

# Selective Disclosure in Overlapping Generations\*

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## Abstract

We develop an overlapping generations model where each agent observes a verifiable private signal about the state and, with positive probability, also receives signals disclosed by his predecessor. The agent then takes an action and decides which signals to pass on. Each agent's action has a positive externality on his predecessor and his optimal action increases in his belief about the state. We show that as the communication friction vanishes, agents become increasingly selective in disclosing information. As the probability that messages reach the next generation approaches one, all signals except those with the highest likelihood ratio will be concealed in equilibrium.

**Keywords:** information disclosure, hard information, overlapping generations.

## 1 Introduction

Research shows that the intergenerational transmission of anecdotes and stories can shape individuals' preferences, beliefs, and behaviors, with important implications for culture and welfare (Bisin and Verdier, 2011, 2023). Information may be lost in this transmission process, not only because of physical communication frictions, but also because of deliberate omissions, as different generations may have misaligned interests (Hirshleifer, 2020; Stubbersfield, 2022). For example, parents who wish to encourage effort may emphasize narratives suggesting that hard work leads to a better life (Benabou and Tirole, 2006; Doepke and Zilibotti, 2017).

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This paper analyzes the intergenerational transmission of information using an overlapping generations model that incorporates both exogenous communication frictions and misaligned interests across generations. As in [Dye \(1985\)](#), we restrict attention to the transmission of *hard information*, so that information can be concealed but cannot be fabricated or falsified. We show that as communication frictions vanish, agents become increasingly selective in disclosing information, and in the limit, all signals except for the most favorable ones will be concealed.

We study steady state equilibria of a doubly infinite time horizon game. The state of the world is constant over time and is either *high* or *low*. In each period, only one agent is active. He observes a private signal about the state and, with some probability, also receives the signals his immediate predecessor passed on to him. The agent then chooses both an effort level on the real line and a subset of the signals he possesses—including his private signal and any inherited signals—to pass on to his immediate successor. With some probability, communication succeeds and his successor observes this subset of signals. However, agents cannot directly observe others' actions, whether and when communication failed, and the exact sequence of the disclosed signals.

Each agent's optimal effort strictly increases in his belief about the state and his action has a positive externality on his predecessor. Therefore, when deciding which subset of signals to disclose, each agent's objective is to maximize his successor's belief about the state.

Due to the plethora of equilibria, some of which are driven by unreasonable off-path beliefs,<sup>1</sup> we restrict attention to equilibria that satisfy at least one of the following two refinements, both are satisfied in all equilibria of [Dye \(1985\)](#)'s model. The first refinement requires that under agents' equilibrium strategy, they will disclose all available signals with likelihood ratios above a threshold and will conceal all other signals, where the threshold can depend on agents' private signals and signals they inherited. The second refinement requires agents' beliefs to be *monotone*, in the sense that their belief about the state increases when their predecessor reveals one fewer lower-likelihood-ratio signal and reveals one more higher-likelihood-ratio signal.

[Theorem 1](#) shows that when communication fails with low probability, disclosure becomes extremely selective in the sense that in all strict equilibria that satisfy at least one of our two refinements, all signals other than the ones with the highest likelihood ratio are concealed.

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<sup>1</sup>For example, when there are three or more signal realizations, there may exist equilibria where signal realizations with the highest likelihood ratio are concealed while those with the second-highest likelihood ratio are revealed. Such equilibria can be sustained by off-path beliefs in which an agent who observes the disclosure of any signal with the highest likelihood ratio infers that many signals with low likelihood ratios are concealed.

To better understand how disclosure selectiveness depends on communication frictions, Theorem 2 allows for arbitrary communication failure rates and focuses on equilibria in which agents disclose a signal if and only if its likelihood exceeds a cutoff that is common across all information sets (i.e., *constant-threshold equilibrium*). The theorem shows that, for every signal with a likelihood ratio greater than one, there exists a constant-threshold equilibrium in which the signal is revealed if and only if the communication failure rate exceeds a cutoff. This cutoff is strictly decreasing in the signal’s likelihood ratio. Consequently, as communication frictions vanish, disclosure becomes more selective, in the sense that a smaller set of signals is revealed.

Our findings contrast with a well-known result in Dye (1985), which shows that a higher probability that the sender possesses evidence leads to less selective disclosure. This difference arises from the different forms of uncertainty agents face about others’ information structures. In our model, consider the sequence of realized signals following the last communication failure. This sequence consists of multiple good signals (those that are disclosed), interspersed with subsequences of bad signals (which may be of zero length). That is, there is one bad sequence preceding the first good signal, one between each pair of adjacent good signals, and one following the last good signal. Moreover, conditional on the state, the expected length of each bad subsequence is independent of the disclosed good signals. Hence, by disclosing another good signal, the agent also reveals the existence of another *bad subsequence*. This negative effect is absent in Dye (1985) as any disclosure proves that no information is hidden. As communication frictions vanish, the expected length of each bad subsequence increases while the likelihood ratio of each good signal remains unchanged. Disclosure becomes more selective since each disclosed good signal needs to have a higher likelihood ratio in order to overcome the stronger negative effect of another bad subsequence.

We illustrate our model with an application to the “belief in a just world” framework of Benabou and Tirole (2006). We add intergenerational selective disclosure to their environment in which young agents choose effort based on beliefs about the returns to effort, while the old agents benefit from young agents’ effort and thus have incentives to shape their beliefs by selectively transmitting verifiable anecdotes. Different steady-state disclosure conventions (e.g., transmitting no signals versus transmitting only the strongest evidence of high returns to effort) generate distinct long-run belief distributions and effort outcomes, naturally echoing the “American Dream” versus “Europe” comparison.

Our work contributes to the literature on disclosure games where the sender has unknown

amount of signals.<sup>2</sup> The models of [Shin \(1994, 2003\)](#) and [Dziuda \(2011\)](#) focus on signals with only two realizations, which cannot answer questions related to the selectiveness of disclosure. [Gao \(2025\)](#) studies a model where the sender has a continuum of signals and focuses on equilibria satisfying the *truth-leaning refinement* of [Hart, Kremer, and Perry \(2017\)](#), which conflicts with our refinements, as we explain in Section 3. She shows that the sender replicates the signal distribution under a more favorable state by disclosing some unfavorable signals. In contrast, we find that agents disclose only the most favorable signals as communication frictions vanish.

[Rappoport \(2025\)](#) focuses on the receiver’s optimal equilibrium and shows that when the sender is expected to have more information, the receiver will take actions that are worse for the sender in every state. Our analysis instead focuses on the selectiveness of the sender’s disclosure behavior rather than the receiver’s action choices. Moreover, we study threshold equilibria and equilibria with monotone beliefs, which may not include the receiver’s optimal equilibrium.

A growing literature studies selective disclosure in dynamic settings.<sup>3</sup> In [Guttman, Kremer, and Skrzypacz \(2014\)](#), a sender may receive up to two signals over two periods and chooses both what to disclose and when. In [Felgenhauer and Schulte \(2014\)](#), [Felgenhauer and Loerke \(2017\)](#), [Lou \(2023\)](#), [Arieli and Stewart \(2025\)](#), and [Dai, Fudenberg, and Pei \(2026\)](#), a sender conducts a sequence of experiments before selecting a subset of outcomes to disclose to a receiver. In [Gieczewski \(2022\)](#) and [Squintani \(2025\)](#), a subset of agents observe the state and decide whether to disclose it to their neighbors. [Pei \(2025\)](#) studies repeated games with random matching where players can voluntarily disclose signals about their past actions. To the best of our knowledge, none of these existing works examine the selectiveness of players’ disclosure behavior.

Our work also contributes to the study of overlapping generations models starting from [Samuelson \(1958\)](#). While many papers in this literature ([Bhaskar, 1998](#); [Acemoglu and Jackson, 2015](#)) focus on the observability of agents’ actions, we study settings where actions are not

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<sup>2</sup>[Di Tillio, Ottaviani, and Sørensen \(2021\)](#) study an agent who observes the realizations of the  $k$  ( $\leq n$ ) highest draws among  $n$  conditionally i.i.d. signals, which contrasts to our model where the number of signals disclosed is endogenous. In [Bardhi and Bobkova \(2023\)](#), each sender decides whether to disclose his signal without observing its realization. In [Antić and Chakraborty \(2025\)](#), the state is  $n$ -dimensional and the sender selects which of  $k$  ( $\leq n$ ) realized values of those dimensions to disclose (typically not the highest ones). [Onuchic and Ramos \(2025\)](#) consider multiple senders who decide whether to disclose a multi-dimensional state via a collective decision rule.

<sup>3</sup>In static disclosure games where the number of signals available to the sender is drawn from a geometric distribution, whether the conclusion of our Theorem 1 continues to hold remains an open question. We will explain the challenges in analyzing the static disclosure game in the conclusion section.

observed, so that agents can influence their successors' behaviors only by disclosing verifiable signals. Niehaus (2011) studies the sharing of skills among a sequence of agents, which contrasts to our model where agents share verifiable signals about some payoff-relevant state.

More closely related is the work of Anderlini, Gerardi, and Lagunoff (2012) that also analyzes an overlapping generations model with strategic communication and payoff externalities, and examines whether agents learn the state in the long run. Different from our model, communication in their model takes the form of cheap talk rather than the disclosure of hard evidence.

