frac{db^*_{41}}{d\Delta}g'\left(b^*_{41}\right) > 0$. Hence $g\left(b^*_{31}\right) - g\left(b^*_{41}\right)$ increases with Δ as long as $b^*_{32} = b^*_{21} > 0$. This proves the second part. Using $b^*_{21} = b^*_{32}$, $b^*_{22} = b^*_{31}$ and rearranging the terms we write the gap as $q\left(\Delta\right) = \frac{2}{\Delta}\left[p\left[g\left(b^*_{21}\right) - g\left(b^*_{11}\right)\right] + \left(p+\Delta\right)\left[g\left(b^*_{31}\right) - g\left(b^*_{41}\right)\right]\right] + \frac{2}{\Delta}\left[c\left(b^*_{11}\right) + c\left(b^*_{41}\right) - 2\,c\left(\frac{b^*_{21}+b^*_{31}}{2}\right)\right]$. Let us define the sum of terms in the bracket as A. Therefore $q\left(\Delta\right) = \frac{2A}{\Delta}$. Observe that the gap is increasing if $A'\Delta > A$.

$$\begin{aligned} A' &= \frac{db^*_{41}}{d\Delta}pg'\left(b^*_{21}\right) - \frac{db^*_{21}}{d\Delta}c'\left(\frac{b^*_{21}+b^*_{31}}{2}\right) + \left(p+\Delta\right)g'\left(b^*_{31}\right)\frac{db^*_{31}}{d\Delta} \\ &- \frac{db^*_{31}}{d\Delta}c'\left(\frac{b^*_{21}+b^*_{31}}{2}\right) - \left(p+\Delta\right)g'\left(b^*_{41}\right)\frac{db^*_{41}}{d\Delta} + c'\left(b^*_{41}\right)\frac{db^*_{41}}{d\Delta} + g\left(b^*_{31}\right) - g\left(b^*_{41}\right) \\ &= g\left(b^*_{31}\right) - g\left(b^*_{41}\right) \end{aligned}$$

So the gap is increasing only if $\Delta\left[g\left(b_{31}^{*}\right)-g\left(b_{41}^{*}\right)\right]>A$, i.e. $p\,g\left(b_{11}^{*}\right)-c\left(b_{11}^{*}\right)-\left[p\,g\left(b_{21}^{*}\right)\right.$ $\left.-\,c\left(\frac{b_{21}^{*}+b_{31}^{*}}{2}\right)\right]>p\,g\left(b_{31}^{*}\right)-c\left(\frac{b_{21}^{*}+b_{31}^{*}}{2}\right)-\left[p\,g\left(b_{41}^{*}\right)-c\left(b_{41}^{*}\right)\right]$. At $\Delta=0$ both sides of the inequality are the same. If we show that for every $\Delta>0$ the derivative of the left hand side is strictly greater than the derivative for the right hand side then the inequality holds and the claim is proved. The derivative of the left hand side is

$$\begin{aligned}\frac{d\,L}{d\,\Delta}&=-p\,g'\left(b_{21}^{*}\right)+\frac{1}{2}c'\left(\frac{b_{21}^{*}+b_{31}^{*}}{2}\right)\left(\frac{d\,b_{31}^{*}}{d\,\Delta}+\frac{d\,b_{21}^{*}}{d\,\Delta}\right)\\&=\frac{1}{2}c'\left(\frac{b_{21}^{*}+b_{31}^{*}}{2}\right)\left(\frac{d\,b_{31}^{*}}{d\,\Delta}-\frac{d\,b_{21}^{*}}{d\,\Delta}\right)\end{aligned}$$

and the derivative of the right hand side is

$$\begin{aligned}\frac{d\,R}{d\,\Delta}&=(p+\Delta)\;g'\left(b_{31}^{*}\right)\frac{d\,b_{31}^{*}}{d\,\Delta}-\frac{1}{2}c'\left(\frac{b_{21}^{*}+b_{31}^{*}}{2}\right)\left(\frac{d\,b_{31}^{*}}{d\,\Delta}+\frac{d\,b_{21}^{*}}{d\,\Delta}\right)\\&\quad-\Delta\;g'\left(b_{31}^{*}\right)\frac{d\,b_{31}^{*}}{d\,\Delta}+(p+\Delta)\;g'\left(b_{41}^{*}\right)\frac{d\,b_{41}^{*}}{d\,\Delta}-c'\left(b_{41}^{*}\right)\frac{d\,b_{41}^{*}}{d\,\Delta}+\Delta\;g'\left(b_{31}^{*}\right)\frac{d\,b_{31}^{*}}{d\,\Delta}\\&=\frac{1}{2}c'\left(\frac{b_{21}^{*}+b_{31}^{*}}{2}\right)\left(\frac{d\,b_{31}^{*}}{d\,\Delta}-\frac{d\,b_{21}^{*}}{d\,\Delta}\right)-\Delta\left(g'\left(b_{31}^{*}\right)\frac{d\,b_{31}^{*}}{d\,\Delta}-g'\left(b_{41}^{*}\right)\frac{d\,b_{41}^{*}}{d\,\Delta}\right)\end{aligned}$$

