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A comment on: "Revisiting dynamic duopoly with consumer switching costs"

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1. Introduction

In this note, we prove that the equilibrium proposed by Padilla [2, Theorem 1] is not an equilibrium for $c < c^*$. We then characterize a Markov perfect equilibrium (MPE) for all values of c and show that findings on the sustainability of tacit collusion [2, Theorem 3] are unchanged for this MPE. We further show that neither the equilibrium proposed by Padilla nor our MPE is an equilibrium if consumers are forward looking.

2. Erratum

In the game considered by Padilla [2], a unit mass of New Customers enters the market each period and each customer lives for two periods. In each period, two generations of customers co-exist, Old Locked-In Customers and New Customers, and each customer maximizes their current utility. New Customers buy from the lowest price firm; Old Locked-In Customers have switching cost c and buy from the

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firm they previously purchased from provided the price difference is at most c. The MPE proposed in [2, Theorem 1] has two possible states: $s = 0$ and 1. If a firm is in state 1 in period t then the firm sold to a unit mass of New Customers in period $t-1$ and in period t these customers are now Old Locked-In Customers with switching cost c. The competing firm is in state $s = 0$ and has zero Old Locked-In Customers in period t. The proposed MPE is in mixed strategies and has the property that both firms mix over the same support $[\underline{p}, \bar{p}]$. Furthermore, if $c < c^*$ [2, Theorem 1a] then $\bar{p} - \underline{p} = c$ and the mixing distribution of the firm with Old Locked-In Customers (i.e., in state 1) has a mass point of $\frac{1}{2}$ at $\bar{p}$. The proposed strategies do not constitute an equilibrium. To see this note that in equilibrium, if a firm in state 0 offers $\underline{p} + \varepsilon$, where $\varepsilon > 0$, the expected payoff is $\approx \underline{p} + \delta V_1$. A deviation by a firm in state 0 to $p = \underline{p} - \varepsilon$ yields an expected payoff $\approx (3/2)\underline{p} + \delta V_1$. Due to the mass point of $\frac{1}{2}$ at $\bar{p}$ in the competitor's mixing distribution, there is a discrete increase in profits. Therefore, for $c < c^*$ deviations to $p < \underline{p}$ are always profitable for the firm in state 0 and [2, Theorem 1a] is not an equilibrium.

3. Results

We proceed by describing the unique symmetric MPE for the game. A similar problem, in a different setting and different notation, is analyzed in [1]. As we show below, while the support of the equilibrium price distributions in the two states overlap they cannot be identical as assumed in [2]. Thus, in our equilibrium when $c < c^*$ firms not only compete for the New Customers but also for the Old Locked-In Customers. Given this distinction from the MPE in [2], we simply refer to the customers who exhibit switching costs as "Old Customers" rather than "Old Locked-In Customers" for the remainder of this note. Each customer maximizes their current period utility and the continuation payoffs of both firms are given by Eqs. (1a), (1b) and (2) in [2]. Our MPE admits two strategies not considered by [2]. First, for $c < c^*$ we find that a firm in state 1 may charge prices that exceed the upper bound of the competing firm (i.e., $\bar{p}_1 > p_1 > \bar{p}_0$). When offering prices that exceed $\bar{p}_0$, the firm forgoes demand from New Customers, increases its margin on Old Customers, and sells to the Old Customers with probability strictly less than 1. Clearly the reason to offer prices in this range is the increased margin on Old Customers and we refer to this as "harvesting." In the second strategy admissible in our equilibrium for $c < c^*$, a firm in state 0 may charge prices below the lower bound of the competing firm (i.e., $\underline{p}_1 > p_0 > \underline{p}_0$). When offering prices below $\underline{p}_1$, a firm sells to New Customers with certainty and sells to the Old Customers of the competing firm with probability strictly less than 1. The firm in state 0 offers prices in this range to acquire Old Customers from the competing firm, and hence we refer to these strategies as "stealing."