## 2 Model

Consider a game with a doubly infinite time horizon  $t \in \mathbb{Z} \equiv \{\dots - 1, 0, 1, \dots\}$ . Agent  $t$  is the only active player in period  $t$ . He observes a *private signal*  $s^t \in S$  about the state  $\theta \in \{\underline{\theta}, \bar{\theta}\}$ , and possibly, some signals disclosed to him by agent  $t - 1$ . Then he chooses an action  $a^t \in \mathbb{R}$  and a set of signals to reveal to agent  $t + 1$ . We assume that  $\theta$  is constant over time, the set of signal realizations  $S \equiv \{s_1, s_2, \dots, s_l\}$  is finite with  $l \geq 2$ , and that no signal perfectly reveals any state. Let  $\pi_0 \in (0, 1)$  denote the prior probability that  $\theta = \bar{\theta}$ . Let  $f_{\theta,i}$  denote the probability of  $s_i$  conditional on state  $\theta$  and let  $\mathbf{f} \equiv \{f_{\bar{\theta},i}, f_{\underline{\theta},i}\}_{i=1}^l$ . To simplify notation, we write  $\bar{f}_i$  instead of  $f_{\bar{\theta},i}$  and  $\underline{f}_i$  instead of  $f_{\underline{\theta},i}$ . Let  $L_i \equiv \bar{f}_i/\underline{f}_i$  denote the *likelihood ratio* of signal  $s_i$ . We assume that  $L_i \neq L_j$  for every  $i \neq j$  and we index signals so that  $L_i$  is strictly decreasing in  $i$ :

$$\infty > L_1 > L_2 > \dots > L_l > 0.^4 \tag{2.1}$$

Agent  $t$ 's payoff is  $u(\theta, a^t) + v(\theta, a^{t+1})$ , which depends on the state  $\theta$ , his action, and his immediate successor's action. We assume that (i)  $v$  is strictly increasing in  $a^{t+1}$ ,<sup>5</sup> (ii)  $u$  is strictly concave in  $a^t$ , and that (iii)  $\arg \max_{a^t \in \mathbb{R}} \{\pi u(\bar{\theta}, a^t) + (1 - \pi)u(\underline{\theta}, a^t)\}$  is strictly increasing in  $\pi \in [0, 1]$ . A concrete example is a setting similar to the one in Benabou and Tirole (2006): Each agent  $t$  chooses effort  $a^t \in \mathbb{R}_+$  at a strictly increasing and convex cost  $c(a^t)$ . He produces output  $\theta a^t + (1 - \theta)\varepsilon^t$ , where  $\varepsilon^t$  stands for a random shock that is independent of effort and  $\theta$  stands for the marginal product of effort that is either high ( $\theta = \bar{\theta}$ ) or low ( $\theta = \underline{\theta}$ ) with  $0 < \underline{\theta} < \bar{\theta} < 1$ . As in Samuelson (1958), he consumes an exogenous fraction of his own output and transfers the rest to his immediate predecessor through some social security system.

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<sup>4</sup>Analogous of our results can be derived when multiple signal realizations share the same likelihood ratio.

<sup>5</sup>Similar results can be derived in the case where  $v$  is strictly decreasing in  $a^{t+1}$ .

The information agent  $t - 1$  reveals to agent  $t$  is characterized by  $\mathbf{h} \equiv (h_1, h_2, \dots, h_l) \in H \equiv \mathbb{N}^l$  where  $h_i$  stands for the number of signal  $s_i$  revealed. With probability  $\delta \in (0, 1)$ , communication succeeds, in which case agent  $t$  observes the number of each signal revealed by agent  $t - 1$  but not the exact order of these signals, i.e., agent  $t$ 's *history* is  $\mathbf{h}$ . With probability  $1 - \delta$ , communication fails in which case agent  $t$ 's history is  $\mathbf{0} \equiv (0, \dots, 0)$ . Agents observe their histories and private signals but cannot directly observe the calendar times at which communication fails. We interpret  $1 - \delta$  as a communication friction and refer to  $h_1 + \dots + h_n$  as the *length* of history  $(h_1, \dots, h_n)$ .

We assume that signals are verifiable so that they can be concealed but not falsified (Grossman, 1981; Milgrom, 1981; Dye, 1985). Formally, let  $\mathbf{1}_i \in \mathbb{N}^l$  denote a vector where the  $i$ th entry is 1 and all other entries are 0. After observing history  $\mathbf{h}$  and receiving private signal  $s_i$ , agent  $t$  can only disclose vectors that are no more than  $\mathbf{h} + \mathbf{1}_i$ . Hence, an agent can only reveal a subset of the signals he possesses, which include his private signal and the signals he inherited (if any).

A *pure strategy*  $\sigma : H \times S \rightarrow \mathbb{R} \times H$  maps an agent's history and private signal to his action and the information he discloses to his immediate successor, subject to the feasibility constraint. Our solution concept is *steady state Bayesian equilibrium* (or *equilibrium* for short), which extends the notion of steady state equilibrium in Clark, Fudenberg, and Wolitzky (2021) and Pei (2025) to incomplete information games. An equilibrium of our game consists of (i) a pure strategy  $\sigma$ , (ii) two distributions  $\underline{\mu}, \bar{\mu} \in \Delta(H)$  over histories, and (iii) a belief map  $\pi : H \rightarrow [0, 1]$  which represents the probability that an agent assigns to state  $\bar{\theta}$  after observing his history but before observing his own private signal. These three components satisfy (i) when all agents use strategy  $\sigma$ , there exists a steady state in which agents' histories are distributed according to  $\underline{\mu}$  conditional on  $\theta = \underline{\theta}$  and are distributed according to  $\bar{\mu}$  conditional on  $\theta = \bar{\theta}$ , (ii)  $\pi(\mathbf{h})$  is derived via Bayes rule from  $(\underline{\mu}, \bar{\mu})$  at every  $\mathbf{h}$  that occurs with positive probability, and that (iii)  $\sigma$  maximizes an agent's expected payoff when all agents' beliefs after observing their histories are given by the mapping  $\pi$ . A *strict equilibrium* is one in which the strategy  $\sigma$  is strictly optimal for the agents at each history-signal pair  $(\mathbf{h}, s) \in H \times S$ .

### 3 Main Results

Our game admits many equilibria because each agent may possess arbitrarily many signals, allowing their successor's belief to become arbitrarily pessimistic at off-path histories. This in turn discourages agents from revealing those off-path histories, making those beliefs self-fulfilling.

Proposition 1 characterizes some of those equilibria:

**Proposition 1.** *Fix  $(u, v, \mathbf{f}, \pi_0)$ . For every  $\delta \in (0, 1)$ ,  $i \in \{1, 2, \dots, l\}$  with  $L_i > 1$ , and  $n \in \mathbb{N} \cup \{\infty\}$ , there is a strict equilibrium where (i) agents never disclose any  $s \neq s_i$  and (ii) the number of  $s_i$  disclosed at history  $\mathbf{h} \equiv \{h_1, \dots, h_l\}$  and private signal  $s_j$  is  $\min\{n, h_i + \mathbb{I}\{i = j\}\}$ .*

The proof is in Appendix A. Proposition 1 implies that regardless of  $\delta$ , there is always an equilibrium where agents disclose all realized  $s_1$  and nothing else. However, it also implies that for any  $s_i$  with a likelihood ratio more than 1, there is also a strict equilibrium in which agents never disclose anything other than  $s_i$  and disclose up to  $n$  realizations of  $s_i$ . Such equilibria exist even if there are other signals with strictly higher likelihood ratios than  $s_i$ , as they can be sustained by the off-path belief that as long as any signal other than  $s_i$  (including those with higher likelihood ratios) is disclosed, the agent who disclosed it must have concealed many signals with low likelihood ratios, which justifies their successors' pessimistic beliefs about  $\theta$ .

We view such beliefs as unreasonable since agents should become more *optimistic* rather than *pessimistic* once their predecessors replace a signal with a lower likelihood ratio with one that has a higher likelihood ratio. We are unaware of standard refinements in signaling games that can rule out those unreasonable equilibria. Hence, we propose two refinements, each of which is sufficient to rule out those unreasonable equilibria.<sup>6</sup> Analogs of our refinements are also used in other dynamic disclosure models such as the one in Guttman, Kremer, and Skrzypacz (2014).

Our first refinement is on agents' equilibrium strategy, and in particular, the component of their strategy that specifies the set of signals disclosed to the next agent upon observing each history  $\mathbf{h}$  and private signal  $s$ , which we denote by  $\sigma_D(\mathbf{h}, s) \in H$ . For every vector  $\mathbf{h} \in H$  and  $j \in \{0, 1, \dots, l\}$ , let  $\mathbf{h}[j] \in H$  denote an  $l$ -dimensional vector where the first  $j$  entries coincide with  $\mathbf{h}$  and the other entries are all 0. By definition,  $\mathbf{h}[0] = \mathbf{0}$  and  $\mathbf{h}[l] = \mathbf{h}$  for every  $\mathbf{h} \in H$ .

**Threshold Strategy.** *A strategy  $\sigma$  is a threshold strategy if there exists a function  $J : H \times S \rightarrow \{0, 1, \dots, l\}$  such that  $\sigma_D(\mathbf{h}, s_i) = (\mathbf{h} + \mathbf{1}_i)[J(\mathbf{h}, s_i)]$  for every  $\mathbf{h} \in H$  and  $i \in \{1, 2, \dots, l\}$ .*

We view our threshold strategy as a natural extension of the equilibrium strategies in Dye (1985) to our dynamic setting. Intuitively, a threshold strategy is one where at every history-signal pair, agents reveal all available signals (including his private signal and the signals contained in his history) with likelihood ratios above some cutoff and conceal all other signals. It

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<sup>6</sup>By the end of this section, we will discuss the relationship between our refinements and the *truth-leaning refinement* in Hart, Kremer, and Perry (2017) as well as the receiver-optimal equilibria in Rappoport (2025).

allows the disclosure threshold to vary across histories and private signals but rules out strategies where at some history-signal pair, agents disclose  $s_2$  but conceal  $s_1$ . For a concrete example, suppose  $J(\mathbf{0}, s) = 1$  for every  $s \in S$  and  $J(\mathbf{h}, s) = 0$  for every  $\mathbf{h} \neq \mathbf{0}$  and  $s \in S$ , then agents disclose  $s_1$  when the length of his history is 0 and disclose nothing otherwise.

Our next refinement is on the belief map  $\pi : H \rightarrow [0, 1]$ , which stands for the probability agents assign to state  $\bar{\theta}$  after observing their history but before observing their private signal.

