Selective Disclosure in Overlapping Generations*

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Abstract

We develop an overlapping-generations model with a constant state. With some probability, each agent observes a private signal about the state and with another independent probability, receives the set of signals disclosed by his predecessor. The agent then takes an action and selects a subset of signals to disclose to the next agent. Each agent's action has a positive externality on his immediate predecessor and his optimal action increases in his belief about the state. We show that when agents are more likely to observe private signals or receive disclosed signals, they become increasingly selective in their disclosure decisions. When the probability that disclosed signals are transmitted to the next generation exceeds a threshold, all signals except the most favorable one will be concealed.

Keywords: information disclosure, hard information, overlapping generations.

JEL Codes: C72, C73, D82, D83.

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1 Introduction

Information is often transmitted through a succession of agents, each of whom can edit the record inherited from earlier generations. This paper studies how better access to, and transmission of, information affects agents' disclosure behavior. We show that agents become more selective in disclosing information when they are more likely to have private information or to inherit signals disclosed by others, in contrast to the finding in Dye (1985). This difference arises because in our setting, disclosing an additional signal is informative about the amount of evidence available for selection: it suggests a longer chain of successful communication and, consequently, more opportunities for unfavorable signals to have been observed and concealed.

To fix ideas, consider an organization with overlapping generations of workers. Old agents are managers, who have incentives to motivate young agents (i.e., new recruits) to work hard without internalizing their effort cost. The managers may achieve that by affecting the recruits' beliefs about the returns to hard work: They can update the organization's official training materials, by adding newly available cases about past projects or employees or removing cases inherited from earlier managers.[1] Over time, some of these cases could be lost due to negligence, accidental data loss, or the managers' strategic omissions. The recruits who later become managers can then revise the materials they inherited before passing them to the following cohort.

We analyze steady-state equilibria of an overlapping-generations (OLG) game. The state of the world is constant and is either *high* or *low*. Only one agent is active in each period. With some probability, he observes a private signal about the state, and with another independent probability, he receives the set of signals disclosed by his immediate predecessor. The agent then chooses his effort and discloses to the next agent a subset of *available signals*, which includes his private signal and any signals he inherited. Agents cannot directly observe others' actions, the exact times at which the disclosed signals arrived, and whether the lack of inherited signals is caused by communication failure or their predecessors' deliberate choice to disclose nothing.[2]

[1] As documented in Kipping and Clark (2012)'s handbook, consulting firms often motivate new recruits by transmitting the firm's culture through stories of successful projects, client impact, and exemplary consultants.

[2] In our example, it seems hard for managers to add fabricated cases to the organization's official training material since newly added cases are typically subject to internal reviews. The cases contained in the training material may not include precise information about the exact time at which each case occurred, since client confidentiality and past employees' privacy concerns may require the distributed files to omit identifying information. Section 5.2 discusses cases in which agents observe the time stamps of disclosed signals or communication failures.

Each agent's optimal effort is strictly increasing in his belief that the state is high and this effort generates a positive externality for his predecessor. Therefore, when choosing the subset of signals to disclose, each agent's objective is to maximize his successor's belief about the state.

As in other disclosure games with multiple signals, there is a plethora of strict equilibria, some of which are sustained by unreasonable off-path beliefs. For example, when there are three signal realizations, there exist strict equilibria where signal realizations with the highest likelihood ratio are concealed while those with the second-highest likelihood ratio are revealed. Such equilibria can be sustained by the off-path belief that the disclosure of any signal with the highest likelihood ratio indicates the sender concealed signals with the lowest likelihood ratio.

Hence, we restrict attention to strict equilibria in which agents use *monotone strategies*. Monotonicity requires that whenever a signal is disclosed at an information set, all available signals with strictly higher likelihood ratios are also disclosed at that information set.

Our theorem shows that a signal is disclosed in some strict monotone equilibrium *if and only if* a measure of the informational environment falls below a signal-specific cutoff. The measure corresponds to the expected number of private signals observed since the most recent communication failure and increases both when agents are more likely to observe their private signals and when disclosures are more likely to reach the next generation. The signal-specific cutoff is infinite only for the signal with the highest likelihood ratio, smaller for signals with lower likelihood ratios, and positive if and only if the signal's likelihood ratio exceeds 1. Consequently, each signal other than the strongest one is disclosed in equilibrium if and only if its likelihood ratio exceeds 1 and agents' chances of observing private and disclosed signals are sufficiently low.

This result implies that when agents receive private signals or disclosed signals with higher probability, they become *more selective* in disclosing information in the sense that a smaller set of signals can be disclosed in equilibrium. It also implies that when communication frictions are small enough, all signals except for the strongest possible one will be concealed in equilibrium.

Our finding contrasts with Dye (1985)'s that a higher probability of the sender having information leads to *less selective* disclosure. In Dye's model, when this probability is close to one, all signal realizations will be disclosed except for the one that has the lowest likelihood ratio.[3]

To build intuition, consider a *constant-threshold equilibrium* where at all information sets,

[3] One can show that in Dye (1985)'s setting, if the receiver fails to receive the information disclosed with some probability, then disclosure becomes *less selective* when this probability decreases. This is similar to the finding in the original Dye (1985)'s model where the sender does not have information with positive probability.

agents disclose a signal if and only if its likelihood ratio is above some cutoff (i.e., *good signals*). The sequence of observed private signals after the latest communication failure has some good signals (the number of which can be 0), and in between each pair of adjacent good signals, a number of bad signals (could be anything from 0 to ∞). Hence, if an agent inherits one more good signal from his predecessor, he also infers from the equilibrium strategy that there are more bad signals in expectation.[4] Such an inference lowers his belief about the state. This effect is absent in Dye (1985)'s model where any disclosure implies that nothing is hidden.

If agents are more likely to observe private signals or more likely to receive disclosed signals, then, in expectation, there will be more bad signals between each pair of adjacent good signals. As a result, the negative effect of disclosing a good signal becomes more pronounced. Disclosure therefore needs to be more selective in equilibrium, since each disclosed signal needs to have a high enough likelihood ratio in order to overcome the stronger negative effect.

Focusing on the most informative constant-threshold equilibrium, we show that the amount of information contained in the disclosed signals is strictly increasing in the probabilities that agents observe private signals and receive disclosed signals, even though higher probabilities make disclosure more selective. However, even as both probabilities approach 1, learning about the state remains incomplete. The reason is that agents can also observe a large number of disclosed good signals when the most recent communication failure occurred far in the past.

Our work contributes to the literature on disclosure games where the sender has an unknown amount of signals.[5] Most of the existing literature on this topic study one-shot games.[6] Shin

[4]This argument may not generalize to all monotone strategies as the disclosure threshold can be history-dependent. Under these strategies, the effect of disclosing another good signal will depend on the entire set of signals disclosed. Our proof uses a *gradualism property* of our OLG game, which is stated in Section 3. This property also distinguishes our OLG setting from analogous one-shot disclosure games considered in Section 5.5.

[5]Di Tillio, Ottaviani, and Sørensen (2021) study an agent who observes the realizations of the k ($\leq n$) highest draws among n conditionally i.i.d. signals, which differs from our model where the number of signals disclosed is endogenous. In Bardhi and Bobkova (2023), each sender decides whether to disclose his signal without observing its realization. In Antić and Chakraborty (2025), the state is n-dimensional and the sender selects which of k ($\leq n$) realized values of those dimensions to disclose (typically not the highest ones). Onuchic and Ramos (2025) consider multiple senders who decide whether to disclose a multi-dimensional state via a collective decision rule.

[6]Bull and Watson (2019) examine a one-shot disclosure game where the sender's evidence is correlated with some unverifiable signal. In their setting, Dye (1985)'s unraveling result fails since disclosing good evidence may lead to unfavorable inference about the unverifiable signal.

(1994, 2003) and Dziuda (2011) focus on binary signals, which are not well-suited to studying the selectiveness of disclosure. Gao (2025) assumes that the sender has a continuum of signals and analyzes equilibria satisfying the *truth-leaning refinement* of Hart, Kremer, and Perry (2017). Rappoport (2025) focuses on receiver-optimal equilibria and shows that the receiver will take worse actions for the sender when the sender is expected to have more information. We instead examine how the selectiveness of the sender's disclosure behavior changes with the informational environment. We also explain in Section 5.1 why the truth-leaning refinement and the receiver-optimality refinement may conflict with our monotonicity refinement.[7]

A growing literature studies disclosure in other dynamic settings. In Guttman, Kremer, and Skrzypacz (2014), a sender may receive up to two signals over two periods and chooses both what to disclose and when. In Felgenhauer and Schulte (2014), Felgenhauer and Loerke (2017), Lou (2023), Arieli and Stewart (2025), and Dai, Fudenberg, and Pei (2026), a sender conducts a sequence of experiments before selecting a subset of outcomes to disclose to a receiver. In Gieczewski (2022) and Squintani (2026), some agents know the state and can disclose it to their neighbors. In Kremer, Schreiber, and Skrzypacz (2024), a potentially informed sender decides whether to disclose the current value of a random walk. Our paper complements this literature by examining how the informational environment affects selectiveness of disclosure.

Our work also contributes to the study of overlapping-generations models starting from Samuelson (1958). While many papers in this literature such as Bhaskar (1998) and Acemoglu and Jackson (2015) focus on the observability of agents' actions, we study settings where actions are not observed, so that agents can influence their successors' behaviors only by disclosing information. Anderlini, Gerardi, and Lagunoff (2012) analyze an overlapping-generations model with strategic communication and payoff externalities. Communication in their model is cheap talk rather than the disclosure of hard information. They also focus on a different question – whether agents learn the state in the long run – while we study the selectiveness of disclosure.

Our model is also related to the social learning literature, such as Banerjee (1992), Bikhchandani, Hirshleifer, and Welch (1992), and Smith and Sørensen (2000).[8] Unlike those models, our setting has payoff externalities and agents learn from signals disclosed by others rather than

[7] Gieczewski and Titova (2024) propose a coalitional proofness refinement and provide conditions under which their refinement coincides with the truth-leaning refinement and the receiver-optimality refinement.

[8] Niehaus (2011) studies the sharing of complementary or substitutable skills among a sequence of agents, which contrasts with our model, where agents share noisy signals about a payoff-relevant state of the world.

from their predecessors' actions. Bénabou and Vellodi (2025) incorporate strategic disclosure in a three-period social learning model. They show that disclosure becomes less selective as communication frictions vanish, which contrasts with our finding.

2 Baseline Model

Consider a game with a doubly infinite time horizon $t \in \mathbb{Z} = \{\dots, -1, 0, 1, \dots\}$ and a constant state of the world $\theta \in \Theta = \{\underline{\theta}, \overline{\theta}\}$. Let $\pi_0 \in (0,1)$ denote the prior probability that $\theta = \overline{\theta}$.

At the beginning of each period $t \in \mathbb{Z}$, a signal s^t is drawn from a finite set $S = \{s_1, s_2, \dots, s_l\}$ where $f_{\theta,i}$ is the probability that $s^t = s_i$ conditional on state θ. To simplify notation, we sometimes write $\overline{f}_i$ instead of $f_{\overline{\theta},i}$ and $\underline{f}_i$ instead of $f_{\underline{\theta},i}$. Let $\mathbf{f} = \{\overline{f}_i, \underline{f}_i\}_{i=1}^{l}$. Let $L_i = \overline{f}_i / \underline{f}_i$ denote the *likelihood ratio* of s_i. We assume that $l \geq 2$, $f_{\theta,i} > 0$ for every θ and i, $L_i \neq L_j$ for every $i \neq j$,[9] and that $\{s^t\}_{t \in \mathbb{Z}}$ are i.i.d. conditional on θ. We index the signal realizations so that

$$\infty > L_1 > L_2 > \cdots > L_l > 0. \tag{2.1}$$

In each period $t \in \mathbb{Z}$, agent t is the only active player. With probability $p \in (0,1]$, he observes s^t. We refer to s^t as agent t's *private signal*. With probability $\delta \in (0,1)$, independent of whether he observes s^t, he receives the (multi)set of signals disclosed by agent $t-1$. Then agent t takes an action $a^t \in \mathbb{R}$ and selects a set of signals to disclose to agent $t+1$.

We assume that agents can observe the count of signals disclosed to them but cannot directly observe their predecessors' actions, whether and when communication failures occurred, or the exact dates and sequence of the disclosed signals. Formally, in addition to his private signal, each agent only observes his *history* $\mathbf{h} = (h_1, h_2, \dots, h_l) \in H = \mathbb{N}^l$. If agent $t-1$ reveals h_1 copies of s_1, h_2 copies of s_2, $\dots$, and h_l copies of s_l, then agent t receives this set of signals with probability δ, in which case agent t's history is $(h_1, h_2, \dots, h_l)$. Agent t fails to receive this set of signals with probability $1-\delta$, in which case agent t's history is $\mathbf{0} = (0, \dots, 0)$. We interpret $1-\delta$ as a communication friction and refer to δ as the communication success rate.

As in Grossman (1981), Milgrom (1981), and Dye (1985), we assume that signals can be concealed but not falsified. As a result, each agent can only reveal a subset of the *available*

[9] Analogs of our results can be derived when multiple signal realizations share the same likelihood ratio, or when there is a continuum of signal realizations. Details of these extensions are available upon request.

signals, including those he received from his immediate predecessor and his own private signal. Formally, let $\mathbf{1}_j \in \mathbb{N}^l$ denote a vector where the jth entry is 1 and all other entries are 0. If agent t's history is $\mathbf{h}$, then (i) when he observes private signal $s_j \in \{s_1, \ldots, s_l\}$, he can only disclose vectors that are less than or equal to $\mathbf{h} + \mathbf{1}_j$ (in the vector order on $\mathbb{N}^l$) and (ii) when he does not observe a private signal, he can only disclose vectors that are no greater than $\mathbf{h}$.

Agent t's payoff is $u(\theta, a^t) + v(\theta, a^{t+1})$, which depends on the state θ, his action a^t, and his immediate successor's action a^{t+1}. We assume that (i) v is strictly increasing in a^{t+1}, (ii) for each $\pi \in [0,1]$, there is a unique $a \in \mathbb{R}$ that maximizes $\pi u(\overline{\theta}, a) + (1-\pi) u(\underline{\theta}, a)$, and this unique maximizer is strictly increasing in π.

Our solution concept is *steady-state Bayesian equilibrium* (or *equilibrium*), which extends the notion of (strict) pure-strategy steady-state equilibrium in Clark, Fudenberg, and Wolitzky (2021) to an incomplete information game.[10] An *equilibrium* consists of (i) a pure strategy $\sigma : H \times \Big(S \cup \{\emptyset\}\Big) \to \mathbb{R} \times H$ that maps an agent's history and private signal (or the absence of) to his action and the set of signals he discloses to his immediate successor, subject to the feasibility constraint,[11] (ii) two distributions $\underline{\mu}, \overline{\mu} \in \Delta(H)$ over histories, and (iii) a belief map $\pi : H \to [0,1]$ which represents the probability that an agent assigns to state $\overline{\theta}$ after observing his history but before observing his private signal. These components satisfy (i) when all agents use strategy σ, the steady-state history distribution is $\underline{\mu}$ conditional on $\theta = \underline{\theta}$ and is $\overline{\mu}$ conditional on $\theta = \overline{\theta}$,[12] (ii) $\pi(\mathbf{h})$ is derived via Bayes rule from $(\underline{\mu}, \overline{\mu})$ at every $\mathbf{h}$ that occurs with positive probability,[13] and (iii) σ maximizes an agent's expected payoff when everyone's belief after observing their histories is given by π.