So $\frac{d\,L}{d\,\Delta}>\frac{d\,R}{d\,\Delta}$ only if $g'\left(b_{31}^{*}\right)\frac{d\,b_{31}^{*}}{d\,\Delta}-g'\left(b_{41}^{*}\right)\frac{d\,b_{41}^{*}}{d\,\Delta}>0$, i.e. $g\left(b_{31}^{*}\right)-g\left(b_{41}^{*}\right)$ is an increasing function. So, $\frac{d\,L}{d\,\Delta}>\frac{d\,R}{d\,\Delta}$ which implies that $q\left(\Delta\right)$ is increasing as long as $\delta_2>0$. It is easy to show that $\Delta\frac{d\,\varepsilon_2}{d\,\Delta}+\varepsilon_2=\Delta g'\left(b_{31}^{*}\right)\frac{d\,b_{31}^{*}}{d\,\Delta}$. So $\frac{d\,\varepsilon_2}{d\,\Delta}=g'\left(b_{31}^{*}\right)\frac{d\,b_{31}^{*}}{d\,\Delta}-\frac{\varepsilon_2}{\Delta}$. Hence $\frac{d\,\delta_2}{d\,\Delta}=-\frac{\varepsilon_2}{\Delta}<0$. So δ_2 decreases as Δ increases and eventually may reach 0. This proves the third part. Together, this implies that if conditions C1 and C2, hold, and the effort function, g, is such that $g\left(b_{31}^{*}\right)$ $-\,g\left(b_{41}^{*}\right)$ increases with Δ, then increasing Δ expands the range $(q(\Delta))$ over which the dealer changes effort level based on the market signals of the other manufacturer. This proves the first part of the claim. Next, we analyze the situation when both the manufacturers offer a different per unit payment on exceeding the threshold. Suppose manufacturer 1 offers an additional payment of Δ_1 on crossing the threshold and the other manufacturer offers Δ_2 on crossing the threshold. Under these conditions the ε's change accordingly: $\varepsilon_1\equiv\frac{1}{\Delta_2}$ $[p(g(b_{11}^{*})-g(b_{21}^{*}))+\ (pg(b_{12}^{*})-g(b_{22}^{*}))+2\ c(\frac{b_{21}^{*}+b_{22}^{*}}{2})-2\ c(\frac{b_{11}^{*}+b_{12}^{*}}{2})]$, $\varepsilon_2\equiv\frac{1}{\Delta_1}\ [p(g(b_{11}^{*})-g(b_{31}^{*}))+p(g(b_{12}^{*})-g(b_{32}^{*}))+2\ c(\frac{b_{31}^{*}+b_{32}^{*}}{2})-2\ c(\frac{b_{11}^{*}+b_{12}^{*}}{2})]$, $\varepsilon_3\equiv\frac{1}{\Delta_2}[(p+\Delta_1)(g(b_{31}^{*})-g(b_{41}^{*}))+p(g(b_{32}^{*})-g(b_{42}^{*}))+2\ c(\frac{b_{41}^{*}+b_{42}^{*}}{2})-2\ c(\frac{b_{31}^{*}+b_{32}^{*}}{2})]$, and $\varepsilon_4\equiv\frac{1}{\Delta_1}[p(g(b_{21}^{*})-g(b_{41}^{*}))+(p+\Delta_2)(g(b_{22}^{*})-g(b_{42}^{*}))+2\ c(\frac{b_{41}^{*}+b_{42}^{*}}{2})-2\ c(\frac{b_{21}^{*}+b_{22}^{*}}{2})]$. In this case it is easy to verify that $\frac{\Delta_2}{\Delta_1}=\frac{\varepsilon_4-\varepsilon_2+g(b_{31}^{*})-g(b_{41}^{*})}{\varepsilon_3-\varepsilon_1+g(b_{22}^{*})-g(b_{42}^{*})}=\frac{\delta_4-\delta_2}{\delta_3-\delta_1}$. If manufacturer 2 increases Δ_2 then the gap for manufacturer 1 increases. At the same time δ_1 decreases. As a result manufacturer 1 will experience sales fluctuations for wider range of market signals. At the same time the range over which manufacturer 2 experiences fluctuations decreases. Furthermore, the fluctuations are experienced at lower values of x_1 than before. This proves the second part of the Claim. ■