The resulting equilibrium has three regions that are summarized in Table 1. Analogous to [2], these can be characterized by threshold values of switching cost (c^{**} and c^*). For very low values of switching cost ($c \leqslant c^{**}$), a firm in state 1

Table 1
Characterization of equilibrium regions

Region	Lower bound of support	Upper bound of support	Length of support
Region I $(0<c\leqslant c^{**})$	$\underline{p}_1>\underline{p}_0$	$R>\bar{p}_1>\bar{p}_0$	$\bar{p}_s-\underline{p}_s=c$
Region II $(c^{**}<c\leqslant c^{*})$	$\underline{p}_1>\underline{p}_0$	$R=\bar{p}_1>\bar{p}_0$	$\bar{p}_0-\underline{p}_0=c, \bar{p}_1-\underline{p}_1<c$
Region III $(c>c^{*})$	$\underline{p}_1=\underline{p}_0=\underline{p}$	$R=\bar{p}_1=\bar{p}_0=\bar{p}$	$\bar{p}_s-\underline{p}_s<c$

"harvests" and a firm in state 0 "steals" and both firms have a continuous support of length c (Region I). For moderate values of switching cost $(c^{**}<c\leqslant c^{*})$, a firm in state 1 "harvests" but the support has length less than c and an upper bound of R, which is each customer's reservation price. A firm in state 0 "steals" but because stealing is only feasible for $p_0<R-c$ the support has no mass in the range $[R-c, \underline{p}_1]$. For large values of switching cost $(c>c^{*})$ there is neither stealing nor harvesting and the support of both firms is identical and has length less than c (Region III). Region III is identical to [2, Theorem 1b] and further analysis of this region is in the Appendix. The remainder of this note focuses on Regions I and II.

In the MPE, there is exactly one firm in state 0 and one firm in state 1 each period. Firms play mixed strategies $\Gamma_s(p)$ in state s, New Customers buy from the lowest price firm, and Old Customers buy from the firm they purchased from in the previous period as long as its current price does not exceed that of its rival's by more than c. The firm that offers the lowest price in period t is in state 1 in period $t+1$ and the competing firm is in state 0. New Customers in period t become Old Customers in period $t+1$ and exit the market after purchasing in period $t+1$.

The closed form solutions for the equilibrium in Regions I and II are cumbersome and are available from the authors. However, the complete equilibrium can be characterized by the mixing distributions in each region (Tables 2 and 3) and a system of six equations for $V_0, V_1, \underline{p}_1, \underline{p}_2, \bar{p}_1, \bar{p}_2$, where V_s is the continuation payoff in state s.

In both Regions I and II, the following four equations hold:

$$V_0-\delta V_1=\underline{p}_1, \tag{1}$$

$$V_1-\delta V_0=(\underline{p}_0+c), \tag{2}$$

Table 2
Mixing distributions in Region I

Price	$\Gamma_0(p)$	$\Gamma_1(p)$
$\underline{p}_0\leqslant p<\underline{p}_1$	$1-\left[\dfrac{V_1-\delta V_0}{p+c}\right]$	0
$\underline{p}_1\leqslant p<\bar{p}_0$	$\dfrac{2p-V_1(1-\delta)}{p+\delta(V_1-V_0)}$	$\dfrac{p+\delta V_1-V_0}{p+\delta(V_1-V_0)}$
$\bar{p}_0\leqslant p<\bar{p}_1$	1	$2-\left[\dfrac{V_0-\delta V_1}{p-c}\right]$

