DELEGATION WITH STRINGS ATTACHED: UNDERFUNDING A BIASED AGENT

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Abstract. We study optimal contracting between a principal and an agent who can contract on both the content of a policy and its implementation scale. We assume content and scale are not separable, as the value of expanding the scale of implementation increases when the policy is close to a player’s preferred policy. In the optimal separating contract, an agent with an upward policy bias can only choose higher policies by reducing the scale of implementation. The solution differs qualitatively from standard quasilinear models and is ex-post inefficient, as the highest policies are too low for both parties and are under-implemented.

1. Introduction

A central problem in economics is how to design institutions and organizations so that agents have incentives to share relevant information with the actors that are in charge of making decisions. The effort to cope with this informational asymmetry fundamentally defines the relationships between investors and asset managers, CEOs and lower level managers, and politicians and bureaucrats. The problem is particularly challenging when the principal cannot use transfers to alleviate incentive problems, as it is often the case in politics, and in interactions among economic agents within firms and non-profit organizations.

Some variants of this problem are well understood. This is the case, for example, when the policy space over which the principal contracts with the agent is unidimensional. In this situation, a principal who can choose among a large space of contracts will simply delegate decision-making authority to the agent over a set of possible actions (Holmström

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The principal’s problem is then reduced to determining how much discretion to delegate to the agent.\footnote{Baron (2000) and Krishna and Morgan (2008) analyzed the problem in which the principal can contract over both policy and money transfers (where both principal and agent have quasilinear preferences over money), as in the standard screening problem (Baron and Myerson (1982)). With full commitment, the model here is a relatively standard screening model, except for a limited liability constraint on nonnegative transfers to the agent.}

The situation in which the principal is so seriously handicapped, however, is extreme. In fact, a common feature in many agency relationships is that the principal can decide not only the content of policy but also its scope of applicability, or scale of implementation. In the investor/asset manager context, for example, the investor can choose the level of risk for the portfolio, but also how much to invest with the asset manager. In politics, elected politicians must not only determine how “harsh” they will allow the CIA’s interrogation techniques to be, but also the scope of the so called black site operations (should they house only confirmed terrorist elites, or any suspect who might have relevant information).

A similar logic applies to resources dedicated to the monitoring and enforcement of a given policy. As chaotic traffic in many Latin American countries shows, strict traffic laws only matter if they are enforced. Similarly, environmental regulations and carbon pollution standards that aim to reduce greenhouse gas emissions can only be effective if the agency in charge of monitoring and enforcement (the EPA in the United States) is endowed with the resources necessary to accomplish these goals in the first place.

In this context, the principal can use the scale of the project in lieu of transfers, providing incentives by distorting both the scale and content of policy outcomes. This new distortion arises because the value of increasing the scale of the project is inexorably linked to the content of the underlying policy. Thus, differently to a transfer of “money”, the value of increasing the scale of the project will naturally be a function of how much each actor values the associated policy in each state of the world. If the asset manager uncovers an investment opportunity that can attain an extraordinarily high expected return with a larger risk, the investor will be willing to invest a larger fraction of its wealth in this high risk portfolio. If the CIA can in fact obtain actionable intelligence from detainees with an intense interrogation treatment, politicians will be more willing to allow the agency to apply these harsher techniques broadly.

In this paper we characterize the optimal institutional arrangement for the principal in this setting. While there has been an extensive literature on delegation, the solution to this class of problems has not yet been explored. Baron (2000) and Krishna and Morgan

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(2008) analyze the unidimensional policy space with transfers assuming quasilinear preferences and quadratic policy payoffs. The multidimensional case without transfers is less common in the literature; Koessler and Martimort (2012) study a two-dimensional policy space with separable quadratic payoffs, while Frankel (2014) considers an $N$-dimensional policy space with separable preferences and a non-Bayesian (max-min) principal. In our case, instead, the content and scope of policy are complements in the utility function of principal and agent. Thus, we have what Koessler and Martimort call “externalities across decisions”.

Multidimensional mechanism design problems are generally difficult to solve because the order in which incentive constraints bind is often endogeneous to the problem (Rochet and Stole (2003)). We appeal to a generalized single crossing condition which abstracts from this difficulty, yet still the standard techniques to deal with screening problems do not readily apply. In particular, with non-separable preferences across policy dimensions, the common procedure of reparametrizing the problem in terms of information rents is not helpful.3 In spite of this, we are able to make considerable progress. First, we solve the optimal contract in the two type case, and present a graphical analysis that makes the logic and results transparent. The graphical analysis also allows us to relate our results with the standard quasilinear setting easily. We then characterize the optimal separating contract in the continuum with a parametric assumption on payoffs (exponential payoffs).

Our analysis leads to a number of new insights. First, we show that whenever conflicts of interests are binding (always in the continuum; for sufficiently large bias in the two type case), the principal will overfund “low types” and underfund “high types”. In our environmental policy example, this says that when climate change is indeed occurring at a fast pace, the resources dedicated to enforcing regulations curbing carbon emissions are too low relative to the first-best: it is in this sense that the principal optimally chooses a toothless policy.

The possibility of tinkering with the scale of implementation induces distortions in the content of policy that can be different from what would result in a comparable model with quasilinear payoffs (i.e., with transfers in lieu of project size). The qualitative nature of these distortions depends on the level of conflict between the agent and the principal relative to the “smallest” possible deviations. When the agent’s bias is sufficiently large relative to the smallest possible deviation (which is always the case in the continuum) the distortions in policy direction in fact are different than what appear in a comparable model with quasilinear payoffs.

3In general, we are dealing with a singular control problem of the type that does not admit the sort of straightforward bang-bang solutions commonly used in other contexts (e.g., the continuous-time literature often exploits bang-bang solutions).
Indeed, both in the continuum and in the two type model with a large bias, the optimal separating contract partitions the state space in a “low” and “high” set of states, such that the principal overfunds and distorts towards the agent in low states, but underfunds and distorts against the agent in high states. In our EPA example, this implies that the optimal contract sets overly stringent regulations that are heavily enforced when climate change is mild, and relatively weak regulations which are under-enforced in the states in which climate change is accelerating more heavily. Thus, the solution is ex-post inefficient, as both Congress and the EPA would both prefer to set more stringent, heavily enforced environmental regulations in the high states. This strong form of ex-post inefficiency in the optimal contract does not appear in standard quasi-linear models where utility is perfectly transferable between parties.

The solution for the continuum illustrates that in choosing the optimal policy function the principal faces a tradeoff between inducing distortions in the content and the scale of implementation of the policy. This is because in order to make the content of policy responsive to the state of the world (reducing distortions in policy direction), the scale of implementation needs to be responsive to the state as well, increasing distortions in project size.

The particular resolution of this tradeoff depends crucially on the relative sensitivity to policy loss of the agent vis a vis the principal. When this ratio is low enough (when the agent does not care too much about policy losses relative to the principal) it is relatively cheap for the principal to compensate the agent with changes in project size to obtain a policy that is very close to her first best, and the optimal contract is fully separating. When the agent is very sensitive to policy losses, on the other hand, attaining a policy close to the first best is very costly for the principal, and the principal would rather take a relatively unresponsive policy than introduce large distortions in project size. In fact, for sufficiently high sensitivity of the agent to policy loss, the principal will be better off with a pooling contract.

The rest of the paper is organized as follows. We review the related literature in Section 2, and describe the model in Section 3. The main results are in Section 4. We begin in Section 4.1 with the two type case, and consider the model with a continuum of states in Section 4.2. In Section 5 we explore in detail a version of our model with quasilinear payoffs for comparison. We conclude in Section 6. All proofs are in the Appendix.
2. Related Literature

This paper contributes to the optimal delegation literature initiated by Holmström (1977). Holmström (1977) considers a problem in which an uninformed principal contracts with an informed agent over a unidimensional policy space, and the principal cannot use transfers. In this setting, the optimal mechanism for the principal makes policy either completely unresponsive to the agent’s type, or equal to his ideal point. This outcome can be achieved by simply delegating decision-making power to the agent over an appropriately chosen set of policies. Melumad and Shibano (1991), and Alonso and Matouschek (2008) then fully characterize the solution to the delegation problem in the absence of restrictions on feasible delegation sets. The optimal delegation set trades off the benefits of making the policy responsive to the state of the world against the loss of decision-making power to a biased agent. When the conflict of interest between principal and agent is sufficiently large the optimal contract is a pooling contract, in which the principal commits to a policy equal to her expected ideal point (delegation is not valuable).

Baron (2000) and Krishna and Morgan (2008) analyze the problem in which the policy space is unidimensional, but the principal can use transfers to alleviate incentive constraints, assuming quasilinear preferences and quadratic policy payoffs (see also Walsh (1995)). Krishna and Morgan (2008) show that in the solution policy outcomes are systematically distorted to favor the agent’s preferences. Thus, even if the principal can use transfers to fully align incentives, in the solution she will choose not to do this to the full extent. The unidimensional policy space with transfers is a natural benchmark for comparison with our model. We relegate this comparison to Section 5, where we develop

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4At a more general level, our work builds on the classic mechanism design and screening literature (see Laffont and Martimort (2009) for a review). Unlike almost all of the literature, we study a two-dimensional screening problem with non-separable preferences. In particular, we deviate from the standard setting with quasilinear preferences, in which one of the policy dimensions is a transfer of money.

5See Ambrus and Egorov (2012) and Amador and Bagwell (2012, 2013) for a variant of this problem with money burning.

6The optimal delegation literature presupposes that the principal can commit to a mechanism (or set of institutions) regulating her interaction with the agent. At the opposite extreme of the spectrum, Crawford and Sobel (1982) assume that the principal cannot commit to a policy choice. In this context, the principal will always choose her preferred policy given the information provided by the agent, and as a result cannot reward the agent with policy concessions after the agent reveals her information.

7The delegation solution has been studied extensively in political science in the context of congressional control of the bureaucracy and executive/executive/legislative relations (see Epstein and O’Halloran (1994), Huber and Shipan (2002)). Bendor and Meirowitz (2004) and Gailmard and Patty (2012) provide an overview of the theoretical literature in political science and a general framework for this family of models. In addition to these theoretical contributions, there is also a large empirical literature on congressional control of the bureaucracies, which often relies at least informally on the principal agent setup. See for example Weingast and Moran (1983) Wood and Anderson (1993), Wood and Waterman (1991), Carpenter (1996), Shipan (2004).
two alternative versions of the model with transfers in the two type case that are directly comparable to our model (one introducing an individual rationality constraint as in the standard screening problems, the other with nonnegative transfers, as in Krishna and Morgan (2008)).

Koessler and Martimort (2012) consider a two-dimensional policy space with separable quadratic payoffs where the agent has the same ideal point in each dimension. They show that interval delegation sets are generally not optimal in this setting, as the optimal decisions on each dimension are never equal to the agent’s ideal points. Two results are particularly relevant for comparison with the results in our paper. First, in this setting pooling is never optimal for the principal. This is true in the two-type version of our model, but not in the continuum. Second, as in our model, the optimal contract can be ex-post inefficient. While in the optimal contract the distance between the policy outcomes in each dimension increases for lower types to induce information revelation, the spread between outcomes at low types can be ex-post too large for both principal and agent.

We are aware of only two papers in political science which consider a model of the interaction of “budget” and policy choice in a principal-agent context. Both papers have fundamental differences with this paper. In particular, both of these papers posit a given sequence of play, and do not consider the optimal mechanism for the principal. In Ting (2001), the agency can choose a more right winged policy at a cost, which enters its quasi-linear utility function as a transfer. Congress initially chooses a budget for the agency and, after observing a signal of the agency’s choice of policy, an auditing level. In McCarty (2004), the agency needs resources to move policy away from the status quo. The President appoints the agent, while Congress chooses the agency’s budget, and thus effectively a range of discretion for the agency around the status quo.

