Conflict Games with Payoff Uncertainty[1]

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Abstract

Stag hunt and chicken games are canonical representations of two kinds of strategic interactions. In stag hunt, aggression feeds on itself, and mutual fear escalates into conflict. Chicken is a model of preemption and deterrence. With complete information, these games have multiple Nash equilbria. Using standard arguments from the Industrial Organization literature, we find sufficient conditions for payoff uncertainty to generate a unique Bayesian Nash equilibrium. These conditions encompass information structures ranging from independent types (as in our previous work) to highly correlated types (as in global games). *Keywords*: conflict, global games, strategic complements, strategic substitutes.

1 Introduction

Simple two-by-two games are frequently used to represent strategic interactions in political science and international relations (see, for example, Jervis [12]). In this literature, the prisoner's dilemma plays a prominent role. But in many instances, stag hunt and chicken games seem more apt metaphors. Stag hunt captures Hobbes's "state of nature", where conflict is caused by lack of trust.[1] Chicken is a model of preemption and deterrence. The prominence of prisoner's dilemma games in the literature may be due to analytical convenience: the prisoner's dilemma has a unique Nash equilibrium, whereas stag hunt and chicken have multiple equilibria.

The prisoner's dilemma can be thought of as a degenerate stag hunt or chicken game, where extreme levels of hostility have made "war" a dominant strategy. And even if this scenario is not very likely, a player who is himself not intrinsically hostile may be unable to completely rule out the possibility that the opponent is extremely hostile, or that the opponent thinks he is very hostile... As is well known, this type of reasoning may produce "spirals" of fear and aggression. The most useful way to think about these spirals is to formally introduce payoff uncertainty. This not only makes the model more realistic, but it may also generate a unique equilibrium.

Consider a two-player game, where each player must choose either *hawk* (H) or *dove* (D). The hawkish action H might represent an act of war, accumulation of weapons, or some other aggressive action. In the payoff matrix, the row represents the choice of player i, and the column the choice of player j. Only player i's payoff is indicated.

	H	D
H	$h_i - c$	h_i
D	$-d$	0

The payoff from the peaceful outcome (D, D) is, without loss of generality,

[1] The "state of nature" is sometimes described as a prisoner's dilemma, but a close reading of Hobbes [11] reveals that this may be misleading. Hobbes's *fundamental law of nature* says that "every man ought to endevor peace, as far as he has hope of obtaining it; and when he cannot obtain it, that he may seek, and use, all helps, and advantages of war" (Hobbes [11], p. 68). Hobbes stresses that the law has two parts, the first "to seek peace, and follow it", and the second "by all means we can, to defend ourselves" (Hobbes [11], p. 68). Hobbes clearly believes that what a man should do depends on what other men are doing, so his "state of nature" is more easily interpreted as a stag-hunt than as a prisoner's dilemma.

normalized to zero. The parameter h_i incorporates player i's benefits or costs from choosing H. If the opponent chooses H then player i suffers a cost c if player i also chooses H, and a cost d if player i chooses D. We assume, for convenience, that the two parameters c and d are the same for both players. However, in general we will have $h_1 \neq h_2$. We refer to h_i as player i's *hostility parameter* or *type*.

Suppose, for the moment, that there is no payoff uncertainty: the players know everything about the game, including each others types. If $h_i > \max\{c-d, 0\}$ for each $i \in \{1,2\}$ then the game is a *prisoner's dilemma* with a unique Nash equilibrium: (H,H). If $c - d < h_i < 0$ for each $i \in \{1,2\}$ then the game is a *stag-hunt* with two Nash equilibria: (H,H) and (D,D). If $0 < h_i < c - d$ for each $i \in \{1,2\}$ then it is a game of *chicken* with two Nash equilibria: (H,D) and (D,H). In games with multiple Nash equilibria, the criterion of *risk-dominance* is sometimes used to select a unique outcome. In the stag-hunt game, the risk-dominant Nash equilibrium is (H,H) if $h_1 + h_2 > c - d$ and (D,D) if $h_1 + h_2 < c - d$. That is, if the combined hostility levels are not too big, a conflict can be avoided. In chicken, the risk-dominant Nash equilibrium is (H,D) if $h_1 > h_2$ and (D,H) if $h_1 < h_2$. That is, the most hostile player is aggressive, the other backs down.

Carlsson and van Damme [8] showed that if the players do not know the true payoff matrix, but receive noisy signals of it, then as the noise vanishes the players coordinate on the risk-dominant outcome. This insight triggered a large literature on *global games*. However, when the noise vanishes, each player becomes very sure of the opponent's true payoffs, and we believe this is not a plausible assumption for many applications. In reality, payoff uncertainty can be large. Therefore, unlike the global games literature, we do not focus on "small" perturbations of the payoff matrix. We will derive conditions under which a unique equilibrium exists when there is significant uncertainty about the opponent's true payoff function. (Unlike Carlsson and van Damme [8] we assume each player knows his own payoff function when choosing an action.)

Realistically, any parameter in the payoff matrix could be stochastic. For convenience, we will assume the uncertainty only relates to the hostility parameters, while c and d are common knowledge. We consider two possibilities. If $c < d$, then actions are strategic complements, as in a stag-hunt game: each player is more inclined to choose H, the more likely it is that that his opponent will choose H. If $c > d$, then actions are strategic substitutes, as in a chicken game: each player is more inclined to choose H, the more likely it is that the opponent will choose D.

Players with very big hostility parameters are "dominant strategy hawks" who behave as in a prisoner's dilemma, with "war" (H) as a dominant strategy. Conversely, players with very small hostility parameters are "dominant strategy doves" for whom "peace" (D) is dominant. The remaining "moderate" types, who do not have any dominant strategy, must form beliefs about the opponent's action before deciding what to do. With strategic complements, these moderates are coordination types who want to match the action of their opponent. With strategic substitutes, the moderates are "opportunists" who want to choose the opposite of their opponent.

In Industrial Organization theory, the contrast between strategic complements and substitutes organizes the way we think about strategic interaction (e.g., Bertrand versus Cournot competition). But these concepts are equally useful in International Relations and conflict. Two kinds of conflicts are often distinguished: those where aggression feeds on itself in a cycle of fear, as in stag hunt; and those where toughness forces the opponent to back down, as in chicken.[2] In stag hunt, actions are strategic complements: the incentive to choose H is increasing in the probability that the opponent chooses H. This can trigger an escalating spiral of aggression as in the classic work of Schelling [17] and Jervis [12]. In a chicken game, actions are strategic substitutes: the incentive to choose H is decreasing in the probability that the opponent chooses H. This captures a scenario where players will back down in the face of aggression. For example, suppose the hawkish action represents sending soldiers to a disputed territory. If only one country sends soldiers, then it will control the territory at little cost. But if both countries send their soldiers, a war could easily break out. If the value of the territory is not large enough to justify the risk of war, it is a game of chicken.

By using the standard approach from the Industrial Organization literature, we obtain rather general conditions for a unique equilibrium to exist. The conditions are stated in terms of properties of the distribution of types. But they are equivalent to the well-known condition that the slopes of (appropriately defined) reaction functions should be less than one in absolute value. The conditions are satisfied if types are independently drawn but diffuse - "large idiosyncratic uncertainty". For example, even if the *ordi-*

[2] "World Wars I and II are often cast as two quite different models of war.. World War I was an unwanted spiral of hostility... World War II was not an unwanted spiral of hostility-it was a failure to deter Hitler's planned aggression." Joseph Nye (p. 111, [16]). *Understanding International Conflict (6th Edition).*

nal payoffs are those of a stag hunt game with probability close to one, but there is a large amount of uncertainty about idiosyncratic cardinal payoffs, then there is a unique equilibrium. However, independence is not always a reasonable assumption. In a conflict over a common resource, types are determined not only by idiosyncratic preferences, but also by the value of the contested resource. In this case, a player's type contains information about his opponent's type, so types are affiliated. Our conditions cover this case as well.

Consider the case where actions are strategic complements. Informally, a player is "almost a dominant strategy hawk" if he is not quite hostile enough to be a dominant strategy hawk, but he is willing to play D only when the opponent is almost sure to play D. If he cannot rule out the possibility that the opponent is a very hostile dominant strategy hawk (who surely plays H), then he feels compelled to play H in self-defense. Thus, the fear of dominant strategy hawks induces these "almost dominant strategy hawks" to play H, which in turn creates more fear, inducing even less hostile "almost-almost dominant strategy hawks" to play H etc. This vicious cycle is a formalization of Schelling's [17] "reciprocal fear of surprise attack" and Jervis's [12] "spiral model". But a more benign spiral might counteract the vicious cycle: the possibility that the opponent is dominant strategy dove (who surely plays D) induces "almost dominant strategy doves" to play D, which in turn induces "almost-almost dominant strategy doves" to play D etc. In Section 3, we derive a sufficient condition for these spirals to produce a unique equilibrium. In particular, this happens when types are independent but diffuse, or are determined by a common shock and small noise as in the global games literature. In general, affiliation has an ambiguous effect on the spirals. On the one hand, affiliation of types makes "almost dominant strategy hawks" think it is quite likely that the opponent is a dominant strategy hawk, which intensifies their fear, and so on. In this sense, affiliation promotes uniqueness when actions are strategic complements. However, if types are strongly affiliated, then there is not much (interim) payoff uncertainty. In this case, multiple equilibria cannot be ruled out.

A different kind of spiral leads to a unique equilibrium in games with strategic substitutes. This spiral is created by types who back down in the face of aggression, triggering more aggression which causes more types to back down, etc. In Section 4, we derive a sufficient condition for uniqueness in such games. With strategic complements, player i's fear of dominant strategy hawks induces "almost dominant strategy doves" to play D. This in turn induces player j's "almost dominant strategy hawks" to play H etc. Again, with independently drawn diffuse types there is a unique equilibrium. Affiliation prevents this type of spiral from gaining ground, because "almost dominant strategy doves" will think it is quite unlikely that the opponent is a dominant strategy hawk, which mitigates their fear, and so on. Therefore, affiliation unambiguously works against uniqueness when actions are strategic substitutes.

