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DEFAULT AND RENEGOTIATION: A DYNAMIC MODEL OF DEBT*

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We analyze the role of debt in persuading an entrepreneur to pay out cash flows, rather than to divert them. In the first part of the paper we study the optimal debt contract—specifically, the trade-off between the size of the loan and the repayment—under the assumption that some debt contract is optimal. In the second part we consider a more general class of (nondebt) contracts, and derive sufficient conditions for debt to be optimal among these.

I. INTRODUCTION

Although there is a vast literature on capital structure, economists do not yet have a fully satisfactory theory of debt finance (or of the differences between debt and equity). One of the reasons for this is that debt is a security with several characteristics: a debtor typically promises a creditor a noncontingent payment stream, provides the creditor with the right to foreclose on the debtor's assets in a default state, and gives the creditor priority in bankruptcy. It is unclear whether all these characteristics are equally important, and whether they necessarily have to go together. In this paper we develop a model based on the second characteristic of debt—the foreclosure right—although our model implicitly has something to say about the other two characteristics as well.

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We consider an entrepreneur who needs funds from an investor (e.g., a bank) to finance an investment project. The project will on average generate returns in the future, but these returns accrue to the entrepreneur in the first instance, and cannot be allocated directly to the investor. We consider the stark—and extreme—case where the entrepreneur can "divert" or "steal" the project returns on a one-for-one basis. However, the entrepreneur cannot "steal" the assets underlying the project. Under these conditions we show that a debt contract of the following form has value. The entrepreneur promises to make a fixed stream of payments to the investor. As long as he makes these payments, the entrepreneur continues to run the project. However, if the entrepreneur defaults, the investor has the right to seize and liquidate the project assets. At this stage the entrepreneur and investor can renegotiate the contract.

Our model supposes symmetric information between the entrepreneur and investor both when the contract is written and once the relationship is under way. However, many of the variables of interest, such as project returns and asset liquidation value, are assumed not to be verifiable by outsiders, e.g., a court; hence contracts cannot be conditioned (directly) on these. The symmetry of information between the parties means that renegotiation of the debt contract following default is relatively straightforward to analyze. However, renegotiation does not necessarily lead to first-best efficiency. The reason is that situations can arise where even though the value to the entrepreneur of retaining assets exceeds their liquidation value, there is no credible way for the entrepreneur to compensate the investor for not liquidating the assets. The point is that the entrepreneur may not have sufficient current funds for such compensation (particularly if his loan default was involuntary), and while he may promise the investor a large fraction of future receipts, the investor will worry that when the time comes, she will not be able to get her hands on these: the entrepreneur will default again. Thus, inefficient liquidation may occur in equilibrium.

To simplify matters, we restrict attention to the case where the entrepreneur-investor relationship lasts for just two periods (or three dates). That is, we suppose that the entrepreneur requires funds at date 0, a return is realized at date 1, and if the project is continued, a further return is earned at date 2. We also assume that part or all of the project can be liquidated at date 1 and that project returns can be reinvested. Let the entrepreneur's wealth be w and the cost of the project be $I > w$. Then a debt contract is characterized by two numbers (P,T), $T \geq 0$, where $I - w + T$ is the amount the entrepreneur borrows at date 0 and P is the promised repayment at date 1. (It is easy to show that the entrepreneur will pay nothing at date 2.)

In Section III we explore the trade-off between P and T. The more the entrepreneur borrows at date 0 (the higher T is), the more he must repay at date 1; i.e., there is a positive relationship between the two variables. Each instrument has a different role to play, however. The advantage of a low value of P is that it strengthens the entrepreneur's position in good states of the world by giving him the right to continue using the assets in exchange for a small repayment. This prevents the investor from using her bargaining power to liquidate assets when they are worth a lot to the entrepreneur. The advantage of a high value of T is that it strengthens the entrepreneur's position in bad states of the world, i.e., default states, by giving him additional liquidity. This allows the entrepreneur to repurchase assets from the investor in the renegotiation process.

In general, it is optimal to use both instruments. However, in Propositions 1—3 we obtain sufficient conditions for just one instrument to be used. We show that, depending on the stochastic structure of the problem, the *fastest* debt contract or the *slowest* debt contract will be optimal. The fastest debt contract is one where $T = 0$; that is, the entrepreneur borrows the minimum amount necessary to finance the project (in other words, there is "maximum equity participation"). At the other extreme, the slowest debt contract is one where the entrepreneur borrows the maximum amount possible at date 0 and defaults with certainty at date 1 (in effect, the entrepreneur "rents" the assets between dates 0 and 1).

Sections II and III are based on the assumption that a debt contract is optimal for the entrepreneur and investor. In Section IV we examine this assumption. Are there other contracts that can solve the cash diversion problem with greater efficiency? In general, the answer is yes. One interpretation of a debt contract is that it provides the entrepreneur with the right to continue the project if he makes a prespecified payment at date 1. An alternative contract would give the investor an option (or right) to liquidate the project if *she* makes a prespecified payment at date 1. More complicated contracts may also be useful. For example, the right to continue the project could be a (stochastic) function of how much the entrepreneur pays. More generally, the entrepreneur and investor could agree to play a message game whereby the amount each party has to pay, and the allocation of the right to control the project assets, are functions of verifiable messages sent by the two parties at date 1.

In Section IV we show that, under some reasonable assumptions, the additional complexity provided by messages is unnecessary. That is, a debt contract is optimal within a large class of (message-game) contracts. The conditions required for this result are that reinvestment in the project at date 1 yields the same rate of return as the project itself, that is, the project exhibits constant returns to scale at date 1; and that the project returns at dates 1 and 2 and the liquidation value are positively related. (In fact, under these conditions, we show that the *fastest* debt contract is optimal.)

There is a simple intuition for the optimality of debt. Ex post, every dollar that the investor receives is a dollar that the entrepreneur cannot reinvest. Under the assumption that the project exhibits constant returns to scale at date 1, and that the key return and liquidation variables are correlated, it is desirable to maximize the entrepreneur's resources in "good" (high return) states of the world, and—given that the investor must be repaid—maximize the investor's payoff in "bad" (low return) states of the world. The reason is that this enables the entrepreneur to reinvest as much as possible when reinvestment is most valuable. Debt does a good job of achieving this since it puts a cap P on the investor's payoff by giving the entrepreneur the right to continue using the assets if he pays P. This cap will be binding in good states of the world, thus limiting the investor's payoff and maximizing the entrepreneur's resources. In contrast, a contract that, say, gives the investor the option to liquidate the project has exactly the opposite (and wrong) effect: the investor will buy out the entrepreneur when the project assets are worth a lot, which means that profitable reinvestment fails to occur.

We have visited some of the themes of this paper in previous work. Hart and Moore [1989] provide an early version of the model and a preliminary extension to the case of more than two periods. Unfortunately, the multiperiod case is far from straightforward except when there is perfect certainty. For an analysis of the multiperiod certainty case, and a discussion of its empirical implications for the maturity structure of debt contracts, see Hart and Moore [1994] and Hart [1995]. The former contains a variant of the model presented here: the entrepreneur can quit, that is, withdraw his human capital from the project, rather than divert the project returns.

The paper is organized as follows. The model is presented in Section II. Section III analyzes the optimal choice of P and T. Section IV considers more general contracts. Section V allows for the possibility of variable project scale at date 0. Finally, Section VI discusses the relationship of our work to the literature, and contains some concluding remarks.

II. The Model

We consider a risk-neutral entrepreneur who requires finance for an investment project at date 0. The project costs I, and the entrepreneur's initial wealth is $w < I$. There is a competitive supply of risk-neutral investors. The task for the entrepreneur is to design a payback agreement that persuades one of them to put up at least $(I - w)$ dollars.[1]

The project lasts two periods, with (uncertain) returns R_1 and R_2 being generated at dates 1 and 2. These returns are specific to this entrepreneur; that is, they cannot be generated without his cooperation. For simplicity, however, we ignore any actions taken by the entrepreneur to generate them; that is, the returns are produced simply by his being in place.

As emphasized in the Introduction, the project returns accrue to the entrepreneur in the first instance. Thus, the payback agreement must be designed to give the entrepreneur an incentive to hand over enough of these returns to the investor to cover her initial cost. We take the entrepreneur's and investor's discount rates both to be zero, which is also the market interest rate.

The investment funds are used to purchase assets which at date 1 have a second-hand or liquidation value $L > 0$, whose expectation EL is less than I. We suppose that the assets are worthless at date 2.

We also assume that any funds not paid over to the investor at date 1 can be reinvested in the project. These funds earn a rate of return equal to s between dates 1 and 2, where $1 \le s \le R_2/L$. That is, at worst reinvestment yields the market rate of interest, and at best it yields the same rate of return as the initial project itself. We allow both s and L to be random variables as of date 0 (along

1. We ignore agreements with several investors. But see Section VI.

with R_1 and R_2). Note that the assumption $1 \leq s \leq R_2/L$ implies that the project's going-concern value R_2 is at least as high as its liquidation value L at date 1.

We make some further assumptions. First, the assets are divisible at date 1. If a fraction $1 - f$ of the assets is sold off at date 1, then the date 1 liquidation receipts will be $(1 - f)L$, and the date 2 project return will be fR_2.[2]

Second, all uncertainty about R_1, R_2, L, and s is resolved at date 1.[3] Third, as a result of their close postinvestment relationship, both parties learn the realizations R_1, R_2, L, and s at this date (so they have symmetric information). However, these realizations are not verifiable to outsiders, and so date 0 contracts cannot be conditioned on them (at least not directly).[4]

Finally, we assume that the project is productive, in the sense that it would be carried out in a first-best world. If $s > 1$ with positive probability, this is always the case since the project is a "money pump" at date 1 (1 dollar at date 1 yields $s > 1$ dollars at date 2). If $s \equiv 1$, then the required condition is $E[R_1 + R_2] > I$; i.e., the project has positive expected net present value in the absence of reinvestment.

Feasible Contracts

We assume that, as the cash flows R_1 and R_2 accrue to the entrepreneur, he can divert them for his own benefit.[5] In contrast, the physical assets (those purchased with the initial investment funds) are fixed in place and can be seized by the investor in the

2. A natural interpretation of the model is that there are constant returns to scale beween dates 1 and 2: the unit cost of assets at date 1 is I_1 (say), with a unit return of R_2 at date 2, where $R_2 \geq I_1$. A "unit" is defined to be the size of the initial project at date 0. However, *dis*investment of the assets carried over from date 0 incurs a deadweight loss (a liquidation cost) of $I_1 - L$ per unit, which we assume is always nonnegative. Interpreting the model in this way, we have $s \equiv R_2/I_1$. When there are no liquidation costs, we have the boundary case $s \equiv R_2/L$.

3. This is without loss of generality since we can always replace the realization of a random variable by its expected value.

4. The assumption that L is nonverifiable is not uncontroversial, because in practice the value of L might be ascertained by putting the assets up for sale at date 1. However, to make the opposite assumption—that L is perfectly verifiable—is not innocuous either, given that to get informative bids for the assets, it may be necessary to commit to consummate the sale, and this may be inefficient: the assets may be worth more to the entrepreneur and investor than to the market. We should add that all of our results have force in the case in which L is nonstochastic, where nonverifiability is not an issue.

