Skewness and time-varying second moments in a nonlinear production network: theory and evidence

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Abstract

This paper studies asymmetry in economic activity over the business cycle. It develops a tractable multisector model of the economy in which complementarity across inputs causes aggregate activity to be left skewed with countercyclical volatility. We then examine implications of the model regarding the time-series skewness of activity at the sectoral level, cyclicality of dispersion and skewness across sectors, and the conditional covariances of sector growth rates, finding support for each in the data. In the data, the skewness of employment growth, industrial production growth, and stock returns increases with the level of aggregation, which is consistent with the model's implication that it is the nonlinearity in the production structure of the economy that generates the skewness. Other prominent models of asymmetry are not able to simultaneously match the range of empirical facts that the production network model can.

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1 Introduction

A defining feature of the business cycle is the existence of recessions as distinct episodes. Rather than simply experiencing symmetric random fluctuations around a trend, output, employment, and other aggregate measures of the state of the economy display sharp declines and relatively smooth expansions. In other words, levels and growth in real activity are skewed left.[1] At the same time, many measures of volatility are countercyclical. As a mathematical matter, there is a mechanical link between skewness and countercyclical volatility—high volatility in bad times leads to a long left tail of outcomes—but few models capture that feature of the economy.

This paper takes that asymmetry as its starting point and develops a tractable multisector production network model in which intermediate inputs are gross complements. We show that the model not only matches the skewness of aggregate variables mentioned above, but also yields a number of other testable predictions that we study empirically.

Our theoretical framework builds on the tradition of multisector production models, such as Long and Plosser (1983). In almost all past work, the elasticity of substitution across inputs in production has been assumed to be 1, which yields a log-linear solution. Baqaee and Farhi (2019), however, argue that allowing non-unitary elasticities is important for understanding a range of facts about the economy, including business cycle asymmetry.

To build intuition, consider an extreme case in which aggregate output is Leontief in a range of inputs and that production of the inputs depends only on sector-specific productivity shocks. Then aggregate output is a function of the minimum of the technology shocks, which is in general skewed left. With network effects, that minimum—the productivity of the worst-performing sector—becomes a common component that affects all sectors.

We study a relatively stylized production network that has the advantage that it is analytically tractable, yielding three sets of empirical predictions:

1. *Unconditional skewness:* Aggregate and sector activity are both skewed left, but the effect is stronger at the aggregate level. The sector-specific component of activity is unskewed.

2. *Cross-sectional moments:* The cross-sectional variance of output is countercyclical and cross-sectional skewness is procyclical.

3. *Conditional covariances:* When a sector receives a negative shock, it subsequently covaries more strongly with other sectors and with aggregate activity.

[1] See Sichel (1993), McKay and Reis (2008), Morley and Piger (2012), Berger, Dew-Becker, and Giglio (2020), and Dupraz, Nakamura, and Steinsson (2020), among others.

The key mechanism in the model is that, due to complementarities in production, aggregate output is a concave function of sector-level shocks, while sector output is equal to aggregate output plus an independent shock. The concavity of aggregate output generates left skewness and countercyclical volatility. The fact that sector output has an independent and symmetrically distributed component explains why it is less negatively skewed than aggregate output.

Countercyclical cross-sectional variance and procyclical skewness are generated simply because aggregate output, as a concave function of the sector-level shocks, tends to be lower in periods when the (sample) cross-sectional standard deviation happens to be higher or the skewness is lower. The model is thus able to generate countercyclical volatility— which is sometimes taken as evidence for countercyclical uncertainty—even though all shocks are homoskedastic. The analysis is consistent with the finding of Dew-Becker and Giglio (2020) that the conditional variance of firm specific shocks from option data is acyclical, and it also calls into question empirical analyses of uncertainty based on cross-sectional standard deviations of realized output or employment (e.g. see the discussion in Bloom (2014)).

The third set of results on conditional covariances directly addresses the key mechanism in the model. In a model with complementarities, when the supply of one input shrinks, downstream output becomes relatively more sensitive to it and less sensitive to other inputs. Because the sectors that receive negative shocks become more relevant, they also covary more strongly with any other sector that buys their output, including final production.

A single simple idea, then, that production features complementarities, generates a wide range of testable predictions that can be used to compare the network production model both to the data and other models.

We next test the model's predictions. We first show that point estimates for skewness are negative for a wide range of aggregate time-series, including employment, GDP, consumption, and investment, both in levels and growth rates.

In testing the additional predictions, we use monthly data on industrial production, employment, and stock returns. For all three measures, skewness is negative at both the aggregate and sector level. Consistent with the model, the magnitude of the skewness increases with the level of aggregation: aggregate activity is at least twice as negatively skewed as sector-level activity. However, when we examine residuals from regressions of sector on aggregate activity, skewness is near zero with tight confidence intervals: all the skewness observed at the sector level is explained by exposure to an aggregate factor. These observations suggest that skewness seems to be an aggregate phenomenon, rather than being due to, for example, skewed micro shocks.

As to prediction 2, we show that cross-sectional moments of real activity are cyclical, but, as predicted by the model, so are sector-specific shocks.

Finally, we show that, following a positive statistical innovation in output in a given sector and month (which, in the model, identifies a sector productivity shock), that sector's industrial production and employment covary less strongly with other sectors and with aggregate activity for the next three to 12 months. The same holds for stock returns—where we can use high-frequency data to better measure covariances—and TFP from the NBER-CES manufacturing database, which is most directly connected to the model.

Beyond those results, it should also be noted that the model generates countercyclical volatility for aggregate and sector output, consistent with a large literature, e.g. Bloom (2007) and Jurado, Ludvigson, and Ng (2015), among many others.

While we derive our theoretical predictions using a highly stylized model, using numerical simulations we show that a more realistic model is quantitatively consistent with the empirical results.

Since our starting point is business cycle asymmetry, our last question is whether other models that generate such asymmetry can also match the additional predictions that this paper generates and tests.

To generate aggregate skewness, one might naturally assume that there are skewed aggregate shocks, such as rare disasters,[2] or perhaps skewness in a universal input (e.g. Brunnermeier and Sannikov (2014)). But such a model does not necessarily generate cyclical cross-sectional moments. Conversely, models of micro uncertainty shocks, such as Bloom (2009) and Christiano, Motto, and Rostagno (2016), imply that cross-sectional moments are cyclical, but they do not have any implications for aggregate skewness.

Finally, one might also consider a model in which skewness arises due to concave decision rules at the *micro* level, but without any nonlinearity in interactions across economic units, as in Ilut, Kehrig, and Schneider (2018). Their model can generate skewness, cyclical moments, and even the conditional covariance results, but it counterfactually predicts that the magnitude of skewness is greater at the micro than the macro level. Specifically, linear aggregation in their model causes micro skewness to wash out.

In other words, all three alternative models we consider fail to match the data on at least one dimension, whereas the network production model can match all three sets of facts parsimoniously. The failures of the models are instructive: the results on time-series skewness point to the existence of a skewed common component in output—that is, skewness arises at the aggregate rather than micro level—while the cross-sectional results imply that the

[2]E.g. Barro (2006), Gourio (2012, 2013), and Wachter (2013)

common component is *endogenous* to sector shocks. The explicit aggregation emphasized by Baqaee and Farhi (2019) creates that endogeneity.

Taken together, the results in this paper have important implications for how to think about volatility and skewness in both the real economy and financial markets.[3] For recent work on time-varying volatility in the real economy, see Justiniano and Primiceri (2008), Clark and Ravazzolo (2015), and Schorfheide, Song, and Yaron (2018). Work on time-varying cross-sectional moments includes Guvenen, Ozkan, and Song (2014), Salgado, Guvenen, and Bloom (2020), and Dew-Becker and Giglio (2020b). A large literature studies stochastic volatility, often implicitly assuming that because the dispersion in observed distributions changes over time, that means that there are "uncertainty shocks", see, e.g., Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018). The results here show that time-variation in sample moments—and their cyclicality—need not have anything to do with uncertainty. Rather, increases in the dispersion of realized shocks may simply be associated with declines in output because of nonlinearities in the economy's production structure.

The model and empirical results also demonstrate that the economy in an important sense does not have a single fixed network structure—in terms of the covariance of output across sectors—and in fact that the variation is important in causing asymmetries in outcomes.

Related work As already mentioned, our paper is related to the literature on time-variation in time-series and cross-sectional moments of output. Our work also belongs to the growing literature that studies the macroeconomic role of production networks as a mechanism for propagation and amplification of shocks (Long and Plosser, 1983; Acemoglu et al., 2012).[4] Particularly relevant is the body of works that emphasizes the importance of complementarities in production for interindustry comovement. Papers such as Horvath (2000), Atalay (2017), and Baqaee and Farhi (2019) study how non-unitary elasticities of substitution can alter shocks' aggregate impact and the induced patterns of comovement. Baqaee and Farhi (2019) additionally show that complementarities in production can generate skewness at the aggregate level from unskewed micro shocks. On the empirical side, Atalay (2017) and Atalay et al. (2018) estimate the elasticity of substitution between various intermediate inputs using sectoral data and find evidence for strong complementarities.[5] We contribute to this literature by showing that complementarities in production not only

[3]For work on unconditional skewness, see Sichel (1993), McKay and Reis (2008), Morley and Piger (2012), and Berger, Dew-Becker, and Giglio (2020).

[4]For other recent related work, see Grassi and Sauvagnat (2019), Frohm and Gunnella (2021), and Liu and Tsyvinski (2020).

[5]See Boehm, Flaaen, and Pandalai-Nayar (2019), Carvalho et al. (2021), and Peter and Ruane (2020) for estimates of elasticities of substitution between intermediate inputs at the firm level.

generate aggregate skewness but are also able to match the four stylized facts on skewness and comparing the model's performance to other leading potential explanations.

More generally, this paper is related to the broader literature that studies the macroeconomic impacts of microeconomic shocks, such as Gabaix's (2011) work on granularity (among many others). More recently, Gourieroux et al. (2020) examine how shocks to systemically important firms may affect aggregate consumption and asset prices (see also Seo and Wachter (2018)).

2 Skewness in aggregate activity

The fact that both levels and growth rates of real activity and stock returns are skewed left has been established in previous work.[6] This section provides a brief overview of some evidence on asymmetry in aggregate activity.

The first column in Table 1 reports skewness coefficients for growth rates (Panel A) and levels (Panel B) of six measures of aggregate activity: industrial production, employment, stock market returns, GDP, consumption, and investment. In all cases, here and below, levels are detrended with an exponentially weighted moving average. p-values for a two-sided test against the null of zero skewness from a block bootstrap are reported in brackets. Across the six series, in both levels and growth rates, the skewness coefficients are negative in all cases. In terms of magnitudes, industrial production features the most skewed distribution (-1.22) followed by investment (-0.86) for growth rates. Investment (-1.20) and stock returns (-1.13) are most negatively skewed in levels.

[Insert Table 1 here.]

