

When Is Price Discrimination Profitable?

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We consider a general model of monopoly price discrimination and characterize the conditions under which price discrimination is and is not profitable. We show that an important condition for profitable price discrimination is that the percentage change in surplus (i.e., consumers' total willingness to pay, less the firm's costs) associated with a product upgrade is increasing in consumers' willingness to pay. We refer to this as an increasing percentage differences condition and relate it to many known results in the marketing, economics, and operations management literatures.

Key words: marketing; pricing; segmentation; economics; econometrics; price discrimination; segmented pricing; product line

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1. Introduction

The marketing, economics, and operations management literatures have recognized many ways in which a firm can price discriminate. Examples include product line pricing (e.g., BMW 3 and 5 series), damaged goods (e.g., Intel 486 SX and DX), intertemporal pricing (e.g., Talbot's semiannual sale), service queues (e.g., priority customer support), advance purchase discounts (e.g., airline, rail, and hotel tickets), and coupons (e.g., free-standing inserts). In these examples, firms offer a menu of choices at different prices and allow consumers to select the offer of their choice (i.e., second-degree price discrimination). For example, a customer who shops at Talbot's has the option of purchasing today at the regular price or waiting for the semiannual sale, where waiting may delay consumption or increase the risk that the product will become unavailable.

Given the numerous means available to price discriminate, the fundamental questions a firm faces are whether and how to optimally price discriminate. Within specific applications of price discrimination, the question of how to price discriminate has received considerable attention (e.g., how to price a product line), but much less attention has been paid to the question of whether it is optimal to price discriminate. Yet there are numerous instances where a firm does not offer multiple products, does not use intertemporal pricing, offers a single service queue, does not offer advance purchase discounts, or does not offer coupons.

In this paper, we show that three conditions are necessary for price discrimination to be profitable. Two

of these conditions are well known in marketing and economics literatures: the monotone hazard rate and single crossing conditions. The third condition is that the percentage change in social surplus from product upgrades is increasing in consumers' willingness to pay. We refer to this as an increasing percentage differences condition. This condition depends on both consumer preferences and firm costs and is distinct from the single crossing property, which focuses only on consumer preferences, and the monotone hazard rate, which focuses on the distribution of consumer types.

We present a model in which a firm chooses the prices and qualities of the products it sells, and show that the model seamlessly extends to other forms of price discrimination. We relate the increasing percentage differences condition to existing results in the literatures on product line pricing, intertemporal pricing, damaged goods, and priority service queues. With a single unifying condition, we are able to both replicate and generalize existing results from these literatures.

Our analysis explicitly recognizes the importance that quality constraints play in the profitability of price discrimination, which has not been previously recognized in these literatures. Upper bounds on quality are a natural assumption in many models of price discrimination for at least two reasons: First, firms are endowed with a given product technology or service level, which bounds the maximum level of quality. For example, past research and development investments limit the fastest processor that Intel can offer. Second, and perhaps more importantly, the technologies available for lowering product quality (e.g., coupons, travel

restrictions, disabling product features, and delaying delivery times) are often much richer and more diverse than the technologies available for raising quality. We show that quality constraints are essential because when there are no bounds on quality then the increasing percentage differences condition is always satisfied.

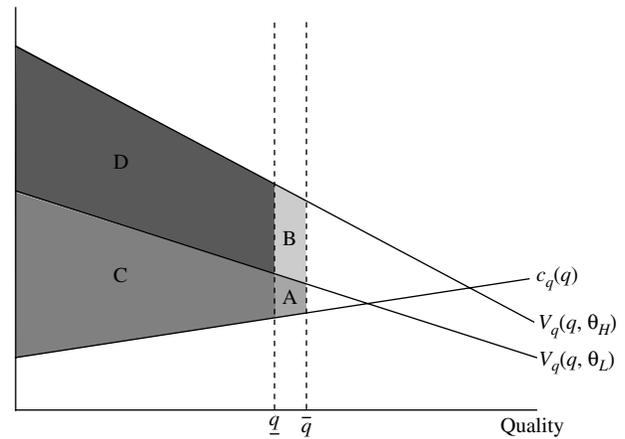
Before we proceed, it is important to note that we take only one of many potential views on what constitutes price discrimination. To explain our view, we first ask, “What would a firm offer if it could directly, or perfectly, segment its customers?” Our model assumptions guarantee that in this situation a firm would offer all customers the same product or service, but at different prices. The ability to perfectly segment its customers means that there is no leakage—customers are unable to purchase at lower prices intended for other customers. We then ask, “What would a firm offer if it could only indirectly, or imperfectly, segment its customers?” In this case, the price the firm can charge to each segment is constrained by its customers’ willingness to buy the product that it is offering for its other segments. If the firm chooses to offer multiple products we interpret this as price discrimination. This definition of price discrimination is appealing because it corresponds to asking whether the solution to the monopolist’s problem is separating or pooling. However, this definition is not always appropriate. For example, when a firm sells different quantities to consumers using uniform pricing, this is not considered price discrimination even when it induces separation.

The remainder of this paper is organized as follows. In §2, we offer a simple example with two types of consumers and two exogenously given product qualities to illustrate the intuition of our increasing percentage differences condition. We extend this example to a monopolist selecting two product qualities subject to an upper bound on quality. Section 3 considers the more general problem in which a monopolist sells to a continuum of consumers and shows that our increasing percentage differences condition is sufficient for price discrimination to be optimal, and that decreasing percentage differences is sufficient for a single price strategy to be optimal. Section 4 relates our increasing percentage differences condition to the literatures on product line pricing, intertemporal price discrimination, damaged goods, and priority service queues. In §5, we conclude the paper with a brief summary and a discussion of when it is reasonable to expect functions to satisfy either our increasing or decreasing percentage differences condition.

2. Two Consumer Types

Consider a single firm that chooses the quality q and the price $p(q)$ of its products. Buyers demand a single

Figure 1 Example of the Two-Type Case



Note. For a given q , the area below the V_q represents the consumer’s willingness to pay, the area below c_q represents the firm’s production cost, and the difference is the surplus created.

unit of the good and their willingness to pay, $V(q, \theta)$, is a function of the product quality q and the buyers’ type θ . There are n_L buyers of type θ_L and n_H buyers of type θ_H . Given the prices, buyers purchase the product that maximizes their consumer surplus, $V(q, \theta) - p(q)$. The firm’s unit cost is $c(q)$. Let $S(q, \theta) = V(q, \theta) - c(q)$ denote the total surplus from selling a product of quality q to a single buyer of type θ . We assume that V and c are continuously differentiable with respect to q , that $\partial V(q, \theta) / \partial q = V_q(q, \theta) > 0$, and that $c_q(q) \geq 0$. Finally, we assume $V(0, \theta) - c(0) \leq 0, \forall \theta$, so the firm will always choose a strictly positive quality for its products.