5. This (admittedly extreme) assumption is meant to capture the idea that the entrepreneur has discretion over cash flows. One way the entrepreneur might divert cash flows is by selling the output from this project to another firm he owns at an artificially low price or by buying input from another firm at an artificially high price.

event of default.[6] If the investor does seize the physical assets, the entrepreneur cannot undertake any reinvestment. In addition, seizure is the worst outcome that can befall the entrepreneur. That is, we rule out jail or physical punishment as ways of disciplining a nonperforming entrepreneur.[7]

Given that the entrepreneur can divert the cash flows, but not the project assets, it is natural to consider the following *debt contract*. The entrepreneur (henceforth known as the debtor D) borrows $B \geq I - w$ at date 0 and agrees to make fixed payments at dates 1 and 2; and if he fails to do so, the investor (henceforth known as the creditor C) can seize the project assets.[8]

We will find it convenient to write $B = I - w + T$, where $T \geq 0$ can be interpreted as the "transfer" that D receives from C, over and above what he needs to finance the project. It is assumed that D places this transfer in a private savings account: T represents *nonrecourse financing* (it cannot be seized by the creditor).[9] If $T < w$, then an equivalent way to think of this is that D puts only $w - T$ of his initial wealth into the project, and keeps the rest in his private savings account.

It is clear that there is no way to persuade D to pay anything at date 2, since at that stage the assets are worthless and so C has no leverage over D. Hence, we can set the date 2 payment equal to zero. From now on, we write the date 1 payment as P and denote a debt contract by a pair (P,T).

C and D's payoffs conditional on the state (R_1, R_2, L, s).

Suppose that a debt contract is in place and a particular realization (R_1,R_2,L,s) of the return streams and liquidation value

6. In practice, the distinction between cash flows (which can be diverted) and physical assets (which cannot) may not be as stark as we assume. What is important for the analysis that follows is that the investor can get her hands on something of value in a default state: the physical assets represent this source of value. Obviously, if the entrepreneur can divert everything, including the assets that generate future cash flows, then the investor has no leverage at all.

7. One justification for ruling out jail is that there is always enough background uncertainty so that the entrepreneur can claim that $R_1 = R_2 = 0$ (recall that R_1 and R_2 are not verifiable). Hence it would be difficult to persuade a judge or jury to convict the entrepreneur of theft. A justification for ruling out (private) physical punishment—apart from the fact that it is probably illegal—is that the investor has no incentive to administer the punishment ex post (after diversion has occurred) if it is at all costly; i.e., punishing the entrepreneur is not credible.

8. Another interpretation is that the investor is a preferred shareholder, who obtains control rights over the project assets if she does not receive a specified dividend payment.

9. If T is put in a public rather than a private savings account, i.e., if it can be seized by the investor, then one can show that a positive T is equivalent to a lower value of P. Thus, this case does not have to be considered.

occurs at date 1. How will D react? Note that D's wealth at date 1 is $T + R_1$, since he carries over T from date 0 and the project has earned R_1. Moreover, all of this is in a private savings account; i.e., it can be diverted. In contrast, the project has assets, with a liquidation value of L, which can potentially be seized by C.

We will assume that D can pay C *either* from his private savings account *or* by liquidating project assets. That is, even though D cannot divert or steal project assets for his own purposes, he can use them for debt repayment purposes. Inter alia, this assumption implies that C never receives more than P; for further discussion see footnote 15 below. Note that, since $s \leq R_2/L$ (the initial project has a higher rate of return than does reinvestment), D will never liquidate assets if he has cash in hand. That is, liquidation is a last resort.

Thus, if $T + R_1 + L \geq P$, D has two choices: either he can make the payment P, or he can default (voluntarily), i.e., pay zero.[10] In contrast, if $T + R_1 + L < P$, D has only one choice: to default (involuntarily).

In the event of default, C has the right to seize the project assets. However, seizure is only a threat point. If the liquidation value L is low, C may prefer to renegotiate the debt contract.

Figure I illustrates the situation facing the two parties, following default by D and seizure of the assets by C. Their *gross* payoffs—i.e., their payoffs from date 1 onward—are indicated on the axes. In the absence of renegotiation, C's payoff would be L, which is what she would get if she liquidated the assets; and D's payoff would be $T + R_1$, his cash holding. That is, the point $(L, T + R_1)$ in Figure I represents the status quo point of any renegotiation.

In a first-best world the Pareto frontier would have slope -1.[11] By contrast, in Figure I the frontier is steeper. The reason is that D is wealth-constrained at date 1. And there is no credible way for D to compensate C out of his additional earnings at date 2: C knows that, whatever promises are made at date 1, D will default at date 2 since by then the assets are worthless.

Moreover, the frontier is kinked, reflecting the fact that the return s from reinvestment is less than the return R_2/L from the assets in place. For values of C's payoff below $T + R_1$, she can be

10. It is easy to show that it is never in D's interest to make a partial payment.

11. In fact, if $s > 1$, the "frontier" would be at infinity, since the project would be a money pump.

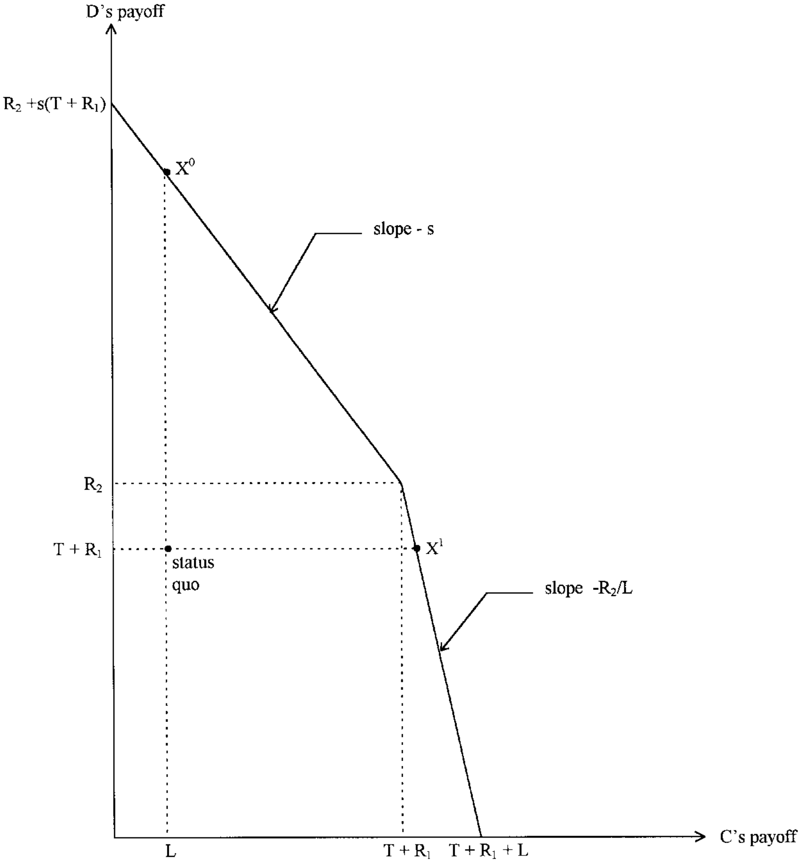

FIGURE I

paid out of D's cash holding, and so there is no need to liquidate assets. Along this portion of the frontier, every dollar less that C is paid out of D's date 1 cash holding can be reinvested by D to generate s dollars at date 2; hence the frontier has slope $-s$. For values of C's payoff higher than $T + R_1$, she has to be paid partly from liquidation receipts. Along this portion of the frontier, every dollar more that C is paid entails the further liquidation of $1/L$ units of assets, which reduces D's date 2 payoff by R_2/L dollars; hence the frontier has slope $-R_2/L$.

Since either party can refuse to renegotiate, the relevant portion of the frontier lies between point X^0 (corresponding to the outcome if D had all the bargaining power) and X^1 (corresponding to the outcome if C had all the bargaining power). Note that, as drawn, X^0 lies above and to the left of the kink in the frontier, whereas X^1 lies below and to the right. However, this need not be the case. If $T + R_1 < L$ (D is "very poor"), then the status quo point lies in the triangle to the southeast of the kink, so that even if D had all the bargaining power there would be some liquidation. And if $T + R_1 > R_2$ (D is "very wealthy"), then the status quo point lies in the triangle to the northwest of the kink, so that even if C had all the bargaining power, there would be no liquidation.

The exact point along X^0X^1 to which the parties renegotiate is moot. We adopt the following simple form of renegotiation. We suppose that with probability $(1 - \alpha)$ D makes a take-it-or-leave-it offer to C, and with probability α C makes a take-it-or-leave-it offer to D.[12] Because the set of feasible payoffs is convex (on account of the kink), the randomness in this game might lead to inefficiency. With this in mind, we augment the renegotiation game by allowing D to make C an offer before the game starts: the potential inefficiency is thereby eliminated.

To analyze this renegotiation game, it is easiest to start by computing C's payoff.

If D gets to make a take-it-or-leave-it offer to C, then the outcome is at point X^0, where C's payoff equals L.

If C gets to make a take-it-or-leave-it offer to D, then the outcome is at point X^1. The calculation of C's payoff in this case is more complicated, because it depends on whether X^1 lies below and to the right of the kink in the frontier ($T + \mathrm{R}_1 < R_2$), or whether X^1 lies above and to the left ($T + R_1 > R_2$).

Suppose first that $T + R_1 < R_2$. Then C will ask for all of D's cash $T + R_1$, and will also insist that a fraction $1 - ((T + R_1)/R_2)$ of the assets be liquidated. In return, D will be handed back the remaining fraction $f = (T + R_1)/R_2$. This makes D's date 2 payoff $T + R_1$, which is equivalent to what he would get if he rejected C's offer. C's return is given by

$$T + R_1 + [1 - ((T + R_1)/R_2)]L.$$

Suppose next that $T + R_1 > R_2$. Then C will agree to sell back

12. In Hart and Moore [1989] a different bargaining process was considered: this turned out to imply that $\alpha = 1$. Throughout the paper we take the division of bargaining power α to be exogenous. For an analysis of the design of bargaining games, see Harris and Raviv [1995].

the project assets to D in return for a cash payment of

$$T + R_1 - ((T + R_1 - R_2)/s).$$

This leaves D with cash equal to $(T + R_1 - R_2)/s$, which when reinvested at the rate of return s, and added to the project return R_2, gives D a total date 2 payoff of $T + R_1$. Again, this is equivalent to what D would get if he rejected C's offer.

We can combine these two subcases to write C's payoff, when C gets to make a take-it-or-leave-it offer to D, as[13]

$$\min\left\{T + R_1 + \left[1 - \left(\frac{T + R_1}{R_2}\right)\right]L, \quad T + R_1 - \left(\frac{T + R_1 - R_2}{s}\right)\right\}.$$

To obtain C's overall (expected) payoff $\overline{P}$, say, in the renegotiation game, we weight C's payoff when D has all the bargaining power and C's payoff when C has all the bargaining power by the probabilities with which they occur. This yields

$$(1) \quad \overline{P}(R_1,R_2,L,s;T) = (1 - \alpha)L$$
$$+ \alpha \min\left\{T + R_1 + \left[1 - \left(\frac{T + R_1}{R_2}\right)\right]L, \quad T + R_1 - \left(\frac{T + R_1 - R_2}{s}\right)\right\}.$$

Note that this is C's actual payoff (rather than expected payoff) from the renegotiation game, given that D makes an offer before the game starts.

Instead of defaulting, D may pay his debt P so as to keep control of the assets. Because D's payoff rises as C's falls (the frontier in Figure I is downward sloping), *D will pay P if and only if $P \leq \overline{P}$*.[14] The point is that if $P \leq \overline{P}$ it is always feasible for D to keep C's payoff down to P.[15]

13. To understand the min formula, note that the two terms are equal when $T + R_1 = R_2$, and that the coefficient of $T + R_1$ is smaller in the second term than in the first. Hence, the second term is bigger than the first term when $T + R_1$ is low. However, this is when C's payoff is given by the first term.