Table 3
Mixing distributions in Region II

Price	$\Gamma_0(p)$	$\Gamma_1(p)$
$\underline{p}_0 \leqslant p < R - c$	$1 - \left[\frac{V_1 - \delta V_0}{p + c}\right]$	0
$R - c \leqslant p < \underline{p}_1$	$1 - \left[\frac{V_1 - \delta V_0}{\bar{p}_1}\right]$	0
$\underline{p}_1 \leqslant p < \bar{p}_0$	$\frac{2p - V_1(1-\delta)}{p + \delta(V_1 - V_0)}$	$\frac{p + \delta V_1 - V_0}{p + \delta(V_1 - V_0)}$
$\bar{p}_0 \leqslant p < R$	1	$2 - \left[\frac{V_0 - \delta V_1}{p - c}\right]$

$$[V_0 - \delta V_1 - \underline{p}_0][\bar{p}_0 + \delta(V_1 - V_0)] - \underline{p}_0 V_0(1-\delta) = 0, \tag{3}$$

$$[\underline{p}_1 + \delta(V_1 - V_0)][V_1 - \delta V_0] = \bar{p}_1[V_1 - \delta V_0 - \underline{p}_1]. \tag{4}$$

In Region I, $\bar{p}_s - \underline{p}_s = c$ and in Region II $\bar{p}_0 - \underline{p}_0 = c$ and $\bar{p}_1 = R$. Threshold values of c are given by $R - \underline{p} = c^*$ (for $\underline{p}_0 = \underline{p}_1 = \underline{p}$ in Region III) and $R - \underline{p}_1 = c^{**}$ (for $\underline{p}_1$ in Region II).

Padilla [2, Theorem 3] also considers how switching costs affect tacit collusion when firm's punishment strategies can only revert to the MPE. Theorem 3 holds for the MPE characterized in this paper as well. To illustrate, we set $R = 1$ and plot the regions where collusion is sustainable for all values of δ and c (Fig. 1). The dotted line corresponds to Region I, the thin solid line corresponds to Region II, and the thick solid line corresponds to Region III. For $c < 0.25$, the optimal strategy when deviating is $p = 1 - c$ with short-run payoff $\pi_d = 2(1-c)$. Further, $\partial(\pi_d + \delta V_1)/\partial c < 0$ and collusion is easier to sustain as c increases because deviating is less profitable. For $c > 0.25$, the optimal deviation is $p = R - \varepsilon$ and therefore the short-run profit from deviating is independent of c. However, the continuation payoff, V_1, is increasing in c making it harder to sustain collusion as c increases.

Consistent with the assumptions in [2], we analyzed a game with myopic customers that maximize their current period utility. Neither the MPE in this paper nor the MPE proposed by Padilla [2, Theorem 1] is an equilibrium for forward-looking customers (contrary to the claim in [2, footnote 3]). To prove this, note that forward-looking customers will anticipate that the firm with the lowest price in period t is expected to offer a higher price in period $t+1$. For firms i and j define the expected future prices as $E(p_{i,t+1} \mid p_{i,t} < p_{j,t}) \equiv E(p_1)$ and $E(p_{j,t+1} \mid p_{i,t} < p_{j,t}) \equiv E(p_0)$. Then $k \equiv E(p_1) - E(p_0) > 0$ follows from stochastic dominance [2, Corollary 2]. A customer that follows the equilibrium strategy receives a discount of ε in period t but expects to pay a premium of k in period $t+1$. Recognizing this, a forward-looking customer will deviate from the proposed equilibrium and purchase from the *higher* priced firm in period t if $\varepsilon < \delta k$. An equilibrium in this game with forward-looking customers is open to future research.