We have previously studied the logic of mutual fear and escalation under the assumption that types are independent (Baliga and Sjöström [5], [6], Baliga, Lucca and Sjöström [4]). In complementary work, Chassang and Padro-i-Miguel [9], [10] use the theory of global games and the concept of risk-dominance to formalize the logic of mutual fear. In this article, we consider a more general model of affiliated types, where both independent types and highly correlated types are special cases. Morris and Shin ([15]) study games with strategic complements and a broader class of information structures than global games. They argue that uniqueness results from either large idiosyncratic uncertainty (as in Baliga and Sjöström [5], [6], Baliga, Lucca and Sjöström [4]) or from highly correlated types (as in global games). We draw a similar conclusion for the case of strategic complements but also consider the case of strategic substitutes.. By exploiting sufficient conditions that are well known from the Industrial Organization literature, we derive fairly general results in a simple and familiar way.

In Section 5, we use the information structure of global games to contrast our approach with Carlsson and van Damme [8]. Our general sufficient condition for uniqueness implies that as types become highly correlated, games with strategic complements have a unique Bayesian Nash equilibrium. This mirrors the well-known result of Carlsson and van Damme [8]. However, when actions are strategic substitutes, our general sufficient condition requires that types are not too highly correlated - "large idiosyncratic uncertainty". Alternatively, with strategic substitutes a unique equilibrium exists if the players are (sufficiently) asymmetric ex ante. In equilibrium, the ex ante asymmetry is magnified: the ex ante less hostile player is intimidated, the ex ante more hostile player is very aggressive.

2 The Model

As discussed above, the payoff matrix is

$$\begin{array}{ccc} & H & D \\ H & h_i - c & h_i \\ D & -d & 0 \end{array}$$

The two parameters c and d are fixed and the same for both players. However, player i's true hostility parameter (or "type") h_i is his private information.

The hostility parameter h_i has a fixed publicly observed component k_i as well as a random privately observed component η_i. Thus, player i's type is

$$h_i = k_i + \eta_i.$$

The game of incomplete information is played as follows. First η_1 and η_2 are drawn from a symmetric joint distribution with support $[\underline{\eta}, \bar{\eta}] \times [\underline{\eta}, \bar{\eta}]$. Then player 1 is informed about η_1, but not about η_2, while player 2 is informed about η_2 but not about η_1. Finally, the two players make their choices simultaneously (H or D).

When the players make their choices, everything except η_1 and η_2 is commonly known. In particular, there is no uncertainty about the fixed parameters k_1 and k_2. The introduction of k_1 and k_2 is a convenient way to allow for *ex ante* asymmetries in the distribution of hostilities. If $k_1 = k_2$ then the two players are ex ante symmetric (and it would be without loss of generality to assume $k_1 = k_2 = 0$). But if $k_1 \neq k_2$ then there is a commonly known ex ante asymmetry.

If η_1 and η_2 are correlated, then player i's knowledge of η_i can be used to update his beliefs about η_j. Formally, the cumulative distribution of η_j conditional on $\eta_i = y$ (where $i \neq j$) is denoted $F(\cdot|y)$. We assume $F(x|y)$ is a continuous function of x and y. On the interior of its domain it is continuously differentiable, with partial derivatives $F_1(x|y) \equiv \frac{\partial F(x|y)}{\partial x}$ and $F_2(x|y) \equiv \frac{\partial F(x|y)}{\partial y}$. Notice that $F_1(\cdot|y)$ is the density of η_j conditional on $\eta_i = y$. Since player j's type is $h_j = k_j + \eta_j$, uncertainty about η_j directly translates into uncertainty about player j's type. The types h_1 and h_2 are correlated if and only if η_1 and η_2 are correlated. If player i's type is $h_i = y$, then player i assigns probability $F(x - k_j|y - k_i)$ to the event that $h_j \leq x$. Indeed, $h_i = y$ if and only if $\eta_i = y - k_i$, and $h_j \leq x$ if and only if $\eta_j \leq x - k_j$.

The least (resp. most) hostile type of player i has hostility parameter $\underline{h}_i = k_i + \underline{\eta}$ (resp. $\bar{h}_i = k_i + \bar{\eta}$). Notice that, for any y,

$$F(\underline{h}_j - k_j|y - k_i) = F(\underline{\eta}|y - k_i) = 0$$

and

$$F(\bar{h}_j - k_j|y - k_i) = F(\bar{\eta}|y - k_i) = 1$$

since it is impossible for player j to have hostility parameter below $\underline{h}_j$ or above $\bar{h}_j$.

We make the following assumption:

Assumption 1 (i) $F_1(x|y) > 0$ for all $x, y \in \left(\underline{\eta}, \bar{\eta}\right)$ and (ii) $F_2(x|y) \leq 0$ for all $x, y \in (\underline{\eta}, \bar{\eta})$.

Part (i) says there is positive density everywhere. Part (ii) says that $F(x|y)$ is not increasing in y. Therefore, as a player becomes more hostile, he becomes no less pessimistic about his opponent's hostility. Part (ii) holds if η_1 and η_2 are affiliated (Milgrom, 2004, Theorem 5.4.3). Affiliation is a natural assumption if the conflict is over some contested resource such as an oil field, where there is uncertainty about the value of the resource. As a special case, part (ii) holds if η_1 and η_2 are independent. Independence is a natural assumption if the uncertainty is over the players' innate attitudes towards conflict. For future reference, we note that Assumption 1 implies that if $y > x$ then $F(y|x) - F(x|y) \geq F(y|y) - F(x|y) > 0$.

We classify types into four categories.

Definition 1 *Player i is a* dominant strategy hawk *if $h_i - c \geq -d$ and $h_i \geq 0$ with at least one strict inequality. Player i is a* dominant strategy dove *if $h_i - c \leq -d$ and $h_i \leq 0$ with at least one strict inequality. Player i is a* coordination type *if $c - d \leq h_i \leq 0$. Player i is an* opportunistic type *if $0 \leq h_i \leq c - d$,.*

Notice that coordination types can exist only in games with strategic complements. For them, H is a best response to H and D a best response to D. Opportunistic types can exist only in games with strategic substitutes. For them, D is a best response to H and H a best response to D.

With strategic complements, there is a positive probability that player i is a dominant strategy hawk (resp. dove) if and only if $\bar{h}_i > 0$ (resp.

$\underline{h}_i < c - d$). With strategic substitutes, there is a positive probability that player i is a dominant strategy hawk (resp. dove) if and only if $\bar{h}_i > c - d$ (resp. $\underline{h}_i < 0$). A positive probability of dominant strategy types creates the "spirals" discussed in the introduction. At this point, we are not assuming that dominant strategy types exist (but it will be implied by the condition for uniqueness - see below).

2.1 Bayesian Nash Equilibrium

Suppose player i is of type h_i, and thinks player j will choose D with probability $\delta_j(h_i)$. (This probability can depend on h_i if types are not independent). Type h_i's expected payoff from H is $h_i - (1 - \delta_j(h_i))c$, while his expected payoff from D is $-(1 - \delta_j(h_i))d$. Thus, if type h_i chooses H instead of D, his *net* gain is

$$h_i + (d - c)(1 - \delta_j(h_i)) \tag{1}$$

A *strategy* for player i is a function $\sigma_i : [\underline{h}_i, \bar{h}_i] \to \{H, D\}$ which specifies an action $\sigma_i(h_i) \in \{H, D\}$ for each type $h_i \in [\underline{h}_i, \bar{h}_i]$. In Bayesian Nash equilibrium (BNE), all types maximize their expected payoff. Therefore, $\sigma_i(h_i) = H$ if the expression in (1) is positive, and $\sigma_i(h_i) = D$ if it is negative. (If expression (1) is zero then type h_i is indifferent and can choose either H or D.) We say that player i uses a *cutoff strategy* if there is a *cutoff point* $x \in [\underline{h}_i, \bar{h}_i]$ such that $\sigma_i(h_i) = H$ for all $h_i > x$ and $\sigma_i(h_i) = D$ for all $h_i < x$. A *cutoff equilibrium* is a BNE in cutoff strategies. Cutoff equilibria seem very natural. They capture the intuition that when a player becomes more hostile he becomes more likely to show aggression.

If player j uses a cutoff strategy with cutoff point x, then $\delta_j(y) = F(x - k_j | y - k_i)$, so player i's net gain from choosing H instead of D when his type is $h_i = y$ is

$$\Psi^i(x, y) \equiv y + (d - c)\left(1 - F(x - k_j | y - k_i)\right). \tag{2}$$

For a cutoff strategy to be a best response, player i should be more inclined to choose H the more hostile he is. That is, $\Psi^i(x, y)$ should be increasing in y :

$$\Psi^i_2(x, y) = 1 - (d - c)F_2(x - k_j | y - k_i) > 0 \tag{3}$$

Figure 1 illustrates this property.

In view of Assumption 1, (3) holds if $d > c$. It also holds if $d < c$ and the two types are not very highly correlated. However, if c is much bigger than d and the two types are highly correlated, then (3) may be violated. Intuitively, if types are highly correlated and the players use cutoff strategies, then a very hostile type thinks it is very likely that the opponent chooses H. If in addition c is much bigger than d, then the (H, H) outcome is very costly. In this situation, the very hostile type may be more inclined to choose D than a less hostile type, and a cutoff strategy might not be a best response against a cutoff strategy.

