Cooperative Investments and the Value of Contracting

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Recent articles have shown that contracts can support the efficient outcome for bilateral trade, even in the face of specific investments and incomplete contracting. These studies typically considered "selfish" investments that benefit the investor (e.g., the seller's investment reduces her production costs). We find very different results for "cooperative" investments that directly benefit the investor's partner (e.g., the seller's investment improves the buyer's value of the good). Most importantly, if committing not to renegotiate the contract is impossible, then contracting has no value, i.e., the parties cannot do better than to abandon contracting altogether in favor of ex post negotiation. (JEL C70, J41, K12, L22)

Holdups arise in bilateral trade when specific investments are involved and contracting is incomplete. Specific investments, which create more value inside a relationship or a transaction than outside, render market forces unreliable in curbing the parties' opportunistic behavior. Contracts could be used to correct opportunism if investment-related information can be specified. That is often difficult, though. The resulting incompleteness of contracts, together with opportunistic expropriation by the trading partner, means that parties will invest at less than the socially optimal levels (Paul Grout, 1984; Oliver Williamson, 1985; Jean Tirole, 1986; Jerry Green and Jean-Jacques Laffont, 1988; Oliver Hart and John Moore, 1988). The holdup problem has led many authors to propose various organizational interventions as remedies, including vertical integration (Benjamin Klein et al., 1978; Williamson, 1979), exchanging hostages (Williamson, 1983), shifting property rights (Sanford Grossman and Hart, 1986; Hart and Moore, 1990), allocating control rights (Philippe Aghion and Patrick Bolton, 1992), and designing an authority relationship (Aghion and Tirole, 1997).2

Recently, however, the incomplete contracting paradigm has been challenged by several authors who argue that simple (incomplete) contracts can solve the holdup problem. Tai-Yeong Chung (1991), Aghion et al. (1994), and Georg Nöldeke and Klaus Schmidt (1995) analyze contracts that, while incomplete, can achieve the efficient outcome. In these papers, form, such as human capital investment. Also, the investor might be able to shift costs from one investment to another. Contracting on the gains to trade is also problematic because specific investments often deliver benefits that are nonstandard and idiosyncratic, making objective measures of the benefits unlikely to be available.

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1 The difficulty is for many reasons. For instance, specific investments often take a nonmonetary, intangible form, such as human capital investment. Also, the investor might be able to shift costs from one investment to another. Contracting on the gains to trade is also problematic because specific investments often deliver benefits that are nonstandard and idiosyncratic, making objective measures of the benefits unlikely to be available.

2 The focus of this paper is contracting, but our conclusions section briefly discusses implications of our results for some of these organizational responses.
if the contract specifies an ex post inefficient level of trade, the parties renegotiate to the efficient level. An important aspect of renegotiation in all of these papers is that one party holds the entire bargaining power. That party, by essentially becoming a residual claimant to the transaction, has the incentive to invest efficiently. While the schemes developed in these papers are ingenious, the assumption that bargaining power can be manipulated ex ante and enforced seems incongruous with an environment that renders contracting incomplete. For example, in Chung’s model, the initial contract stipulates that one party has the right to make a take-it-or-leave-it offer in the renegotiation phase. The other party can accept the offer or reject it, which ends the game. Such a scheme may not work if the parties cannot commit to end the game in the event of a rejected offer (and not engage in counteroffers).

Aaron Edlin and Stefan Reichelstein (1996a, b) avoid criticism about ex ante manipulation of bargaining power. They suppose that the parties have exogenously specified bargaining power. Their scheme begins with a simple contract that specifies a fixed trade price and quantity. After realizing the gains to trade, the parties renegotiate to the quantity that is ex post efficient, with the bargaining surplus divided between them according to their relative bargaining strengths. This renegotiation process by itself generates less than full incentives for the investments, leading to underinvestment. However, each party has an additional incentive to invest in order to improve its status quo (disagreement) outcome. (Increased investment by the seller to reduce cost, for example, improves her status quo payoff by lowering her cost of producing the status quo quantity.) Edlin and Reichelstein show that, with certain conditions, an appropriately chosen initial contract can provide the right incentives for investments.

Edlin and Reichelstein’s efficiency result is limited by their restriction on the nature of the specific investments. They, and most of the literature on specific investments, suppose that a seller invests to reduce her cost or a buyer invests to increase his benefit from the procured good or service (henceforth called ‘‘selfish investments’’). Our interest is in ‘‘cooperative investments’’ that generate a direct benefit for the trading partner. A cooperative investment is ‘‘pure’’ if it offers no (or negative) accompanying direct benefits to the investor, and it is ‘‘hybrid’’ if it offers direct benefits to both parties, i.e., has both cooperative and selfish elements.

Cooperative investments have received little attention, despite being common in practice and present in several classic settings. For example, the famous General Motors-Fisher Body example deals with Fisher Body’s decision of whether to build a plant adjacent to General Motors. Such an arrangement, by lowering shipping costs and improving supply reliability, offered benefits to both parties (i.e., a hybrid investment). Another important example is the principal–agent literature, which, while not explicit about it, generally analyzes cooperative investments (i.e., effort made by the agent that directly benefits the principal).

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5 W. Bentley MacLeod and James Malcomson (1993) and Che and Chung (1999) are the first theoretical treatments of cooperative investments. The latter paper derives a special case of the irrelevance of contracting result that we develop here. It assumes a binary production technology and deals only with fixed-price contracting schemes. (Its focus is on alternative legal rules for breach of a contract.) B. Douglas Bernheim and Michael Whinston (1998) also observe difficulties with motivating cooperative investments.

6 In the standard principal–agent setting, the agent’s effort is unobserved by the principal but there exists a verifiable signal of that investment (e.g., quantity or quality of output) which is contractible (see, for example, Bengt Holmstrom [1979]). In our setting, both parties observe several signals of investments, including their levels, but none of them are verifiable.
Other common examples of cooperative investments include quality-enhancing R & D efforts by suppliers and workers simply “paying attention” to their jobs.

Cooperative investments are critically important in modern manufacturing, where the adoption of quick-response inventory systems and flexible manufacturing approaches has increased the need for coordination across different production stages. Such coordination often requires investments of time and resources that have cooperative elements. For example, Banri Asanuma (1989 p. 14) describes how suppliers customize parts for buyers even when “specific investments ... have to be incurred to implement such customization.” Also, Toshihiro Nishiguchi (1994 p. 138) reports that suppliers “send engineers to work with [automakers] in design and production. They play innovative roles in ... gathering information about [the automakers'] long-term product strategies.” David Burt (1989) describes how demands on suppliers can go beyond just designing components to developing specifications to test equipment and acquiring test software.

It can also be the buyer who makes a cooperative investment. Xerox incorporates supplier-designed components into many of its products, which requires idiosyncratic adaptations of production lines and procedures to individual suppliers (Burt, 1989). An electrical utility can customize its turbines to accommodate the lower grade of coal produced by a nearby mine. Some Japanese automakers pay for consultants to work with suppliers, possibly for months, to improve production methods (Jeffrey Dyer and William Ouchi, 1993). After Honda chose Donnelly Corporation as its sole supplier of mirrors for its U.S.-manufactured cars, “Honda sent engineers swarming over the two Donnelly plants, scrutinizing the operations for kinks in the flow. Honda hopes Donnelly will reduce costs about 2% a year, with the two companies splitting the savings” (Myron Magnet, 1994). Vauxhall “regularly works in partnerships with suppliers to improve efficiency and trim costs. It reckons to have helped suppliers reduce costs by 30-40% of late” (Engineer, 1996).

Retail examples of cooperative investments abound, too. Kraft created cross-functional business teams for its major retail customers. After a six-month study, one team recommended a reorganization of a retailer’s dairy case and the introduction of new products. The retailer experienced a 22-percent increase in volume and fewer stock-outs, and Kraft realized a similar increase in its sales through better positioning of its high-demand products (Nirmalya Kumar, 1996).

In political lobbying or other settings involving exchange of favors, the party who first executes its favor can be viewed as making a cooperative investment if there is a relationship-specific element to its action. For instance, when an individual contributes to the campaign of a politician in the hope of gaining political influence, the contribution is a “sunk investment” since there is no binding assurance that the favor will be repaid.

Cooperative and selfish forms of specific investments both face the threat of postcontractual holdup. While complete contracts can solve the holdup problem for either form of investment, we show that incomplete contracts perform very differently with these two forms. Our findings are as follows. First, regardless of the degree to which the investments are cooperative, efficiency can be realized if the parties credibly commit not to renegete their contract. Our second and more important result treats the case in which a commitment not to renegotiate is impossible for the parties. Like Edlin and Reichelstein (1996a, b), we suppose that the bargaining shares of the parties are exogenously specified. We show that if investments are sufficiently cooperative, then there exists an intermediate range of bargaining shares for which contracting has no value; i.e., contracting offers the parties no advantages over ex post negotiation. As the investments become more cooperative, the range for which contracting is worthless expands; and it covers the entire range if both investments are purely cooperative. Lastly, even if the cooperative nature of the investments is sufficiently weak that contracting has value, an efficient outcome may not be possible. The reason that contracts perform differently for the two types of investments can be traced to the different impacts that they have on the
status quo position of the investor. Selfish investments improve the status quo position of the investor. Cooperative investments have the opposite effect: they worsen the investor’s bargaining position by improving the status quo position of the partner (e.g., a higher investment by the seller to increase the buyer’s value simply raises the buyer’s value from the default transaction).