**Monotone Belief.** *A belief map  $\pi : H \rightarrow [0, 1]$  is monotone if  $\pi(\mathbf{h}) > \pi(\mathbf{h}^*)$  for every pair of histories  $\mathbf{h} \equiv (h_1, \dots, h_l)$  and  $\mathbf{h}^* \equiv (h_1^*, \dots, h_l^*)$  that have the same length and satisfy*

$$\sum_{j=1}^n h_j \geq \sum_{j=1}^n h_j^* \text{ for every } n < l \text{ with strict inequality for some.} \quad (3.1)$$

Intuitively, a belief map is monotone if fixing the total number of signals disclosed, agent  $t + 1$  will become more optimistic about the state once agent  $t$  replaces a signal that has a lower likelihood ratio with one that has a higher likelihood ratio. This property is satisfied in the equilibria of [Dye \(1985\)](#)'s model where the sender has at most one signal.

Among the equilibria described in [Proposition 1](#), the one where agents reveal all realized  $s_1$  but nothing else satisfies both of our refinements. The ones where agents disclose  $s_i$  for some  $i \geq 2$  but conceal  $s_1$  fail both refinements. However, it remains unclear whether there are equilibria where agents reveal  $\{s_1, s_2, \dots, s_i\}$  for some  $i \geq 2$  and conceal other signals, or whether there are equilibria with a monotone belief map where agents reveal signals other than  $s_1$ .

Our main result, [Theorem 1](#), shows that when communication frictions are small enough, none of the above can happen in any strict equilibrium that satisfies at least one of our refinements.

**Theorem 1.** *Fix any  $(u, v, \mathbf{f}, \pi_0)$ . There exists  $\delta^* \in (0, 1)$  such that for every  $\delta > \delta^*$ :*

1. *Signals  $s_2, \dots, s_l$  are never revealed in any strict equilibrium with a threshold strategy.*<sup>7</sup>
2. *Signals  $s_2, \dots, s_l$  are never revealed in any strict equilibrium with a monotone belief map.*

[Theorem 1](#) implies that as the communication friction vanishes, agents become extremely selective in disclosing information, in the sense that they will conceal all signals except for  $s_1$ , the signal that has the highest likelihood ratio. The required magnitude of  $\delta^*$  depends on the signal structure  $\mathbf{f}$ . For example, when  $L_1 - L_2$  is close to 0, the value  $\delta^*$  is close to 1.

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<sup>7</sup>In any equilibrium with a threshold strategy, agents will use a threshold strategy in equilibrium but need to have incentives not to deviate to any other strategy, including strategies that are not threshold strategies.

Together with Proposition 1, Theorem 1 leads to a full characterization of strict equilibria that satisfies either one of our refinements when  $\delta$  is close to 1:

**Corollary 1.** *Fix any  $(u, v, \mathbf{f}, \pi_0)$ . There exists  $\delta^* \in (0, 1)$  such that for every  $\delta > \delta^*$ :*

1. *There are only two strict equilibria in threshold strategies: one where agents never disclose anything and one where agents only disclose signal  $s_1$  and disclose all realized  $s_1$ .*
2. *Every strict equilibrium with a monotone belief map is characterized by an integer  $n \in \mathbb{N} \cup \{\infty\}$  such that  $\sigma_D(\mathbf{h}, s_i) = \min\{n\mathbf{1}_1, (\mathbf{h} + \mathbf{1}_i)[1]\}$  for every  $\mathbf{h} \in H$  and  $i \in \{1, 2, \dots, l\}$ , i.e., agents conceal all signals other than  $s_1$  and disclose up to  $n$  realizations of  $s_1$ .*

The proof of Theorem 1 and Corollary 1 are both in Section 4.

In order to better visualize how communication frictions affect the selectiveness of agents' disclosure, our next result allows for any value of  $\delta$  and provides a full characterization of equilibria in which agents use *constant threshold strategies*. These are strategies where the disclosure threshold is independent of agents' histories and private signals.

**Constant Threshold Strategy.** *A strategy  $\sigma$  is a constant threshold strategy if there exists  $j \in \{0, 1, \dots, l\}$  such that  $\sigma_D(\mathbf{h}, s_i) = (\mathbf{h} + \mathbf{1}_i)[j]$  for every  $\mathbf{h} \in H$  and  $i \in \{1, 2, \dots, l\}$ .*

For example, agents will never disclose any signal when the threshold is constantly 0, and will disclose all signals when the threshold is constantly  $l$ . Theorem 2 focuses on equilibria in constant threshold strategies and shows that as  $\delta$  increases, agents become increasingly selective in disclosing information, in the sense that a smaller set of signals can be disclosed in equilibrium.

**Theorem 2.** *Fix any  $(u, v, \mathbf{f}, \pi_0)$  and  $i \in \{2, \dots, l\}$ .*

1. *If  $L_i > 1$ , then there exists  $\delta_i \in (0, 1)$  such that there is an equilibrium where agents use a constant threshold  $i$  if and only if  $\delta \leq \delta_i$ . Furthermore,  $\delta_i$  is strictly decreasing in  $i$ .*
2. *If  $L_i \leq 1$ , then agents will never use a constant threshold  $i$  regardless of  $\delta$ .*

Theorem 2 implies that (i) signals with likelihood ratio no more than 1 will never be disclosed in any equilibrium with a constant threshold strategy and (ii) any signal with likelihood ratio more than 1 will be disclosed in some equilibria with constant threshold strategies if and only if the communication success rate  $\delta$  is below some cutoff. Furthermore, the cutoff  $\delta$  is strictly increasing in the likelihood ratio of the signal. Since constant threshold strategies are threshold

strategies, we know that when  $\delta > \delta_2$ , the only equilibrium where agents use a constant threshold strategy is the one where agents disclose a signal if and only if it is  $s_1$ .

Our finding that disclosure becomes more selective as  $\delta$  increases stands in contrast to the results in [Dye \(1985\)](#). In [Dye \(1985\)](#)'s model, as the probability that the sender has evidence increases, he becomes less selective in the sense that a larger set of realized signals are disclosed in equilibrium. In the limiting case studied by [Grossman \(1981\)](#) and [Milgrom \(1981\)](#) where the sender has evidence for sure, no disclosure leads to the receiver to infer that the sender has the worst possible signal, which motivates the sender to disclose even the second worst signal.

In our model, a larger  $\delta$  increases the chances that agents observe the signals passed on to them by their predecessors, so absent strategic concerns, their successors should expect them to have more signals. However, disclosure becomes more selective due to the nature of the uncertainty their successors face about their information structures, which differs from that in [Dye \(1985\)](#).

To understand the differences, suppose agents are expected to disclose signals in  $\{s_1, \dots, s_i\}$  (i.e., *good signals*) and to conceal signals in  $\{s_{i+1}, \dots, s_l\}$  (i.e., *bad signals*). After the last communication failure, the sequence of realized signals consists of a number of good signals as well as some subsequences of bad signals (i.e., *bad subsequences*), where the length of each bad subsequence can be anything from 0 to  $+\infty$ . Each agent observes the number of good signals after the latest communication failure but not the lengths of the bad subsequences. When he observes  $k$  good signals, he expects there to be  $k + 1$  bad subsequences: one before the first good signal, one after the last good signal, and one in between each pair of adjacent good signals. An example of such a sequence of signals is depicted in [Figure 1](#), which helps to understand why the number of bad subsequences equals the number of good signals plus 1.

An important observation is that, *conditional on the state*, agents' belief about the length of each bad subsequence is independent of the number of good signals. Therefore, upon disclosing another good signal, there are two effects on the next agent's belief. There is a direct effect that is positive whenever the likelihood ratio of that good signal exceeds 1, but there is also an indirect effect since revealing another good signal also implies that there is an additional bad subsequence, and bad subsequences are more likely to arise when the state is  $\underline{\theta}$ . This indirect effect is absent in [Dye \(1985\)](#)'s model where disclosing a signal shows to the receiver that no information is hidden.

Therefore, the agents in our model have incentives to disclose a signal if and only if its

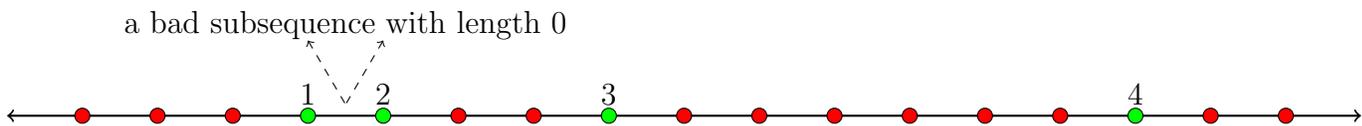


Figure 1: A sequence of realized private signals after the most recent communication failure, with red circles representing bad signals and green circles representing good signals. In this example, there are 4 good signals and 5 bad subsequences. The bad subsequence before good signal 1 has length 3, the bad sequence between good signals 1 and 2 has length 0, the bad subsequence between good signals 2 and 3 has length 2, the bad subsequence between good signals 3 and 4 has length 6, and the bad subsequence after good signal 4 has length 2.

likelihood ratio is large enough so that the direct effect dominates the indirect effect. As  $\delta$  increases, the expected length of each bad subsequence increases conditional on each state, so the indirect effect of an additional bad subsequence becomes stronger yet the direct effect remains unchanged. Hence, agents become more selective in disclosing signals as fewer signals will have high enough likelihood ratios to overcome the increasingly pronounced indirect effect.

In what follows, we present a proof of Theorem 2, which helps readers to visualize the two effects of disclosing an additional signal, as well as how they depend on  $\delta$ .

*Proof of Theorem 2:* Since  $v(\theta, a^{t+1})$  is strictly increasing in  $a^{t+1}$  and agent  $t + 1$ 's action is strictly increasing in the probability with which his belief assigns to  $\bar{\theta}$ , we know that at every  $(\mathbf{h}, s_i) \in H \times S$ , agent  $t$  will choose a vector  $\mathbf{h}' \leq \mathbf{h} + \mathbf{1}_i$  in order to maximize  $\pi(\mathbf{h}')$ .