2.1. Two Exogenous Products

We first consider a simple example in which the firm chooses whether to sell either one or two products with exogenous qualities, q and \bar{q} . We assume that buyers of type θ_H are willing to pay strictly more than buyers of type θ_L , i.e., $V(q, \theta_H) > V(q, \theta_L), \forall q$. Furthermore, we assume that the total surplus is strictly positive for both buyer types and that the total surplus is strictly greater for product \bar{q} than for product q . Therefore, a social planner would produce just the high quality product \bar{q} and sell it to both types. Given our assumptions, the only reason for the firm to offer both products is to extract more surplus from the buyers (i.e., to price discriminate).

A necessary condition for the firm to want to price discriminate is that buyers of type θ_H are willing to pay more for an increase in product quality than buyers of type θ_L or $V(\bar{q}, \theta_H) - V(q, \theta_H) > V(\bar{q}, \theta_L) - V(q, \theta_L)$. Figure 1 depicts an example with this property.¹ Buyers of type θ_H are willing to pay $A + B$

¹ Two other assumptions that are implicit in Figure 1 are not required for our analysis. First, we depict the costs and valuations

(in addition to the extra cost) for the higher quality, but buyers of type θ_L are only willing to pay A (in addition to the extra cost).

If the firm depicted in Figure 1 served all consumers with a single quality \bar{q} at a single price, it would charge $V(\bar{q}, \theta_L)$ and earn $C + A$ on each sale. The type θ_H buyers would capture surplus $D + B$, whereas the type θ_L buyers would capture zero surplus. If, instead, the firm chose to offer both a high and a low quality product, it would charge $V(q, \theta_L)$ for the low quality product (and earn C on each sale) and $V(q, \theta_L) + V(\bar{q}, \theta_H) - V(q, \theta_H)$ for the high quality product (and earn $C + A + B$ on each sale). So relative to selling a single good to all consumers, it would forgo A on each sale to a low type but gain an additional B on each sale to a high type. So the firm's profits would increase as long as $Bn_H > An_L$ or $A/(A + B) < n_H/(n_L + n_H)$.

If the firm depicted in Figure 1 served only the high-type buyers, it would charge $V(\bar{q}, \theta_H)$ and earn $A + B + C + D$ on each sale. The high-type buyers would capture zero surplus. If, instead, the firm chose to offer both high and low quality, its profits would increase as long as the profit earned on the new low-type buyers covered the lower margin on high-type buyers, that is, $Cn_L > Dn_H$ or $C/(C + D) > n_H/(n_L + n_H)$.

It follows that the firm is willing to offer both products if and only if $C/(C + D) > n_H/(n_L + n_H)$ and $A/(A + B) < n_H/(n_L + n_H)$, and is willing to offer both products only if $C/(C + D) > A/(A + B)$ or $(A + B)/(C + D) > A/C$. Hence both products are offered *only* if the ratio of the high-type buyers' total surplus to the low-type buyers' total surplus is increasing in quality. Equivalently, the incremental surplus from an increase in quality as a percentage of the total surplus must be increasing in the buyers' type. We call this condition increasing percentage differences.

More formally, if $(f(x_1, y) - f(x_2, y))/f(x_2, y)$ is increasing in y for all $x_1 > x_2$, then f satisfies increasing percentage differences. Also, if either $f(x_1, y)/f(x_2, y)$ or $\ln f(x_1, y) - \ln f(x_2, y)$ is increasing in y for all $x_1 > x_2$, then f satisfies increasing percentage differences. Because of the close relationship to supermodularity, the increasing percentage differences condition is more often referred to as log supermodularity, and its opposite as log submodularity, and we will use these terms extensively throughout the paper. Finally, note that if $f(x, y)$ is continuously differentiable, then $f(x, y)$ is log supermodular if and only if $f_{xy}f - f_x f_y > 0$.

as continuous functions satisfying $V_{qq} < 0$, and $c_{qq} > 0$, that is, V_q slopes down and c_q slopes up. Second, we treat the area under the functions V_q as the total willingness to pay, and the area under the function and c_q as total cost, so in the figure we are also implicitly assuming $V(0, \theta_H) = V(0, \theta_L) = 0$ and $c(0) = 0$.

A simple numerical example illustrates the increasing percentage differences condition. Suppose $c(q) = 2$, $c(\bar{q}) = 4$, $V(\bar{q}, \theta_H) = 16$, $V(q, \theta_H) = 8$, $V(\bar{q}, \theta_L) = 10$, and $V(q, \theta_L) = 5$, then upgrading quality doubles social surplus for both consumer types: $(16 - 4)/(8 - 2) = 2$ versus $(10 - 4)/(5 - 2) = 2$. Because the percentage increase is the same, price discrimination neither increases nor lowers profits. But, if type θ_H 's valuation for the high quality product is slightly higher, $V(\bar{q}, \theta_H) > 16$, or the cost of high quality is slightly higher, $c(\bar{q}) > 4$, then the increasing percentage differences condition holds and price discrimination increases profits. The example also illustrates that a large disparity in consumers' valuations for a quality increase, in this case eight for type θ_H and five for type θ_L , does not by itself imply that price discrimination is profitable.

The above example demonstrates that the profitability of price discrimination depends on (i) buyers with higher valuations be willing to pay more for an increase in quality, (ii) the distribution of consumer preferences, and (iii) the increasing percentage differences condition or equivalently, the log supermodularity of the total surplus function. In the next section, we formalize the role of quality constraints and show that the increasing percentage differences condition is always satisfied when quality is unconstrained.

2.2. Two Products with a Quality Constraint

We now consider a model in which the firm is free to choose the quality of its products, optimally subject to a constraint that quality be less than or equal to one. As before, we assume that $V(q, \theta_H) > V(q, \theta_L)$, $\forall q$, so high-type buyers value the good more than low-type buyers. We also assume $V(1, \theta) - c(1) > 0$ and $V(1, \theta) - c(1) \geq V(q, \theta) - c(q)$, $\forall q < 1$, so the high quality product is efficient for both types.

As discussed previously, price discrimination is only profitable if the marginal willingness to pay for quality is higher for the type θ_H buyers. This is emphasized in the following lemma:

LEMMA 1. *If $V_q(q, \theta)$ is weakly decreasing in θ and quality is constrained (i.e., $q \leq 1$), then the firm never price discriminates.*

The assumption about consumer preferences in Lemma 1 is the well-known single crossing property, which is distinct from our increasing percentage differences condition. When the single crossing property fails, it is not feasible to charge the high-type buyers a higher price by selling a lower quality good to the low-type buyers. The consumers who value the good more care less about quality, so prices that induce the high-type buyer to purchase the high quality good will also induce the low-type buyer to buy the high quality good (if the low-type buyer purchases at all).