A *strict equilibrium* is one where agents' equilibrium strategy is strictly optimal at every information set.[14] A strict equilibrium always exists, as will be implied by Proposition 1.

[10] We discuss mixed-strategy equilibria in Section 5.3. We allow $p = 1$ but require that $\delta < 1$ since steady-state equilibria are not well-defined when $\delta = 1$. Similarly, existing steady-state analysis such as Clark, Fudenberg, and Wolitzky (2021) and Pei (2026) assume that agents die with positive probability after each period.

[11] The notation $\emptyset$ denotes the event that an agent does not observe his private signal.

[12] As in earlier works that study steady-state equilibria such as Clark et al. (2021) and Pei (2026), we require all agents to use the same mapping $\sigma : H \times \Big(S \cup \{\emptyset\}\Big) \to \mathbb{R} \times H$. We show in Appendix A.1 that for every pure strategy σ and conditional on every state $\theta \in \{\overline{\theta}, \underline{\theta}\}$, there is a unique steady-state distribution over histories.

[13] Our results extend if we additionally require that, for every $\mathbf{h} \in H$, the belief $\pi(\mathbf{h})$ be derived from some conditional probability system that is consistent with the agents' equilibrium strategy.

[14] Our theorem extends when we only require players to have strict incentives at on-path information sets.

Interpretation: Consider an organization with overlapping cohorts. In period t, agent t is a junior member of the organization (e.g., a worker) who chooses effort a^t at a strictly convex cost. Agent $t-1$ is a senior member (e.g., a manager) who benefits from the worker's effort but does not internalize its cost. The manager can motivate the worker by influencing the worker's belief about the returns to effort (i.e., the state θ), e.g., by selecting the body of evidence presented to workers in the organization's training material.

This evidence may include earlier consulting projects done by the company that had a significant impact on their clients' businesses, or stories of successful workers in earlier cohorts (see Kipping and Clark (2012) for how consulting firms motivate new recruits). The manager may add or remove cases from the training material he inherited before presenting it to workers. The newly added cases could be the ones based on the manager's own observations, such as stories about his peers and coworkers, or projects done by the company when he was a junior.

It is difficult for managers to include fabricated cases since newly added materials are typically subject to internal review before being incorporated into any official training material. Because the training materials can be copied, summarized, and anonymized as they are passed on over time, workers may be unable to know precisely when each case occurred and the exact individuals involved, even if they can reliably learn its substantive content.[15] Information transmission may break down for exogenous reasons, such as accidental data loss, managerial negligence, or other unforeseen disruptions, which is captured by $1-\delta$. In period $t+1$, a subset of the workers from period t are promoted to managerial positions. These newly promoted managers can then revise the training materials that are presented to the next cohort of workers, i.e., agent $t+1$.

Section 5 discusses our modeling choices in more detail, as well as several alternative formulations and extensions. These include settings in which agents can observe the exact time stamps of disclosed signals or the exact time at which communication failed. We also discuss settings in which the successful transmission of different signals is independent, the communication success rate depends on the number of signals disclosed, and when agents face constraints when disclosing information.

[15]For example, in consulting firms, cases are checked before being included in the case library or training material. Client confidentiality and privacy concerns may require the distributed files to omit identifying information, including the clients and the former employees involved, and the exact times at which the cases happened.

3 Main Result

Our game admits many strict equilibria because each agent may possess arbitrarily many signals, allowing his successor's belief to be arbitrarily pessimistic at off-path histories. This, in turn, discourages agents from inducing those off-path histories, making those beliefs self-fulfilling. We start with some preliminary observations that describe some of those strict equilibria:

Proposition 1. *For any $j \in \{1, \ldots, l-1\}$ that satisfies $L_j > 1$, there is a strict equilibrium where at each history $\mathbf{h} = (h_1, \ldots, h_l)$, agents disclose all available s_j and nothing else, i.e., they disclose $(h_j + 1)\mathbf{1}_j$ if they observe private signal s_j, and disclose $h_j \mathbf{1}_j$ otherwise.*

The proof is in Appendix A.2. Proposition 1 implies that an equilibrium always exists in which agents disclose all realized s_1 and nothing else. However, for any $s_j \neq s_1$ with a likelihood ratio greater than 1, there is also a strict equilibrium in which agents disclose only s_j and conceal the rest, including those with strictly higher likelihood ratios. Intuitively, these equilibria can be sustained by the off-path belief that whenever any signal other than s_j (including those with higher likelihood ratios) is disclosed, the agent who disclosed it must have concealed signals with low likelihood ratios, which justifies their successors' pessimistic off-path beliefs about θ.

Equilibria where agents conceal higher-likelihood-ratio signals while revealing lower-likelihood-ratio ones seem unreasonable. For example, compare two scenarios in which agent t receives only one signal from agent $t-1$. Intuitively, agent t should be more *optimistic* about the state if this signal is s_1 instead of s_2, since s_1 has a higher likelihood ratio. More generally, agent t should be more optimistic about the state when agent $t-1$ replaces a low-likelihood-ratio signal with a high-likelihood-ratio signal in the set of signals disclosed. If so, agent $t-1$ will have no incentive to disclose any s_j while concealing signals with likelihood ratios strictly greater than L_j.

The above logic motivates our *monotonicity refinement*.[16] Let $\sigma_D(\mathbf{h}, s) \in H = \mathbb{N}^l$ denote the set of signals disclosed to the next agent when the current agent's history is $\mathbf{h}$ and private signal is $s \in S \cup \{\emptyset\}$, which is a component of agents' strategy σ.

Monotone Strategy & Monotone Equilibrium. *A strategy σ is monotone if for every $\mathbf{h} = (h_1, \ldots, h_l) \in H$, $s \in S \cup \{\emptyset\}$, and $j \in \{2, \ldots, l\}$, the jth entry of $\sigma_D(\mathbf{h}, s)$ being strictly*

[16]In Section 5.1, we explain why standard refinements such as requiring agents' off-path beliefs to be derived from a conditional probability system or satisfying the intuitive criterion and D1, have little bite in our setting. We also relate our monotonicity refinement to other refinements in the voluntary disclosure literature that restrict off-path beliefs, such as belief monotonicity in Guttman, Kremer, and Skrzypacz (2014).

positive implies that for every $i < j$, the ith entry of $\sigma_D(\mathbf{h}, s)$ equals h_i when $s \neq s_i$ and equals $h_i + 1$ when $s = s_i$. An equilibrium is monotone if agents use a monotone strategy.

Hence, a strategy is monotone if when at least one copy of s_j is disclosed at an information set, all available signals with likelihood ratios strictly greater than L_j are also disclosed at that information set. All equilibria in Dye (1985)'s model are monotone. In our dynamic setting, monotonicity allows, for example, agents to conceal s_1 at history-signal pair $(\mathbf{h}, s)$ and reveal s_2 at $(\mathbf{h}', s') \neq (\mathbf{h}, s)$, as long as all copies of s_1 contained in $(\mathbf{h}', s')$ are also revealed at $(\mathbf{h}', s')$.

A special class of monotone strategies are *constant-threshold strategies*. Formally, for every $\mathbf{h} \in H$ and $j \in \{0, 1, \ldots, l\}$, let $\mathbf{h}[j] \in H$ denote an l-dimensional vector whose first j entries coincide with $\mathbf{h}$ and whose other entries are all 0. By definition, $\mathbf{h}[0] = \mathbf{0}$ and $\mathbf{h}[l] = \mathbf{h}$.

Constant-Threshold Strategy & Constant-Threshold Equilibrium. *A strategy σ is a constant-threshold strategy if there exists $j \in \{0, \ldots, l\}$ such that (i) $\sigma_D(\mathbf{h}, s_i) = (\mathbf{h} + \mathbf{1}_i)[j]$ for every $\mathbf{h} \in H$ and $i \in \{1, \ldots, l\}$ and (ii) $\sigma_D(\mathbf{h}, \emptyset) = \mathbf{h}[j]$. A constant-threshold equilibrium is an equilibrium in which agents use a constant-threshold strategy.*

Compared to monotone strategies, constant-threshold strategies require a fixed disclosure threshold that does not vary with agents' histories or realized private signals. They also preclude partial disclosure of identical signals, e.g., if an agent has two copies of s_2 available, a constant-threshold strategy must prescribe either disclosing both copies of s_2 or concealing both.[17]

Among the equilibria in Proposition 1, those in which agents disclose s_j for some $j \geq 2$ and conceal s_1 are not monotone. The equilibrium in which agents reveal only s_1 is a constant-threshold equilibrium, so it is also a monotone equilibrium.

Some natural follow-up questions arise. Are there other strict monotone equilibria or constant-threshold equilibria beyond the ones described in Proposition 1, such as equilibria in which agents reveal s_1, s_2, s_3 and conceal all other signals, or in which agents reveal all copies of s_1 when they have s_1 available and reveal all copies of s_2 when they have no s_1 available? More generally, how does agents' equilibrium disclosure behavior depend on the signal structure $\mathbf{f}$, the probability p with which they receive private signals, and the communication friction $1 - \delta$?

[17] Section 5.1 extends our result to equilibria where agents use general threshold strategies. Such strategies allow the disclosure threshold to depend on the history and private signal, but require that at every history-signal pair, either all available copies of a signal realization are disclosed or all available copies of it are concealed.

Our main theorem characterizes, for any given signal $s_j \in S$, conditions under which s_j is disclosed in some strict monotone equilibrium (or constant-threshold equilibrium). Let

$$\eta(\delta, p) = \frac{\delta p}{1-\delta}. \tag{3.1}$$

Intuitively, in the steady state, $\delta/(1-\delta)$ is the expected number of periods since the most recent communication failure. Therefore, $\eta(\delta, p)$ is the expected number of private signals observed along an uninterrupted sequence of successful communication. By definition, $\eta(\delta, p) > 0$ for every $(\delta, p) \in (0,1) \times (0,1]$, increases in both δ and p, and diverges to ∞ as $\delta \to 1$.

We set $\eta_1 = \infty$, and for every $j \in \{2, \ldots, l\}$, let

$$\eta_j = \frac{\overline{f}_j - \underline{f}_j}{\underline{f}_j \sum_{i=1}^{j-1} \overline{f}_i - \overline{f}_j \sum_{i=1}^{j-1} \underline{f}_i} \quad \text{or, equivalently,} \quad \eta_j = \frac{L_j - 1}{\sum_{i=1}^{j-1} \underline{f}_i (L_i - L_j)}. \tag{3.2}$$

For every $j \geq 2$, the denominators in (3.2) are strictly positive because $L_i > L_j$ for every $i < j$. Hence, $\eta_j > 0$ if and only if $L_j > 1$. Moreover, one can show that $\infty = \eta_1 > \eta_2 > \cdots > \eta_l = -1$.

Theorem 1. *For any $j \in \{1, \ldots, l\}$:*

1. *If $\eta(\delta, p) \geq \eta_j$, then in any strict monotone equilibrium, signal s_j is never disclosed at any history that occurs with positive probability.*

2. *Signal s_j is disclosed in some strict constant-threshold equilibrium if and only if $\eta(\delta, p) < \eta_j$. Signal s_j is disclosed in some constant-threshold equilibrium if and only if $\eta(\delta, p) \leq \eta_j$.*

The proof is in Section 4. Since every constant-threshold equilibrium is also a monotone equilibrium, Theorem 1 implies that signal s_j is disclosed in some strict monotone (or strict constant-threshold) equilibrium if and only if $\eta(\delta, p) < \eta_j$, and is disclosed in some constant-threshold equilibrium if and only if $\eta(\delta, p) \leq \eta_j$. Because $\eta_1 = \infty$, there always exists a strict constant-threshold equilibrium in which agents disclose s_1. Since $\eta_j \leq 0$ for every j satisfying $L_j \leq 1$, agents never disclose any signal with likelihood ratio less than 1 in any equilibrium. Since η_j is strictly decreasing in j, an increase in $\eta(\delta, p)$ reduces the set of signal realizations that can be disclosed. Since $\eta(\delta, p) \to \infty$ as $\delta \to 1$, when the communication friction $1 - \delta$ is sufficiently small, all signals except for the strongest one s_1 will be concealed in equilibrium.

Applying (3.1) and (3.2), Theorem 1 implies that signal $s_j \neq s_1$ is disclosed in some strict monotone equilibrium only if (i) communication frictions are large enough (i.e., δ is small), (ii) agents do not observe private signals too frequently (i.e., p is small), and (iii) signals with

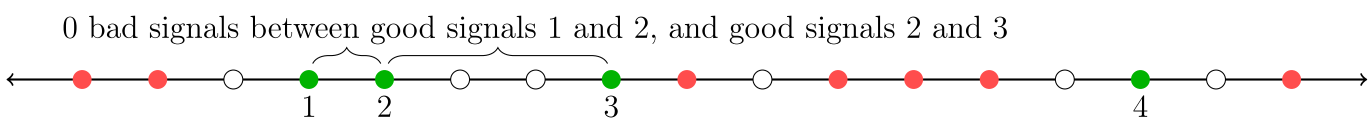

Figure 1: A sequence of realized signals after the most recent communication failure. Red circles represent observed bad signals, green circles represent observed good signals, and white circles represent unobserved signals. In this example, the number of bad signals is 0 between good signals 1 and 2 as well as between good signals 2 and 3, and is 4 between good signals 3 and 4.

likelihood ratios greater than L_j either arrive with low probability, or have likelihood ratios close to L_j. As p and δ become larger, i.e., agents are more likely to observe private signals and to inherit signals from their predecessors, $\eta(\delta, p)$ increases and by Theorem 1, disclosure becomes *more selective* in the sense that a smaller set of signals can be disclosed in equilibrium.

Our finding contrasts with that of Dye (1985), who shows that as the probability that the sender has information increases, disclosure becomes *less selective*, in the sense that a larger set of signals is disclosed in equilibrium. Similarly, one can show that in Dye (1985)'s setting, when the receiver fails to receive the information disclosed by the sender with probability $1-\delta$, disclosure becomes *less selective* as δ increases. In the limiting case considered by Grossman (1981) and Milgrom (1981) where the sender has information and can successfully transmit it with probability close to 1, no disclosure leads the receiver to infer that the sender has the worst possible signal, which motivates the sender to disclose even the second-worst signal.

This difference between our result and Dye (1985)'s arises due to the nature of the uncertainty the receivers face about the senders' information structure. To see this, let us fix an arbitrary signal s_i that satisfies $L_i > 1$, and let us call $s_1, \dots, s_i$ *good signals* and $s_{i+1}, \dots, s_l$ *bad signals*.

Fix any period $t \in \mathbb{Z}$ and consider the sequence of realized signals $\{s^k\}_{k=\tau}^{t-1}$, where τ denotes the last period before t in which communication failed. As shown in Figure 1, this sequence has a number of (observed) good signals,[18] which ranges from 0 to ∞, and between each pair of adjacent good signals, a number of (observed) bad signals, which also ranges from 0 to ∞.

Suppose that, in equilibrium, agents use a constant-threshold strategy of disclosing all good signals and concealing all bad ones. Each agent can then observe the number of copies of each

[18] Private signal s^k is an *observed signal* if and only if it is observed by agent k. If agent k does not observe s^k, then it is an *unobserved signal*, counted as neither good nor bad, which we depict as a white dot in Figure 1.

good signal in the sequence but not the number or content of bad signals. Recall that in between each pair of adjacent good signals, there is a number of bad signals, which from the receiving agent's perspective, is a random variable $\widetilde{n}$. Since signals are conditionally i.i.d., the distribution of $\widetilde{n}$, conditional on state θ, does not depend on the good signals realized.