Suppose all agents use a constant threshold  $n$ , then under the steady state history distribution in state  $\theta$ , the probability that an agent observes history  $\mathbf{h} \equiv (h_1, h_2, \dots, h_n, 0, \dots, 0)$  is

$$(1 - \delta)\delta^{\bar{h}} \cdot X_{\theta}(\mathbf{h}) \cdot \prod_{i=1}^n (f_{\theta,i})^{h_i}, \quad (3.2)$$

where  $\bar{h} \equiv h_1 + \dots + h_n$  denotes the length of history  $\mathbf{h}$  and  $X_{\theta}(\mathbf{h})$  denotes the coefficient in front of the term  $\prod_{i=1}^n (p_i)^{h_i}$  in the following polynomial of  $p_1, p_2, \dots, p_n$ :

$$\sum_{j=\bar{h}}^{\infty} \left( p_1 + p_2 + \dots + p_n + \delta(1 - f_{\theta,1} - \dots - f_{\theta,n}) \right)^j. \quad (3.3)$$

A series of algebraic manipulations reveals that

$$X_{\theta}(\mathbf{h}) = \left( 1 - \delta(1 - f_{\theta,1} - \dots - f_{\theta,n}) \right)^{-(\bar{h}+1)} \cdot \frac{\bar{h}!}{h_1!h_2!\dots h_n!}. \quad (3.4)$$

Therefore,

$$\frac{\pi(\mathbf{h})}{1 - \pi(\mathbf{h})} = \frac{\pi_0}{1 - \pi_0} \cdot \left\{ \frac{1 - \delta(1 - \underline{f}_1 - \dots - \underline{f}_n)}{1 - \delta(1 - \overline{f}_1 - \dots - \overline{f}_n)} \right\}^{\bar{h}+1} \cdot \prod_{i=1}^n (L_i)^{h_i}. \quad (3.5)$$

Equation (3.5) clearly shows the two effects of disclosing an additional signal. As we mentioned earlier, when the total number of signals disclosed is  $\bar{h}$ , there are  $\bar{h} + 1$  bad subsequences (some of them may have length zero) after the most recent communication failure, which explains why the power of the second term on the right-hand-side is  $\bar{h} + 1$ . By disclosing an additional signal  $s_i$ , the direct effect multiplies the likelihood ratio by  $L_i$  (the likelihood ratio of  $s_i$ ) and the indirect effect multiplies the likelihood ratio by

$$\frac{1 - \delta(1 - \underline{f}_1 - \dots - \underline{f}_n)}{1 - \delta(1 - \overline{f}_1 - \dots - \overline{f}_n)}, \quad (3.6)$$

which is the contribution of an additional bad subsequence. Crucially, as long as  $\delta > 0$  and  $n < l$ , the indirect effect, captured by (3.6), is always strictly less than 1. This is because each bad subsequence only consists of signals with likelihood ratios below some cutoff. The value of (3.6) decreases in  $\delta$  since a larger  $\delta$  increases the expected length of each bad subsequence conditional on each state. Therefore, the value of (3.6) is bounded below by its limit where  $\delta \rightarrow 1$ , which equals

$$\frac{\underline{f}_1 + \dots + \underline{f}_n}{\overline{f}_1 + \dots + \overline{f}_n} \quad (3.7)$$

This lower bound is more than the inverse of  $L_1$  but is less than the inverse of  $L_n$ , i.e., disclosing the signal with the highest likelihood ratio is sufficient to overcome the negative indirect effect but as long as signals other than  $s_1$  are disclosed, disclosing the worst one among those signals is never sufficient to overcome to negative indirect effect when  $\delta$  is large enough.

For a strategy with a constant threshold  $n$  to be part of an equilibrium, a necessary condition is that agents prefer to disclose  $(h_1, \dots, h_{n-1}, h_n, 0, \dots, 0)$  to  $(h_1, \dots, h_{n-1}, h_n - 1, 0, \dots, 0)$  for any  $h_n \geq 1$  and  $h_1, \dots, h_{n-1} \geq 0$ . By (3.5), this incentive constraint is satisfied if and only if

$$L_n \cdot \frac{1 - \delta(1 - \underline{f}_1 - \dots - \underline{f}_n)}{1 - \delta(1 - \overline{f}_1 - \dots - \overline{f}_n)} \geq 1. \quad (3.8)$$

This incentive constraint is also sufficient since  $L_1 > L_2 > \dots > L_n$ , so (3.8) implies that

$$L_i \cdot \frac{1 - \delta(1 - \underline{f}_1 - \dots - \underline{f}_n)}{1 - \delta(1 - \overline{f}_1 - \dots - \overline{f}_n)} \geq 1 \text{ for every } i \leq n.$$

Since

$$\frac{1 - \delta(1 - \underline{f}_1 - \dots - \underline{f}_n)}{1 - \delta(1 - \overline{f}_1 - \dots - \overline{f}_n)} \leq 1$$

with strict inequality for every  $n < l$ , we know that  $L_n > 1$  is necessary for (3.8). Since the left-hand-side of (3.8) is strictly decreasing in  $\delta$ , which converges to  $L_n$  as  $\delta \rightarrow 0$  and converges to (3.7) times  $L_n$  as  $\delta \rightarrow 1$ , we know that for every  $n \geq 2$  that satisfies  $L_n > 1$ , there exists  $\delta_n \in (0, 1)$  such that inequality (3.8) holds if and only if  $\delta \leq \delta_n$ . Since  $\delta_n$  is pinned down by the equation

$$\underline{f}_n / \overline{f}_n = \frac{1 - \delta_n(1 - \underline{f}_1 - \dots - \underline{f}_n)}{1 - \delta_n(1 - \overline{f}_1 - \dots - \overline{f}_n)},$$

or equivalently,

$$\delta_n^{-1} - 1 = \frac{\underline{f}_n \sum_{j=1}^n \overline{f}_j - \overline{f}_n \sum_{j=1}^n \underline{f}_j}{\overline{f}_n - \underline{f}_n}. \quad (3.9)$$

After some algebraic manipulations, and using the fact that  $L_n > 1$ , which implies that  $\overline{f}_n > \underline{f}_n$  and  $\overline{f}_{n-1} > \underline{f}_{n-1}$ , we obtain that  $\delta_{n-1} > \delta_n$  if and only if

$$\left\{ \sum_{j=1}^n (\overline{f}_j - \underline{f}_j) \right\} \cdot \left\{ \underline{f}_n \overline{f}_{n-1} - \overline{f}_n \underline{f}_{n-1} \right\} > 0,$$

which is true since  $L_{n-1} > L_n$  and  $\sum_{j=1}^n (\overline{f}_j - \underline{f}_j) > 0$  for any  $n < l$ .  $\square$

Theorem 2 implies that once  $\delta$  exceeds the cutoff  $\delta_n$  defined in equation (3.9), there is no equilibrium in constant threshold strategies where agents use threshold  $n$  across all histories and private signals. This seems to suggest that once we focus on equilibria in constant threshold strategies, the average amount of information an agent receives is discontinuous in  $\delta$  in the sense that the average amount of information drops at the cutoff values of  $\delta$ .

Proposition 2 shows that there is no such discontinuity. Focusing on the most informative equilibrium in constant threshold strategies, i.e., the one with the highest disclosure threshold in terms of the signal index, the average amount of information an agent receives, measured by difference in Kullback–Leibler divergence between the agent’s prior and his belief after observing his history (but before observing his private signal), is continuous and strictly increasing in  $\delta$ .

Formally, let  $\mathbf{h}_\delta^{(n)} \in H$  denote the disclosed history observed by an agent in the steady state when all agents use constant threshold strategy  $n$ , which is a random variable. For any  $\delta \in (\delta_{n+1}, \delta_n]$ , let  $\mathbf{h}_\delta \equiv \mathbf{h}_\delta^{(n)}$ , so that  $\mathbf{h}_\delta \in H$  is the disclosed history observed by an agent in the steady state when we select the equilibrium with the largest threshold  $n$  that exists for each  $\delta$ . Theorem 2 implies  $\mathbf{h}_\delta$  is well-defined. We denote the difference in the expected Kullback–Leibler divergence between an agent’s prior belief about the state and his posterior belief by

$$I(\theta; \mathbf{h}_\delta) = g(\pi_0) - \mathbb{E}[g(\Pr(\theta = \bar{\theta} | \mathbf{h}_\delta))], \quad (3.10)$$

where  $g(p) \equiv -p \log p - (1-p) \log(1-p)$  is the entropy function. We refer to  $I(\theta; \mathbf{h}_\theta)$  as *mutual information*. Intuitively,  $I(\theta; \mathbf{h}_\theta)$  is greater when agents learn more about the state from their histories. Proposition 2 shows that the mutual information is continuous and strictly increasing in  $\delta$ , even when the number of signals disclosed changes at the cutoff values of  $\delta$ .

**Proposition 2.** *Focusing on the most informative equilibrium in constant threshold strategies,  $I(\theta; \mathbf{h}_\delta)$  is a continuous and strictly increasing function of  $\delta$ .*

*Proof.* For any  $n \geq 2$  fix  $\delta' < \delta$  so that  $\delta', \delta \in (\delta_{n+1}, \delta_n]$ . We first show that  $I(\theta; \mathbf{h}_{\delta'}) \leq I(\theta; \mathbf{h}_\delta)$ . Observe that both  $\mathbf{h}_\delta$  and  $\mathbf{h}_{\delta'}$  involve disclosing signals  $\{s_1, \dots, s_n\}$ . We refer to the sequence of signals after the last communication failure (including the ones concealed) as the *complete history* of game. Since the expected length of the complete history in the steady state equals  $\delta/(1-\delta)$ , the expected number of signals disclosed is increasing in  $\delta$ . So if  $\delta' < \delta$  the random variable  $\mathbf{h}_{\delta'}$  can be represented as a garbling of the random variable  $\mathbf{h}_\delta$  and hence  $I(\theta; \mathbf{h}_{\delta'}) \leq I(\theta; \mathbf{h}_\delta)$ . Observe also that for such  $\delta$ ,  $I(\theta; \mathbf{h}_\delta)$  varies continuously since  $\delta/(1-\delta)$  is continuous in  $\delta$ .