If condition (3) holds then player i's best response to player j's cutoff x is to use a cutoff point denoted $\beta_i(x)$. The best-response function $\beta_i(x)$ is defined as follows. (i) If $\Psi^i(x, \underline{h}_i) \geq 0$ then $\beta_i(x) = \underline{h}_i$ (so player i plays H with probability one). (ii) If $\Psi_i(x, \bar{h}_i) \leq 0$ then $\beta_i(x) = \bar{h}_i$ (so player i plays D with probability one). (iii) Otherwise, $\beta_i(x) \in \left(\underline{h}_i, \bar{h}_i\right)$ is the unique solution to the equation $\Psi^i(x, \beta_i(x)) = 0$ (all types above $\beta_i(x)$ play H, and all types below $\beta_i(x)$ play D). As long as $\Psi^i(x, y)$ is increasing in y, $\beta_i(x)$ is a well-defined continuous function (by the implicit function theorem), and the slope of β_i is obtained by totally differentiating $\Psi^i(x, \beta_i(x)) = 0$. Thus,

$$\beta_i'(x) = -\frac{\Psi_1^i(x, \beta_i(x))}{\Psi_2^i(x, \beta_i(x))} = -\frac{(c-d)F_1(x - k_j|\beta_i(x) - k_i)}{1 - (d-c)F_2(x - k_j|\beta_i(x) - k_i)}. \tag{4}$$

A cutoff equilibrium is an intersection of the two best response curves. That is, if h_i^* denotes player i's equilibrium cutoff point, then we must have $h_1^* = \beta_1(h_2^*)$ and $h_2^* = \beta_2(h_1^*)$. Notice that $\beta_i'(x) > 0$ if $d > c$ (strategic complements) and $\beta_i'(x) < 0$ if $d < c$ (strategic substitutes). Also notice that if both equilibrium cutoff points h_1^* and h_2^* are interior, $\underline{h}_i < h_i^* < \bar{h}_i$, then $\Psi^1(h_2^*, h_1^*) = \Psi^2(h_1^*, h_2^*) = 0$. Using equation (2), the condition $\Psi^i(h_j^*, h_i^*) = 0$ for an interior equilibrium becomes

$$h_i^* + (d-c)\left(1 - F(h_j^* - k_j|h_i^* - k_i)\right) = 0. \tag{5}$$

Proposition 2 *If $\Psi^i(x, y)$ is increasing in y for each $i \in \{1, 2\}$ and all $x \in [\underline{h}_i, \bar{h}_i]$, then a cutoff equilibrium exists.*

Proof. Since the function $\Psi^2(\beta_1(x), x)$ is continuous in x, one of the following three cases must occur:

(i) $\Psi^2(\beta_1(\underline{h}_2), \underline{h}_2) \geq 0$. In this case the cut-off points $(\beta_1(\underline{h}_2), \underline{h}_2)$ form a BNE.

(ii) $\Psi^2(\beta_1(\bar{h}_2), \bar{h}_2) \leq 0$. In this case the cut-off points $(\beta_1(\bar{h}_2), \bar{h}_2)$ form a BNE.

(iii) there is $x \in [\underline{h}_2, \bar{h}_2]$ such that $\Psi^2(\beta_1(x), x) = 0$. In this case the cut-off points $(\beta_1(x), x)$ form a BNE. ■

3 Strategic Complements

Actions are strategic complements when $d > c$. In this case, (3) holds, so $\beta_i(x)$ is a well-defined continuous function, and a cutoff equilibrium exists by Proposition 2. We first derive a sufficient condition for this to be the *unique* BNE.

3.1 The uniqueness result

Our main result for the case of strategic complements is the following.

Theorem 3 *Suppose $d > c$ and for all $s, t \in (\underline{\eta}, \bar{\eta})$,*

$$F_1(s|t) + F_2(s|t) < \frac{1}{d-c}. \tag{6}$$

There is a unique BNE. This BNE is a cutoff equilibrium.

In the appendix, we prove that all BNE must be cutoff equilibria. At least one cutoff equilibrium exists by Proposition 2. To complete the proof of Theorem 3, we only need to show that there cannot be more than one cutoff equilibrium.

It suffices to show that the slope of the best response function β_i is less than one in absolute value. As is well known, this guarantees that the two best response curves can intersect only once (for example, Vives [20]). Equation (4) can be manipulated as follows:

$$\begin{aligned} \beta_i'(x) &= \frac{(d-c)F_1(x-k_j|\beta_i(x)-k_i)}{1-(d-c)F_2(x-k_j|\beta_i(x)-k_i)} \\ &= 1 - \frac{1-(d-c)\left\{F_1(x-k_j|\beta_i(x)-k_i)+F_2(x-k_j|\beta_i(x)-k_i)\right\}}{\Psi_2^i(x,\beta(x))} \\ &< 1. \end{aligned}$$

The inequality is due to (3) and (6). Since we already know that $\beta_i'(x) > 0$ when $d > c$, we conclude that $0 < \beta_i'(x) < 1$. Thus, the cutoff equilibrium is unique.[3]

3.2 Diagrammatic illustration

We will illustrate Theorem 3 diagrammatically. Consider the function $Q : [\underline{\eta}, \bar{\eta}] \to [0,1]$ defined by $Q(\eta) \equiv F(\eta|\eta)$. Notice that $Q(\underline{\eta}) = 0$ and $Q(\bar{\eta}) = 1$. By calculus,

$$\begin{aligned} \frac{\bar{\eta} - \underline{\eta}}{d - c} &= \int_{\underline{\eta}}^{\bar{\eta}} \left(\frac{1}{d-c} \right) \, d\eta > \int_{\underline{\eta}}^{\bar{\eta}} \{F_1(\eta|\eta) + F_2(\eta|\eta)\} \, d\eta \\ &= \int_{\underline{\eta}}^{\bar{\eta}} Q'(\eta) \, d\eta = Q(\bar{\eta}) - Q(\underline{\eta}) = 1 \end{aligned}$$

where the inequality is due to (6). Therefore, for each $i \in \{1,2\}$,

$$\bar{h}_i - \underline{h}_i = \bar{\eta} - \underline{\eta} > d - c.$$

This implies that either $\bar{h}_i > 0$ or $\underline{h}_i < c - d$ (or both). That is, for each player, there is a positive probability of being a dominant strategy type. Whether it is dominant strategy hawks or doves (or both) which have positive probability will determine the nature of the equilibrium.

First, suppose $\underline{h}_i < c - d < 0 < \bar{h}_i$ for each $i \in \{1,2\}$, so the support of each player's types includes both dominant strategy hawks and dominant strategy doves. Notice that $\Psi^i(\underline{h}_j, y) = y + d - c$, so $\beta_i\left(\underline{h}_j\right) = c - d$. That is, if player j plays H with probability 1 (his cutoff point is $\underline{h}_j$) then player i's best response is to choose H whenever he is not a dominant strategy

[3] If $k_1 = k_2 = 0$ (so the players are ex ante symmetric) and $\bar{h}_1 = \bar{h}_2 > 0$ and $\underline{h}_1 = \underline{h}_2 < c - d$ (so dominant strategy types of both kinds exist), then another sufficient condition for a unique equilibrium in cutoff strategies is that the function $Q(x) \equiv F(x|x)$ is concave. To prove this, define the modified best response function $\hat{\beta}$ by using the function

$$\hat{\Psi}(x,y) \equiv y + (d-c)(1 - Q(x))$$

instead of the function Ψ. This yields $\hat{\beta}(x) = -(d-c)(1-Q(x))$. If Q is concave, then $\hat{\beta}$ intersects the 45 degree line exactly once. Moreover, β coincides with $\hat{\beta}$ on the 45 degree line, so β intersects the 45 degree line in a unique point as well. This shows that our previous results that relied on concavity (see Baliga, Lucca and Sjöström [4]) can be generalized to allow correlated types.

dove. Similarly, $\Psi^i(\bar{h}_j, y) = y$, so $\beta_i\left(\bar{h}_j\right) = 0$. That is, if player j plays H with probability 0 (his cutoff point is $\bar{h}_j$) then player i's best response is to choose H whenever he is a dominant strategy hawk. If $\underline{h}_j < x < \bar{h}_j$, then $\beta_i(x) \in \left[\underline{h}_i, \bar{h}_i\right]$ is the unique solution to the equation $\Psi^i(x, \beta_i(x)) = 0$, and we know that $0 < \beta_i'(x) < 1$.

If there are both dominant strategy hawks and dominant strategy doves, then certainly some types choose H and some types choose D. That is, the unique equilibrium must be interior: each player $i \in \{1, 2\}$ chooses a cut-off point $h_i^* \in \left(\underline{h}_i, \bar{h}_i\right)$ which is a best response to the opponent's cutoff h_j^*, i.e., which satisfies (5). The equilibrium (h_1^*, h_2^*) is illustrated in Figure 2.

Now suppose some player, say player 1, can be a dominant strategy hawk but not a dominant strategy dove: $c - d < \underline{h}_1 < 0 < \bar{h}_1$. It is now possible that in equilibrium player 1 chooses H with probability one, but the equilibrium is still unique.

If both players can be dominant strategy hawks but not dominant strategy doves, i.e., $\underline{h}_i \geq c - d$ for $i \in \{1, 2\}$, then there surely exists an equilibrium where each player chooses H, regardless of type. By Theorem 3, there can be no other equilibrium in this case. Thus, if each player can be a dominant strategy hawk, but not a dominant strategy dove, then peaceful coexistence is impossible, and each player chooses H with probability one. Intuitively, the positive probability of dominant strategy hawks triggers a spiral, where more and types choose H (as discussed in the introduction). In the absence of dominant strategy doves, there is no benign spiral spreading from the other direction. Accordingly, all types get swept away by the "hawkish" spiral. This is an extreme case of Schelling's [17] dilemma, where conflict is triggered by reciprocal fear of surprise attack.[4]

3.3 Interim rationalizability

The spirals we have discussed correspond to the procedure of "interim rationalizability" (for a characterization and discussion of this and related procedures, see Battigalli et. al. [7]). Thus, the unique BNE is the unique (interim) rationalizable outcome. To see this, suppose the joint distribution of types is commonly known. For each $i \in \{1, 2\}$, action D can be eliminated for the dominant strategy hawks, i.e., for types such that $h_i \geq c - d$,

[4] Conversely, if $\bar{h}_i \leq 0$ for $i \in \{1, 2\}$, so there are no dominant-strategy hawks, then the unique equilibrium is for each player to choose D, regardless of type.

because it cannot be a best response to any beliefs. In the next "round" of elimination, player i eliminates D for all types h_i which are not dominant strategy types, but which satisfy $\Psi^i(c-d, h_i) > 0$. Indeed, for these "almost dominant strategy hawks", D cannot be a best response to any beliefs which assign probability one to player j's dominant strategy hawks choosing H. And so on.