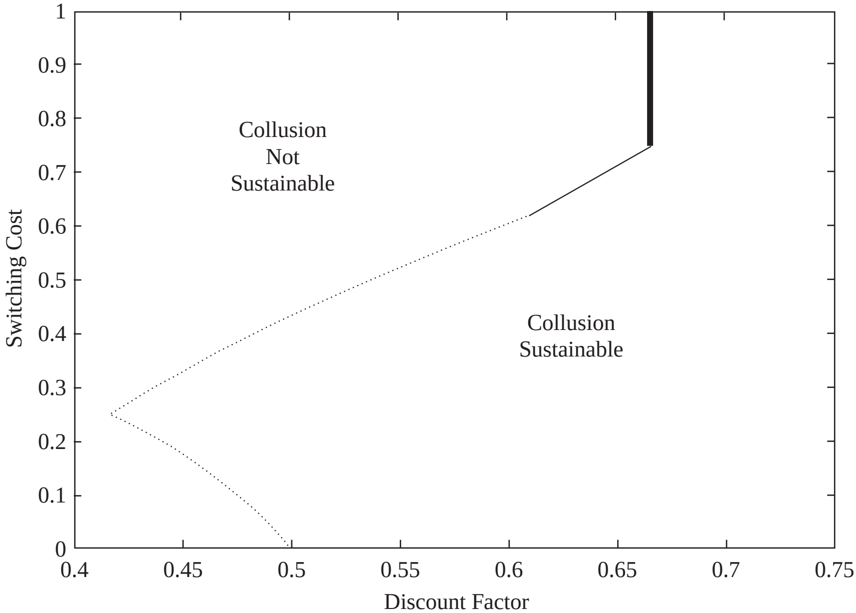

Fig. 1. Regions where collusion is feasible.

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Appendix

Proof of equilibrium

The proof proceeds in three steps. First, we establish an ordering of the maximum and minimum prices in each state (Claim 1). Second, we show that there are no points of discontinuity in the support of the equilibrium price distributions (Claim 2). Third, the equilibrium is characterized in Claim 3.

Claim 1. (a) $\underline{p}_0 \leqslant \underline{p}_1$, (b) $\underline{p}_1 \leqslant \bar{p}_0$, (c) $\bar{p}_0 \leqslant \bar{p}_1$.

Proof. Available from the authors. $\square$

Claim 2. *In any symmetric mixed strategy equilibrium, there are no mass points at:* (a) *points of differentiability*, (b) $\underline{p}_s$ *in* $\Gamma_s(p)$, (c) $\bar{p}_1 - c$ *in* $\Gamma_0(p)$, $\underline{p}_0 + c$ *in* $\Gamma_1(p)$.

Proof. Available from the authors. $\square$

Claim 3. *Let* $\Omega = \{V_0, V_1, \underline{p}_0, \bar{p}_0, \underline{p}_1, \bar{p}_1\}$. *Claims* 1 *and* 2 *and the following region-specific equations fully characterize the unique MPE. In Region* III ($\forall c > c^* = R - \underline{p}$), *the MPE is characterized by* $\Gamma_s(p)$ *in* (A.1) *and* (A.2), *where* Ω *is defined by* (A.3)–(A.5), $\bar{p}_0 = \bar{p}_1 = R$, *and* $\underline{p}_0 = \underline{p}_1 = \underline{p}$. *In Region* II ($\forall c \in (c^{**}, c^*]$, *where* $c^{**} = R - \underline{p}_1$), *the MPE is characterized by* $\Gamma_s(p)$ *in Table* 3, *where* Ω *is defined by* (1)–(4), $\bar{p}_0 - \underline{p}_0 = c$ *and* $\bar{p}_1 = R$. *In Region* I ($\forall c \in (0, c^{**}]$), *the MPE is characterized by* $\Gamma_s(p)$ *in Table* 2, *where* Ω *is defined by* (1)–(4), $\bar{p}_0 - \underline{p}_0 = c$ *and* $\bar{p}_1 - \underline{p}_1 = c$.

$$\Gamma_0(p) = \frac{2p - V_1(1-\delta)}{p + \delta(V_1 - V_0)}, \quad \underline{p} \leqslant p < R, \tag{A.1}$$

$$\Gamma_1(p) = \frac{p + \delta V_1 - V_0}{p + \delta(V_1 - V_0)}, \quad \underline{p} \leqslant p < R, \tag{A.2}$$

$$V_0 = \underline{p}(1+\delta)/(1-\delta), \tag{A.3}$$

$$V_1 = 2\underline{p}/(1-\delta), \tag{A.4}$$

$$\underline{p} = R/(2+\delta). \tag{A.5}$$

Proof. We characterize the equilibrium in Regions I–III; the proof of uniqueness is available from the authors. We first derive $\Gamma_s(p)$ (Tables 2 and 3, (A.1) and (A.2)) and then derive the region-specific equations for Ω. The payoffs for a firm in state $s \in \{0, 1\}$ from charging any price $p \in [\underline{p}_s, \bar{p}_s]$ is