Hence, disclosing one more good signal has two effects on the next agent's belief. First, it reveals that another good signal has been realized, which increases the next agent's belief, since good signals have likelihood ratios more than 1. Second, the agent can infer from the equilibrium strategy that $\widetilde{n}$ additional bad signals have been realized and concealed, which has a negative effect on belief. This differs from Dye (1985)'s model, where the receiver believes that nothing is concealed after any disclosure.

As (δ, p) increases, the distribution of $\widetilde{n}$, conditional on any θ, shifts to the right, so the negative effect of disclosing another good signal becomes more pronounced. Disclosure therefore becomes more selective, since each good signal disclosed in equilibrium must have a high enough likelihood ratio to offset the stronger negative effect. This logic is reflected in our result since

$$\eta(\delta, p) < \eta_j \quad \Leftrightarrow \quad L_j > \frac{1 + \eta(\delta, p) \sum_{i=1}^{j-1} \overline{f}_i}{1 + \eta(\delta, p) \sum_{i=1}^{j-1} \underline{f}_i}. \tag{3.3}$$

Nevertheless, the *net effect* of disclosing the strongest possible signal s_1 is always positive. To see why, consider an auxiliary scenario where agents observe only the total number of good signals $s_1, \ldots, s_i$ in the sequence, but not the identity of each one. If they learn that there is no good signal, their posterior is strictly less than their prior π_0. Suppose by way of contradiction that disclosing an additional good signal in this auxiliary game has a negative net effect on the next agent's belief, then agents' posteriors would be strictly lower than their prior regardless of the number of good signals disclosed, which cannot happen since agents' belief is a martingale. Since the likelihood ratio of s_1 is no less than that of an "average" good signal in the set $\{s_1, \ldots, s_i\}$, the net effect of disclosing an additional s_1 on the next agent's belief must be positive.

The above logic does not generalize to all monotone strategies, since the set of signals disclosed can be history-dependent. In this case, even conditional on θ, the expected number of undisclosed signals between each pair of adjacent disclosed signals may depend on the entire set of signals disclosed, and so too may the net effect of disclosing another copy of a given signal.

Our proof uses a *gradualism property* that distinguishes our overlapping-generations game from one-shot disclosure games in which receivers face uncertainty about the number of signals:

Gradualism Property. *For any $t \in \mathbb{Z}$ and any period $t+1$ history $\mathbf{h}^{t+1} = (h_1^{t+1}, \ldots, h_l^{t+1})$*

that comes after period t history $\mathbf{h}^t = (h_1^t, \ldots, h_l^t)$, there exists at most one $j \in \{1, \ldots, l\}$ such that $h_j^{t+1} > h_j^t$. If such j exists, then $h_j^{t+1} = h_j^t + 1$.

Intuitively, this property is driven by our modeling assumption that each agent receives at most one private signal in addition to the signals disclosed by his immediate predecessor.

Now, fix any strict monotone equilibrium and suppose s_j is the weakest signal disclosed on path. A consequence of our gradualism property is that there exist on-path histories with exactly one copy of s_j. We pick the shortest among those on-path histories, and if there are multiple, pick any one that minimizes the next agent's belief. We denote the chosen history by $\mathbf{h}^*$.

The key technical step is to show that when an agent's history is $\mathbf{h}^*$, his immediate predecessor's history is either $\mathbf{h}^*$ or $\mathbf{h}^* - \mathbf{1}_j$ (see Lemma 1).[19] That is, in the period right before history reaches $\mathbf{h}^*$ for the first time, the additional signal included must be s_j.

We then examine the marginal effect of disclosing s_j at history $\mathbf{h}^* - \mathbf{1}_j$ by comparing agents' beliefs at $\mathbf{h}^* - \mathbf{1}_j$ and at $\mathbf{h}^*$. As in constant-threshold equilibria, disclosing s_j reveals to the next agent that one copy of s_j occurred, which multiplies the likelihood ratio of their belief by L_j. However, when $s_j \neq s_1$, observing $\mathbf{h}^*$ can be bad news since it shows that no signal better than s_j has been realized after the disclosed s_j. This is because if an agent observes a private signal better than s_j at $\mathbf{h}^*$, he will no longer reveal $\mathbf{h}^*$ to the next agent as it requires disclosing s_j while hiding his private signal that has a higher likelihood ratio. This cannot happen when the agent uses a monotone strategy.

Hence, agents will disclose s_j at $\mathbf{h}^* - \mathbf{1}_j$ only if L_j is large enough to overcome the negative effect of no better signal arriving after s_j. This negative effect is stronger when signals better than s_j are more likely to arrive, such as when communication frictions are smaller (i.e., δ is larger) and when agents observe their private signals with higher probability (i.e., p is larger).

We conclude this section by discussing the implications of Theorem 1. First, when $\eta(\delta, p) \geq \eta_2$, no signal other than s_1 can be disclosed in a strict monotone equilibrium. This leads to a characterization of all strict monotone equilibria when $\eta(\delta, p)$ is large enough:[20]

[19] In the special case of constant-threshold equilibrium, history $\mathbf{h}^*$ is $\mathbf{1}_j$. However, our proof for *strict* monotone equilibrium does not apply to constant-threshold equilibrium as it requires players to have strict incentives (Lemma 1), whereas our result for constant-threshold equilibria does not require the equilibria to be strict.

[20] It seems hard to characterize *all* strict monotone equilibria when $\eta(\delta, p) < \eta_2$, since according to Theorem 1, multiple signals, including at least s_1 and s_2, can be disclosed on the equilibrium path, and a general monotone strategy allows agents' disclosure threshold to depend on the history and realized private signal.

Corollary 1. *If $\eta(\delta, p) \geq \eta_2$, every strict monotone equilibrium is characterized by $n \in \mathbb{N} \cup \{\infty\}$ such that agents disclose up to n copies of s_1 and conceal all other signals, i.e., at on-path history $\mathbf{h} = (h_1, \ldots, h_l)$, agents disclose $\min\{n, h_1 + 1\}$ copies of s_1 and nothing else when they observe private signal s_1, and disclose $\min\{n, h_1\}$ copies of s_1 otherwise.*

The proof is in Appendix A.2. Our theorem also leads to a characterization of all constant-threshold equilibria for any parameter configuration.

Corollary 2. *For any $i \geq 2$, if $\eta(\delta, p) \in (\eta_i, \eta_{i-1}]$, then for each $j \in \{0, \ldots, i-1\}$, there is a constant-threshold equilibrium in which agents conceal $s_{j+1}, \ldots, s_l$ and disclose all other signals, and for each $j \geq i$, there is no constant-threshold equilibrium in which agents disclose $s_1, \ldots, s_j$.*

We omit the proof of this corollary as it follows directly from the second part of Theorem 1.

Focusing on the most informative constant-threshold equilibrium (i.e., the one in which the largest set of signals is disclosed), Corollary 2 seems to suggest that there is a discontinuity at $\eta(\delta, p) = \eta_j$ since when $\eta(\delta, p) \leq \eta_j$, signal s_j is disclosed in the most informative equilibrium, whereas it can no longer be disclosed when $\eta(\delta, p) > \eta_j$. Proposition 2 shows that no such discontinuity exists. Let $\mathbf{h}_{\delta,p}$ denote the steady-state history observed by an agent in the most informative constant-threshold equilibrium, which is a random variable that takes values in H. We measure the information contained in agents' histories according to

$$\mathcal{I}(\delta, p) = g(\pi_0) - \mathbb{E}\left[g\left(\pi(\mathbf{h}_{\delta,p})\right)\right],$$

where $\pi(\mathbf{h}) = \Pr(\theta = \overline{\theta} \mid \mathbf{h})$ is their posterior after observing the disclosed history but before observing their private signal, and $g(q) = -q \log q - (1-q)\log(1-q)$ is the entropy function. That is, $\mathcal{I}(\delta, p)$ is the expected Kullback–Leibler divergence of the agent's posterior from his prior. We show that $\mathcal{I}$ is jointly continuous in (δ, p) and strictly increasing in each parameter.

Proposition 2. *The function $\mathcal{I} : (0,1) \times (0,1] \to \mathbb{R}_+$ is continuous. Moreover, $\mathcal{I}(\delta, p)$ depends on (δ, p) only through $\eta(\delta, p)$ and is strictly increasing in $\eta(\delta, p)$.*

The proof is in Appendix A.4. The amount of information transmitted is continuous, since when $\eta(\delta, p) = \eta_j$, the number of copies of s_j disclosed does not affect the receiver's posterior belief in the most informative constant-threshold equilibrium, as the positive effect of observing another s_j exactly cancels out the negative effect of expecting more bad signals.

Since the amount of information contained in the disclosed history is strictly increasing in $\eta(\delta, p)$, it is natural to ask whether agents can learn the state with arbitrarily high precision as $\eta(\delta, p) \to \infty$. Proposition 3 shows that this is not the case. In particular, even when $p = 1$ and $\delta \to 1$, the limiting distribution over the agents' steady-state posterior beliefs in the most informative constant-threshold equilibrium converges to neither the Dirac measure on 1 (conditional on $\theta = \overline{\theta}$) nor the Dirac measure on 0 (conditional on $\theta = \underline{\theta}$).

Proposition 3. *Fix any $p \in (0, 1]$ and $\theta \in \Theta$. As $\delta \to 1$, the posterior belief $\pi(\mathbf{h}_{\delta,p})$ in the most informative constant-threshold equilibrium, conditional on θ, converges in distribution to a nondegenerate random variable that takes values strictly between 0 and 1 almost surely.*

The proof is in Appendix A.5. Proposition 3 implies that agents do not fully learn the state in the most informative constant-threshold equilibrium, even when $p = 1$ and $\delta \to 1$. Intuitively, although the expected number of disclosed copies of s_1 diverges to infinity as $\delta \to 1$, agents do not observe how many periods have elapsed since the most recent communication failure. As a result, a large number of disclosed copies of s_1 may reflect either that s_1 is relatively likely, which suggests that the state is high, or that the time since the last communication failure is long. This uncertainty about the time since the last communication failure does not vanish as communication frictions disappear. As a result, the limiting distributions of disclosed histories under the two states continue to overlap, and agents do not fully learn the state even in the limit.

4 Proof of Theorem 1

Section 4.1 establishes that, for any $j \geq 2$, $\eta(\delta, p) < \eta_j$ is necessary for s_j to be disclosed at any on-path history in any strict monotone equilibrium. Section 4.2 provides necessary and sufficient conditions for any $s_j \neq s_1$ to be disclosed in some (strict) constant-threshold equilibrium. Throughout this section, we replace $\eta(\delta, p)$ with η in order to avoid cumbersome notation.

4.1 Proof of Theorem 1: Part 1

We state two properties of strict equilibria. First, each agent tries to maximize his immediate successor's belief when deciding what to disclose. Since he can ignore his private signal and reveal only his history $\mathbf{h}$, from which he will induce the same belief when communication succeeds, the history he discloses in equilibrium must lead to a weakly higher belief than $\pi(\mathbf{h})$. This leads to an *improvement principle* similar to the one in Banerjee and Fudenberg (2004).

Improvement Principle. *For any equilibrium and any $t \in \mathbb{Z}$, if the realized history in period t is $\mathbf{h}^t$ and agent t chooses to disclose $\mathbf{h}^{t+1}$ after receiving private signal $s^t \in S \cup \{\emptyset\}$ at $\mathbf{h}^t$, then $\pi(\mathbf{h}^{t+1}) \geq \pi(\mathbf{h}^t)$. If the equilibrium is strict and $\mathbf{h}^{t+1} \neq \mathbf{h}^t$, then $\pi(\mathbf{h}^{t+1}) > \pi(\mathbf{h}^t)$.*

An implication of our improvement principle is the following *no-turning-back property.*

No-Turning-Back Property. *In any strict equilibrium, for any $\mathbf{h}$ that occurs with positive probability and any $\mathbf{h}' < \mathbf{h}$, $\mathbf{h}'$ will not occur after $\mathbf{h}$ before the next communication failure.*

This is because whenever agents can disclose $\mathbf{h}$, they can disclose any $\mathbf{h}' < \mathbf{h}$, so their strict incentive to disclose $\mathbf{h}$ implies that $\pi(\mathbf{h}) > \pi(\mathbf{h}')$. Iterating this argument, we know that no agent will disclose $\mathbf{h}'$ after $\mathbf{h}$ until communication fails.

We now return to the proof of Theorem 1. Fix any strict monotone equilibrium $(\sigma, \underline{\mu}, \overline{\mu}, \pi)$. For convenience, let $\mu_\theta \in \Delta(H)$ denote the history distribution conditional on θ. Let $H_\sigma \subset H$ denote the set of histories that occur with positive probability under $\underline{\mu}$ (or, equivalently, under $\overline{\mu}$). Let j denote the largest integer in $\{0, 1, \ldots, l\}$ such that there exists $\mathbf{h} = (h_1, \ldots, h_l) \in H_\sigma$ with $h_j \geq 1$ (if $H_\sigma = \{\mathbf{0}\}$, then set $j = 0$). By definition, s_j has the lowest likelihood ratio among the signals disclosed on the equilibrium path. Our gradualism property implies that there exists $\mathbf{h} \in H_\sigma$ with $h_j = 1$.

For any $j \geq 2$, we derive necessary conditions for strict monotone equilibria with s_j being the weakest signal disclosed on path to exist. Let $H^{**} \subset H_\sigma$ denote the set of positive-probability histories under σ such that the jth entry is 1, i.e., on-path histories that contain exactly one copy of s_j. Let $H^* \subset H^{**}$ denote the subset of H^{**} such that a history $\mathbf{h}$ belongs to H^* if and only if it minimizes $h_1 + \cdots + h_{j-1}$ among all histories in H^{**}. By definition, all histories in H^* have the same length, and hence, H^* is a finite set. Let $\mathbf{h}^* = (h_1^*, \ldots, h_l^*)$ denote an arbitrary solution to the minimization problem $\arg\min_{\mathbf{h} \in H^*} \pi(\mathbf{h})$. By construction, $h_{j+1}^* = \cdots = h_l^* = 0$ and $h_j^* = 1$. The following lemma shows that $\mathbf{h}^*$ can only originate from $\mathbf{h}^* - \mathbf{1}_j$.

Lemma 1. *In any strict monotone equilibrium, if an agent's history is $\mathbf{h}^*$, then his immediate predecessor's history is either $\mathbf{h}^*$ or $\mathbf{h}^* - \mathbf{1}_j$.*

The proof is in Appendix A.3.