The limited nature of contracting has been noted by others. Stewart Macaulay (1963) observed that business transactions often do not resort to explicit contracts. Kathryn Spier (1992), Mathias Dewatripont and Eric Maskin (1995), and Bernheim and Whinston (1998) show that, while contracting is valuable, the parties may rationally choose to leave the contract incomplete. Closest to our result is Ilya Segal (1999), who shows that contracting becomes worthless as the complexity of the environment increases without bound. His focus on selfish investments distinguishes the current paper.\footnote{Segal assumes that investment affects only the efficient trade choice, so the Edlin and Reichelstein (1996a, b) approach is not applicable.}

An example in the next section illustrates the difficulties of contracting with cooperative investments, as compared with selfish investments. Section II describes our model. Section III develops two benchmark outcomes: the efficient outcome and the outcome resulting from ex post negotiation with no initial contract. Assuming that the parties can prevent renegotiation, Section IV establishes an efficiency result. Section V studies the case of renegotiation and develops our main result on the irrelevance of contracting. Conclusions appear in Section VI.

I. An Illustrative Example

The basic differences between cooperative and selfish investments can be illustrated with a simple example. Suppose that a buyer procures 0 or 1 units of a good from a seller. The seller’s cost of production is zero. The buyer’s value of procuring the good is \( v(e) \), where \( e \) is a relationship-specific investment (measured in the monetary unit). The specific investment is selfish if the buyer makes the investment and cooperative if the seller makes the investment. Assume that \( v(e) \) is continuously differentiable, bounded above, strictly concave, and satisfies \( v(0) = 0 \) and \( v'(0) > 1 \). Then, the first-best outcome has trade occur after an investment, \( e^* \), that maximizes \( v(e) - e \).

Suppose first that there is no contract before the investment; instead, the parties evenly split their surplus \( v(e) \). Then, each party receives \( \frac{1}{2}v(e) \), and the investing party (whether it is the buyer or the seller) chooses \( e^* \) to maximize \( \frac{1}{2}v(e) - e \). Clearly, \( e^* \) falls short of \( e^* \).

Next suppose that the parties sign a contract, prior to the investment decision, that specifies a simple fixed price \( p \in [0, v(e^*) - e^*] \) at which trade is to occur (as in Aghion et al., 1994; Edlin and Reichelstein, 1996a, b). If the buyer makes the investment (i.e., a "purely selfish investment"), such a contract induces the first-best outcome, since the buyer receives \( v(e) - p - e \), which is maximized at \( e = e^* \).

However, if the seller makes the investment (i.e., a "purely cooperative investment"), her payoff of \( p - e \) leads her to choose zero investment.

The first-best outcome can be achieved in the cooperative investment case through the use of a more sophisticated contract, assuming that the parties can commit not to renegotiate the contract. Suppose that the contract specifies an option for the buyer to purchase the good at the price of \( p = v(e^*) \). The buyer will exercise that option if \( v(e) \geq v(e^*) \) and reject the good if \( v(e) < v(e^*) \). Given this response by the buyer, the seller chooses \( e = e^* \). This first-best outcome relies on the parties believing that the buyer’s rejection of the good results in no trade; i.e., there is no possibility of renegotiation.

Suppose, however, that the parties cannot credibly commit not to renegotiate, and let \( p \geq 0 \) be the option price specified in the contract. Suppose that the buyer rejects the good at the contract price. Then, since trade is mutually beneficial ex post, the parties will renegotiate to reverse the no-trade decision and split the surplus, giving the buyer a payoff of \( \frac{1}{2}v(e) \). Comparing this payoff from rejecting the good to the payoff of \( v(e) - p \) from accepting the good, we see that the buyer will
reject the good if and only if \( \frac{1}{2} \nu(e) < p \). Given this response, the seller will receive \( \frac{1}{2} \nu(e) \) if \( \frac{1}{2} \nu(e) < p \) and \( p \) if \( \frac{1}{2} \nu(e) > p \). Hence, the seller chooses \( e \) to maximize

\[
\min \{ \frac{1}{2} \nu(e), p \} - e.
\]

Clearly, the investment level that solves this problem cannot exceed \( e^* \), regardless of \( p \). In fact, the best contract must specify \( p = \frac{1}{2} \nu(e^*) \), in which case \( e^* \) is attained. As is shown here, the combination of cooperativeness and renegotiation undermines the value of contracting completely. Below we establish this irrelevance of contracting result much more generally.

II. Model

We consider a two-stage game between a buyer and a seller, both of whom are risk neutral. In the first stage, the buyer and the seller make relation-specific investments \( b \geq 0 \) and \( s \geq 0 \), respectively. In the second stage, the level of trade, \( q \geq 0 \), is determined. The buyer's (gross) value from purchasing \( q \) is \( v(q, e, b, s) \), and the seller's cost of producing \( q \) is \( c(q, e, b, s) \), where \( e \) is a random variable drawn from \([0, 1]\) by the continuously differentiable cumulative distribution function \( F(\cdot) \). It is useful to define the state as

\[
\theta = (e, b, s) \in \Theta = [0, 1] \times \mathbb{R}_+^2.
\]

Many existing models restrict attention to selfish investments (see Chung, 1991; Nödeke and Schmidt, 1995; Edlin and Reichelstein, 1996a, b). Our specifications of the value and cost functions are more general, though, allowing investment by the seller (respectively buyer) to affect both the buyer's value and the seller's cost of production.\(^8\) We make the following assumptions.

\(^8\)Che and Hausch (1997) show that the results in this paper extend to the setting in which investment choices not only directly affect the buyer's value and the seller's cost, but also indirectly affect them through the random variable, i.e., the distribution of the random variable is a function of \( s \) and \( b \).

ASSUMPTION 1: \( v \) and \( c \) are continuously differentiable in all arguments.

ASSUMPTION 2: For any \( \theta \in \Theta \), \( v(0, \theta) = c(0, \theta) = 0 \), \( v(q, \theta) \) and \( c(q, \theta) \) are nondecreasing in \( q \), and \( v(q, \theta) - c(q, \theta) \) is negative for \( q > M \) for some \( M < \infty \).

ASSUMPTION 3: For any \( q > 0 \), \( v(q, \theta) - c(q, \theta) \) is nondecreasing in \( \theta \), and bounded above by some \( N < \infty \) for any \( (q, \theta) \in \mathbb{R}_+ \times \Theta \).

Assumptions 1–3 imply that, for any realized state \( \theta \in \Theta \), the maximum net joint surplus,

\[
\phi(\theta) = \max_{q \geq 0} v(q, \theta) - c(q, \theta),
\]

is well defined, differentiable, and is bounded above for all \( \theta \in \Theta \).

The Nature of Investments. —The nature of the specific investments can be characterized by the identities of the parties that directly benefit from the investments. The seller's investment, \( s \), is cooperative if \( v_s(q, e, b, s) > 0 \) for \( q > 0 \), and selfish if \( c_s(q, e, b, s) < 0 \) for \( q > 0 \). (Throughout, a subscript is used to denote a partial derivative.) Likewise, the buyer's investment is cooperative if \( c_e(q, e, b, s) < 0 \) for \( q > 0 \), and selfish if \( v_e(q, e, b, s) > 0 \) for \( q > 0 \). Note that an investment can be hybrid, i.e., simultaneously cooperative and selfish.

Timing. —Figure 1 depicts the sequence of events. The parties contract at date 0. At date 1, the parties invest, followed by the realization of \( e \) at date 2. At date 3, if the initial contract involves a menu, the specific contract terms are chosen (by the initially designated party, such as the buyer, seller or both). The chosen contract terms are then enforced, unless they are renegotiated. We consider two possibilities for renegotiation. First, the parties can commit not to renegotiate the contract, so the terms of contract chosen at date 3 are enforced at date 4. In the second possibility, the parties cannot commit not to renegotiate the contract, in which case renegotiation can take
place between date 3 and date 4. Throughout, we focus on subgame-perfect equilibrium as a solution concept.

**Information.**—We assume that both parties observe \((e, b, s)\) after date 2, but that these variables are not verifiable, so no other parties can observe them. This assumption precludes contracts directly contingent on these variables. The quantity of trade and transfer payments exchanged between the parties, and the parties’ reports about \((e, b, s)\) are verifiable, however. The most general form of contracting can therefore specify a menu of quantity-transfer pairs as functions of the parties’ reports about \((e, b, s)\).\(^9\)

**Renegotiation.**—We assume that renegotiation results in an efficient quantity of trade and that both parties appropriate the entire net trade surplus (i.e., budget balancing). This latter requirement means that the parties cannot credibly give away resources to a third party.\(^10\) The bargaining positions of the parties are exogenously determined. Specifically, the parties share the surplus from bargaining, with the seller receiving a fraction \(\alpha \in [0, 1]\).\(^11\)

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**III. Two Benchmark Outcomes**

This section determines both the first-best and Williamson outcomes. The latter assumes ex post negotiations without an initial contract. Since the parties receive less than full marginal returns to their investments, they will be seen to make inefficient investment choices. These two regimes provide useful benchmarks against which later results may be compared.