The remaining columns in Table 1 report results for a nonparametric measure of asymmetry. Denote the mean and standard deviation of some variable x by μ_x and σ_x, respectively, and the empirical cumulative distribution function as $\hat{F}_x(z)$. The table reports the following tail probability ratios:

$$\text{tail probability ratio} = \frac{\hat{F}_x(\mu_x - k\sigma_x)}{1 - \hat{F}_x(\mu_x + k\sigma_x)}$$

[6] Berger, Dew-Becker, and Giglio (2020) show that growth rates of employment, capacity utilization, industrial production, GDP, durable and non-durable consumption, and residential and nonresidential investment are all skewed left. Furthermore, returns on the S&P 500 are skewed left, as is their option-implied distribution. Morley and Piger (2012) provide a much more thorough analysis of asymmetry in the output gap—that is, on skewness in levels, rather than growth rates—and finding similar results— while Sichel (1993) provides an earlier analysis distinguishing asymmetry in levels from growth rates. See also references therein for the literature on business cycle asymmetry.

for various values of k. This ratio measures the relative probability that x is k standard deviations below its mean compared to the probability it is k standard deviations above its mean. If the left tail is asymptotically heavier than the right, in the sense that the probability ratio diverges to ∞, then choosing a large value of k will produce a larger ratio. At the same time, though, when k is larger, the probability ratio is calculated based on fewer observations. So the ability to reject the null that the ratio is equal to 1 will, heuristically, peak for some finite k.[7]

The cutoff k ranges in the table between 1 and 3, with power appearing to peak around values of 1.5 and 2. In all cases, as k rises, the relative probability of negative deviations rises. In other words, large negative deviations in both levels and growth rates of the six variables studied in Table 1 have been far more common than large positive deviations. At $k = 1.5$ and $k = 2$, a number of the ratios are statistically significantly larger than 1 at conventional levels, but that statistical significance is not uniform across all estimates. Compared to the results for the coefficient of skewness above, the tail probability ratios are relatively more weakly measured statistically. So while the tail measures are attractive for being simple and nonparametric, they face the usual tradeoff of also having somewhat lower power. The tail measures have the advantage, though, that they will map directly into the theoretical results developed below.

To summarize, Table 1 shows that in the empirical sample we study, major measures of aggregate activity are consistently skewed left according to a range of different measures. While the results are individually only marginally statistically significant, they overall tell a consistent story of a long left tail, and we take that left skewness as the basic starting point for the remainder of the analysis.

Time-series skewness is closely related to countercyclical volatility. If volatility rises when a variable falls, it will tend to have a long left tail (since $\mathbb{E}[x^3] = \mathbb{E}[x \cdot x^2]$, negative skewness means that the level of a series covaries negatively with its square). There is a large literature studying countercyclicality of aggregate volatility. Jurado, Ludvigson and Ng (2015), for example, show that the volatility of forecast errors for aggregate outcomes is significantly countercyclical.

3 Nonlinear network model

This section presents a production network model that can generate the negative skewness in aggregate activity presented in the previous section. The model builds on the work of Long

[7] For example, for $k = 10$ the sample CDFs will both be equal to zero and the ratio undefined—we have no 10-standard-deviation events in our sample.

and Plosser (1983) and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) and allows for nonlinearity in production. Baqaee and Farhi (2019) and Dew-Becker and Tahbaz-Salehi (2020) examine the behavior of nonlinear production networks under very general conditions, but both papers use approximations—analyzing small shocks in the former case and large shocks in the latter. In both cases, it is shown that when the elasticities of substitution in production are less than 1, aggregate output is a concave function of sectoral shocks and skewed left (assuming symmetrically distributed micro shocks).

Since it is already established that in general complementarity leads to negative skewness, here we focus on other predictions of the model. To generate clear implications, we set the model up to be as simple as possible, which will allow us to obtain transparent analytic solutions and formally derive testable predictions. Section 6 examines results from a numerical solution of a richer and more realistic specification.

Consider an economy consisting of n sectors, each producing a distinct product according to the following constant-returns technology

$$y_{i,t} = z_{i,t}\zeta_i \ell_{i,t}^{1-\alpha} \left(\sum_{j=1}^{n} a_j^{1/\xi} x_{ij,t}^{(\xi-1)/\xi} \right)^{\alpha\xi/(1-\xi)}, \tag{1}$$

where $z_{i,t}$ is sector i's productivity on date t, ℓ_i is sector i's use of labor, $x_{i,j}$ is sector i's use of material input j, and ξ is the elasticity of substitution across material inputs. $\zeta_i = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}$ is a normalization constant.

The nonnegative constants $\{a_j\}$ determine the importance of inputs in the intermediate input bundle and are normalized such that $\sum_{j=1}^{n} a_j = 1$. Note that these weights do not depend on the identity of the using sector i, implying that all sectors use the same mix of inputs for production. While restrictive, we impose this symmetry assumption in order to get a closed-form characterization of equilibrium. The quantitative exercise in Section 6 relaxes this assumption and allows for sector-specific intermediate input bundles.

Final consumption, which constitutes all of GDP, is

$$\text{GDP}_t = \left(\sum_{j=1}^{n} a_j^{1/\xi} c_{j,t}^{(\xi-1)/\xi} \right)^{\xi/(\xi-1)}, \tag{2}$$

where $c_{j,t}$ is consumption of good j at time t. The elasticity of substitution across goods and the weight of each good is the same in final consumption as in sector production. In addition to consuming, households inelastically supply a single unit of labor to the firms.[8]

[8] In Appendix A.2, we show that our results continue to hold if we instead assume that labor is sector-

The competitive equilibrium of the economy is a collection of prices and quantities such that (i) firms in all sectors maximize their profits taking all prices as given, (ii) the representative household maximizes utility, taking prices as given, and (iii) all markets clear, with the market-clearing condition for good j given by

$$c_{j,t} + \sum_{i=1}^{n} x_{ij,t} = y_{j,t}. \tag{3}$$

Since the model is fully static, we drop the time subscripts where there is no risk of confusion. The appendix shows that, in equilibrium, aggregate and sectoral outputs are given by

$$\log y_i = (1 - \xi + \alpha\xi)\log(\mathrm{GDP}) + \xi \log z_i + \delta_i \tag{4}$$

$$\log(\mathrm{GDP}) = \frac{1}{(\xi - 1)(1 - \alpha)} \log \sum_{i=1}^{n} a_i z_i^{\xi - 1}, \tag{5}$$

respectively, where δ_i is a constant that is independent of the shocks. Two observations are immediate.

First, equation (5) illustrates that aggregate output is a CES aggregate over the sector productivities with elasticity ξ. When the inputs are gross complements—i.e., when $\xi < 1$—aggregate output is a concave function of the sector productivities.

Second, equation (4) shows that sector output has a factor structure: log output in each sector is simply equal to a multiple of aggregate output plus an independent shock. That factor structure will be the source of some of the key results, but the core feature is not the factor structure but rather that the common factor—aggregate output—is a concave function of the sector shocks.

The restrictions in the model—identical elasticities and production weights across sectors—are chosen so that we can obtain the closed-form characterization in (4) and (5). Those restrictions are not required to generate the result that aggregate output is concave in the sector shocks. All that is required is for all elasticities of substitution to be less than 1. Nonetheless, the closed-form expressions in (4) and (5) are useful for making the relationship between the shocks, aggregate output, and sector output transparent.

For the rest of the paper we use $\epsilon_{i,t} = \log z_{i,t}$ to denote the log productivity shock to sector i. We assume that the $\epsilon_{i,t}$ are distributed symmetrically around zero with full support on $\mathbb{R}$, both conditionally and unconditionally. This assumption guarantees that any skewness

specific (in the sense that it cannot be relocated among sectors) and is supplied elastically.

in log output will then arise endogenously through the production and aggregation process.

4 Testable predictions of the model

This section uses the closed-form solution to the model to develop a series of empirical predictions. The predictions are divided into three basic categories, all of which take advantage of cross-sector information: the unconditional distribution of sector activity; time-variation in the cross-sectional distribution of activity; and conditional variances and covariances.

4.1 The unconditional distribution of real activity

Let $\mu = \mathbb{E}[\log \text{GDP}]$ and $\sigma = \text{stdev}(\log \text{GDP})$

Proposition 1. *Suppose log productivity shocks are drawn from a common distribution $F(\cdot)$ with density $f(\cdot)$ such that $\lim_{\tau\to\infty} \frac{f(\tau+k)}{F(-\tau)f(\tau)} = \infty$ for all k. If $\xi < 1$, then $\log \text{GDP}$ is negatively skewed in the sense that*

$$\lim_{\tau\to-\infty} \frac{\mathbb{P}(\log \text{GDP} < \mu - \tau\sigma)}{\mathbb{P}(\log \text{GDP} > \mu + \tau\sigma)} = \infty. \tag{6}$$

If $\xi > 1$, then aggregate output is positively skewed, in the sense that above limit is equal to zero.

Note that proposition 1 uses a concept of skewness different from the more common scaled third moment (though we note that there are many measures of skewness studied in the literature). The limit we analyze describes the tails of the distribution of aggregate output. The advantage of this approach is that it is possible to derive formal results that hold irrespective of the specific details of intermediate input mix and elasticities, and for a wide range of shock distributions.[9] Furthermore, due to its asymptotic nature, the result in (6) is also insensitive to changes in any finite moment of the distribution: for example, any change in the mean or variance of any of the productivity shocks has no effect on the limit.

Proposition 1 refers to levels, due to the static setup of the model, but results can also be obtained in growth rates, which is relevant in our empirical applications. Appendix A.2.3 provides a full characterization.

[9]For example, the distributional restriction in Proposition 1 holds for the entire family of Weibull-tailed distributions (such as normal, exponential, and subexponential Weibull distributions), as well as Pareto-tailed distributions with finite mean and standard deviation.

Under the assumption that intermediate inputs are gross complements, Proposition 1 leads to the following prediction:

Prediction 1a. Log aggregate and sector output are both skewed left, and the magnitude is greater for aggregate output.

The intuition for this result is simple: log sector output is equal to log aggregate output plus a symmetrically distributed shock. As a result, the skewness of $\log y_i$ has to be less than that of $\log \mathrm{GDP}$.

To push this result further, note that the decomposition in equation (4) indicates that sector output inherits its skewness from the that of the aggregate output. This is because it is fundamentally the aggregation process (through the nonlinearities in the economy's production structure) that generates skewness in the first place, as sectoral shocks themselves are not skewed. So if we extract the sector-specific component of activity, we should not observe any skewness. We formalize this in our next prediction.

Prediction 1b. The residual from a regression of log sector output on log aggregate output is not skewed.

Recall from equation (4) that, in the model, the regression of $\log y_i$ on $\log \mathrm{GDP}$ identifies $\epsilon_i = \log z_i$ (up to an affine transformation). Therefore, examining the relative skewness of the ϵ and $\log \mathrm{GDP}$ tests a core prediction of the model: that macro skewness is caused by the aggregation process (i.e. it is systemic), rather than by skewness in sector- or firm-level shocks. That fact allows us to directly test the model against micro-based explanations of skewness, such as the model of Ilut, Kehrig, and Schneider (2018).

4.2 The cross-sectional distribution of activity

There is a long-running literature that studies the cyclicality of cross-sectional dispersion in outcomes. Our model has strong implications for dispersion. Since aggregate output is a concave function of the sector-level shocks, an increase in their dispersion mechanically reduces sector output, simply through a Jensen's inequality effect. To see that, a cumulant-type expansion for output yields,

$$\log \mathrm{GDP} \approx \frac{1}{(1-\alpha)} \sum_{m=1}^{3} \frac{1}{m!} (\xi - 1)^{m-1} \mu_m (\{\epsilon_j\}), \tag{7}$$

where $\mu_m(\{\epsilon_j\})$ is the sample mth moment of the cross-sectional distribution of productivity.[10] The approximation shows that high dispersion and negative skewness in the cross-sectional distribution of productivity reduce GDP when $\xi < 1$. That can be used to derive the following formal result:

Proposition 2. *Suppose log sectoral shocks are drawn from an unskewed distribution with positive excess kurtosis. Let μ_2 and μ_3 denote the cross-sectional variance and skewness of the realized log productivity shocks. If $\xi < 1$, then, to a first-order approximation in $\xi - 1$,*

$$\operatorname{cov}(\mu_2, \log \mathrm{GDP}) < 0 \tag{8}$$

$$\operatorname{cov}(\mu_3, \log \mathrm{GDP}) > 0. \tag{9}$$

In other words, when inputs are complements, the cross-sectional variance and skewness of realized shocks are, respectively, countercyclical and procyclical. Furthermore, when coupled with equation (4), Proposition 2 also implies that the same cyclical behavior also holds for the cross-sectional distribution of sectoral output. This is a simple consequence of the fact that, according to (4), the cross-sectional distribution of sectoral output is a scaled version of the cross-sectional distribution of shocks.