$$\begin{aligned}\text{State 0: } V_0 = {} & p[(1 - \Gamma_1(p)) + (1 - \Gamma_1(p + c))] \\ & + \delta[(1 - \Gamma_1(p))V_1 + \Gamma_1(p)V_0].\end{aligned} \tag{A.6}$$

$$\begin{aligned}\text{State 1: } V_1 = {} & p[(1 - \Gamma_0(p)) + (1 - \Gamma_0(p - c))] \\ & + \delta[(1 - \Gamma_0(p))V_1 + \Gamma_0(p)V_0].\end{aligned} \tag{A.7}$$

Consider $\underline{p}_1 \leqslant p < \bar{p}_0$. For prices in this interval, $\Gamma_1(p + c) = 1$ and $\Gamma_0(p - c) = 0$. After substituting in (A.6) and (A.7) and solving for $\Gamma_0(p)$ and $\Gamma_1(p)$ we get

$$\Gamma_0(p) = \frac{2p - V_1(1-\delta)}{p + \delta(V_1 - V_0)}, \quad \underline{p}_1 \leqslant p < \bar{p}_0, \tag{A.8}$$

$$\Gamma_1(p) = \frac{p + \delta V_1 - V_0}{p + \delta(V_1 - V_0)}, \quad \underline{p}_1 \leqslant p < \bar{p}_0. \tag{A.9}$$

Next, consider $p_1 \in [\underline{p}_0 + c, \bar{p}_1)$ and $p_0 \in [\underline{p}_0, \bar{p}_1 - c)$. In this range, $\Gamma_1(p_0) = 0$ and $\Gamma_0(p_1) = 1$. After substituting into (A.6) and (A.7) and solving for $\Gamma_0(p)$ and $\Gamma_1(p)$

we get

$$\Gamma_0(p) = 1 - \left[\frac{V_1 - \delta V_0}{p+c}\right], \quad \underline{p}_0 \leqslant p < \bar{p}_1 - c, \tag{A.10}$$

$$\Gamma_1(p) = 2 - \left[\frac{V_0 - \delta V_1}{p-c}\right], \quad \underline{p}_0 + c \leqslant p < \bar{p}_1. \tag{A.11}$$

We now derive conditions that define V_0, V_1, $\underline{p}_0$, $\bar{p}_0$, $\underline{p}_1$ and $\bar{p}_1$ for each region.

Region I: By definition $\bar{p}_0 - \underline{p}_0 = c$, $\bar{p}_1 - \underline{p}_1 = c$. From $\Gamma_0(\underline{p}_0) = 0$, and $\Gamma_1(\underline{p}_1) = 0$ we get $V_1 - \delta V_0 = (\underline{p}_0 + c)$ and $V_0 - \delta V_1 = \underline{p}_1$, which are (2) and (1). In equilibrium the expected pay-off of a firm in state s must at least weakly dominate the pay-off from deviations outside the support of the distribution in that state. We now consider such deviations outside the support of the distribution to derive (3) and (4). Consider, $p_0 > \bar{p}_0$. For $p_0 > \bar{p}_0$, $\Gamma_1(p_0 + c) = 1$. In equilibrium, to prevent deviations to $p_0 > \bar{p}_0$ the following must hold:

$$p_0[1 - \Gamma_1(p_0)] + \delta[(1 - \Gamma_1(p_0))V_1 + \Gamma_1(p_0)V_0] \leqslant V_0. \tag{A.12}$$