**The First-Best Outcome.**—The first-best outcome requires that, for each state \(\theta \in \Theta\), trade be at an efficient level \(q \in \text{argmax}_{q'} q' = 0 \quad \nu(q', \theta) = c(q', \theta)\). Given efficient trading, the first-best outcome further requires that the investments, \((b, s)\), maximize the total expected gains to trade:

\[
W^*(b, s) = \Phi(b, s) - b - s,
\]

where

\[
\Phi(b, s) = \int_0^B \phi(e, b, s) dF(e).
\]

We make the following additional assumption:

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\(^9\) This most general contract can be enforced with a specific performance damage clause (see Edlin and Reichelstein, 1996b). While the judicial cost associated with enforcing the most general menu contract may limit its feasibility, we allow it to establish the most general statement about irrelevance of contracting.

\(^10\) A commitment to make payments to a third party may enable the buyer and seller to avoid renegotiation, which undermines our subsequent result on the irrelevance of contracting. We thank Ray Deneckere for raising this issue.

\(^11\) It is possible to specify an underlying bargaining game that corresponds to a constant bargaining share, e.g., a generalized Nash bargaining game or a Rubinstein bargaining game with different discount factors. Appendix A of Edlin and Reichelstein (1996b) develops a more fully specified model of renegotiation with specific performance, using Myerson’s bargaining model, which also applies here. For some part of the paper, they consider a monotone sharing rule, which is more general than the constant sharing rule. All of our results hold with the monotone sharing rule as long as the share is sufficiently close to being constant, but only with a considerable loss of expository clarity.
ASSUMPTION 4: $W^*(\cdot, \cdot)$ is strictly concave.\footnote{$W^*(\cdot, \cdot)$ is strictly concave if, for example, $v(q, e, b, s) - c(q, e, b, s)$ is sufficiently concave in $(b, s)$ for all $(q, e)$ so that $\phi(e, b, s)$ is strictly concave in $(b, s)$.} Its unique maximizer, $(b^*, s^*)$, has at least one investment that is strictly positive.

The pair of first-best investments, $(b^*, s^*)$, can be characterized as follows. Let $b = B^*(s)$ denote the efficient response of $b$ given $s$, i.e.,

\begin{equation}
B^*(s) = \arg \max_{\tilde{b}} \Phi(\tilde{b}, s) - \tilde{b},
\end{equation}

and let $s = S^*(b)$ denote the efficient response of $s$ given $b$, i.e.,

\begin{equation}
S^*(b) = \arg \max_{\tilde{s}} \Phi(b, \tilde{s}) - \tilde{s}.
\end{equation}

(By Assumptions 3 and 4, these functions are well defined and bounded above.) Then, the efficient pair satisfies $b^* = B^*(s^*)$ and $s^* = S^*(b^*)$. These response functions and the efficient investment pair are graphically illustrated in Figure 2.

**No Contracting "Williamson" Game.**—Now suppose that the parties do not write a contract initially and instead simply bargain *ex post* to determine the terms of trade after the realization of $\theta$. This situation is precisely what Williamson considered as the market transaction. Since bargaining is efficient, an efficient quantity will be chosen. The buyer and the seller internalize $1 - \alpha$ and $\alpha$ fractions of the net surplus, respectively. At date 0, each party chooses his or her investment level to maximize his or her expected gains from trade, given that the other party does the same, i.e., their choices form an equilibrium.

Let $b = B(s)$ be the buyer’s best response to $s$, which maximizes

\begin{equation}
U_b^*(b, s) = (1 - \alpha)\Phi(b, s) - b,
\end{equation}

\begin{equation}
U_s^*(b, s) = \Phi(b, s) - \beta s.
\end{equation}
given \( s \), and let \( s = S(b) \) denote the seller's best response to \( b \), which maximizes

\[
U^s(b, s) = \alpha \Phi(b, s) - s,
\]

given \( b \). Both \( B(\cdot) \) and \( S(\cdot) \) are well defined by Assumptions 3 and 4, and an equilibrium pair of investments exists. For simplicity, we make the following assumption.

ASSUMPTION 5: There exists a unique equilibrium pair of investments, \((b^*, s^*)\), in the Williamson game.

Functions \( B(\cdot) \) and \( S(\cdot) \) and the equilibrium investment pair are graphed in Figure 2. Comparing (3) and (4) with (1) and (2), it is clear that \( B(\cdot) = B^*(\cdot) \) and \( S(\cdot) = S^*(\cdot) \). Furthermore, for all \( \alpha \in (0, 1) \), \( b^* < B^*(s^*) \) and \( s^* < S^*(b^*) \), whenever \( B^*(s^*) > 0 \) and \( S^*(b^*) > 0 \). In this sense, the investments are less than socially optimal. [This does not necessarily imply that \((b^*, s^*) < (b^*, s^*)\), although strict inequality must hold for at least one component.] This underinvestment result is well known and directly follows from the parties receiving only a fraction of the marginal returns to their investments. For the Williamson game, total expected gains to trade are \( W^*(b^*, s^*) < W^*(b^*, s^*) \).

IV. Contracting with Commitment

This section assumes that the parties can commit themselves not to renegotiate a contract. As we show, given this assumption, there is a contract that can implement the efficient outcome.

For each state \( \theta \in \Theta \), an outcome of trade is denoted as a pair, \((q(\theta), t(\theta))\), where \( q \) is quantity traded and \( t \) is a transfer from the buyer to the seller. An outcome function is a mapping, \((q, t) : \Theta \to \mathbb{R}_+ \times \mathbb{R}_+ \). With the commitment not to renegotiate, any outcome function that generates nonnegative surplus to each party in every state can be implemented as an equilibrium. A simple mechanism that works is the so-called "shoot the liar mechanism": both parties are asked to report their observed state \( \theta \in \Theta \), and if their reports match, then the outcome for that state is enforced; otherwise, both parties are penalized by zero trade and zero transfer. In this mechanism, each party has nothing to gain from lying, if the other party reports truthfully, so the chosen outcome function can be implemented as an equilibrium. This mechanism may admit multiple equilibria, some of which may implement an unintended outcome. Virtually all such outcome functions can be implemented as unique subgame-perfect equilibria, though, by using the sequential mechanism suggested by Moore and Rafael Repullo (1988). The important point is that, with the commitment not to renegotiate the initial contract, the information that the parties commonly observe can be costlessly revealed to a third party. Hence, a complete contract that depends on the true (more precisely truthfully revealed) state is possible. To achieve the first-best outcome, it therefore suffices to find an outcome function that provides the parties with the correct marginal returns to their investments.

PROPOSITION 1. Suppose that Assumptions 1–4 hold. If the parties can commit not to renegotiate their original contract, then the efficient outcome can be implemented as a subgame-perfect equilibrium.

PROOF:

Let \( q^*(\theta) \in \arg \max_q v(q, \theta) - c(q, \theta) \), and recall \( \phi(\theta) \), the maximum net joint surplus. Consider an outcome function in which \( q(e, b, s) = q^*(e, b, s) \) and \( t(e, b, s) = v(q^*(e, b, s), e, b, s) - \Phi(b, s^*) + T \), for an arbitrary constant \( T \). Such a profile satisfies

\[
\frac{1}{1 - \alpha} \Phi_s(b, s) - 1 < \Phi_s(b, s) - 1 \leq 0,
\]

which implies that, unless \( B^*(s^*) = 0 \), \( b^* < B^*(s^*) \). Similarly, \( s^* < S^*(b^*) \), unless \( S^*(b^*) = 0 \).

\[13\]

In this sense, the current result generalizes the efficiency results in the selfish investment contexts of William Rogerson (1992) and Benjamin Hermalin and Michael Katz (1993). After establishing our result, we learned of the efficiency result by Maskin and Tirole (1999). While the scope of their result is broader, encompassing unforeseen contingencies, our proof is more explicit in the construction of the equilibrium outcome profile.
the conditions of Moore and Repullo (1988) and thus can be implemented as a (unique) subgame-perfect equilibrium. Since the equilibrium quantity is efficient in the profile, it suffices to show that the outcome function yields the efficient investment choices \((b^*, s^*)\) as an equilibrium.

Suppose that the buyer chooses \(b^*\). Then, the seller's \(ex \ ante\) expected payoff associated with \(s\) is:

\[
\int_0^1 \left[ t(e, b^*, s) - c(q^*(e, b^*, s), e, b^*, s) \right] dF(e) - s
\]

\[
= \int_0^1 \left[ v(q^*(e, b^*, s), e, b^*, s) - c(q^*(e, b^*, s), e, b^*, s) \right] dF(e) - \Phi(b^*, s^*) + T - s
\]

\[
= \Phi(b^*, s) - \Phi(b^*, s^*) + T - s.
\]

Clearly, the seller will choose \(s^* = S^*(b^*)\) (if she initially participates in the contract).

Likewise, if the seller chooses \(s^* = S^*(b^*)\), the buyer has the \(ex \ ante\) expected payoff:

\[
\int_0^1 \left[ v(q^*(e, b, s^*), e, b, s^*) - t(e, b, s^*) \right] dF(e) - b
\]

\[
= \Phi(b, s^*) - T - b.
\]

Clearly, the buyer will also choose \(b^* = B^*(s^*)\), his first-best investment. Since \((b^*, s^*)\) are the first-best investments, there exists a \(T\) that induces participation of both parties.

The first-best outcome is especially easy to implement if only one party, say the seller, makes an investment. Then, a simple two-stage scheme works in which the investing party (the seller) announces a transfer and a quantity in the first stage (after the realization of the state), and the other party (the buyer) accepts or rejects the offer in the second stage, with rejection resulting in no trade and zero transfer. This scheme essentially makes the investing party a residual claimant, who then naturally has the incentive to choose the efficient quantity and investment.