Proposition 2 then leads to the following pair of predictions:

Prediction 2a. The cross-sectional variance of log sector output (and the sector-specific residuals) is countercyclical.

Prediction 2b. The cross-sectional skewness of log sector output (and the sector-specific residuals) is procyclical.

Proposition 2 has a certain remarkable feature: even though the sectors have different weights in production, Proposition 2 expresses aggregate output in terms of unweighted cross-sectional moments. The fact that we are able to obtain such a relationship helps justify the usual practice of looking at unweighted cross-sectional moments and their relationship with aggregate output.

As above, the cyclicality of the cross-sectional distribution is not caused by changes in conditional distributions or "time-varying uncertainty." The cross-sectional distribution is a random variable that is correlated with output, even though there are no shocks to the volatility of sector-level shocks. Proposition 2 shows that there is a mechanical relationship between the cross-sectional sample moments and output.

[10] One may note that the unweighted moments appear here. Formally, this is an expansion conditional on only knowing the cross-sectional moments. A more accurate approximation can naturally be obtained if one weights the shocks depending on the importance of the sectors.

Finally, note that the predictions here rely on the fact that aggregate output is a function of the sector shocks. If aggregate output were independent of the sector-specific shocks—e.g. if the sector shocks "washed out" through linear aggregation—then the cross-sectional distribution of shocks would be uncorrelated with aggregate output.

4.3 Conditional covariances

Our results thus far are about the contemporaneous relationship between sectoral and aggregate variables as a function of the elasticity of substitution ξ. In our next result, we explore the time-series implications of the model. To this end, and just in this section, we impose a further assumption that the log productivities, ϵ_{it}, follow AR(1) processes. This assumption allows us to analyze how conditional moments relate to the current realization of shocks.

Proposition 3. *Suppose $\xi < 1$ and that log productivity shocks follow AR(1) processes with positive persistence. Then, for all distinct pairs of sectors $i \neq k$,*

$$\frac{\mathrm{d}\Delta_t^{ik}}{\mathrm{d}\epsilon_{i,t}} < 0 \qquad \textit{and} \qquad \frac{\mathrm{d}\Xi_t^{ik}}{\mathrm{d}\epsilon_{i,t}} < 0, \tag{10}$$

where

$$\begin{aligned}
\Delta_t^{ik} &= \sum_{j \neq i} \mathrm{cov}_t(\log y_{i,t+1}, \log y_{j,t+1}) - \sum_{j \neq k} \mathrm{cov}_t(\log y_{k,t+1}, \log y_{j,t+1}) \\
\Xi_t^{ik} &= \mathrm{cov}_t(\log y_{i,t+1}, \log \mathrm{GDP}_{t+1}) - \mathrm{cov}_t(\log y_{k,t+1}, \log \mathrm{GDP}_{t+1}),
\end{aligned}$$

and cov_t *denotes the covariance conditional on information available on date t.*

We thus have the following predictions.

Prediction 3a. Following a negative innovation in $\epsilon_{i,t}$, sector i's conditional covariances with other sectors rise relative to other sectors' covariances with one another.

Prediction 3b. Following a negative innovation in $\epsilon_{i,t}$, sector i's conditional covariance with aggregate activity rises.

Intuitively, Proposition 3 and predictions 3a and 3b follow from the fact that when a sector receives a negative shock, it becomes relatively more important in determining variation in aggregate output when $\xi < 1$.[11]

[11] Note that levels and growth rates generate the same predictions since they have the same conditional distributions up to a level shift.

Proposition 3 formalizes the idea that when inputs are gross complements, when a sector receives a negative shock it becomes more "central" than other sectors in the sense that sectoral and aggregate outputs covary more strongly with sector i's output. This is yet another core prediction of the model, in that it directly tests the idea that aggregate output is a concave function of the sector shocks. When aggregation is concave, it is exactly the sectors that receive negative shocks that should rise most in importance. We show formally below that most other models of aggregate skewness do not generate such a prediction for time-varying centrality.

4.4 Extensions

Appendix A.2 reports results for two variations of the baseline model that relax some of the strict assumptions above. In particular, Appendix A.2.1 shows that the characterization in equations (4) and (5) remain valid with minor modifications in the presence of (i) elastic labor supply or (ii) sector-specific factors of production (say, capital or inflexible labor). Those results show that Predictions 1–3 are robust to the specific assumptions made on factor elasticities and mobility. Furthermore, Appendix A.2.2, shows that sectoral and aggregate payments to any fixed factor share the same characteristics as sectoral and aggregate output. To the extent that stock returns move with payments to capital, then, the predictions in Section 4 also apply to stock returns.

5 Empirical analysis

We now test the empirical predictions developed in the last section.

5.1 Data

Macroeconomic data: We focus on measures of activity that have data at the monthly frequency or higher and are measured at a high level of sectoral detail. The two monthly series are industrial production (IP), available from the Federal Reserve website, which is measured at up to the five-digit NAICS level of detail in manufacturing industries, and employment, available from the Current Employment Survey of the U.S. Bureau of Labor Statistics (BLS), which is measured up to the six-digit NAICS level and covers the entire economy. For industrial production, we follow Foerster, Sarte, and Watson (2011) and study data since 1972. For employment, the sample with detailed NAICS coverage begins in 1990, while data on two-digit sectors BLS-defined supersectors is available since 1972.

Stock returns: Stock returns have the drawback that they do not directly measure activity, being driven not just by current conditions but also by expectations for the future (and discount rates). However, returns are measured at much higher frequencies—we use daily data—which is useful for estimating time-variation in conditional moments. For sector-level measures of stock returns, we construct value-weighted portfolios according to SIC sectors, requiring at least five firms in a given sector/month pair to include it in the analysis.

5.2 Prediction 1: Time-series skewness

Figure 1 documents skewness across different levels of aggregation for industrial production, employment, and stock returns. At a given level of aggregation, we calculate the skewness coefficient (the scaled third moment) in each sector's time-series, and then report the average of those skewness coefficients at each level of aggregation. 90-percent confidence bands are plotted for each estimate.[12]

5.2.1 Prediction 1a: Skewness declines with the level of aggregation

The first and second columns of panels in Figure 1 plot estimates of time-series skewness at the aggregate and sector levels for IP, employment, and stock returns, in both growth rates and levels. While most of the analysis of the model is in levels, Appendix A.2.3 shows conditions under which skewness also appears in growth rates. In the figure, squares are point estimates and circles depict the difference between each sector's average skewness and overall aggregate skewness.

[Insert Figure 1 here.]

The top panels report results for industrial production. The skewness of total industrial production growth is -1.22. At the two-digit level—just three sectors: durable and nondurable manufacturing and mining—average skewness is -0.96. At the three- and four-digit levels, where there are 43 and 81 total sectors, respectively, skewness declines in magnitude to -0.55 then -0.45. Finally, at the five-digit level skewness is only -0.41.

We conclude that skewness in IP growth at the aggregate level is, at the point estimates, *three times* greater than at the most disaggregated level. The red series show confidence bands for those differences, and three of the four are statistically significant.

[12] The standard errors are calculated with a blockwise jackknife that clusters by date. Specifically, each jackknife replication removes 50 consecutive months of data from the sample—the same 50 months for all sectors—and we iterate over all possible starting months for the excluded dates; see Lahiri (2003).

The second and third rows report results for employment growth and stock returns. For aggregate employment, skewness is -1.49, compared to -0.30 at the five-digit level. The pattern is similar for stock returns, where aggregate skewness is -0.65, while average skewness at the five-digit level is only -0.30. The results for employment are statistically slightly weaker than for industrial production, while those for stock returns are the strongest. Since returns are uncorrelated over time, while employment and IP are substantially serially correlated, the number of effective observations for returns is much larger, increasing statistical power.

To summarize, we find strong evidence for the first prediction of the model: skewness is greater—by a factor of two to five—at higher levels of aggregation.

5.2.2 Prediction 1b: Sector residuals are unskewed

In the model, sector shocks can be identified from a regression of sectoral on aggregate activity. To that end, we estimate the regressions

$$y_{i,t} = a_i + b_i y_t + \nu_{i,t}, \tag{11}$$

$$\Delta y_{i,t} = a_{\Delta,i} + b_{\Delta,i} \Delta y_t + \nu_{\Delta,i,t}, \tag{12}$$

where $y_{i,t}$ denotes some measure of activity in sector i and Δ is the first-difference operator.The third and fourth columns of panels in Figure 1 plot skewness for the residuals $\nu_{i,t}$ and $\nu_{\Delta,i,t}$ as well as the difference between skewness for the residuals and skewness for the original data, $y_{i,t}$, at the same level of aggregation. Recall that the prediction of the theoretical model is that $\nu_{i,t}$ and $\nu_{\Delta,i,t}$ are unskewed, since skewness in sector activity in the model is due to exposure to the aggregate component.

Figure 1 shows that the skewness of the residuals is, across all variables, economically close to zero. In each case it is less negative than the skewness for the original data, and it is typically substantially smaller than -0.5. For the returns data, skewness for residuals is indistinguishable from zero, and for industrial production it is less than half the magnitude that is observed in the original data.

We conclude that while there is strong evidence for negative skewness in raw growth rates in line with prediction 1a, the distribution of residuals displays little to no skewness, consistent with prediction 1b. This finding further strengthens the notion that skewness is an endogenous outcome: sector-specific shocks add little to asymmetry across sectors.

5.3 Prediction 2: The cross-sectional distribution

The second set of predictions of the model is about the cyclicality of the cross-sectional distribution of sector activity. In each month t, we calculate the cross-sectional variance and skewness of monthly growth rates of industrial production and employment at the four-digit NAICS level. We also calculate the mean across days within each month of the cross-sectional variance of sector-level stock returns.[13] In all three cases, we calculate cross-sectional moments not just for total sector activity, but also from residuals from a regression of sector activity on aggregate activity ($\nu_{\Delta,i,t}$ in (12)). That eliminates the effects of exposure to any common component, and in the model corresponds to sector-level technology shocks.

It is important to emphasize that these are *realized* sample moments—they do not measure a conditional distribution, so they do not tell us whether or not the conditional probability density from which the sector growth rates are drawn changes over time. We are simply measuring sample moments—which are random variables—and examining their contemporaneous relationship with the state of the business cycle. To measure the cyclicality of cross-sectional variance and skewness, we estimate the regressions

$$\text{variance}_t = \alpha_v + \beta_v \times \text{economic activity}_t + \eta_{v,t}, \tag{13}$$

$$\text{skewness}_t = \alpha_s + \beta_s \times \text{economic activity}_t + \eta_{s,t}, \tag{14}$$

where economic activity_t is proxied either by an NBER recession indicator or aggregate employment growth and the ηs are residuals.

Panel A of Table 2 reports results from univariate regressions of the cross-sectional variances of both levels and growth rates of industrial production and employment on an NBER recession indicator and aggregate employment growth as two different measures of the state of the business cycle. The cross-sectional variance, skewness, and aggregate employment growth are normalized to have unit variance to help in interpreting the coefficients.

[Insert Table 2 here.]