In Region I, $\bar{p}_0 = \underline{p}_0 + c$, from Table 2, $\Gamma_1(p) = 2 - \left[\frac{V_0 - \delta V_1}{p-c}\right]$ $\forall p \in [\underline{p}_0 + c, \bar{p}_1)$. Substituting this expression for $\Gamma_1(p)$ in Eq. (A.12) we get

$$[p_0 + \delta(V_1 - V_0)]\left[\frac{V_0 - \delta V_1}{p_0 - c} - 1\right] \leqslant V_0(1-\delta). \tag{A.13}$$

The LHS of Eq. (A.13) has a negative derivative, so it is sufficient to verify Eq. (A.13) at $\bar{p}_0$:

$$[V_0 - \delta V_1 - \underline{p}_0][\bar{p}_0 + \delta(V_1 - V_0)] \leqslant \underline{p}_0 V_0(1-\delta). \tag{A.14}$$

Now consider $p_0 < \underline{p}_0$. For small deviations $p_0 < \underline{p}_0$, $\underline{p}_1 \leqslant p_0 + c < \bar{p}_0$. In equilibrium, the expected payoff to a firm in state 0 must be weakly greater than charging $p_0 < \underline{p}_0$, so that

$$p_0[1 + (1 - \Gamma_1(p_0 + c))] + \delta V_1 \leqslant V_0. \tag{A.15}$$

From Table 2, $\Gamma_1(p) = \frac{p + \delta V_1 - V_0}{p + \delta(V_1 - V_0)}$ $\forall p \in [\underline{p}_1, \bar{p}_0)$. Substituting this for $\Gamma_1(p)$ in Eq. (A.15):

$$p_0\left[\frac{V_0(1-\delta)}{p_0 + c + \delta(V_1 - V_0)}\right] + \delta V_1 + p_0 \leqslant V_0. \tag{A.16}$$

The LHS of Eq. (A.16) has a positive derivative, so it is sufficient to verify it at $\underline{p}_0$. This leads to

$$[V_0 - \delta V_1 - \underline{p}_0][\bar{p}_0 + \delta(V_1 - V_0)] \geqslant \underline{p}_0 V_0(1-\delta). \tag{A.17}$$

Combining Eqs. (A.14) and (A.17), we get (3). Now consider $p_1 > \bar{p}_1$. For $p_1 > \bar{p}_1$, $\Gamma_0(p_1) = 1$. To prevent deviations to $p_1 > \bar{p}_1$ we must have

$$p_1[1 - \Gamma_0(p_1 - c)] + \delta V_0 \leqslant V_1. \tag{A.18}$$

For small deviations to $p_1 > \bar{p}_1$ we have $p_1 - c \in [\underline{p}_1, \bar{p}_0)$. From Table 2, $\Gamma_0(p) = \frac{2p - V_1(1-\delta)}{p + \delta(V_1 - V_0)}$ $\forall p \in [\underline{p}_1, \bar{p}_0)$. Substituting for $\Gamma_0(p)$ in Eq. (A.18) we get

$$p_1 \frac{V_1 - \delta V_0 - (p_1 - c)}{\delta(V_1 - V_0) + (p_1 - c)} \leqslant (V_1 - \delta V_0). \tag{A.19}$$

The derivative of the left-hand side of Eq. (A.19) with respect to price will be negative if

$$\delta(V_1 - V_0)(V_1 - \delta V_0 - 2p_1 + c) - (V_1 - \delta V_0)c - (p_1 - c)^2 \leqslant 0.$$

The last 2 terms are negative for all p. We now show that $(V_1 - \delta V_0 - 2p_1 + c) < 0$ at $\bar{p}_1$. Substituting for $p_1 = \bar{p}_1$ and using $\bar{p}_1 - c = \underline{p}_1$, this becomes $\delta V_0 + \bar{p}_1 + \underline{p}_1 > V_1$. Finally, we recognize that when offering $p_1 = \bar{p}_1$ in equilibrium, the payoff is $\bar{p}_1 K + \delta V_0 = V_1$, where $K < 1$. Therefore, $\bar{p}_1 + \delta V_0 > V_1$. Since $\underline{p}_1 > 0$, we have $\delta V_0 + \bar{p}_1 + \underline{p}_1 > V_1$. Hence, the derivative of LHS of Eq. (A.19) with respect to price is negative at $\bar{p}_1$ so it is sufficient to verify Eq. (A.19) at $\bar{p}_1$. This leads to

$$\bar{p}_1[(V_1 - \delta V_0) - \underline{p}_1] \leqslant [V_1 - \delta V_0][\underline{p}_1 + \delta(V_1 - V_0)]. \tag{A.20}$$