3.4 The uniform independent case

Suppose η_1 and η_2 are independently drawn from a uniform distribution on $[\underline{\eta}, \bar{\eta}]$. In this case,

$$F(s|t) = \frac{s - \underline{\eta}}{\bar{\eta} - \underline{\eta}} \tag{7}$$

so $F_1(s|t) = 1/\left(\bar{\eta} - \underline{\eta}\right)$ and $F_2(s|t) = 0$. Therefore, (6) holds if and only if

$$\bar{\eta} - \underline{\eta} > d - c. \tag{8}$$

Theorem 3 implies that (8) is sufficient condition for a unique BNE to exist. Thus, with a uniform distribution, there is a unique equilibrium as long as the support $[\underline{\eta}, \bar{\eta}]$ is big enough.

To simplify the calculations, suppose $k_2 = k \geq 0 = k_1$. Thus, player 2 is ex ante (weakly) more hostile than player 1. If only one kind of dominant strategy type exists, say only dominant strategy hawks, then the unique equilibrium is for both players to choose H regardless of type. For a more interesting interior equilibrium, assume $\bar{\eta} > 0$ and $\underline{\eta} + k < c - d$. This guarantees that the support of each player's types includes dominant strategy types of both kinds. Using (7), we solve (5) to get the equilibrium cutoff points $h_1^* = A + \left(\bar{\eta} - \underline{\eta}\right) B$ and $h_2^* = A + (d - c)B$, where

$$A \equiv \frac{(c-d)\bar{\eta}}{\left(\bar{\eta} - \underline{\eta}\right) - (d - c)}$$

and

$$B \equiv \frac{(c-d)k}{\left(\bar{\eta} - \underline{\eta}\right)^2 - (d - c)^2}$$

It can be verified that $\bar{\eta} > 0$ and $\underline{\eta} + k < c - d$ guarantee $h_i^* \in \left(\underline{h}_i, \bar{h}_i\right)$. If $k > 0$ then $h_1^* < h_2^*$, since $\bar{\eta} - \underline{\eta} > d - c$ and $B < 0$. Thus, if it should happen that both players' true types lie in the interval (h_1^*, h_2^*), then player 1 chooses H (because $h_1 > h_1^*$) but player 2 chooses D (because $h_2 < h_2^*$). In this sense, the ex ante less hostile player becomes ex post the most aggressive one - because he feels more threatened by the opponent.

3.5 Discussion

Suppose the players are symmetric: $k_1 = k_2 = 0$. Now the best response curves must intersect at the 45 degree line, and the unique equilibrium is a symmetric cut-off equilibrium, $h_1^* = h_2^* = h^*$. The symmetric cut off point is the unique solution in $[\underline{h}_i, \bar{h}_i]$ to the equation

$$h^* + (d - c)\,(1 - F(h^*|h^*)) = 0 \tag{9}$$

The intuition behind Theorem 3 may be brought out by a standard "stability" argument. Starting at the symmetric cutoff equilibrium h^*, suppose both simultaneously reduce their cut-off by ε (so a few more types use H). Then, consider type $h^* - \varepsilon$. If type $h^* - \varepsilon$ now prefers D, the initial equilibrium is stable, and this is what we want to verify. In fact there are two opposing effects. First, at the original cut-off h^*, type $h^* - \varepsilon$ strictly preferred D, so there is reason to believe he still prefers D. However, the opponent has now become more hostile. At the initial equilibrium, cutoff type h^* thought that the opponent would choose H with probability $1 - F(h^*|h^*)$. But after the perturbation, the new cutoff type $h^* - \varepsilon$ thinks that the opponent will choose H with probability $1 - F(h^* - \varepsilon|h^* - \varepsilon)$. If

$$F_1(h^*|h^*) + F_2(h^*|h^*) < \frac{1}{d - c} \tag{10}$$

then the first effect dominates, and type $h^* - \varepsilon$ will strictly prefer D after the perturbation, as required by stability. But (10) follows from (6). As usual, the stability condition guarantees intuitive comparative statics. By (9) and (10), an increase in $d - c$ will lead to more aggressive behavior (a reduction in equilibrium h^*).

If types are independent, then $F_2 \equiv 0$ and (6) simply requires that the density of the random variable η_i is sufficiently spread out, i.e., that there is "enough uncertainty" about types. Now suppose types are affiliated. This impacts the stability of the equilibrium via the expression $F_1(h^*|h^*) + F_2(h^*|h^*)$. There are two contradictory effects. On the one hand, affiliation causes type

h^* to think the opponent is likely to be similar to himself, so $F_1(h^*|h^*)$ is large. This effect makes uniqueness less likely. On the other hand, affiliation causes $F_2(h^*|h^*)$ to be negative. This effect makes uniqueness more likely. While the first effect is easy to understand, in terms of concentrating the density of types in a smaller area, the second effect is more subtle. Intuitively, in the above stability argument, affiliation causes type $h^* - \varepsilon$ to be less pessimistic about the opponent's hostility than type h^*, making him more likely to prefer D. That is, the best response curves are more likely to have the slopes that guarantee stability and uniqueness.

4 Strategic Substitutes

Actions are strategic substitutes when $d < c$. In this case, we need to make sufficient assumptions to guarantee that (3) holds, otherwise a cutoff equilibrium may not exist.

4.1 The uniqueness result

Our main result for the case of strategic substitutes is the following.

Theorem 4 *Suppose $d < c$ and for all $x, s, t \in (\underline{\eta}, \bar{\eta})$,*

$$F_1(s|t) - F_2(s|t) < \frac{1}{c-d} \tag{11}$$

and

$$F_1(s|x) - F_2(x|t) < \frac{1}{c-d}. \tag{12}$$

There is a unique BNE. This BNE is a cutoff equilibrium.

In the appendix, we prove that all BNE must be cutoff equilibria. The inequality (11) implies

$$(c-d)F_2(s|t) > (c-d)F_1(s|t) - 1 \geq -1.$$

Therefore, (3) holds, so a cutoff equilibrium exists by Proposition 2. To complete the proof of Theorem 4, we only need to show that there can be at most one cutoff equilibrium.

It again suffices to show that the slope of the best response function β_i is less than one in absolute value (Vives [20]). Equation (4) implies

$$\begin{aligned} 1+\beta_i'(x) &= 1+\frac{(d-c)F_1(x-k_j|\beta_i(x)-k_i)}{1-(d-c)F_2(x-k_j|\beta_i(x)-k_i)} \\ &= \frac{1+(c-d)\left(F_2(x-k_j|\beta_i(x)-k_i)-F_1(x-k_j|\beta_i(x)-k_i)\right)}{\Psi_2^i(x,\beta_i(x))} \\ &> 0 \end{aligned}$$

The inequality follows from (3) and (11). Since we already know that $\beta_i'(x) < 0$ when $d < c$, we conclude that $-1 < \beta_i'(x) < 0$. Thus, the cutoff equilibrium is unique. The unique BNE is, by familiar arguments, the unique interim rationalizable outcome.

If the inequality (11) holds, then (3) holds, so a cutoff equilibrium exists, and since the slope of the best response function is less than one in absolute value, there is only one cutoff equilibrium. However, this does not quite guarantee that no non-cutoff equilibrium exists. If types are affiliated, then when player 1 is a hostile type he expects player 2 to be a hostile type as well. When player 1 is less hostile, he expects player 2 to be less hostile. This opens up the possibility of coordination on non-monotonic strategies. Player 1's type h_1 backs down, expecting player 2 to be aggressive. But type $h_1' < h_1$, although he is inherently less hostile, may behave more aggressively, because his expectations about player 2 are different from the expectations of type h_1. (Clearly, this cannot happen if types are independent, or if types are affiliated but actions are strategic complements). Mathematically, inequality (12) (which has no analogue in the case of strategic complements) rules out such non-monotonicities (see the Appendix).

4.2 Diagrammatic illustration

We will illustrate Theorem 4 diagrammatically. Again let $Q(\eta) \equiv F(\eta|\eta)$. By calculus,

$$\begin{aligned}
\frac{\bar{\eta}-\underline{\eta}}{c-d} &= \int_{\underline{\eta}}^{\bar{\eta}} \left(\frac{1}{c-d}\right) \, d\eta \\
&> \int_{\underline{\eta}}^{\bar{\eta}} \{F_1(\eta|\eta) - F_2(\eta|\eta\} \, d\eta \geq \int_{\underline{\eta}}^{\bar{\eta}} \{F_1(\eta|\eta) + F_2(\eta|\eta\} \, d\eta \\
&= \int_{\underline{\eta}}^{\bar{\eta}} Q'(\eta) \, d\eta = Q(\bar{\eta}) - Q(\underline{\eta}) = 1
\end{aligned}$$

where the first inequality is due to (11) and the second is due to the fact that $F_2(\eta|\eta) \leq 0$. Therefore, for each $i \in \{1, 2\}$,

$$\bar{h}_i - \underline{h}_i = \bar{\eta} - \underline{\eta} > c - d.$$

This implies that either $\bar{h}_i > c - d$ or $\underline{h}_i < 0$ (or both). That is, for each player, there is a positive probability of being a dominant strategy type. Figure 3 illustrates the case where the support of each player's types includes both dominant strategy hawks and dominant strategy doves, so the unique equilibrium must be interior. Thus, player i's equilibrium cut-off point solves (5). Since $-1 < \beta_i'(x) < 0$ for all $x \in [\underline{h}_j, \bar{h}_j]$, the two best-response functions cross exactly once, as shown in Figure 3.