3. The Model

A principal is to contract with an agent who has private information about a payoff-relevant state variable. An outcome \((y, m) \in \mathbb{R} \times \mathbb{R}^+ =: X\), comprises a “policy” \(y\), and a scale of implementation or scope \(m\). The policy \(y\) is the result of an action \(x \in \mathbb{R}\) and a random state variable \(\omega \in \Omega\), for a compact set \(\Omega \subset \mathbb{R}\). In particular, we let \(y = x - \omega\). It is common knowledge that \(\omega \sim F\), where we assume that \(\text{supp}(F) = \Omega\). However, the realization of the random state variable \(\omega\) is private information of the agent.

\(^8\)Banks (1989) considers a model in which an agency has private information about the cost of providing a service, while Congress decides the agency’s budget and whether to audit the agency or not.
Let $z_j$ denote $j$’s ideal policy, $j \in \{P, A\}$. Without loss of generality, we fix $z_P = 0$ and $z_A = b > 0$. We say that $b$ is the bias of the agent relative to the principal. The principal and agent have state-contingent preferences. For any action $x$, state $\omega$, and program size $m$, the principal’s payoff is $U^p(x, m|\omega) := u^p(\ell^p(x, \omega), m) - \gamma(m)$, and the agent’s payoff is $U^a(x, m|\omega) = u^a(\ell^a(x, \omega), m)$, where $\ell^j(x, \omega) := (x - \omega - z_j)^2$. We assume that $\gamma(\cdot)$ is increasing and convex, and that for $j = P, A$, (i) $u^j_x \leq 0$ and $u^j_m \leq 0$, (ii) $u^j_{xx} \geq 0$ and $u^j_{mm} \leq 0$, and (iii) $u^j_{mx} \leq 0$. This last assumption says that the value for player $j$ of an extra dollar invested in the program is decreasing in $\ell^j$. Our assumptions imply that for each state $\omega \in \Omega$ and player $j \in \{P, A\}$, the “better than” sets $B^j(u|\omega) := \{(x, m) : U^j(x, m|\omega) \geq u\}$ are convex. Moreover, they imply that the agent’s preferences satisfy the generalized single crossing condition (SCC):

$$\frac{\partial}{\partial \omega} \left( \frac{U^a_u(x, m|\omega)}{U^a_m(x, m|\omega)} \right) \geq 0.$$ 

We consider the problem of maximizing the principal’s payoff by choosing a state-contingent contract with the agent. To allow a rich contract space, we take a mechanism design approach. Without loss of generality, we consider direct truthful mechanisms, in which the principal proposes a menu of contracts $\{(x(\omega), m(\omega))\}_{\omega \in \Omega}$ to the agent, and is committed to implementing the policy $(x(\hat{\omega}), m(\hat{\omega}))$ if the agent announces that the realized state is $\hat{\omega} \in \Omega$. By the revelation principle, any equilibrium outcome of a contract with arbitrary communication protocols between the principal and the agent is implementable by a truthful direct mechanism. Thus, while we will not recover the particular protocol that principal and agent might be using, the solution will capture the equilibrium relation between states and outcomes. Throughout, we will restrict to deterministic mechanisms. We discuss this issue in our concluding remarks.

## 4. Optimal Delegation with Strings Attached

In this section we present our main results. We begin with the binary state space $\Omega = \{0, 1\}$, which presents our results in a highly tractable setting. In Section 4.2 we extend our analysis to allow for a continuum of states, i.e., $\Omega = [0, 1]$.

### 4.1. Two Types

Our first order of business is to characterize the first best policy for the principal in state $\omega$, $(\hat{x}_\omega, \hat{m}_\omega)$. This is straightforward. Since the principal wants policy to match the state of the world, $\hat{x}_\omega = \omega$. The optimal scale of the project with full information, on the other
hand, is such that the marginal benefit of project expansion *at the ideal policy for the principal* equals the marginal cost; i.e., \( u^p_m(0, \hat{m}_\omega) = \gamma_m(\hat{m}_\omega) \). Thus \( \hat{m}_0 = \hat{m}_1 =: \hat{m} \).

When the agent is privately informed about the realization of the state the principal’s problem is to choose \((x_0, m_0)\) and \((x_1, m_1)\) to maximize

\[
\sum_{\omega \in \{0, 1\}} f(\omega) U^p(x_\omega, m_\omega | \omega)
\]

subject to the incentive compatibility (IC) constraints:

\[
U^a(x_\omega, m_\omega | \omega) \geq U^a(x_{\omega'}, m_{\omega'} | \omega) \quad \text{for} \ \omega, \omega' \in \{0, 1\}.
\]

The nature of the solution depends on the level of conflict of interests between the principal and the agent. First, as usual in these type of problems, if the conflict of interests between the principal and the agent is sufficiently low \((b \leq 1/2 \text{ in our case})\), the incentive constraints will not be binding in the solution and the principal will be able to achieve her first-best policy in each state (see Lemma A.7.1). Since the agent has an upward policy bias \((b > 0)\), the state 1 incentive constraint is trivially not binding at the first best. And given \(b < 1/2\), \((\hat{x}_0, \hat{m})\) is preferred to \((\hat{x}_1, \hat{m})\) for the agent in state 0, since \(|\hat{x}_0 - b| < |\hat{x}_1 - b|\) (see Figure 1).

When \(b > 1/2\), instead, the principal will not be able to implement the first-best. In this case, the optimal solution for the principal implies trading-off losses in the two states to achieve a policy function that is incentive compatible for the agent. We begin by showing that *generically*, it is optimal for the principal to give the agent some discretion over policy outcomes.\(^9\)

**Proposition 4.1 (No Pooling).** A pooling contract \((x^*_p, m^*_p)\) is generically suboptimal for the principal.

The intuition for the result is illustrated in Figure 4.1. The principal’s indifference curves in state 0 and 1 are depicted in blue and red, respectively. The set of points where the indifference curves in the two states are tangent to one another is shown by the green line. Note that if an optimal pooling contract \((x^*_p, m^*_p)\) is proposed, it will be somewhere on this line, for otherwise we can improve the principal’s utility by proposing a pooling contract in this set. (In particular, the optimal pooling contract for \(f(0) = 2/5\) is shown by the black circle.) Note however that if \((x^*_p, m^*_p)\) is an optimal contract, it must be that the agent’s indifference curve in state 0 is also tangent to the principal’s indifference

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\(^9\)We consider the topological notion of genericity — where a property is generic if it is satisfied in an open dense set; and not generic if it is satisfied only in a closed nowhere dense set. We show that the utility functions which satisfy a necessary condition for pooling are nowhere dense in the appropriate Sobolev space, \(W^{1,p}(X)\).
curves at this point. Otherwise utility can be improved by moving “inside” the principal’s better-than sets in each state, as shown by the black triangles in the figure. It follows that a pooling contract \((x_p^*, m_p^*)\) can only be optimal if a triple tangency of indifference curves is satisfied, a property that only holds in a closed nowhere dense set of utility functions. Given that the second-best solution involves granting the agent discretion, the principal has to design the policy function to ensure that the agent has incentives to report truthfully. Achieving incentive compatibility in this setup will necessarily imply policy distortions, which are themselves costly to the principal. The principal will therefore shape the policy function to achieve its objective in the least costly manner. Given that the binding incentive constraint is that of state 0, this entails making policy in state 1 less attractive to the agent and/or policy in state 0 more attractive to the agent in the least costly manner for the principal.

Formally, we define the state-contingent contract curves, a reward curve in state 0, \(CC(0)\), and a discipline curve in state 1, \(CC(1)\). Let \(V := [U^a(\tilde{x}_0, \tilde{m}_0|0), U^a(\tilde{x}_1, \tilde{m}_1|0)]\). Then

\[
\begin{align*}
U^a(x, m|0) &= u_0 \\
U^a(x, m|1) &= u_0 \\
U^p(x, m|0) &= v_0 \\
U^p(x, m|1) &= v_0
\end{align*}
\]
Definition 4.2. The reward curve is the set of points \( CC(0) := \{(\tilde{x}^0(u), \tilde{m}^0(u)) : u \in V\} \), where
\[
(\tilde{x}^0(u), \tilde{m}^0(u)) := \arg \max_{(x,m)} U^p(x,m|0) \text{ s.t. } U^a(x,m|0) \geq u.
\]
The discipline curve is the set of points \( CC(1) := \{(\tilde{x}^1(u), \tilde{m}^1(u)) : u \in V\} \), where
\[
(\tilde{x}^1(u), \tilde{m}^1(u)) := \arg \max_{(x,m)} U^p(x,m|1) \text{ s.t. } U^a(x,m|0) \leq u.
\]
Because policies on the contract curves reward the agent in state 1 and discipline the agent in state 0 efficiently, a policy lying anywhere outside the contract curves can be improved with an alternative policy that preserves incentives and increases the principal’s utility. As a result,

Lemma 4.3. The optimal incentive compatible policy for the principal lies on the contract curves; i.e., \((x^*_\omega, m^*_\omega) \in CC(\omega)\), and thus
\[
\frac{U^p_x(x^*_\omega, m^*_\omega|\omega)}{U^p_m(x^*_\omega, m^*_\omega|\omega)} = \frac{U^a_x(x^*_\omega, m^*_\omega|0)}{U^a_m(x^*_\omega, m^*_\omega|0)} \text{ for } \omega = 0, 1.
\]
Lemma 4.3 allows us to characterize the nature of the distortions in the scope and direction of policy through the shape of the contract curves. To do this it is useful to distinguish two cases. We say that the agent is a moderate if his ideal policy in state 0 is below the first best policy for the principal in state 1; i.e., if \( b < 1 \). We say that the agent is a zealot if \( b > 1 \).

The left panel of Figure 4.2 plots representative contract curves for a moderate agent. As the figure illustrates, the reward curve is increasing, and the discipline curve is decreasing.
This means that policy will be distorted in the direction of the agent’s bias in both states, and that the scale of implementation will be larger than the first best for the principal in state $\omega = 0$, but smaller than the first best for the principal in state $\omega = 1$.

The intuition for the result can be seen graphically in Figure 4.1. When $b < 1$, the state 1 indifference curves for the principal and the state 0 indifference curves for the agent (blue) are tangent below and to the right of the principal’s ideal point in state 1. Because the ideal policy of the agent in state 0 is still lower than the ideal policy of the principal in state 1, the least costly way to leave the agent at some utility level $u$ below what he would obtain at $(\hat{x}_1, \hat{m}_1)$ is to reduce the scale of the program $m$ and increase the policy direction $x$ (achieving this payoff for the agent with some point $x < 1$ would be more costly to the principal, as any point on the blue indifference curve with $x < 1$ is in a lower state 1 indifference curve for the principal).

How much does policy need to adjust relative to project size depends on the strength of the agent’s bias. Consider a point in the disciplining curve $CC(1)$ for an agent with bias $b' < 1$, as illustrated in Figure 4.1. At any such point, the relative value of changes in policy direction and scale in state 1 for a state-0 agent is given by the agent’s state-0 MRS between policy and project size at this point. Now suppose that we increase the agent’s bias to $\tilde{b} \in (b', 1)$. This agent would have a flatter indifference curve through the point. Because in state 0 the $\tilde{b}$ bias agent’s preferred policy is closer to the ideal policy of the principal in state 1, this agent is willing to give away a larger loss in policy direction to get a given amount of additional project size. Thus, the contract curve for an agent $\tilde{b} \in (b', 1)$ will be steeper than for $b'$. In the extreme, for $b = 1$, the ideal policy of the agent in state 0 coincides with the ideal policy of the principal in state 1. Thus, the most
efficient way to punish the (state-0) agent in state 1 is to reduce the implementation scale without changing policy.