And selling a lower quality good to the consumers who value the good more is not profitable because it lowers the gains from trade with the high-type buyers and does not increase the price the firm can charge to the low-type buyers.²

Henceforth, we assume that $V_q(q, \theta)$ is strictly increasing in θ , $V_{qq}(q, \theta) - c_{qq}(q) < 0$ and that $V_{qq}(q, \theta)$ is weakly increasing in θ . These assumptions guarantee that it is possible to sort consumers into different quality products at different prices but do not guarantee that this is profitable. The firm chooses the quality levels, q_L and q_H , and the prices, p_L and p_H , to maximize its profit subject to the consumers' incentive compatibility and participation constraints. This problem can be written as

$$\begin{aligned} \max_{q_L, q_H, p_L, p_H} \quad & I(V(q_L, \theta_L) - p_L)n_L(p_L - c(q_L)) \\ & + I(V(q_H, \theta_H) - p_H)n_H(p_H - c(q_H)) \quad (1) \\ \text{subject to} \quad & V(q_H, \theta_H) - p_H \geq V(q_L, \theta_H) - p_L, \quad (\text{IC-1}) \\ & V(q_L, \theta_L) - p_L \geq V(q_H, \theta_L) - p_H, \quad (\text{IC-2}) \end{aligned}$$

and $q_L \leq q_H \leq 1$, where I is the indicator function (consumers purchase only if their surplus is nonnegative).

Clearly, any solution to (1) satisfies $q_H = 1$. Suppose the firm chooses to serve both buyer types. It follows that the type θ_L buyer pays $p_L = V(q_L, \theta_L)$ for quality $q_L \leq 1$, and the type θ_H buyer pays $p_H = V(1, \theta_H) - (V(q_L, \theta_H) - V(q_L, \theta_L))$ for quality $q_H = 1$. The firm's choice of quality, \hat{q} , for the type θ_L buyers solves

$$\begin{aligned} \max_{\hat{q} \leq 1} \quad & n_L(V(\hat{q}, \theta_L) - c(\hat{q})) \\ & + n_H(V(1, \theta_H) - c(1) - (V(\hat{q}, \theta_H) - V(\hat{q}, \theta_L))). \quad (2) \end{aligned}$$

The associated first-order condition,

$$\begin{aligned} G(q) = \quad & n_L(V_q(q, \theta_L) - c_q(q)) \\ & + n_H(V_q(q, \theta_L) - V_q(q, \theta_H)) = 0, \quad (3) \end{aligned}$$

is clearly continuous and differentiable. Because $V_{qq} - C_{qq} < 0$ and V_{qq} is weakly increasing in θ , it follows that $G'(q) < 0$. So (3) uniquely defines $\hat{q} \in (0, 1)$ if and only if $G(0) > 0$ and $G(1) < 0$. So selling different products to both types of buyers dominates selling a single product to both types of buyers if and only if $G(\hat{q}) = 0$ for some $\hat{q} \in (0, 1)$.

² The lemma also highlights the importance of the quality constraint. If the consumers with higher valuations valued incremental quality less and there was no quality constraint, then price discrimination would still be profitable. However, the firm would sell a higher quality product to the consumer with the lower valuation. This can easily be shown with a change of variables (e.g., let $\tilde{q} = -q$).

Selling different products to both types of buyers dominates selling a single product to just the type θ_H buyers if and only if

$$\begin{aligned} & n_L(V(\hat{q}, \theta_L) - c(\hat{q})) \\ & + n_H(V(1, \theta_H) - c(1) - (V(\hat{q}, \theta_H) - V(\hat{q}, \theta_L))) \\ & > n_H(V(1, \theta_H) - c(1)), \end{aligned}$$

or equivalently

$$n_L(V(\hat{q}, \theta_L) - c(\hat{q})) - n_H(V(\hat{q}, \theta_H) - V(\hat{q}, \theta_L)) > 0 \quad (4)$$

for some $\hat{q} < 1$.

Equation (4) and $V(0, \theta) - c(0) < 0, \forall \theta$ imply $G(0) > 0$, so when (4) holds, $G(\hat{q}) = 0$ for some $\hat{q} \in (0, 1)$ is equivalent to $G(\hat{q}) = 0$ for some $\hat{q} < 1$, or simply, $G(1) < 0$. So price discriminating dominates both selling a single product to just the type θ_H buyers and selling a single product to both the type θ_L and the type θ_H buyers if and only if (4) holds for some $\hat{q} < 1$, and $G(1) < 0$, or

$$\begin{aligned} G(1) = \quad & n_L(V_q(1, \theta_L) - c_q(1)) \\ & + n_H(V_q(1, \theta_L) - V_q(1, \theta_H)) < 0. \quad (5) \end{aligned}$$

Equations (4) and (5) both hold if and only if

$$\frac{S_q(1, \theta_L)}{S_q(1, \theta_H)} = \frac{V_q(1, \theta_L) - c_q(1)}{V_q(1, \theta_H) - c_q(1)} < \frac{n_H}{n_H + n_L} \quad \text{and} \quad (6)$$

$$\frac{S(\hat{q}, \theta_L)}{S(\hat{q}, \theta_H)} = \frac{V(\hat{q}, \theta_L) - c(\hat{q})}{V(\hat{q}, \theta_H) - c(\hat{q})} > \frac{n_H}{n_H + n_L} \quad (7)$$

for some $\hat{q} < 1$, so a necessary condition for price discrimination to be optimal is

$$\frac{S_q(1, \theta_L)}{S_q(1, \theta_H)} < \frac{S(\hat{q}, \theta_L)}{S(\hat{q}, \theta_H)} \quad \text{for some } \hat{q} < 1. \quad (8)$$

If $S(q, \theta) = V(q, \theta) - c(q)$ is everywhere log submodular, then $S(q, \theta_H)/S(q, \theta_L)$ is decreasing in q and $S_q(1, \theta)/S(1, \theta)$ is decreasing in θ , and it follows that

$$\frac{S_q(1, \theta_L)}{S_q(1, \theta_H)} > \frac{S(1, \theta_L)}{S(1, \theta_H)} > \frac{S(q, \theta_L)}{S(q, \theta_H)} \quad (9)$$

for all $q < 1$, so (8) cannot hold. If $S(q, \theta)$ is everywhere log supermodular, then it follows that $S(q, \theta_H)/S(q, \theta_L)$ is increasing in q and $S_q(1, \theta)/S(1, \theta)$ is increasing in θ , and it follows that (8) does hold. This implies:

PROPOSITION 1. *Suppose $V_q(q, \theta)$ is strictly increasing in θ . Let $N^*(\tilde{q})$ denote the open interval*

$$\left(\frac{V_q(1, \theta_L) - c_q(1)}{V_q(1, \theta_H) - c_q(1)}, \frac{V(\hat{q}, \theta_L) - c(\hat{q})}{V(\hat{q}, \theta_H) - c(\hat{q})} \right),$$

where

$$\hat{q} = \arg \max_{q \leq 1} n_L S(q, \theta_L) + n_H S(1, \theta_H) - n_H (V(q, \theta_H) - V(q, \theta_L)), \quad (10)$$

then the firm will offer multiple qualities if $V(q, \theta) - c(q)$ is everywhere log supermodular and $n_H/(n_H + n_L) \in N^*(\hat{q})$. If $V(q, \theta) - c(q)$ is everywhere log submodular, then the firm offers a single product quality.