Fix any cutoff  $\delta_n$ . By definition, signals  $\{s_1, \dots, s_n\}$  are disclosed in the most informative constant threshold equilibrium when  $\delta$  is slightly below  $\delta_n$  and signals  $\{s_1, \dots, s_{n-1}\}$  are disclosed in the most informative constant threshold equilibrium when  $\delta$  is slightly above  $\delta_n$ . At the cutoff  $\delta_n$ , the incentive constraint to disclose signal  $n$  is binding, so that by equation (3.8) we have

$$L_n \cdot \frac{1 - \delta_n(1 - \underline{f}_1 - \dots - \underline{f}_n)}{1 - \delta_n(1 - \overline{f}_1 - \dots - \overline{f}_n)} = 1.$$

But because of the binding constraint, disclosing an additional realization of  $s_n$  does not change the likelihood ratio of the public belief. Hence, when  $\delta = \delta_n$ , we have  $\pi(h_1, \dots, h_{n-1}, 0, \dots, 0) = \pi(h_1, \dots, h_{n-1}, h_n, 0, \dots, 0)$  for any  $h_n \in \mathbb{N}$  and  $(h_1, \dots, h_{n-1}) \in \mathbb{N}^{n-1}$ . Therefore, we have  $I(\theta; \mathbf{h}_{\delta_n}^{(n)}) = I(\theta; \mathbf{h}_{\delta_n}^{(n-1)}) = I(\theta; \mathbf{h}_{\delta_n})$  and  $I(\theta; \mathbf{h}_\delta)$  is continuous at  $\delta = \delta_n$ .  $\square$

We conclude this section by discussing our refinements and comparing them with the ones in Guttman, Kremer, and Skrzypacz (2014), Hart, Kremer, and Perry (2017), and Rappoport (2025). First, Guttman, Kremer, and Skrzypacz (2014) define *monotone equilibria* and *threshold equilibria* when a sender can receive up to two signals in two periods and can decide what to disclose and when. We view our notions of monotone beliefs and threshold strategies as natural extensions of their refinements to our setting where the sender can have any number of signals.

Second, our threshold strategy refinement and our monotone belief refinement do not imply one another. For example, strategies where agents disclose only one realization of a signal even

when multiple of them are available are not threshold strategies but are allowed in equilibria with monotone belief maps. Agents' incentives to use a threshold strategy  $J : H \times S \rightarrow H$  imply that they have no incentive to conceal any signal with an index below  $J(\mathbf{h}, s)$  at any  $(\mathbf{h}, s)$ . Deviations where they conceal some of those signals will also reduce the length of the next agent's history (conditional on communication succeeds). The monotonicity refinement on the belief map has no bite in ruling out such deviations as it only imposes restrictions on histories with the same length.

Third, as long as  $L_2 > 1$  and  $\delta$  is large enough, both of our refinements conflict with the *truth leaning refinement* in [Hart, Kremer, and Perry \(2017\)](#). In our setting, their refinement requires that (A0) an agent will disclose all of his information if doing so maximizes his payoff and (P0) agents believe that their predecessors did not hide any information upon observing any  $\mathbf{h} \in H$  that occurs with zero probability in equilibrium. See page 700 of their paper for details.

We explain why there is no strict equilibrium that satisfies both our threshold strategy refinement and their (P0) requirement. The conflict between our monotone belief refinement and their (P0) requirement follows from the same logic. From [Corollary 1](#), we know that if  $\delta$  is large, then in every strict equilibrium in threshold strategies, agents will disclose  $\mathbf{0}$  when their history is  $\mathbf{0}$  and their private signal is  $s_2$ . According to Bayes rule, their belief about the state upon observing  $\mathbf{0}$  is no more than their prior  $\pi_0$ . If such an equilibrium also satisfies (P0), then given that  $\mathbf{1}_2$  occurring with zero probability,  $\pi(\mathbf{1}_2)$  must equal the agent's posterior belief when they know that their predecessor's history is  $\mathbf{0}$  and their private signal is  $s_2$ . Since  $L_2 > 1$ , this leads to a posterior belief that is strictly greater than the posterior belief from disclosing nothing, which contradicts our earlier conclusion that agents will disclose  $\mathbf{0}$  upon receiving private signal  $s_2$  at history  $\mathbf{0}$ .

Fourth, our refinements are neither stronger nor weaker relative to the receiver's optimal equilibrium, which is the refinement used in [Rappoport \(2025\)](#). First, the equilibrium where the sender never discloses any signal satisfies both of our refinements. However, it is not the receiver-optimal equilibrium as the one where the sender only discloses  $s_1$  leads to higher receiver welfare. Second, suppose  $L_1 > L_2 > 1 > \dots$  but  $\overline{f}_1$  and  $\underline{f}_1$  are much smaller relative to  $\overline{f}_2$  and  $\underline{f}_2$ . When  $\delta$  is close to 1, only signal  $s_1$  can be disclosed in any strict equilibrium that satisfies any of our two refinements, whereas the receiver is strict better off in the equilibrium where only signal  $s_2$  is disclosed. The latter equilibrium exists for any  $\delta$  according to our [Proposition 1](#).

## 4 Proof of Theorem 1

We begin by stating several useful properties of strict equilibria of our game. We start from a gradualism property.

**Gradualism Property.** *For any  $t \in \mathbb{Z}$ , period  $t$  history  $\mathbf{h}^t \equiv (h_1^t, \dots, h_l^t)$ , and period  $t + 1$  history  $\mathbf{h}^{t+1} \equiv (h_1^{t+1}, \dots, h_l^{t+1})$  that comes after  $\mathbf{h}^t$ , there exists at most one  $i \in \{1, 2, \dots, l\}$  such that  $h_i^{t+1} > h_i^t$ . If such an  $i$  exists, then  $h_i^{t+1} = h_i^t + 1$ .*

Intuitively, this property is driven by our modeling assumption that each agent receives only one private signal in addition to the signals passed on to him by his predecessor.

Second, when an agent decides what information to disclose, his objective is to maximize his immediate successor's belief. Since he has the option to ignore his private signal and reveal his history  $\mathbf{h}$ , from which he can induce the same public belief when communication succeeds, the history he discloses in equilibrium must lead to a weakly higher belief compared to  $\pi(\mathbf{h})$ . This leads to an *improvement principle* similar to the one in [Banerjee and Fudenberg \(2004\)](#).

**Improvement Principle.** *For any equilibrium and any two adjacent periods  $t$  and  $t + 1$ , if the realized history in period  $t$  is  $\mathbf{h}^t$  and the history disclosed by period  $t$  agent is  $\mathbf{h}^{t+1}$ , then  $\pi(\mathbf{h}^{t+1}) \geq \pi(\mathbf{h}^t)$ . If this equilibrium is strict, then  $\pi(\mathbf{h}^{t+1}) > \pi(\mathbf{h}^t)$  as long as  $\mathbf{h}^{t+1} \neq \mathbf{h}^t$ .*

An implication of our improvement principle is the following *no-turning-back property*.

**No-Turning-Back Property.** *For any strict equilibrium, any history  $\mathbf{h}$  that occurs with positive probability in this equilibrium, and any  $\mathbf{h}' < \mathbf{h}$ , history  $\mathbf{h}'$  will not occur with positive probability after history  $\mathbf{h}$  but before the next communication failure.*

This is because agents can disclose any  $\mathbf{h}' < \mathbf{h}$  as long as they can disclose  $\mathbf{h}$ , so their incentive to disclose  $\mathbf{h}$  in a strict equilibrium implies that  $\pi(\mathbf{h}) > \pi(\mathbf{h}')$ . Iteratively apply the improvement principle, we know that no one will disclose  $\mathbf{h}'$  after  $\mathbf{h}$  until communication fails.

Using these properties, one can see why [Corollary 1](#) is a direct implication of [Theorem 1](#). Since only  $s_1$  can be disclosed when  $\delta$  is large enough and any strict equilibrium needs to respect the no-turning-back property, there are only two threshold strategies that meet both requirements: one where no signal is disclosed and one where agents disclose all realized  $s_1$ .<sup>8</sup>

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<sup>8</sup>The no-turning-back property rules out strategies where at histories with a positive number of  $s_1$ , agents set the disclosure threshold to be 0, i.e., conceal all realized signals.

Strict equilibria with monotone belief maps turn out to be more permissive: any strategy where agents disclose up to  $n$  realizations of  $s_1$  can be part of such equilibria as long as the belief map  $\pi$  assigns a low value at any history with length more than  $n$ .

We now return to the proof of Theorem 1. Fix any strict equilibrium  $(\sigma, \underline{\mu}, \bar{\mu}, \pi)$ . For convenience, we write  $\mu_\theta$  as the probability measure over histories induced by  $\sigma$  conditional on state  $\theta$ . Let  $H_\sigma \subset H$  denote the set of histories that occur with positive probability under  $\underline{\mu}$ . Let  $j$  denote the largest integer in  $\{0, 1, \dots, l\}$  such that there exists  $\mathbf{h} \equiv (h_1, \dots, h_l) \in H_\sigma$  with  $h_j \geq 1$ . Our gradualism property implies that there exists  $\mathbf{h} \in H_\sigma$  with  $h_j = 1$ .

Let  $H(j) \subset H_\sigma$  denote the set of positive-probability histories under  $\sigma$  such that the  $j$ th entry is 1. Let  $H(j-1) \subset H(j)$  denote the subset of  $H(j)$  such that a history  $\mathbf{h}$  belongs to  $H(j-1)$  if and only if it minimizes  $h_1 + \dots + h_{j-1}$  among all vectors in  $H(j)$ . Iteratively define  $H(j-2), \dots, H(1)$  with  $H(1) \subset H(2) \subset \dots \subset H(j-2) \subset H(j-1)$  such that for every  $i \in \{1, 2, \dots, j-2\}$ ,  $\mathbf{h}$  belongs to  $H(i)$  if and only if it minimizes  $h_1 + \dots + h_i$  among all vectors in  $H(i+1)$ . This leads to a unique vector in  $H(1)$ , which we denote by  $\mathbf{h}^* \equiv (h_1^*, \dots, h_l^*)$ .

**Equilibria in Threshold Strategies:** Suppose for some  $\delta$  is close enough to 1, there exists a strict equilibrium where agents use threshold strategy  $\sigma$  under which  $j \geq 2$ . We establish the following lemma, which shows that  $\mathbf{h}^*$  can only originate from  $\mathbf{h}^* - \mathbf{1}_j$ .

**Lemma 1.** *Suppose in some strict equilibrium, agents use a threshold strategy. If an agent's history is  $\mathbf{h}^*$ , then his immediate predecessor's history is either  $\mathbf{h}^*$  or  $\mathbf{h}^* - \mathbf{1}_j$ .*

The proof is in Appendix B.