Next consider $p_1 < \underline{p}_1$. To prevent deviations to $p_1 < \underline{p}_1$, we must have

$$p_1[1 + (1 - \Gamma_0(p_1))] + \delta[(1 - \Gamma_0(p_1))V_1 + \Gamma_0(p_1)V_0] \leqslant V_1, \tag{A.21}$$

which simplifies to

$$[p_1 + \delta(V_1 - V_0)](1 - \Gamma_0(p_1)) \leqslant V_1 - \delta V_0 - p_1. \tag{A.22}$$

In Region I, $\bar{p}_1 - c = \underline{p}_1$, and from Table 2, $\Gamma_0(p) = 1 - \left[\frac{V_1 - \delta V_0}{p + c}\right]$ $\forall p \in [\underline{p}_0, \bar{p}_1 - c)$. Substituting this expression for $\Gamma_0(p)$ in Eq. (A.22). we get

$$[p_1 + \delta(V_1 - V_0)]\left[\frac{V_1 - \delta V_0}{p + c}\right] \leqslant V_1 - \delta V_0 - p_1. \tag{A.23}$$

The LHS of this equation has a positive derivative, so it is sufficient to verify Eq. (A.23) at $\underline{p}_1$. This leads to

$$[\underline{p}_1 + \delta(V_1 - V_0)][V_1 - \delta V_0] \leqslant \bar{p}_1[V_1 - \delta V_0 - \underline{p}_1]. \tag{A.24}$$

Combining Eqs. (A.20) and (A.24) we get (4).

Region II: By definition, $\bar{p}_0 - \underline{p}_0 = c$ and $\bar{p}_1 - \underline{p}_1 < c$. Derivation of Eqs. (1)–(3) are identical to Region I. For Eq. (4), we first consider small deviations, $p_1 < \underline{p}_1$ such that p_1 lies in the flat region of $\Gamma_0(p)$ i.e. $p_1 \in [\bar{p}_1 - c, \underline{p}_1)$. Such deviations must be weakly dominated by the equilibrium payoffs:

$$p_1[1 + (1 - \Gamma_0(p_1))] + \delta[(1 - \Gamma_0(p_1))V_1 + \Gamma_0(p_1)V_0] \leqslant V_1 \tag{A.25}$$

which simplifies to

$$[p_1 + \delta(V_1 - V_0)](1 - \Gamma_0(p_1)) + p_1 \leqslant V_1 - \delta V_0. \tag{A.26}$$

Substituting for $\Gamma_0(p)$, where $p \in [\bar{p}_1 - c, \underline{p}_1)$, we get

$$[p_1 + \delta(V_1 - V_0)]\left[\frac{V_1 - \delta V_0}{\bar{p}_1}\right] + p_1 \leqslant V_1 - \delta V_0. \tag{A.27}$$

The LHS of Eq. (A.27) has a positive derivative, so it is sufficient to verify it at $\underline{p}_1$. This leads to

$$[\underline{p}_1 + \delta(V_1 - V_0)](V_1 - \delta V_0) \leqslant \bar{p}_1(V_1 - \delta V_0 - \underline{p}_1). \tag{A.28}$$

Monotonicity in $\Gamma_0(p)$ at $\underline{p}_1$ implies $\Gamma_0(\underline{p}_1) \geqslant \Gamma_0(\bar{p}_1 - c)$, or

$$\frac{2\underline{p}_1 - V_1(1-\delta)}{\underline{p}_1 + \delta(V_1 - V_0)} \geqslant \frac{\bar{p}_1 - (V_1 - \delta V_0)}{\bar{p}_1}. \tag{A.29}$$