A critical condition for the mechanism in Proposition 1 to work is a credible commitment not to renegotiate an initial contract. Without the commitment power, an efficient outcome may not arise. For instance, the desirable performance of the two-stage scheme relies on the enforcement of "no trade" if the buyer rejects the seller's offer. Yet, if zero trade is inefficient, the parties will have an incentive to renegotiate the outcome, with the resulting surplus divided according to their relative bargaining strengths. Hence, the mechanism fails to provide sufficient incentives for the seller to invest. In general, the renegotiation possibility can impede the parties' ability to generate contractual incentives for investments. We now address this issue for cooperative investments.

V. Contracting with Renegotiation

In this section, we suppose that the parties cannot credibly commit not to renegotiate. This assumption is consistent with the current judicial system under which any mutually agreed-upon modifications to a contract are enforceable; i.e., "those who make a contract, may unmake it."\(^{16}\) Although the

\(^{16}\) Beatty v. Guggenheim Exploration Co., 122 N.E. 378, pp. 387–88 (N.Y. 1919). This opinion by Justice Cardozo focuses on the chronology of agreements, as pointed out by Christine Jolls (1997). An original contract and a subsequent renegotiation of the contract represent two agreements by the parties, distinguished only in chronology. According to this view, the last one in time prevails. The Uniform Commercial Code (Sec. 2-209) allows any modifications to a contract, given a showing that they are motivated by (for example) "a market shift that makes performance come to involve a loss" (Jolls, 1997).
parties may still circumvent renegotiation by specifying a clause that penalizes themselves for attempting to renegotiate,\textsuperscript{17} courts are generally unwilling to enforce such nonmodification clauses (see Jolls, 1997). We do not argue here that renegotiation is unavoidable in all circumstances. Indeed, we believe that the ability to avoid renegotiation may be supported under some organizational forms. (See a remark on this point in our conclusion section.) Rather, our approach here is to take the possibility of renegotiation as a practical reality and explore its implications.

As mentioned, the possibility of renegotiation may undermine the incentives for investment. This problem can be overcome with purely selfish investments, as evidenced by Edlin and Reichelstein’s efficiency result. In stark contrast to their result, we show that, if investments are sufficiently cooperative, then not only is the first-best outcome unachievable, but contracting is irrelevant.

Before proceeding with the general analysis, we illustrate the intuition of the main result by exploring the contract scheme suggested by Edlin and Reichelstein (1996a, b).

A. The Value of Noncontingent Contracts

Edlin and Reichelstein suppose that the parties agree to a noncontingent initial contract (henceforth called a \textit{simple} contract) at date 0 that specifies \( \langle \bar{q}, \bar{t}, \rangle \), where \( \bar{q} \) denotes quantity and \( \bar{t} \) denotes a lump-sum transfer payment by the buyer. These contract terms can be renegotiated to an \textit{ex post} efficient quantity after the parties realize the state of nature, \( \theta \in \Theta \). Since our purpose here is mainly illustrative, we consider a special case where only the seller makes an investment. Specifically, assume that \( v(q, e, b, s) = V(q, e, s) \) and \( c(q, e, b, s) = C(q, e, s) \). Redefine \( \phi(e, s) \) and \( \Phi(s) \) correspondingly, with a slight abuse of notation.

Using backward induction, we begin with the last period. After date 3, the parties renegotiate the initial contract \( \langle \bar{q}, \bar{t}, \rangle \) to an \textit{ex post} efficient outcome.\textsuperscript{18} Given the initial contract and the subsequent renegotiation, the seller’s expected payoff from choosing \( s \) at date 1 is:

\[
U^E_s(s; \bar{q}) = E_v \left\{ \bar{t} - C(\bar{q}, e, s) + \alpha[\phi(e, s) - \{ V(\bar{q}, e, s) - C(\bar{q}, e, s) \}] \right\} - s.
\]

The term \( \bar{t} - C(\bar{q}, e, s) \) represents the seller’s status quo payoff under the initial contract. The next term represents the seller’s share of the surplus arising from renegotiation. As mentioned before, the seller captures \( \alpha \) fraction of the gains from renegotiation, which is the difference between the joint surplus under the original contract terms, \( V(\bar{q}, e, s) - C(\bar{q}, e, s) \), and the joint surplus that would result from renegotiation, \( \phi(e, s) \). The seller chooses her investment \( s \) to maximize \( U^E_s(s; \bar{q}) \). The equilibrium investment, if positive, satisfies the first-order condition:

\[
(5) \quad \frac{\partial U^E_s(s; \bar{q})}{\partial s} = \alpha \Phi'(s) - E_v \left\{ \alpha V_s(\bar{q}, e, s) + (1 - \alpha)C_s(\bar{q}, e, s) \right\} - 1 = 0.
\]

It is instructive to analyze the terms in (5). The first term, \( \alpha \Phi'(s) \), represents the marginal return to the investment generated through an \textit{ex post} bargaining process. This would be the only marginal return to the investment, had there been no contract. The term inside the expectation operator captures the effect that the investment has on the relative status quo position of the seller in renegotiation. Specifically, an investment by the seller increases her status quo position by \( -C_s(\bar{q}, e, s) \) and it increases the buyer’s status quo payoff by \( V_s(\bar{q}, e, s) \). The former effect \textit{improves} the seller’s bargaining position by \( - (1 - \alpha)C_s(\bar{q}, e, s) \).\textsuperscript{19} The latter

\textsuperscript{17} For example, Maskin and Tirole (1999) suggest a contract that specifies a large penalty that can be collected by a party upon evidence that his partner attempted to renegotiate. Such a contract creates a prisoner’s dilemma situation in which neither party initiates renegotiation in equilibrium.

\textsuperscript{18} Date 3 is irrelevant in this scheme since the initial contract is a single transfer-quantity pair.

\textsuperscript{19} An increase in a status quo position by a dollar reduces the gains from renegotiation by a dollar, the \( \alpha \) fraction of which the seller is entitled to, so the net gain for the seller is the \( (1 - \alpha) \) fraction of a dollar.
effect worsens the seller's bargaining position by $\alpha V_s(\bar{q}, e, s)$ since the improved status quo position of the buyer means a smaller gain from renegotiation, the $\alpha$ fraction of which the seller receives. The expectation term thus captures the net gain in the seller's bargaining position resulting from her investment.

A part of the analysis in this subsection can be simplified by the following assumption.

**ASSUMPTION 6**: $E_x\{\alpha V_s(\bar{q}, e, s) + (1 - \alpha)C_s(\bar{q}, e, s)\}$ is nondecreasing in $s$ whenever $E_x\{\alpha V_s(\bar{q}, e, s) + (1 - \alpha)C_s(\bar{q}, e, s)\} < 0.$

To see why the nature of specific investments matters for the simple contract, consider two extreme cases. Suppose first a purely selfish investment with $V_s = 0$ and $C_s < 0$ for all $(q, e, s)$. Then, the expectation term reduces to $E_x\{\alpha C_s(\bar{q}, e, s)\}$. As Edlin and Reichenstein (1996a, b) point out, if there exists $\bar{q}$ such that $E_x\{-C_s(\bar{q}, e, s^*)\} = \Phi'(s^*)$, then the first-best outcome can be achieved with a default quantity of $\bar{q}$. To see this, from (5) and Assumption 6, we have for $\bar{q}$ that

$$\frac{\partial U^S(s; \bar{q})}{\partial s} = \alpha\Phi'(s) - E_x\{\alpha C_s(\bar{q}, e, s)\} - 1 \equiv \alpha\Phi'(s) + (1 - \alpha)\Phi'(s^*) - 1 \equiv 0 \Leftrightarrow s \equiv s^*.$$ 

Therefore, the seller chooses the first-best investment level.

Next suppose a purely cooperative investment with $V_s > 0$ and $C_s = 0$ for all $(q, e, s)$. Then, the expectation term reduces to $E_x\{\alpha V_s(\bar{q}, e, s)\} \geq 0$. Thus, for all $\bar{q} \geq 0$ and for all $s > s^*$,

$$\frac{\partial U^S(s; \bar{q})}{\partial s} \leq \alpha\Phi'(s) - 1 < 0.$$ 

This implies that the seller's investment choice will not be more than $s^*$, for all $\bar{q} \geq 0$. Therefore, in this case, setting $\bar{q} = 0$ is optimal, which yields the Williamson outcome. In contrast to the Edlin and Reichenstein's result, the simple contract has no value in this case.

We now consider the hybrid case where the seller's investment has both cooperative and selfish elements (i.e., $V_s > 0$ and $C_s < 0$). Inspecting (5) reveals how the seller's bargaining power affects her incentive for investment. As the seller's bargaining share $\alpha$ increases, she becomes more sensitive to the change in the buyer's status quo position and less sensitive to the change in her own status quo position, as the former influences the bargaining outcome more. Hence, the cooperative element of the investment becomes more important for the seller, which implies that the simple contract becomes less effective in inducing the seller's investment. This intuition is confirmed in the following proposition.

**PROPOSITION 2.** Suppose that Assumptions 1–4 and 6 hold. If only the seller makes an investment, then there exist $\kappa^*$ and $\kappa$ with $0 \leq \kappa^* \leq \kappa = 1$, such that: (i) a simple contract can attain the first-best outcome if $\alpha \leq \kappa^*$; (ii) a simple contract is valuable (i.e., $\bar{q} > 0$ is optimal) but unable to achieve the first-best outcome if $\kappa^* < \alpha < \kappa$; and (iii) the simple contract has no value (i.e., it is optimal to set $\bar{q} = 0$) if $\alpha \geq \kappa$.