With strategic substitutes the presence of both kinds of dominant strategy types is not necessary for an interior equilibrium, however. Suppose the two players are symmetric, and each can be a dominant strategy hawk or an opportunistic type, but not a dominant strategy dove. The unique equilibrium must be symmetric and interior. Indeed, while the dominant strategy hawks surely play H, not all opportunistic types can play H in equilibrium, because if they did, the opportunistic types would be better off switching to D. Thus, there is an interior cutoff point. Corner equilibria can occur with asymmetric players. If, for example, player 1 cannot be a dominant strategy hawk, while player 2 cannot be a dominant strategy dove, in the unique equilibrium all of player 1's types play D, all of player 2's types play H.

4.3 The uniform independent case

Suppose η_1 and η_2 are independently drawn from a uniform distribution with support $[\underline{\eta}, \bar{\eta}]$. Conditions (11) and (12) both reduce to

$$\bar{\eta} - \underline{\eta} > c - d \tag{13}$$

(c.f. inequality (8) for the case of strategic complements). If (13) holds then a unique BNE exists. This BNE may be at a corner, e.g., if k_i is high then player i might choose H with probability one. If the equilibrium is interior, it can be computed exactly as in Section 3.4, by first substituting (7) into (5), and then solving explicitly for h_1^* and h_2^*. The solutions are, again, $h_1^* = A + (\bar{\eta} - \underline{\eta}) B$ and $h_2^* = A + (d - c)B$. But now we have $d < c$, so if $k > 0$ then $h_1^* > h_2^*$. Thus, with strategic substitutes, the ex ante less hostile player 1 is intimidated and chooses a high cutoff point, while the ex ante more hostile player 2 is emboldened and chooses a low cutoff point. Recall that with strategic complements (Section 3.4), the situation was the opposite: the ex ante most hostile player chose the highest cutoff point.

4.4 Discussion

It is interesting to compare Theorems 3 and 4. If types are independent, then $F(s|t)$ is independent of its second argument, so (6), (11) and (12) all reduce to the same inequality:

$$F_1(s|t) < \left| \frac{1}{d - c} \right| \tag{14}$$

for all $s, t \in (\underline{\eta}, \bar{\eta})$. Thus, with independent types, (14) is a sufficient condition for uniqueness of equilibrium, whether actions are strategic complements or substitutes. However, if types are affiliated, so that $F_2(s|t) < 0$, then the hypothesis of Theorem 4 becomes harder to satisfy, because $F_2(s|t)$ enters with a *negative* sign in (11) and (12).

When actions are strategic substitutes, opportunistic types want to *mis*-match the opponent's action. Opportunistic types with h_i close to $c - d$ are almost indifferent between H and D when the opponent plays D. But these "almost dominant strategy doves" will back down and play D if there is positive probability that the opponent is a dominant strategy hawk. This encourages opportunistic types who are "almost dominant strategy hawks" to play H, which in turn causes "almost-almost dominant strategy doves" to back down, and so on. But this spiral may be too weak to determine the actions of all types when types are affiliated, since "almost dominant strategy hawks" will think it quite unlikely that the opponent is an "almost dominant strategy dove", and so on. Thus, in games with strategic substitutes, affiliation promotes multiple equilibria.

5 Global Games

In this section we consider the global games information structure of Carlsson and van Damme [8]. For simplicity, we start by assuming the players are symmetric ex ante. Thus, $k_1 = k_2 = 0$, and player i's type is $h_i = \eta_i$. Types are derived from a common shock θ plus idiosyncratic shocks. The common shock θ has a density which is strictly positive, continuously differentiable and bounded on its support $\Theta \equiv [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}$. Player i's type is $\eta_i = \theta + \varepsilon e_i$, where εe_i is an idiosyncratic shock. Here e_i is a random variable and $\varepsilon > 0$ is a fixed parameter which calibrates the importance of the idiosyncratic shocks. The vector (e_1, e_2) is independent of θ and admits a continuous, symmetric joint density. The support of e_i is $[-\frac{1}{2}, \frac{1}{2}]$. Player i knows his own type but not the opponent's. Neither player knows the true θ.

In this section, the distribution $F^{\varepsilon}(\eta_j|\eta_i)$ of η_j conditional on η_i, where $i \neq j$, is indexed by ε. Let ψ^{ε} denote the density of $\eta_i - \eta_j = \varepsilon e_i - \varepsilon e_j$. It can be shown (see Van Zandt and Vives [19]) that Assumption 1 holds when ε is small.

Carlsson and van Damme [8] show that a unique BNE exists if ε is small enough, so there is very little idiosyncratic noise and the players' types are highly correlated. Unlike Carlsson and van Damme [8], we assume each player knows his own payoff function (in the Carlsson and van Damme [8] model, payoffs depend on θ directly, whereas we assume player i's payoff function only depends on θ via h_i). We will show that if ε is small then our sufficient condition for a unique BNE holds if actions are strategic complements, but not if actions are strategic substitutes.

Lemma 5 *For any η_i there is $k(\eta_i)$ such that for all sufficiently small $\varepsilon > 0$,*

$$|F^{\varepsilon}(\eta_j|\eta_i) - \int_{t \leq \eta_j - \eta_i} \psi^{\varepsilon}(t)dt| \ \leq \ k(\eta_i)\varepsilon.$$

Lemma 5 follows from the argument of Lemma 4.1 in Carlsson and van Damme [8]. The proof is omitted.

Proposition 6 $F_1^\varepsilon(\eta_j|\eta_i) + F_2^\varepsilon(\eta_j|\eta_i) \to 0$ *as* $\varepsilon \to 0$, *for any* η_i, η_j.

Proof. Fix ε, η_i and η_j and define the function R_ε as follows:

$$R_\varepsilon(\delta) \equiv F^\varepsilon(\eta_j + \delta|\eta_i + \delta)$$

Then R_ε is a continuously differentiable function of δ, with derivative

$$R'_\varepsilon(\delta) = F_1^\varepsilon(\eta_j + \delta|\eta_i + \delta) + F_2^\varepsilon(\eta_j + \delta|\eta_i + \delta) \tag{15}$$

We need to show that $R'_\varepsilon(0) \to 0$ as $\varepsilon \to 0$. Fix $\delta > 0$ and $\varepsilon > 0$. Lemma 5 implies that

$$|R_\varepsilon(\delta) - R_\varepsilon(0)| \le (k(\eta_i + \delta) + k(\eta_i))\,\varepsilon \tag{16}$$

By the mean value theorem, there is $\delta_\varepsilon \in (0, \delta)$ such that

$$|R_\varepsilon(\delta) - R_\varepsilon(0)| = |R'_\varepsilon(\delta_\varepsilon)| \times \delta \tag{17}$$

Combining (16) and (17), holding $\delta > 0$ fixed and letting $\varepsilon \to 0$, yields $R'_\varepsilon(\delta_\varepsilon) \to 0$. Since $0 < \delta_\varepsilon < \delta$ where δ is arbitrary, and $R'_\varepsilon(\delta)$ is a continuous function of δ for each $\varepsilon > 0$, we have $R'_\varepsilon(0) \to 0$. ■

Proposition 6 implies that the inequality (6) holds for $\varepsilon > 0$ small enough. Hence, by Theorem 3 there is a unique BNE if actions are strategic complements and ε is small.

We will now show that the sufficient condition for uniqueness in games with strategic substitutes (Theorem 4) is in general *not* satisfied when ε is small. To show this, we consider the case where all distributions are uniform. If player i draws $\eta_i \in [\underline{\theta} + \varepsilon, \bar{\theta} - \varepsilon]$, then his posterior beliefs about θ are given by a uniform distribution on $[\eta_i - \varepsilon, \eta_i + \varepsilon]$. Player i's beliefs about η_j are given by a symmetric, triangular distribution around η_i with support $[\eta_i - 2\varepsilon, \eta_i + 2\varepsilon]$. If $s, t \in [\underline{\theta} + \varepsilon, \bar{\theta} - \varepsilon]$ then

$$F^\varepsilon(t|s) = \begin{cases} 1 & \text{if} \quad t \ge s + 2\varepsilon \\ 1 - \frac{1}{2}\left(1 - \frac{t-s}{2\varepsilon}\right)^2 & \text{if} \quad s \le t \le s + 2\varepsilon \\ \frac{1}{2}\left(1 - \frac{s-t}{2\varepsilon}\right)^2 & \text{if} \quad s - 2\varepsilon \le t \le s \\ 0 & \text{if} \quad t \le s - 2\varepsilon \end{cases}$$

which implies

$$F^{\varepsilon}(t|s) + F^{\varepsilon}(s|t) = 1. \tag{18}$$

We also compute

$$F_1^{\varepsilon}(t|s) = \begin{cases} 0 & \text{if} \quad t > s + 2\varepsilon \\ \frac{1}{4\varepsilon^2}\left(s - t + 2\varepsilon\right) & \text{if} \quad s \le t \le s + 2\varepsilon \\ \frac{1}{4\varepsilon^2}\left(t - s + 2\varepsilon\right) & \text{if} \quad s - 2\varepsilon \le t \le s \\ 0 & \text{if} \quad t \le s - 2\varepsilon \end{cases}$$

Notice that

$$F_1^{\varepsilon}(t|s) = F_1^{\varepsilon}(s|t). \tag{19}$$

Differentiating (18) with respect to s yields[5]

$$F_2^{\varepsilon}(t|s) + F_1^{\varepsilon}(s|t) = 0. \tag{20}$$

For $|s - t| < 2\varepsilon$ we have, using (19) and (20),

$$F_1^{\varepsilon}(s|t) - F_2^{\varepsilon}(s|t) = 2F_1^{\varepsilon}(s|t) = \frac{1}{2\varepsilon^2}\left(|s - t| + 2\varepsilon\right)$$

which reaches a maximum $2/\varepsilon$ when $|s - t| = 2\varepsilon$. Also,

$$F_1^{\varepsilon}(s|x) - F_2^{\varepsilon}(x|t) = F_1^{\varepsilon}(s|x) + F_1^{\varepsilon}(t|x)$$

by (20), which also reaches a maximum $2/\varepsilon$. The inequalities (11) and (12) require

$$\frac{2}{\varepsilon} < \frac{1}{c - d}$$

which is certainly not satisfied when ε is small. Therefore, when ε is small the hypothesis of Theorem 4 is violated. Instead, Theorem 4 implies the following.