When the agent is a zealot, instead, the nature of the optimal policy changes. In this case, both the reward curve $CC(0)$ and the discipline curve $CC(1)$ are increasing (right panel of Figure 4.2). This means that while the implementation scale of the policy in state $\omega = 1$ will be smaller than in the first-best as in the previous case, the direction of the policy outcome in state $\omega = 1$ will now be distorted against the direction of the agent’s bias. The reasoning is symmetric to the previous case. When $b > 1$, as for $b''$ in the figure, the ideal policy of a state 0 agent is larger than the ideal policy of the principal in state 1. Thus, the least costly way to leave the agent at some utility level $u$ below what he would obtain at $(\hat{x}_1, \hat{m}_1)$ is now to decrease policy direction $x$ and reduce the implementation scale $m$ as before. Furthermore, as before (in logic if not in direction), as we continue to increase the bias of the agent above $b = 1$, increases in the value of the state 1 implementation scale become less valuable for the state 0 agent relative to gains in policy, and the most efficient way to punish this agent is through small reductions in project size and sharp distortions in policy (a flatter discipline curve).

The next proposition summarizes the previous discussion.

**Proposition 4.4.** Suppose $b > 1/2$. Then the optimal incentive compatible solution entails distortions in both states; i.e., $(x_\omega^*, m_\omega^*) \neq (\hat{x}_\omega, \hat{m}_\omega)$ for $\omega = 0, 1$. Moreover,

1. The principal overfunds the agent relative to first best in state 0 ($m_0 > \hat{m}_0$) and underfunds the agent in state 1 ($m_1 < \hat{m}_1$).

2. The optimal contract for a moderate agent distorts policy towards the agent in both states; i.e., $x_\omega > \omega$ for all $\omega \in \{0, 1\}$. The optimal contract for a zealot
distorts policy in favor of the agent in state 0 but against the agent in state 1; i.e., $0 < x_0 < x_1 < 1$. When $b = 1$, $x_1 = 1$.

(3) In the optimal contract for a moderate (a zealot) the distortion in state 1 project size increases (decreases) continuously with the agent’s bias $b$ and the distortion in policy direction decreases (increases) continuously with $b$.

In the context of our environmental policy example, for instance, Proposition 4.4 says that Congress underfunds the EPA relative to the first best level precisely when climate change is occurring rapidly, and overfunds the agency relative to the first best if climate change turns out not to be a grave concern. Overfunding in the “low” state always comes together with an environmental policy that is overly aggressive for the median legislator. The distortions in policy in the “high” state, however, depend on the extent of conflict of interests between Congress and the agency: when the EPA is only moderately biased relative to the median legislator, environmental policy does more to curb emissions than what Congress would want, but when the conflict of interests between the EPA and Congress is high, the optimal incentive compatible plan sets a lax environmental policy when climate change is accelerating. This implies that when climate change is indeed occurring at a fast pace, both regulations and resources dedicated to curbing carbon emissions are too low relative to Congress’ first best policy. Thus, ex-post, in these cases both Congress and the agency would favor more stringent regulations and an increase of resources to the EPA.

Proposition 4.4 characterizes the qualitative nature of the distortions and was entirely independent of the principal’s prior, $f$. How much the principal distorts policy in each state depends on the likelihood of each state. Note that since the principal chooses between pairs of points on the contract curves, we can rewrite the principal’s problem as:

$$
\max_{u \in V} f(0)U^p(\tilde{x}^0(u), \tilde{m}^0(u)|0) + f(1)U^p(\tilde{x}^1(u), \tilde{m}^1(u)|1)
$$

In particular, the first order conditions at the optimal level $u^*$ (interior by Proposition 4.4) imply that:

$$
\frac{f(0)}{1 - f(0)} = -\frac{\partial U^p(\tilde{x}^1(u), \tilde{m}^1(u)|1)/\partial u}{\partial U^p(\tilde{x}^0(u), \tilde{m}^0(u)|0)/\partial u}.
$$

Note that the optimal trade-off between distortions in state $\omega = 1$ and state $\omega = 0$ depends on the likelihood of each state. In fact, since $\partial U^p(\tilde{x}^0(u), \tilde{m}^0(u))/\partial u < 0$ and $\partial U^p(\tilde{x}^1(u), \tilde{m}^1(u))/\partial u > 0$, as $f(0)$ increases we “move down the contract curves” in Figure 4.2, reducing the size of the distortion in state $\omega = 0$ in exchange for an increased policy distortion in state $\omega = 1$. Therefore, as state 0 becomes more probable, the magnitude of the distortions in the direction and implementation scale of policy in state 1 will be more severe.
4.2. Continuum of Types

We now extend our analysis to the case in which there is a continuum of states. In this context, the principal offers the agent a menu of incentive compatible contracts \( \{x(\omega), m(\omega)\}_{\omega \in [0,1]} \). Letting \( U^a(\hat{\omega}, \omega) := U^a(x(\hat{\omega}), m(\hat{\omega})|\omega) \), the principal’s problem is:

\[
(PP) \quad \max_{\{x(\omega), m(\omega)\}} \int_0^1 U^p(x(\omega), m(\omega)|\omega)f(\omega)d\omega
\]

subject to:

\( U^a(\omega, \omega) \geq U^a(\hat{\omega}, \omega) \) for all \( \omega, \hat{\omega} \in [0,1] \).

Our main goal is to establish whether the nature of the distortions in policy we obtained in the two-type model extend naturally to the case in which there are multiple states. With this goal in mind, we will focus on characterizing the optimal fully separating contract, which we assume to be differentiable.

For our richer results, we will assume that principal and agent have exponential payoffs;\(^{10}\)

\[ U^a(x, m|\omega) = m \exp(-\beta|x - \omega - b|), \]

and:

\[ U^p(x, m|\omega) = m \exp(-\eta/2(x - \omega)^2) - \frac{\gamma}{2}m^2. \]

With this assumption, we will be able to characterize the optimal menu of contracts in sufficiently rich detail so as to compare the results with the two-type case. We will also show that in this case the optimal contract is continuous and piecewise differentiable. Thus, the original assumption of differentiability only rules out kinks in the optimal contract. We will then also provide conditions under which the fully separating contract dominates any pooling contract in this context.

Our first step is to reduce the continuum of incentive compatibility constraints in (PP) in the usual way. We show that as in the standard quasilinear model, as long as the policy function has the property that \( x(\cdot) \) is increasing in \( \omega \), only local deviations are relevant.\(^{11}\)

\(^{10}\)Assuming a specific utility function simplifies the analysis considerably and is standard in the literature. Baron (2000) and Krishna and Morgan (2008) assume quadratic policy payoffs in a unidimensional policy space with separability of transfers (i.e., a quasilinear utility function). Melumad and Shibano (1991) assume quadratic payoffs in a unidimensional policy space with no transfers. In the same context, Alonso and Matouschek (2008) assume quadratic payoffs for the principal, and a single-peaked symmetric utility function for the agent. Koessler and Martimort (2012) assume that payoffs are quadratic in each dimension and separable across dimensions. We deviate from the quadratic payoffs assumption that is prevalent in the literature because of the non-separability of payoffs that is at the core of our problem.

\(^{11}\)Part of the complexity of multidimensional mechanism design problems is that the order in which incentive constraints bind is typically endogeneous to the mechanism (e.g., see the review by Rochet and Stole (2003). The fact that the agent’s preferences in our model satisfy a generalized single crossing
This argument has two parts. The local incentive compatibility constraint for type $\omega$ ensures that type $\omega$ can not gain by announcing to be a type arbitrarily close to $\omega$. A necessary condition for no profitable local deviations at $\omega$ is that:

$$\left. \frac{\partial U^a(\hat{\omega}, \omega)}{\partial \hat{\omega}} \right|_{\hat{\omega} = \omega} = 0,$$

or equivalently:

$$m'(\omega) = - \frac{U^a_x(x(\omega), m(\omega)|\omega)}{U^a_m(x(\omega), m(\omega)|\omega)} y(\omega),$$

Thus, incentive compatibility implies that at any point $\omega$, the rate of change in the scope of policy in the optimal contract must be proportional to the rate of change of policy direction by a factor given by the agent’s marginal rate of substitution in that state. Condition (3) is also sufficient to assure no profitable local deviations if $x(\cdot)$ is nondecreasing (see Lemma A.7.2). In fact, because of the single crossing condition, if $x(\cdot)$ is nondecreasing, (3) is necessary and sufficient to rule out both local and global deviations (see Lemma A.7.3).

We can then write the principal’s problem (PP) as:

$$\max_{\{x(\omega), m(\omega)\}} \int_0^1 U^p(x, m|\omega)f(\omega)d\omega$$

subject to the law of motion (3) and the constraints $x'(\omega) \geq 0$, $m(\omega) \geq 0$.

The law of motion defines a functional equation for the project size $m$ in terms of $x$, the only control variable in the above problem is $x$. We can write the above problem in Bolza form by introducing a new function $y$, which will be a ”stand-in” for $x'$. This, of course, requires an additional constraint reflecting that relationship. The optimal control problem\footnote{condition allows us to abstract from these complexities. However we still have to deal with technical issues arising from the lack of quasilinearity.\textsuperscript{12}Ignoring the $x'(\omega) \geq 0$ constraint for now.} is therefore:

$$\max_{y(\omega)} \int_0^1 U^p(x, m|\omega)f(\omega)d\omega$$

subject to:

$$m'(\omega) = - \frac{U^a_x(x(\omega), m(\omega)|\omega)}{U^a_m(x(\omega), m(\omega)|\omega)} y(\omega),$$

$$x'(\omega) = y(\omega), m(\omega) \geq 0.$$
The Hamiltonian for this problem is then:
\[ H = U^p(x, m|\omega) f(\omega) - \lambda_1 \frac{U_a^a(x, m|\omega)}{U_m^a(x, m|\omega)} y + 2\lambda y + \mu m. \]

The associated Euler-Lagrange conditions for a fully separating solution are characterized in Remark A.7.4 in the Appendix.

We can now prove our second substantive result. We have already established that the policy content \( x(\cdot) \) is nondecreasing in type. (In fact, in a separating equilibrium \( x(\cdot) \) must be strictly increasing, for if \( x' = 0 \) in an interval \([a, b]\) \( \subset [0, 1]\), then (3) implies that \( m' = 0 \) in \([a, b]\), which implies pooling.) We next show that the implementation scale \( m(\cdot) \) similarly must be nonincreasing in type. Thus, as in the binary state environment, the scale of implementation of policy decreases with the direction of policy \( x \). To show this result, we must first establish an intermediate step.

**Lemma 4.5.** In the solution to (PP), \( x(\omega) \leq \omega + b \) for all \( \omega \in [0, 1] \).

Thus, in the optimal separating contract, the direction of policy is never larger than the ideal policy of the agent. Our result now follows immediately using Lemma 4.5. Note that given that \( x(\cdot) \) is nondecreasing and \( U_a^a(\cdot) > 0 \), the truth telling condition (3) says that \( m(\cdot) \) will be weakly decreasing at \( \omega \) if and only if \( U_a^a(\cdot) \geq 0 \). But this happens if and only if \( x(\omega) \leq \omega + b \). We then have

**Corollary 4.6.** In the optimal incentive compatible plan \((x, m)\)
\[ \frac{dm}{dx}(\omega) = - \frac{U_a^a(x(\omega), m(\omega)|\omega)}{U_m^a(x(\omega), m(\omega)|\omega)} \leq 0 \quad \text{for all } \omega \in [0, 1], \]
with strict inequality whenever \( x(\omega) < \omega + b \).