This result is perfectly consistent with a two-type version of Mussa and Rosen (1978). In particular, if the quality constraint does not bind, then $q^*(\theta_H) = 1$ implies $V_q(1, \theta_H) - c_q(1) = 0$, and $V(q, \theta) - c(q)$ is log supermodular at $q = 1$. In the absence of a quality constraint, there always exists a distribution of consumer types such that the firm will price discriminate.³

The endogenous quality model emphasizes the importance of the quality constraint in models of price discrimination. Absent a constraint on quality, the firm would match the quality of its products to consumers' needs. But, when quality is constrained, price discrimination may not be profitable because it is more likely that the efficient quality is the same for all consumers.

When quality is constrained, we again see that three conditions are necessary for price discrimination to be profitable. First, the surplus function must be log supermodular (increasing percentage differences). Second, consumer preferences must satisfy the single crossing condition. Third, there must neither be too few nor too many low-type consumers. If there are too few low valuation consumers, the firm sells only to the high valuation consumers, and if there are too many, the firm chooses to sell the same product to both the low and high valuation buyers.

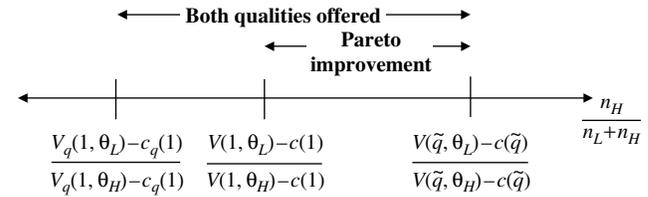
2.3. Welfare

We focus on a specific aspect of welfare: Does price discrimination lead to a Pareto improvement? Both Deneckere and McAfee (1996) and Anderson and Song (2004) showed that indirect, or second-degree, price discrimination could lead to a Pareto improvement. A necessary condition for a Pareto improvement is that price discrimination increases the number of buyers served.

We first characterize necessary and sufficient conditions for a Pareto improvement to occur when there are two types of buyers. A Pareto improvement occurs when both types of buyers are served when price discrimination is feasible, but only the high-type buyer is served when price discrimination is banned.

³ $V_q(1, \theta_H) - c_q(1) = 0$ implies $V_q(1, \theta_L) - c_q(1) < 0$, so the minimum value in N^* is negative or equivalently $[0, \hat{n}] \subset N^*$ for some $\hat{n} \in (0, 1)$. So in the absence of a quality constraint, price discrimination is always profitable if n_L is sufficiently large.

Figure 2 Welfare Analysis



Note. The welfare impact of price discrimination as a function of fraction of high-type buyers in the market.

(If both types are served without discrimination, then any profitable discrimination makes the high-type buyers worse off.) From Proposition 1, when $V - c$ is log supermodular, the range of $n_H/(n_L + n_H)$ for which price discrimination is optimal is

$$\left(\frac{V_q(1, \theta_L) - c_q(1)}{V_q(1, \theta_H) - c_q(1)}, \frac{V(\hat{q}, \theta_L) - c(\hat{q})}{V(\hat{q}, \theta_H) - c(\hat{q})} \right), \quad (11)$$

and when price discrimination is banned, it is optimal to sell only to the high-type buyers when

$$\frac{n_H}{n_L + n_H} > \frac{V(1, \theta) - c(1)}{V(1, \bar{\theta}) - c(1)}, \quad (12)$$

which implies:

PROPOSITION 2. *Given two consumer types, if $V(q, \theta) - c(q)$ is log supermodular in a neighborhood of $q = 1$, $n_H/(n_H + n_L) \in N^*$, and $n_H/(n_H + n_L) > (V(1, \theta_L) - c(1))/(V(1, \theta_H) - c(1))$, then offering multiple qualities results in a Pareto improvement.*

The conditions in Proposition 2 are illustrated graphically in Figure 2 and make it clear that a Pareto improvement is more likely when there are a moderate number of high-type consumers. This occurs whenever a firm shifts from selling to only high-type consumers at a uniform pricing to selling multiple qualities at different prices. Type θ_H consumers enjoy a lower price for the same level of quality, and type θ_L consumers now have access to an affordable low price, or, low quality product, or both.

3. Continuous-Consumer-Type Model

In this section, we analyze a more general model in which there are a continuum of heterogeneous buyers. Buyers' types, θ , are distributed with probability distribution $f(\theta)$ and a continuous cumulative distribution $F(\theta)$ on the interval $[\underline{\theta}, \bar{\theta}]$. We assume that $J(\theta) = (1 - F(\theta))/f(\theta)$ is monotonically decreasing, that is, that the distribution F has a monotone hazard rate. Consumers maximize their consumer surplus, equal to $V(q, \theta) - p(q)$. We assume that $V > 0$, $V_q > 0$, $V_\theta > 0$, and $V_{\theta\theta} < 0$ for all $q \in [0, 1]$ and $\theta \in [\underline{\theta}, \bar{\theta}]$. The firm can produce products of any quality, q , at a unit cost of production $c(q)$, subject to

the constraint that $q \leq 1$. Let $S(q, \theta) = V(q, \theta) - c(q)$ denote the total surplus function. We assume that $S(0, \theta) < 0$ for all θ , so the firm always chooses a strictly positive quality, and that $S_{qq} < 0$. Finally we make the technical assumptions on S (or equivalently, on V) that $S_{q\theta} \geq 0$ and $S_{q\theta\theta} \leq 0$, though it is sufficient to assume that $|S_{q\theta}|$ and $|S_{q\theta\theta}|$ are both small.

We assume that consumers with greater willingness to pay value an increase in product quality more than other consumers. That is, $V_{q\theta} > 0$ for all $q \in [0, 1]$ and $\theta \in [\underline{\theta}, \bar{\theta}]$, which implies that the solution, $(\theta_L^*, q^*(\theta))$, to the social planner's problem,

$$\begin{aligned} \max_{\theta_L, q(\theta)} \int_{\theta_L}^{\bar{\theta}} S(q(\theta), \theta) dF(\theta) \\ \text{subject to } q(\theta) \leq 1, \end{aligned} \quad (13)$$

is a nondecreasing function. We also assume that $\theta_L^* > \underline{\theta}$, which implies that it will never be optimal for the firm to sell to all of the consumers. Note that $S_{qq} < 0$ implies that $q^*(\theta)$ is uniquely defined by the first-order conditions of the Lagrangian associated with (13).