Consistent with the predictions of the model and with previous work (Davis and Haltiwanger (1992), Ilut, Kehrig, and Schneider (2018), and Salgado, Guvenen, and Bloom (2020)), we find that there are statistically and economically significant increases in cross-sectional

[13] That is, using daily returns in sector i, $r_{i,d}$, the monthly cross-sectional variance is $\frac{1}{\#(days \in t)} \sum_{d \in t} \text{var}_d\left(r_{i,d}\right)$ where var_d is the cross-sectional standard deviation on day d. We use 140 three-digit SIC sectors for stock returns for data availability reasons—there are not enough firms in the CRSP dataset to calculate sector returns with too much detail.

dispersion when the economy is weak. Across the various estimates, in both levels and growth rates, and for raw measures and sector-specific residuals, variance is on average higher by 0.79 standard deviations during recessions and the correlation with aggregate employment growth is -0.31. Moreover, across all coefficients, the point estimate has the predicted sign in every case (and is statistically significant in all but four).

Beyond what has been documented in past work, Table 2 also shows that the cyclicality of the cross-sectional variance holds not just for total activity at the sector level, but also the sector-specific residuals. That is, the cyclicality in cross-sectional variance is not due just to exposures to a common factor. After extracting that factor, and looking just at the sector-specific part—which corresponds to the sector shock in the model—the cross-sectional variance remains countercyclical. That fact is not consistent with all structural models, as we discuss further in section 7.

Table 2 also reports analogous results for cross-sectional skewness. In this case, we find that skewness is procyclical. On average, the estimates imply that skewness is more negative by 0.28 standard deviations in recessions and has a correlation of 0.10 with aggregate employment growth. The results are statistically weaker than for variances, which is to be expected as skewness is more poorly estimated than variance. Again, while cross-sectional skewness for total activity has been found to be procyclical elsewhere, these results are novel for showing that the same result holds for the sector-specific component of activity.

5.4 Prediction 3: Conditional Moments

This final section tests a core prediction of the network model: since GDP is a concave function of the sector shocks, when a sector receives a negative shock it should become more central.

We examine variation in centrality based on conditional covariances for three datasets: (1) employment and industrial production, directly measuring activity at the monthly level; (2) stock returns, indirectly measuring activity, but at the daily level; and (3) total factor productivity, which is most tightly linked to the model primitives, but available only at the annual level.

5.4.1 Industrial production and employment

Define Σ_t to be the (unobservable) conditional covariance matrix of sector-level growth rates on date t. Define $\Sigma_{t,i}$ to be the average of the i'th column of Σ_t, excluding the (i,i) element. $\Sigma_{t,i}$ is the average of the covariances of sector i with all other sectors. When we say that sector i covaries more strongly with other sectors, we mean $\Sigma_{t,i}$ rises. We also define $\beta_{i,t}$ to be the conditional covariance of activity in sector i with aggregate activity.

The goal is to estimate relationships of the form

$$\tilde{x}_{i,t} = \tilde{a}_i + \sum_{j=0}^{J-1} \tilde{b}_j \epsilon_{i,t-j} + \tilde{c}_t + \tilde{\eta}_{i,t}, \tag{15}$$

for $\tilde{x}_{i,t}$ equal to $\Sigma_{t,i}$ or $\beta_{i,t}$ and where $\epsilon_{i,t}$ measures the innovation to the level of activity in sector i on date t (included up to lag $J-1$). $\tilde{a}_i$, $\tilde{b}_j$ and $\tilde{c}_t$ are coefficients and $\tilde{\eta}_{i,t}$ is a residual. The problem is that $\Sigma_{t,i}$ and $\beta_{i,t}$ are not observable. We therefore proxy for them with date-t products, similar to the literature on heteroskedasticity and feasible generalized least squares.

More specifically, define $\epsilon_{i,t}$ to be the statistical innovation in activity in sector i (i.e. from a forecasting regression).[14] Note that $\epsilon_{i,t}$ here is not meant to capture the sector-specific TFP shock. Rather, it is just the innovation in sector output conditional on date-$(t-1)$ information, which will contain both sector and aggregate components.

We then proxy for $\Sigma_{t,i}$ with $\sum_{j\neq i} \epsilon_{i,t+1}\epsilon_{j,t+1}$ and $\beta_{i,t}$ with $\epsilon_{i,t+1}\epsilon_{agg,t+1}$ (where $\epsilon_{agg,t}$ is the statistical innovation in aggregate activity). Those products are single-observation sample moments when the conditional expectation of $\epsilon_{i,t}$ is zero, with the property that

$$\mathbb{E}_t\left[\sum_{j\neq i} \epsilon_{i,t+1}\epsilon_{j,t+1}\right] = \Sigma_{t,i}, \tag{16}$$

$$\mathbb{E}_t\left[\epsilon_{i,t+1}\epsilon_{agg,t+1}\right] = \beta_{i,t}. \tag{17}$$

This leads to the following regression

$$x_{i,t} = a_i + \sum_{j=0}^{J-1} b_j \epsilon_{i,t-j} + c_t + \eta_{i,t}, \tag{18}$$

$$\text{where } x_{i,t} = \sum_{j\neq i} \epsilon_{i,t+1}\epsilon_{j,t+1} \text{ or } \epsilon_{i,t+1}\epsilon_{agg,t+1}, \tag{19}$$

and $\eta_{i,t}$ captures both the true residual, $\tilde{\eta}_{i,t}$, and also the measurement error in the dependent variable, $x_{i,t} - \tilde{x}_{i,t}$ (e.g., $\epsilon_{i,t+1}\epsilon_{agg,t+1} - \beta_{i,t}$). The errors are therefore in general non-Gaussian. Because there may be common components across sectors in the innovations, $\epsilon_{i,t}$, we include time fixed effects in the estimation (c_t) and cluster the standard errors by date. Similarly,

[14] Specifically, we forecast activity in each sector using four lags of sector activity and the lagged value of the first three principal components of activity across all sectors.

some sectors will covary with others more strongly on average, hence, we also include sector fixed effects, a_i. The inclusion of time fixed effects means that changes in the conditional moments are all interpreted as changes relative to those in other sectors. For example, since the date-t mean of $\Sigma_{t,i}$ is equal to the mean of all pairwise covariances, positive values for the b_j coefficients mean that a positive shock to sector i raises its covariances *relative to those between other sectors*. Because the $x_{i,t}$ variables are functions of date-$t+1$ observations, the regressions all represent forecasts and hence conditional moments. That is, the fitted value of the right-hand side is a date-t conditional expectation.

The top three rows of Table 3 report results of the forecasting regressions for IP and employment. In each case, we use the level of aggregation that yields the largest number of sectors. For IP it is the 4-digit level. For employment, we use 2-digit data that extends to 1972 and 5-digit when using data since 1990 in separate regressions. In all cases, we use three monthly lags of activity on the right-hand side ($J = 3$) and report the sum of the coefficients in the table. Standard errors clustered by date are reported in brackets.

[Insert Table 3 here.]

For the regressions forecasting $\Sigma_{t,i}$ and $\beta_{i,t}$, the estimated coefficients are negative for both IP and employment. For the first two rows, the coefficients are of similar magnitude, about -0.05, while they are close to zero for the short employment sample. To give some intuition, a value of 0.05 implies that when a sector's activity rises by one standard deviation, the product on the left-hand side variable falls by 0.05 standard deviations. The regressions thus give consistent support to the model's prediction that following a negative shock, a sector becomes more central and more correlated with aggregate activity.

5.4.2 Stock returns

Since IP and employment are only available at the monthly frequency, forcing us to use a single observation to proxy for a covariance, one might naturally worry that the proxies for the moments in (16)–(17) would have a substantial amount of measurement error.[15] Stock returns have the advantage that they are available at the daily frequency and thus allow us to measure the covariance matrix for each month more accurately. We denote the sample covariance in month t by $\hat{\Sigma}_t$, and then $\hat{\Sigma}_{t,i}$ is the sum of the i'th row, excluding the diagonal element, as for economic activity. Similarly, the covariance of each sector's returns with

[15] Notice, however, that such measurement error appears in the residual in the regression, and therefore is accounted for in the standard errors.

returns on the overall market can be estimated using daily data within each month, $\hat{\beta}_{i,t}$, and the sector's variance can be estimated from the monthly sample variance. Regression results are gathered in the fourth row of Table 3:

$$x_{i,t} = a_i + \sum_{j=0}^{J-1} b_j r_{i,t-j} + c_t + \eta_{i,t}, \tag{20}$$

for $x_{i,t} = \hat{\Sigma}_{t,i}$, $\hat{\beta}_{i,t}$, or $\hat{\sigma}^2_{i,t}$, where $r_{i,t}$ is the return in sector i in month t. That is, we use the same specification as for IP and employment, just replacing $\sum_{j \neq i} \epsilon_{i,t}\epsilon_{j,t}$ with $\hat{\Sigma}_{t,i}$, etc.

The results are highly similar to those for IP and employment, with coefficients again close to -0.05. While there is less measurement error in this regression, we also have fewer observations since we use a higher level of aggregation (due to the number of firms available), so the magnitude of the standard errors is similar to that for our proxies of economic activity.

5.4.3 TFP shocks

To get a more direct measure of productivity shocks in each sector, this section uses the NBER-CES manufacturing database, which measures productivity at the 6-digit level among manufacturing industries, but is only available at the annual frequency (Acemoglu, Akcigit, and Kerr, 2015).

We estimate regressions using the same specification as for IP and employment above (equations (18) and (19)), but with two modifications. First, we use data on real gross output and hours of production workers instead of IP and employment on the left-hand side. Second, the independent variable, instead of being the lagged statistical innovation in activity, is the lagged statistical innovation in total factor productivity, as reported in the NBER-CES database.

The bottom two rows of Table 3 report the results from this exercise. The results are very similar to what we report for IP and employment. In particular, for gross output, as with IP (which also measures gross output), the coefficients are close to -0.05 and statistically significant, though in this case the coefficient for sector volatility is close to zero. For hours worked, as with employment, the point estimates are negative, and -0.05 is close to boundary of the confidence bands, but the point estimates themselves are close to zero.

In summary, there is strong evidence for gross output following the predictions of the model, in terms of negative shocks increasing covariances, but for employment the results are again weak at best.

5.4.4 Robustness

We have examined a number of robustness tests for the main results. The coefficient estimates are similar when we weight the sectors by their relative size, when we winsorize the dependent variable, and when we estimate the same regressions at alternative levels of aggregation.

6 Quantitative illustration

The stylized nature of the baseline model in Section 3 yields simple testable predictions, but at the cost of strict assumptions. This section develops a more general version of the model that relaxes the various symmetry assumptions and then explores the extent to which a quantitative version of the model can match the actual numbers reported in Section 5. Such a calibration exercise shows that a more realistic version of the model can generate quantitative results that are broadly similar to the empirical results documented in Section 5. We leave the formidable task of full estimation of a large-scale multisector model to future work.

6.1 Model and calibration

The specification closely follows that of Baqaee and Farhi (2019), with three key generalizations from our baseline model. First, we allow for heterogeneity in the intermediate input mix used by various industries. Second, we allow the elasticity of substitution between material input bundles in firms' production technologies (ξ) to be different from the elasticity in the consumption good bundle ($\hat{\xi}$). And third, whereas our baseline model assumed firms combine labor and intermediate inputs using a Cobb-Douglas aggregator, we assume that the elasticity of substitution between labor and material inputs to be equal to some β that may be distinct from 1. Therefore, the following pair of equations replace equations (1) and (2) in Section 3:

$$y_{i,t} = z_{i,t} \left((1-\alpha_i)^{1/\beta} \ell_i^{(\beta-1)/\beta} + \alpha_i^{1/\beta} \left(\sum_{j=1}^{n} a_{ij}^{1/\xi} x_{ij,t}^{(\xi-1)/\xi} \right)^{\frac{(\beta-1)\xi}{\beta(1-\xi)}} \right)^{\beta/(1-\beta)} \tag{21}$$

$$\text{GDP}_t = \left(\sum_{j=1}^{n} a_{cj}^{1/\hat{\xi}} c_{j,t}^{(\hat{\xi}-1)/\hat{\xi}} \right)^{\hat{\xi}/(\hat{\xi}-1)}. \tag{22}$$

As in the baseline economy, the market-clearing condition for good j is given by (3). In another departure from the baseline model, we assume that labor is sector-specific and cannot be reallocated across sectors, with the total supply of labor available to firms in industry i given by $\bar{L}_i$.[16]

Our calibration follows that of Baqaee and Farhi (2019) and draws on their replication files. The production weights are chosen to match the 1982 input-output table (results are similar for other years). TFP shocks are calibrated to match the relative variance of TFP by industry along with the observed correlations. Their overall scale is chosen to match the volatility of industrial production growth (and the moments we will examine will be matched to the IP data). We draw the log productivity shocks from a t-distribution with four degrees of freedom, which generates fat tails consistent with observed sector-specific IP growth. The autocorrelation of sector productivity is set to 0.85 (at the monthly frequency) to match the dynamics of sector-level IP growth.