14. The italicized statement in the text also covers the case where D is forced to default—namely, where $T + R_1 + L < P$—because, in that case, $P > \overline{P}$ (using the fact that, from (1), $\overline{P} \leq T + R_1 + L$).

15. The assumption that D can liquidate project assets by himself to pay C is crucial here. If D could not self-liquidate, then a situation might arise where $T + R_1 < P < \overline{P}$, but C's payoff would be $\overline{P}$ rather than P since D would be forced to default.

There are two justifications for the assumption that D can self-liquidate. The first is that C's loan is secured on the general assets of D's company, rather than on specific assets, and that D can sell these general assets for cash in the normal course of doing business (i.e., it would be prohibitively expensive for C to monitor every transaction in which D is engaged). A second justification is the following. Suppose that the loan is secured on specific project assets (and these are registered

Hence C's gross payoff is min $\{\overline{P},P\}$. And her *net* payoff N, say, net of the initial transfer T, equals

$$N(R_1,R_2,L,s;P,T) = \min\{\overline{P} - T, P - T\}, \tag{2}$$

where $\overline{P}$ is given by (1).

We now use Figure I to calculate D's payoff. C's *gross* payoff (drawn along the horizontal axis) is $T + N$. If this is more than $T + R_1$, then D has to liquidate some of the assets: the outcome of the renegotiation lies below and to the right of the kink, and D gets $R_2 - (N - R_1)R_2/L$. On the other hand, if C's gross payoff $T + N$ is less than $T + R_1$, then D pays C entirely in cash, and there is no liquidation. The outcome of the renegotiation lies above and to the left of the kink, and D gets $R_2 + (R_1 - N)s$. Combining these two cases, D's payoff Π, say, is[16]

$$\Pi\,(R_1,R_2,L,s;P,T) = \min\{R_2 - (N - R_1)R_2/L,\ R_2 + (R_1 - N)s\}. \tag{3}$$

The fraction of the initial project assets that D retains equals

$$f(R_1,R_2,L,s;P,T) = \min\{1,\ 1 - (N - R_1)/L\}. \tag{4}$$

Equations (1)—(4) summarize the situation at date 1, conditional on the state $(R_1,R_2,L,\ s)$ and the debt contract (P,T).

III. Analysis of the Optimal Debt Contract

We turn next to the optimal choice of P and T. Since D and C are risk neutral, an optimal contract will maximize the expectation of D's payoff Π subject to the constraint that C's expected gross return is no less than $I - w + T$ (the amount borrowed by D). Given that we have defined N as C's *net* payoff (i.e., net of the transfer T), an optimal contract solves

$$\max_{P,T\geq 0} E\,\Pi \tag{5}$$

and cannot be sold). Then D could always rent the assets to a third party between dates 1 and 2. C would not need to be aware of this since the third party could ensure that D used the proceeds to pay C at date 1, i.e., D would not be in default. Moreover, if D defaults at date 2 and the assets (which are now worthless) are handed to C, then this does not affect the third party since he has already had the use of them between dates 1 and 2.

16. To understand this min formula, note that the two terms are equal when $T + R_1 = T + N$, and that the coefficient of N is more negative in the first term than in the second. Hence the first term is smaller than the second term when N is high. However, this is when D's payoff is given by the first term.

subject to

$$EN \geq I - w,$$

where N and Π (indexed by the state and the debt contract) are given by (2) and (3), and the expectations are taken with respect to the joint distribution of R_1, R_2, L, and s. Note that C's break-even constraint will hold with equality at the optimum since otherwise D's expected payoff could be increased by lowering P or raising T.[17]

An inspection of (2) and (3) reveals that the two instruments P and T have distinct roles. On the one hand, a reduction in P increases D's payoff in *nondefault* states, that is, in states where $P \leq \overline{P}$. On the other hand, an increase in T increases D's payoff in *all* states.

In fact, there is only one degree of freedom. Any rise in T must be balanced by a rise in P, so as to satisfy C's break-even constraint. The increase in P must actually be *greater* than the increase in T, because if C hands over an extra dollar at date 0 she typically gets only part of it back at date 1 in debt renegotiation.[18] Overall, *a balanced rise in P and T helps D in default states* (the rise in P makes no difference if D defaults), *but harms D in nondefault states* (the rise in P more than offsets the rise in T).

We now present some propositions showing how each instrument can be useful in different circumstances. We define two polar debt contracts.

DEFINITION. The *fastest* debt contract has $T = 0$. The *slowest* debt contract has $P = \infty$.[19]

In the fastest debt contract, D borrows the minimum amount

17. Note that a necessary and sufficient condition for the project to be undertaken in this second-best world is that the constraint set in (5) is nonempty and the maximized value of the objective function exceeds w (which is what D would obtain if the project did not go ahead). Since N is increasing in P and decreasing in T, C's net return is maximized when $P = \infty$ and $T = 0$, that is, it equals $E\overline{P}$. It follows that $E\overline{P} \geq I - w$; i.e.,

$$(*) \qquad (1 - \alpha)EL + \alpha E \min\{R_1 + (1 - R_1/R_2)L, R_1 - (R_1 - R_2)/s\} \geq I - w$$

is a necessary condition for the constraint set to be nonempty. Hence (*) is a necessary condition for the project to take place. When $w = 0$, (*) is also sufficient since D's participation constraint is nonbinding. It is clear from an inspection of (*) that some profitable projects will not be carried out.

18. It is easy to confirm from (1) and (2) that $N(R_1,R_2,L,s;P,T)$ falls when P and T rise by the same amount.

19. We introduced this terminology in Hart and Moore [1994].

necessary to finance the project. To put it another way, D puts in all his wealth, so that there is full equity participation.

In the slowest contract, since clearly D can never pay $P = \infty$ at date 1 and so always defaults, he effectively has the right to use the project assets for only one period: at date 1 control reverts to C. To put it another way, D *rents* the assets from C between dates 0 and 1: at date 1, C is the owner of the project and makes the decision about whether to continue or liquidate the project. Note that in the finite (or bounded) state case all that is required is that P be high; P does not have to equal ∞.[20]

The best contract ensures that, as far as possible, D is well off in those key states where either reinvestment is relatively productive (s is high) or where liquidation would be relatively costly (R_2/L is high), while at the same time guaranteeing that on average C breaks even. The nub of the matter is to find a contract that does a good job of cross-subsidizing D in the key states from the other states.

In certain circumstances, the choice of contract is immaterial.

PROPOSITION 1. Suppose that either (1) R_1, R_2, L, and s are nonstochastic, or (2) $s \equiv R_2/L$ and s is nonstochastic, or (3) L is nonstochastic and $\alpha = 0$. Then all debt contracts that satisfy C's break-even constraint with equality are optimal.

To prove part (3), note that if L is nonstochastic and $\alpha = 0$, then $\overline{P}$ in (1), and hence C's payoff N in (2), are nonstochastic. Thus, given that C breaks even, $N \equiv I - w$, and so D's payoff Π in (3) is independent of P and T. The same argument proves part (1). Part (2) follows from the fact that if $s \equiv R_2/L$ is nonstochastic then all funds invested between dates 1 and 2—irrespective of whether they are used to avoid liquidation or used for reinvestment—yield a *common* return, which is independent of the state of nature. And so it is immaterial how C is reimbursed: provided that she is paid $I - w$ on average, D's expected payoff is the same for all debt contracts.

Apart from these very special cases, different debt contracts will perform different amounts of cross subsidization. We have two classes of results. First, Proposition 2 below relates to the extreme case where $s \equiv 1$ (that is, there is no profitable reinvestment at date 1); second, Proposition 3 relates to the other extreme

20. For some empirical evidence on the use of fast and slow debt contracts, see Section VII of Hart and Moore [1994].

case where $s \equiv R_2/L$ (that is, there are constant returns to scale at date 1).

Suppose first that $s \equiv 1$. In this case, the gross social surplus from the project—the sum of D and C's ex post payoffs, $N + \Pi$—equals $R_1, + fR_2 + (1 - f)L$. Since the solution to (5) must maximize $EN + E\Pi$ subject to $EN = I - w$ (given that $EN = I - w$ at the optimum), it follows that an optimal contract solves

$$\max_{P,T \geq 0} E[f(R_2 - L)] \tag{6}$$

subject to

$$EN = I - w.$$

In other words, when $s \equiv 1$, an optimal contract as far as possible concentrates any liquidation onto those states where the social loss $R_2 - L$ is low.

PROPOSITION 2. Suppose that $s \equiv 1$. Then, among the class of debt contracts: (1) if only R_1 is stochastic, the slowest debt contract is optimal; (2) if only R_2 is stochastic, the fastest debt contract is optimal; and (3) if only L is stochastic and $\alpha = 1$, the slowest debt contract is optimal.

Proof. See Appendix.

Part (1) can be understood from our earlier finding: a balanced rise in P and T (i.e., so that C continues to break even) helps D in the default states (here, the low R_1 states) and hurts D in the nondefault states (here, the high R_1 states). In effect, a balanced increase in P and T serves to cross subsidize D in the bad states from the good states. Total surplus goes up because the better off D is in the bad states the less liquidation is needed; and liquidation tends to occur in the states where R_1 is low.[21]

The intuition for part (2) is slightly more complicated. If D defaults when R_2 is high, C can use her bargaining power in the renegotiation process to force a lot of liquidation, since even a small fraction of the assets is worth a great deal to D. This creates a lot of inefficiency, since these are the states where the social loss $R_2 - L$ is high. The best way to reduce (or eliminate) this inefficiency is to allow D to keep C at bay by making a low debt payment P: in other words, to help D not to default. But this is

21. For example, suppose that $I = 20$, $w = 7$, $R_2 = 18$, $L = 6$, and $R_1 = 21$ or 9 with equal probability. Also suppose that C has all the bargaining power: $\alpha = 1$. Then the slowest debt contract $(P,T) = (\infty,3)$ dominates all other debt contracts, including the fastest, $(P,T) = (14,0)$.

precisely what the fastest debt contract achieves. In contrast, the slowest debt contract helps D in the default states, i.e., the low R_2 states ($\overline{P}$ is increasing in R_2), where liquidation is not socially that costly.[22]

A similar intuition applies to part (3), although the conclusion is reversed: the slowest debt contract is again optimal when only L varies and $\alpha = 1$. The default states are those where L is low ($\overline{P}$ is increasing in L). These are also the states where liquidation is very costly, since $R_2 - L$ is high. Therefore, the slowest debt contract, which helps D in default states, is good. In contrast, the fastest debt contract, which helps D in nondefault states, is less effective.[23]

Now let us turn to the other extreme case where $s \equiv R_2/L$; i.e., where funds that are reinvested yield the same rate of return between dates 1 and 2 as the project itself. In this case, which will be the focus of much of the rest of the paper, we will be able to show that, under a slight strengthening of our assumptions, the fastest debt contract is optimal not only among debt contracts, but also relative to a large class of nondebt contracts.

When $s \equiv R_2/L$, there is no kink in the frontier in Figure I. It follows from (1) that C's gross payoff at date 1 from renegotiation following default by D is given by

$$\overline{P} = L + \alpha(T + R_1)(1 - 1/s). \tag{7}$$

And it follows from (2) that in a debt contract (P,T), C's payoff net of the initial transfer T is given by

$$N = \min\{L + \alpha R_1(1 - 1/s) - T(1 - \alpha(1 - 1/s)),\ P - T\}. \tag{8}$$

In particular, the *maximum* feasible value M, say, of N is

$$M = L + \alpha R_1(1 - 1/s), \tag{9}$$

22. For example, suppose that $I = 14$, $w = 5$, $R_1 = 12$, $L = 6$, and $R_2 = 24$ or 8 with equal probability. Also suppose that C has all the bargaining power: $\alpha = 1$. Then the fastest debt contract, $(P,T) = (10,0)$, achieves first-best; whereas in the slowest debt contract, $(P,T) = (\infty,4)$, there is liquidation when $R_2 = 24$.