The firm chooses $p(q)$ to maximize its profits. Without loss of generality we assume that the firm uses a direct revelation mechanism, $\{q(\theta), p(\theta)\}$. Because of the incentive constraints, the firm can only implement mechanisms that serve every consumer in some interval $[\theta_L, \bar{\theta}]$, so we can write the firm's problem as

$$\max_{\theta_L, q(\theta), p(\theta)} \int_{\theta_L}^{\bar{\theta}} [p(\theta) - c(q(\theta))] dF(\theta) \quad (14)$$

subject to the incentive compatibility constraints, $q(\theta) \in \arg \max_{q \leq 1} V(q, \theta) - p(q)$, $\forall \theta$, the participation constraints, $\max_q V(q, \theta) - p(q) \geq 0$, $\forall \theta \geq \theta_L$, and the quality constraint, $q(\theta) \leq 1$, $\forall \theta$. Using standard methods (see Fudenberg and Tirole 1991, Myerson 1991, Hermalin 2006, and the Technical Appendix, which is provided in the e-companion⁴), we simplify the firm's problem as follows:

LEMMA 2. The firm's problem can be written as

$$\max_{\theta_L, q(\theta)} \int_{\theta_L}^{\bar{\theta}} [S(q(\theta), \theta) - J(\theta)S_\theta(q(\theta), \theta)] dF(\theta) \quad (15)$$

subject to the constraints that $q(\theta)$ is nondecreasing and $q(\theta) \leq 1$ for all θ .

Let $X(q(\theta), \theta) \equiv S(q(\theta), \theta) - J(\theta)S_\theta(q(\theta), \theta)$ be the integrand of the firm's objective function. Ignoring the constraint that $q(\theta)$ is nondecreasing, the solution, $(q(\theta), \theta_L)$, obtained by pointwise constrained

maximization, is implicitly defined by

$$\begin{aligned} X_q(q(\theta), \theta) \\ = S_{qq}(q(\theta), \theta) - J(\theta)S_{q\theta}(q(\theta), \theta) - \lambda(\theta) = 0, \quad \forall \theta, \end{aligned} \quad (16)$$

where the Lagrangian multiplier, $\lambda(\theta)$, is strictly positive if $q(\theta) = 1$ and 0 otherwise; and by

$$X(q(\theta_L), \theta_L) = S(q(\theta_L), \theta_L) - J(\theta_L)S_\theta(q(\theta_L), \theta_L) = 0. \quad (17)$$

Using the implicit function theorem, (16) implies that $q(\theta)$ is either strictly increasing or equal to one for all θ if $X_{qq} < 0$ and $X_{q\theta} > 0$, both of which follow from our stated assumptions on S and J . So the assumption that the first constraint does not bind is without loss of generality. Also, by the envelope theorem the total derivative of X with respect to θ is $(1 - J'(\theta))S_\theta - J(\theta)S_{\theta\theta}$, which is strictly positive since $J'(\theta) < 0$, $S_\theta = V_\theta > 0$ and $S_{\theta\theta} = V_{\theta\theta} < 0$. So (17) implies

$$S(q(\theta), \theta) - J(\theta)S_\theta(q(\theta), \theta) > 0, \quad \forall \theta \in (\theta_L, \bar{\theta}], \quad (18)$$

and Equations (16) and (18) imply that a necessary condition for $q(\theta) < 1$ (and equivalently, $\lambda(\theta) = 0$) for some $\theta \in (\theta_L, \bar{\theta})$ is

$$\frac{S_{q\theta}(q(\theta), \theta)}{S_{qq}(q(\theta), \theta)} = \frac{1}{J(\theta)} > \frac{S_\theta(q(\theta), \theta)}{S(q(\theta), \theta)} \quad (19)$$

or equivalently, that $S(q, \theta)$ is log supermodular at $(q(\theta), \theta)$. Summarizing, we have:

PROPOSITION 3. If $S(q, \theta)$ is log submodular for all $q < 1$ and all θ , then the firm's optimal strategy is to produce a single product, i.e., $q(\theta) = 1$ for all θ .

It also follows that $q(\theta) < 1$ for some θ if $S(q, \theta)$ is log supermodular in a neighborhood of $(1, \theta)$. Equations (16) and (17) imply

$$\frac{S_{q\theta}(q(\theta_L), \theta_L)}{S_q(q(\theta_L), \theta_L) - \lambda(\theta_L)} = \frac{1}{J(\theta_L)} = \frac{S_\theta(q(\theta_L), \theta_L)}{S(q(\theta_L), \theta_L)} \quad (20)$$

or

$$\begin{aligned} S_{q\theta}(q(\theta_L), \theta_L)S(q(\theta_L), \theta_L) - S_q(q(\theta_L), \theta_L)S_\theta(q(\theta_L), \theta_L) \\ = -\lambda(\theta_L)S_\theta(q(\theta_L), \theta_L). \end{aligned} \quad (21)$$

The right-hand side of (21) is nonpositive, so if $S(q, \theta)$ is log supermodular at $(1, \theta_L)$, (21) implies $q(\theta_L) < 1$. That is, the firm produces multiple products.⁵ Formally we state this result as follows:

PROPOSITION 4. If $S(q, \theta)$ is log supermodular at $(1, \theta)$ for all θ , then the firm's optimal strategy is to produce multiple products, i.e., $q(\theta)$ is strictly increasing for some θ .

⁴ An electronic companion to this paper is available as part of the online version that can be found at <http://mansci.journal.informs.org/>.

⁵ Johnson and Myatt (2003) prove a mathematically similar result for the product range of a multiproduct monopolist, but they consider an exogenous and finite product space, and they do not link their result to the existing literature on price discrimination.

As in the two-product model, Propositions 3 and 4 are perfectly consistent with Mussa and Rosen (1978), Moorthy (1984), and related papers on firms' product line decision without a quality constraint. If the quality constraint $q \leq 1$ never binds, that is, if $S_q(1, \theta) \leq 0$ for all θ , then $S_q(1, \theta)S_\theta(1, \theta) \leq 0$. This implies S is log supermodular at $(1, \theta)$ for all θ and the firm produces multiple products. On the other hand, when the constraint is strictly binding then the efficient quality for every consumer is the same. So the firm offers multiple products only as a mechanism for increasing its profits. In this case, it isn't always profitable to price discriminate.