This condition will be satisfied, by the intermediate value theorem, if $C_s < 0$ for all $(q, e, s)$, as is assumed by Edlin and Reichenstein.

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20 This assumption ensures that $\partial^2 U^S(s; \bar{q})/\partial s^2 < 0$ when contracting is valuable. Recall that the same condition holds when contracting is not valuable [i.e., when $E_x\{\alpha V_s(\bar{q}, e, s) + (1 - \alpha)C_s(\bar{q}, e, s)\} = 0$ for all $\bar{q}$], since then $\bar{q}$ will be optimally set equal to zero, in which case $\partial^2 U^S(s; \bar{q})/\partial s^2 = \alpha\Phi'(s) < 0$. Assumption 6 means, for example, that when $\alpha$ is close to zero (so that the expectation term is negative), $C_s(\bar{q}, e, s)$ is strictly convex in $s$ for all $\bar{q} > 0$ and $e$. Note that this assumption is used only for this subsection and not for the general results established in the next subsection.

21 This condition will be satisfied, by the intermediate value theorem, if $C_s < 0$ for all $(q, e, s)$, as is assumed by Edlin and Reichenstein.

22 For purely cooperative investments, $\kappa = 0$ and only case (iii) is relevant. For purely selfish investments, $\kappa^* = 0$ and only case (i) is relevant.
PROOF:
See Appendix A.

It is intuitive that contracting could be more valuable for the party with less bargaining power. It is somewhat surprising, however, that the first-best outcome may be attainable when the investing party has the least bargaining power. The intuition is as mentioned above: a party with a smaller bargaining share is more sensitive to a change in her status quo position and less sensitive to a change in her partner’s status quo position, so the cooperative nature of an investment has less adverse effect on the investment. This point can be illustrated by an example.

Example: Let \( V(q, e, s) = \varepsilon(s + 3) \min\{q, 1\} \) and let \( C(q, e, s) = q/s \) if \( q \leq 2 \) and \( C(q, e, s) = \infty \) if \( q > 2 \), where \( \varepsilon \in \{0, 1\} \) with equal probabilities. It is straightforward that the optimal quantity is \( q^*(e, s) = 1 \) if \( e = 1 \) and \( s \geq 0.30 \), and \( q^*(e, s) = 0 \) otherwise. The efficient level of investment is \( s^* = 1 \). Figure 3A plots the expected joint surpluses without contracting and with the simple contract when \( \bar{q} = 2 \); the value of the contract is the difference between these two surpluses. For \( \alpha < \kappa = 0.77 \), the simple contract is valuable and more so as \( \alpha \) decreases. At \( \alpha = \kappa^* = 0 \), the first best is attained (with the resulting joint surplus equaling 0.5). For \( \alpha \approx 0.77 \), the simple contract adds no value. As \( \alpha \) rises in this region, however, the holdup problem is diminished since the seller appropriates more marginal return and becomes a residual claimant when \( \alpha = 1 \), so the holdup problem is the most serious at \( \alpha \approx 0.76 \).

To illustrate that the region where the simple contract is worthless expands as the investment becomes more cooperative, suppose now that \( V(q, e, s) = \varepsilon(3s/2 + 2) \min\{q, 1\} \) and that \( C(q, e, s) = q/(2s) \) for \( q \leq 2 \), with the rest of the structure remaining the same. (The investment is more cooperative because \( V \) has increased and \( C \) is less negative.) Figure 3B shows that the simple contract has no value for \( \alpha \geq 0.68 \). Further, the first best can never be attained.
by the simple contract. The joint surplus is lowest at $\alpha \approx 0.67$.

B. General Results

We now show that our insights about the difficulties that cooperative investments pose for contracting hold for more general contracting schemes and for the general payoff specifications in Section II. To allow for all possible contracting schemes, we appeal to the revelation principle, which enables us to restrict attention to direct revelation mechanisms. A direct revelation mechanism specifies the prenegotiation contract terms as functions of both parties’ reports about the state of nature, in a way that gives the parties an incentive to report truthfully. Let $\theta = (e, b, s) \in \Theta$ be the true state, and let $\theta_B = (e_B, b_B, s_B) \in \Theta$ and $\theta_S = (e_S, b_S, s_S) \in \Theta$, respectively, denote the parties’ reports about $\theta$. Then, the mechanism determines the quantity to be traded, $q$, and a transfer, $t$, as functions of the reports, $\theta_B$ and $\theta_S$. Formally, the mechanism is a mapping $(q, t): \Theta^2 \rightarrow \mathcal{X} \times \mathcal{R}$.\(^{23}\)

The time line is the same as described in Section II, with the contract terms chosen at date 3 as functions of the parties’ reports about the state. Suppose that at date 3, the realized state is $\theta$ and reports $\theta_B$ and $\theta_S$ are chosen. The contract terms are then renegotiated to an efficient trade level if $q(\theta_B, \theta_S) \in \arg\max_q v(q, \theta) - c(q, \theta)$. This sequence generates payoffs for the buyer and the seller that are:

\(^{23}\) That is, we rule out random mechanisms. Random mechanisms are questionable in terms of enforceability. Furthermore, allowing random mechanisms does not change our results qualitatively (while adding notational clutter). Our main results in Proposition 3 go through with an appropriate modification. [Specifically, the inequalities involved in the later definitions (6)–(9) of the variables $\alpha, \alpha^*, \bar{\alpha}$ and $\bar{\alpha}^*$, will need to hold in expected value terms for every lottery that selects a positive quantity with non-zero probability.] Note that random mechanisms play a nontrivial role if the parties are risk averse (see Maskin and Tirole, 1999).
\[ u_B(\theta_B, \theta_S; \theta) = v(q(\theta_B, \theta_S), \theta) - t(\theta_B, \theta_S) \\
+ (1 - \alpha)[\phi(\theta) - \{ v(q(\theta_B, \theta_S), \theta) \\
- c(q(\theta_B, \theta_S), \theta) \} ], \]

and

\[ u_S(\theta_B, \theta_S; \theta) = t(\theta_B, \theta_S) - c(q(\theta_B, \theta_S), \theta) \\
+ \alpha[\phi(\theta) - \{ v(q(\theta_B, \theta_S), \theta) \\
- c(q(\theta_B, \theta_S), \theta) \} ], \]

respectively. In each of these expressions, the first term represents the relevant party’s status quo payoff under the initial contract, while the bracketed term represents the party’s share of the surplus from renegotiation. Notice that \( u_B(\theta_B, \theta_S; \theta) + u_S(\theta_B, \theta_S; \theta) = \phi(\theta) \) for all \( \theta_B, \theta_S, \theta \in \Theta \), i.e., the game is a fixed-sum game \textit{ex post}. This fixed-sum game feature of the problem plays an important role in the subsequent analysis. When reporting truthfully in state \( \theta \), the two parties receive \( \bar{u}_B(\theta) = u_B(\theta_B, \theta_S; \theta) \) and \( \bar{u}_S(\theta) = u_S(\theta_B, \theta_S; \theta) \), respectively.

The contracting problem facing the parties is to find a direct mechanism that solves:

\[ [\text{R}] \quad \max_{q(.), t(.), \bar{b}, \bar{s}} \Phi(b, s) - b - s \]

subject to

\[ \text{(BIC)} \quad \bar{u}_B(\theta) \geq u_B(\theta_B, \theta_S; \theta) \quad \forall \theta, \theta_B \in \Theta \]

\[ \text{(SIC)} \quad \bar{u}_S(\theta) \geq u_S(\theta_B, \theta_S; \theta) \quad \forall \theta, \theta_S \in \Theta \]

\[ \text{(BI)} \quad b \in \arg \max_{\bar{b}} U_B(\bar{b}, s) - \bar{b} \]

\[ \text{(SI)} \quad s \in \arg \max_{\bar{s}} U_S(b, \bar{s}) - \bar{s}, \]

where

\[ U_i(b, s) = \int_0^s \bar{u}_i(e, b, s) \, dF(e), \]

for \( i = B, S \).

Because of renegotiation, the objective function assumes the first-best trading decision. Conditions (BIC) and (SIC) ensure that both parties report truthfully (knowing that the chosen contract terms will be negotiated to an efficient quantity) as a Nash equilibrium for any subgame following every state \( \theta \in \Theta \) [even for \( (b, s) \) off the equilibrium path].\(^{24}\) Requiring truthful reporting to be a Nash equilibrium on and off the equilibrium path follows from the subgame-perfection requirement. Constraints (SI) and (BI) mean that the parties’ investment choices are mutual best responses, given \textit{ex post} truthful reporting of the realized state by the parties.\(^{25}\)

The revelation principle ensures that the above constraints admit all subgame-perfect equilibrium outcomes resulting from any feasible contract schemes. Suppose that an arbitrary contracting scheme specifies a game to be played by the parties prior to the renegotiation stage. Without loss of generality, we can interpret \( (q(\theta_B, \theta_S), t(\theta_B, \theta_S)) \) as the outcome in this game of the buyer and the seller playing the strategies of states \( \theta_B \) and \( \theta_S \), respectively. Clearly, the subgame-perfect equilibrium of this game requires (BIC) and (SIC) to be satisfied; i.e., each must find it in his best interest to play the strategy of state \( \theta \) in state \( \theta \) when the other does the same.