For any on-path history $\mathbf{h} \in H_\sigma$, let $D(\mathbf{h}) = \{i : \sigma_D(\mathbf{h}, s_i) \neq \mathbf{h}\} \subset \{1, 2, \ldots, l\}$. Intuitively, $i \in D(\mathbf{h})$ if and only if after an agent receives private signal s_i at history $\mathbf{h}$, he will disclose something different from $\mathbf{h}$, i.e., the history in the next period will not be $\mathbf{h}$ when communication succeeds. The no-turning-back property implies that for any $\mathbf{h} \in H_\sigma$, $i \in D(\mathbf{h})$ if and only if the disclosed vector after receiving private signal s_i at history $\mathbf{h}$ is not weakly less than $\mathbf{h}$, or equivalently, the number of copies of s_i disclosed is more than the ith entry of $\mathbf{h}$. Since $\mathbf{h}^* = (h_1^*, \ldots, h_{j-1}^*, 1, 0, \ldots, 0)$ and s_j is defined as the weakest signal disclosed on the equilibrium path, we have

$$\{1, \ldots, j-1\} \subset D(\mathbf{h}^*) \subset \{1, \ldots, j\}. \tag{4.1}$$

This is because if there exists $i < j$ such that $i \notin D(\mathbf{h}^*)$, agents disclose $\mathbf{h}^*$ upon receiving private signal s_i at history $\mathbf{h}^*$, in which case they disclose at least one copy of s_j while concealing at least one copy of s_i. This contradicts the monotonicity of agents' equilibrium strategy.

By Lemma 1, history $\mathbf{h}^*$ can occur only after $\mathbf{h}^*$ or $\mathbf{h}^* - \mathbf{1}_j$, which implies that

$$\mu_\theta(\mathbf{h}^*) = \mu_\theta(\mathbf{h}^* - \mathbf{1}_j) \cdot \delta p f_{\theta,j} \cdot \sum_{n=0}^{\infty} \delta^n (1 - p \sum_{i \in D(\mathbf{h}^*)} f_{\theta,i})^n = \mu_\theta(\mathbf{h}^* - \mathbf{1}_j) \cdot \frac{\delta p f_{\theta,j}}{1 - \delta(1 - p \sum_{i \in D(\mathbf{h}^*)} f_{\theta,i})}.$$

Hence, by Bayes rule, $\pi(\mathbf{h}^*) > \pi(\mathbf{h}^* - \mathbf{1}_j)$ if and only if

$$L_j \cdot \frac{1 - \delta(1 - p \sum_{i \in D(\mathbf{h}^*)} \underline{f}_i)}{1 - \delta(1 - p \sum_{i \in D(\mathbf{h}^*)} \overline{f}_i)} = L_j \cdot \frac{1 + \eta \sum_{i \in D(\mathbf{h}^*)} \underline{f}_i}{1 + \eta \sum_{i \in D(\mathbf{h}^*)} \overline{f}_i} > 1, \tag{4.2}$$

where the equality follows from the identity $1 - \delta(1 - px) = (1 - \delta)(1 + \eta x)$. Expression (4.1) implies that $D(\mathbf{h}^*)$ is either $\{1, \ldots, j-1\}$ or $\{1, \ldots, j\}$. Therefore, (4.2) becomes either

$$L_j \cdot \frac{1 + \eta \sum_{i=1}^{j-1} \underline{f}_i}{1 + \eta \sum_{i=1}^{j-1} \overline{f}_i} > 1 \text{ or } L_j \cdot \frac{1 + \eta \sum_{i=1}^{j} \underline{f}_i}{1 + \eta \sum_{i=1}^{j} \overline{f}_i} > 1 \tag{4.3}$$

Applying (3.2) and (3.3), we know that either of the above two inequalities in (4.3) is satisfied if and only if $L_j > 1$ and $\eta < \eta_j$. Since η_j is strictly decreasing in j, it follows that when $L_j \leq 1$ or when $L_j > 1$ but $\eta \geq \eta_j$, there is also no strict monotone equilibrium in which the weakest signal disclosed has a likelihood ratio strictly less than L_j. This completes the proof of Part 1.

4.2 Proof of Theorem 1: Part 2

Suppose that in equilibrium, agents disclose $s_1, \ldots, s_j$ and conceal all other signals. Under the steady-state history distribution in state θ, the probability of history $\mathbf{h} = (h_1, h_2, \ldots, h_j, 0, \ldots, 0)$

is

$$(1-\delta)(\delta p)^{|\mathbf{h}|} \cdot X_\theta(\mathbf{h}) \cdot \prod_{i=1}^{j} (f_{\theta,i})^{h_i}, \tag{4.4}$$

where $|\mathbf{h}| = h_1 + \cdots + h_j$ denotes the length of history $\mathbf{h}$ and $X_\theta(\mathbf{h})$ denotes the coefficient in front of the term $\prod_{i=1}^{j}(q_i)^{h_i}$ in the following power series of $q_1, q_2, \ldots, q_j$:

$$\sum_{k=|\mathbf{h}|}^{\infty} \Big(q_1 + q_2 + \cdots + q_j + \delta(1 - p\sum_{i=1}^{j} f_{\theta,i})\Big)^k. \tag{4.5}$$

A series of algebraic manipulations reveals that

$$X_\theta(\mathbf{h}) = \Big(1 - \delta(1 - p\sum_{i=1}^{j} f_{\theta,i})\Big)^{-|\mathbf{h}|-1} \cdot \frac{|\mathbf{h}|!}{h_1!h_2!\ldots h_j!}. \tag{4.6}$$

Since $1-\delta(1-px) = (1-\delta)(1+\eta x)$, the posterior likelihood ratio after observing $\mathbf{h}$ is

$$\frac{\pi(\mathbf{h})}{1-\pi(\mathbf{h})} = \frac{\pi_0}{1-\pi_0} \cdot \Big(\frac{1+\eta\sum_{i=1}^{j}\underline{f}_i}{1+\eta\sum_{i=1}^{j}\overline{f}_i}\Big)^{|\mathbf{h}|+1} \cdot \prod_{i=1}^{j}(L_i)^{h_i}. \tag{4.7}$$

Since $\eta > 0$ for any (δ, p) and $\sum_{i=1}^{j}\overline{f}_i > \sum_{i=1}^{j}\underline{f}_i$ for every $j < l$, we have

$$\frac{1+\eta\sum_{i=1}^{j}\underline{f}_i}{1+\eta\sum_{i=1}^{j}\overline{f}_i} < 1. \tag{4.8}$$

For a constant-threshold strategy in which agents only disclose $s_1, \ldots, s_j$ to be an equilibrium, a necessary condition is that agents weakly prefer to disclose $(h_1, \ldots, h_{j-1}, h_j, 0, \ldots, 0)$ to $(h_1, \ldots, h_{j-1}, h_j - 1, 0, \ldots, 0)$ for any $h_j \geq 1$ and $h_1, \ldots, h_{j-1} \geq 0$. By (4.7), this incentive constraint is satisfied if and only if

$$L_j \cdot \frac{1+\eta\sum_{i=1}^{j}\underline{f}_i}{1+\eta\sum_{i=1}^{j}\overline{f}_i} \geq 1. \tag{4.9}$$

For $j \geq 2$, expression (3.2) implies that (4.9) is equivalent to $\eta \leq \eta_j$. Similarly, for a constant-threshold strategy where agents only disclose $s_1, \ldots, s_j$ to be a strict equilibrium, we need

$$L_j \cdot \frac{1+\eta\sum_{i=1}^{j}\underline{f}_i}{1+\eta\sum_{i=1}^{j}\overline{f}_i} > 1 \quad \Leftrightarrow \quad \eta < \eta_j. \tag{4.10}$$

We next establish necessity for the existence of a constant-threshold equilibrium in which s_j is disclosed. Suppose $j \geq 2$ and that s_j is disclosed in a constant-threshold equilibrium with threshold k. Then $k \geq j$, and the preceding argument implies that $\eta_k \geq \eta$. Since $\eta > 0$, this requires $\eta_k > 0$, and therefore $L_k > 1$. Since $j \leq k$, we have $\eta_j \geq \eta_k \geq \eta$. If the constant-threshold equilibrium is strict, then $\eta_k > \eta$, and hence $\eta_j > \eta$.

We next show that the strategy in which agents only disclose $s_1, \ldots, s_j$ is a constant-threshold equilibrium when $\eta \leq \eta_j$, and is a strict constant-threshold equilibrium when $\eta < \eta_j$. First, if (4.9) is satisfied,

$$L_k \cdot \frac{1+\eta \sum_{i=1}^{j} \underline{f}_i}{1+\eta \sum_{i=1}^{j} \overline{f}_i} > L_j \cdot \frac{1+\eta \sum_{i=1}^{j} \underline{f}_i}{1+\eta \sum_{i=1}^{j} \overline{f}_i} \geq 1 \text{ for every } k < j,$$

where the strict inequality follows from $L_k > L_j$. This implies that agents also have an incentive to disclose signals $s_1, \ldots, s_{j-1}$ when $\eta \leq \eta_j$. Similarly, when (4.10) is satisfied, agents will have strict incentives to disclose $s_1, \ldots, s_j$. Agents have no incentive to disclose $s_{j+1}, \ldots, s_l$ as long as their belief at any off-path history $\mathbf{h}$, i.e., those where $\mathbf{h} \neq \mathbf{h}[j]$, satisfies $\pi(\mathbf{h}) < \pi(\mathbf{0})$.[21]

5 Discussions & Extensions

This section discusses several aspects of our model and results. Section 5.1 connects our monotonicity and constant-threshold refinements to those proposed in the disclosure literature. Section 5.2 discusses two modeling assumptions, that agents cannot directly observe the exact time stamps of the disclosed signals and that all disclosed signals are lost when communication fails. Section 5.3 discusses non-strict equilibria and mixed-strategy equilibria. Section 5.4 considers several extensions, such as when agents face disclosure constraints and when the communication success rate depends on the number of signals disclosed. Section 5.5 explains the difference between our OLG game and an analogous one-shot disclosure game.

5.1 Alternative Equilibrium Refinements

All the constant-threshold equilibria in Corollary 2, as well as the equilibria that violate monotonicity in Proposition 1, are *Perfect Bayesian equilibria* as defined in Fudenberg and Tirole (1991), which requires players' beliefs to be derived from a conditional probability system induced by their equilibrium strategies.[22] This is because, when agents only disclose signals in

[21]Similarly, a strict constant-threshold equilibrium in which no signal is disclosed always exists, as long as we set agents' beliefs at any history $\mathbf{h} \neq \mathbf{0}$ to be strictly less than their prior belief about the state.

[22]Part 1 of Theorem 1 characterizes a common property of all strict monotone equilibria, so it trivially extends once we impose the conditional probability system refinement. Part 2 of Theorem 1 establishes the existence of a (strict) constant-threshold equilibrium, so imposing the conditional probability system refinement makes the result stronger. This is why our discussion in this paragraph focuses on constant-threshold equilibria.

$S' \subset S$, any history that contains signals in $S \backslash S'$ occurs with zero probability conditional on any $(\mathbf{h}, s) \in H \times (S \cup \{\emptyset\})$, so the conditional probability system requirement has no bite.

Standard forward-induction refinements, such as the intuitive criterion and D1 in Cho and Kreps (1987), also have little bite, e.g., both monotone and non-monotone equilibria described in Proposition 1 satisfy those refinements. Intuitively, the sender's payoff is independent of the information he observes, so these refinements cannot distinguish among sender-types who disclose the same set of signals in equilibrium and differ only in the signals they conceal.

Guttman, Kremer, and Skrzypacz (2014) introduced a refinement on off-path beliefs called *belief monotonicity* in a dynamic disclosure game with two periods and the sender having at most two signals. Their refinement requires that, at histories where only one signal is disclosed, the receiver's posterior belief is strictly higher when the disclosed signal has a higher likelihood ratio. We extend their refinement to our setting where senders can have arbitrarily many signals.

Monotone Belief. *A belief map $\pi : H \to [0,1]$ is monotone if $\pi(\mathbf{h}) > \pi(\mathbf{h}^*)$ for every pair of histories $\mathbf{h} = (h_1, \dots, h_l)$ and $\mathbf{h}^* = (h_1^*, \dots, h_l^*)$ that satisfy $\sum_{j=1}^{l} h_j = \sum_{j=1}^{l} h_j^*$ and*

$$\sum_{j=1}^{n} h_j \geq \sum_{j=1}^{n} h_j^* \text{ for every } n < l \text{ with strict inequality for some.} \tag{5.1}$$

Intuitively, a belief map is monotone if agents have a strictly higher belief about the state when their predecessors disclose one more higher-likelihood-ratio signal and one fewer lower-likelihood-ratio signal. As in Guttman et al. (2014), it only imposes restrictions on the receiver's posterior beliefs at pairs of histories with the same length, i.e., $\sum_{j=1}^{l} h_j = \sum_{j=1}^{l} h_j^*$.

Theorem 1 extends once we replace *monotone strategy* with *monotone belief*. This is because when the belief map is monotone, any non-monotone strategy has a strictly profitable deviation, by replacing a disclosed low-likelihood-ratio signal with a concealed high-likelihood-ratio signal. This implies that if s_j is never disclosed in any equilibrium with a monotone strategy, then it is also never disclosed in any equilibrium with a monotone belief map. Moreover, for the constant-threshold equilibrium we constructed in the proof of Theorem 1, there exist off-path beliefs that satisfy agents' incentive constraints as well as the belief monotonicity requirement.[23]

Guttman et al. (2014) also proposed a *threshold strategy* refinement, which requires the sender to disclose a signal if and only if its likelihood ratio is above some cutoff. Extending it to our

[23]Similar to the off-path beliefs in Section 4.2, for every $\mathbf{h}$ that occurs with zero probability in equilibrium, let $\pi(\mathbf{h}) < \pi(\mathbf{0})$. For any $n \in \mathbb{N}$, there is a finite number of off-path histories with length n, which means that there exists a mapping π from length-n off-path histories to the open interval $(0, \pi(\mathbf{0}))$ that satisfies (5.1).

setting leads to a refinement that is more restrictive than monotone equilibrium but is more permissive than constant-threshold equilibrium as the threshold can be history-dependent.

Threshold Strategy & Threshold Equilibrium. *A strategy σ is a threshold strategy if there exists a function $J : H \times (S \cup \{\emptyset\}) \to \{0, 1, \ldots, l\}$ such that for every $\mathbf{h} \in H$ and $i \in \{1, 2, \ldots, l\}$, we have $\sigma_D(\mathbf{h}, s_i) = (\mathbf{h} + \mathbf{1}_i)[J(\mathbf{h}, s_i)]$ and $\sigma_D(\mathbf{h}, \emptyset) = \mathbf{h}[J(\mathbf{h}, \emptyset)]$. An equilibrium is a threshold equilibrium if agents use a threshold strategy.*

Our results for strict monotone equilibria extend to strict threshold equilibria. Part 1 of Theorem 1 extends since the set of strict monotone equilibria includes the set of strict threshold equilibria. Part 2 of Theorem 1 extends because, when $\eta(\delta, p) < \eta_j$, there exists a constant-threshold equilibrium where s_j is disclosed, which by definition, must also be a threshold equilibrium.

Hart, Kremer, and Perry (2017) introduced a truth-leaning refinement, which in our setting, requires that (A0) an agent will disclose all available signals if doing so maximizes his payoff and (P0) agents believe that their predecessors did not hide any information upon observing any $\mathbf{h} \in H$ that occurs with zero probability in equilibrium. See page 700 of their paper for details.

We explain why there is no strict equilibrium that satisfies both our monotonicity refinement and their (P0) requirement when $L_2 > 1$ and $\eta(\delta, p) > \eta_2$. Suppose by way of contradiction that there exists an equilibrium that satisfies both refinements. Our theorem implies that in every strict monotone equilibrium, agents will disclose $\mathbf{0}$ when their history is $\mathbf{0}$ and their private signal is s_2, so $\pi(\mathbf{0}) \geq \pi(\mathbf{1}_2)$. According to Bayes rule, the probability with which their belief assigns to $\overline{\theta}$ upon observing history $\mathbf{0}$ is no more than their prior, so $\pi_0 \geq \pi(\mathbf{0})$. If such an equilibrium also satisfies (P0), then given that history $\mathbf{1}_2$ occurs with zero probability, $\pi(\mathbf{1}_2)$ must equal the agent's posterior belief when they know that their predecessor's history is $\mathbf{0}$ and their private signal is s_2. Since $L_2 > 1$, we have $\pi(\mathbf{1}_2) > \pi(\mathbf{0})$, which contradicts $\pi(\mathbf{0}) \geq \pi(\mathbf{1}_2)$.