Next, consider the agent's disclosure behavior at history  $\mathbf{h}^*$ . Since the equilibrium is strict,  $\pi(\mathbf{h}^*) > \pi(\mathbf{h})$  for any  $\mathbf{h} < \mathbf{h}^*$ . The definition of  $j$  implies that upon receiving any signal  $s_k$  with  $k > j$  at  $\mathbf{h}^*$ , the disclosure threshold must be  $j$ , i.e., signals with index more than  $j$  are all concealed and those with index no more than  $j$  are all revealed. Upon receiving any signal  $s_i$  with  $i \leq j$  at  $\mathbf{h}^*$ , the history in the next period will not be  $\mathbf{h}^*$  regardless of the threshold the agent uses. Hence, conditional on state  $\theta$  and the current-period history being  $\mathbf{h}^*$ , the probability with which history remains at  $\mathbf{h}^*$  in the next period is  $\delta(1 - \sum_{i=1}^j f_{\theta,i})$ .

Recall from Lemma 1 that when an agent's history is  $\mathbf{h}^*$ , his immediate predecessor's history is either  $\mathbf{h}^*$  or  $\mathbf{h}^* - \mathbf{1}_j$ . The probability with which history  $\mathbf{h}^*$  occurs conditional on state  $\theta$  is

$$\mu_\theta(\mathbf{h}^*) = \mu_\theta(\mathbf{h}^* - \mathbf{1}_j) \cdot \delta f_{\theta,j} \cdot \sum_{k=0}^{\infty} \delta^k \left(1 - \sum_{i=1}^j f_{\theta,i}\right)^k$$

$$= \mu_\theta(\mathbf{h}^* - \mathbf{1}_j) \cdot \frac{\delta f_{\theta,j}}{1 - \delta(1 - \sum_{i=1}^j f_{\theta,i})}. \quad (4.1)$$

After receiving private signal  $s_j$  at history  $\mathbf{h}^* - \mathbf{1}_j$ , an agent has an incentive to disclose history  $\mathbf{h}^*$  to his successor only if  $\pi(\mathbf{h}^*) \geq \pi(\mathbf{h}^* - \mathbf{1}_j)$ , which according to (4.1), translates into

$$L_j \cdot \frac{1 - \delta(1 - \sum_{i=1}^j \frac{f_i}{f_i})}{1 - \delta(1 - \sum_{i=1}^j \frac{f_i}{f_i})} \geq 1. \quad (4.2)$$

When  $\delta$  is close to 1, the above inequality fails for any  $j \geq 2$ , which leads to a contradiction.

**Equilibria with Monotone Belief Maps:** Suppose for some  $\delta$  is close enough to 1, there exists a strict equilibrium with a monotone belief map  $\pi$  under which  $j \geq 2$ . The monotonicity of  $\pi$  implies that  $\pi(\mathbf{h}^*) \leq \pi(\mathbf{h})$  for any  $\mathbf{h} \in H(j-1)$ . We establish the following lemma.

**Lemma 2.** *Suppose the belief map is monotone in some strict equilibrium. If an agent's history is  $\mathbf{h}^*$ , then his immediate predecessor's history is either  $\mathbf{h}^*$  or  $\mathbf{h}^* - \mathbf{1}_j$ .*

The proof is in Appendix C.

For every  $\mathbf{h} \in H_\sigma$ , let  $D(\mathbf{h}) \subset \{1, 2, \dots, l\}$  be such that  $i \in D(\mathbf{h})$  if and only if the disclosed vector after receiving private signal  $s_i$  at history  $\mathbf{h}$  is not weakly dominated by  $\mathbf{h}$ , or equivalently, the number of signal  $s_i$  disclosed is more than the  $i$ th entry of  $\mathbf{h}$ . Let  $\bar{d}(\mathbf{h}) \equiv \max D(\mathbf{h})$ .

Since the equilibrium is strict, we know that  $\pi(\mathbf{h}) > \pi(\mathbf{h}')$  for every  $\mathbf{h} \in H_\sigma$  and  $\mathbf{h}' < \mathbf{h}$ . Hence, if the agent receives private signal  $s_i$  at history  $\mathbf{h}$  with  $i \notin D(\mathbf{h})$ , he will disclose history  $\mathbf{h}$  to his immediate successor, so the history will remain the same as long as communication succeeds. This together with Lemma 2 leads to

$$\mu_\theta(\mathbf{h}^*) = \mu_\theta(\mathbf{h}^* - \mathbf{1}_j) \cdot \delta f_{\theta,j} \cdot \sum_{n=0}^{\infty} \delta^n (1 - \sum_{i \in D(\mathbf{h}^*)} f_{\theta,i})^n = \mu_\theta(\mathbf{h}^* - \mathbf{1}_j) \cdot \frac{\delta f_{\theta,j}}{1 - \delta(1 - \sum_{i \in D(\mathbf{h}^*)} f_{\theta,i})}. \quad (4.3)$$

Equation (4.3) implies that  $\pi(\mathbf{h}^*) \geq \pi(\mathbf{h}^* - \mathbf{1}_j)$  if and only if

$$L_j \cdot \frac{1 - \delta(1 - \sum_{i \in D(\mathbf{h}^*)} \frac{f_i}{f_i})}{1 - \delta(1 - \sum_{i \in D(\mathbf{h}^*)} \frac{f_i}{f_i})} \geq 1. \quad (4.4)$$

When  $\delta$  is above some cutoff, inequality (4.4) is true only if there exists  $i \in D(\mathbf{h}^*)$  such that  $i > j$ , which contradicts the definition of  $j$  under which there exists no  $\mathbf{h} \in H_\sigma$  such that  $h_i \geq 1$  for any  $i > j$ . When  $\delta$  is above some cutoff, this contradiction rules out strict equilibria with monotone belief maps where signals other than  $s_1$  are disclosed.

## 5 Application: Beliefs in a Just World

We now present an application of our model to the problem of influencing future generations' beliefs about the returns to effort in the style of [Benabou and Tirole \(2006\)](#). The key difference is that in our model, the strategic choice of the old generation is which verifiable signals to disclose to the young. This is relevant because in our overlapping generations setting, there are potentially many signals that are available for transmission, which can be interpreted as various anecdotes that they have experienced or have heard from their predecessors.

In each period  $t \in \mathbb{Z}$ , there is a unit mass of young agents and a unit mass of old agents. A representative young agent works in period  $t$  by choosing an action  $a^t \in \mathbb{R}$  given some private signal  $s^t \in S$  and with probability  $\delta \in (0, 1)$ , the signals disclosed to him by the old agent, i.e., the agent who was young in period  $t - 1$ .<sup>9</sup> The signals are informative about a constant state of the world  $\theta \in \{\underline{\theta}, \bar{\theta}\}$ , which captures the true returns to effort. We assume that  $0 < \underline{\theta} < \bar{\theta} < 1$ . The prior probability that  $\theta = \bar{\theta}$  is  $\pi_0 \in (0, 1)$ . The old agent chooses which signals from his history to disclose to the young agent. Within each period  $t$  the timing is as follows.

1. The old agent begins period  $t$  with a history of verifiable signals. He chooses which subset of these signals to disclose to the young agent.
2. The young agent observes the disclosed history with probability  $\delta$  and then privately observes a new verifiable signal  $s^t \in S$  about the state  $\theta$ .
3. The young agent chooses effort  $a^t \in \mathbb{R}$ .
4. Output  $y^t \in \{0, 1\}$  is realized. Redistribution and consumption occur according to some exogenous redistribution rule.
5. The young agent becomes the old agent in the next period. He has access to his own private signal and with probability  $\delta$ , the signals passed on to him from the older generation.

As in [Piketty \(1995\)](#) and [Benabou and Tirole \(2006\)](#), a representative young agent in period  $t$  produces output according to the following production technology

$$y^t = \begin{cases} 1 & \text{with probability } \theta a^t \\ 0 & \text{with probability } 1 - \theta a^t \end{cases},$$

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<sup>9</sup>When not in the informative e interpret this as a continuum of dynasties where the old agent in a dynasty passes on information at time  $t$  to the young agent in that dynasty.

where  $a^t$  is the level of effort chosen by young agent in period  $t$ .

There is a fixed tax rate denoted by  $\tau$  in the economy, and everybody gets a fixed, basic transfer  $b$ . The period  $t$  young agent obtains utility

$$u(y^t(1 - \tau) + b) - \frac{1}{2}(a^t)^2,$$

where  $\frac{1}{2}(a^t)^2$  is the cost of effort.<sup>10</sup> Each old agent obtains utility  $u(b) + \lambda v(y^t)$ , where  $\lambda > 0$  captures the old agents' altruism motives in the sense that they benefit from the consumption of their children. We assume that  $u(\cdot)$  is strictly increasing and concave, so that agents are risk-averse. Each agent's *lifetime utility* equals the sum of his utilities across the two periods.

A young agent's belief about  $\theta$ , before observing his private signal, but after seeing the history  $\mathbf{h} \in H$  disclosed by the old agent, is denoted by  $\pi(\mathbf{h})$ . Denote by  $\pi(\mathbf{h}, s_i)$  the young agent's posterior belief about  $\theta$  after he also observes private signal  $s_i$ , so that

$$\pi(\mathbf{h}, s_i) = \frac{\pi(\mathbf{h}) \overline{f}_i}{\pi(\mathbf{h}) \overline{f}_i + (1 - \pi(\mathbf{h})) \underline{f}_i}.$$

Let  $\widehat{\theta}(\pi) = \pi \bar{\theta} + (1 - \pi) \underline{\theta}$  denote the expected value of the state conditional on an agent having posterior belief  $\pi$ . We let  $\widehat{\theta}(\mathbf{h}, s) = \widehat{\theta}(\pi(\mathbf{h}, s))$  denote the agent's expectation of the state after he observes the disclosed history as well as his private signal.