Claim 20. We show the steps to prove the first inequality. The remaining inequalities can be proved using similar steps and convexity arguments. Notice that

$$\Pi_{LE} - \Pi_{LG} = p\left(g(b_1^{le}) - g(b_{21}^*)\right) + p\left(g(b_2^{le}) - g(b_{22}^*)\right) - 2c\left(\frac{b_1^{le} + b_2^{le}}{2}\right) + 2c\left(\frac{b_{21}^* + b_{22}^*}{2}\right) - \Delta\left(x_2 + g(b_{22}^*) - K\right).$$

We know that $g(b_1^{le}) - g(b_{21}^*) \leq g^{'}(b_{21}^*)\left(b_1^{le} - b_{21}^*\right)$, $g(b_2^{le}) - g(b_{22}^*) \leq g^{'}(b_{22}^*)\left(b_2^{le} - b_{22}^*\right)$, and $c\left(\frac{b_{21}^*+b_{22}^*}{2}\right) - c\left(\frac{b_1^{le}+b_2^{le}}{2}\right) \leq c^{'}\left(\frac{b_{21}^*+b_{22}^*}{2}\right)\left(\frac{b_{21}^*+b_{22}^*}{2} - \frac{b_1^{le}+b_2^{le}}{2}\right)$. Hence

$$\begin{aligned}
\Pi_{LE} - \Pi_{LG} &\leq p\, g^{'}(b_{21}^*)\left(b_1^{le} - b_{21}^*\right) + p\, g^{'}(b_{22}^*)\left(b_2^{le} - b_{22}^*\right) \\
&\quad + c^{'}\left(\frac{b_{21}^* + b_{22}^*}{2}\right)\left(b_{21}^* + b_{22}^* - b_1^{le} - b_2^{le}\right) - \Delta\ \left(x_2 + g(b_{22}^*) - K\right) \\
&= p\, g^{'}(b_{21}^*)\left(b_1^{le} - b_{21}^*\right) + (p + \Delta)\, g^{'}(b_{22}^*)\left(b_2^{le} - b_{22}^*\right) \\
&\quad - c^{'}\left(\frac{b_{21}^* + b_{22}^*}{2}\right)\left(b_1^{le} - b_{21}^*\right) - c^{'}\left(\frac{b_{21}^* + b_{22}^*}{2}\right)\left(b_2^{le} - b_{22}^*\right) \\
&\quad - \Delta\left(x_2 + g(b_{22}^*) - K\right) - \Delta\, g^{'}(b_{22}^*)\left(b_2^{le} - b_{22}^*\right) \\
&= -\Delta\left(x_2 + g(b_{22}^*) + g^{'}(b_{22}^*)\left(b_2^{le} - b_{22}^*\right) - K\right) \leq 0
\end{aligned}$$

This proves that $\Pi_{LG} \geq \Pi_{LE}$. Similarly, by symmetry we must have $\Pi_{GL} \geq \Pi_{EL}$. To prove the last part let us suppose $D \geq \Delta\,\varepsilon_2$.Then

$$\begin{aligned}
\Pi_{LG} - \Pi_{LL} &= p\ \left(g(b_{21}^*) - g(b_{11}^*)\right) + p\ \left(g(b_{22}^*) - g(b_{12}^*)\right) \\
&\quad - 2c\left(\frac{b_{21}^* + b_{22}^*}{2}\right) + 2c\left(\frac{b_{11}^* + b_{12}^*}{2}\right) + D \\
&\quad + \Delta\left(x_2 + g(b_{22}^*) - K\right) = -\Delta\,\varepsilon_2 + D + \Delta\ \left(x_2 + g(b_{22}^*) - K\right)
\end{aligned}$$

So $\Pi_{LG} - \Pi_{LL} \geq 0$ for $x_2 \geq K - g(b_{22}^*)$. ■