Inspecting (BIC) and (SIC), it is instructive to note that the standard first-order approach does not work since the payoff functions, \( u_B \)

\(^{24}\) Our equilibrium concept may permit multiple equilibria, some of which may involve untruthful reporting. The equilibrium payoffs are unique, however, since the game becomes a fixed-sum game \textit{ex post}. Moreover, the lack of unique implementation is not a concern, since our aim is to prove a negative result that contracting has limited value, and requiring uniqueness cannot increase the value of contracting.

\(^{25}\) The above program does not require participation constraints. Since not contracting is feasible and offers nonnegative expected surplus to both parties, the contract that solves [R] induces participation by both parties for some fixed side payment.
and $u_s$, do not satisfy the single-crossing property (see Paul Milgrom and Chris Shannon, 1994), given the arbitrariness of the functional forms of $(q, t)$. In the current setting, however, the fixed-sum game feature of the problem allows us to solve the problem without the first-order approach.

Before proceeding, we first characterize the extent to which an investment is cooperative. For the seller’s investment, $s$, define the following two measures:

$$\alpha = \inf \{ k \in [0, 1] | kv_s(q, \theta) + (1-k)c_s(q, \theta) \geq 0, \forall (q, \theta) \in \mathcal{R}_+ \times \Theta \}$$

and

$$\alpha^* = \inf \{ k \in [0, 1] | kv_s(q, \theta) + (1-k)c_s(q, \theta) \leq 0, \forall (q, \theta) \in \mathcal{R}_+ \times \Theta \}$$

if the sets are nonempty, and each measure equals 1 if the respective set is empty. By Assumption 3 and the envelope theorem, the right-hand side of the inequality in the second measure is nonpositive, so $\alpha^* = \alpha$. Small values of $\alpha$ and $\alpha^*$ mean that the seller’s investment is highly cooperative. Since $v_s$ will be relatively large if the seller's investment is highly cooperative, even if $c_s$ is negative (selfish element), a small $k$ can cause $kv_s + (1-k)c_s$ to be nonnegative for all $(q, \theta)$. In the extreme case of a purely cooperative investment by the seller [i.e., $(c_b, v_b) > (0, 0)$ for all $(q, \theta)$], then $\alpha = \alpha^* = 0$. In the other extreme of a purely selfish investment [i.e., $(c_b, v_b) < (0, 0)$ for all $(q, \theta)$], then $\alpha = 1.$

Similarly, we define the corresponding two measures for the buyer’s investment:

$$\bar{\alpha} = \sup \{ k \in [0, 1] | kv_b(q, \theta) + (1-k)c_b(q, \theta) \leq 0, \forall (q, \theta) \in \mathcal{R}_+ \times \Theta \}$$

and

$$\bar{\alpha}^* = \sup \{ k \in [0, 1] | kv_b(q, \theta) + (1-k)c_b(q, \theta) \leq k\phi_b(\theta), \forall (q, \theta) \in \mathcal{R}_+ \times \Theta \}$$

if the sets are nonempty, and each measure equals 0 if the respective set is empty. As before, the condition in the second measure is less onerous than that in the first measure, so $\bar{\alpha}^* \geq \bar{\alpha}$. High values of $\bar{\alpha}$ and $\bar{\alpha}^*$ mean that the buyer’s investment is highly cooperative. A purely cooperative investment by the buyer [with $(c_b, v_b) < (0, 0)$ for all $(q, \theta)$] has $\bar{\alpha} = \bar{\alpha}^* = 1$ and a purely selfish investment [with $(c_b, v_b) > (0, 0)$ for all $(q, \theta)$] has $\bar{\alpha} = 0$.

We now begin our analysis of $[R]$ by showing the degree to which the parties can be provided incentives for investments.

**LEMMA 1:** Suppose that Assumptions 1–4 hold. For any $(b, s) \in \mathcal{R}_+^2$,

$$\frac{\partial U_s(b, s)}{\partial s} = \lim_{s' \to s} \frac{U_s(b, s) - U_s(b, s')}{s - s'}$$

$$\begin{cases}
\leq \alpha \phi_s(b, s), & \text{if } \alpha \leq \alpha; \\
< \phi_s(b, s), & \text{if } \alpha^* < \alpha,
\end{cases}$$

and

$$\frac{\partial U_b(b, s)}{\partial b} = \lim_{b' \to b} \frac{U_b(b, s) - U_b(b', s)}{b - b'}$$

$$\begin{cases}
\leq (1-\alpha)\phi_b(b, s), & \text{if } \bar{\alpha} \geq \alpha; \\
< \phi_b(b, s), & \text{if } \bar{\alpha}^* > \alpha.
\end{cases}$$

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26 These measures are similar to $\kappa$ and $\kappa^*$ in Appendix A, although it can be verified that $\kappa^* = \alpha^*$ and $\kappa = \alpha$.

27 If, in addition, $c_s < 0$, $v_s < 0$, as was assumed in Chung (1991) and Edlin and Reichelstein (1996a, b), then $\alpha^* = 1.$
PROOF:
See Appendix B.

Lemma 1 implies that, if the investments are sufficiently cooperative (in the sense that $\alpha$ and $\alpha^*$ are small, and $\bar{\alpha}$ and $\bar{\alpha}^*$ are large, relative to $\alpha$), then there exists an upper bound on the marginal return to the investment that each party internalizes. Such a bound also means that there is a limit on the extent to which incentives can be provided to each party for his or her investment through contracting. In particular, we show that, if $\alpha \in [\alpha, \bar{\alpha}]$ (i.e., the investments are sufficiently cooperative), then investments attainable under any feasible mechanism are bounded by $B(\cdot)$ and $S(\cdot)$ — the best-response curves in the Williamson game. This set is formally defined as

$$\Omega \equiv \{ (b', s') : b' \leq B(s') \text{ and } s' \leq S(b') \},$$

and is the shaded area in Figure 2.

PROPOSITION 3: Suppose that Assumptions 1–5 hold and that committing not to renegotiate the contract is impossible for the parties. (i) If $\alpha > \alpha^*$ or $\alpha < \bar{\alpha}^*$, then the first-best outcome cannot be achieved by any contract. (ii) If $\alpha \in [\alpha, \bar{\alpha}]$, then any feasible investment pair $(b, s)$ belongs to $\Omega$. If, in addition, $(b^\prime, s^\prime)$ yields the highest joint surplus in $\Omega$ [i.e., $W^*(b^\prime, s^\prime) = W^*(b, s)$ for all $(b, s) \in \Omega$], then the solution to [R] generates the Williamson outcome. That is, the optimal contract is no contract.\(^{28}\)

PROOF:
For the proof of (i), we show that, if $\alpha > \alpha$, then for any feasible contract, $s < S^*(b)$, which is sufficient for unattainability of the first-best outcome. The case of $\alpha < \bar{\alpha}$ is symmetric and thus omitted. Assume that $\alpha > \alpha$ and suppose, to the contrary, that $s \geq S^*(b)$.

Then, for any such $s$,

$$\frac{\partial U_s(b, s)}{\partial s} - 1 < \Phi_s(b, s) - 1 \equiv 0,$$

where the first inequality follows from Lemma 1, and the second follows from $s \geq S^*(b)$, and Assumption 4. The inequality implies that the seller has an incentive to lower her investment whenever $s \geq S^*(b)$, which implies that $s < S^*(b)$ in any feasible outcome.

We next show that, if $\alpha \in [\alpha, \bar{\alpha}]$, then $s \leq S(b)$ and $b \leq B(s)$. Suppose, to the contrary, that $s > S(b)$. Then,

$$\frac{\partial U_s(b, s)}{\partial s} - 1 \leq \alpha \Phi_s(b, s) - 1 < 0,$$

where the first inequality follows from Lemma 1, and the second follows from $s > S(b)$, and Assumption 4. The above inequality implies that it pays the seller to lower her investment, which provides the contradiction. Therefore, we conclude that $s \leq S(b)$. The proof for $b \geq B(s)$ is similarly obtained.

If, in addition, $W^*(b^\prime, s^\prime) = W^*(b, s)$ for all $(b, s) \in \Omega$, then $W^*(b^\prime, s^\prime)$ is the upper bound for the value of [R]. This upper bound is clearly attained by the Williamson mechanism, $(q, t) = (0, 0)$, i.e., no contracting. Since this mechanism trivially satisfies (BIC) and (SIC) (inducing truthful reporting as a weakly dominant strategy), writing no initial contract solves [R] in this case.

The first part of the proposition implies the following.

COROLLARY 1: Suppose that Assumptions 1–4 hold. If $\alpha^* \leq \bar{\alpha}^*$, then the first-best outcome cannot be attained for any $\alpha \in [0, 1]$.\(^{29}\)

The condition for the corollary holds if one investment is sufficiently cooperative relative to the other (e.g., if one investment is purely cooperative: $\alpha^* = 0$ or $\bar{\alpha}^* = 1$).

\(^{28}\) Maskin and Tirole (1999) establish an efficiency result in the presence of renegotiation. Their result, however, depends on conditions, including risk aversion, that do not hold in our setting.

\(^{29}\) Aghion et al. (1994) argue that the first-best outcome cannot be attained by a noncontingent contract that assigns the entire bargaining power to one party in the renegotiation game, if both investments have cooperative elements. Proposition 3(i) confirms this observation, since, if both investments are cooperative, $\alpha^* < 1$ and $\bar{\alpha}^* > 0$, so the first-best outcome cannot be achieved if $\alpha = 0$ or $\alpha = 1$. Corollary 1 makes a more general point: the first best is unattainable for any $\alpha \in [0, 1]$ if $\alpha^* \leq \bar{\alpha}^*$. 