23. For example, suppose that $I = 70$, $w = 49$, $R_1 = 18$, $R_2 = 72$, and $L = 36$ or 18 with equal probability. Also suppose that C has all the bargaining power: $\alpha = 1$. Then the slowest debt contract, $(P,T) = (\infty,46)$, dominates all other debt contracts, including the fastest, $(P,T) = (21,0)$.

Note that Proposition 2(3) requires $\alpha = 1$. When $\alpha < 1$, another effect becomes important. A fall in L may reduce $\overline{P}$ so much that D can buy back the assets even when $T = 0$; i.e., there may be no liquidation in low L states. But then a positive T does not improve efficiency in default states, and it is better to target the nondefault states through a reduction in P. Consider the above numerical example, except suppose that D, not C, has all the bargaining power; i.e., suppose that $\alpha = 0$. Then the fastest debt contract, $(P,T) = (24,0)$, strictly dominates all other debt contracts, including the slowest, $(P,T) = (\infty,6)$.

which is the value of $\overline{P}$ when $T = 0$. This new derived variable M will play an important role in the analysis of Section IV. The significance of M is that, since D can always default, M is the upper bound on C's return in any given date 1 state *whatever* contract has been written at date 0.

When $s \equiv R_2/L$, D's payoff is given by

$$\Pi = sL + sR_1 - sN. \tag{10}$$

Hence program (5) reduces to

$$\min_{P,T \geq 0} E[sN] \tag{11}$$

subject to

$$EN = I - w,$$

where N is given by (8). In other words, an optimal contract as far as possible concentrates C's payoff onto those states where s is low.

PROPOSITION 3. Suppose that $s \equiv R_2/L$ and that a higher value of s increases the distribution of M conditional on s, in the sense of first-order stochastic dominance. Then among the class of debt contracts the fastest debt contract is optimal.

Proof. See Appendix.

Proposition 3 assumes not only that $s \equiv R_2/L$, but also that increases in s go together with increases in M (and hence $\overline{P}$). This implies that high s states are the nondefault states. Given that high s states are also "good" states where the project assets and reinvestment yield a high rate of return, the fastest debt contract, which helps D in nondefault states, works well.

IV. MORE GENERAL CONTRACTS

The analysis in Sections II and III placed considerable restrictions on the class of admissible contracts. We looked only at debt contracts, where D borrows $I - w + T$ from C at date 0, and promises to repay a fixed amount P at date 1. In this section we consider a much broader class of contracts. The following example illustrates the power of alternative contracts.

Example. In this example there are no profitable reinvestment opportunities ($s \equiv 1$), and only L is stochastic. Suppose that

$I = 90, w = 80, R_1 = 0, R_2 = 100$, and $L = 20$, 60, or 100 with equal probability. Also suppose that D has all the bargaining power, i.e., $\alpha = 0$.

It is straightforward to show that here the best debt contract is the slowest debt contract $(P,T) = (\infty,50)$. Under this contract, D always defaults; and, since $\alpha = 0$, C's gross payoff $\overline{P}$ equals L in each of the three states, which gives her an average gross return of 60. That is, D borrows 60 at date 0; he uses 10 to finance the difference between I and w; and he retains a transfer T of 50. In state 1, where $L = 20$, D pays $\overline{P}$ out of his cash holding, $T + R_1 = 50$. In state 2, where $L = 60$, D pays $\overline{P}$ by liquidating $1/6$ of the assets. Assets are also liquidated in state 3, where $L = 100$, but there is no efficiency loss since $R_2 = L$ in that state. Overall, the first-best is not achieved, since there is inefficient liquidation in state 2.

The first-best can be achieved, however, by an *option-to-buy* contract under which C has an option to buy the project assets from D at date 1 at a price $Q = 70$. If C exercises her option in any state, the parties will renegotiate, and C's gross payoff will be $\overline{P} = L$. If C does not exercise her option, then she gets nothing, and D keeps control over the assets. In states 1 and 2, L is less than the option price Q, and so C will not exercise her option. In state 3 she will exercise her option, making a net return, $L - Q$, equal to 30. Hence at date 0 she is willing to pay D 10 in order to hold the option; and D uses this to finance $I - w$. Notice that under this option-to-buy contract, liquidation occurs only in state 3, when there is no efficiency loss.

The conclusion one draws from this Example is that debt contracts can be strictly inferior to other kinds of contract.[24]

To make further progress, we need to characterize the set of feasible contracts. The option-to-buy contract can be viewed as a special example of a *message-game contract,* where C sends one of two possible messages at date 1. "Exercising my option" is one of

24. This conclusion does not depend on the fact that L is stochastic, or on the facts that $s \equiv 1$ and $\alpha = 0$. Consider another three-state example: $I = 33$, $w = 30$, $L = 20$, and (R_1,R_2) takes values (43,100), (0,320), and (0,20) in states 1, 2, and 3, which have equal probability. Assume that C and D have equal bargaining power: $\alpha = 1/2$. s may take *any* values in the permitted range $[1,R_2/L]$. It is straightforward to show that the best debt contract is the fastest, $(P,T) = (3,0)$, under which there is inefficient liquidation in state 2. However, this debt contract is strictly dominated by an option-to-buy contract with $Q = 47$ (and no transfer T). C exercises her option only in state 1, which is when D has enough cash to be able to buy back all of the assets; in the other two states D keeps control without having to pay anything.

C's messages, the upshot of which is that she owes Q to D, and if she pays, she gets control over the assets. "Not exercising my option" is the other message, which leads to D keeping control, and nothing is owed by either party. It is important that the message is public, in that it can be verified by a court in the event of a dispute.

Notice that the contract is effective because the messages provide an indirect way of conditioning on the state of the world. The contract is designed so as to give C the incentive to send different messages in different states. That is, even though a court cannot directly verify which state has occurred, C's behavior—her choice of message—reveals information about the state.

Once publicly verifiable messages are admitted, the contractual possibilities become rich. There is no reason to limit the set of messages to just two. Also, it need not be the case that only C sends messages: C and D have common information, and so in principle either of them is in a position to inform the court (indirectly, at least) about the state.

For example, D could send a numerical message: the meaning of message "σ," say, is that he will pay the amount $P = \sigma$ and that, provided he pays, there is then a probability $\rho = \rho(\sigma)$ that he retains control. The lottery $\rho(\cdot)$, which is publicly held, is specified in the date 0 contract. Clearly, there is no loss of generality in restricting attention to nondecreasing functions $\rho(\cdot)$, since D would have no incentive to pay more for a lower probability of keeping control. The more familiar version of this contract is a nonlinear pricing schedule, where D chooses how much to pay, P, and $\rho(P)$ is the probability that he then keeps control (the contract can be thought of as "smoothed debt").

The most general message-game contract we consider is where *both C and D* send abstract messages—σ_C and σ_D, say—at date 1, on the strength of which there is some amount $P = P(\sigma_C, \sigma_D)$ that D owes C. (P may be negative, in which case C owes $-P$ to D.) If the money is paid, then D keeps control over the assets with probability $\rho = \rho(\sigma_C, \sigma_D)$. The mappings $P(\cdot,\cdot)$ and $\rho(\cdot,\cdot)$ are specified in the date 0 contract.[25]

Crucially, however, we continue to assume that, even after message(s) have been sent, D can refuse to pay and choose instead

25. There are yet other possible mechanisms, played in stages, which screen on D's cash holdings by requiring him to put up money before he plays a particular branch of the game tree. Such mechanisms exploit infeasibility off the equilibrium path. We postpone discussing these until the end of this section.

to default.[26] The worst sanction that can be imposed on him is that he loses control of the assets. That is, whatever moves the parties may have made as part of a contractually specified mechanism (whatever messages may have been sent), once some terminal node (P,ρ) has been reached, D in effect always has the choice between paying P (if he can afford to) or defaulting, i.e., choosing the pair (0,0). And if D does default, he can always then renegotiate with C.[27]

We will see in Proposition 5 below that the fact that D can default and renegotiate a contract dramatically reduces the set of message-game contracts which one needs to consider.

First, we should observe that parts (1) and (2) of Proposition 1 extend to include message-game contracts. That is, if either there is no uncertainty, or if there are constant returns at date 1 and the return happens to be the same across all states, then the choice of contract is immaterial.

PROPOSITION 4. Suppose that either (1) R_1, R_2, L, and s are nonstochastic, or (2) $s \equiv R_2/L$ and s is nonstochastic. Then all message-game contracts that satisfy C's break-even constraint with equality are optimal.[28]

Proposition 5 below deals with the case $s \equiv R_2/L$. *This is the case we shall deal with for the rest of the paper.*

Substituting (9) into (8), we see that C's net payoff N under a debt contract (P,T) is a function of M and s only. In Lemma 1 we prove that under *any* message-game contract, C's equilibrium payoff across different states of nature can be expressed in terms of M, s, and V, where V is defined by

$$V \equiv L + R_1. \tag{12}$$

26. As C is a deep pocket, she can commit herself not to default by putting up a bond at date 0 which she forfeits if she defaults.

27. In this respect, we depart from much of the literature on implementation, where it is tacitly assumed that agents can be forced to abide by the outcome of a mechanism. It is also usually supposed that the agents can agree in advance not to renegotiate once the mechanism has been played, even though there is no asymmetry of information between them (a usual source of breakdown in bargaining). Maskin and Moore [1987] characterize what can be implemented when agents cannot precommit not to renegotiate. For an introduction to the literature on implementation in environments with complete information, see Moore [1992].

28. Interestingly, part (3) of Proposition 1 does *not* extend to include message-game contracts, because, if $s < R_2/L$, the kink in the frontier can be exploited to permit the design of games with desirable *mixed-strategy* equilibria. However, our belief is that if D is able to purchase outside insurance against the outcome of mixed-strategy equilibria, these constructions do not help—in which case debt contracts *are* always optimal when L is nonstochastic and $\alpha = 0$.

Accordingly, we can write C's net equilibrium payoff as $N(M,s,V)$.[29]

Notice that M is the *most* that C can get in the event of D defaulting (and the upper bound is attained only when $T = 0$). Since D can always default, a corollary is that no message-game contract can give C a net equilibrium payoff greater than M. We formally prove this in (13) of Lemma 1.

We actually prove more than this. C's payoff is *nondecreasing* in each of the three variables M, s, and V: see (14) in Lemma 1.

LEMMA 1. Assume that $s \equiv R_2/L$. In any message-game contract, C's equilibrium payoff, net of any transfer T, can be expressed as a function of the three derived variables M, s, and V (where M and V are given in (9) and (12)). Moreover, C's payoff $N(M,s,V)$, say, must satisfy

(13) $$N(M,s,V) \leq M;$$

(14) $N(M,s,V)$ is nondecreasing in M, s, and V;

(15) $N(M,s,V)$ is independent of s and V if $\alpha = 0$.

Proof. See Appendix.