Simplifying and rearranging the terms in the above equation we get

$$[\underline{p}_1 + \delta(V_1 - V_0)](V_1 - \delta V_0) \geqslant \bar{p}_1(V_1 - \delta V_0 - \underline{p}_1). \tag{A.30}$$

Combining Eqs. (A.28) and (A.30) gives Eq. (4). $\bar{p}_1 = R$ follows from the condition that is required to deter deviation $p_1 > \bar{p}_1$. To prevent deviations to $p_1 > \bar{p}_1$, we have

$$p_1[1 - \Gamma_0(p_1 - c)] + \delta V_0 \leqslant V_1.$$

For small deviations $p_1 > \bar{p}_1$, $p_1 - c$ will lie in the flat region of the support of the firm in state 0. Substituting, $\Gamma_0(p)$, in the above condition, we get $p_1 \leqslant \bar{p}_1$. This yields a contradiction. The only solution of $\bar{p}_1$ that can deter such deviations is $\bar{p}_1 = R$.

Region III: By definition $\bar{p}_0 - \underline{p}_0 < c$, $\bar{p}_1 - \underline{p}_1 < c$. Let $\bar{p}_0 < \bar{p}_1$ and consider two cases: (a) Assume $\underline{p}_0 + c \leqslant \bar{p}_1$. For all $p_1 \in [\bar{p}_0, \underline{p}_0 + c)$ the firm in state 1 loses the new customers but serves the Old Customers with certainty. Hence, $p_1 = \underline{p}_0 + c$ dominates all $p_1 \in [\bar{p}_0, \underline{p}_0 + c)$. This implies that in state 0, $p_0 = \underline{p}_0 + c$ dominates $\bar{p}_0$, which contradicts $\bar{p}_0 - \underline{p}_0 < c$. (b) Assume $\underline{p}_0 + c > \bar{p}_1$. $p_1 = \bar{p}_1$ dominates all $p_1 \in [\bar{p}_0, \bar{p}_1)$. Given this, the firm in state 0 will strictly prefer $p_0 = \bar{p}_1$ to $\bar{p}_0$, which contradicts $\bar{p}_0 < \bar{p}_1$. Since neither case is possible, following Claim 1c, we conclude that $\bar{p}_0 = \bar{p}_1$. Further, since $\bar{p}_s - \underline{p}_s < c$, firms only compete for New Customers, implying $\underline{p}_0 = \underline{p}_1$. To deter $p_1 > \bar{p}_1$ the following must hold:

$$V_1 \geqslant p_1[1 + (1 - \Gamma_0(p_1))] + \delta[(1 - \Gamma_0(p_1))V_1 + \Gamma_0(p_1)V_0].$$

For all $p_1 > \bar{p}_1$, $\Gamma_0(p_1) = 1$. The above condition simplifies to $V_1 - \delta V_0 \geqslant p_1$. The RHS of this expression is increasing in p_1 implying $\bar{p}_1 = R$. $\Gamma_0(\underline{p}) = 0$ and $\Gamma_1(\underline{p}) = 0$ imply

$$V_1 = 2\underline{p}/(1-\delta), \tag{A.31}$$

$$V_0 = \underline{p}(1+\delta)/(1-\delta). \tag{A.32}$$

In state 1, $p_1 = R$ leads to profits $R + \delta V_0$. Thus $p_1 = R$ dominates all $2p + \delta V_1 \leqslant R + \delta V_0$, which holds with equality at $p = \underline{p}$. Using (A.31) and (A.32) yields

$$\underline{p} = R/(2+\delta). \tag{A.33}$$

Hence, $\bar{p}_0 = \bar{p}_1 = R$, $\underline{p}_0 = \underline{p}_1 = \underline{p}$, together with (A.31)–(A.33) complete the characterization of the equilibrium in Region III. □

References

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