The second part of Proposition 3 specifies two conditions that are sufficient to render contracting worthless. The first condition, that $\alpha \in [\underline{\alpha}, \bar{\alpha}]$, means that the feasible set of investments is $\Omega$ (the shaded area in Figure 2). This condition holds if both investments are sufficiently cooperative, and it becomes less restrictive as the investments become more cooperative since, as was previously noted, $\underline{\alpha}$ decreases and $\bar{\alpha}$ increases as both investments become more cooperative. If both investments are purely cooperative, for example, $\underline{\alpha} = 0$ and $\bar{\alpha} = 1$. The second condition is that the equilibrium pair in the Williamson game, $(b^w, s^w)$, yields the highest joint surplus among all investment pairs in $\Omega$. In principle, this condition can fail. For example, if the seller's investment is relatively more important than the buyer's investment but the seller has little bargaining power to internalize the social return to her investment through the market arrangement, then another investment pair in $\Omega$ may be socially preferred to $(b^w, s^w)$. Investment pair $C$ in Figure 4 illustrates this case. Proposition 3 implies that a contract can only discourage an investment relative to the Williamson outcome. It is conceivable that discouraging the buyer's investment (through contracting) may have the beneficial effect of encouraging the seller's investment, if the two investments are substitutes, as depicted in Figure 4. Thus, there may exist a contract that implements $C$, in which case contracting is clearly valuable.

We provide two sufficient conditions for the above situation not to occur, i.e., for $(b^w, s^w)$ to give the highest joint surplus in $\Omega$. One sufficient condition is that the best-response curves, $B(\cdot)$ and $S(\cdot)$, are nondecreasing (as occurs in Figure 2). This holds if $\Phi(b, s)$ is supermodular in $(b, s)$ [i.e., $\Phi_m(b, s) > 0$; see Donald Topkis, 1978], which means that $U^N(b, s) = (1 - \alpha)\Phi(b, s) - b$ and $U^S(b, s) = \alpha\Phi(b, s) - s$ are both supermodular in $(b, s)$.

A weaker, yet more cumbersome, condition would be that $U^N(b, s)$ and $U^S(b, s)$ satisfy the single-crossing property in $(b, s)$ and in $(s, b)$, respectively. See Theorem 4 of Milgrom and Shannon (1994).
COROLLARY 2: Suppose that Assumptions 1–5 hold. If \( \alpha \in [\underline{\alpha}, \bar{\alpha}] \) and \( \Phi(b, s) \) is supermodular in \((b, s)\), then contracting has no value.

PROOF:
Since both \( U_P(b, s) \) and \( U_S(b, s) \) are supermodular in \((b, s)\), the best-response curves, \( B(\cdot) \) and \( S(\cdot) \), in the Williamson game are nondecreasing. For any \((b, s) \in \Omega\), define a mapping \( \tau: \Omega \to \Omega \).

\[
\tau(b, s) = (B(S(b)), S(b)).
\]

We first show that \( \tau(b, s) \in \Omega \) if \((b, s) \in \Omega\). Fix a \((b, s) \in \Omega\). Trivially, \( B(S(b)) \leq B(S(b)) \). So, it suffices to show that \( S(b) \leq S(B(S(b))) \). Since \((b, s) \in \Omega\) and \( B(\cdot) \) is monotonic, \( b \leq B(s) \) implies \( B(S(b)) \), which in turn implies, by the monotonicity of \( S(\cdot) \), that \( S(b) \leq B(S(b)) \). Combining the two arguments yields \( \tau(b, s) \in \Omega \), as desired.

Now observe that, for any \((b, s) \in \Omega\),

\[
W^*(\tau(b, s)) \geq W^*(b, S(b)) \geq W^*(b, s),
\]

where the first inequality holds since \( b \leq B(S(b)) \), \( B(\cdot) \leq B^*(\cdot) \), and \( W^*(\cdot, \cdot) \) is strictly concave, and the second inequality holds analogously. Since \( \tau(b, s) \in \Omega \) for \((b, s) \in \Omega\), applying the inequality recursively, \( W^*(\tau^T(b, s)) \) is nondecreasing in \( T \) for any \((b, s) \in \Omega\).

Finally, supermodularity and the uniqueness of the equilibrium in the Williamson game implies that, for any \((b, s) \in \Omega\), \( \tau^w(b, s) = \lim_{T \to \infty} \tau^T(b, s) \) exists and equals \((b^w, s^w)\).

Combining all arguments, \( W^*(b^w, s^w) = W^*(\tau^w(b, s)) \geq W^*(b, s) \) for all \((b, s) \in \Omega\). Hence, the result follows from Proposition 3(ii).

In words, supermodularity of \( \Phi(\cdot, \cdot) \) means that an investment by one party increases the marginal social return to the other party’s investment. Even if neither investment directly impacts the other, supermodularity may arise indirectly through the trade decision. For example, if the seller’s investment promotes more trade, then, in turn, this may increase the marginal value of the buyer’s investment. The supermodularity of \( \Phi(\cdot, \cdot) \) covers a large class of cases studied in the literature. For example, the payoff functions studied by Chung (1991), Hermalin and Katz (1993), Aghion et al. (1994), and Edlin and Reichelstein (1996a, b) all exhibit supermodularity.

One-sided investment offers another sufficient condition. Suppose, without loss of generality, that only the seller makes an investment. One way to assume this in our model is to fix \( b = 0 \). Then, \( W^*(0, s^w) \geq W^*(0, s) \) for all \( s \leq s^w \), since \( s^w < s^* \) and since \( W^*(0, \cdot) \) is strictly concave. Therefore, we have the following result.

COROLLARY 3: Suppose that Assumptions 1–4 hold. If only the seller makes an investment and \( \alpha \geq \alpha \), then the optimal contract is no contract. A symmetric result holds for a one-sided investment by the buyer if \( \alpha \leq \bar{\alpha} \).

VI. Conclusions

Several recent articles have shown that the first-best outcome for bilateral trade, even in the face of specific investments and incomplete contracting, can be supported with appropriately designed contracts. These results, for the most part, hold only for selfish invest-

\[ \frac{\partial^2 \Phi(b, s)}{\partial b \partial s} = E \left( \frac{\partial^2 V(q^*, b, s)}{\partial q^* \partial q^*} \right) \geq 0, \]

since \( q^* \) is nondecreasing in \( b \), which in turn follows from the positive cross-partial derivative conditions.
ments that directly benefit the investor. This paper finds very different results for cooperative investments which render direct benefits to the investor's partner.

With cooperative investments, the value of contracting varies depending on whether the parties can or cannot commit not to renegotiate the contract. With a commitment not to renegotiate, there exist schemes that achieve efficiency. If the parties have difficulty committing not to renegotiate and if the investments are sufficiently cooperative, however, then not only is the efficient outcome unachievable, but the parties may not do better than limiting themselves to ex post negotiation.

By specifying conditions under which contracts are not effective, this paper shows that the holdup problem remains a valid concern. One response to our negative result is to abandon contracting altogether in favor of spot markets with their attendant underinvestment consequences. This point may explain the oft-observed paucity of explicit contracting in business transactions (for example, see Macaulay, 1963).

More constructively, we can ask whether the organizational safeguards suggested in the literature provide effective remedies against the holdup problem in the presence of cooperative investments. The arguments for some of the well-known safeguards such as vertical integration (Klein et al., 1978; Williamson, 1979) and asset ownership allocation (Grossman and Hart, 1986; Hart and Moore, 1990) do not hinge critically on the nature of specific investments, so their prescriptions appear valid even in the cooperative setting. It is the necessity of these safeguard arrangements for which the nature of investments matters and our theory contributes. With selfish investments, these safeguard arrangements are superfluous since contracting provides a superior and perhaps less costly remedy. With cooperative investments, however, the current theory validates the necessity of the suggested organizational safeguards.

Our theory also identifies the ability to avoid renegotiation as an important condition for achieving efficiency. Since the extent to which such an ability can be supported can vary across organizational forms, this notion may offer a useful new perspective on the theory of organization. For instance, intrafirm trade resulting from vertical integration may be less susceptible to holdups than interfirm trade if committing not to renegotiate is more easily supported within the firm (perhaps through headquarters) than outside. This can also provide some insight on the internal organization of the firm. For example, a division's decisions that involve highly cooperative elements, such as those having significant impact on the rest of the firm's image and strategy, may be more efficiently governed by corporate headquarters (i.e., centralization), whereas it may be effective to decentralize decisions that have mainly selfish elements. (See Aghion and Tirole [1997] for a similar remark.) Finally, an implication can also be drawn about the judicial attitude towards enforcing nonmodification clauses in contracting. The courts

value is only \( V(q, e, 0) \), i.e., he does not benefit from the seller's specific investment. Alternatively, suppose that the seller owns both assets. In this case, suppose that the seller can produce according to \( C(q, e) \) and can generate a value of \( \bar{V}(q, e, s) \), where \( 0 < \bar{V}(q, e, s) < V(q, e, s) \) for all \( q > 0 \) and for all \( (e, s) \). Then, in all of the ownership arrangements, contracting has no value; and at the same time, the seller-control regime is the best at overcoming the holdup problem, which confirms the general principle developed in Grossman and Hart (1986) and Hart and Moore (1990). Note, however, Stephen Chiu (1998) shows that this principle is sensitive to the nature of the bargaining game.