Fix a communication success rate  $\delta \in (0, 1)$  and a constant threshold strategy with cutoff  $j \in \{0, 1, \dots, l\}$ , so that the stationary distributions of histories conditional on states  $\bar{\theta}$  and  $\underline{\theta}$  are given by  $\bar{\mu}$  and  $\underline{\mu}$ , respectively. We later confirm that these strategies indeed form an equilibrium.

Given information  $(\mathbf{h}, s)$ , the young agent in period  $t$  chooses  $a^t$  to maximize expected utility

$$\max_{a^t \in \mathbb{R}} \widehat{\theta}(\mathbf{h}, s_i) a^t u(1 - \tau + b) + \left(1 - \widehat{\theta}(\mathbf{h}, s_i) a^t\right) u(b) - \frac{1}{2}(a^t)^2.$$

Let the difference in consumption utility between success and failure be  $\Delta c(b) = u(1 - \tau + b) - u(b)$ . The first-order condition for an interior solution implies the following linear rule in choosing effort

$$a^t(\mathbf{h}, s_i) = \widehat{\theta}(\mathbf{h}, s_i) \Delta c(b). \tag{5.1}$$

In a stationary, symmetric equilibrium all young agents take an action that is a function of  $(\mathbf{h}, s_i)$ . We assume that the law of large numbers holds for the continuum and that the distribution of agent histories and signals in period  $t$  matches the stationary steady-state distribution

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<sup>10</sup>Our conclusions in this section extend qualitatively to more general convex effort cost functions.

so that the stationary expected income in the economy is

$$\begin{aligned} Y &= \mathbb{E}_{\theta,(\mathbf{h},s_i)} \left[ \widehat{\theta}(\mathbf{h}, s_i) a_k^t(\mathbf{h}, s_i) \right] \\ &= \mathbb{E}_{\theta,(\mathbf{h},s_i)} \left[ \widehat{\theta}(\mathbf{h}, s_i)^2 \right] \Delta c(b), \end{aligned}$$

where conditional on the states  $\bar{\theta}$  and  $\underline{\theta}$ , the distributions of histories  $\mathbf{h}$  are  $\bar{\mu}$  and  $\underline{\mu}$ , respectively. We also assume that government transfers are ex-ante budget balanced so that  $\tau Y = 2b + \psi x$ , where  $x$  is the amount spent on public goods and, like the tax rate  $\tau$ , is exogenous.

The fact that government budgets are ex-ante balanced means that the value of the basic transfer  $b$  does not provide any additional information about the state of the world to the agents.

Let  $\kappa = \mathbb{E}_{\theta,(\mathbf{h},s_i)} \left[ \widehat{\theta}(\mathbf{h}, s_i)^2 \right]$  and observe that since  $b = (\tau Y - \psi x) / 2$ , the stationary expected income  $Y$  satisfies the following equation in equilibrium

$$Y = \kappa \Delta c \left( \frac{\tau Y - \psi x}{2} \right). \quad (5.2)$$

Because  $u$  is strictly concave, the right-hand side is decreasing in  $Y$  while the left-hand side is strictly increasing. Hence, there is at most one fixed point. Furthermore, since at  $Y = 0$ , the left-hand side is smaller, while at  $Y = \max \{1, \kappa u(1 - \tau)\}$ , the left-hand side is bigger than the right-hand side. Thus, the intermediate value theorem implies the existence of a fixed point for  $Y$  that satisfies (5.2).

At this value of  $Y$ , we have preferences for the agents that satisfy the assumptions of our model from Section 2. Observe that period  $t$  effort by the young agents is increasing in their belief that the state  $\theta = \bar{\theta}$ , and that in turn the old generation's welfare is increasing in this effort since  $Y$  is increasing in  $a^t$ . Note that since agents are atomistic, if  $\lambda = 0$  an old agent is not directly impacted by the actions of the young agent from the same dynasty. If  $\lambda > 0$ , the old agent in period  $t$  can be thought of as caring about the young agent's utility but discounting the effort cost, as in [Benabou and Tirole \(2006\)](#), or just enjoying a fraction of the young person's income. These preferences motivate the old agent to convince the young that returns to effort are high, which is what we assumed in Section 2.

Proposition 1 and Theorem 2 then imply that an equilibrium in constant threshold strategies exists for all  $\delta$ . We let  $j$  denote a threshold that can arise in one of those equilibria and note that at this value of  $\delta$  and  $j$ , the above effort choices are consistent with the average income  $Y$  that balances ex ante the government budget via the above fixed-point argument.

Note that this setup shares a key feature of [Piketty \(1995\)](#) and [Benabou and Tirole \(2006\)](#), namely that holding fixed the fundamentals (such as the signal structure  $\mathbf{f}$ , agents' preferences, and prior belief  $\pi_0$ ), there are multiple steady-state “beliefs”, which are due to different possible equilibria (i.e., intergenerational disclosure conventions. In particular, there is always an uninformative steady-state equilibrium in which old agents disclose no signals (i.e., the threshold is  $j = 0$ ). In this equilibrium, young agents' public belief remains at the prior and their effort responds only to the privately observed signal. Proposition 1 implies that for any  $\delta$  there also exists a steady-state equilibrium in which the only signals ever transmitted are the most favorable ones (i.e., the threshold is  $j = 1$ ). For lower  $\delta$ , Theorem 2 shows that additional equilibria with higher cutoffs ( $j > 1$ ). These equilibria correspond to the [Benabou and Tirole \(2006\)](#) discussion of the United States versus Europe. That is, a society in which the norm is that elders pass on only the strongest evidence supporting high returns to effort has more dispersed equilibrium beliefs and higher equilibrium effort (a belief in the “American Dream”), than a society with more muted disclosure norms.

**Comparative Statics on  $\delta$ :** Consider a planner who cares about maximizing the sum of utilities of the young and old agent. When writing the planner's problem, we ignore the preferences for the old over the income of the young by considering the case where  $\lambda = 0$ . Old agents in society would still want to convince the young that returns to effort are high, in order to motivate effort and have a higher basic transfer, if there was a finite number of dynasties.

Like in [Benabou and Tirole \(2006\)](#), there is a natural rate of information recall,  $\delta_0$ , but this could be increased at a cost  $x$ . Earlier in Section 5, we saw that  $x$  was the amount spent on provision of public goods and we interpret it now as the amount spent on education. Spending more on education increases the effective  $\delta$  in society. We assume that  $\delta(x) = 1 - (1 - \delta_0)e^{-x}$ .

The planner's problem is to choose  $\delta$  to maximize

$$\begin{aligned} \max_{\delta} \quad & \mathbb{E}_{\theta, (\mathbf{h}, s_i)} \left[ u((1 - \tau)y(\mathbf{h}, s_i) + b) - \frac{1}{2} a(\mathbf{h}, s_i)^2 + u(b) \right] \\ \text{s.t.} \quad & a(\mathbf{h}, s_i) = \widehat{\theta}(\mathbf{h}, s_i) \Delta c(b), \\ & y(\mathbf{h}, s_i) = \theta a(\mathbf{h}, s_i), \\ & b = \frac{\tau Y - \psi x}{2}, \quad \text{where } Y = \mathbb{E}_{\theta, (\mathbf{h}, s_i)} \left[ \widehat{\theta}(\mathbf{h}, s_i)^2 \right] \Delta c(b). \end{aligned}$$

With  $\psi = 0$ , the planner is free to adjust  $\delta$  in any way she likes. Because expected output  $Y$  is increasing in  $\kappa$  and thus the variance in the agents posterior over  $\theta$ , Proposition 2 implies that

the planner always prefers a strictly larger  $\delta$ . For sufficiently high  $\psi$ , the optimal choice of the planner is to leave  $\delta$  at  $\delta_0$ . While for intermediate cases, some intermediate  $\delta$  will be optimal.

## 6 Concluding Remarks

This paper studies the intergenerational transmission of verifiable information in an overlapping generations model that incorporates both physical communication frictions as well as agents' strategic incentives to conceal unfavorable information. We show that lowering the communication frictions will make the agents more selective in disclosing information. This is because disclosing an additional good signal has two effects on the next agent's belief: a direct effect which shows the existence of another good signal as well as an indirect effect which shows the existence of another subsequence of bad signals. As communication frictions vanish, the expected length of each bad subsequence increases. Hence, the indirect effect will become stronger, so that fewer signals can have high enough likelihood ratios to overcome the more pronounced indirect effect. The indirect effect is absent in [Dye \(1985\)](#)'s model where increasing the probability that the sender has information makes his disclosure less selective.

A natural question is whether our conclusions extend to a one-shot disclosure game between a sender and a receiver, where the number of signals the sender has follows a geometric distribution with parameter  $\delta$ . For equilibria in constant threshold strategies considered in [Theorem 2](#), our conclusion that agents become more selective in disclosing information as  $\delta$  increases will extend. However, it is unclear whether [Theorem 1](#) extends to this one-shot game, namely, when the equilibrium disclosure threshold could vary across histories and private signals or when the restriction on the equilibrium strategy is replaced with one on the belief map.

Intuitively, our overlapping generations model has a gradualism property stated in [Section 4](#) that makes the analysis tractable. It leads to [Lemma 1](#) and [Lemma 2](#), which makes it tractable to compute the change in the receiver's belief once the sender discloses one fewer signal  $s_j$ , the signal with the lowest likelihood ratio among those being disclosed.

In contrast, more possibilities could arise in the one-shot disclosure game, which makes the change in the receiver's belief harder to compute. For example, it could be the case that agents never disclose  $s_2$  when there is only one such signal available but will disclose all  $s_2$  when at least two such signals are available. This can be sustained by the off-path belief that upon observing only one  $s_2$ , the receiver believes that the sender has hidden many signals with low likelihood

ratios but will not entertain this adverse belief upon observing no  $s_2$  or two or more  $s_2$ .

Such a possibility is ruled out in our overlapping generations model since without any agent disclosing the first realization of  $s_2$  at histories that do not contain any  $s_2$ , no agent will have two  $s_2$  available to disclose. Understanding the connections between our overlapping generations model and the one-shot disclosure game is an important question left for future research.