A second implication of Lemma 4.5 is that the equilibrium payoff of the agent is decreasing in type. Thus, the lowest type (type 0) makes the largest informational rent. To see this, note that by the envelope theorem (or substituting eq. 3), \( U_a^a(x(\omega), m(\omega)|\omega)x'(\omega) + U_m^a(x(\omega), m(\omega)|\omega)m'(\omega) = 0 \). Then
\[ \frac{d}{d\omega} U_a^a(x(\omega), m(\omega)|\omega) = U_a^a(x(\omega), m(\omega)|\omega) < 0, \]
where the inequality follows from the fact that \( x(\omega) \leq \omega + b \).

Providing a more detailed characterization of the solution at this level of generality is difficult. However, in order to evaluate whether the lessons we learned in the two-type case extend to this environment, we need to characterize the nature of the distortions relative to the first best. As a first step in this direction, we will assume hereafter that
principal and agent have exponential payoffs. A key property of this payoff specification is that from the truth telling condition (3) we can obtain a closed form for the project size function $m(\cdot)$ as a function of the policy $x(\cdot)$ and the state $\omega$ itself. This, in turn, allows us to take a direct approach to solve PP without solving for the multipliers $\lambda_1(\cdot)$ and $\lambda_2(\cdot)$ in the constrained optimization formulation of the Euler-Lagrange conditions (20-25).\footnote{This simplification does come at a cost in generality. In particular, we want to point out that in this case the single crossing condition is only satisfied weakly, as $\frac{\partial}{\partial \omega} \left( \frac{U^a(x,m(\omega))}{U^a(x,m(\omega))} \right) = 0$. As a result, the optimal contract will remove the incentive for the agent to misrepresent his type leaving him indifferent over sending alternative reports.}

Note that since in the solution $x(\omega) \leq \omega + b$ for all $\omega$ by Lemma 4.5, we can write $U^a(x, m|\omega) = m \exp(\beta[x - \omega - b])$. Then (3) becomes $\frac{m(\omega)}{m(\omega)} = -\beta x'(\omega)$, and thus

$$m(\omega) = m_0 \exp(-\beta[x(\omega) - x_0])$$

It follows that the optimal project size $m(\cdot)$ is a strictly decreasing function of $\omega$, which decreases faster the steeper $x(\omega)$ is and the more responsive the agent is to policy loss, as measured by $\beta$.

Given that expression (4) incorporates the IC constraints (3), we can now directly substitute (4) into the objective function of the principal. Since $m(\omega) > 0$ for all $\omega$, the

![Figure 5](image-url)

**Figure 5.** In the optimal separating contract, policy $x(\cdot)$ is a strictly increasing function of the state $\omega$ and project size is a weakly decreasing function of the state $\omega$ (strictly decreasing whenever $x(\omega) < \omega + b$).
constraint \( m(\omega) \geq 0 \) is not binding. In addition, as it is common in the literature, we will ignore the constraint that \( x'(\omega) \geq 0 \) and check that the solution satisfies the constraint ex-post. With these remarks, we can rewrite the principal’s problem as

\[
\max_{x_0, m_0, x(\cdot)} J(x_0, m_0, x(\cdot)) = \int_0^1 U^p(x(\omega), m_0 \exp(-\beta[x(\omega) - x_0]|\omega) f(\omega) d\omega \quad \text{s.t.} \quad x(0) = x_0.
\]

The necessary first order condition with respect to \( x(\omega) \) then gives

\[
(5) \quad MRS_{xp}^p(\omega) \equiv \frac{U^p_x(x(\omega), m(\omega)|\omega)}{U^p_m(x(\omega), m(\omega)|\omega)} = \beta m(\omega) = MRS_{xm}^a(\omega) \quad \forall \omega \in [0, 1].
\]

Note that since \( m(\cdot) \) is strictly decreasing, this implies that the principal’s marginal rate of substitution in the optimal contract is decreasing in \( \omega \), so that increases in policy are relatively less valuable for the principal in higher states, while increases in implementation scale are relatively more valuable for the principal in higher states.

The optimality condition (5) leads to two immediate implications. First, note that since \( U^p_x(\cdot) \leq 0 \) if and only if \( x \geq \omega \), the optimal contract will overfund any project that distorts policy towards the agent and underfund projects that distort policy against the agent.

**Proposition 4.7.** In the optimal separating solution, \( m(\omega) \geq \hat{m} = 1/\gamma \Leftrightarrow x(\omega) \geq \omega \).

Second, using this result we can show that the optimal separating solution is continuous and piecewise differentiable. Thus, in particular, the optimal differentiable contract that we characterize here dominates any solution with discontinuities, and the original assumption only rules out kinks in the optimal contract.

**Proposition 4.8.** Suppose principal and agent have exponential payoffs. Then fully separating solutions to the principal’s problem are continuous and piecewise differentiable.

Proposition 4.8 builds on Lemma A.7.5 in the Appendix, which shows that in general (for any utility functions satisfying the assumptions of the model) the equilibrium payoffs of principal \( U^p(x(\omega), m(\omega)|\omega) \) and agent \( U^a(x(\omega), m(\omega)|\omega) \) are continuous in \( \omega \). This rules out all but a specific kind of discontinuity, which we can then rule out with (5).

It is important to note that the equality of the marginal rates of substitution of the principal and the agent in (5) does not imply that the optimal incentive compatible contract is efficient. Consider for example Figure 6, which plots a possible solution \((x(\cdot), m(\cdot))\) in the \((x, m)\) space. Note that at each point in the curve, the marginal rates of substitution of the principal and the agent are equal. However, the contract underfunds and distorts policy against the agent in “high” states (states \( \omega > \hat{\omega} \)), as in the zealot solution of the two type model. In these high states, the indifference curve of the agent is tangent to
the indifference curve of the principal from below, and both the principal and the agent would prefer to increase funding and choose a higher policy.

**Figure 6.** Optimal Contract with Exponential Payoffs.

The result sketched in the figure is interesting because it represents the natural generalization of the results for the two-type model. Since in the continuum local deviations are “small” relative to the size of the bias, the agent is always a “zealot”, and thus, based on the logic of the two-type model, we would expect that the optimal contract will overfund and distort policy in favor of the agent for “low types” and underfund and distort policy against the agent for “high types”, leading to the ex-post inefficiency.

Our main goal is to confirm whether in fact the solution has this strong inefficiency ex-post, or if instead the optimal incentive compatible contract eliminates all gains from trade (as is the case for “low” states in the contract depicted in the figure). The result is stated in Theorem 4.9. For this it will be convenient to define

\[ r(\omega) \equiv (\eta/2)[(x(\omega) - \omega)^2 - x_0^2] - \beta[x(\omega) - x_0]. \]

**Theorem 4.9.** The optimal fully separating incentive compatible policy \((x(\cdot), m(\cdot))\) is such that there exists a \(\tilde{\omega} \in (0, 1)\) such that \(x(\omega) > \omega\), and \(m(\omega) > \hat{m}\) for \(\omega \in [0, \tilde{\omega})\) and

\[ \text{Consider first a finite state space with typical element } \omega_t, \ t = 1, \ldots, T, \text{ such that } \omega_{t+1} - \omega_t = \Delta \text{ for some } \Delta > 0. \text{ A direct extrapolation of the two-type results to the finite state case would imply that (i) } x(\omega_t) = \omega_t \text{ for all } t \text{ whenever } b < \Delta/2, \text{ as in the “low bias” case, (ii) } x(\omega_t) > \omega_t \text{ whenever } \Delta/2 < b < \Delta, \text{ as in the “moderate” case, and that (iii) when } b > \Delta \text{ there exists a } \overline{t} \text{ such that } x(\omega_t) > \omega_t \text{ for } t \leq \overline{t} \text{ and } x(\omega_t) < \omega_t \text{ for } t > \overline{t}, \text{ so that policy distorts in favor of the agent for “low types” and against the agent for “high types”, as in the “zealot” case. Now as } \Delta \to 0, \text{ eventually } b > \Delta, \text{ and only the “zealot” case has bite in the continuum.} \]
\[ x(\omega) < \omega, \text{ and } m(\omega) < \hat{m} \text{ for } \omega \in (\hat{\omega}, 1]. \] Moreover, for all \( \omega \in [0, 1], \)

\begin{equation}
(6) \quad x'(\omega) = \frac{(x(\omega) - \omega) \exp (r(\omega)) - \frac{1}{(\beta + \eta x_0)}}{(x(\omega) - \omega) - \frac{\beta}{\eta} \exp (r(\omega)) - \frac{1}{(\beta + \eta x_0)}}.
\end{equation}

The proof of Theorem 4.9 builds on two key facts. First, we show from the transversality condition in the Euler-Lagrange equations that (i) the optimal policy \( x(\cdot) \) cannot be always above or always below \( \omega \). Second, a direct examination of (6) shows that \( x'(\omega) < 1 \) whenever \( x(\omega) < \omega \). Thus, (ii) if \( x(\omega) < \omega \) at some \( \omega \in [0, 1] \), then \( x(\omega') < \omega' \) for all \( \omega' \geq \omega \). This in turn implies that \( x_0 > 0 \), for otherwise \( x \) would always be below \( \omega \), which contradicts (i). So \( x(\omega) \) starts above \( \omega \) and then must cross \( \omega \) at least once. But by (ii), if \( x \) is ever below \( \omega \), it will not go back up. Thus it must cross \( \omega \) only once.

Fact (i) above captures the resolution of a tradeoff for the principal. Recall that in the first best the policy matches the state, \( x(\omega) = \omega \) for all \( \omega \in [0, 1] \), and the project size \( m(\cdot) \) is flat at \( \hat{m} = 1/\gamma \). Now, in order to provide incentives without using transfers, the principal needs to introduce distortions in policy and/or implementation scale. In the exponential case, the tradeoff between these distortions is given by the expression \( m'(\omega) = -\beta m(\omega) x'(\omega) \). This reflects that in order to make \( x(\cdot) \) responsive to the state (reducing distortions in policy), \( m(\cdot) \) needs to be responsive to the state as well (increasing distortions in implementation scale). Thus, in choosing how close \( x(\cdot) \) can trace \( \omega \), the principal faces a tradeoff between inducing distortions in policy direction versus inducing distortions in implementation scale.\(^{15}\)

The particular resolution of the tradeoff between distortions in policy and project size depends crucially on the relative sensitivity to policy loss of the agent vis a vis the principal, captured here by the ratio \( \beta/\eta \). Note that the denominator of (6) is equal to the numerator minus the term \( \frac{\beta}{\eta} \exp (r(\omega)) \). It follows that \( x'(\omega) \to 1 \) for all \( \omega \) as \( \beta/\eta \to 0 \). In this situation the agent does not care too much about policy losses relative to the principal. Thus, it is relatively cheap for the principal to compensate the agent with changes in project size to obtain a policy that is very close to her first best. In particular, if \( \beta \to 0 \), the agent does not care much about policy losses (not only relative to the principal) and thus is cheap to compensate with small changes in project size. As we can see from (4),

\(^{15}\)This simple logic is due in part to the exponential/linear payoffs. In this case, the agent’s MRS does not depend on the distance between policy \( x(\omega) \) and the agent’s preferred policy, \( \omega + b \). Thus, if \( x(\cdot) \) is always below or always above \( \omega \) the principal can do better in all states by just shifting \( x(\cdot) \) above or below. Now, in general, the agent’s MRS can be decreasing as \( x \) gets closer to \( \omega + b \). When this is the case, the principal has two competing ways of reducing distortions in project size: by making \( x(\cdot) \) flatter, and by making \( x(\cdot) \) close to \( \omega + b \).
in this case \( m(\cdot) \) will be relatively flat. This of course is excellent for the principal, who can achieve an outcome close to her first best.