Lastly, our refinements are neither stronger nor weaker than the receiver-optimal equilibrium used in Rappoport (2025). First, equilibria where the sender discloses nothing satisfy both of our refinements, but they are dominated by the one where s_1 is disclosed in terms of receiver welfare. Second, suppose for example $l = 3$, $L_1 > L_2 > 1 > L_3$, and $\overline{f}_1$ and $\underline{f}_1$ are much smaller than $\overline{f}_2$ and $\underline{f}_2$. When $\eta(\delta, p) > \eta_2$, signals s_2 and s_3 are never disclosed in any strict monotone equilibrium. However, the receiver will be strictly better off in the non-monotone equilibrium where only signal s_2 is disclosed, as signal s_2 occurs with much higher probability than s_1. According to our Proposition 1, the latter equilibrium exists regardless of the parameters.

5.2 Discussions of Modeling Assumptions

A key assumption of our model is that agents can only observe the number of each signal disclosed but cannot directly observe (i) the exact date at which each disclosed signal was realized and (ii) the time at which communication failed in the past. Our assumption fits when senders (e.g., managers who can edit training materials presented to new recruits) can credibly disclose the substantive content of the signals (e.g., fabricating cases is hard due to organizations' internal review processes), but find it hard to reveal the exact time at which the signals arrived due to, for example, privacy regulations that prevent them from revealing too much identifying information.

Alternatively, one may consider settings in which agents cannot credibly disclose hard information, so communication takes the form of cheap talk. Such an environment is studied by Anderlini, Gerardi, and Lagunoff (2012) in an OLG model where the sender's payoff is single-peaked in the receiver's action. By contrast, we focus on environments where the sender's payoff is strictly increasing in the receiver's action, in which case cheap talk is uninformative. Our assumption on the sender's payoff seems reasonable when the receiver's action is interpreted as his costly effort, which can strictly benefit the sender (e.g., the sender is the receiver's manager).

There are also situations where agents can credibly disclose the signals' arrival times in addition to their substantive content. In the special case where each agent observes his private signal for sure ($p = 1$), observing the exact time at which communication failed in the past leads to unraveling, since each agent can infer the exact number of bad signals concealed.

If agents can observe the time stamps of disclosed signals but not the time at which communication failed in the past, unraveling may not happen since there is still uncertainty regarding the number of realized signals after the latest communication breakdown, which, depending on the equilibrium strategies, could be correlated with the set of signals disclosed.

In the more general case where $p < 1$, the absence of a disclosed signal in a given period may also be caused by the agent in that period failing to observe his private signal. Consequently, agents remain uncertain about the number of signals observed since the most recent communication breakdown, even when they can observe the time stamps of the disclosed signals or the time at which communication failed in the past. As a result, depending on the equilibrium strategies, disclosing an additional good signal may still lead the subsequent agent to infer a larger expected number of bad signals, so the forces identified in our analysis may continue to operate. A related question is: What if agents can voluntarily disclose the time stamps of the disclosed signals? It is unclear whether agents have incentives to disclose the time stamps, which may depend on the equilibrium being played. Characterizing how changes in δ, p, and $\mathbf{f}$ affect the selectiveness of disclosure in richer environments is an interesting direction for future research.

Our baseline model also assumes that either all signals disclosed by an agent are received by his successor, or all signals disclosed are lost. Section 5.4 extends our result to the case where the probability of communication failure depends on the number of signals disclosed. Alternatively, one may consider a model in which each disclosed signal is transmitted independently. For example, if an agent discloses two signals, then his successor receives both with probability δ^2, exactly one with probability $2\delta(1-\delta)$, and neither with probability $(1-\delta)^2$.

In this alternative setting, for every $\delta \in (0,1)$ and $p \in (0,1]$, there exists a constant-threshold equilibrium in which all signals with likelihood ratios greater than 1 are disclosed, which contrasts with our setting where only the strongest possible signal can be disclosed when δ and p are large.

To see why, suppose the constant disclosure threshold is j, where $L_j > 1$. Conditional on state θ, agent $t \in \mathbb{Z}$ fails to observe the signal realized in period $\tau(< t)$ with probability $1 - \delta^{t-\tau} p \sum_{i=1}^{j} f_{\theta,i}$. This can occur for three reasons: s^τ is lost during transmission, agent τ fails to observe his private signal, or the realized signal has a likelihood ratio below L_j and is therefore concealed. Thus, failing to observe a past signal is bad news. Because signals are transmitted independently, disclosing s_j both reveals one additional realization of s_j and eliminates one potential concealed signal. When $L_j > 1$, both effects increase the next agent's posterior belief.

By contrast, in our baseline model, where either all disclosed signals are successfully transmitted or all are lost, agents can only infer the sequence of realized signals following the most recent communication failure, which occurred in some random period τ. Observing one additional disclosed good signal leads them to infer that τ is smaller. Consequently, they expect the post-τ sequence to be longer and, in expectation, to contain more bad signals as well. Although our result is derived in the extreme case where communication successes are perfectly correlated across disclosed signals, the negative effect of disclosing good signals seems to be relevant as long as there is some positive correlation between the successful transmission of different signals.

5.3 Non-Strict Equilibria & Mixed-Strategy Equilibria

As in Clark et al. (2021), Part 1 of Theorem 1 focuses on *strict* equilibria. We do not know whether this result extends to all pure-strategy equilibria, in particular, those where agents do not have strict incentives. This is because the *no-turning-back property* stated in Section 4.1 breaks down, so Lemma 1 and the calculations that follow no longer apply.

To see intuitively where our argument breaks down, let s_j denote the weakest signal disclosed on path and recall the definition of $\mathbf{h}^*$ in Section 3. Suppose $\pi(\mathbf{h}^*) = \pi(\mathbf{h}^* - \mathbf{1}_j)$, so that agents are indifferent between revealing and concealing s_j upon receiving it at $\mathbf{h}^* - \mathbf{1}_j$. They may then disclose $\mathbf{h}^*$ upon receiving s_j at $\mathbf{h}^* - \mathbf{1}_j$, while disclosing $\mathbf{h}^* - \mathbf{1}_j$ upon receiving some bad signal s_i, with $i > j$, at $\mathbf{h}^*$. This, in turn, raises the next agent's belief upon observing history $\mathbf{h}^*$, since observing $\mathbf{h}^*$ implies that the bad signal s_i did not arrive after the disclosure of s_j. Consequently, the argument leading to the contradiction $\pi(\mathbf{h}^*) < \pi(\mathbf{h}^* - \mathbf{1}_j)$ no longer goes through.

Nevertheless, all pure-strategy equilibria are strict under generic parameter values in an extension we consider in Section 5.4 where each agent can disclose at most $N \in \mathbb{N}$ signals. This is because for any finite N, the set of histories is finite, as is the set of signal profiles that can be disclosed. If two histories lead to the same expected payoff for the sender, then the indifference condition defines a polynomial equation in δ. Since there is a finite number of signal profiles, there can be at most a finite number of values of δ for every $(u, v, \mathbf{f}, \pi_0, p)$ under which the sender is indifferent. Hence, the set of parameters under which non-strict pure-strategy equilibria exist has Lebesgue measure zero.

We now turn to mixed-strategy equilibria. Since we assumed that there is a unique $a \in \mathbb{R}$ that maximizes $\pi u(\overline{\theta}, a) + (1 - \pi) u(\underline{\theta}, a)$ for every $\pi \in [0, 1]$, agents always have strict incentives when choosing their actions. Consequently, the only source of randomization is their disclosure decisions. As discussed earlier, we do not know whether our result on monotone equilibria extends to mixed-strategy equilibria. We focus on a class of mixed-strategy equilibria that are tractable to analyze and are natural generalizations of constant-threshold equilibria:

Constant-Threshold Mixed Strategy. *A constant-threshold mixed strategy is characterized by $j \in \{1, \ldots, l\}$ and $q \in (0, 1)$ such that agents disclose all $s_1, \ldots, s_{j-1}$, disclose all s_j in their histories, disclose their private signal s_j with probability q, and never disclose $s_{j+1}, \ldots, s_l$.*

Intuitively, this class of strategies requires agents to disclose all signals with likelihood ratio strictly above a cutoff as well as all inherited signals with likelihood ratio at the cutoff, to conceal all signals with likelihood ratio strictly below the cutoff, and to randomize, with probability strictly between 0 and 1, over disclosing private signals with likelihood ratio at the cutoff.

One can show that for any $j \in \{0, 1, \ldots, l\}$ and $q \in (0, 1)$, a constant-threshold mixed strategy equilibrium parameterized by (j, q) exists if and only if (i) $j \geq 2$, (ii) $L_j > 1$, and (iii) $\eta(\delta, p) = \eta_j$. This is because agents are indifferent between disclosing private signal s_j and concealing it if and only if

$$L_j = \frac{1 - \delta(1 - p(q\overline{f_j} + \sum_{i=1}^{j-1} \overline{f_i}))}{1 - \delta(1 - p(q\underline{f_j} + \sum_{i=1}^{j-1} \underline{f_i}))}. \tag{5.2}$$

Regardless of $q \in (0, 1)$, the RHS of (5.2) is strictly greater than 1 and strictly less than L_1, which leads to the requirements that $j \geq 2$ and $L_j > 1$. Furthermore, regardless of q, equation (5.2) holds if and only if $\eta(\delta, p) = \eta_j$. Hence, the class of mixed-strategy equilibria we considered exists only under knife-edge parameters. When agents also randomize over the cutoff signals contained in their histories, or when their disclosure rule depends non-trivially on their information set, the marginal effect of disclosing another s_j depends on the entire set of signals disclosed. Characterizing such mixed-strategy equilibria is left for future research.

5.4 Extensions

Theorem 1 extends when each agent t observes s^t after choosing his action a^t. This is because the result only hinges on agents' beliefs after observing the set of disclosed signals. Similarly, our result extends when agents have heterogeneous payoffs, as long as they are supermodular with respect to (θ, a^t), are separable between a^t and a^{t+1}, and are strictly increasing in a^{t+1}.

By contrast, new complications arise when agents' payoffs are not separable between a^t and a^{t+1}. In this case, an agent's action depends not only on his belief about the state, but also on his belief about his successor's action. As a result, when agent t decides which set of signals to disclose, his objective is not only to maximize agent $t+1$'s belief, but also to equip agent $t+1$ with signals to increase (or decrease) agent $t+2$'s belief, depending on whether agents' actions are complements or substitutes. A similar complication arises when agent t's payoff depends also on $\{a^{t+2}, a^{t+3}, \dots\}$, or when the distribution of agent t's private signal s^t depends on agent t's action. Whether our theorem extends to these more general cases remains an open question.

We then discuss two other extensions. First, Part 1 of Theorem 1 extends when agents face disclosure constraints. For example, suppose each agent can disclose at most $N \in \mathbb{N}$ signals.[24] That is, after receiving private signal $s_i \in S \cup \{\emptyset\}$ at history $\mathbf{h}$, the agent can disclose

[24] Constant-threshold equilibria with threshold $j \geq 1$ are not well-defined under disclosure constraints, since agents cannot disclose $N+1$ copies of s_1 after receiving s_1 at history $N\mathbf{1}_1$. So Part 2 does not extend as stated.

$\mathbf{h}' = (h'_1, \ldots, h'_l)$ if and only if $\mathbf{h}' \leq \mathbf{h} + \mathbf{1}_i$ and $\sum_{j=1}^{l} h'_j \leq N$. This is because the gradualism property, the improvement principle, and the no-turning-back property all continue to apply. Lemma 1 also extends since it only rules out the possibility that the current-period history is $\mathbf{h}^*$ yet the previous-period history is neither $\mathbf{h}^*$ nor $\mathbf{h}^* - \mathbf{1}_j$, and such a possibility cannot re-appear due to disclosure constraints. The calculation after Lemma 1 compares agents' posterior beliefs at $\mathbf{h}^*$ and at $\mathbf{h}^* - \mathbf{1}_j$, which is also unaffected.

Second, we consider an extension in which the communication success rate depends on the number of signals disclosed. Suppose when agent t discloses a set of signals $\mathbf{h}$, the probability that agent $t+1$ receives $\mathbf{h}$ is $\delta(\mathbf{h}) = \exp(-\Delta \cdot q(\mathbf{h}))$ where $\Delta > 0$ is a parameter and $q : H \to [\underline{q}, \overline{q}]$ for some $0 < \underline{q} < \overline{q} < \infty$ such that $q(\mathbf{h})$ depends on $\mathbf{h}$ only through its length and $q(\mathbf{h})$ weakly increases as the length of $\mathbf{h}$ increases. In this setting, communication is weakly more likely to fail when more signals are disclosed. As $\Delta \to 0$, the communication success rate converges to 1.

When Δ is below some cutoff, we show that no signal other than s_1 is ever disclosed in any strict monotone equilibrium. To start, the gradualism property and the no-turning-back property extend. A modified version of the improvement principle holds: when in equilibrium, agents disclose $\mathbf{h}^{t+1} (\neq \mathbf{h}^t)$ at history $\mathbf{h}^t$ following some private signal $s \in S$, the expected value of $v(\theta, a^*(\pi))$ when the agent discloses $\mathbf{h}^{t+1}$ is greater than the expected value when he discloses $\mathbf{h}^t$. We explain why Lemma 1 extends to this more general setting at the end of Appendix A.3.

Now suppose by way of contradiction that no matter how small Δ is, there exists a strict monotone equilibrium in which the weakest signal disclosed on path is $s_j \neq s_1$. By Lemma 1, $\mathbf{h}^*$ can only occur after $\mathbf{h}^*$ or $\mathbf{h}^* - \mathbf{1}_j$, so we have the following equation that relates the probability of history $\mathbf{h}^*$ conditional on θ to the probability of history $\mathbf{h}^* - \mathbf{1}_j$ conditional on θ:

$$\mu_\theta(\mathbf{h}^*) = \mu_\theta(\mathbf{h}^* - \mathbf{1}_j) \cdot \delta(\mathbf{h}^*) \cdot p f_{\theta,j} \cdot \sum_{n=0}^{\infty} \delta(\mathbf{h}^*)^n \cdot (1 - p \sum_{i \in D(\mathbf{h}^*)} f_{\theta,i})^n, \tag{5.3}$$

where $D(\mathbf{h}^*) \subset S$ is the set of private signals such that the next period history is no longer $\mathbf{h}^*$ once an agent receives such a private signal at $\mathbf{h}^*$. As in the baseline model, $D(\mathbf{h}^*)$ is either $\{1, \ldots, j-1\}$ or $\{1, \ldots, j\}$. Equation (5.3) implies that $\pi(\mathbf{h}^*) > \pi(\mathbf{h}^* - \mathbf{1}_j)$ if and only if

$$L_j \cdot \frac{1 - \delta(\mathbf{h}^*)(1 - p \sum_{i \in D(\mathbf{h}^*)} \underline{f}_i)}{1 - \delta(\mathbf{h}^*)(1 - p \sum_{i \in D(\mathbf{h}^*)} \overline{f}_i)} > 1, \tag{5.4}$$

which is analogous to (4.2) in the baseline model. Since $\delta(\mathbf{h}^*) = \exp(-\Delta \cdot q(\mathbf{h}^*))$ and the function $q(\cdot)$ is uniformly bounded above and below, we know that there exists $\Delta^* > 0$ such that when $\Delta < \Delta^*$, inequality (5.4) fails for all j and $D(\mathbf{h}^*)$ that satisfy $j \geq 2$ and $j \geq \max D(\mathbf{h}^*)$.