Proposition 7 *If distributions are uniform and $c > d$ then there is a unique BNE for any $\varepsilon > 2\,(c - d)$.*

[5] Combining (19) and (20) yields $F_1(t|s) + F_2(t|s) = 0$ so (6) certainly holds. Therefore, for the case of strategic complements, there is a unique BNE for *any* $\varepsilon > 0$, not just for small ε. But here we are considering strategic substitutes.

Thus, Theorem 4 implies that when actions are strategic substitutes, a unique BNE exists if ε is big enough. That is, there has to be enough idiosyncratic uncertainty. When $\varepsilon > 2(c-d)$, type $h_i = 0$ the "weakest" opportunistic type puts positive probability on player j being a dominant strategy hawk. This implies that type 0 and types just above 0 play D. Similarly, type $c-d$ the "toughest" opportunistic of player i puts positive probability on player j being a dominant strategy dove. This implies type $c-d$ and types just below $c-d$ play H. The process of iterated deletion of dominated strategies can be completed to obtain a unique equilibrium. Uniqueness here is a stark consequence of "deterrence by fear." The possibility of conflict with dominant strategy hawks makes "weak" opportunistic types behave dovishly, which in turn encourages hawkish behavior by "tough" opportunistic types, etc.

We will end by considering the possibility of ex ante asymmetries. If player 2, say, has an a priori reputation for hostility, and actions are strategic substitutes, then it is intuitive that player 1 will be quite cautious, which will embolden player 2, making player 1 even more cautious, etc. The ex ante asymmetry may feed on itself and create a unique BNE, where player 2 will have his way while player 1 retreats. For this process to be triggered, the toughest opportunistic type $c-d$ of player 1 must put significant probability on facing a dominant strategy hawk of player 2. If ε is small, this will be the case and there is a unique BNE. This is the case of "small noise" and is the analog of the global games argument with strategic substitutes. It is formalized in the following proposition.

Proposition 8 *Suppose distributions are uniform and $c > d$. Also suppose $k_2 = k > 2\varepsilon > 0 = k_1$ (so player 2 is* ex ante *significantly more hostile than player 1). Moreover, suppose $k + \underline{\theta} + \varepsilon < 0$ and $\bar{\theta} - \varepsilon > c - d$ (so dominant strategy hawks and doves exist for each player). There is a unique BNE, where player 1 plays H iff $h_1 \geq c-d$ and player 2 plays H iff $h_2 \geq 0$.*

The proof is in the Appendix.

To complete the discussion of strategic substitutes, we show that if there is neither significant idiosyncratic uncertainty (i.e. ε is small) nor significant ex ante asymmetry (i.e. k is small), then multiple equilibria exist. First, if ε is small, the toughest type $c-d$ of player 1 does not put positive probability on player 2 being a dominant strategy dove. As k is small, type $c-d$ puts significant probability on player 2 being an opportunistic type. Hence, his optimal strategy can depend on whether the opportunistic types of player 2 he faces plays D or H. This can imply the possibility of multiple equilibria including in the scenario when there is no asymmetry:

Proposition 9 *Suppose distributions are uniform and $c > d$. Also suppose $k_2 = k > 0 = k_1$ and $\varepsilon < (c-d)/8$. If $k < 2\varepsilon\left(1 - \frac{4\varepsilon}{c-d}\right)$ then multiple BNE exist.*

The proof is in the Appendix. Notice that $2\varepsilon\left(1 - \frac{4\varepsilon}{c-d}\right) > 0$ as long as $\varepsilon < (c-d)/8$.

To summarize our discussion of games with strategic substitutes, a unique equilibrium exists if either ε or k (or both) are large. Intuitively, deterrence by fear leads to a unique equilibrium if the fear of conflict triggers dovish behavior among the "weak", while opportunism triggers aggression among the "strong". For this to occur, either there must be enough idiosyncratic uncertainty (ε is big), so neither player is convinced that the opponent is similar to himself, or it must be common knowledge that one player is inherently more hostile (k is big). If both ε and k are small then multiple equilibria exist, and the playerθs face a genuine coordination problem.

6 Conclusion

Formal models are most useful if they make unambiguous predictions. Well-known games such as chicken and stag-hunt have multiple Nash equilibria when the payoff matrix is common knowledge. But the assumption that payoffs are common knowledge cannot even be approximately satisfied in practise. How can the leader of a country be sure of the enemy's cost and benefits of going to war? When payoffs are uncertain, equilibria typically have a "cutoff" property: each player is aggressive only when his hostility exceeds the cutoff. Under quite general conditions, these cutoff equilibria are unique. The sufficient conditions for uniqueness of equilibrium are equivalent to the "small slope" conditions, on appropriately defined best-response functions, familiar from the Industrial Organization literature.

As is well known, the slope conditions are useful for comparative statics. In Baliga, Lucca and Sjöström (2009), we considered how the domestic political system affects the leader's payoffs, and thereby influences the equilibrium probability of conflict. In order to obtain a unique equilibrium, we made rather stringent assumptions, including independent types. The current paper shows that uniqueness may prevail under much more general conditions.

In certain situations, multiple equilibria cannot be ruled out. In particular, multiple equilibria can exist if there is "not enough uncertainty". If actions are strategic complements then all equilibria must at least be cutoff equilibria. The equilibrium set might be analyzed using techniques of monotone comparative statics, along the lines of, say, Bertrand competition with product differentiation. This is left for future research. However, if actions are strategic substitutes, then non-cutoff equilibria may exist, and these might represent an "irreducible" multiplicity which may limit the usefulness of the model.

7 Appendix

7.1 Proof of non-existence of non-cutoff equilibria

To complete the proofs of Theorem 3 (strategic complements) and Theorem 4 (strategic substitutes), we will show that, under the hypotheses of these theorems, any BNE must be a cutoff equilibrium. Since the proofs for strategic complements and substitutes are in part parallel, it is convenient to combine them. Recall that the hypotheses of either theorem imply that (3) holds.

It suffices to show that *one* player must use a cutoff strategy, because (3) guarantees that the best response to a cutoff strategy must be a cutoff strategy. Notice that a constant strategy (always D or always H) is a special case of a cutoff strategy, so if either player uses a constant strategy we are done. From now on, assume neither player uses a constant strategy: some types play D and some types play H.

For $i \in \{1, 2\}$, define

$$x_i \equiv \inf \{h_i : \sigma_i(h_i) = H\} \tag{21}$$

and

$$y_i \equiv \sup \{h_i : \sigma_i(h_i) = D\} \tag{22}$$

By definition, $x_i \leq y_i$. Notice that if $x_i = y_i$ then player i uses a cutoff strategy. We will show that there is always some player i such that $x_i = y_i$.

Recall that $\delta_j(h_i)$ denotes the probability that player $j \neq i$ plays D, conditional on player i's type being h_i. By definition of x_i, player i's type x_i

weakly prefers H, so (1) implies

$$x_i + (1 - \delta_j(x_i))(d - c) \geq 0 \tag{23}$$

Similarly, player i's type y_i weakly prefers D, so

$$y_i + (1 - \delta_j(y_i))(d - c) \leq 0 \tag{24}$$

By definition, $\sigma_j(h_j) = D$ for all $h_j < x_j$ and $\sigma_j(h_j) = H$ for all $h_j > y_j$. Therefore, for $j \neq i$,

$$F(x_j - k_j | x_i - k_i) \leq \delta_j(x_i) \leq F(y_j - k_j | x_i - k_i) \tag{25}$$

and

$$F(x_j - k_j | y_i - k_i) \leq \delta_j(y_i) \leq F(y_j - k_j | y_i - k_i). \tag{26}$$

From now on we consider the two cases separately.

Strategic complements. Without loss of generality, assume

$$y_1 - k_1 \geq y_2 - k_2. \tag{27}$$

We will show that $y_1 = x_1$ so player 1 uses a cutoff strategy..

Since $c < d$, (23) and the first inequality of (25) (with $i = 1$) imply

$$x_1 + (1 - F(x_2 - k_2 | x_1 - k_1))(d - c) \geq 0 \tag{28}$$

Similarly, (24) and the first inequality of (26) imply

$$y_1 + (1 - F(y_2 - k_2 | y_1 - k_1))(d - c) \leq 0 \tag{29}$$

Combining (28) and (29) yields

$$F(y_2 - k_2 | y_1 - k_1) - F(x_2 - k_2 | x_1 - k_1) \geq \frac{1}{d - c}(y_1 - x_1) \tag{30}$$

By (27), the inequality (30) implies

$$F(y_1 - k_1 | y_1 - k_1) - F(x_1 - k_1 | x_1 - k_1) \geq \frac{1}{d - c}(y_1 - x_1). \tag{31}$$

By the mean value theorem, there is $z \in (\underline{\eta}, \bar{\eta})$ such that

$$(F_1(z|z) + F_2(z|z))(y_1 - x_1) = F(y_1 - k_1 | y_1 - k_1) - F(x_1 - k_1 | x_1 - k_1)$$

and substituting this in (31) yields

$$\left(F_1(z|z) + F_2(z|z)\right)(y_1 - x_1) \geq \frac{1}{d-c}(y_1 - x_1)\,. \tag{32}$$

But we know that $y_1 \geq x_1$, and the hypothesis of Theorem 3 says that

$$F_1(z|z) + F_2(z|z) < \frac{1}{d-c}$$

Therefore, (32) implies $y_1 = x_1$. Therefore, the proof of Theorem 3 is complete.