As Lemma 1 is key to Proposition 5 below, we should sketch the intuition behind it. In any given state, the two parties are playing a "message/default" game—after which they will, if necessary, renegotiate their way onto the payoff frontier. The *compound* game (that is, including the subsequent renegotiation) is akin to a zero-sum game, since the parties' payoffs are perfectly negatively correlated. (It is not a zero-sum game per se, because, unlike in a zero-sum game, the payoff frontier has slope $-s$, not -1.) In any given state, one can think of this compound game in terms of a reduced-form matrix, where the messages σ_C and σ_D, respectively, identify the row and column, and the corresponding entry in the matrix specifies a pair of payoffs lying on the frontier. Clearly, C's equilibrium payoff in this compound game cannot be greater than her maximum payoff M in any entry of the matrix: hence the upper bound constraint (13). Conditions (14) and (15) relate to how C's equilibrium payoff varies with the state. Here we appeal to the fact that the value of the compound game is given by the min-max formula for zero-sum games. Now C's payoff in each

29. The third variable V separately enters C's payoff only if $\alpha \neq 0$, and for at least one pair of messages (σ_C,σ_D), the contract specifies $0 < \rho(\sigma_C,\sigma_D) < 1$.

entry of the matrix can be shown to be nondecreasing in M, s, and V: the point is that an increase in any one of M, s, or V increases the surplus, and, for a given (P,ρ), both parties share in the increase. It follows immediately from the min-max formula that C's equilibrium payoff in the compound game is also nondecreasing in M, s, and V: hence the monotonicity condition (14). Likewise, since C's payoff in each entry of the matrix can be shown to be independent of s and V if $\alpha = 0$, the same is true of her equilibrium payoff in the compound game: hence the independence condition (15).

To sum up what we have learned so far in this section: message-game contracts can be both realistic (e.g., options to buy, or nonlinear pricing contracts) and effective. However, D's ability to default and renegotiate places considerable restrictions on C's equilibrium net payoff. In particular, if $s \equiv R_2/L$, then C's payoff is a nondecreasing function of the three derived variables M, s, and V; is bounded above by M; and is independent of s and V if $\alpha = 0$.

Substituting (12) into (10), D's payoff Π equals $sV - sN(M,s,V)$. Since $E[sV]$ is independent of $N(\cdot,\cdot,\cdot)$, an optimal message game contract solves the following program, which is akin to (11):

$$\text{(16)} \qquad \underset{N(\cdot,\cdot,\cdot)}{\text{minimize}} \; E[sN(M,s,V)]$$

subject to

$$E[N(M,s,V)] \geq I - w;$$
$$N(M,s,V) \leq M;$$
$$N(M,s,V) \text{ is nondecreasing in } M,\ s, \text{ and } V;$$
$$N(M,s,V) \text{ is independent of } s \text{ and } V \text{ if } \alpha = 0.$$

The first constraint in (16) is C's participation constraint. And by Lemma 1, the last three constraints, (13)—(15), are necessary conditions on C's equilibrium payoff arising from a message-game contract.[30]

It is revealing to graph the N-functions that derive from the two contracts which we considered earlier: a debt contract, and an option-to-buy contract. First, consider a debt contract (P,T). Substitute (9) into (8) to give C's payoff N in terms of M and s. In Figure IIa, this N is graphed against M, holding s constant. As we saw in Section III, in order to satisfy C's participation constraint,

30. As we are about to prove that, in certain circumstances, the fastest debt contract yields an N function which solves (16), the question of sufficiency will not detain us here.

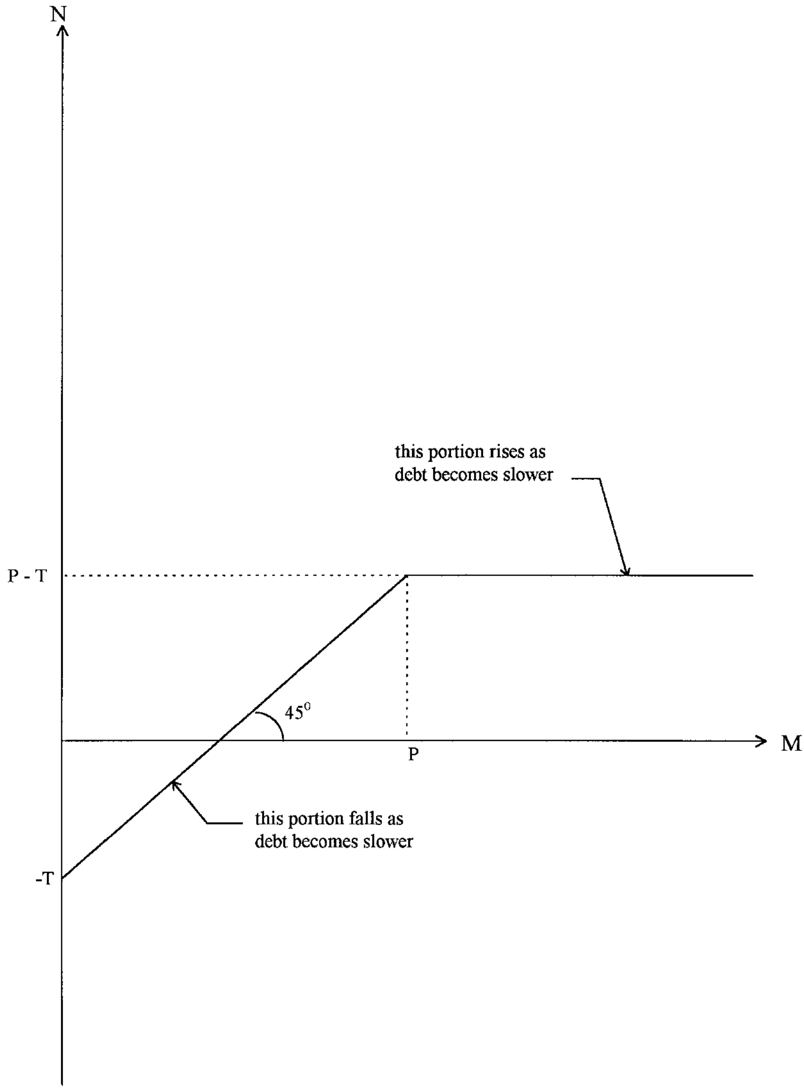

FIGURE IIa
A Debt Contract

if T rises by a dollar, then P has to rise by more than a dollar. Accordingly, as T rises, the flat portion of the graph rises, but the vertical intercept falls. Roughly speaking, a rise in T makes the graph less flat.

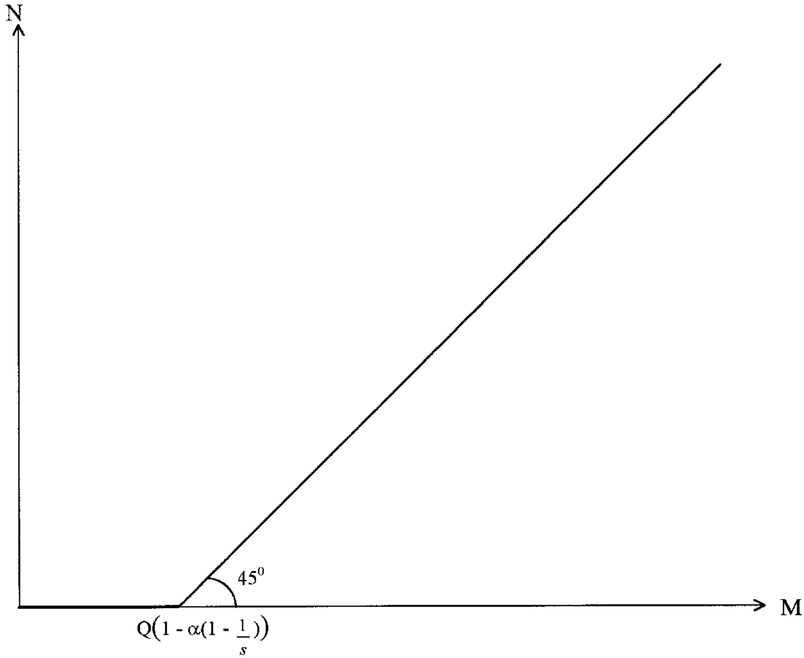

FIGURE IIb
An Option-to-Buy Contract

Second, consider an option-to-buy contract where C can buy the assets at date 1 by paying Q to D (and there is no transfer T). From (7), we know that if C exercises her option she can sell the assets back to D for $\overline{P} = M + \alpha Q(1 - 1/s)$. Therefore, C will exercise the option if and only if $M + \alpha Q(1 - 1/s) > Q$, and her payoff is given by

$$N(M,s,V) = \max\{M - Q[1 - \alpha(1 - 1/s)], 0\}. \tag{17}$$

See Figure IIb, where the function N in (17) is graphed against M, holding s constant. Notice that this graph is less flat than the graph for the fastest debt contract in Figure IIa.[31]

To return to the analysis, consider the minimand in (16). Given C's participation constraint, it would be best if her payoff N were to decrease in s because this would help D in those states

31. Note that, in all the graphs, $N(M,s,V)$ satisfies conditions (13)—(15).

when s is high. Moreover, if s is affiliated with M, then we would like N to decrease in M too.[32] Unfortunately, we have to contend with the monotonicity conditions in (16). In particular, N has to be nondecreasing in M. The best we can hope for is a flat N, equal to $I - w$; this corresponds to the riskless fastest debt contract $P = I - w$. However, if there are values of M which are less than $I - w$, we must respect the upper bound constraint in (16). Figures IIa and IIb suggest that the flattest N corresponds to the fastest debt contract. Proposition 5 confirms that this is true, given some additional assumptions.

PROPOSITION 5. Assume that $s \equiv R_2/L$. Then the fastest debt contract is optimal in the class of message-game contracts if

either (1) $\alpha > 0$, and M, s, and V are affiliated;

or (2) $\alpha = 0$, and L and s are affiliated.[33]

Proof. See Appendix.

It is worth rehearsing the intuition for this result given in the Introduction. Every dollar that C receives at date 1 is a dollar that D cannot reinvest. Under the assumption that $s \equiv R_2/L$, and that the key variables are affiliated, it is desirable to minimize C's payoff N in "good" states of the world, since this enables D to reinvest as much as possible when reinvestment is valuable. A debt contract works well, since it puts a cap P on N, which binds in good states. Moreover, the fastest debt contract works best since the cap P is smallest if $T = 0$.

Part (2) of Proposition 5 tells us that when $\alpha = 0$, and L and $s \equiv R_2/L$ are affiliated, the fastest debt contract is optimal irrespective of R_1.[34]

This section might be summarized by saying that our exploration of more general contracts—message-game contracts—turns out to have been a digression in the case $s \equiv R_2/L$, at least when certain key variables are affiliated. Under these circumstances, we can restrict attention to the fastest debt contract after all.

There is a caveat. Although message-game contracts are quite

32. For a definition and discussion of affiliation, see the Appendix of Milgrom and Weber [1982]. Affiliation can be viewed as an extension of the monotone likelihood ratio property to more than two random variables. That is, two random variables X and Y, with joint density function $f(x,y)$, are affiliated iff $f(x^+,y^+)\, f(x^-,y^-) \geq f(x^-,y^+)\, f(x^+,y^-)$ for all x^+, x^-, y^+, and y^- with $x^+ \geq x^-$ and $y^+ \geq y^-$.

33. One can show that these are far from necessary conditions for the fastest debt contract to be optimal in the class of message-game contracts.

34. Affiliation is implied if either L or s is nonstochastic.

general, there are other forms of mechanism that are played in stages and that screen on D's date 1 cash holdings by requiring him to put up money early on, before he plays a particular branch of the game tree. In effect, these mechanisms exploit infeasibility off the equilibrium path.