We complete our analysis by briefly considering the case in which consumers with greater willingness to pay value an increase in product quality less than other consumers, that is $V_{q\theta} < 0$. If the single crossing property fails and the quality constraint binds, then a firm is unable to profitably price discriminate. Offering a lower quality product to the consumer with a greater willingness to pay is never profitable. Once again, this emphasizes the role of the quality constraint because price discrimination would still be profitable absent the constraint.⁶

The other important necessary condition is that the distribution of consumer types satisfies the monotone hazard rate property. At first glance, this assumption appears far less restrictive than the analogous assumption in the two-type model because the monotone hazard rate property is satisfied for many distributions. However, this is misleading because we are also assuming that the distribution is continuous (i.e., no mass points). Continuity together with the monotone hazard rate precludes bunching of consumers at $q < 1$. We recognize that the bunching of consumers may provide another rationale for uniform pricing and that our results are limited to distributions that satisfy the monotone hazard rate condition.⁷

Also, it is important to emphasize that not all functions are everywhere log supermodular or everywhere log submodular, so some functions may not be covered by Propositions 3 or 4. We elaborate on this point in the conclusion.

The welfare results in the continuous-type case are more subtle than in the two-type model. Suppose all

⁶ Formally, the firm's problem can be written as (15) subject to the constraints that $q(\theta)$ is *nonincreasing* and $q(\theta) \leq 1$ for all θ . Equations (15) and (16) follow, but now $X_{q\theta} < 0$, so $q(\theta)$ is either strictly decreasing or equal to one for all θ . Equations (17)–(19) also follow, but since $S_{q\theta} < 0$, $S(q, \theta)$ is log submodular and the firm always produces a single product.

⁷ In many other models the monotone hazard rate assumption prevents bunching. In our model, even with this assumption, bunching still occurs at $q = 1$. However, the monotone hazard rate assumption prevents bunching between $q(\theta_L)$ and 1. That is, if type θ purchases product $q(\theta)$ and $q(\theta) < 1$, then no other consumer purchases the same product.

buyer types $\theta > \theta^*$ are served under uniform pricing. If fewer consumers are served under price discrimination than under uniform pricing, then price discrimination cannot be a Pareto improvement. But, if the market expands and more customers are served, then it seems plausible that price discrimination could be Pareto improving. Although it is clear that all consumer types $\theta \leq \theta^*$ are weakly better off with price discrimination, consumers with types $\theta > \theta^*$ may or may not be better off. Establishing sufficient conditions for a Pareto improvement in the continuous-type model remains a task for future work.

4. Applications

Although our results are developed in the context of product line pricing, they readily generalize to other types of indirect, or second-degree, price discrimination. These include versioning information goods (Varian 1995, 2001; Shapiro and Varian 1998; Bhargava and Choudhary 2001, 2008), advance purchase discounts (Shugan and Xie 2001; Gale and Holmes 1992, 1993), and coupons (Anderson and Song 2004, Nevo and Wolfram 2002, Gerstner and Hess 1991). In this section, we illustrate the relevance of our increasing percent differences condition (i.e., log supermodularity) by revisiting some important contributions in the price discrimination literature on product line pricing, intertemporal pricing, damaged goods, and priority queuing systems.

4.1. Product Line Price Discrimination

Our model is directly applicable to an extensive literature in marketing on product line design. Much of this literature has focused on the question of how to optimally price a product line (Reibstein and Gatignon 1984, Dobson and Kalish 1988, Moorthy 1984, Zenor 1994) or how to develop an optimal quantity discount schedule (Oren et al. 1984). These papers begin with the premise that offering a product line or quantity discount is optimal and then tackle the question of pricing. In contrast, our paper seeks to address the question of whether to offer a single product or a product line.

We illustrate how our log supermodularity condition relates to Villas-Boas' (1998) research on product line design in a channel. We relate Propositions 1 and 2 from Villas-Boas (1998) to our increasing percentage differences condition, then derive a corollary that provides a necessary condition for product line price discrimination in a vertical channel.

Villas-Boas (1998) considers a vertical channel with a single manufacturer and a single retailer and investigates how vertical channel incentives affect the product line offering. For a given set of manufacturer product qualities, \bar{q} and \underline{q} , and manufacturer prices,

$\bar{\omega}$ and $\underline{\omega}$, the problem faced by the retailer is identical to the problem faced by the firm in our model. The retailer faces consumers with valuations $V(q, \theta)$, where $q \in (\bar{q}, \underline{q})$ and $\theta \in (\bar{\theta}, \underline{\theta})$. There are γ consumers of type $\underline{\theta}$ and $1 - \gamma$ consumers of type $\bar{\theta}$.

Proposition 1 of Villas-Boas (1998) shows that if a naive manufacturer offers a product line that is optimal for the vertically integrated channel, then the retailer does not adopt both qualities (i.e., retail price discrimination is not profitable). To relate this finding to our model, note that the manufacturer offers product \bar{q} at price $\bar{\omega} = V(\bar{q}, \theta_H) - V(q, \theta_H) + V(q, \theta_L)$ and product \underline{q} at price $\underline{\omega} = V(\underline{q}, \theta_L)$. This implies that $S(q, \theta) = V(q, \theta) - \omega$ is log submodular, which is readily seen from Equation (8). The numerator on the right-hand side of Equation (8) is zero and so the surplus function is always log submodular.

Proposition 2 of Villas-Boas (1998) characterizes conditions for a retailer to adopt both manufacturer products (see Equations (3) and (4) in Villas-Boas 1998). It is straightforward to show these conditions are equivalent to the conditions $Bn_H > An_L$ and $Cn_L > Dn_H$ in our two consumer-type example and imply log supermodularity as a necessary condition.

More generally, our increasing percentage differences condition implies the following corollary:

COROLLARY 1. *In a vertical channel with one manufacturer and one retailer selling two products, the manufacturer's qualities, \bar{q} and \underline{q} , and wholesale prices, $\bar{\omega}$ and $\underline{\omega}$, must satisfy*

$$\frac{V(\bar{q}, \theta_H) - \bar{\omega}}{V(\underline{q}, \theta_H) - \underline{\omega}} > \frac{V(\bar{q}, \theta_L) - \bar{\omega}}{V(\underline{q}, \theta_L) - \underline{\omega}}$$

for a retailer to offer the full product line.

4.2. Intertemporal Price Discrimination

Another common type of price discrimination is intertemporal discounts. Firms charge higher prices to their less patient customers and lower prices to their more patient customers simply by decreasing price over time. We illustrate how our results relate to two seminal papers in this literature: Stokey (1979) and Salant (1989).

Stokey (1979) considers a monopolist with unit cost of production $k(t) = \kappa\delta^t$ selling to consumers with utility functions $U(\theta, t) = \theta\delta^t$. A well-known result from this model is that intertemporal price discrimination is never optimal. We now show that this important result follows immediately from Proposition 3.