5.5 OLG Game versus One-Shot Disclosure Game

To understand the role of our OLG structure, we consider a one-shot disclosure game between a sender and a receiver in which the receiver does not know the number of signals the sender has.

Suppose the state is binary $\theta \in \{\overline{\theta}, \underline{\theta}\}$. The sender observes $\widetilde{n}$ conditionally i.i.d. signals $\{s^t\}_{t=1}^{\widetilde{n}}$, where $s^t \in \{s_1, \ldots, s_l\}$ and $\Pr(\widetilde{n} = n) = (1-\delta)\delta^n$ for every $n \geq 0$. We assume that signal realizations with lower indices have higher likelihood ratios. The sender discloses a subset of the signals in order to maximize the probability that the receiver's posterior belief assigns to $\overline{\theta}$. The receiver only observes the number of each signal the sender discloses. Hence, both players' information sets belong to $H = \mathbb{N}^l$. This is analogous to our OLG game where $p = 1$.

The sender's strategy is $\sigma_D : \mathbb{N}^l \to \mathbb{N}^l$ subject to $\sigma_D(\mathbf{h}) \leq \mathbf{h}$ for every $\mathbf{h} \in \mathbb{N}^l$. We extend the notions of monotone and constant-threshold strategies to this setting. The sender's strategy is *monotone* if for every $\mathbf{h} = (h_1, \ldots, h_l)$ and $j \in \{2, \ldots, l\}$, if the jth entry of $\sigma_D(\mathbf{h})$ is not 0, then the ith entry of $\sigma_D(\mathbf{h})$ equals h_i for every $i < j$.[25] The sender's strategy is a *constant-threshold strategy* if there exists $j \in \{0, 1, \ldots, l\}$ such that $\sigma_D(\mathbf{h}) = \mathbf{h}[j]$ for every $\mathbf{h} \in H$.

Our results for (strict) constant-threshold equilibria, Part 2 of Theorem 1 and Corollary 2, extend to this one-shot game once we set $p = 1$. The intuition remains the same: suppose in equilibrium, the sender uses a constant-threshold strategy of disclosing $s_1, \ldots, s_i$ (good signals) and concealing $s_{i+1}, \ldots, s_l$ (bad signals). Disclosing another good signal shows to the receiver not only that there is one more good signal, but also there are more bad signals in expectation, so the sender's disclosure incentives depend on the relative magnitudes of the two effects.

However, Part 1 of Theorem 1, which concerns strict monotone equilibria, does not extend to this one-shot game, thereby distinguishing this game from our OLG game. To highlight the difference, we introduce a class of strategies parameterized by a positive integer $k \in \mathbb{N}$ where s_2 is sometimes disclosed. We show that no matter how close δ is to 1, there always exists $k \in \mathbb{N}$ such that Strategy k is part of a strict monotone equilibrium in the one-shot game:

- Strategy k: For every $j \neq k+1$, if the sender observes j copies of signal s_1, then he discloses all the s_1 he has and nothing else. If the sender has $k+1$ copies of s_1 but no s_2, then he discloses k copies of s_1 and nothing else. If the sender has $k+1$ copies of s_1 and at least one copy of s_2, then he discloses $k+1$ copies of s_1, one copy of s_2, and nothing else.

[25] Our proposition in this section extends if we replace strategy monotonicity with belief monotonicity, that the receiver's belief increases when the sender replaces a lower-likelihood-ratio signal with a higher one.

Proposition 4. *For any $\delta \in (0,1)$, there exists $k \in \mathbb{N}$ such that there exists a strict monotone equilibrium in the one-shot disclosure game where the sender uses Strategy k.*[26]

The proof is in Appendix A.6. Compared to our OLG game, the gradualism property no longer holds in the one-shot disclosure game, so Lemma 1 is no longer true. To see intuitively where the argument for Theorem 1 breaks down, note that under Strategy k, s_2 is the weakest signal disclosed on the equilibrium path, so history $\mathbf{h}^*$ is $(k+1)\mathbf{1}_1 + \mathbf{1}_2$, and history $\mathbf{h}^* - \mathbf{1}_j$ is $(k+1)\mathbf{1}_1$. Since the sender never discloses $(k+1)\mathbf{1}_1$ on the equilibrium path, when he has $k+1$ copies of s_1 and one copy of s_2 available, his decision regarding whether to disclose s_2 depends on the comparison between the receiver's posterior beliefs at $k\mathbf{1}_1$ and at $(k+1)\mathbf{1}_1+\mathbf{1}_2$, instead of between the receiver's beliefs at $(k+1)\mathbf{1}_1$ and at $(k+1)\mathbf{1}_1+\mathbf{1}_2$. If the sender discloses $(k+1)\mathbf{1}_1 + \mathbf{1}_2$ instead of $k\mathbf{1}_1$, he shows to the receiver that (i) there is one more s_1, (ii) there is one more s_2, and (iii) there is no more s_1 beyond the $k+1$ copies disclosed. In our OLG game, the first effect, which is in favor of disclosing s_2, is absent. Instead of disclosing $(k+1)\mathbf{1}_1 + \mathbf{1}_2$, a more attractive option is to disclose $(k+1)\mathbf{1}_1$, which happens on the equilibrium path thanks to the gradualism property. It shows to the receiver that there are $k+1$ copies of s_1 without showing her that there is also an s_2 as well as some bad signals between the disclosed signals.

6 Concluding Remarks

We study an overlapping-generations model in which each agent receives a private signal and a set of disclosed signals, each with positive probability, before taking an action and disclosing a subset of signals to his immediate successor. Focusing on strict monotone equilibria, our main result shows that agents disclose a smaller set of signals when they are more likely to observe private signals or receive disclosed signals. The intuition is that when an agent observes an additional "good" signal that has been disclosed, he also infers from the equilibrium strategy that, in expectation, more bad signals have been concealed. This adverse inference from disclosure is absent in Dye (1985), where any disclosure reveals that no information has been withheld. As communication frictions vanish or new information arrives more frequently, this adverse inference becomes stronger, making agents more selective in their disclosure decisions.

[26]Strategy k is *not* a threshold strategy since only one copy of s_2 is disclosed when the sender has $k+1$ copies of s_1 and 2 copies of s_2 available. In fact, one can show that there exists $\delta^* \in (0,1)$ such that when $\delta > \delta^*$, the sender will never disclose $s_2, \dots, s_l$ in any strict threshold equilibrium in the one-shot disclosure game.

For tractability, our analysis focuses on the case in which agents observe the substantive content of disclosed signals but not their exact arrival times, and in each period either all signals disclosed by an agent are received by his immediate successor or all of them are lost. As discussed in Section 5.2, however, the adverse inference from disclosure that we identify is likely to arise whenever agents face uncertainty about the number of available signals (for example, when agents have some information about the time stamps of disclosed signals but each agent receives his private signal with probability less than 1) and the successful transmission of each disclosed signal is positively correlated with that of the other disclosed signals. Characterizing equilibrium disclosure behaviors in these more general settings is left for future research.

By contrast, the effect identified in our main result is absent when agents know the exact number of signals after the latest communication failure, such as when $p = 1$ and agents can observe the exact time at which communication failed in the past, or when the successful transmission of different disclosed signals is independent.

As in the classic OLG model of Samuelson (1958), and in the subsequent work of Bhaskar (1998) and Acemoglu and Jackson (2015), we assume that each agent's payoff depends only on his own action and that of his immediate successor,[27] but not on the actions of later generations. If agent t's payoff also depends on the actions of agents $t+2, t+3, \ldots$, then agent t's objective is no longer simply to maximize agent $t+1$'s belief about the state; he also has an incentive to equip agent $t+1$ with signals that can be passed on to agent $t+2$, $t+3$, etc. Characterizing monotone equilibria in this more general setting is left for future research.

[27] Allowing agents' payoffs to depend on their predecessors' actions does not change our results, since such dependence does not affect agents' incentives.

A Omitted Proofs

A.1 Uniqueness of the Steady State

We show that conditional on state θ, any pure strategy σ induces a unique steady-state distribution over the agents' histories.[28] Let $\sigma_D : H \times (S \cup \{\emptyset\}) \to H$ denote the component of σ that maps an agent's history and private signal (if any) to the set of signals disclosed to the next agent. For any $\mathbf{h}, \mathbf{h}' \in H$, define the probability of moving from history $\mathbf{h}$ to history $\mathbf{h}'$ conditional on the selected history being successfully observed by the next generation as $Q_\theta^\sigma(\mathbf{h}, \mathbf{h}') = (1-p)\mathbb{1}_{\sigma_D(\mathbf{h},\emptyset)=\mathbf{h}'} + p\sum_{i=1}^{l} f_{\theta,i}\mathbb{1}_{\sigma_D(\mathbf{h},s_i)=\mathbf{h}'}$ and observe that it defines a Markov process. We then define the probability of transitioning from history $\mathbf{h}$ to history $\mathbf{h}'$ as

$$P_\theta^\sigma(\mathbf{h}, \mathbf{h}') = (1-\delta)\mathbb{1}_{\mathbf{h}'=\mathbf{0}} + \delta Q_\theta^\sigma(\mathbf{h}, \mathbf{h}'). \tag{A.1}$$

We show existence and uniqueness of a steady-state distribution using the Banach fixed point theorem. Let $T_\theta^\sigma : \Delta(H) \to \Delta(H)$ be the transition operator implied by P_θ^σ, so that $(T_\theta^\sigma(\mu))(\mathbf{h}') = \sum_{\mathbf{h}\in H} \mu(\mathbf{h}) P_\theta^\sigma(\mathbf{h}, \mathbf{h}')$. We show that this is a contraction mapping when equipped with the total variation distance.

To see that applying only the Q_θ^σ transition does not increase total variation distance, take any $\mu, \mu' \in \Delta(H)$, and observe that

$$\begin{aligned}
\Big\| \sum_{\mathbf{h}} \mu(\mathbf{h}) Q_\theta^\sigma(\mathbf{h}, \cdot) - \sum_{\mathbf{h}} \mu'(\mathbf{h}) Q_\theta^\sigma(\mathbf{h}, \cdot) \Big\|_{\mathrm{TV}} &= \frac{1}{2} \sum_{\mathbf{h}'\in H} \Big| \sum_{\mathbf{h}\in H} \big(\mu(\mathbf{h}) - \mu'(\mathbf{h})\big) Q_\theta^\sigma(\mathbf{h}, \mathbf{h}') \Big| \\
&\le \frac{1}{2} \sum_{\mathbf{h}'\in H} \sum_{\mathbf{h}\in H} \big|\mu(\mathbf{h}) - \mu'(\mathbf{h})\big| \, Q_\theta^\sigma(\mathbf{h}, \mathbf{h}') \\
&= \frac{1}{2} \sum_{\mathbf{h}\in H} \big|\mu(\mathbf{h}) - \mu'(\mathbf{h})\big| \sum_{\mathbf{h}'\in H} Q_\theta^\sigma(\mathbf{h}, \mathbf{h}') \\
&= \frac{1}{2} \sum_{\mathbf{h}\in H} \big|\mu(\mathbf{h}) - \mu'(\mathbf{h})\big| \\
&= \|\mu - \mu'\|_{\mathrm{TV}},
\end{aligned}$$

where we used that $\sum_{\mathbf{h}'\in H} Q_\theta^\sigma(\mathbf{h}, \mathbf{h}') = 1$ for each $\mathbf{h}$.

By (A.1) and the above inequality, we have that for any $\mu, \mu' \in \Delta(H)$, $\|T_\theta^\sigma(\mu) - T_\theta^\sigma(\mu')\|_{\mathrm{TV}} \le \delta \|\mu - \mu'\|_{\mathrm{TV}}$. Since $\delta \in (0,1)$, T_θ^σ is a contraction on $(\Delta(H), \|\cdot\|_{\mathrm{TV}})$. By the Banach fixed point theorem, there exists a unique steady-state distribution over histories $\mu_\theta \in \Delta(H)$.

[28] The proof also applies to mixed strategies by defining $P_\theta^\sigma(\mathbf{h}, \mathbf{h}') = (1-\delta)\mathbb{1}_{\mathbf{h}'=\mathbf{0}} + \delta\big((1-p)\sigma_D(\mathbf{h}, \emptyset)(\mathbf{h}') + p\sum_{i=1}^{l} f_{\theta,i}\sigma_D(\mathbf{h}, s_i)(\mathbf{h}')\big)$, where $\sigma_D(\mathbf{h}, s)(\mathbf{h}')$ denotes the probability of disclosing $\mathbf{h}'$ at information set $(\mathbf{h}, s)$.

A.2 Proof of Proposition 1 and Corollary 1

We show that for any $n \in \mathbb{N} \cup \{\infty\}$ and $i \in \{1, \ldots, l-1\}$ that satisfies $L_i > 1$, there exists a strict equilibrium where agents disclose up to n copies of s_i and nothing else.

Suppose in equilibrium, all agents use the strategy of disclosing no signal other than s_i and disclosing up to n copies of s_i. For any $\mathbf{h}$ that occurs with zero probability under this strategy, let $\pi(\mathbf{h}) < \pi(\mathbf{0})$. Then conditional on state θ, for every $m < n$, the probability of history $m\mathbf{1}_i$ is $(1-\delta)\delta^m \cdot (1-\delta(1-pf_{\theta,i}))^{-(m+1)} \cdot (pf_{\theta,i})^m$. Therefore,

$$\frac{\pi(m\mathbf{1}_i)}{1-\pi(m\mathbf{1}_i)} = \frac{\pi_0}{1-\pi_0} \cdot \left(\frac{1-\delta(1-p\underline{f}_i)}{1-\delta(1-p\overline{f}_i)}\right)^{m+1} \cdot L_i^m. \tag{A.2}$$

If $L_i > 1$, then the value of (A.2) is strictly increasing in m regardless of δ. The probability of history $n\mathbf{1}_i$ is

$$\sum_{k=n}^{\infty}(1-\delta)\delta^k \cdot (1-\delta(1-pf_{\theta,i}))^{-(k+1)} \cdot (pf_{\theta,i})^k, \tag{A.3}$$

so

$$\begin{aligned}
\frac{\pi(n\mathbf{1}_i)}{1-\pi(n\mathbf{1}_i)} &\geq \frac{\pi_0}{1-\pi_0} \cdot \min_{k \geq n}\left\{\frac{(1-\delta)\,\delta^k\big(1-\delta(1-p\overline{f}_i)\big)^{-(k+1)}\overline{f}_i^{\,k}}{(1-\delta)\,\delta^k\big(1-\delta(1-p\underline{f}_i)\big)^{-(k+1)}\underline{f}_i^{\,k}}\right\} \\
&\geq \frac{\pi_0}{1-\pi_0} \cdot \frac{(1-\delta)\,\delta^n\big(1-\delta(1-p\overline{f}_i)\big)^{-(n+1)}\overline{f}_i^{\,n}}{(1-\delta)\,\delta^n\big(1-\delta(1-p\underline{f}_i)\big)^{-(n+1)}\underline{f}_i^{\,n}} \\
&= \frac{\pi_0}{1-\pi_0} \cdot \left(\frac{1-\delta(1-p\underline{f}_i)}{1-\delta(1-p\overline{f}_i)}\right)^{n+1} L_i^n.
\end{aligned} \tag{A.4}$$

The RHS of (A.4) is strictly greater than the value of (A.2) for any $m < n$. These calculations together with our earlier specifications of off-path beliefs verify that agents have strict incentives to disclose up to n copies of s_i and nothing else, which establishes Proposition 1.