For example, suppose that $I = 3$, $L = 2$, and (R_1,R_2) takes values (0,2) and (3,10) in states 1 and 2, which have equal probability. Also suppose that $s \equiv R_2/L$, and that D has all the bargaining power: $\alpha = 0$. (Note that, since L is nonstochastic, Proposition 5(2) applies.) Finally, suppose that $2 < w < 3$. Consider (16), but without the constraint that N is nondecreasing in M and s. It is easy to see that the solution is $N = 2$ in state 1 and $N = 4 - 2w$ in state 2. This solution violates the constraint that N is nondecreasing in M and s, and hence cannot be achieved by any of the message games described in this section.

However, the following mechanism *does* achieve the above solution. Let the contract specify that if D pays 3 to C at date 1, then C must pay $2w - 1$ to D. However, if D fails to pay 3, then C obtains control of the assets. This contract achieves the desired outcome because in state 2 D can pay 3, whereas in state 1 he cannot.

There is an obvious problem with a mechanism like this. In state 1, D could approach a third party and borrow (short-term) using as collateral the payment he is about to receive from C. We suspect that, if this kind of borrowing is allowed, all that matters is the *net* amount P that D is required to pay—which brings us back to message games with final outcomes (P,ρ), which we have considered. However, these matters require further investigation.

V. Initial Project Scale

We complete our analysis by considering briefly the choice of project *scale* at date 0. Until now, we have taken the size of the investment, I, to be fixed. Suppose instead that the initial investment can be varied; in particular, suppose that the project exhibits constant returns to scale. It is easiest to think in terms of the "unit project," costing 1 at date 0, with cash returns r_1 and r_2 at dates 1 and 2, and with liquidation value l at date 1. That is, $R_1 = Ir_1$, $R_2 = Ir_2$, and $L = Il$. Our concern in this section is with the choice of I.

Given constant returns to scale at date 0, a natural case to consider is where there are also constant returns at date 1: $s \equiv$

r_2/l. In addition, assume that

(18) $$m, s \text{ and } v \text{ are affiliated,}$$

where

$$m \equiv l + \alpha r_1(1 - l/r_2),$$
$$s \equiv r_2/l,$$

and

$$v \equiv l + r_1.$$

(In the light of Proposition 5(2), if $\alpha = 0$ it is enough to assume that l and s are affiliated.) Given (18), we know from Proposition 5(1) that the fastest debt contract, where $T = 0$, is optimal for *any* I. Thus, for each I that can be financed, let $P = P(I)$ be the (smallest) debt level at which C just breaks even: $E[\min\{Im, P\}] = I - w$.

We are concerned with the optimal choice of I. The economics of the problem are revealed by separating D's objective into benefits and costs:

(19) $$\max_{I \geq 0} Ib - c(I),$$

where

$$b \equiv E[vs] \quad \text{and} \quad c(I) \equiv E[\min\{Im, P(I)\}s].$$

Notice that D's benefits are linear in I. We show in Proposition 6 below that, among other things, his cost function $c(I)$ is convex.

The case of perfect certainty is particularly simple. Here, $P(I) = I - w \leq Im$. I then solves

(20) $$\max_{I \geq 0} (Iv + w - I)s$$

subject to

$$Im \geq I - w.$$

There are two cases to consider: $m < 1$ and $m \geq 1$.

(1) $m < 1$. In this case, the constraint in (20) eventually binds.

(1a) If $v > 1$, the objective function is increasing in I, and there is an interior optimum; $I = w/(1 - m)$. Notice the multiplier if $m > 0$: the optimal scale of the project is proportional to D's initial wealth with a constant of proportionality that exceeds one.

(1b) If $v \leq 1$, the objective function is decreasing in I, and it is optimal to set $I = 0$.

(2) $m \geq 1$. In this case, the constraint in (20) never binds and, since $v \geq m$, the objective function is (weakly) increasing in I. In effect, every increase in I of one dollar increases C's potential payoff by at least one dollar, so the project is a money pump, and it is optimal to set $I = \infty$.

We now see how these findings generalize when there is uncertainty.

PROPOSITION 6. Assume that (18) holds, and that there is a finite number of states.

(1) If $Em < 1$, the cost function $c(I)$ defined in (19) is increasing, piecewise linear, and convex in the interval $w \leq I \leq w/(1 - Em)$, with a slope no less than Es.
  (1a) If $E[vs] > Es$, then some $I \geq w/(1 - \underline{m})$ is optimal, where $\underline{m}$ is the minimum of m.
  (1b) If $E[vs] \leq Es$, it is optimal to set $I = 0$.

(2) If $Em \geq 1$, then it is optimal to set $I = \infty$.

Proof. See Appendix.

Proposition 6 tells us that, under assumption (18), the problem of choosing I is well-behaved. As with the nonstochastic case, there is a multiplier (at least if $\underline{m} > 0$): provided that D invests his initial wealth w in the project, he will borrow to make some additional investment.

If we were willing to *assume* the fastest debt contract (rather than prove that it is optimal within the class of message-game contracts), assumption (18) could be dropped. All the results of the proposition would hold if $E[s|m]$ were nondecreasing in m. The proof in the Appendix makes use of this weaker assumption.

VI. SUMMARY, RELATIONSHIP TO THE LITERATURE, AND CONCLUDING REMARKS

A brief summary of the paper may be useful. We have analyzed the role of debt in persuading an entrepreneur to pay out cash flows, rather than to divert them. In the first part of the paper, we studied the optimal debt contract—specifically, the trade-off between the size of the loan and the repayment—under the assumption that some debt contract was optimal. In the second part we considered a more general class of (nondebt) contracts and derived sufficient conditions for debt to be optimal among these.

Our paper can be seen as part of the recent literature that analyzes financial decisions from an "incomplete contracting" perspective.[35] This literature starts with Aghion and Bolton [1992]. Aghion and Bolton analyze debt in terms of the allocation of residual control rights over assets (along the lines of Grossman and Hart [1986] and Hart and Moore [1990]). They consider a situation where a project yields private benefits to an entrepreneur as well as (verifiable) monetary benefits. It is assumed that some project actions must be taken in the future, but these cannot be contracted on initially. (One such action might concern the liquidation decision.) If the entrepreneur has all the residual control rights, he will take actions that increase his private benefits, but at the expense of the return to investors. On the other hand, if the investor has control, she will take actions that do not respect the investor's private benefits. Aghion and Bolton study the optimal balance of control between the entrepreneur and the investor. Of particular interest, they show that the optimal allocation is state contingent: the entrepreneur should have residual control rights in states of the world where his private benefits are relatively high, and the investor should have control in states where the entrepreneur's private benefits are relatively low.

There are two important differences between Aghion and Bolton's [1992] work and ours. First, although Aghion and Bolton show that control will shift from the debtor to the creditor in certain states of the world, they do not provide general conditions under which these states can naturally be interpreted as "default" or "bankruptcy" states (for example, they could be high-profit rather than low-profit states). Second, and related, Aghion and Bolton ignore the role of debt as a mechanism for getting a debtor to pay up. That is, Aghion and Bolton assume that control shifts are triggered by a verifiable state of the world (e.g., the state might be that profits are low). In contrast, in our model the shift in control is endogenous—it occurs because the debtor fails to make a promised repayment.

Our paper also has similarities to Bolton and Scharfstein's [1990] analysis of predation and the costly state verification (CSV) models of Townsend [1979] and Gale and Hellwig [1985]. Bolton

35. For a fuller discussion of the literature, see Chapter 5 of Hart [1995].

and Scharfstein develop a model where the penalty for nonpayment of debt is that the creditor withholds future finance rather than liquidating existing assets. They are more concerned with how debt can be used strategically to influence competition in product markets than with a general characterization of debt contracts. In the costly state verification models there is also a penalty for nonpayment, but it is that the debtor is inspected. The CSV models additionally assume that information is asymmetric, tend to rule out ex post renegotiation, and take the cost of bankruptcy as given (it is the cost of monitoring).[36] In contrast, our model is based on symmetric information, allows for ex post renegotiation, and endogenizes the cost of default.[37]

There is also a parallel between this paper and the work of Bulow and Rogoff [1989] on sovereign debt. Bulow and Rogoff analyze a model in which a debtor country borrows from a creditor country for current consumption but cannot commit to repay the loan out of future production. If the debtor repudiates the loan, the creditor can retaliate by blockading the debtor country's trade. In the Bulow and Rogoff paper, there is nothing corresponding to irreversible liquidation, and, as a result, there is never any ex post inefficiency (no blockade occurs in equilibrium). In contrast, in our model there can be inefficient liquidation ex post. Also, because of their concern with sovereign debt, Bulow and Rogoff do not study the role of legally enforceable contracts in sustaining repayment paths.

We conclude by noting some directions for future research. Probably the most interesting extension of the model is to the case of more than two periods, which would permit an analysis of the maturity of debt contracts. As noted in the Introduction, Hart and Moore [1994] and Hart [1995] carry out such an extension, but only for the case of perfect certainty. A preliminary discussion of the uncertainty case was contained in our earlier paper [Hart and Moore 1989]. However, the analysis in that paper was intricate; we were unable to go beyond a three-period model, and there were relatively few clear-cut results. There were some general findings,

36. Gale and Hellwig [1989] do include a discussion of renegotiation, however.

37. The work of Allen [1983] and Kahn and Huberman [1988] should also be mentioned. Allen studies a model in which the penalty for not repaying a loan is the seizure of assets and future exclusion from the capital market. However, Allen focuses on inefficiencies with respect to the initial size of the project, rather than on control issues or the cost of default. Kahn and Huberman [1988] investigate the role of asset seizure in encouraging a debtor to repay a loan, but in a context where renegotiation always leads to ex post efficiency.

however, which we believe would broadly apply to any intertemporal model of debt based on control. We found that a key tension between short-term and long-term debt is the following. On the one hand, short-term debt gives the creditor early leverage over the project's return stream, which is good because it can keep total indebtedness low. On the other hand, short-term debt may give too much control to the creditor in certain states and lead to premature liquidation; that is, the creditor may liquidate early because the debtor cannot credibly promise to repay later. In this sense, long-term debt contracts protect the debtor from the creditor. An important next step in the research is to formulate a tractable, multiperiod model of debt with uncertainty.

Even in the two-period model there are a number of further avenues to explore. In the first part of the paper, we focused on the trade-off between P and T, and showed that T could be used to limit C's bargaining power in bad states. However, as Section IV makes clear, contracts other than debt contracts may be useful when the conditions of Propositions 4 and 5 do not hold. It is an open question as to what is the nature of an optimal contract in these circumstances.

It would be interesting to relax some of the assumptions we have made about renegotiation. We have supposed that the parties can choose from a large class of mechanisms for allocating control, but that the parties cannot control the division of bargaining power in the renegotiation game. We have also ruled out the presence of third parties to the contract. All these assumptions are worth dropping. For an analysis of how the renegotiation process might be designed to achieve a better outcome, see Harris and Raviv [1995].

In addition, we have studied a one-shot situation. An interesting generalization is to a repeated relationship where parties may acquire a reputation for repaying their debts, or for liquidating assets rather than renegotiating. (A long-lived bank might acquire a reputation for renegotiating only when a default is involuntary.) It would be interesting to know whether under these conditions debt still has a role to play, or whether other instruments might substitute for debt. For an analysis of this and related issues, see Fluck [1996] and Gomes [1996], and for a more general discussion of debt and reputation, see Diamond [1989].

A further extension is to the case of multiple investors. If there are multiple creditors, then it is plausible that the process of renegotiating a debt contract becomes more difficult (e.g., because the creditors have different information). This brings benefits as well as costs. The benefit is that strategic default by the debtor is less attractive, which means that the constraint that the creditor be paid back is relaxed. The cost is that, if default is involuntary, the project may be liquidated when it should be continued. The trade-off between the two effects is studied in Bolton and Scharfstein [1996].