## A Proof of Proposition 1

Suppose agents disclose no signal other than  $s_i$  and disclose up to  $n$  realizations of  $s_i$ . Then conditional on state  $\theta$ , for every  $m < n$ , the probability of history  $m\mathbf{1}_i$  is

$$(1 - \delta)\delta^m \cdot (1 - \delta(1 - f_{\theta,i}))^{-(m+1)} \cdot f_{\theta,i}^m. \quad (\text{A.1})$$

Therefore,

$$\frac{\pi(m\mathbf{1}_i)}{1 - \pi(m\mathbf{1}_i)} = \frac{\pi_0}{1 - \pi_0} \cdot \left( \frac{1 - \delta(1 - \underline{f}_i)}{1 - \delta(1 - \overline{f}_i)} \right)^{m+1} \cdot L_i^m. \quad (\text{A.2})$$

If  $L_i > 1$ , then the value of (A.2) is strictly increasing in  $m$  regardless of  $\delta$ . The probability of history  $n\mathbf{1}$  is

$$\sum_{k=n}^{\infty} (1 - \delta)\delta^k \cdot (1 - \delta(1 - f_{\theta,i}))^{-(k+1)} \cdot f_{\theta,i}^k, \quad (\text{A.3})$$

so

$$\begin{aligned} \frac{\pi(n\mathbf{1}_i)}{1 - \pi(n\mathbf{1}_i)} &\geq \frac{\pi_0}{1 - \pi_0} \cdot \min_{k \geq n} \frac{(1 - \delta)\delta^k \cdot (1 - \delta(1 - \overline{f}_i))^{-(k+1)} \cdot \overline{f}_i^k}{(1 - \delta)\delta^k \cdot (1 - \delta(1 - \underline{f}_i))^{-(k+1)} \cdot \underline{f}_i^k} \\ &\geq \frac{\pi_0}{1 - \pi_0} \cdot \frac{(1 - \delta)\delta^n \cdot (1 - \delta(1 - \overline{f}_i))^{-(n+1)} \cdot \overline{f}_i^n}{(1 - \delta)\delta^n \cdot (1 - \delta(1 - \underline{f}_i))^{-(n+1)} \cdot \underline{f}_i^n} = \frac{\pi_0}{1 - \pi_0} \cdot \left( \frac{1 - \delta(1 - \underline{f}_i)}{1 - \delta(1 - \overline{f}_i)} \right)^{n+1} \cdot L_i^n. \end{aligned} \quad (\text{A.4})$$

The right-hand-side of (A.4) is strictly greater than the value of (A.2) for any  $m < n$ . These calculations verified that regardless of  $\delta \in (0, 1)$ , there exists an equilibrium where agents only disclose up to  $n$  realizations of  $s_i$ . The resulting belief map  $\pi : H \rightarrow [0, 1]$  can be constructed so that  $\pi(\mathbf{h}) < \pi(\mathbf{0})$  for every  $\mathbf{h} \in H$  that occurs with zero probability under the agents' equilibrium strategy.

## B Proof of Lemma 1

Let  $\mathbf{h}' \equiv (h'_1, \dots, h'_l)$  denote his immediate predecessor's history and suppose by way of contradiction that  $\mathbf{h}' \notin \{\mathbf{h}^*, \mathbf{h}^* - \mathbf{1}_j\}$ . We consider three cases separately:

1. If  $h'_j = 0$ , then in order for history to evolve from  $\mathbf{h}'$  to  $\mathbf{h}^*$  in one period, the current-period private signal must be  $s_j$  and  $s_j$  needs to be disclosed after receiving it at history  $\mathbf{h}'$ . Since  $\sigma$  is a threshold strategy, the fact that  $s_j$  is disclosed after receiving private signal  $s_j$  at history  $\mathbf{h}'$  implies that existing signals  $s_1, \dots, s_{j-1}$  are all disclosed. This implies that  $\mathbf{h}' = (h_1^*, \dots, h_{j-1}^*, 0, \dots, 0)$ , which contradicts our hypothesis that  $\mathbf{h}' \neq \mathbf{h}^* - \mathbf{1}_j$ .

2. If  $h'_j = 1$ , then the construction of  $\mathbf{h}^*$  requires that  $\sum_{i=1}^{j-1} h'_i \geq \sum_{i=1}^{j-1} h_i^*$  and our hypothesis that  $\mathbf{h}' \neq \mathbf{h}^*$  implies that  $(h'_1, \dots, h'_{j-1}) \neq (h_1^*, \dots, h_{j-1}^*)$ . The two together imply that there exists  $i \in \{1, 2, \dots, j-1\}$  such that  $h'_i > h_i^*$ . This implies that after receiving some signal at history  $\mathbf{h}'$ , an agent will disclose signal  $s_j$  while concealing at least one realized  $s_i$  for some  $i < j$ . This contradicts the hypothesis that agents use a threshold strategy.
3. If  $h'_j \geq 2$ , then after receiving some signal at  $\mathbf{h}'$ , agents will conceal at least one realized  $s_j$  while reveal at least one  $s_j$ . This contradicts the hypothesis that  $\sigma$  is a threshold strategy.

## C Proof of Lemma 2

Suppose by way of contradiction that the realized history in period  $t-1$  was  $\hat{\mathbf{h}} \equiv (\hat{h}_1, \dots, \hat{h}_l)$ , which is neither  $\mathbf{h}^*$  nor  $\mathbf{h}^* - \mathbf{1}_j$ . We obtain a contradiction in three cases:

1. If  $\hat{h}_j = 0$ , then  $h_j^* - \hat{h}_j = 1$ , so the gradualism property implies that  $\sum_{i=1}^{j-1} \hat{h}_i \geq \sum_{i=1}^{j-1} h_i^*$ . Since  $\hat{\mathbf{h}} \neq \mathbf{h}^* - \mathbf{1}_j$ , we have  $(\hat{h}_1, \dots, \hat{h}_{j-1}) \neq (h_1^*, \dots, h_{j-1}^*)$ . Hence, there exists  $1 \leq i \leq j-1$  such that  $\hat{h}_i > h_i^*$ . This implies that after observing signal  $s_j$  at history  $\hat{\mathbf{h}}$ , the agent discloses  $s_j$  yet conceals at least one realization of  $s_i$ . When the belief map  $\pi$  is monotone, disclosing  $\mathbf{h}^*$  leads to a strictly lower belief relative to disclosing  $\mathbf{h}^* - \mathbf{1}_j + \mathbf{1}_i$ . The latter is feasible by construction. This contradicts the agent's incentive constraint.
2. If  $\hat{h}_j = 1$ , then the construction of  $\mathbf{h}^*$  implies that  $\sum_{i=1}^{j-1} \hat{h}_i \geq \sum_{i=1}^{j-1} h_i^*$ . Since  $\hat{\mathbf{h}} \neq \mathbf{h}^*$ , we have  $(\hat{h}_1, \dots, \hat{h}_{j-1}) \neq (h_1^*, \dots, h_{j-1}^*)$ , so there exists  $1 \leq i \leq j-1$  such that  $\hat{h}_i > h_i^*$ . Again, the monotonicity of the belief map  $\pi$  implies that disclosing  $\mathbf{h}^*$  leads to a strictly lower belief relative to disclosing  $\mathbf{h}^* - \mathbf{1}_j + \mathbf{1}_i$ . By construction, vector  $\mathbf{h}^* - \mathbf{1}_j + \mathbf{1}_i$  is also feasible for the agent, so disclosing  $\mathbf{h}^*$  violates his incentive constraint.
3. If  $\hat{h}_j \geq 2$ , then consider the sequence of agents' realized histories starting from the latest communication failure, which we normalize as period 0. Let  $\mathbf{h}(\tau)$  denote the realized history in period  $\tau$ . By definition,  $\mathbf{h}(0) = (0, 0, \dots, 0)$ ,  $\mathbf{h}(t-1) = \hat{\mathbf{h}}$ , and  $\mathbf{h}(t) = \mathbf{h}^*$ . The gradualism property implies the existence of  $\tau < t-1$  such that the  $j$ th entry of  $\mathbf{h}(\tau)$  is 1. Let  $\mathbf{h}(\tau) \equiv \mathbf{h}^{**} \equiv (h_1^{**}, \dots, h_{j-1}^{**}, 1, 0, \dots, 0)$ . The definition of vector  $\mathbf{h}^*$  implies that at least one of the following two cases is true. The first case is  $\sum_{i=1}^{j-1} h_i^{**} = \sum_{i=1}^{j-1} h_i^*$ , which by the

construction of history  $\mathbf{h}^*$ , implies that

$$\sum_{i=1}^k h_i^{**} \geq \sum_{i=1}^k h_i^* \text{ for every } i \in \{1, 2, \dots, j-1\}.$$

The second case is  $\sum_{i=1}^{j-1} h_i^{**} > \sum_{i=1}^{j-1} h_i^*$ , in which case there exists  $\mathbf{h} \equiv (h_1, \dots, h_{j-1}, 1, 0, \dots, 0)$  that is weakly less than  $\mathbf{h}^{**}$  such that

$$\sum_{i=1}^k h_i \geq \sum_{i=1}^k h_i^* \text{ for every } i \in \{1, 2, \dots, j-1\}.$$

The monotonicity of the belief map  $\pi$  then implies the existence of  $\mathbf{h} \leq \mathbf{h}^{**}$  that satisfies  $\pi(\mathbf{h}) \geq \pi(\mathbf{h}^*)$ . Since history  $\mathbf{h}^{**}$  occurs with positive probability in equilibrium, we have  $\pi(\mathbf{h}^{**}) \geq \pi(\mathbf{h})$  for every  $\mathbf{h} \leq \mathbf{h}^{**}$ . Therefore,  $\pi(\mathbf{h}^{**}) \geq \pi(\mathbf{h}^*)$ . However  $\mathbf{h}(t) = \mathbf{h}^*$  and  $\mathbf{h}(\tau) = \mathbf{h}^{**}$  with  $t > \tau$  along a realized path of histories, and a history in between them  $\mathbf{h}(t-1) = \hat{\mathbf{h}} \neq \mathbf{h}^{**}$ . The conclusion that  $\pi(\mathbf{h}^{**}) \geq \pi(\mathbf{h}^*)$  contradicts the improvement principle we established earlier, which requires that  $\pi(\mathbf{h}^{**}) < \pi(\mathbf{h}^*)$ .

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