For Corollary 1, let $i = 1$ and note that when $\eta(\delta, p) \geq \eta_2$, Theorem 1 implies that only s_1 can be disclosed on the equilibrium path. Therefore, every strict monotone equilibrium is characterized by $n \in \mathbb{N} \cup \{\infty\}$ such that agents disclose up to n copies of s_1 and nothing else.

A.3 Proof of Lemma 1

Suppose by way of contradiction that the realized history in period $t-1$ was $\hat{\mathbf{h}} = (\hat{h}_1, \ldots, \hat{h}_l)$, which is neither $\mathbf{h}^*$ nor $\mathbf{h}^* - \mathbf{1}_j$. We obtain a contradiction in each of the following three cases:

1. If $\hat{h}_j = 0$, then $h_j^* - \hat{h}_j = 1$, so the gradualism property implies that $\sum_{i=1}^{j-1} \hat{h}_i \geq \sum_{i=1}^{j-1} h_i^*$. Since $\hat{\mathbf{h}} \neq \mathbf{h}^* - \mathbf{1}_j$, we have $(\hat{h}_1, \ldots, \hat{h}_{j-1}) \neq (h_1^*, \ldots, h_{j-1}^*)$. Hence, there exists $1 \leq i \leq j-1$ such that $\hat{h}_i > h_i^*$. This implies that after observing signal s_j at history $\hat{\mathbf{h}}$, the agent discloses s_j yet conceals at least one realization of s_i, which contradicts the monotonicity of their equilibrium strategy.

2. If $\hat{h}_j = 1$, then the construction of $\mathbf{h}^*$ implies that $\sum_{i=1}^{j-1} \hat{h}_i \geq \sum_{i=1}^{j-1} h_i^*$. Since $\hat{\mathbf{h}} \neq \mathbf{h}^*$, we have $(\hat{h}_1, \ldots, \hat{h}_{j-1}) \neq (h_1^*, \ldots, h_{j-1}^*)$, so there exists $1 \leq i \leq j-1$ such that $\hat{h}_i > h_i^*$. This again implies that after observing signal s_j at history $\hat{\mathbf{h}}$, the agent discloses s_j yet conceals at least one realization of s_i, which contradicts the monotonicity of their equilibrium strategy.

3. If $\hat{h}_j \geq 2$, then consider the sequence of agents' realized histories starting from the latest communication failure, which we normalize as period 0. Let $\mathbf{h}(\tau)$ denote the realized history in period τ. By definition, $\mathbf{h}(0) = (0, 0, \ldots, 0)$, $\mathbf{h}(t-1) = \hat{\mathbf{h}}$, and $\mathbf{h}(t) = \mathbf{h}^*$. The gradualism property implies the existence of $0 < \tau < t-1$ such that (i) the jth entry of $\mathbf{h}(\tau)$ is 1, and (ii) the jth entry of $\mathbf{h}(\hat{\tau})$ is weakly greater than 1 for every $\hat{\tau} \in [\tau, t-1]$. Let $\mathbf{h}(\tau) = \mathbf{h}^{**} = (h_1^{**}, \ldots, h_{j-1}^{**}, 1, 0, \ldots, 0)$. The definition of vector $\mathbf{h}^*$ implies that at least one of the following two cases is true.

   The first case is $\sum_{i=1}^{j-1} h_i^{**} = \sum_{i=1}^{j-1} h_i^*$, which by the construction of history $\mathbf{h}^*$, implies that $\pi(\mathbf{h}^*) \leq \pi(\mathbf{h}^{**})$. The construction of τ implies that there is no communication breakdown between period τ to t, so the conclusion that $\pi(\mathbf{h}^*) \leq \pi(\mathbf{h}^{**})$ contradicts the improvement principle, which requires that $\pi(\mathbf{h}^*) = \pi(\mathbf{h}(t)) > \pi(\mathbf{h}(t-1)) > \pi(\mathbf{h}(\tau)) = \pi(\mathbf{h}^{**})$, as $\mathbf{h}(t) \neq \mathbf{h}(t-1)$ and $\mathbf{h}(t-1) \neq \mathbf{h}(\tau)$.

   The second case is $\sum_{i=1}^{j-1} h_i^{**} > \sum_{i=1}^{j-1} h_i^*$, i.e., the total number of signals $s_1, \ldots, s_{j-1}$ is strictly more in $\mathbf{h}(\tau)$ than in $\mathbf{h}(t)$. By construction of τ, the jth entry of $\mathbf{h}(\hat{\tau})$ is weakly greater than 1 for every $\hat{\tau} \in [\tau, t-1]$. This together with $\hat{h}_j \geq 2$ implies that there exists at least one period from τ to $t-1$ in which signal s_j is disclosed while some signals in $\{s_1, \ldots, s_{j-1}\}$ are concealed. This contradicts agents using a monotone strategy.

Remark: We extend this proof to the case where communication success rate weakly decreases with the number of signals disclosed. We revisit the three cases considered in the proof. The first two cases only involve comparison between beliefs at two histories of the same length, namely, $\mathbf{h}^*$

and $\mathbf{h}^*-\mathbf{1}_j+\mathbf{1}_i$. Since the communication success rate depends only on the length of the disclosed history, an agent prefers to disclose $\mathbf{h}^*$ to $\mathbf{h}^*-\mathbf{1}_j+\mathbf{1}_i$ if and only if $\pi(\mathbf{h}^*) \geq \pi(\mathbf{h}^*-\mathbf{1}_j+\mathbf{1}_i)$. In the third case, recall the construction of history $\mathbf{h}^{**}$ as well as the existence of history $\mathbf{h}$ such that $\mathbf{h} \leq \mathbf{h}^{**}$, $\mathbf{h}$ and $\mathbf{h}^*$ share the same length, and $\pi(\mathbf{h}) \geq \pi(\mathbf{h}^*)$. Therefore, the expected payoff from disclosing $\mathbf{h}$ is weakly greater than the expected payoff from disclosing $\mathbf{h}^*$, as the communication success rates are the same yet disclosing $\mathbf{h}$ leads to a weakly higher belief. This leads to a contradiction since disclosing $\mathbf{h}$ is available when the history is $\mathbf{h}^{**}$ yet history $\mathbf{h}^*$ is reached after $\mathbf{h}^{**}$ before the next time communication breaks down.

A.4 Proof of Proposition 2

For every $n \geq 1$ and $\theta \in \{\underline{\theta}, \overline{\theta}\}$, let $q_{\theta,n} = \sum_{i=1}^n f_{\theta,i}$, and write $\underline{q}_n = q_{\underline{\theta},n}$, $\overline{q}_n = q_{\overline{\theta},n}$. By Theorem 1, the largest threshold in a constant-threshold equilibrium depends on (δ, p) only through η. We introduce a new function $I(\cdot;\cdot)$ based on $\mathcal{I}(\cdot,\cdot)$ according to $I(\theta; \mathbf{h}_{\delta,p}) = \mathcal{I}(\delta, p)$.

For any $n \geq 1$, let $\mathbf{h}_{\delta,p}^{(n)}$ denote the steady-state history when agents use the constant-threshold strategy with threshold n. Let K be the number of periods since last communication failure. Then $\Pr(K=k) = (1-\delta)\delta^k$ for every $k \in \mathbb{N}$. Fixing $K=k$, we have a multinomial experiment with $n+1$ possible outcomes (no signal, as well as each of the n possible signals). Because of independence across periods, conditional on $K=k$, the probability-generating function becomes $(1-pq_{\theta,n}+p\sum_{i=1}^n f_{\theta,i}z_i)^k$. Thus, the probability-generating function of $\mathbf{h}_{\delta,p}^{(n)}$ is

$$\begin{aligned} G_{\theta,n}^{\delta,p}(z_1,\ldots,z_n) = \mathbb{E}\left[\prod_{i=1}^n z_i^{h_i}\right] &= \sum_{k=0}^{\infty}(1-\delta)\delta^k\left(1-pq_{\theta,n}+p\sum_{i=1}^n f_{\theta,i}z_i\right)^k \\ &= \frac{1-\delta}{1-\delta\left(1-pq_{\theta,n}+p\sum_{i=1}^n f_{\theta,i}z_i\right)} \\ &= \frac{1}{1+\eta(\delta,p)\sum_{i=1}^n f_{\theta,i}(1-z_i)}. \end{aligned} \tag{A.5}$$

Since the last expression depends on (δ, p) only through $\eta(\delta,p)$, we write $G_{\theta,n}^{\eta}$ for this generating function, $\mathbf{h}_{\eta}^{(n)}$ for a history with this distribution, and η for $\eta(\delta,p)$ throughout the remainder of the proof. Let $\pi_{\eta}^{(n)}(\mathbf{h})$ denote the posterior belief after history $\mathbf{h}$ under threshold n.

To show that $I\left(\theta; \mathbf{h}_{\eta}^{(n)}\right)$ is strictly increasing in η for every fixed threshold n, fix $\eta' < \eta$ such that the largest equilibrium threshold is n at both values. Generate random history $\mathbf{H} \in H$ according to the distribution of $\mathbf{h}_{\eta}^{(n)}$. Intuitively, in the steady-state, reducing the effective information from η to η' is equivalent to independently deleting each disclosed signal with probability

$1 - \eta'/\eta$. We now verify that this indeed reproduces the entire history distribution at η', rather than only its expected length. Conditional on $\mathbf{H}$, construct random history $\mathbf{H}'$ by retaining each signal in $\mathbf{H}$ with probability $r = \frac{\eta'}{\eta}$, where the retention of each signal is independent of both the retention of all other signals and the state. By (A.5), conditional on state θ,

$$\mathbb{E}_\theta\left[\prod_{i=1}^n z_i^{H_i'}\right] = G_{\theta,n}^{\eta}\left(1 - r + rz_1, \ldots, 1 - r + rz_n\right) = \frac{1}{1 + \eta r \sum_{i=1}^n f_{\theta,i}(1 - z_i)} = G_{\theta,n}^{\eta'}(z_1, \ldots, z_n).$$

Hence, $\mathbf{H}'$ has the same conditional distribution as $\mathbf{h}_{\eta'}^{(n)}$. Thus, the history at η' can be obtained from the history at η through a state-independent random deletion process. Let $\Pi = \Pr(\theta = \bar{\theta} \mid \mathbf{H})$ and $\Pi' = \Pr(\theta = \bar{\theta} \mid \mathbf{H}')$ and observe that $\Pi' = \mathbb{E}\left[\mathbb{1}_{\{\theta = \bar{\theta}\}} \mid \mathbf{H}'\right] = \mathbb{E}\left[\mathbb{E}\left[\mathbb{1}_{\{\theta=\bar{\theta}\}} \mid \mathbf{H}\right] \,\middle|\, \mathbf{H}'\right] = \mathbb{E}[\Pi \mid \mathbf{H}']$. Since the binary entropy function g is strictly concave, Jensen's inequality implies that $\mathbb{E}[g(\Pi) \mid \mathbf{H}'] \leq g\left(\mathbb{E}[\Pi \mid \mathbf{H}']\right) = g(\Pi')$. Taking expectations gives $I\left(\theta; \mathbf{h}_{\eta'}^{(n)}\right) \leq I\left(\theta; \mathbf{h}_{\eta}^{(n)}\right)$. We now show that

$$I\left(\theta; \mathbf{h}_{\eta'}^{(n)}\right) < I\left(\theta; \mathbf{h}_{\eta}^{(n)}\right). \tag{A.6}$$

Conditional on $\mathbf{H}' = \mathbf{0}$, both $\mathbf{H} = \mathbf{0}$ and $\mathbf{H} = \mathbf{1}_1$ occur with positive probability. Moreover, the posterior beliefs following histories $\mathbf{H}$ and $\mathbf{H}'$ differ. By (4.7), their posterior odds satisfy

$$\frac{\Pr(\bar{\theta} \mid \mathbf{H} = \mathbf{1}_1)/\Pr(\underline{\theta} \mid \mathbf{H} = \mathbf{1}_1)}{\Pr(\bar{\theta} \mid \mathbf{H} = \mathbf{0})/\Pr(\underline{\theta} \mid \mathbf{H} = \mathbf{0})} = L_1 \frac{1 + \eta \underline{q}_n}{1 + \eta \bar{q}_n} > 1,$$

where the inequality follows since $L_1(1 + \eta\underline{q}_n) - (1 + \eta\bar{q}_n) = (L_1 - 1) + \eta \sum_{i=1}^n \underline{f}_i(L_1 - L_i) > 0$. Thus, Π is not constant conditional on $\mathbf{H}' = \mathbf{0}$. Strict concavity of g therefore implies (A.6).

We now show that $I\left(\theta; \mathbf{h}_{\eta}^{(n)}\right)$ is continuous in η. Let $\varepsilon = \Pr(\mathbf{H} \neq \mathbf{H}')$. Conditional on state θ, the expected number of deleted signals is $\left(1 - \frac{\eta'}{\eta}\right)\mathbb{E}_\theta[|\mathbf{H}|] = (\eta - \eta')q_{\theta,n}$. Therefore, $\varepsilon \leq (\eta - \eta')\max_\theta q_{\theta,n} \leq \eta - \eta'$. Since $\mathbf{H}'$ is constructed from $\mathbf{H}$ using a state-independent randomization, the chain rule for mutual information gives $I(\theta; \mathbf{H}) - I(\theta; \mathbf{H}') = I(\theta; \mathbf{H} \mid \mathbf{H}')$.

Let $E = \mathbb{1}_{\{\mathbf{H} \neq \mathbf{H}'\}}$. Since $\mathbf{H} = \mathbf{H}'$ conditional on the event that $E = 0$ and when $E = 1$, the conditional mutual information is at most $\log 2$, we have

$$I(\theta; \mathbf{H} \mid \mathbf{H}') \leq I(\theta; \mathbf{H}, E \mid \mathbf{H}') \leq H(E) + \Pr(E = 1)\log 2 \leq g(\varepsilon) + \varepsilon \log 2.$$

Since the right-hand side approaches zero as $\eta' \to \eta$, $I\left(\theta; \mathbf{h}_{\eta}^{(n)}\right)$ is continuous in η for fixed n.