Strategic substitutes. Assume without loss of generality that

$$x_1 - k_1 \leq x_2 - k_2 \tag{33}$$

Since $c > d$, (23) and the second inequality of (25) imply

$$x_i + (1 - F(y_j - k_j|x_i - k_i))(d-c) \geq 0 \tag{34}$$

By the same reasoning,

$$y_i + (1 - F(x_j - k_j|y_i - k_i))(d-c) \leq 0 \tag{35}$$

Set $i = 1$ in (34) and (35) and combine the two inequalities to get

$$F(y_2 - k_2|x_1 - k_1) - F(x_2 - k_2|y_1 - k_1) \geq \frac{1}{c-d}(y_1 - x_1)\,. \tag{36}$$

Set $i = 1$ in (34) and $i = 2$ in (35) and combine the two inequalities to get

$$F(y_2 - k_2|x_1 - k_1) - F(x_1 - k_1|y_2 - k_2) \geq \frac{1}{c-d}(y_2 - x_1)\,. \tag{37}$$

Set $i = 2$ in (34) and $i = 1$ in (35) and combine the two inequalities to get

$$F(y_1 - k_1|x_2 - k_2) - F(x_2 - k_2|y_1 - k_1) \geq \frac{1}{c-d}(y_1 - x_2)\,. \tag{38}$$

Now we need to consider several cases.

Case A: $y_1 - k_1 \geq y_2 - k_2$. Using this inequality and (33), the inequality (36) implies

$$F(y_1 - k_1|x_1 - k_1) - F(x_1 - k_1|y_1 - k_1) \geq \frac{1}{c-d}(y_1 - x_1) \tag{39}$$

Applying the mean value theorem twice,

$$\begin{aligned} & F(y_1 - k_1|x_1 - k_1) - F(x_1 - k_1|y_1 - k_1) \\ = \; & [F(y_1 - k_1|x_1 - k_1) - F(x_1 - k_1|x_1 - k_1)] + [F(x_1 - k_1|x_1 - k_1) - F(x_1 - k_1|y_1 - k_1)] \\ = \; & F_1(t|x_1 - k_1)\,(y_1 - x_1) + F_2(x_1 - k_1|s)(x_1 - y_1) \\ = \; & (F_1(t|x_1 - k_1) - F_2(x_1 - k_1|s))\,(y_1 - x_1) \end{aligned}$$

for some $s, t \in \left(\underline{\eta}, \overline{\eta}\right)$. Substituting this in (39) yields

$$(F_1(t|x_1 - k_1) - F_2(x_1 - k_1|s))\,(y_1 - x_1) \geq \frac{1}{c-d}\,(y_1 - x_1)$$

But we know that $y_1 \geq x_1$, so the hypothesis of Theorem 4 implies $x_1 = y_1$. (Notice that inequality (12) was required for this step.)

Case B: $y_1 - k_1 < y_2 - k_2$. In this case, the inequality (38) implies

$$F(y_2 - k_2|x_2 - k_2) - F(x_2 - k_2|y_2 - k_2) \geq \frac{1}{c-d}\,(y_1 - x_2)\,. \tag{40}$$

Case B has three subcases.

Sub-case B1: $y_1 \geq y_2$. Here, (40) implies

$$F(y_2 - k_2|x_2 - k_2) - F(x_2 - k_2|y_2 - k_2) \geq \frac{1}{c-d}\,(y_2 - x_2)\,.$$

This is symmetric with inequality (39), but for player 2 instead of player 1. Applying the mean-value theorem therefore yields $x_2 = y_2$.

Sub-case B2: $y_2 > y_1$ and $k_2 \geq k_1$. Here, from (37) we obtain

$$F(y_2 - k_1|x_1 - k_1) - F(x_1 - k_1|y_2 - k_1) \geq \frac{1}{c-d}\,(y_2 - x_1)$$

This is symmetric with inequality (39), but with y_2 replacing y_1. Applying the mean-value theorem therefore yields $x_1 = y_2$. But this contradicts $y_2 > y_1 \geq x_1$, so sub-case B2 is impossible.

Sub-case B3: $y_2 > y_1$ and $k_2 < k_1$. Here, from (38), we obtain

$$F(y_1 - k_1|x_2 - k_1) - F(x_2 - k_1|y_1 - k_1) \geq \frac{1}{c-d}\,(y_1 - x_2)$$

This is symmetric with inequality (39), but with x_2 replacing x_1. Applying the mean-value theorem therefore yields $x_2 = y_1$. Substituting this into (38), we obtain

$$F(y_1 - k_1|y_1 - k_2) - F(y_1 - k_2|y_1 - k_1) \geq 0. \tag{41}$$

However, recall that $y > x$ implies $F(y|x) - F(x|y) > 0$. Since $y_1 - k_2 > y_1 - k_1$, the inequality (41) cannot hold, so sub-case B3 is impossible.

Thus, in both case A and case B, some player uses a cutoff strategy. Therefore, the proof of Theorem 4 is complete.

7.2 Proof of Proposition 8

We use the triangular distribution to determine the beliefs of opportunistic types.

Conditional on θ, player 1's type $h_1 = \eta_1$ is uniformly distributed on $[\theta - \varepsilon, \theta + \varepsilon]$, while player 2's type $h_2 = k + \eta_2$ is uniformly distributed on $[k + \theta - \varepsilon, k + \theta + \varepsilon]$. A strategy $\sigma_i : [\underline{h}, \bar{h}] \to \{H, D\}$ specifies a choice $\sigma_i(h_i) \in \{H, D\}$ for each type h_i. If player i's type h_i thinks player j will choose D with probability $\delta_j(h_i)$, then his net gain from choosing H instead of D is

$$h_i + (d - c)(1 - \delta_j(h_i)). \tag{42}$$

Remark 10 *If $\eta_1 < -k - 2\varepsilon$, then player 1 knows that $h_2 = k + \eta_2 < 0$, so player 2 must be a dominant strategy dove. If $\eta_1 > c - d - k + 2\varepsilon$, then player 1 knows that $h_2 = k + \eta_2 > c - d$, so player 2 must be a dominant strategy hawk. If $\eta_2 < -2\varepsilon$, then player 2 knows that $h_1 = \eta_1 < 0$, so player 1 must be a dominant strategy dove. If $\eta_2 > c - d + 2\varepsilon$, then player 2 knows that $h_1 = \eta_1 > c - d$, so player 1 must be a dominant strategy hawk.*

Consider the process of eliminating (interim) dominated strategies. In the first "round" of elimination, D is eliminated for dominant strategy hawks ($h_i \geq c - d$) and H for dominant strategy doves. Now consider the second round.

By hypothesis, $k > 2\varepsilon$. When $c - d - k + 2\varepsilon < \eta_1 < c - d$, player 1 knows that player 2 is a dominant strategy hawk (see Remark 10). Hence, H can be eliminated for player 1. Indeed, even if η_1 is slightly below $c - d - k + 2\varepsilon$, H can be eliminated, because player 2 is highly likely to be a dominant strategy hawk. Let η_1' be the largest η_1 such that H cannot be eliminated for player 1's type η_1 in round 2. Notice that $\eta_1' < c - d - k + 2\varepsilon$. Now if $h_2 = \eta_2 + k$ is slightly below $c - d$, then player 2 knows that player 1 has a positive probability of having a type between η_1' and $c - d$. Such types of player 1 had H removed in round 2 of the elimination of interim dominated strategies. Therefore, in round 3, D must be eliminated for types of player 2 slightly below $c-d$. Let η_2' be the largest η_2 such that D cannot be eliminated for player 2 in round 3. In round 4, player 1's types slightly below η_1' will be able to remove D, etc.

We claim that this process must eventually eliminate H for all $h_1 \in (0, c-d)$, and D for all $h_2 \in (0, c-d)$. If this were not true, then the process cannot proceed below some $h_1^* > 0$ and $h_2^* > 0$. Now, $h_2^* > h_1^* + k - 2\varepsilon$, otherwise type h_1^* knows that $h_2 \geq h_2^*$, and all such types have eliminated D, but then H must be eliminated for types slightly below h_1^*.

Consider player 2's type $h_2^* = \eta_2^* + k$. He knows that $h_1 = \eta_1 \leq h_2^* - k + 2\varepsilon < c - d - k + 2\varepsilon \leq c - d$. Now H has been eliminated for all $h_1 \in (h_1^*, c-d)$, and according to type h_2^*, the probability that player 1's type lies in this interval is $1 - F^\varepsilon(h_1^* | h_2^* - k)$. Therefore, if D cannot be eliminated for type h_2^*, it must be the case that type h_2^* weakly prefers D when the opponent uses H with probability at most $F^\varepsilon(h_1^* | h_2^* - k)$. This implies

$$h_2^* + (d-c) F^\varepsilon(h_1^* | h_2^* - k) \leq 0$$

By a similar argument, if H cannot be eliminated for type h_1^*, then type h_1^* must prefer H when the opponent uses H with probability at least $1 - F^\varepsilon(h_2^* - k | h_1^*)$. That is,

$$h_1^* + (d-c)\left(1 - F^\varepsilon(h_2^* - k | h_1^*)\right) \geq 0$$

Subtracting the first inequality from the second yields

$$h_1^* - h_2^* \geq (c-d)\left(1 - F^\varepsilon(h_2^* - k | h_1^*) - F^\varepsilon(h_1^* | h_2^* - k)\right) = 0$$

where the last equality uses 18. However, this contradicts $h_2^* > h_1^* + k - 2\varepsilon$.

7.3 Proof of Proposition 9

By hypothesis, $\varepsilon < (c-d)/8$. Suppose $k < 2\varepsilon\left(1 - \frac{4\varepsilon}{c-d}\right)$. Notice the right hand side of the inequality is strictly positive, by the hypothesis. We will first show that the process of iterated elimination of dominated strategies stops after the first "round" of elimination.

Consider type $h_1 = c - d$. In the second "round", he cannot rule out the possibility that player 2 will choose D when $h_2 < c - d$. Moreover, the event that $h_2 = \eta_2 + k < c - d$ has positive probability when $\eta_1 = h_1 = c - d$

and $k < 2\varepsilon$. Therefore, we cannot eliminate H for type $h_1 = c - d$. Player 1's type $h_1 = 0$ cannot eliminate H, because H has not been eliminated for $h_2 > 0$. Since neither of the "boundary" opportunistic types can eliminate H, no opportunistic type at all can eliminate H. Clearly, they cannot eliminate D either. Thus, no opportunistic type of player 1 can eliminate any action in round 2. A similar argument applies to player 2.