By the same token, vertical integration may not solve the holdup problem without the commitment ability. Robert Eccles and Harrison White (1988 p. 547) report that division managers often prefer external trade over internal trade, partly based on internal transactions being "fraught with more difficulties than were external transactions."

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Footnotes:

33 One can incorporate the issue of asset ownership allocation in our model. To illustrate, consider the special case where only the seller invests, and recall \( V(q, e, s) \) and \( C(q, e) \) are the buyer's value and the seller's cost of performance, when the transaction takes place between the two parties. Suppose that each party requires an asset for operation. If the assets are owned separately by the respective parties, then no trade between the two parties results in no trade and zero surplus, as we assumed throughout. Suppose instead that the buyer owns both assets. Then, in the event of no trade, we can assume that the buyer can still produce according to \( C(q, e) \) but his

34 By the same token, vertical integration may not solve the holdup problem without the commitment ability. Robert Eccles and Harrison White (1988 p. 547) report that division managers often prefer external trade over internal trade, partly based on internal transactions being "fraught with more difficulties than were external transactions."
current reluctance to enforce such clauses makes the commitment to avoid renegotiation difficult to achieve in a trade between nonintegrated parties. Enforcing nonmodification clauses can therefore enhance efficiency if cooperative investments are important.\textsuperscript{35}

Another implication of our result relates to the foundations of incomplete contracting. Incompleteness is often attributed to an inability to write contracts contingent on events that, while observable to the parties, cannot be verified by the court. A criticism of this approach is that such incompleteness can be "completed" by having the parties generate verifiable signals, as occurs in the Moore and Repullo (1988) scheme, for example. With selfish investments, designing such signals can cure the problem in the sense that efficiency can be achieved.\textsuperscript{36} With sufficiently cooperative investments and renegotiation, however, such a process of generating verifiable signals add nothing, as we have shown. Thus, for this setting, the assumption of observable but unverifiable contingencies remains a reasonable foundation for incomplete contracting.

APPENDIX A: PROOF OF PROPOSITION 2

Let

\begin{equation}
\kappa = \inf \left\{ k \in [0, 1] \right\} \{ kV_s(q, \varepsilon, s^w) + (1 - k)C_s(q, \varepsilon, s^w) \} \geq 0, \forall q \geq 0 \}
\end{equation}

and let

\begin{equation}
\kappa^* = \inf \left\{ k \in [0, 1] \right\} \{ kV_s(q, \varepsilon, s^*) + (1 - k)C_s(q, \varepsilon, s^*) \} \geq -(1 - k)\Phi'(s^*), \forall q \geq 0 \}.
\end{equation}

When the condition in (A1) [respectively (A2)] is unsatisfied for all \( k \), let \( \kappa = 1 \) (respectively \( \kappa^* = 1 \)). Note that \( 0 \leq \kappa \leq \kappa^* \leq 1 \).

Suppose first that \( \alpha < \kappa^* \). Then, since the left-hand side of the inequality in (A2) is zero when \( q = 0 \) and is continuous in \( q \), there exists a \( q > 0 \) that makes both sides of (A2) equal. At such a \( q \),

\[
\frac{\partial U^E_s(s; q)}{\partial s} = \alpha \Phi'(s) - E \{ \alpha V_s(q, \varepsilon, s) + (1 - \alpha)C_s(q, \varepsilon, s) \} - 1 \equiv \alpha \Phi'(s)
\]

\[
+ (1 - \alpha)\Phi'(s^*) - 1 \equiv 0 \iff s \equiv s^*,
\]

given the concavity of \( \Phi(\cdot) \) and Assumption 6. Hence, the seller will choose the first-best investment level.

Now suppose that \( \alpha \equiv \kappa \). Then, for any \( q \equiv 0 \),

\[
\frac{\partial U^E_s(s; q)}{\partial s} \leq \alpha \Phi'(s) - 1 < 0,
\]

for all \( s \equiv s^w \). To see that the first inequality holds, suppose to the contrary that \( \partial U^E_s(s; q)/\partial s > \alpha \Phi'(s) - 1 \) for some \( s > s^w \). Then, by Assumption 6, \( \partial U^E_s(s^w; q)/\partial s > \alpha \Phi'(s^w) - 1 \), which contradicts the fact that \( \alpha \equiv \kappa \). The above result implies that \( s \equiv s^w \) for all \( q > 0 \), so the simple contract can do no better than the Williamson outcome. In this case, therefore, it is optimal to set \( q = 0 \).

Finally suppose that \( \kappa^* < \alpha < \kappa \). Then, the first inequality implies that, for all \( q \equiv 0 \), the inequality in (A2) holds with strict inequality. Therefore, for all \( q \equiv 0 \),

\[
\frac{\partial U^E_s(s^*; q)}{\partial s} < \alpha \Phi'(s^*) + (1 - \alpha)\Phi'(s^*)
\]

\[
- 1 = 0,
\]

\textsuperscript{35} See Jolls (1997) for a similar view.

\textsuperscript{36} Rogerson (1992) shows that the first-best outcome can be implemented with selfish investments using the Moore-Repullo scheme, if renegotiation can be prevented. For the renegotiation case, Edlin and Reichelstein (1996a, b) show that the result holds for some circumstances.
so the first-best outcome cannot be attained. On the other hand, the second inequality \((\alpha < \kappa)\) implies that there exists \(\bar{q} > 0\) such that 
\(E_{\bar{q}} \{ \alpha V(\bar{q}, \varepsilon, s^{-}) + (1 - \alpha) C_{\bar{q}}(\bar{q}, \varepsilon, s^{-}) \} < 0\). Along with Assumption 6, this implies that at such a \(\bar{q}\),
\[
\frac{\partial U_{S}^{ER}(s; \bar{q})}{\partial s} > \alpha \Phi'(s) - 1 \geq 0,
\]
for all \(s \leq s^{-}\). It therefore follows that a simple contract with such a \(\bar{q}\) can induce a cooperative investment level \(s > s^{-}\). By the intermediate value theorem, then there exists a simple contract with \(\bar{q}' \leq \bar{q}\) that induces \(s' \in (s^{-}, s^{*})\). Such a contract is clearly valuable, although it does not induce the first-best outcome.

APPENDIX B: PROOF OF LEMMA 1

We prove the first statement. The second statement follows analogously, using a symmetric argument. Fix any \(\theta, \theta' \in O\). Then, (SIC) implies that \(\bar{u}_{S}(\theta) \equiv u_{S}(\theta, \theta'; \theta)\). Meanwhile, (BIC) implies that \(\bar{u}_{S}(\theta) = u_{S}(\theta', \theta; \theta)\), which, by the fixed-sum game property [i.e., \(\bar{u}_{S}(\theta) + \bar{u}_{S}(\theta) = u_{S}(\theta', \theta; \theta) + u_{S}(\theta', \theta; \theta) = \Phi(\theta)\)], in turn implies that \(\bar{u}_{S}(\theta) \equiv u_{S}(\theta', \theta; \theta)\). Combining the two results, we have

\[\bar{u}_{S}(\theta, \theta'; \theta) = u_{S}(\theta, \theta'; \theta).\]

Following the same approach for state \(\theta'\), we have

\[u_{S}(\theta', \theta; \theta') \equiv \bar{u}_{S}(\theta') \equiv u_{S}(\theta, \theta', \theta').\]

Combining (B1) and (B2) yields

\[\bar{u}_{S}(\theta) - \bar{u}_{S}(\theta') \leq u_{S}(\theta', \theta; \theta)\]

\[- u_{S}(\theta', \theta; \theta') = \alpha[\Phi(\theta) - \phi(\theta')]\]

\[- \{ \alpha[v(q(\theta', \theta), \theta) - v(q(\theta', \theta), \theta')]\}

\[+ (1 - \alpha)[c(q(\theta', \theta), \theta)\]

\[- c(q(\theta', \theta), \theta')]\}].

Fixing \(\theta' = (\varepsilon, b, s')\), (B3) implies that

\[
\frac{\partial U_{S}(\varepsilon, b, s)}{\partial s} = \lim_{s' \rightarrow s} \frac{\bar{u}_{S}(\varepsilon, b, s) - \bar{u}_{S}(\varepsilon, b, s')}{s - s'} = \alpha \Phi_{\varepsilon}(\varepsilon, b, s)
\]

\[- \lim_{s' \rightarrow s} \{ \alpha v(q(\theta', \theta), \theta)

\[+ (1 - \alpha)c(q(\theta', \theta), \theta)\}

\[\leq \alpha \Phi_{\varepsilon}(\varepsilon, b, s), \quad \text{if } \kappa \geq \alpha, \text{ and}

\[< \Phi_{\varepsilon}(\varepsilon, b, s), \quad \text{if } \kappa^* < \alpha.\]

From (B4), if \(\alpha \geq \kappa\), then

\[\frac{\partial U_{S}(b, s)}{\partial s} \leq \int_{0}^{1} \frac{\partial U_{S}(\varepsilon, b, s)}{\partial s} dF(\varepsilon) \leq \alpha \int_{0}^{1} \Phi_{\varepsilon}(\varepsilon, b, s) dF(\varepsilon) = \alpha \Phi_{\varepsilon}(b, s).\]

The first inequality follows from a (generalized) version of Fatou's lemma (see H. L. Royden, 1968 p. 90). The second inequality follows from (B4). By similar reasoning, if \(\alpha > \kappa^*\), then

\[\frac{\partial U_{S}(b, s)}{\partial s} < \Phi_{\varepsilon}(b, s).\]

A similar method proves the second result.

REFERENCES