The elasticities of substitution are set to $\xi = 0.1$, $\hat{\xi} = 0.9$, and $\beta = 0.5$, implying that goods are less substitutable in firms' production technology than in the consumption good bundle, which is close to a Cobb-Douglas specification. The strong complementarity in sectoral production technologies—which is in line with the empirical estimates of Atalay (2017)—means that the mix of material inputs is not amenable to adjustment.

6.2 Results

Table 4 reports moments of the model corresponding to the results from Figure 1 and Tables 2–3 along with the associated empirical results for industrial production. The top section shows that, as in the data, skewness is higher at the aggregate than the sector level, though in the model sector skewness is somewhat higher than in the data. Residual and aggregate skewness in the model is well within the empirical confidence bands.

[Insert Table 4 here.]

The middle section examines the cyclicality of cross-sectional variance and skewness. NBER recessions in the model are defined as periods when aggregate output growth is in the bottom 15 percent of the unconditional distribution, to replicate the empirical frequency of recessions. The signs and magnitudes of the coefficients are highly similar between the model and the data, both for skewness and variance. The model replicates the empirical result that cross-sectional variance is countercyclical and cross-sectional skewness is procyclical, and the magnitudes are empirically realistic.

[16] See Appendix A.2, where we show our theoretical results in Propositions 1–3 extend to economies with fixed factors.

Finally, the bottom section reports results for the conditional covariance regressions. Similar to the data, the coefficients in the two versions of the regressions are similar. In the model, they are equal to -0.025, compared to approximately -0.05 in the data. The magnitude of that difference is about the same size as the empirical standard errors, so the data and model again yield quantitatively similar results.

Overall, the quantitative model performs well in matching the time-series, cross-sectional, and conditional moments, given that we made few choices in the calibration. Table 4 therefore shows that a richer version of the model, designed to be closer to quantitative realism than the highly restricted setup analyzed theoretically above, is able to broadly match the empirical behavior of the economy documented in Figure 1 and Tables 2–3.

7 Alternative models of skewness

This section examines alternative models of skewness. We examine relatively stylized forms of models meant to capture different potential economic mechanisms that could be driving aggregate or cross-sectional skewness. The analysis shows that none of the alternatives is able to generate all of the same predictions—or match all of the empirical results—developed above. The table at the end of the section summarizes the results for the various models.

7.1 Skewed aggregate shocks

Consider the following simple reduced-form specification for aggregate and sector output, y_t and $y_{i,t}$:

$$y_t = \eta_t, \tag{23}$$

$$y_{i,t} = b_i y_t + \epsilon_{i,t}, \tag{24}$$

where η_t is a shock to aggregate output that is skewed left and $\epsilon_{i,t}$ is a symmetrically distributed idiosyncratic shock. We assume that there are many individual sectors that aggregate linearly so that $y_t = \text{xE}\left[y_{i,t}\right]$ (with the average of $b\{i\}$ equal to 1), where xE denotes the cross-sectional mean. This type of reduced-form could be generated by a number of different structural models.[17]

[17]Technology or policy shocks, represented by μ_t, could be skewed left, due to rare disasters (Rietz (1988), Barro (2006)), or with less extreme but still left-tilter asymmetry as in Berger, Dew-Becker, and Giglio (2020). Alternatively, some input to production used by all sectors, e.g., the output of the financial sector (financial intermediation) could be skewed to the left. For example, the financial sector might face occasionally binding constraints (see, e.g., Kocherlakota (2000) or Brunnermeier and Sannikov (2014)).

Time-series skewness: The skewness of η_t and symmetry of $\epsilon_{i,t}$ immediately implies that $skew\left(y_t\right) < skew\left(y_{i,t}\right) < 0$, as in the data.

Cyclicality of cross-sectional moments: On any date, the cross-sectional variance of sector output is

$$\text{xvar}\left(y_{i,t}\right) = \eta_t^2\,\text{xvar}\left(b_i\right) + \text{xvar}\left(\epsilon_{i,t}\right) + 2\,\text{xcov}\left(b_i, \epsilon_{i,t}\right), \tag{25}$$

where $\text{xvar}\left(y_{i,t}\right)$ denotes the cross-sectional variance of $y_{i,t}$ (on date t). It then immediately follows that

$$\text{cov}\left(y_t, \text{xvar}\left(y_{i,t}\right)\right) = \text{xvar}\left(b_i\right)\mathbb{E}\left[\eta_t^3\right]. \tag{26}$$

In other words, cross-sectional variance is countercyclical as long as aggregate output is skewed left. Recall, though, that the empirical results in Table 2 apply not just to sector growth rates, but also to residuals from regressions of sector growth rates on aggregate growth. That is, in the data the cyclicality of cross-sectional variance is not due to exposure to a common shock, but rather to cyclicality in the variance of the sector-specific component. In the model with skewed aggregate shocks described in (23)–(24), the assumption that η_t and $\epsilon_{i,t}$ are independent means that the cross-sectional moments of the $\epsilon_{i,t}$ are acyclical:

$$\text{cov}\left(y_t, \text{xvar}\left(\epsilon_{i,t}\right)\right) = 0 \tag{27}$$

which is inconsistent with the empirical results.

The failure of the model described in (23)–(24) to match the cyclicality of the cross-sectional distribution of the residuals is instructive. Its basic structure is superficially similar to the equilibrium of our model in (4)–(5), but it has a critical difference: the common component, η_t, is exogenous and independent of the sector-specific shocks, $\epsilon_{i,t}$, whereas in the aggregative model the common component is endogenous to the sector-specific shocks.

The model also fails to match the variation in the conditional covariances across sectors, again simply due to the independence of the sector and aggregate shocks.

7.2 Sector output is a concave function of symmetric shocks

Ilut, Kehrig, and Schneider (2018) (henceforth, IKS) study a model in which firms have concave responses to economic shocks, such that firm output or employment takes the form

$$y_{i,t} = g\left(\eta_t + \epsilon_{i,t}\right) \tag{28}$$

for a concave and increasing function g, where η_t is an aggregate shock and $\epsilon_{i,t}$ is an idiosyncratic shock. The shocks are mean-zero and independent with symmetrical distributions. IKS then assume that aggregate output is simply the sum over many firms of $y_{i,t}$—i.e. it is an expectation across values of $\epsilon_{i,t}$, conditional on the value of η_t:

$$y_t = \text{xE}\left[y_{i,t}\right], \tag{29}$$

where again $\text{xE}\left[y_{i,t}\right]$ denotes a mean across values of i on date t.

Unlike our network production model, the model of IKS generates skewness through micro decisions, so it provides a useful benchmark for understanding the difference between skewness arising from concave *aggregation* of symmetric micro shocks and skewness arising from concave micro *responses* to shocks.

Time-series skewness: Sector (or firm) output, $y_{i,t}$, in this model is skewed left due to the concavity of g. However, in this case skewness *decreases* with the level of aggregation.[18] To see why, consider a second-order approximation to sector output around the point $\eta_t = \epsilon_{i,t} = 0$

$$y_{i,t} \approx g(0) + g'(0)(\eta_t + \epsilon_{i,t}) + \frac{1}{2}g''(0)(\eta_t + \epsilon_{i,t})^2, \tag{30}$$

$$y_t \approx g(0) + g'(0)\eta_t + \frac{1}{2}g''(0)\left(\eta_t^2 + \text{xvar}(\epsilon_{i,t})\right). \tag{31}$$

The quadratic term in $y_{i,t}$ is $\frac{1}{2}g''(0)(\eta_t + \epsilon_{i,t})^2$, while in y_t it is $\frac{1}{2}g''(0)(\eta_t^2 + \text{xvar}(\epsilon_{i,t}))$. The skewness of $y_{i,t}$ is larger than the skewness of y_t essentially because there is more variability in what is being squared at the sector than at the aggregate level—$\eta_t + \epsilon_{i,t}$ instead of just η_t.

These results would also obtain in a granular model, as in Gabaix (2011), in which sector shocks are skewed left. That is, suppose there is effectively a small number of sectors, so that the sector shocks have nontrivial effects on aggregate output. Then even if they are skewed, after (linear) aggregation, aggregate output will be less skewed than sector output, due to simple averaging. In other words, the fact that skewness increases with aggregation is inconsistent with a simple form of micro granularity.

Cyclicality of cross-sectional moments: IKS give a formal derivation of the result that cross-sectional variance is countercyclical. A simple, though informal, way to see it is

[18] Indeed, this is what IKS find in their simulation (see their Table 9).

to examine a linear approximation to sector output (i.e. use the delta method),

$$\text{xvar}_t\left(y_{i,t}\right) \approx g'\left(\eta_t\right)^2 \text{xvar}\left(\epsilon_{i,t}\right). \tag{32}$$

By assumption, $g'\left(\eta_t\right)$ strictly increases as η_t declines, making cross-sectional variance countercyclical.

The model also generates countercyclicality for the variance of the sector-specific component of output, as in the data. In the first-order approximation above, the sector-specific component, after removing aggregate output, is $g'\left(\eta_t\right)\epsilon_{i,t}$. The cross-sectional variance of those residuals is then the same as the cross-sectional variance of output itself, and thus has the same cyclicality.[19]

Variation in conditional covariances: Consider a linear approximation of $y_{i,t}$ around the point that the variance of the aggregate shock, η_t, is negligible compared to that of the idiosyncratic shocks: $y_{i,t} \approx g(\epsilon_{i,t}) + g'(\epsilon_{i,t})\eta_t$. Under such an approximation,

$$\text{cov}_t(y_{i,t+1}, y_{j,t+1}) \approx \mathbb{E}_t[g'(\epsilon_{i,t+1})]\mathbb{E}_t[g'(\epsilon_{j,t+1})]\text{var}_t(\eta_{t+1}). \tag{33}$$

As long as shocks exhibit positive persistence, a negative shock to sector i increases $g'(\epsilon_{i,t+1})$. Therefore, a negative shock to sector i raises the covariance of sector i's output with all other sectors, without having any effect on covariances between other sectors. This result is not noted by IKS, but it represents an additional pair of empirical facts that the model matches.

Summary: The model of concave decision rules generates negative skewness, and can match our empirical findings on the cyclicality of cross-sectional moments and the changes in conditional covariances. However, it fails to generate the result that skewness increases with the level of aggregation. The key difference between this model and the model of complementarity in production is the underlying origin of skewness. With concave decision rules, the skewness arises at the firm or sector level. Under linear aggregation, that skewness washes out at the aggregate level. In the network model with complementarity, it is fundamentally *created by* aggregation, leading to agreater skewness at the aggregate than the firm level.

[19] IKS also show that cross-sectional skewness is procyclical under a restriction on g.