Now, recall that we solved the principal’s problem PP assuming that in the solution \( x(\cdot) \) would be nondecreasing. The previous discussion makes clear that when the agent is sufficiently insensitive to policy loss relative to the principal in fact \( x'(\omega) > 0 \) for all \( \omega \in [0, 1] \), and thus the optimal incentive compatible contract will be fully separating.

**Proposition 4.10.** There exists \( \delta > 0 \) such that if \( \beta/\eta < \delta \), the optimal incentive compatible contract is a fully separating contract.

When the agent is very sensitive to policy losses, on the other hand (\( \beta \) large), attaining a policy \( x(\cdot) \) close to the first best is very costly in terms of project size distortions, and the principal would rather take a relatively unresponsive policy than introduce large distortions in project size. In fact, for sufficiently high \( \beta \), the principal will be better off with a pooling contract. To see this, note that with \( \beta \to \infty \) the numerator of (6) goes to \( x(\omega) - \omega \) and the denominator to \( -\infty \). Since \( x(\omega) > \omega \) for some \( \omega \), this implies that \( x' < 0 \), which violates the monotonicity constraint. When the agent is much more sensitive to policy loss than the principal the agent is too costly to compensate, and it is better for the principal to pool types. It is worth noting here that while this result is in line with many papers in the literature, it is in contrast with Koessler and Martimort (2012), who show that in a model with separability across policy dimensions pooling is never optimal.\(^{16}\)

### 5. Discussion: The Quasilinear Model

To put our results in the context of the previous literature with transferable utility, we consider a version of the model in which “project size” enters into the utility of principal and agent simply as transfers in a quasilinear utility function. For simplicity, we do this here for the two type model. For any action \( x \), state \( \omega \), and program size \( m \), the principal’s payoff is now \( U^p(x, m|\omega) := v^p(\ell^p(x, \omega)) - m \), and the agent’s payoff is \( U^a(x, m|\omega) = v^a(\ell^a(x, \omega)) + m \). As before, we assume that for \( j = P, A \), (i) \( v^j_\ell < 0 \) and (ii) \( v^j_{\ell\ell} \leq 0 \).

In order for this problem to be well defined, we need to ensure that it is bounded in some way (otherwise the principal could ask for infinite transfers from the agent). There are two possible ways to do this, and the choice makes a large difference to the qualitative results. The first approach is to introduce the standard individual rationality constraint in

\(^{16}\)The result holds for any level of conflict of interest (bias) between principal and agent, provided the conflict of interest between the principal and the agent are different on each dimension (Martimort and Semenov (2006)).
principal-agent models, where the contract must assure the agent a minimum utility level determined by an outside option. The second is to impose the constraint that transfers are non-negative, which puts a hard lower-bound on what can be achieved. This corresponds to the model of delegation with transfers in Krishna and Morgan (2008), Section 3.

In both cases the Principal chooses \((x_0, m_0)\) and \((x_1, m_1)\) to maximize expected utility
\[
E[v^p(ℓ^p(x_ω, ω)) - m_ω] \text{ subject to the incentive compatibility constraints that the plan } (x_ω, m_ω) \text{ is optimal for the agent in state } ω; \text{ i.e., that } v^a(ℓ^a(x_ω, ω)) + m_ω \geq v^a(ℓ^a(x_ω', ω)) + m_ω' \text{ for } ω, ω' \in \{0, 1\}. \]
\[
(7) \quad v^a(ℓ^a(x_ω, ω)) + m_ω \geq 0.
\]

In this formulation, the problem boils down to a standard screening problem. When bias is “low” \((b \leq \hat{b} \text{ for some } \hat{b})\), the first-best is incentive compatible (left panel of Figure 5). When \(b > \hat{b}\), the first-best is unattainable, and the solution has many of the features of the textbook screening problem,. This is illustrated in the right panel of Figure 5. The contract curves (in green) characterize the efficient frontier of a tradeoff between extracting more surplus in state 0 or state 1 with incentive compatible contracts. Points to the south and southeast of the contract curves in state \(ω = 0\) and \(ω = 1\) lead to a higher payoff for the principal in state 0, and points to the north and northwest of the contract curves leading to a higher payoff for the principal in state 1. Since utility is perfectly transferable between the principal and agent, both \(x_0\) and \(x_1\) are increasing in the agent’s bias \(b\), so unlike in our model, there is no recoil effect. As in our model, though, in the quasilinear model with an IR constraint pooling is generically suboptimal.

Consider now the case of non-negative transfers, as in Krishna and Morgan (2008). Now we replace the IR constraint (7) with
\[
(8) \quad m_ω \geq 0, \text{ for } ω \in \{0, 1\}.
\]
If the agent’s bias is sufficiently low \((b \leq 1/2)\) we can implement the first best, so assume \(b > 1/2\). In a separating solution, the IC constraint in state \(ω = 1\) will not bind, and the IC constraint in state \(ω = 0\) will hold with equality. From here we can obtain \(m_0 = m_1 + [v^a(ℓ^a(x_1, 0)) - v^a(ℓ^a(x_0, 0))]\). Substituting in the objective function and noting that in the solution \(m_1^* = 0\), we can write the principal’s problem as
\[
\max_{x_0, x_1} f(0)[v^p(ℓ^p(x_0, 0)) - v^a(ℓ^a(x_1, 0)) + v^a(ℓ^a(x_0, 0))] + f(1)v^p(ℓ^p(x_1, 1))
\]
From the first order condition with respect to \(x_0\) we obtain
\[
U^p_\omega(x_0^*, m_0^*|0) = -U^a_\omega(x_0^*, m_0^*|0).
\]
Since \(U^p_\omega(x, m|ω) = -1\) and \(U^a_\omega(x, m|ω) = 1\), this implies that \((x_0^*, m_0^*) \in CC(0)\). From
the state 1 FOC, however, $f(1)U^p(x_1^*, m_1^*|1) = f(0)U^a(x_1^*, m_1^*|0)$, so $(x_1^*, m_1^*) \notin CC(1)$: the contract curve in state 1 is constrained so that it has to be on the $m$-axis, i.e., $m = 0$.

The solution is illustrated in Figure 5. The left panel depicts the case of moderate bias, $1/2 < b < 1$. The points A and B in the figure lie on the state 0 indifference curve for the agent that goes through the principal’s ideal point in state 1. Point A includes positive transfers to the agent, and is on the state 0 contract curve, at the tangency of the agent’s indifference curve with a state 0 indifference curve for the principal. Point B includes zero transfers, and is in the constrained contract curve. The contract $(A, B)$ maximizes the principal’s payoff in state 1 among incentive compatible contracts but implies large transfers to the agent in state 0. If the likelihood of state 0 is high, the principal is better off moving to a point like $(A', B')$, trading transfers in state 0 for upward distortions in policy in state 1. For a sufficiently large likelihood on state 0, the contract must be in the constrained contract curve in both states, moving to a point like $(A'', B'')$, with no transfers and distortions in the policy in both states.

The right panel illustrates the case of $b \in (1, 2)$. Note that distorting policy in state 1 upwards is never optimal for the principal. Here the tradeoff is achieved distorting policy downwards in state 1 (against the agent’s preferred policy) and reducing transfers in state

\[17\] If $b \geq 2$ we get pooling at the prior as the solution to the problem.
Figure 8. Quasilinear Model with Non-negative Transfers. The figure shows the principal’s indifference curves in state 0 (in blue) and state 1 (in red), and the agent’s indifference curves in state 0 (in light blue).

0, as in a point like \((A', B')\). Thus, differently than the IR constraint version of the model, the quasilinear model with non-negative transfers does have a recoil effect. Differently than in our model, though, this recoil effect is discontinuous in the bias \(b\).

For a sufficiently high likelihood of state 0, no separating contract exists, and the principal is reduced to a pooling contract along the yellow constrained contract curve. To see why pooling obtains in this context, consider point C on the yellow curve. Note that the triple tangency condition argument against pooling in our model (Proposition 4.1) does not apply here, because given the nonnegative constraint on transfers, the principal’s indifference curves will not be tangent at the optimal pooling point. Thus pooling is optimal if the agent’s state 0 indifference curve runs through the non-trivial gap between the principal’s indifference curves.

6. Conclusion

A common feature in many agency relationships is that the principal can decide both the direction and the scope or implementation scale of a policy. In such cases, there is a natural complementarity between these dimensions of policy, as the value of expanding the scale of implementation increases for both principal and agent the closer the implemented policy is to their preferred policy. In this paper we characterize the optimal contract for the principal in this environment when she cannot count on transfers to alleviate incentive problems.
Because of the non-separability across policy dimensions that is at the core of our problem, the common procedure of re-parametrizing the problem in terms of information rents is not helpful. In general we are dealing with a singular control problem of the type that does not admit the sort of straightforward bang-bang solutions commonly used in other contexts. However, we are able to make considerable progress. First, we solve the optimal contract in the two type case, and present a graphical analysis that makes the logic and results transparent. We then characterize the optimal separating contract in the continuum with a parametric assumption on payoffs (exponential payoffs).

We show that the optimal separating contract is equivalent to delegation “with strings attached”: an agent with an upward policy bias can only choose higher policies by reducing the scale of the project. The possibility of tinkering with the scale of implementation induces distortions in the content of policy that can be different from what would result in a comparable model with quasilinear payoffs, and that are in fact different when the agent’s bias is sufficiently large relative to the smallest possible deviation (which is always the case in the continuum). Indeed, in this case the optimal separating contract partitions the state space in a “low” and “high” set of states, such that the principal overfunds and distorts towards the agent in low states, but underfunds and distorts against the agent in high states.

This strong form of ex-post inefficiency in the optimal contract does not appear in the standard model with quasilinear payoffs where utility is perfectly transferable between parties, and leads to new insights in applications. In the environmental regulation case, for example, this implies that the optimal contract sets overly stringent regulations that are heavily enforced when climate change is mild, and relatively weak regulations which are under-enforced in the states in which climate change is accelerating more heavily. Thus, both Congress and the EPA would both prefer to set more stringent, heavily enforced environmental regulations in the high states.

While we have made significant progress in analyzing this problem, much work remains. As much of the literature before us, we have restricted the principal to choose among deterministic mechanisms. A key direction for future work would be to extend the analysis in this paper to stochastic mechanisms (e.g., Strausz (2006), Kovác and Mylovanov (2009)). For example, if the principal is less risk averse than the agent this kind of contract can be useful for the principal in this context, allowing her to relax incentive constraints without utility loss. Alternatively, if the agent’s risk attitudes change throughout the policy space these differences can be exploited (e.g., if the agent is risk loving near the principal’s state 0 optimal point, but risk averse near the principal’s state 1 optimal point the principal
may be able to reward and/or punish through random mechanisms depending on the principal’s preferences).

To see this, consider the binary state model. For incentive compatibility, the principal needs to make the policy in state $\omega = 0$ more attractive, and the policy in state $\omega = 1$ less attractive to the state 0 agent. Now suppose, for simplicity, that the principal is risk neutral, while the agent is risk averse. Then the principal can gain substituting the state 1 policy in the optimal deterministic contract $(x^*_1, m^*_1)$ with a lottery that plays a point $(x'_1, m'_1)$ with a probability $\mu' \in (0, 1)$ and a point $(x''_1, m''_1)$ with probability $(1 - \mu)$ in state $\omega = 1$.