A monopolist chooses a menu of prices paid at time 0 and delivery times, subject to the constraint that $t \geq 0$, to maximize profits. Similar to Salant (1989), we use a change of variables, $q = \delta^t$, so that $V(\theta, q) = \theta q$ and costs $c(q) = cq$. With this transformation, the firm's problem is to choose the profit-maximizing menu of prices and qualities subject to the

constraint that $q \leq 1$. Clearly, $V(\theta, q) - c(q) = \theta q - cq$ is not log supermodular, and $q = 1$ is the optimal quality for all θ . So by Proposition 3, intertemporal price discrimination is never optimal, even though it is clearly feasible.

Salant (1989) sought to explain the apparently contradictory findings that product line price discrimination is always optimal (Mussa and Rosen 1978) and that intertemporal price discrimination is not optimal (Stokey 1979). Salant's was the first paper to emphasize that upper bounds on quality may cause firms to forgo price discrimination.

Salant (1989) made it easier to see that intertemporal price discrimination is optimal with more general cost functions, such as $k(t) = \kappa(t)\delta^t$. After a change of variables, this implies $c(q) = \kappa(\log q/\log \delta)q$, and Salant showed that $c_q(q) > c(q)/q$ was a necessary condition for price discrimination. To relate this to our model, we note that the surplus function, $V(\theta, q) - c(q) = \theta q - c(q)$, is log supermodular if and only if $c_q(q) > c(q)/q$. So if the marginal cost of quality is positive and greater than the average cost of quality, then intertemporal price discrimination is profitable.

Proposition 1 implies the following corollary, which generalizes Salant's (1989) sufficient conditions for price discrimination:

COROLLARY 2. *If $V(\theta, q) = \theta q$ and if $c_q(q) > c(q)/q$ for all $q \in (0, 1]$ then offering multiple products is optimal if and only if n_L and n_H satisfy*

$$\frac{\theta_L \bar{q} - c_q(\bar{q})}{\theta_H \bar{q} - c_q(\bar{q})} > \frac{n_H}{n_H + n_L} > \frac{\theta_L - c_q(1)}{\theta_H - c_q(1)}, \quad (22)$$

where \bar{q} is given by (10).

In the Supplemental Appendix (provided in the e-companion), we show that the conditions in Salant (1989) can also be stated in terms of the fraction of high-type buyers in the market. The lower bound in (22) is equivalent to Salant's condition, but the upper bound in (22) is strictly greater than Salant's upper bound. In the Supplemental Appendix, we prove that our sufficient conditions are weaker than Salant's sufficient conditions. In addition, we show that Propositions 3 and 4 enable us to generalize Salant's results for discrete types to a market with a continuum of consumer types.

4.3. Price Discrimination with Damaged Goods

A damaged good is one for which $c_q(q) \leq 0$, that is, it is weakly more expensive to produce lower quality goods. Deneckere and McAfee (1996) derive conditions for optimal price discrimination with damaged goods and demonstrate that it can be both profitable and Pareto improving to offer a damaged good. They assume a continuum of types with unit demands and

restrict attention to two product qualities, q_L and q_H . Consumers have quasilinear utilities $V(q_H, \theta) = \theta$ and $V(q_L, \theta) = \lambda(\theta)$.

The necessary and sufficient condition derived by Deneckere and McAfee (1996) is a special case of our more general condition. Specifically, in Deneckere and McAfee's model, $V(q, \theta) - c(q)$ is log supermodular if and only if

$$\frac{1}{\theta - c_H} > \frac{\lambda'(\theta)}{\lambda(\theta) - c_L}$$

or $\lambda(\theta) - c_L - (\theta - c_H)\lambda'(\theta) > 0$. To see how this is related to the condition derived by Deneckere and McAfee, note that the price a single product firm would charge is $p = V(q_H, \theta) = \theta$, where θ is defined by $\theta - c_H - (1 - F(\theta))/f(\theta) = 0$. So $V(q, \theta) - c(q)$ is log supermodular if and only if $\lambda(\theta) - c_L - (1 - F(\theta))/f(\theta)\lambda'(\theta) > 0$, which is the necessary and sufficient condition for the provision of damaged goods derived by Deneckere and McAfee (1996).⁸

4.4. Price Discrimination with Priority Queuing Systems

Much of the research on priority queuing systems focuses on using them to increase efficiency, but more recent work has emphasized their ability to price discriminate. Afèche (2006) shows that a profit-maximizing firm may be able to benefit from a strategy of deliberately delaying delivery to some consumers in order to increase the price it can charge to others. He considers a model in which type $x \in \{x_1, x_2\}$ consumers derive value $U(x, t) = v - xt$ from consuming the good delivered with delay t (Afèche allows v to be stochastic, but independently distributed). Doing a change of variables, $\theta = -x$ and $q = \delta^t$, yields $V(\theta, q) = v + \theta \ln q / \ln \delta$, which is always log supermodular (and satisfies $V_\theta > 0$, $V_q > 0$ and $V_{q\theta} > 0$). It follows from Proposition 1 that as long as there are enough consumers of each type, and no other source of delay, the firm would always find it profitable to offer the low θ customer a product that is deliberately delayed as this increases the price it can charge the high value customer. However, in Afèche's (2006) paper, demand uncertainty and congestion are major reasons for delays, so the conditions under which deliberate delay is profitable are more restrictive.

The queuing system literature has considered many models in which this price discrimination strategy would not be profitable. In the Supplemental Appendix, we use our results to show that deliberate delay is not profitable in the queuing models of Afèche and Mendelsen (2004) and Van Mieghem

(2000). More generally, although deliberate delay is potentially profitable, whether it is profitable depends on the precise model specification. Our results contribute to the literature on using queuing systems as a mechanism for price discrimination by showing that log supermodularity of the surplus function is a necessary condition (and in the absence of congestion delays, a sufficient condition) for deliberate delay to be profitable.

5. Conclusion

Even when price discrimination is feasible, a firm may forgo this option because it is unprofitable. A firm may choose to sell a single version of their product, choose a price that is constant over time, choose to serve all consumers with a single queue, choose not to offer advance purchase discounts, or choose not to offer coupons. In this paper, we identify three conditions that determine whether these forms of price discrimination are profitable. The first two conditions (i.e., single-crossing condition and monotone hazard condition) are well known in the literature and standard assumptions in models of price discrimination. Derivation of the third condition, which we refer to as an increasing percentage differences condition, is a key contribution of this paper. The condition focuses on the total surplus available in a transaction and, as such, integrates both supply side and demand side conditions that are necessary for price discrimination. We illustrate the relevance of this condition by revisiting some important contributions in the price discrimination literature on product line pricing, intertemporal pricing, damaged goods, and priority queuing systems.