7.3 Time-varying uncertainty

Consider a simple model of time-varying cross-sectional volatility,

$$y_t = \sigma_{t-1}\eta_t - k\sigma_t^2, \tag{34}$$

$$y_{i,t} = y_t + \sigma_{t-1}\epsilon_{i,t}, \tag{35}$$

where $y_{i,t}$ and y_t denote sectoral and aggregate variables (such as output or log output), η_t and $\epsilon_{i,t}$ are the corresponding innovations with variances normalized to 1, and k is a constant that determines how output responds to variation in cross-sectional volatility relates to the level of output. Once again, we assume that η_t and $\epsilon_{i,t}$ are independent with unskewed distributions and that aggregation is linear over many sectors so that $y_t = \mathbb{E}_t[y_{i,t}]$.

Time-series skewness: Time-series skewness can occur in this model either if shocks to σ_t^2 are skewed to the right—so that $-k\sigma_t^2$ is skewed left—or if η_t and σ_t are correlated, so that the distribution of $\sigma_t\eta_t$ is skewed left. These features are not universal characteristics of models of uncertainty shocks. For example, the model of Bloom et al. (2018) does not generate skewness in aggregate output growth, because both the term $\sigma_t\eta_t$ and changes in volatility are symmetrically distributed. That said, if the conditions necessary for y_t to be skewed left are satisfied, then $y_{i,t}$ is also skewed left, and again by less than y_t, because of the fact that $\epsilon_{i,t}$ is symmetrically distributed. So the model *can*, though does not necessarily, match the data on time-series skewness of aggregate and sector output.

Time-varying cross-sectional moments: Cross-sectional variance in the model described by (35) is time-varying by assumption. Furthermore, cross-sectional variance is countercyclical for $k > 0$. This result holds both for total sector output and for the sector specific component, $\epsilon_{i,t}$.

To generate time-varying cross-sectional skewness, as in the data, an additional skewness process would need to be added to the model. Salgado, Guvenen, and Bloom (2020) examine such a setup.

Time-series skewness in the model is determined by the parameter k. Procyclical cross-sectional skewness, on the other hand, would require an extra free parameter. So the model requires a new parameter (or assumption) for each of the empirical results it matches. In that sense, it is less parsimonious than the network model, which just requires an assumption about a single parameter (i.e. that inputs are complements).

Conditional moments: The covariance of output between sectors is

$$\text{cov}\left(y_{i,t}, y_{j,t}\right) = \text{var}\left(\epsilon_t\right) = \sigma_{t-1}^2. \tag{36}$$

In other words, the covariances are all identical on any date. They change over time due to σ_t^2, and they are all countercyclical, but there is no variation across sectors. In both the network and IKS models, the source of the increased covariance following negative shocks is that a sector loads more heavily on the common component. This illustrates the importance of the common component being endogenous to the sector shocks, unlike here, where it is purely exogenous.

Summary: In a model where aggregate output responds negatively to uncertainty, it is possible to generate negative skewness. The model also, by assumption, generates countercyclical cross-sectional and time-series volatility. However, the model has no prediction for differences in covariances across sectors. The analysis in this section is based on a reduced-form representation, but it is possible that fully nonlinear solutions of structural models, like that of Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018), might yield different results. We examined simulations of that model, however (helpfully provided by the authors), and find that output and employment, in both levels and growth rates, are significantly *positively* skewed, implying that at least the baseline calibration of Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018) does not generate the starting point of our empirical analysis, that levels and growth rates of output and employment are skewed left.

7.4 Implications

Table ?? summarizes the empirical results and the ability of the models to qualitatively match them.

The equilibrium of the network model has two key features that allow it to match the empirical facts: there is skewness in the common but not sector-specific components, and the common component is endogenous to the sector shocks.

8 Conclusion

The goal of this paper is to understand the sources of asymmetries in aggregate output. It studies a network model in which inputs are complements and shows that it naturally

Summary Alternative Models

	Model			
Fact	Complementary network	Skewed shocks	Concave responses	Uncertainty shocks
Increasing skewness with aggregation	✓	✓	×	?
No skewness for residuals	✓	✓	×	✓
Cyclicality of cross-sectional variance	✓	✓	✓	✓
Cyclicality of cross-sectional resid. var.	✓	×	✓	✓
Centrality rises after negative shocks	✓	×	✓	×

generates time-series skewness in aggregate and sector output. It has a range of additional empirical predictions, all of which have support in the data. The idea of complementarity, advanced most recently by Baqaee and Farhi (2019), is powerful in understanding both the aggregate and cross-sectional behavior of the economy.

The model implies that second and third moments change over time and are cyclical. In the past, it has sometimes been argued that the observed cyclicality of those moments implies that there are exogenous shocks to *uncertainty*, and that uncertainty then has negative effects on the economy. The model advanced here, though, is one in which changes in volatility are a result of fundamental productivity shocks and have no independent effect on the level of output.

A second important implication, which is supported by our empirical contributions, is that the centrality of sectors changes over time. In some models, recessions have common causes, e.g. technology shocks. Here, however, every episode is different. When a sector receives a negative shock, it becomes relatively more important. So in a period where oil stocks are low, shocks to the oil sector become a major driving force (e.g., Hamilton (2003), Kilian (2008)), whereas in periods when the financial sector is highly constrained, financial shocks become most relevant (e.g., Brunnermeier and Sannikov (2014)). A key insight of this paper is that complementarity means that the aggregate effects of shocks change in important ways over time, those changes can be measured from the covariances of sector growth rates, and many models fail to match them.

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Table 1: **Measures of Aggregate Time-Series Skewness**

		Tail Probability Ratios				
	Skewness	$k = 1$	$k = 1.5$	$k = 2$	$k = 2.5$	$k = 3$
		Panel A: Growth Rates				
IP	-1.22	1.03	0.92	1.75	2.50	9.00
	[0.01]	[0.84]	[0.70]	[0.27]	[0.20]	[0.10]
Employment	-0.52	1.16	1.78	3.43	6.00	4.00
	[0.25]	[0.43]	[0.08]	[0.07]	[0.15]	[0.43]
Stock Returns	-0.57	1.08	1.56	1.50	2.00	4.00
	[0.06]	[0.34]	[0.01]	[0.32]	[0.25]	[0.22]
GDP	-0.36	0.92	1.30	2.00	3.00	2.00
	[0.45]	[0.72]	[0.42]	[0.32]	[0.41]	[0.70]
Consumption	-0.78	0.74	0.80	2.33	4.00	∞
	[0.14]	[0.14]	[0.45]	[0.14]	[0.24]	[0.52]
Investment	-0.86	0.79	1.3.0	1.17	∞	∞
	[0.16]	[0.18]	[0.32]	[0.78]	[0.22]	[0.52]
		Panel B: Levels				
IP	-0.95	0.79	1.78	∞	∞	∞
	[0.08]	[0.27]	[0.27]	[0.08]	[0.34]	[0.49]
Employment	-0.68	1.32	1.81	3.14	∞	∞
	[0.12]	[0.38]	[0.36]	[0.48]	[0.53]	[0.43]
Stock Returns	-1.14	1.03	3.29	∞	∞	∞
	[0.04]	[0.88]	[0.02]	[0.05]	[0.26]	[0.40]
GDP	-0.45	0.86	1.75	8.00	∞	.
	[0.18]	[0.53]	[0.30]	[0.21]	[0.57]	[.]
Consumption	-0.44	1.30	1.56	6.00	∞	.
	[0.14]	[0.28]	[0.48]	[0.45]	[0.43]	[.]
Investment	-1.20	0.96	5.00	∞	∞	.
	[0.06]	[0.88]	[0.02]	[0.12]	[0.44]	[.]

Notes: The first column reports skewness for growth rates (Panel A) and levels (Panel B) for six variables: Industrial production, employment, stock returns, GDP, consumption, and investment. p-values against a symmetric null from a block bootstrap are reported in brackets. Data is monthly (quarterly for GDP, consumption, and investment) and runs from January 1972 to December 2019.

Table 2: **Cross-Sectional Moments and Cyclicality**

	IP	IP resid	Employ	Employ resid	Returns resid	Returns
	Panel A: Growth Rates					
	Variance					
NBER	0.48***	0.41***	0.95***	0.56***	1.00**	1.00**
	[0.16]	[0.14]	[0.23]	[0.16]	[0.41]	[0.41]
Employment growth	−0.14	−0.11	−0.39***	−0.28***	−0.36**	−0.36**
	[0.09]	[0.08]	[0.06]	[0.07]	[0.15]	[0.15]
	Skewness					
NBER	−0.23**	−0.02	−0.44***	−0.40***	−0.17	−0.16
	[0.09]	[0.10]	[0.13]	[0.14]	[0.16]	[0.16]
Employment growth	0.07*	0.00	0.13***	0.11**	0.05	0.05
	[0.04]	[0.05]	[0.05]	[0.05]	[0.06]	[0.06]
# Obs	566	566	352	352	588	588
	Panel B: Levels					
	Variance					
NBER	0.75***	1.00***	0.66	1.18		
	[0.20]	[0.27]	[0.41]	[0.73]		
Employment growth	−0.23***	−0.30***	−0.32***	−0.57***		
	[0.07]	[0.10]	[0.11]	[0.20]		
# Obs	566	566	352	352		
	Skewness					
NBER	0.09	0.08	−0.38***	−1.21***		
	[0.23]	[0.22]	[0.08]	[0.27]		
Employment growth	0.00	0.00	0.13***	0.41***		
	[0.07]	[0.07]	[0.03]	[0.11]		
# Obs	566	566	352	352		

Notes: This table reports regression results from regressing cross-sectional variance or skewness from growth rates (Panel A) or levels (Panel B) on economic activity. Each entry in the table is a univariate regression coefficient. NBER is a dummy variable which takes a value of one in recessions and zero otherwise. Employment growth is standardized to have unit variance, as is the dependent variable in each regression (cross-sectional variance or skewness). Standard errors, reported in brackets, are calculated using the Newey–West (1987) method with 12 monthly lags. The columns labeled residuals use the cross-sectional variance of residuals from regressions of sector growth rates on aggregate growth. * indicates significance at the 10- ** 5-, and *** 1-percent level, respectively.

Table 3: **Conditional Covariance**

	$\Sigma_{t,i}$		$\beta_{t,i}$	
IP	-0.047^{**}	[0.020]	-0.053^{***}	[0.021]
Employment (1972-2019)	-0.109^{**}	[0.046]	-0.062^{*}	[0.039]
Employment (1990-2019)	-0.004	[0.017]	-0.014	[0.016]
Stock Returns	-0.059^{***}	[0.022]	-0.075^{***}	[0.027]
Shipments	-0.043^{**}	[0.018]	-0.040^{**}	[0.019]
Hours	-0.010	[0.014]	-0.012	[0.014]

Notes: Each row reports results of regressions measuring the response of conditional covariances to lagged innovations. For each variable, we use the level of aggregation that yields the largest number of sectors. $\Sigma_{t,i}$ reports results where the dependent variable is the covariance of each sector's growth rate with the sum of those for all other sectors. $\beta_{t,i}$ reports results for covariances with aggregate growth rates. For the first four rows, the independent variable is the lagged statistical innovation in the sector's growth rate. In the bottom section it is the lagged statistical innovation in TFP. All regressions include time and sector fixed effects and standard errors, reported in brackets, are clustered by date. The first four rows use monthly data and report the sum of the coefficients on three monthly lags. The final two rows use annual data and report the coefficient on a single annual lag. * indicates significance at the 10- ** 5-, and *** 1-percent level, respectively.