This is illustrated in Figure 9. Choose a point $(x'_1, m'_1)$ that is a convex combination between $(x^*_1, m^*_1)$ and the principal’s state 1 ideal point, $(\hat{x}_1, \hat{m}_1)$, choose a point $(x''_1, m''_1)$ and a probability $\mu \in (0, 1)$ such that $(x^*_1, m^*_1) = \mu(x'_1, m'_1) + (1 - \mu)(x''_1, m''_1)$. Note that the risk neutral principal is indifferent between $(x^*_1, m^*_1)$ and the lottery $\mu[(x'_1, m'_1)] + (1 - \mu)[(x''_1, m''_1)]$, but the state 0 agent is strictly worse-off. Then letting $\mu' = \mu + \varepsilon$ for $\varepsilon > 0$ small, the agent is still strictly worse-off but the principal is better-off in $\mu'[(x'_1, m'_1)] + (1 - \mu')[(x''_1, m''_1)]$ than in $(x^*_1, m^*_1)$.

This simple example illustrates that if the principal is less risk averse than the agent, there is space to improve outcomes by considering stochastic mechanisms. This also suggests that deterministic mechanisms can be optimal when the principal is at least as risk averse as the agent. This, we believe, is at the heart of the comment in Koessler and Martimort (2012) regarding the optimality of deterministic mechanisms in that context. A full analysis of contracting with stochastic mechanisms in our setup is beyond the scope of this paper, and is left for future work.
REFERENCES


Lemma A.7.1. \((x^*_ω, m^*_ω) = (\widehat{c}_ω, \widehat{m}_ω)\) for \(ω \in \{0, 1\}\) if and only if \(b \leq 1/2\).

Proof of Lemma A.7.1. The Lagrangian for the principal is:
\[
\sum_ω U^p(x_ω, m_ω|ω)f(ω) + \lambda_0 [U^a(0, m_0|0) - U^a(x_1, m_1|0)] + \lambda_1 [U^a(x_1, m_1|1) - U^a(x_0, m_0|1)]
\]

The first order conditions are:
\[
\begin{align*}
\frac{\partial L}{\partial x_0} &= U^p_x(x_0, m_0|0)f(0) + \lambda_0 U^a_x(x_0, m_0|0) - \lambda_1 U^a_x(x_0, m_0|1) = 0, \\
\frac{\partial L}{\partial x_1} &= U^p_x(x_1, m_1|1)f(1) - \lambda_0 U^a_x(x_1, m_1|0) + \lambda_1 U^a_x(x_1, m_1|1) = 0, \\
\frac{\partial L}{\partial m_0} &= U^p_m(x_0, m_0|0)f(0) + \lambda_0 U^a_m(x_0, m_0|0) - \lambda_1 U^a_m(x_0, m_0|1) = 0, \\
\frac{\partial L}{\partial m_1} &= U^p_m(x_1, m_1|1)f(1) - \lambda_0 U^a_m(x_1, m_1|0) + \lambda_1 U^a_m(x_1, m_1|1) = 0, \\
\frac{\partial L}{\partial \lambda_0} &= U^a(x_0, m_0|0) - U^a(x_1, m_1|0) \geq 0, \quad \lambda_0 \geq 0, \quad \frac{\partial L}{\partial \lambda_0} = 0, \\
\frac{\partial L}{\partial \lambda_1} &= U^a(x_1, m_1|1) - U^a(x_0, m_0|1) \geq 0, \quad \lambda_1 \geq 0, \quad \frac{\partial L}{\partial \lambda_1} = 0.
\end{align*}
\]

Suppose that neither constraint is binding. Then we have \(\lambda^*_0 = \lambda^*_1 = 0\). Thus (9) becomes \(U^p(x_0, m_0|0) = 0 \iff x^*_0 = \widehat{x}_0 = 0\), and (10) becomes \(U^p(x_1, m_1|1) = 0 \iff x^*_1 = \widehat{x}_1 = 1\). Then (11) becomes \(U^p_m(x_0, m_0|0) = 0\), and since \(x_0 = 0\), then \(U^p_m(x_0, m_0|0) = 0\). Therefore \(u^p_m(0, m_0) = \gamma_m(m_0)\), or \(m_0 = \widehat{m}\). Similarly, from (12), \(m_1 = \widehat{m}\). Then (13) and (14) are reduced to:
\[
\begin{align*}
u^a((b^2, \widehat{m})) &\geq u^a(((1 - b)^2, \widehat{m})) \iff b \leq \frac{1}{2}, \\
\end{align*}
\]
and
\[
\begin{align*}
u^a((b^2, \widehat{m})) &\geq u^a(((1 + b)^2, \widehat{m})) \iff b \geq \frac{1}{2}.
\end{align*}
\]
This concludes the proof of the lemma. 

Proof of Proposition 4.1. We will show that the utility functions which satisfy a necessary condition for pooling are nowhere dense in the appropriate Sobolev space, \(W^{1, p}(X)\). In particular, we show that if a pooling contract \(x^*_p, m^*_p\) is optimal for the Principal, then the following triple tangency condition must be satisfied:
\[
\gamma'_{p,0}(t^*_{p,0}) = \pm \gamma'_{p,1}(t^*_{p,1}) = \pm \gamma'_{a,0}(t^*_{a,0}),
\]
where \( \gamma_{j,\omega} : I \to \mathbb{R}^2 \) is the parametrization by arclength of \( IC^j(x_o^*, m_o^*|\omega) \), \( \gamma_{j,\omega}(t_{j,\omega}) = (x_o^*, m_o^*) \) and \( I \subseteq \mathbb{R} \) is a non-empty interval.

First, the Principal’s indifference curves have to be tangent, for otherwise \((x_o^*, m_o^*)\) does not solve the optimal pooling problem,

\[
\max_{(x_o, m_o)} \sum_{\omega \in \{0,1\}} f(\omega)U_p(x_o, m_o|\omega).
\]

To see this, note that the first-order condition of the above problem implies:

\[
\nabla U_p(x_o^*, m_o^*)|0 = \frac{-f_1}{f_0} \nabla U_p(x_o^*, m_o^*)|1.
\]

It follows that for any \((x, m)\):

\[
\nabla U_p(x_o^*, m_o^*)|0 \cdot [(x, m) - (x_o^*, m_o^*)] = 0
\]

\(\Leftrightarrow\)

\[
\nabla U_p(x_o^*, m_o^*)|1 \cdot [(x, m) - (x_o^*, m_o^*)] = 0.
\]

Now, by definition of \(\gamma_{j,\omega}\):

\[
\nabla U_p(x_o^*, m_o^*)|0 \cdot [\gamma_{p,0}'(t_{p,0}) - (x_o^*, m_o^*)] = 0, \text{ and}
\]

\[
\nabla U_p(x_o^*, m_o^*)|1 \cdot [\gamma_{p,1}'(t_{p,1}) - (x_o^*, m_o^*)] = 0,
\]

and since \(\|\gamma_{p,1}'(t_{p,1})\| = \|\gamma_{p,0}'(t_{p,0})\| = 1\) (because \(\gamma_{j,\omega}\) is the natural parametrization), it follows that \(\gamma_{p,0}'(t_{p,0}) = \pm \gamma_{p,1}'(t_{p,1})\). This proves the first equality.

We will show the second equality by contradiction. Assume that \(\gamma_{p,1}'(t_{p,1}) \neq \pm \gamma_{a,0}'(t_{a,0})\).

Note that for any \(\varepsilon\), the menu \((\varepsilon \gamma_{a,0}'(t_{a,0}^-), -\varepsilon \gamma_{a,0}'(t_{a,0}^+))\) is incentive compatible\(^{19}\), since both contracts are on \(IC^a(x_o^*, m_o^*)|0\) [alternatively, could move along IC curve by proposing menu \((\gamma_{a,0}'(t_{a,0}^- - \varepsilon), \gamma_{a,0}'(t_{a,0}^- + \varepsilon))\)]. Because \(\gamma_{p,1}'(t_{p,1}) \neq \pm \gamma_{a,0}'(t_{a,0})\), either

\[
\varepsilon \gamma_{a,0}'(t_{a,0}^-) \in int B^p(x_o^*, m_o^*)|0, -\varepsilon \gamma_{a,0}'(t_{a,0}^-) \in int B^p(x_o^*, m_o^*)|1
\]

or

\[
\varepsilon \gamma_{a,0}'(t_{a,0}^+) \in int B^p(x_o^*, m_o^*)|1, -\varepsilon \gamma_{a,0}'(t_{a,0}^+) \in int B^p(x_o^*, m_o^*)|0.
\]

Without loss of generality, assume the first holds. This implies that \(U^p(\varepsilon \gamma_{a,0}'(t_{a,0}^-)|0) > U^p(x_o^*, m_o^*)|0\) and \(U^p(-\varepsilon \gamma_{a,0}'(t_{a,0}^+)|1) > U^p(x_o^*, m_o^*)|1\), thus the separating menu dominates the pooling menu state by state, which contradicts the optimality of pooling. ■

---

\(^{18}\)Note that the sign depends on the direction of the parametrization; reversing the direction would change the sign.

\(^{19}\)Note that we are using the convention in differential geometry that \(\gamma_{j,\omega}'(t)\) is a vector with base point at \(\gamma_{j,\omega}(t)\) and that \(\varepsilon \gamma_{j,\omega}'(t)\) is a tangent vector length \(\varepsilon\), instead of the unit tangent vector (this is a slight abuse of notation).
Proof of Lemma 4.3. Suppose in the solution of the Principal’s optimization problem $\lambda_0^* > 0$ and $\lambda_1^* = 0$. Then equations (9) and (11) boil down to:

$$\lambda_0 = -f(0) \frac{U^p_x(x_0, m_0|0)}{U^a_x(x_0, m_0|0)} = -f(0) \frac{U^p_m(x_0, m_0|0)}{U^a_m(x_0, m_0|0)} \Rightarrow \frac{U^p_x(x_0, m_0|0)}{U^a_x(x_0, m_0|0)} = \frac{U^a_m(x_0, m_0|0)}{U^a_m(x_0, m_0|0)}$$

so that $(x^*_0, m^*_0) \in CC (0)$, and equations (10) and (12) boil down to:

$$\lambda_0 = f(1) \frac{U^p_x(x_1, m_1|1)}{U^a_x(x_1, m_1|0)} = f(1) \frac{U^p_m(x_1, m_1|1)}{U^a_m(x_1, m_1|0)} \Rightarrow \frac{U^p_x(x_1, m_1|1)}{U^a_x(x_1, m_1|1)} = \frac{U^a_m(x_1, m_1|1)}{U^a_m(x_1, m_1|0)}$$

and thus $(x^*_1, m^*_1) \in CC (1)$.

Proof of Proposition 4.4. Consider equation (15). Note that since $\lambda_0 > 0$ and $U^a_m(x_0, m_0|0) > 0$, then (15) implies that $U^p_m(x_0, m_0|0) < 0$. Thus, there is overfunding in state 0; i.e., $m_0 > \hat{m}$. Also, since $U^p_x(x_0, m_0|0) \geq 0$ iff $x_0 \leq 0$ and $U^a_x(x_0, m_0|0) \geq 0$ iff $x_0 \leq b$, (15) implies that $x_0 \in (0, b)$, so the optimal policy in state 0 distorts in favor of the agent.

Consider next expression (16). Note that $\lambda_0 > 0$ and $U^a_m(x_1, m_1|0) > 0$ in (16) imply that $U^p_m(x_1, m_1|1) > 0$. Thus there is underfunding in state 1; i.e., $m_1 < \hat{m}$. And from the first equality, we have that $U^p_x(x_1, m_1|1)$ and $U^a_x(x_1, m_1|0)$ have to have the same sign, so either $x_1 < \min\{1, b\}$ or $x_1 > \max\{1, b\}$. So suppose first that $b < 1$. Then either $x_1 < b$ or $x_1 > 1$. However, it cannot be that $x_1 < b$. To see this, note that in this case the symmetric point about the ideal point of the agent $(2b - x_1, m_1)$ would give the agent the same payoff but would increase the utility of the principal. Thus such $(x_1, m_1) \notin CC (1)$. It follows that if $b < 1$, then $x_1 > 1$. Suppose next that $b > 1$. Then either $x_1 < 1$ or $x_1 > b$, but by a similar argument as before, it must be that $x_1 < 1$.