Finally, in a richer model where the entrepreneur cannot "steal" all the cash flows, (nonvoting) equity becomes a feasible claim as well as debt, since dividends can be paid. Dewatripont and Tirole [1994] have shown that under these conditions the entrepreneur's budget constraint can be "hardened" by allocating debt to one outside investor and equity to another. (In a similar vein, Berglof and von Thadden [1994] have shown that it is sometimes optimal to allocate short-term debt to one investor and long-term debt to another.) Incorporating equity into a model like the one described here would greatly enrich the analysis and is an important topic for future research.

APPENDIX

Proof of Proposition 2

Consider the set of feasible transfers T. (T is feasible if there exists a debt contract (P,T) satisfying C's participation constraint. We may assume that $T = 0$ is feasible, otherwise the project could not be financed at date 0.) For each feasible T, let $P(T)$ denote the smallest debt level at which C breaks even: $E[N(R_1,R_2,L,s;P(T),T)] = I - w$.

Given $s \equiv 1$, (1) reduces to

$$(1') \quad \overline{P}(R_1,R_2,L;T) = (1-\alpha)L + \alpha \min\left\{T + R_1 + \left[1 - \left(\frac{T+R_1}{R_2}\right)\right]L,\ R_2\right\}.$$

Note that $\overline{P}(R_1,R_2,L;T) - T$ is nonincreasing in T. Also, recall from the text that $P(T) - T$ is nondecreasing in T.

First, we prove part (1) of the proposition, where only R_1 is stochastic. $\overline{P} = \overline{P}(R_1;T)$ is nondecreasing in R_1. So find $R_1^*(T)$ (which may be infinite) such that $\overline{P}(R_1;T) \leq P(T)$ for $R_1 \leq R_1^*(T)$ and $\overline{P}(R_1;T) \geq P(T)$ for $R_1 \geq R_1^*(T)$. $R_1 - \overline{P}(R_1;T)$ is also nondecreasing in R_1; so from (2) the function $R_1 - N(R_1;P(T),T)$

is nondecreasing in R_1. And $R_1 - N(R_1;P(T),T)$ is nondecreasing in T for $R_1 \leq R_1^*(T)$, and is nonincreasing in T for $R_1 \geq R_1^*(T)$.

By (4), $f(R_1;P(T),T)$ is a positive affine transformation of $R_1 - N(R_1;P(T),T)$, truncated above by 1 for high R_1. Without the truncation, Ef would be independent of T, since $E[R_1 - N] = ER_1 - I + w$ is independent of T. Thus, with the truncation, Ef rises as T rises. Since $E[f(R_2 - L)] = (R_2 - L)Ef$, it follows from (6) that it is optimal to increase T. The slowest debt contract is optimal. Part (1) is proved.

Next we prove part (2) of the proposition, where only R_2 is stochastic. $\overline{P} = \overline{P}(R_2;T)$ is nondecreasing in R_2. So find $R_2^*(T)$ (which may be infinite) such that $\overline{P}(R_2;T) \leq P(T)$ for $R_2 \leq R_2^*(T)$ and $\overline{P}(R_2;T) \geq P(T)$ for $R_2 \geq R_2^*(T)$. From (2), $N(R_2;P(T),T)$ is also nondecreasing in R_2. And $N(R_2;P(T),T)$ is nonincreasing in T for $R_2 \leq R_2^*(T)$, and is nondecreasing in T for $R_2 \geq R_2^*(T)$.

By (4), $f(R_2;P(T),T)$ is a negative affine transformation of $N(R_2;P(T),T)$, truncated above by 1 for low R_2. Without the truncation, Ef would be independent of T, since $EN = I - w$ is independent of T. With the truncation, Ef falls as T rises. Moreover, f falls as T rises when $R_2 - L$ is high. And f rises as T rises when $R_2 - L$ is low. It therefore follows from a standard stochastic dominance argument that $E[f(R_2 - L)]$ falls as T rises. From (6) it is therefore optimal to reduce T. The fastest debt contract is optimal. Part (2) is proved.

Finally, we prove part (3) of the proposition, where only L is stochastic. $\overline{P} = \overline{P}(L;T)$ is nondecreasing in L (when $T + R_1 > R_2$, the second term of the min operator in (1′) is strictly less than the first). So find $L^*(T)$ (which may be infinite) such that $\overline{P}(L;T) \leq P(T)$ for $L \leq L^*(T)$ and $\overline{P}(L;T) \geq P(T)$ for $L \geq L^*(T)$. From (2), $N(L;P(T),T)$ is also nondecreasing in L. And $N(L;P(T),T)$ is nonincreasing in T for $L \leq L^*(T)$, and is nondecreasing in T for $L \geq L^*(T)$.

Given $\alpha = 1$, the only way that $N(L;P(T),T)$ can vary with L is because, for at least some $L \leq L^*(T)$, the first term of the min operator in (1′) is strictly less than the second. In which case it follows that $N(L;P(T),T) > R_1$ for all L. And so, from (4), $f(L;P(T),T) < 1$ for all L, and $fL - L$ is a negative affine transformation of N. $E[fL - L]$ is independent of T, since $EN = I - w$ is independent of T. That is, $E[fL]$ is independent of T. Moreover, fL rises as T rises when L is low; i.e., when $(R_2 - L)/L$ is high. And fL falls as T rises when L is high; i.e., when $(R_2 - L)/L$ is low. It therefore follows from a standard stochastic dominance argument that $E[f(R_2 - L)]$ rises as T rises. From (6) it is therefore optimal to increase T.

The other possibility is that $N(L;P(T),T)$ is independent of L. In which case $N(L;P(T),T) \equiv I - w$, which is independent of T.

In sum, the slowest debt contract is optimal. Part (3) is proved.

QED

Proof of Proposition 3

Take some debt contract (P,T) for which $T > 0$, and denote C's payoff from (8) by $N(M,s)$. Consider replacing this contract by the fastest debt contract $(\hat{P},0)$, where $\hat{P}$ is the smallest solution to $E[\min\{M,\hat{P}\}] = I - w$. C's payoff under the latter contract equals $\min\{\hat{P},M\} \equiv \hat{N}(M)$, say, which is independent of s. Given that (P,T) finances I,

$$E[N(M,s)] \geq I - w = E[\hat{N}(M)].$$

This implies that

$$\text{(i)} \qquad E[\Delta(M,s)] \leq 0,$$

where $\Delta(M,s) \equiv \hat{N}(M) - N(M,s)$. It follows that $\hat{P} \leq P - T$.

Now

$$\Delta(M,s) = \begin{cases} T\left(1 - \alpha + \dfrac{\alpha}{s}\right) & \text{for } M \leq \hat{P} \\ \hat{P} - M + T\left(1 - \alpha + \dfrac{\alpha}{s}\right) & \text{for } \hat{P} < M < P - T\alpha\left(1 - \dfrac{1}{s}\right) \\ \hat{P} - P + T & \text{for } M \geq P - T\alpha\left(1 - \dfrac{1}{s}\right). \end{cases}$$

By inspection, $\Delta(M,s)$ is nonincreasing in M and s. Hence taking any $s^- \leq s^+$, we have

$$E[\Delta(M,s^+)|s^+] \leq E[\Delta(M,s^+)|s^-] \leq E[\Delta(M,s^-)|s^-].$$

Here, the first inequality follows from a standard dominance argument: $\Delta(M,s^+)$ is nonincreasing in M, and, for all M, the distribution function of M conditional on s^- is no less than the distribution function of M conditional on s^+. The second inequality reflects the fact that $\Delta(M,s)$ is nonincreasing in s.

Thus, we have shown that $E[\Delta(M,s)|s]$ is nonincreasing in s, and so there exists some s^*, say, where $1 \leq s^* \leq \infty$, for which $E[\Delta(M,s)|s]$ is nonnegative for all $s \leq s^*$ and is strictly negative for all $s > s^*$. This implies that

$$(s - s^*)E[\Delta(M,s)|s] \leq 0 \qquad \text{for all } s.$$

Taking expectations over s and appealing to the law of iterated expectations, we have

$$E[s\Delta(M,s)] \leq s^*E[\Delta(M,s)],$$

which is nonpositive by (i). Thus, $E[s\hat{N}(M)] \leq E[sN(M,s)]$. That is, from (11), the fastest debt contract $(\hat{P},0)$ (weakly) dominates the debt contract (P,T).

QED

Proof of Lemma 1

We first need to calculate C's net payoff ν, say, in some state (R_1,L,s) if *after* having played some mechanism, the parties reach a particular (P,ρ) node with $0 < \rho < 1$. In what follows, we suppose that D carries over an amount of cash $T \geq 0$ from date 0.

There are three regions to consider, depending on the size of P: $L + R_1 + T < P$ (region 1); $L < P \leq L + R_1 + T$ (region 2); and $P \leq L$ (region 3).

In region 1, D does not have enough cash to pay P, and so must default. From (7), C's payoff, net of T, is

$$M - T(1 - \alpha(1 - 1/s)) \equiv \nu^1.$$

In region 2, D can only pay P by augmenting L from his private cash holdings $R_1 + T$. (It is clear that, given $s \equiv R_2/L$ and $\rho < 1$, D will always use the firm's assets L in preference to his own.) If D pays P, then with probability ρ he keeps control over the assets, which is an efficient outcome. With probability $1 - \rho$, C gets control, in which case they may renegotiate. The renegotiation starts from the status quo: (a) the liquidation value of the remaining assets is zero (since D used them all to contribute L toward the payment P); and (b) D's private cash holdings have gone down to $R_1 + T - (P - L) = V + T - P$. From (7) we deduce that, provided D pays P, C's net payoff is[38]

$$P + (1 - \rho)(\alpha(V + T - P)(1 - 1/s)) - T \equiv \nu^2.$$

If $\nu^2 > \nu^1$, D defaults, and C gets ν^1.

In region 3, D can pay P from the firm's assets L. Again, if D pays P, then with probability ρ he keeps control over the assets, which is an efficient outcome. And with probability $1 - \rho$, C gets control, in which case they may renegotiate. The renegotiation starts from the status quo: (a) the liquidation value of the remaining assets is $L - P$ (since D used the remainder to pay the P); and (b) D's private cash holdings are intact at $R_1 + T$. From (7) we deduce that, provided D pays P, C's net payoff is

$$P + (1 - \rho)(L - P + \alpha(R_1 + T)(1 - 1/s)) - T$$
$$= \rho P + (1 - \rho)(M + \alpha T(1 - 1/s)) - T \equiv \nu^3.$$

If $\nu^3 > \nu^1$, D defaults, and C gets ν^1.

Observe that ν^1 is independent of P, whereas ν^2 and ν^3 are both nondecreasing in P. Also, $\nu^1 \leq \nu^2$ at the boundary of regions 1 and 2. Finally, $\nu^2 - \nu^3$ is nondecreasing in P, with $\nu^2 = \nu^3$ at the boundary of regions 2 and 3. Putting these facts together, we conclude that, in all three regions, C's net payoff ν is given by

$$\nu = \min(\nu^1, \max\{\nu^2, \nu^3\}).$$

From this we can deduce three things about ν. First, ν is never more than M (since $\nu \leq \nu^1 \leq M$). Second, ν is a function of the three variables M, s, and V, and is nondecreasing in all of them. Third, if $\alpha = 0$, ν is independent of s and V.

In effect, we can identify a state by the realization of the triplet $(M, \alpha s, \alpha V) \equiv z$, say. For a given terminal node (P,ρ) of the message game, denote C's payoff in state z by $\nu((P,\rho)|z)$. We have shown that $\nu((P,\rho)|z) \leq M$ and that $\nu((P,\rho)|z)$ is nondecreasing in $z = (M, \alpha s, \alpha V)$.