We now show that multiple equilibria exist.

Let $h^* = \frac{(c-d)(2\varepsilon-k)^2}{8\varepsilon^2}$. Notice that

$$\frac{(c-d)(2\varepsilon-k)}{8\varepsilon^2} > 1$$

because, by hypothesis, $\varepsilon < (c-d)/8 < (c-d)/4$ and $k < 2\varepsilon\left(1 - \frac{4\varepsilon}{c-d}\right) < 2\varepsilon$. Therefore,

$$\begin{aligned} h^* + k - 2\varepsilon &= \frac{(c-d)(2\varepsilon-k)^2}{8\varepsilon^2} - (2\varepsilon - k) \\ &= \left(\frac{(c-d)(2\varepsilon-k)}{8\varepsilon^2} - 1\right)(2\varepsilon - k) > 0 \end{aligned}$$

Also,

$$h^* + k + 2\varepsilon = \frac{(c-d)(2\varepsilon-k)^2}{8\varepsilon^2} + k + 2\varepsilon < \frac{c-d}{2} + 4\varepsilon < c - d$$

as, by hypothesis, $\varepsilon < (c-d)/8$ and $k < 2\varepsilon\left(1 - \frac{4\varepsilon}{c-d}\right) < 2\varepsilon$.

Consider the following strategies: player 1 plays D if $h_1 \leq h^*$; player 2 plays D if either $h_2 \leq 0$ or $h_2 \in [h^*, c-d]$. Notice that player 2 does not use a cutoff strategy.

Consider player 1 first. For player 1 of type h^*, the probability that player 2 plays H is $F^\varepsilon(h^* - k|h^*) = \frac{(2\varepsilon-k)^2}{8\varepsilon^2}$ and he is indifferent between H and D. Higher types are more aggressive and assess a lower probability that player 2 plays H. These types strictly prefer to play H and, by a symmetric argument, lower types prefer to play D.

We must also show player 2's strategy is a best response. For player 2, if he is a dominant strategy type, the specified strategy is clearly optimal. For $h_2 \in [h^* + k - 2\varepsilon, h^*]$,

$$\Pr\{h_1 < h^*|h_2\} = 1 - \frac{1}{8\varepsilon^2}\left((h_2 - k + 2\varepsilon) - h^*\right)^2 = \delta_1(h_2).$$

Substituting this into (42), the net gain from playing H rather than D becomes

$$h_2 + \frac{(d-c)}{8\varepsilon^2}\left(h_2 - h^* - k + 2\varepsilon\right)^2. \tag{43}$$

This is quadratic in h_2 and equals zero when $h_2 = h^*$. It reaches a maximum at

$$\hat{h} = h^* + k - 2\varepsilon + \frac{4\varepsilon^2}{c-d}$$

which is interior to the interval $[h^* + k - 2\varepsilon, h^*]$ because by assumption $k < 2\varepsilon\left(1 - \frac{2\varepsilon}{c-d}\right)$. In fact, (43) is clearly strictly positive for $h_2 \in [h^* + k - 2\varepsilon, h^*)$. For $h_2 \in [0, h^* + k - 2\varepsilon]$, player 2 knows his opponent plays D and then it is optimal to play H as (42) is equal to $h_2 \geq 0$. There is a similar argument for $h_2 \in (h^*, c-d]$ and so the entire $\Psi^2(h^*, h_2)$ picture is as in Figure 4. There is another equilibrium with the roles of players 1 and 2 reversed. There is also an equilibrium where player 2 plays H when $h_2 \geq 0$ and player 1 plays H when $h_1 \geq c-d$.[6]

[6] Let $h^* = (c-d)\left(1 - \frac{(2\varepsilon-k)^2}{8\varepsilon^2}\right)$. Notice that

$$\begin{aligned} h^* - k - 2\varepsilon &= (c-d)\left(1 - \frac{(2\varepsilon-k)^2}{8\varepsilon^2}\right) - 2\varepsilon - k \\ &> (c-d)/2 - 4\varepsilon > 0 \end{aligned}$$

as $2\varepsilon > k$ and $(c-d)/8 > \varepsilon$. Also, by $k < 2\varepsilon\left(1 - \frac{4\varepsilon}{c-d}\right)$,

$$\begin{aligned} h^* - k + 2\varepsilon &= (c-d)\left(1 - \frac{(2\varepsilon-k)^2}{8\varepsilon^2}\right) + 2\varepsilon - k \\ &< (c-d)\left(1 - \frac{(2\varepsilon-k)}{c-d}\right) + 2\varepsilon - k \\ &= c-d. \end{aligned}$$

Players' strategies are as follows: player 2 plays D iff $h_2 \leq h^*$; player 1 plays D iff $h_1 \leq 0$ or $h_1 \in [h^*, c-d]$.

Consider player 2 first. For player 2 of type h^*, the probability that player 1 plays H is $F(h^*|h^*-k) = 1 - \frac{(2\varepsilon-k)^2}{8\varepsilon^2}$ and he is indifferent between H and D. Higher types are more aggressive and assess a lower probability that player 1 plays H. These types strictly prefer to play H and, by a symmetric argument, lower types prefer to play D.

We must also show player 1's strategy is a best response. For player 1, if he is a dominant strategy type, the specified strategy is clearly optimal. For $h_1 \in [h^*, h^* + 2\varepsilon - k]$, $\Pr\{h_2 < h^*|h_1\} = \frac{1}{8\varepsilon^2}\left(h^* - k - (h_1 - 2\varepsilon)\right)^2 = \delta_2(h_1)$. Substituting this into (42), the net

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gain from playing H rather than D becomes

$$h_1 + (d - c) \left(1 - \frac{1}{8\varepsilon^2} \left(h^* - k - (h_1 - 2\varepsilon)\right)^2\right). \quad (44)$$

This is quadratic in h_1 and equals zero when $h_1 = h^*$. It reaches a minimum at

$$\hat{h} = h^* - k + 2\varepsilon - \frac{4\varepsilon^2}{c - d}$$

which is interior to the interval $[h^*, h^* + 2\varepsilon - k]$ as long as $k < 2\varepsilon \left(1 - \frac{2\varepsilon}{c-d}\right)$, which it is by assumption. In fact, (44) is clearly strictly negative for $h_1 \in (h^*, h^* + 2\varepsilon - k]$. For $h_1 \in (h^* + 2\varepsilon - k, c - d]$, player 1 knows his opponent plays H and then it is optimal to play D as (42) is negative. There is a similar argument for $h_1 \in [0, h^*)$.

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Figure 1: $\Psi^{i}(x,y)$

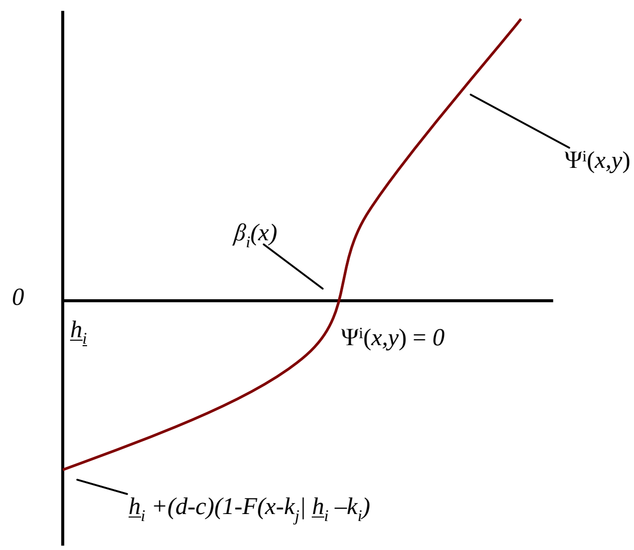

Figure 2: Equilibrium for Strategic Complements

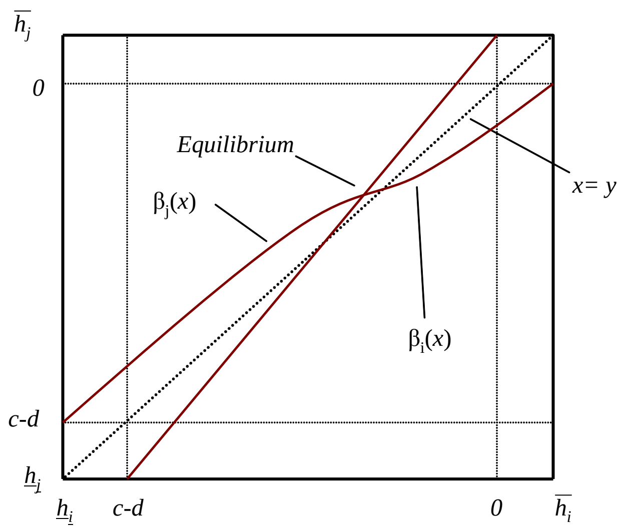

Figure 3: Equilibrium for Strategic Substitutes

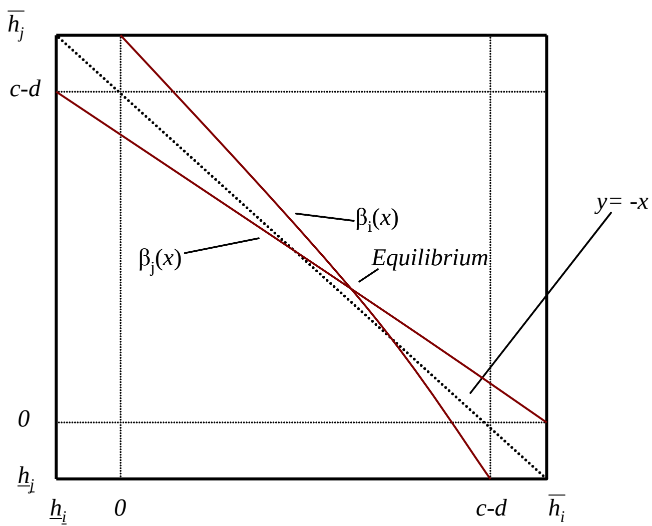

Figure 4: $\Psi^2(h^*,h_2)$