Finally, we show that if $f(0) \in (0, 1)$, the optimal incentive compatible solution entails distortions in both states: $(x^*_0, m^*_0) \neq (\hat{x}_0, \hat{m}_0)$ for $\omega = 0, 1$. Equivalently, we need to show that if $f(0) \in (0, 1)$, the solution to Problem 2 satisfies $u^* \in (U^a(\hat{x}_0, \hat{m}_0|0), U^a(\hat{x}_1, \hat{m}_1|0))$. We will show that if $f(0) \neq 0$ then $u < U^a(\hat{x}_1, \hat{m}_1|0)$. A similar argument proves the opposite direction. We have that:

$$\frac{\partial}{\partial u} U^p(\tilde{x}^\omega(u), \tilde{m}^\omega(u)|\omega) = u^p(\ell^P(\tilde{x}^\omega(u), \omega), \tilde{m}^\omega(u))2(\tilde{x}^\omega(u) - \omega) \tilde{x}^\omega(u)$$

$$+ \left[ u^p(\ell^P(\tilde{x}^\omega(u), \omega), \tilde{m}^\omega(u)) - \gamma_m(\tilde{m}^\omega(u)) \right] \tilde{m}^\omega(u),$$

and

$$\frac{\partial}{\partial u} U^p(\tilde{x}^1(u), \tilde{m}^1(u)|1) \bigg|_{U^a(\hat{x}_1, \hat{m}_1|0)} = [u^p_m(0, \hat{m}_1) - \gamma_m(\hat{m}_1)] \hat{m}_u^0(U^a(\hat{x}_1, \hat{m}_1|0)) = 0,$$
where the last part follows by the definition of \( \hat{m}_1 \), i.e., the FOC that this first-best has to satisfy is \( u''_m(0, \hat{m}_1) - \gamma_m(\hat{m}_1) = 0 \). Next, note that:

\[
\frac{\partial}{\partial u} U^p(\tilde{x}^0(u), \tilde{m}^0(u)|0) \bigg|_{U^a(\tilde{x}_1, \tilde{m}_1|0)} = 2u''_m(1, \hat{m}_1)\tilde{x}_u(U^a(\tilde{x}_1, \hat{m}_1|0)) + [u''_m(1, \hat{m}_1) - \gamma_m(\hat{m}_1)] \tilde{m}_u(U^a(\tilde{x}_1, \hat{m}_1|0)).
\]

We have that \( 2u''_m(1, \hat{m}_1) < 0 \), and since we are overfunding always in state 0, \( u''_m(1, \hat{m}_1) - \gamma_m(\hat{m}_1) < 0 \). Furthermore, since the indifference curve moves in the north-east direction, we have that \( \tilde{x}_u(U^a(\tilde{x}_1, \hat{m}_1|0)) > 0 \) and \( \tilde{m}_u(U^a(\tilde{x}_1, \hat{m}_1|0)) > 0 \). All of this implies that:

\[
\frac{\partial}{\partial u} U^p(\tilde{x}^0(u), \tilde{m}^0(u)|0) \bigg|_{U^a(\tilde{x}_1, \hat{m}_1|0)} < 0,
\]

which means utility can be improved by decreasing \( u \) if \( f(0) \neq 0 \); thus \( u^* < U^a(\tilde{x}_1, \hat{m}_1|0) \).

\[\blacksquare\]

**Lemma A.7.2.** If a policy function \( q(\cdot) = (x(\cdot), m(\cdot)) \) is implementable, \( x(\cdot) \) is nondecreasing.

**Proof of Lemma A.7.2.** The proof follows the standard line, and is included here for completeness. The first-order condition for truth-telling is:

\[
\frac{\partial U^a(\omega, \omega)}{\partial \omega} \bigg|_{\omega = \omega} = U^a_x(x(\omega), m(\omega)|\omega)x'(\omega) + U^a_m(x(\omega), m(\omega)|\omega)m'(\omega) \bigg|_{\omega = \omega} = 0,
\]

or equivalently,

\[
m'(\omega) = -\frac{U^a_x(x(\omega), m(\omega)|\omega)}{U^a_m(x(\omega), m(\omega)|\omega)}x'(\omega).
\]

The second-order condition for no (local) profitable deviations is:

\[
\frac{\partial^2 U(\omega, \omega)}{\partial \omega^2} \bigg|_{\omega = \omega} \leq 0
\]

Differentiating (17) gives

\[
\left[ \frac{\partial^2 U(\omega, \omega)}{\partial \omega^2} + \frac{\partial^2 U(\omega, \omega)}{\partial \omega \partial \omega} \right] dw = 0 \Rightarrow \frac{\partial^2 U(\omega, \omega)}{\partial \omega \partial \omega} = -\frac{\partial^2 U(\omega, \omega)}{\partial \omega^2}
\]

so that (19) is

\[
\frac{\partial^2 U(\omega, \omega)}{\partial \omega \partial \omega} \geq 0
\]

From (17), this is

\[
U^a_{x\omega}(x(\omega), m(\omega)|\omega)x^{a}_{m\omega}(x(\omega), m(\omega)|\omega)m'(\omega) \geq 0
\]

Substituting \( m'(\omega) \) from (18), this is
We want to show that:

**Proof of Lemma A.7.3.**

\[
x'(\omega) \left[ U^a_{x\omega}(x(\omega), m(\omega)|\omega) - U^a_{m\omega}(x(\omega), m(\omega)|\omega) \right] \frac{U^a_x(x(\omega), m(\omega)|\omega)}{U^a_m(x(\omega), m(\omega)|\omega)} \geq 0
\]

Since the bracket is nonnegative from the SCC \( \frac{\partial}{\partial \omega} \left( \frac{U^a_{x\omega}(x, m|\omega)}{U^a_m(x, m|\omega)} \right) \geq 0 \), then \( x'(\omega) \geq 0 \). 

**Lemma A.7.3.** If \( x(\cdot) \) is nondecreasing and (3) holds for all \( \omega \in \Omega \),

\[
U^a(\omega, \omega) \geq U^a(\hat{\omega}, \omega) \quad \text{for all} \quad \omega, \hat{\omega} \in [0, 1].
\]

**Proof of Lemma A.7.3.** We want to show that:

\[
0 \geq \frac{\partial U^a(\hat{\omega}, \omega)}{\partial \hat{\omega}} = U^a_m(x(\hat{\omega}), m(\hat{\omega})|\omega)x'(\hat{\omega}) + U^a_m(x(\hat{\omega}), m(\hat{\omega})|\omega)m'(\hat{\omega}) \quad \forall \omega, \omega' \in [0, 1]
\]

Dividing and multiplying by \( U^a_m(x(\hat{\omega}), m(\hat{\omega})|\omega) \), we have

\[
\frac{\partial U^a(\hat{\omega}, \omega)}{\partial \hat{\omega}} = U^a_m(x(\hat{\omega}), m(\hat{\omega})|\omega) \left[ U^a_x(x(\hat{\omega}), m(\hat{\omega})|\omega)x'(\hat{\omega}) + m'(\hat{\omega}) \right]
\]

By the SCC, if \( \hat{\omega} > \omega \),

\[
\frac{\partial U^a(\hat{\omega}, \omega)}{\partial \hat{\omega}} \leq U^a_m(x(\hat{\omega}), m(\hat{\omega})|\omega) \left[ U^a_x(x(\hat{\omega}), m(\hat{\omega})|\omega)x'(\hat{\omega}) + m'(\hat{\omega}) \right] = 0
\]

A similar argument holds for \( \hat{\omega} < \omega \). 

**Remark A.7.4.** The Hamiltonian for problem (PP) is

\[
\mathcal{H} = U^p(x, m|\omega)f(\omega) - \lambda_1 \frac{U^a_x(x, m|\omega)}{U^a_m(x, m|\omega)} y + \lambda_2 y
\]

The necessary and sufficient conditions for a fully separating solution are that there exist \( \lambda_1 \geq 0 \) and \( \lambda_2 \geq 0 \) such that:

\[
m' = \mathcal{H}_{\lambda_1} = -\frac{U^a_x(x, m|\omega)}{U^a_m(x, m|\omega)} y
\]

\[
x' = \mathcal{H}_{\lambda_2} = y
\]

\[
0 = \mathcal{H}_y = -\lambda_1 \frac{U^a_x(x, m|\omega)}{U^a_m(x, m|\omega)} + \lambda_2
\]

\[
\lambda'_1 = -\mathcal{H}_m = -U^p_m(x, m|\omega)f(\omega) + \lambda_1 y \frac{\partial}{\partial m} \left( \frac{U^a_x(x, m|\omega)}{U^a_m(x, m|\omega)} \right)
\]
\( \lambda_2' = -\mathcal{H}_x = -U_x^p(x, m|\omega)f(\omega) + \lambda_1 y \frac{\partial}{\partial x} \left( \frac{U_x^a(x, m|\omega)}{U_m^a(x, m|\omega)} \right), \)

\( 0 = \mu m, \)

with initial conditions \( m(0) = m_0 \) and \( x(0) = x_0 \) and transversality conditions \( \lambda_1(1) = 0 \) and \( \lambda_2(1) = 0 \), and \( \lambda_1(0) = 0 \) and \( \lambda_2(0) = 0 \). From the Pontryagin Maximum Principle (for example, see Zeidler (1985) Theorem 48.C), any optimum for the Principal satisfies the Euler-Lagrange equations above. Moreover, the optimal control problem in equations (20-25) satisfies the weak Mangasarian sufficient condition for a maximum (the problem is in general weakly concave), and thus a solution to (20-25) is a global maximizer.

Proof of Lemma 4.5. The proof follows a similar argument in Krishna and Morgan (2008).

Suppose, to the contrary, that there exists an \( \omega \) such that \( x(\omega) > b + \omega \). Consider (22). Since \( x(\omega) > b + \omega \), we have \( U_x^a(x, m|\omega) < 0 \). Suppose first that \( \lambda_1 > 0 \). Since \( U_m^a(x, m|\omega) > 0 \) and \( \lambda_2 \geq 0 \), we have

\[-\lambda_1 \frac{U_x^a(x, m|\omega)}{U_m^a(x, m|\omega)} + \lambda_2 > 0\]

which is a contradiction (this expression has to equal zero by (22)). Suppose then that \( \lambda_1 = 0 \). Then from (24)

\[ \lambda_2' = -U_x^p(x, m|\omega)f(\omega) > 0 \Rightarrow \lambda_2(\omega) > 0, \]

which again contradicts (22). ■

Lemma A.7.5. The payoffs of principal and agent in the solution \( \{x(\cdot), m(\cdot)\} \) to the principal’s problem, \( U^p(x(\omega), m(\omega)|\omega) \) and \( U^a(x(\omega), m(\omega)|\omega) \), are continuous in \( \omega \).