Consider two states $z = (M, \alpha s, \alpha V)$ and $z' = (M', \alpha s', \alpha V')$, for which $M' \leq M$, $\alpha s' \leq \alpha s$, and $\alpha V' \leq \alpha V$.

For a message-game contract, suppose that C and D play

38. Notice that here we are appealing to the fact that the parties are risk neutral and the technology is linear ($s \equiv R_2/L$), so that there are no gains from negotiating prior to the lottery.

strategies $[\sigma_C,\sigma_D]$ and $[\sigma_C',\sigma_D']$ in states z and z', respectively. (These strategies may be mixed.) And suppose that the mechanism specifies respective lotteries over (P,ρ) pairs $(\tilde{P}[\sigma_C,\sigma_D],\tilde{\rho}[\sigma_C,\sigma_D])$ and $(\tilde{P}[\sigma_C',\sigma_D'],\tilde{\rho}[\sigma_C',\sigma_D'])$. (These are lotteries given that the strategies may be mixed.)

In state z, since C prefers σ_C to σ_C',

$$\text{(i)} \qquad E\nu((\tilde{P}[\sigma_C,\sigma_D],\tilde{\rho}[\sigma_C,\sigma_D])|z) \geq E\nu((\tilde{P}[\sigma_C',\sigma_D],\tilde{\rho}[\sigma_C',\sigma_D])|z),$$

where E is the expectations operator taken over lotteries.

Equally, in state z', D prefers σ_D' to σ_D. Remembering that all outcomes lie on the (constrained) frontier, we may view this in terms of C's payoff:[39]

$$\text{(ii)} \qquad E\nu((\tilde{P}[\sigma_C',\sigma_D'],\tilde{\rho}[\sigma_C',\sigma_D'])|z') \leq E\nu((\tilde{P}[\sigma_C',\sigma_D],\tilde{\rho}[\sigma_C',\sigma_D])|z').$$

Finally, since $\nu((P,\rho)|z)$ is nondecreasing in z,

$$\text{(iii)} \qquad E\nu((\tilde{P}[\sigma_C',\sigma_D],\tilde{\rho}[\sigma_C',\sigma_D])|z) \geq E\nu((\tilde{P}[\sigma_C',\sigma_D],\tilde{\rho}[\sigma_C',\sigma_D])|z').$$

The left-hand sides of (i) and (ii) are C's equilibrium net payoffs in states z and z', respectively. Combining (i), (ii), and (iii), we obtain (14) and (15) in Lemma 1.

Condition (13) is an immediate consequence of the fact that $\nu((P,\rho)|z) \leq M$.

QED

Proof of Proposition 5

We can shorten the proof by bringing parts (1) and (2) together: in the light of (15), let us identify a state by $(M,\alpha s,\alpha V)$, and write C's payoff as $N(M,\alpha s,\alpha V)$, rather than $N(M,s,V)$.

Take any $N^0(.\,,.\,,.)$ satisfying the three constraints:

$$\text{(i)} \qquad E[N(M,\alpha s,\alpha V)] \geq I - w;$$

$$\text{(ii)} \qquad N(M,\alpha s,\alpha V) \leq M;$$

(iii) $N(M,\alpha s,\alpha V)$ is nondecreasing in M, αs, and αV.

39. Here we again appeal to the fact that, given $s \equiv R_2/L$, the frontier is linear (it has no kink)—so that, because the parties are risk neutral, lotteries are (constrained) efficient.

We proceed in two steps. First, we "flatten" N^0 in the $\alpha s - \alpha V$ plane. For each M, consider

$$\text{(iv)} \qquad N^1(M) \equiv E[N^0(M,\alpha s, \alpha V)|M].$$

$N^1(M)$ obviously continues to satisfy (ii) and, by construction, satisfies (i). $N^1(M)$ is nondecreasing in M, thanks to affiliation.[40] Moreover, the minimand in (16) has (weakly) decreased. To see this, use the law of iterated expectations:

$$\begin{aligned} &E[s(N^0(M,\alpha s,\alpha V) - N^1(M))] \\ &\quad = E[E[s(N^0(M,\alpha s,\alpha V) - N^1(M))|M] \\ &\quad \geq E[E[s|M]E[(N^0(M,\alpha s,\alpha V) - N^1(M))|M]] \\ &\quad = 0 \text{ by (iv)}, \end{aligned}$$

where the inequality follows from affiliation.[41]

The second step in the proof is to replace $N^1(M)$ by

$$N^2(M) \equiv \min\{M,P\},$$

where P solves $E[\min\{M,P\}] = I - w$. This implies that

$$\text{(v)} \qquad E[N^1(M)] \geq I - w = E[N^2(M)].$$

$N^2(M)$ obviously satisfies (ii) and (iii) and, by construction, satisfies (i). To confirm that the minimand in (16) has (weakly) decreased, suppose that M takes the J values $M_1 > \ldots > M_j > \ldots > M_J$. For $1 \leq j \leq J$, let $\pi_j(s)$ be the probability that $M = M_j$ conditional on s. By inspection, there exists some j^*, where $1 \leq j^* \leq J$, such that

$$N^1(M_j) > N^2(M_j) \qquad \text{for } 1 \leq j \leq j^*$$

and

$$N^1(M_j) \leq N^2(M_j) \qquad \text{for } j^* + 1 \leq j \leq J.$$

Define

$$\Delta(s) \equiv E[(N^1(M) - N^2(M))|s].$$

40. See Theorem 23(iii) in Milgrom and Weber [1982], with their $Z \equiv (M,\alpha s,\alpha V)$, their $g(Z) \equiv N^0(M,\alpha s,\alpha V)$, and for $M_1 > M_2$, their sublattice $S \equiv A_1 \cup A_2$, where $A_j = \{(M,\alpha s,\alpha V)|\ M = M_j\}$, $j = 1, 2$. Taking $A = A_1$ and $\overline{A} = A_2$, we find that $N^1(M_1) \geq N^1(M_2)$.

41. See Theorem 23(ii) of Milgrom and Weber [1982], with their $Z \equiv (M,\alpha s,\alpha V)$, their $g(Z) \equiv s$, and their $h(Z) \equiv N^0(M,\alpha s,\alpha V) - N^1(M)$, conditioning on the sublattice $S = \{(M,\alpha s,\alpha V)|M\}$. On this sublattice, both g and h are nondecreasing functions.

Then for $s^+ > s^-$,

$$\begin{aligned}\Delta(s^+) &= \sum_{j=1}^{J} \pi_j(s^+)[N^1(M_j) - N^2(M_j)] \\ &= \sum_{j=1}^{J} \pi_j(s^-)\left(\frac{\pi_j(s^+)}{\pi_j(s^-)}\right)[N^1(M_j) - N^2(M_j)] \\ &\geq \left(\frac{\pi_{j*}(s^+)}{\pi_{j*}(s^-)}\right)\sum_{j=1}^{J} \pi_j(s^-)[N^1(M_j) - N^2(M_j)] \\ &= \left(\frac{\pi_{j*}(s^+)}{\pi_{j*}(s^-)}\right)\Delta(s^-),\end{aligned}$$

where the inequality follows from affiliation.[42] Hence $\Delta(s^-) \geq 0$ implies that $\Delta(s^+) \geq 0$. That is, $\Delta(s)$ exhibits single crossing: there exists some s^* such that

$$(s - s^*)[\Delta(s) - \Delta(s^*)] \geq 0 \qquad \text{for all } s.$$

Taking expectations and applying the law of iterated expectations, we have

$$E[s(N^1(M) - N^2(M))] \geq s^* E[N^1(M) - N^2(M)],$$

which is nonnegative by (v). Thus the minimand in (16) has (weakly) decreased.

QED

Proof of Proposition 6

Here we shall first prove that, without any distributional assumptions, $c(I)$ is increasing and piecewise linear in the interval $w \leq I \leq w/(1 - Em)$, and that the slope of $c(I)$ equals Es for $w \leq I < w/(1 - \underline{m})$. Next, we will prove that if $E[s|m]$ is nondecreasing in m, $c(I)$ is convex. The rest of the Proposition then follows directly from (19).

Let m take the values $m_1 > \ldots > m_j > \ldots > m_J$, with associated probabilities $\pi_j > 0, j = 1, \ldots, J$. For $j = 1, \ldots, J$, define

$$\mu_j \equiv 1 - m_j \sum_{k=1}^{j} \pi_k - \sum_{k=j+1}^{J} \pi_k m_k.$$

42. A direction consequence of Theorem 24 of Milgrom and Weber [1982] is that

$$\pi_j(s^+)/\pi_j(s^-)$$

is nonincreasing in j.

Now $\mu_1 < \ldots < \mu_j < \ldots < \mu_J$. Note that $\mu_J = 1 - m_J$ and $\mu_1 = 1 - Em$, which is strictly positive by assumption. For notational convenience, let $m_{J+1} = 0$ and $\mu_{J+1} = 1$.

We partition $[w, w/\mu_1]$ into J regions $R_J \cup \ldots \cup R_j \cup \ldots R_1$, where

$$R_j \equiv [w/\mu_{j+1}, w/\mu_j] \qquad j = 1, \ldots, J.$$

The regions R_j, $1 \leq j \leq J$, are defined so that for $I \in R_j$ the $P = P(I)$, say, that solves $E[\min\{P, Im\}] = I - w$ lies between Im_{j+1} and Im_j. The slope of $P(I)$ in the interior of region R_j is

$$\beta_j = \frac{1 - \Sigma_{k=j+1}^{J} \pi_k m_k}{\Sigma_{k=1}^{j} \pi_k} \qquad i \leq j \leq J.$$

Notice that

(i) $$\beta_j - m_j = \frac{\mu_j}{\Sigma_{k=1}^{j} \pi_k} > 0.$$

And, for future reference, observe that, for $2 \leq j \leq J$,

(ii) $$(\beta_{j-1} - \beta_j) \sum_{k=1}^{j-1} \pi_k - \pi_j(\beta_j - m_j) = 0,$$

which, from (i), implies that $\beta_{j-1} > \beta_j$.

Now for I in the interior of region R_j, $1 \leq j \leq J$, the slope of $c(I)$ equals

$$\beta_j \sum_{k=1}^{j} \pi_k s_k + \sum_{k=j+1}^{J} \pi_k s_k m_k \equiv \phi_j,$$

where $s_j \equiv E[s|m_j]$. Each ϕ_j is a positive constant for $j = 1, \ldots, J$. Hence, since $c(I)$ is continuous across the boundaries of the regions, we deduce that $c(I)$ is increasing and piecewise linear in $w \leq I \leq w/\mu_1$. If $m_j = \underline{m} > 0$, then in the interval $[w, w/(1 - \underline{m}))$ the slope of $c(I)$, ϕ_J, equals $\beta_J \Sigma_{k=1}^{J} \pi_k s_k = Es$.

If $E[s|m]$ is nondecreasing in m, then $s_1 \geq \ldots \geq s_J$. Now for $2 \leq j \leq J$,

(iii) $$\phi_{j-1} - \phi_j = (\beta_{j-1} - \beta_j) \sum_{k=1}^{j-1} \pi_k s_k - \pi_j s_j(\beta_j - m_j).$$

But since $s_1 \geq \ldots \geq s_k \geq \ldots \geq s_j$, and $\beta_{j-1} > \beta_j$, the right-hand side of (iii) is no less than s_j times the left-hand side of (ii). That is, $\phi_{j-1} \geq \phi_j$ for all $2 \leq j \leq J$, and $c(I)$ is convex.

QED

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