Table 4: **Model Simulation**

Panel A: Time-Series Skewness

	Model	Data	std err
Aggregate	-1.72	-1.25	[0.68]
Sector	-0.94	-0.42	[0.20]
Residual	-0.17	-0.27	[0.15]

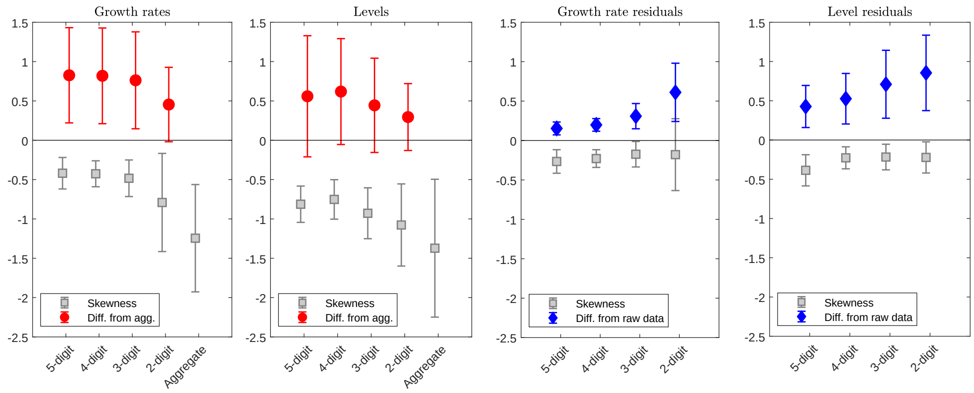
Panel B: Cross-Sectional Moments and Cyclicality

	Growth Rates			Residuals		
	Model	Data	std err	Model	Data	std err
			Variance			
Recession	0.61	0.48	[0.16]	0.40	0.41	[0.14]
IP	-0.24	-0.14	[0.09]	-0.17	-0.11	[0.08]
			Skewness			
Recession	-0.20	-0.23	[0.09]	-0.07	-0.02	[0.10]
IP	0.10	0.07	[0.04]	0.03	0.00	[0.05]

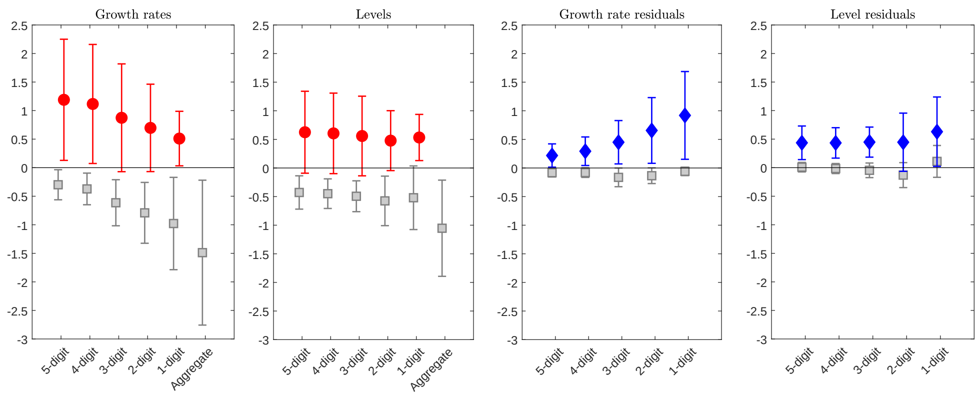
Panel C: Conditional Covariance

	Model	Data	std err	Model	Data	std err
		$\Sigma_{t,i}$			$\beta_{i,t}$	
IP	-0.26	-0.05	[0.02]	-0.02	-0.05	[0.02]

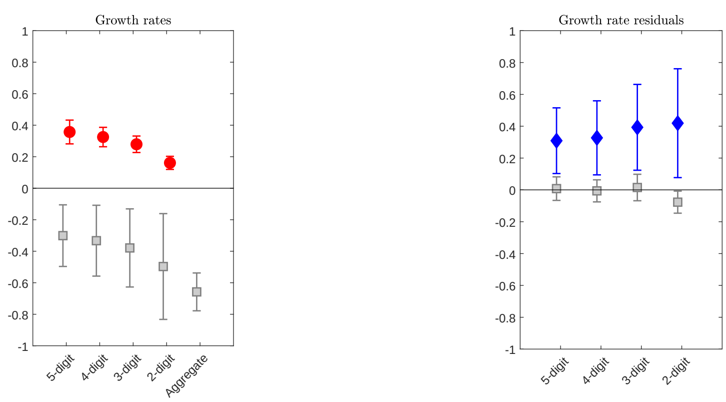
Notes: The "model" columns report moments from a simulation of 10,000 periods from the numerical solution to the model. The "data" columns report corresponding empirical estimates and standard errors. The sector-level estimates in the top and bottom sections are for industrial production for 4-digit industries.



(a) Industrial Production



(b) Employment



(c) Stock Returns

Figure 1: **Time-Series Skewness**

Notes: This figure plots skewness for different levels of aggregation for industrial production, employment, and stock returns for growth rates (left two panels) or in levels (right two panels) together with 90-percent confidence intervals. Data is monthly and starts in January 1972 and ends in December 2019 for industrial production and stock returns and starts in January 1990 and ends in December 2019 for employment.

A.1 Proofs

Equilibrium Characterization

In what follows we characterize the competitive equilibrium of the economy described in Section 3 and derive equations (4) and (5). Throughout, we let the consumption good bundle as the numeraire.

Let p_i denote the equilibrium price of good i and z_i be the productivity shock to sector i. Since all firms are competitive with constant returns technologies, it is immediate that the equilibrium price of good i satisfies

$$p_i = \frac{1}{z_i} w^{1-\alpha} \left(\sum_{j=1}^{n} a_j p_j^{1-\xi} \right)^{\alpha/(1-\xi)} = \frac{1}{z_i} w^{1-\alpha}, \tag{A.1}$$

where the second equality follows from our choice of the consumption good as the numéraire. Consequently,

$$\sum_{i=1}^{n} a_i p_i^{1-\xi} = w^{(1-\alpha)(1-\xi)} \sum_{i=1}^{n} a_i z_i^{\xi-1}.$$

Once again, the choice of the consumption good bundle as the numeraire implies that the left-hand side of the above equation is equal to 1. Furthermore, since we normalized the total supply of labor to 1, the household's budget constraint implies that $\text{GDP} = w$. As a result, log aggregate output in this economy is given by

$$\log(\text{GDP}) = \frac{1}{(\xi - 1)(1 - \alpha)} \log \left(\sum_{i=1}^{n} a_i z_i^{\xi-1} \right).$$

This establishes (5). To establish (4), first note that household's demand for good j is equal to $c_j = a_j p_j^{-\xi} \,\text{GDP}$. Similarly, sector i's demand for good j is given by $x_{ij} = \alpha a_j p_j^{-\xi} p_i y_i$. Consequently, market-clearing for good j implies that

$$y_j = a_j p_j^{-\xi} \,\text{GDP} + \alpha a_j p_j^{-\xi} \sum_{i=1}^{n} p_i y_i.$$

Multiplying both sides of the above equation by p_j, summing over all j, and using the fact that the consumption good bundle is the numeraire, implies that $\sum_{i=1}^{n} p_i y_i = \text{GDP}/(1-\alpha)$.

Plugging this into the previous equation we therefore obtain

$$y_j = \frac{1}{1-\alpha} a_j p_j^{-\xi} \, \mathrm{GDP} \, .$$

Replacing for the equilibrium price from (A.1) and using the fact that the wage is equal to aggregate output then establishes (4). □

Proof of Proposition 1

Let $a_{\min} = \min\{a_1, \ldots, a_n\} > 0$. Since $\xi < 1$, equation (5) implies that

$$\log \mathrm{GDP} \leq \frac{1}{(\xi-1)(1-\alpha)} \left(\log a_{\min} + \log \sum_{i=1}^{n} z_i^{\xi-1} \right) \leq k_0 + \frac{1}{1-\alpha} \epsilon_{\min},$$

where $\epsilon_{\min} = \min\{\epsilon_1, \ldots, \epsilon_n\}$, $\epsilon_i = \log z_i$ is the log productivity shock to sector i, and $k_0 = \frac{\log a_{\min}}{(\xi-1)(1-\alpha)}$ is a positive constant. The above inequality implies that, for any $\tau \geq 0$,

$$\frac{\mathbb{P}(\log \mathrm{GDP} < \mu - \tau\sigma)}{\mathbb{P}(\log \mathrm{GDP} > \mu + \tau\sigma)} \geq \frac{\mathbb{P}(\epsilon_{\min} < -(1-\alpha)(\tau\sigma - \mu + k_0))}{\mathbb{P}(\epsilon_{\min} > (1-\alpha)(\tau\sigma + \mu - k_0))}$$

where $\mu = \mathbb{E}[\log \mathrm{GDP}]$ and $\sigma = \mathrm{stdev}(\log \mathrm{GDP})$ denote the expected value and standard deviation of log aggregate output, respectively. Since all log productivity shocks are symmetrically distributed around their mean of zero, the above inequality implies that

$$\lim_{\tau\to\infty} \frac{\mathbb{P}(\log \mathrm{GDP} < \mu - \tau\sigma)}{\mathbb{P}(\log \mathrm{GDP} > \mu + \tau\sigma)} \geq \lim_{\tau\to\infty} \frac{1 - F^n(\tau + k_1)}{F^n(-\tau + k_1)},$$

where $F(\cdot)$ denotes the common cumulate distribution function of the log productivity shocks and $k_1 = (1-\alpha)(k_0 - \mu)$. As a result,

$$\lim_{\tau\to\infty} \frac{\mathbb{P}(\log \mathrm{GDP} < \mu - \tau\sigma)}{\mathbb{P}(\log \mathrm{GDP} > \mu + \tau\sigma)} \geq \lim_{\tau\to\infty} \frac{f(\tau + k_1)}{F^{n-1}(-(\tau - k_1)) f(\tau - k_1)} \geq \lim_{\tau\to\infty} \frac{f(\tau + k_1)}{F(-(\tau - k_1)) f(\tau - k_1)}$$

where $f(\cdot)$ denotes the density function corresponding to $F(\cdot)$ and the last inequality follows from the fact that $n \geq 2$. Now the assumption on the distribution function $F(\cdot)$ guarantees that the right-hand side of the above inequality diverges to infinity, thus establishing (6). □

Proof of Proposition 2

Fix the realization of log productivity shocks $\epsilon = (\epsilon_1, \dots, \epsilon_n)$ and let ξ denote the elasticity of substitution between intermediate inputs. Define function $h : (0, \infty) \to \mathbb{R}$ as $h(\xi) = (1-\alpha)(\xi-1) \log \text{GDP}$, where GDP, where recall log aggregate output satisfies (5). Therefore,

$$h(\xi) = \log \left(\sum_{i=1}^{n} a_i e^{(\xi-1)\epsilon_i} \right).$$

It is straightforward to verify that a second-order Taylor expansion of $h(\xi)$ around $\xi = 1$ is given by

$$h(\xi) = (\xi - 1)\kappa_1 + \frac{1}{2}(\xi - 1)^2 \kappa_2 + o((\xi - 1)^2), \tag{A.2}$$

where κ_1 and κ_2 denote, respectively, the first and second central moment of log productivity shocks $(\epsilon_1, \dots, \epsilon_n)$ with weights $(a_1, \dots, a_n)$:

$$\kappa_1 = \sum_{i=1}^{n} a_i \epsilon_i \quad , \quad \kappa_2 = \sum_{i=1}^{n} a_i \epsilon_i^2 - \left(\sum_{i=1}^{n} a_i \epsilon_i \right)^2.$$

Therefore, equation (A.2) implies that, to a second-order approximation, log aggregate output can be written as

$$\log(\text{GDP}) = \frac{1}{1-\alpha}\kappa_1 + \frac{1}{2(1-\alpha)} (\xi - 1) \kappa_2 + o(\xi - 1). \tag{A.3}$$