Our results show that log supermodularity of the surplus function affects whether it is profitable to price discriminate. The applicability of our results depends on whether the surplus function is everywhere log supermodular or everywhere log submodular. The applications section of this paper demonstrates that our results apply to many extant models of price discrimination. To illustrate that the results extend to many well-studied utility and cost functions, consider the following example. Let $V(\theta, q) = a + \theta q$ and $c(q) = F + q^b$. We now show that the values of a , b , and F determine whether $S(q, \theta)$ is everywhere log supermodular, everywhere log submodular, or neither. Assume as a base case that $a = 0$, $b = 1$, and $F = 0$; for the base case, $S(q, \theta)$ is weakly supermodular and log submodular. Now consider four alternative cases:

Case 1. If $a > 0$ then $S(q, \theta)$ is strictly log supermodular.

Case 2. If $b > 1$ then $S(q, \theta)$ is strictly log supermodular.

⁸ McAfee (2006) extends Deneckere and McAfee (1996) and explicitly identifies the increasing percentage differences condition as a necessary condition for a damaged goods strategy to be profitable.

Case 3. If $F > 0$, then $S(q, \theta)$ is strictly log submodular.

Case 4. If $b < 1$ then $S(q, \theta)$ is strictly log submodular.

Note, however, that there exist parameter values such that $S(q, \theta)$ is log supermodular for some values of q and θ and log submodular for other values of q and θ (e.g., $F > 0$, $a > 0$), and in these cases Propositions 3 and 4 are less useful for deciding whether price discrimination is optimal.

Although our framework is quite general, there are some important limitations on our analysis. First, we consider only quasilinear utility. Second, we consider only a single dimension of consumer heterogeneity. And finally, we consider only monopoly pricing. Some generalizations to competition are possible (see Johnson and Myatt 2006 for one potential approach), but the most natural models of competition would have both dimensions of horizontal and vertical consumer heterogeneity (see Stole 2005). The last two issues are important because empirical tests of the theory are likely to be performed on firm behavior in competitive environments.

6. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at <http://mansci.journal.informs.org/>.

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References

- Afèche, P. 2006. Incentive-compatible revenue management in queueing systems: Optimal strategic delay and other delaying tactics. Working paper, Rotman School of Management, Toronto.
- Afèche, P., H. Mendelson. 2004. Pricing and priority auctions in queueing systems with a generalized delay cost structure. *Management Sci.* **50**(7) 869–882.
- Anderson, E. T., I. Song. 2004. Coordinating price reductions and coupon events. *J. Marketing Res.* **41**(4) 411–422.
- Bhargava, H. K., V. Choudhary. 2001. Second-degree price discrimination for information goods under nonlinear utility functions. *Proc. 34th Annual Hawaii Internat. Conf. on System Sciences*, Vol. 7. IEEE Computer Society, Washington DC.
- Bhargava, H. K., V. Choudhary. 2008. Research note: When is versioning optimal for information goods? *Management Sci.* **54**(5) 1029–1035.
- Deneckere, R. J., R. P. McAfee. 1996. Damaged goods. *J. Econom. Management Strategy* **5**(2) 149–174.
- Dobson, G., S. Kalish. 1988. Positioning and pricing a product line. *Marketing Sci.* **7**(2) 107–125.
- Fudenberg, D., J. Tirole. 1991. *Game Theory*. MIT Press, Cambridge, MA.
- Gale, I. L., T. J. Holmes. 1992. The efficiency of advance-purchase discounts in the presence of aggregate demand uncertainty. *Internat. J. Indust. Organ.* **10**(3) 413–437.
- Gale, I. L., T. J. Holmes. 1993. Advance-purchase discounts and monopoly allocation of capacity. *Amer. Econom. Rev.* **83**(1) 135–146.
- Gerstner, E., J. D. Hess. 1991. A theory of channel price promotions. *Amer. Econom. Rev.* **81**(4) 872–886.
- Hermalin, B. 2006. Second-degree price discrimination with a continuum of types. Lecture notes, http://www.faculty.haas.berkeley.edu/hermalin/continuous_2nd_degree.pdf.
- Johnson, J., D. Myatt. 2003. Multiproduct quality competition: Fighting brands and product line pruning. *Amer. Econom. Rev.* **93**(3) 748–774.
- Johnson, J., D. Myatt. 2006. Multiproduct cournot oligopoly. *RAND J. Econom.* **37**(3) 583–601.
- McAfee, P. 2007. Pricing damaged goods. Economics Discussion Paper 2007-2, <http://www.economics-ejournal.org/economics/discussionpapers/2007-2>.
- Moorthy, K. S. 1984. Marketing segmentation, self-selection, and product line design. *Marketing Sci.* **3**(4) 288–307.
- Mussa, M., S. Rosen. 1978. Monopoly and product quality. *J. Econom. Theory* **18**(2) 301–317.
- Myerson, R. B. 1991. *Game Theory: Analysis of Conflict*. Harvard University Press, Cambridge, MA.
- Nevo, A., C. Wolfram. 2002. Why do manufacturers issue coupons? An empirical analysis of breakfast cereals. *RAND J. Econom.* **33**(2) 319–339.
- Oren, S., S. Smith, R. Wilson. 1984. Pricing a product line. *J. Bus.* **57**(1) S73–S99.
- Reibstein, D., H. Gatignon. 1984. Optimal product line pricing: The influence of elasticities and cross-elasticities. *J. Marketing Res.* **21**(3) 259–267.
- Salant, S. W. 1989. When is inducing self-selection suboptimal for a monopolist? *Quart. J. Econom.* **104**(2) 391–397.
- Shapiro, C., H. R. Varian. 1998. Versioning: The smart way to sell information. *Harvard Bus. Rev.* (November–December) 92–105.
- Shugan, S., J. Xie. 2001. Electronic tickets, smart cards, and online prepayment: When and how to advance sell. *Marketing Sci.* **20**(3) 219–243.
- Stokey, N. L. 1979. Intertemporal price discrimination. *Quart. J. Econom.* **93**(3) 355–371.
- Stole, L. 2005. Price discrimination and competition. M. Armstrong, R. H. Porter, eds. *Handbook of Industrial Organization*, Vol. 3. Elsevier, Amsterdam, 2221–2299.
- Van Mieghem, J. A. 2000. Price and service discrimination in queueing systems: Incentive compatibility of $Gc\mu$ scheduling. *Management Sci.* **46**(9) 1249–1267.
- Varian, H. R. 1995. Pricing information goods. Research Libraries Group, ed. *Symposium on Scholarship of the New Information Environment Proceedings*. Research Libraries Group, Harvard Law School, Cambridge, MA.
- Varian, H. R. 2001. Versioning information goods. B. Kahin, H. R. Varian, eds. *Internet Publishing and Beyond: The Economics of Digital Information and Intellectual Property*. MIT Press, Cambridge, MA.
- Villas-Boas, J. M. 1998. Product line design for a distribution channel. *Marketing Sci.* **17**(2) 156–169.
- Zenor, M. J. 1994. The profit benefits of category management. *J. Marketing Res.* **31**(2) 202–213.