Proof of Lemma A.7.5. To see that \( U^a(x(\omega), m(\omega)|\omega) \) is continuous, assume by way of contradiction that there exists some \( \varepsilon > 0 \), such that for all \( \delta > 0 \) sufficiently small, \( |U^a(x(\omega), m(\omega)|\omega) - U^a(x(\omega - \delta), m(\omega - \delta)|\omega - \delta)| > \varepsilon \). Then since \( U^a(x, m|\omega) \) is continuous in \( \omega \), we have that for \( \delta > 0 \) sufficiently small:

\[ 0 \leq U^a(x(\omega), m(\omega)|\omega) - U^a(x(\omega), m(\omega)|\omega - \delta) < \varepsilon, \]

\[ 0 \leq U^a(x(\omega - \delta), m(\omega - \delta)|\omega - \delta) - U^a(x(\omega - \delta), m(\omega - \delta)|\omega) < \varepsilon, \]

where the absolute values are not needed by truth-telling. But then, if \( U^a(x(\omega), m(\omega)|\omega) - U^a(x(\omega - \delta), m(\omega - \delta)|\omega - \delta) > \varepsilon \):

\[ U^a(x(\omega - \delta), m(\omega - \delta)|\omega - \delta) < U^a(x(\omega), m(\omega)|\omega) + \varepsilon \]

\[ < U^a(x(\omega), m(\omega)|\omega - \delta), \]
which is a contradiction since truth-telling fails for $\omega - \delta$. Similarly, if $U^a(x(\omega - \delta), m(\omega - \delta) | \omega - \delta) - U^a(x(\omega), m(\omega) | \omega) > \varepsilon$, then truth-telling will fail for type $\omega$.

To see that $U^p(x(\omega), m(\omega) | \omega)$ is continuous in $\omega$, assume by way of contradiction that there exists some $\varepsilon > 0$, such that for all $\delta > 0$,

$$U^p(x(\omega), m(\omega) | \omega) - U^p(x(\omega - \delta), m(\omega - \delta) | \omega - \delta) > \varepsilon$$

(the other case follows similarly). Clearly this implies that either $x$ or $m$ or both are discontinuous at $\omega$. Since $U^a(x(\omega'), m(\omega') | \omega)$ is continuous in $\omega'$ around $\omega' = \omega$, for all $\eta > 0$, there exists a $\delta > 0$ such that for any $\omega' \in (\omega - \delta, \omega)$ there exists some $\tilde{x}(\omega'), \tilde{m}(\omega')$ such that:

$$U^a(x(\omega'), m(\omega') | \omega') = U^a(\tilde{x}(\omega'), \tilde{m}(\omega') | \omega') \text{, and}$$

$$|\tilde{x}(\omega') - x(\omega)| < \eta,$$

$$|\tilde{m}(\omega') - m(\omega)| < \eta,$$

and which preserves local IC. Since local IC implies global IC by the single-crossing property changing $x, m$ to $\tilde{x}, \tilde{m}$ respects incentive constraints. Furthermore, by the continuity of $U^p(x, m|\omega)$ in $x, m$ we have that for some $\delta' > 0$, and $\omega' \in (\omega, \omega - \delta')$ we have:

$$|U^p(x(\omega), m(\omega) | \omega) - U^p(\tilde{x}(\omega'), \tilde{m}(\omega') | \omega)| < \frac{\varepsilon}{2}, \text{ and}$$

$$|U^p(\tilde{x}(\omega'), \tilde{m}(\omega') | \omega') - U^p(\tilde{x}(\omega'), \tilde{m}(\omega') | \omega)| < \frac{\varepsilon}{2},$$

so that by the triangle inequality:

$$|U^p(x(\omega), m(\omega) | \omega) - U^p(\tilde{x}(\omega'), \tilde{m}(\omega') | \omega')| < \varepsilon.$$

But then for all $\omega' \in (\omega, \omega - \delta')$:

$$U^p(\tilde{x}(\omega'), \tilde{m}(\omega') | \omega') > U^p(x(\omega), m(\omega) | \omega) - \varepsilon$$

$$> U^p(x(\omega'), m(\omega') | \omega'),$$

thus since $f$ has full support the proposed $x, m$ cannot be optimal. □

**Proof of Lemma 4.8.** Let $x, m$ be a solution to the principal’s problem. By monotonicity, we know that $x$ and $m$ have to be differentiable and hence continuous almost everywhere. At points of differentiability, we have that:

$$MRS^p_{xm}(\omega) = \frac{U^p_x(x(\omega), m(\omega) | \omega)}{U^p_m(x(\omega), m(\omega) | \omega)} = \frac{U^a_x(x(\omega), m(\omega) | \omega)}{U^a_m(x(\omega), m(\omega) | \omega)} = MRS^a_{xm}(\omega),$$

which is a tangency condition between the indifference curves of the principal and agent.

Note that for our specified utility functions generically there is at most one point of
tangency of indifference curves. This is also generically true for other utility functions. To be precise, note that for the indifference curve \( v_a = U^a(x(\omega), m(\omega)|\omega) \) we have:

\[
m = v_a \exp \left( -\beta (x - \omega - b) \right),
\]

where we are restricting attention to the relevant range, i.e., \( x < \omega + b \). Thus on this indifference curve:

\[
\frac{dm}{dx} = -\beta v_a \exp \left( -\beta (x - \omega - b) \right).
\]

Similarly, for the indifference curve of the principal \( v_p = U^p(x(\omega), m(\omega)|\omega) \), we have:

\[
\frac{dm}{dx} = \frac{\eta(x - \omega) \exp \left( -\frac{\eta}{2}(x - \omega)^2 \right) \left( \pm 1 - \sqrt{1 - 2\gamma v_p \exp(\eta(x - \omega)^2)} \right)}{\gamma \sqrt{1 - 2\gamma v_p \exp(\eta(x - \omega)^2)}}.
\]

It is clear that equality of these expressions cannot hold for multiple \( x \) when \( v_a \) and \( v_p \) are fixed, unless very special choices are made for utility function parameters, e.g., \( \beta = 0 \), \( \eta = 0 \). Thus generically, there is at most one point of tangency. We further note that the tangency condition is differentiable and hence continuous.

Since the tangency condition is continuous, and \( x, m \) is differentiable in a neighborhood above and below \( \tilde{\omega} \):

\[
\begin{align*}
U^p_p(x(\tilde{\omega}_-), m(\tilde{\omega}_-) | \tilde{\omega}_-) & = U^a_p(x(\tilde{\omega}_-), m(\tilde{\omega}_-) | \tilde{\omega}_-), \\
U^p_m(x(\tilde{\omega}_-), m(\tilde{\omega}_-) | \tilde{\omega}_-) & = U^a_m(x(\tilde{\omega}_-), m(\tilde{\omega}_-) | \tilde{\omega}_-), \\
U^p_p(x(\tilde{\omega}_+), m(\tilde{\omega}_+) | \tilde{\omega}_+) & = U^a_p(x(\tilde{\omega}_+), m(\tilde{\omega}_+) | \tilde{\omega}_+), \\
U^p_m(x(\tilde{\omega}_+), m(\tilde{\omega}_+) | \tilde{\omega}_+) & = U^a_m(x(\tilde{\omega}_+), m(\tilde{\omega}_+) | \tilde{\omega}_+).
\end{align*}
\]

But \( U^a(x(\omega), m(\omega)|\omega) \) and \( U^p(x(\omega), m(\omega)|\omega) \) are continuous in \( \omega \) by lemma A.7.5, we have:

\[
\begin{align*}
U^a(x(\tilde{\omega}_+), m(\tilde{\omega}_+) | \tilde{\omega}_+) & = U^a(x(\tilde{\omega}_-), m(\tilde{\omega}_-) | \tilde{\omega}_-), \text{ and} \\
U^p(x(\tilde{\omega}_+), m(\tilde{\omega}_+) | \tilde{\omega}_+) & = U^p(x(\tilde{\omega}_-), m(\tilde{\omega}_-) | \tilde{\omega}_-),
\end{align*}
\]

thus the tangency would have to occur on the same indifference curves in the limit. But since there is a unique tangency point for type \( \tilde{\omega} \), we have that \( x(\tilde{\omega}_-) = x(\tilde{\omega}_+) \) and \( m(\tilde{\omega}_-) = m(\tilde{\omega}_+) \).

**Proof of Theorem 4.9.** Part 1. First we show that in the solution we cannot have either \( x(\omega) > \omega \) for all \( \omega \in [0, 1] \) or \( x(\omega) < \omega \) for all \( \omega \in [0, 1] \). In the exponential case, the Euler-Lagrange equation (24) becomes

\[
(26) \quad \lambda'_2 = \eta(x - \omega)m \exp \left( -\frac{\eta}{2}(x - \omega)^2 \right) f(\omega)
\]
Substituting (4) in (26) we get
\[ \lambda_2' = \eta(x - \omega)m_0 \exp\left( -\frac{\eta}{2}(x - \omega)^2 - \beta[x(\omega) - x_0] \right) f(\omega), \]
so that
\[ \lambda_2(\omega) = \eta m_0 \int_0^\omega (x - v) \exp\left( -\frac{\eta}{2}(x - v)^2 - \beta[x - x_0] \right) f(v)dz \]
Note then that the transversality condition \( \lambda_2(1) = 0 \) gives
\[ \int_0^1 (x - v) \exp\left( -\frac{\eta}{2}(x - v)^2 - \beta[x - x_0] \right) f(v)dv = 0 \]
and the result follows since \( \exp(\cdot) > 0. \)

Part 2. Next, we characterize properties of the optimal separating contract and derive (6). Let \( r(\omega) \equiv (\eta/2)[(x(\omega) - \omega)^2 - x_0^2] - \beta[x(\omega) - x_0] \) and let \( \tilde{r}(\omega) = r(\omega) + (\eta/2)x_0^2. \)
Note that we can write (5) as
\[ \exp(\tilde{r}(\omega)) = \beta + \eta(x(\omega) - \omega) \beta \gamma m_0 \forall \omega \in [0,1]. \]
Imposing the constraint that \( x(0) = x_0 \) in (29), we obtain
\[ m_0 = [1 + (\eta/\beta)x_0] \frac{1}{\gamma} \exp\left( -\frac{\eta}{2}x_0^2 \right) \]
Substituting back in (29), we have
\[ \exp( r(\omega)) = \frac{[\beta + \eta(x(\omega) - \omega)]}{(\beta + \eta x_0)} \forall \omega \in [0,1]. \]

Equation (31) completely characterizes the optimal policy \( x(\cdot) \) as a function of the initial value \( x_0. \) Differentiating (31), we obtain (6). To obtain the optimal \( x_0, \) note that we can now rewrite the principal’s problem as
\[ \max_{x_0} J(x_0) = \int_0^1 U^p \left( x(\omega), \frac{(\beta + \eta x_0)}{\beta \gamma} \exp\left( -\beta[x(\omega) - x_0] - \left(\frac{\eta}{2}\right)x_0^2 \right) |\omega \right) f(\omega)d\omega \quad \text{s.t. (31)}, \]
so at the optimum
\[ \frac{\partial J}{\partial x_0}(x_0) = 0 \]
Equations (4), (30), (31), and (32) completely characterize the optimal fully separating incentive compatible contract for the principal, provided the solution exists.
Part 3. Recall that by (6),
\[
x'(\omega) = \frac{(x(\omega) - \omega) \exp(r(\omega)) - \frac{1}{(\beta + \eta x_0)}}{(x(\omega) - \omega) - \frac{\beta}{\eta} \exp(r(\omega)) - \frac{1}{(\beta + \eta x_0)}}
\]
Note that if \( x(\omega) < \omega \), then the numerator and denominator of (6) are negative. Then \( x'(\omega) < 1 \) if and only if \( \frac{\beta}{\eta} \exp(r(\omega)) > 0 \), which is always the case. It follows that if \( x(\omega') < \omega' \) for some \( \omega' \in [0, 1] \), then \( x(\omega) < \omega \) for all \( \omega \in [\omega', 1] \). Then it must be that \( x_0 > 0 \), for otherwise \( x(\omega) < \omega \) for all \( \omega \in [0, 1] \), contradicting (28). So \( x(\omega) \) starts above \( \omega \) and then must cross \( \omega \) at least once. But note that if it crosses once, it will not go back up. This concludes the proof. ■