It remains to show that mutual information is continuous when n changes. Fix $n \geq 2$ with $\eta_n > 0$ and a pair (δ, p) such that $\eta(\delta, p) = \eta_n$. Let $\pi^{(m)} = \pi_{\eta_n}^{(m)}$ denote the posterior belief under threshold m and the pair (δ, p). At this (δ, p), the incentive constraint for disclosing s_n binds, so equation (4.9) gives $L_n \cdot \frac{1+\eta_n \underline{q}_n}{1+\eta_n \overline{q}_n} = 1$. Since $\overline{f}_n = L_n \underline{f}_n$, this implies $L_n \cdot \frac{1+\eta_n \underline{q}_{n-1}}{1+\eta_n \overline{q}_{n-1}} = 1$. So,

$$\frac{1+\eta_n \underline{q}_n}{1+\eta_n \overline{q}_n} = \frac{1+\eta_n \underline{q}_{n-1}}{1+\eta_n \overline{q}_{n-1}} = \frac{1}{L_n}. \tag{A.7}$$

Let $\mathbf{h} = (h_1, \dots, h_n, 0, \dots, 0)$ and let $\widehat{\mathbf{h}} = (h_1, \dots, h_{n-1}, 0, \dots, 0)$. Using (4.7) and (A.7), the posterior odds under threshold n satisfy

$$\frac{\pi^{(n)}(\mathbf{h})}{1-\pi^{(n)}(\mathbf{h})} = \frac{\pi_0}{1-\pi_0} \left(\frac{1}{L_n}\right)^{|\mathbf{h}|+1} \prod_{i=1}^{n} L_i^{h_i} = \frac{\pi_0}{1-\pi_0} \left(\frac{1}{L_n}\right)^{\sum_{i=1}^{n-1} h_i+1} \prod_{i=1}^{n-1} L_i^{h_i} = \frac{\pi^{(n-1)}(\widehat{\mathbf{h}})}{1-\pi^{(n-1)}(\widehat{\mathbf{h}})}.$$

Therefore, $\pi^{(n)}(\mathbf{h}) = \pi^{(n-1)}(\widehat{\mathbf{h}})$. That is, at the cutoff, the number of disclosed copies of s_n does not affect the posterior belief.

Setting $z_n = 1$ in (A.5) gives $G^{\eta_n}_{\theta,n}(z_1, \dots, z_{n-1}, 1) = \frac{1}{1+\eta_n \sum_{i=1}^{n-1} f_{\theta,i}(1-z_i)} = G^{\eta_n}_{\theta,n-1}(z_1, \dots, z_{n-1})$. Thus, conditional on either state, the first $n-1$ coordinates of $\mathbf{h}^{(n)}_{\delta,p}$ have the same distribution as $\mathbf{h}^{(n-1)}_{\delta,p}$. Together with $\pi^{(n)}(\mathbf{h}) = \pi^{(n-1)}(\widehat{\mathbf{h}})$ this implies $I\left(\theta; \mathbf{h}^{(n)}_{\delta,p}\right) = I\left(\theta; \mathbf{h}^{(n-1)}_{\delta,p}\right)$.

Thus, mutual information is continuous and strictly increasing in η on every region on which the largest equilibrium threshold is constant, and the adjacent expressions coincide whenever that threshold changes. Since there are only finitely many thresholds, the selected mutual information is continuous and strictly increasing in η.

Finally, $\eta(\delta, p)$ is jointly continuous in (δ, p), strictly increasing in δ for every fixed $p > 0$, and strictly increasing in p for every fixed $\delta \in (0, 1)$. It follows that $\mathcal{I}(\delta, p)$ is jointly continuous and strictly increasing in each parameter.

A.5 Proof of Proposition 3

Fix $p \in (0, 1]$ and write $\eta = \eta(\delta, p)$. We know that $\eta \to \infty$ as $\delta \to 1$. By Corollary 2, when δ is sufficiently close to 1, the most informative constant-threshold equilibrium is the one in which agents disclose all available copies of s_1 and conceal all other signals. Hence, every on-path history takes the form $m\mathbf{1}_1$ for some $m \in \mathbb{N}$ (and note that the length of the history is $|\mathbf{h}_{\delta,p}| = m$). Setting $n = 1$ in equation (A.5), conditional on state θ, the probability-generating function of the length of $\mathbf{h}_{\delta,p}$ is

$$\mathbb{E}\left[z^{|\mathbf{h}_{\delta,p}|} \mid \theta\right] = \frac{1}{1+\eta f_{\theta,1}(1-z)}.$$

It follows that for every $m \in \mathbb{N}$, we have

$$\Pr\left(|\mathbf{h}_{\delta,p}| = m \mid \theta\right) = \frac{1}{1+\eta f_{\theta,1}} \left(\frac{\eta f_{\theta,1}}{1+\eta f_{\theta,1}}\right)^m, \tag{A.8}$$

Thus, conditional on any state θ, the number of disclosed copies of s_1 has a geometric distribution with mean $\eta f_{\theta,1}$.

For every $x \geq 0$, equation (A.8) implies

$$\Pr\left(\frac{|\mathbf{h}_{\delta,p}|}{\eta} > x \middle| \theta\right) = \sum_{k=\lfloor \eta x \rfloor+1}^{\infty} \frac{1}{1+\eta f_{\theta,1}} \left(\frac{\eta f_{\theta,1}}{1+\eta f_{\theta,1}}\right)^k = \left(\frac{\eta f_{\theta,1}}{1+\eta f_{\theta,1}}\right)^{\lfloor \eta x \rfloor+1}.$$

Since $\eta \log\left(\frac{\eta f_{\theta,1}}{1+\eta f_{\theta,1}}\right) = -\eta \log\left(1 + \frac{1}{\eta f_{\theta,1}}\right) \to -\frac{1}{f_{\theta,1}}$, and $\frac{\lfloor \eta x \rfloor+1}{\eta} \to x$, we have

$$\log \Pr\left(\frac{|\mathbf{h}_{\delta,p}|}{\eta} > x \middle| \theta\right) = \frac{\lfloor \eta x \rfloor + 1}{\eta} \eta \log\left(\frac{\eta f_{\theta,1}}{1+\eta f_{\theta,1}}\right) \to -\frac{x}{f_{\theta,1}}.$$

Thus, the preceding calculations imply that, for every $x \geq 0$, $\Pr\left(\frac{|\mathbf{h}_{\delta,p}|}{\eta} > x \middle| \theta\right) \to \exp\left(-\frac{x}{f_{\theta,1}}\right)$, or that $\Pr\left(\frac{|\mathbf{h}_{\delta,p}|}{\eta} \leq x \middle| \theta\right) \to 1 - \exp\left(-\frac{x}{f_{\theta,1}}\right)$, which is the CDF of an exponential random variable with mean $f_{\theta,1}$. Therefore, $\frac{|\mathbf{h}_{\delta,p}|}{\eta} \Rightarrow Y_\theta$, where, conditional on state θ, Y_θ is exponentially distributed with mean $f_{\theta,1}$.

We now derive the limiting distribution of agents' posterior belief in the steady state. From equation (4.7), the posterior odds after observing history $m\mathbf{1}_1$ are

$$\frac{\pi(m\mathbf{1}_1)}{1-\pi(m\mathbf{1}_1)} = \frac{\pi_0}{1-\pi_0} \left(\frac{1+\eta \underline{f}_1}{1+\eta \overline{f}_1}\right)^{m+1} L_1^m. \tag{A.9}$$

Taking logarithms, we can write the posterior log likelihood ratio as

$$\log \frac{\pi(m\mathbf{1}_1)}{1-\pi(m\mathbf{1}_1)} = \log \frac{\pi_0}{1-\pi_0} + \log\left(\frac{1+\eta \underline{f}_1}{1+\eta \overline{f}_1}\right) + \frac{m}{\eta} \eta \log\left(L_1 \frac{1+\eta \underline{f}_1}{1+\eta \overline{f}_1}\right).$$

Note that for the second term we have $\log\left(\frac{1+\eta \underline{f}_1}{1+\eta \overline{f}_1}\right) \to -\log L_1$. Using $\overline{f}_1 = L_1 \underline{f}_1$, the expression inside the logarithm in the last term can be rewritten as $L_1 \frac{1+\eta \underline{f}_1}{1+\eta \overline{f}_1} = \frac{L_1+\eta \overline{f}_1}{1+\eta \overline{f}_1} = 1 + \frac{L_1-1}{1+\eta \overline{f}_1}$, so that

$$\eta \log\left(L_1 \frac{1+\eta \underline{f}_1}{1+\eta \overline{f}_1}\right) \to \frac{L_1 - 1}{\overline{f}_1} = \frac{1}{\underline{f}_1} - \frac{1}{\overline{f}_1}.$$

Let $Z_\theta = \log \frac{\pi_0}{1-\pi_0} - \log L_1 + \left(\frac{1}{\underline{f}_1} - \frac{1}{\overline{f}_1}\right) Y_\theta$. The preceding limits and Slutsky's theorem imply that, conditional on state θ, $\log \frac{\pi(\mathbf{h}_{\delta,p})}{1-\pi(\mathbf{h}_{\delta,p})} \Rightarrow Z_\theta$.

Since $L_1 > 1$, $\frac{1}{\underline{f}_1} - \frac{1}{\overline{f}_1} > 0$. Conditional on either state, Y_θ is finite with probability one and has a nondegenerate exponential distribution. Therefore, Z_θ is also finite with probability one and nondegenerate. Because the function $z \mapsto \frac{\exp(z)}{1+\exp(z)}$ is continuous and strictly increasing from $\mathbb{R}$ into $(0,1)$, the continuous mapping theorem implies that, conditional on either state, $\pi(\mathbf{h}_{\delta,p}) \Rightarrow \frac{\exp(Z_\theta)}{1+\exp(Z_\theta)}$, which is a nondegenerate random variable taking values in $(0,1)$. Full learning would require $\pi(\mathbf{h}_{\delta,p})$ to converge in probability to 1 conditional on $\theta = \overline{\theta}$ and to 0 conditional on $\theta = \underline{\theta}$. Convergence in probability to either constant would imply convergence in distribution to that same constant. This is impossible because the limiting posterior distribution is nondegenerate and takes values in $(0,1)$.

A.6 Proof of Proposition 4

If the sender uses Strategy k, then the receiver will only observe $\mathbf{h} \equiv (k+1)\mathbf{1}_1 + \mathbf{1}_2$ and $j\mathbf{1}_1$ for every $j \neq k+1$ with positive probability. The receiver's posterior beliefs $\pi : H \to [0,1]$ at these histories are pinned down by Bayes rule. Therefore, in order to verify that Strategy k is part of a strict equilibrium, we only need to verify that the following three conditions are satisfied:

1. $\pi(j\mathbf{1}_1)$ is strictly increasing in j for every $j \neq k+1$.

2. $\pi((k+2)\mathbf{1}_1) > \pi(\mathbf{h})$.

3. $\pi(\mathbf{h}) > \pi(k\mathbf{1}_1)$.

The first two conditions are satisfied for all $k \in \mathbb{N}$, as implied by our calculations in the OLG game. Next, we show that for any $\delta \in (0,1)$, there exists a large enough $k \in \mathbb{N}$ such that the third condition is also satisfied. If the sender uses Strategy k in the one-shot game, then conditional on state θ, the receiver will observe $k\mathbf{1}_1$ with probability

$$(1-\delta)\delta^k\Big(1-\delta(1-f_{\theta,1})\Big)^{-(k+1)} f_{\theta,1}^k + (1-\delta)\delta^{k+1}\Big(1-\delta(1-f_{\theta,1}-f_{\theta,2})\Big)^{-(k+2)} f_{\theta,1}^{k+1},$$

where the first term is the probability that the sender observes k copies of s_1 and an unknown number of copies of s_2 to s_l, and the second term is the probability that the sender observes $k+1$ copies of s_1, no signal s_2, as well as an unknown number of copies of s_3 to s_l. Still under Strategy k and conditional on state θ, the receiver will observe $\mathbf{h}$ with probability

$$(1-\delta)\delta^{k+1}\Big(1-\delta(1-f_{\theta,1})\Big)^{-(k+2)} f_{\theta,1}^{k+1} - (1-\delta)\delta^{k+1}\Big(1-\delta(1-f_{\theta,1}-f_{\theta,2})\Big)^{-(k+2)} f_{\theta,1}^{k+1},$$

where the first term is the probability that the sender observes $k+1$ copies of s_1 and an unknown number of signals from s_2 to s_l and the second term coincides with the second term in the previous expression except with the opposite sign. According to Bayes rule, we know that $\pi(\mathbf{h}) > \pi(k\mathbf{1}_1)$ if and only if the value of

$$\frac{\Big(1-\delta(1-f_{\theta,1})\Big)^{-(k+1)} + \delta f_{\theta,1}\Big(1-\delta(1-f_{\theta,1}-f_{\theta,2})\Big)^{-(k+2)}}{f_{\theta,1}\Big(1-\delta(1-f_{\theta,1})\Big)^{-(k+2)} - f_{\theta,1}\Big(1-\delta(1-f_{\theta,1}-f_{\theta,2})\Big)^{-(k+2)}} \tag{A.10}$$

strictly decreases once θ increases from $\underline{\theta}$ to $\overline{\theta}$. This is the case if and only if

$$\frac{\Big(\overline{X}^{-(k+1)} + \delta\overline{f}_1 \cdot \overline{Y}^{-(k+2)}\Big) \cdot \Big(\underline{X}^{-(k+2)} - \underline{Y}^{-(k+2)}\Big)}{\Big(\underline{X}^{-(k+1)} + \delta\underline{f}_1 \cdot \underline{Y}^{-(k+2)}\Big) \cdot \Big(\overline{X}^{-(k+2)} - \overline{Y}^{-(k+2)}\Big)} < \frac{\overline{f}_1}{\underline{f}_1} = L_1, \tag{A.11}$$

where

$$\overline{X} \equiv 1-\delta(1-\overline{f}_1), \quad \underline{X} \equiv 1-\delta(1-\underline{f}_1)$$

$$\overline{Y} \equiv 1-\delta(1-\overline{f}_1-\overline{f}_2), \text{ and } \underline{Y} \equiv 1-\delta(1-\underline{f}_1-\underline{f}_2).$$

Let

$$\overline{R} \equiv \overline{X}/\overline{Y} \text{ and } \underline{R} \equiv \underline{X}/\underline{Y}.$$

Inequality (A.11) can then be rewritten as

$$\frac{(\overline{X} + \delta\overline{f}_1 \cdot \overline{R}^{k+2})(1-\underline{R}^{k+2})}{(\underline{X} + \delta\underline{f}_1 \cdot \underline{R}^{k+2})(1-\overline{R}^{k+2})} < L_1 \tag{A.12}$$

For every $\delta \in (0,1)$, we have $\overline{X} < \overline{Y}$ and $\underline{X} < \underline{Y}$, so $\overline{R}, \underline{R} \in (0,1)$. Fix any $\delta \in (0,1)$, we know that both $\overline{R}^{k+2}$ and $\underline{R}^{k+2}$ converge to 0 as $k \to \infty$. Hence, the numerator of the LHS of (A.12) converges to $\overline{X}$ and its denominator converges to $\underline{X}$, so the LHS of (A.12) converges to $\overline{X}/\underline{X}$ as $k \to \infty$. Moreover, since $\overline{f}_1 > \underline{f}_1$, we have

$$L_1 - \frac{\overline{X}}{\underline{X}} = \frac{\overline{f}_1\underline{X} - \underline{f}_1\overline{X}}{\underline{f}_1\underline{X}} = \frac{(1-\delta)\left(\overline{f}_1 - \underline{f}_1\right)}{\underline{f}_1\underline{X}} > 0 \quad \text{for every } \delta \in (0,1). \tag{A.13}$$

Therefore, for every $\delta \in (0,1)$, the LHS of (A.12), converges to $\overline{X}/\underline{X}$ as $k \to \infty$, which is strictly less than L_1. It follows that there exists $K \in \mathbb{N}$ such that inequality (A.12) is satisfied for every $k \geq K$. Hence, for any $\delta \in (0,1)$, there exists $k \in \mathbb{N}$ such that Strategy k is part of a strict monotone equilibrium.

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