Given the realization of the shocks $\epsilon = (\epsilon_1, \dots, \epsilon_n)$, let $\mu_2 = \frac{1}{n} \sum_{i=1}^{n} \epsilon_i^2 - \left(\frac{1}{n} \sum_{i=1}^{n} \epsilon_i \right)^2$ denote the cross-sectional variance of the realized log productivity shocks. Equation (A.3) then implies that the covariance between log aggregate output and the cross-sectional variance of the realized shocks is given by

$$\text{cov}(\mu_2, \log(\text{GDP})) = \frac{\xi - 1}{2(1-\alpha)} \text{cov}(\mu_2, \kappa_2) + o(\xi - 1), \tag{A.4}$$

where we are using the fact that since all shocks have symmetric distributions around their mean of zero, $\mathbb{E}[\epsilon_i^k] = 0$ for all odd k. Note that

$$\mathbb{E}[\mu_2] = \frac{n-1}{n} \mathbb{E}[\epsilon_i^2] \quad , \quad \mathbb{E}[\kappa_2] = \left(1 - \sum_{i=1}^{n} a_i^2 \right) \mathbb{E}[\epsilon_i^2]. \tag{A.5}$$

Furthermore,

$$\mathbb{E}[\mu_2\kappa_2] = \frac{1}{n}\sum_{i,j=1}^{n} a_i\mathbb{E}[\epsilon_i^2\epsilon_j^2] - \frac{1}{n}\sum_{i,j,k=1}^{n} a_j a_k\mathbb{E}[\epsilon_i^2\epsilon_j\epsilon_k] - \frac{1}{n^2}\sum_{i,j,k=1}^{n} a_i\mathbb{E}[\epsilon_i^2\epsilon_j\epsilon_k] + \frac{1}{n^2}\sum_{i,j,k,r=1}^{n} a_i a_j\mathbb{E}[\epsilon_i\epsilon_j\epsilon_k\epsilon_r].$$

Since productivity shocks are independent with symmetric distributions around their mean of zero,

$$\begin{aligned}\mathbb{E}[\mu_2\kappa_2] &= \frac{n-1}{n^2}\left(1-\sum_{j=1}^{n} a_j^2\right)\left(\mathbb{E}[\epsilon_i^4] + (n-1)\mathbb{E}^2[\epsilon_i^2]\right) + \frac{1}{n^2}\sum_{\substack{i,k=1\\ j\neq i, r\neq k}}^{n} a_i a_j\mathbb{E}[\epsilon_i\epsilon_j\epsilon_k\epsilon_r] \\ &= \frac{n-1}{n^2}\left(1-\sum_{j=1}^{n} a_j^2\right)\left(\mathbb{E}[\epsilon_i^4] + (n-1)\mathbb{E}^2[\epsilon_i^2]\right) + \frac{2}{n^2}\sum_{\substack{i=1\\ j\neq i}}^{n} a_i a_j\mathbb{E}[\epsilon_i^2\epsilon_j^2],\end{aligned}$$

which in turn implies that

$$\mathbb{E}[\mu_2\kappa_2] = \frac{1}{n^2}\left(1-\sum_{j=1}^{n} a_j^2\right)\left((n-1)\mathbb{E}[\epsilon_i^4] + ((n-1)^2+2)\mathbb{E}^2[\epsilon_i^2]\right).$$

The above equation together with (A.4) and (A.5) implies that the covariance between the cross-sectional variance of the shocks and log aggregate output is given by

$$\operatorname{cov}(\mu_2, \log \mathrm{GDP}) = \frac{1}{2n^2}(\xi-1)\left(1-\sum_{i=1}^{n} a_i^2\right)\left((n-1)\mathbb{E}[\epsilon_i^4] - (n-3)\mathbb{E}^2[\epsilon_i^2]\right).$$

Since $\mathbb{E}[\epsilon_i^4] \geq \mathbb{E}^2[\epsilon_i^2]$, it is immediate that the right-hand side of the above equation is always negative whenever $\xi < 1$. Therefore, when intermediate inputs are gross complements, the covariance between the cross-sectional variance of the shocks and log aggregate output is negative whenever $\xi < 1$. This establishes (8).

To establish (9), let μ_3 denote the cross-sectional skewness of realized shocks and recall that log aggregate output satisfies (A.3). As a result,

$$\operatorname{cov}(\mu_3, \log \mathrm{GDP}) = \frac{1}{1-\alpha}\mathbb{E}[\mu_3\kappa_1] + o\left(\xi - 1\right), \tag{A.6}$$

where once again we are using the fact that $\mathbb{E}[\epsilon_i^k] = 0$ for all odd k. Next, note that

$$\mathbb{E}[\mu_3\kappa_1] = \frac{1}{n}\sum_{i,j=1}^{n} a_i\mathbb{E}[\epsilon_i\epsilon_j^3] - \frac{3}{n^2}\sum_{i,j,k=1}^{n} a_i\mathbb{E}[\epsilon_i\epsilon_j\epsilon_k^2] + \frac{2}{n^3}\sum_{i,j,k,r=1}^{n} a_i\mathbb{E}[\epsilon_i\epsilon_j\epsilon_k\epsilon_r]$$
$$= \frac{1}{n}\mathbb{E}[\epsilon_i^4] - \frac{3}{n^2}\left(\mathbb{E}[\epsilon_i^4] + (n-1)\mathbb{E}^2[\epsilon_i^2]\right) + \frac{2}{n^3}(\mathbb{E}[\epsilon_i^4] + 3(n-1)\mathbb{E}^2[\epsilon_i^2]).$$

Putting the above together with equation (A.6) therefore implies that

$$\mathrm{cov}(\mu_3, \log \mathrm{GDP}) = \frac{(n-1)(n-2)}{n^3(1-\alpha)}(\mathbb{E}[\epsilon_i^4] - 3\mathbb{E}^2[\epsilon_i^2])$$

to a first-order approximation in $\xi - 1$, which is always strictly positive when $n > 2$ and log productivity shocks have a positive excess kurtosis. □

Proof of Proposition 3

By assumption, log productivity shock to sector i follows an AR(1) process. Let $\rho_i > 0$ denote the persistence parameter of the shock to sector i, i.e., $\epsilon_{i,t+1} = \rho_i\epsilon_{i,t} + \eta_{i,t+1}$, where $\eta_{i,t+1}$ is productivity innovation at sector i. Recall that the log output of sector i has a factor structure given by equation (4). Therefore,

$$\mathrm{cov}_t(\log y_{i,t+1}, \log y_{j,t+1}) = \xi(1-\xi+\alpha\xi)\mathrm{cov}_t(\epsilon_{i,t+1}+\epsilon_{j,t+1}, \log \mathrm{GDP}_{t+1})$$
$$+ (1-\xi+\alpha\xi)^2\mathrm{var}_t(\log \mathrm{GDP}_{t+1}),$$

where we are using the fact that sectoral shocks are independent. Summing both sides of the above equation over all sectors $j \neq i$ implies that

$$\sum_{j\neq i}\mathrm{cov}_t(\log y_{i,t+1}, \log y_{j,t+1}) = (n-2)\xi(1-\xi+\alpha\xi)\mathrm{cov}_t(\epsilon_{i,t+1}, \log \mathrm{GDP}_{t+1})$$
$$+ (n-1)(1-\xi+\alpha\xi)^2\mathrm{var}_t(\log \mathrm{GDP}_{t+1})$$
$$+ \xi(1-\xi+\alpha\xi)\,\mathrm{cov}_t\left(\sum_{j=1}^{n}\epsilon_{j,t+1}, \log \mathrm{GDP}_{t+1}\right).$$

Consequently, for any pair of sectors $k \neq i$,

$$\Delta_t^{ik} = (n-2)\xi(1-\xi+\alpha\xi)\mathrm{cov}_t(\epsilon_{i,t+1}-\epsilon_{k,t+1}, \log \mathrm{GDP}_{t+1}).$$

Differentiating the above equation with respect to $\epsilon_{i,t}$ then implies that

$$\frac{\mathrm{d}\Delta_t^{ik}}{\mathrm{d}\epsilon_{i,t}} = (n-2)\xi(1-\xi+\alpha\xi)\,\mathrm{cov}_t\left(\epsilon_{i,t+1}-\epsilon_{k,t+1}, \frac{\mathrm{d}}{\mathrm{d}\epsilon_{i,t}}\log \mathrm{GDP}_{t+1}\right). \tag{A.7}$$

On the other hand, recall that log aggregate output is given by (5) and log productivity shock to sector i follows an AR(1) process with persistence parameter ρ_i. Therefore,

$$\frac{\mathrm{d}}{\mathrm{d}\epsilon_{i,t}}\log \mathrm{GDP}_{t+1} = \frac{\rho_i a_i}{1-\alpha}\frac{e^{(\xi-1)\epsilon_{it+1}}}{\sum_{j=1}^n a_j e^{(\xi-1)\epsilon_{j,t+1}}}.$$

Since $\xi < 1$, it is immediate that the right-hand side of the above expression is decreasing in $\epsilon_{i,t+1}$ and increasing in $\epsilon_{k,t+1}$ for all $k \neq i$. Therefore,

$$\mathrm{cov}_t\left(\epsilon_{i,t+1}, \frac{\mathrm{d}}{\mathrm{d}\epsilon_{i,t}}\log \mathrm{GDP}_{t+1}\right) < 0 \qquad \text{and} \qquad \mathrm{cov}_t\left(\epsilon_{k,t+1}, \frac{\mathrm{d}}{\mathrm{d}\epsilon_{i,t}}\log \mathrm{GDP}_{t+1}\right) > 0, \tag{A.8}$$

where we are using the fact that if $g(\cdot)$ is strictly increasing, then $\mathrm{cov}(x, g(x)) > 0$ for any non-degenerate random variable x.[1] The above inequalities thus imply that when $\xi < 1$, the right-hand side of (A.7) is strictly negative. This establishes the first inequality in (10).

To establish the second inequality in (10), note that equation (4) implies that

$$\mathrm{cov}_t(\log y_{i,t+1}, \log \mathrm{GDP}_{t+1}) = (1-\xi+\alpha\xi)\mathrm{var}_t(\log \mathrm{GDP}_{t+1}) + \xi\mathrm{cov}_t(\epsilon_{i,t+1}, \log \mathrm{GDP}_{t+1}).$$

Therefore, $\Xi_t^{ik} = \xi\mathrm{cov}_t(\epsilon_{i,t+1}, \log \mathrm{GDP}_{t+1}) - \xi\mathrm{cov}_t(\epsilon_{k,t+1}, \log \mathrm{GDP}_{t+1})$, which in turn implies that

$$\frac{\mathrm{d}\Xi_t^{ik}}{\mathrm{d}\epsilon_{i,t+1}} = \xi\mathrm{cov}_t(\epsilon_{i,t+1}, \frac{\mathrm{d}}{\mathrm{d}\epsilon_{i,t}}\log \mathrm{GDP}_{t+1}) - \xi\mathrm{cov}_t(\epsilon_{k,t+1}, \frac{\mathrm{d}}{\mathrm{d}\epsilon_{i,t}}\log \mathrm{GDP}_{t+1}).$$

The inequalities in (A.8) then imply that $\mathrm{d}\Xi_t^{ik}/\mathrm{d}\epsilon_{i,t+1} < 0$. □

A.2 Extensions and Variations

In our baseline model in Section 3, we assumed that labor can be flexibly reallocated across sectors and is supplied by the representative household inelastically. In this appendix, we show that the characterization in equations (4) and (5) remain valid with minor modifications

[1] To see this, note that $\mathrm{cov}(x, g(x)) = \mathbb{E}[xg(x)] - \mathbb{E}[x]\mathbb{E}[g(x)] = \mathbb{E}[(x-\mathbb{E}[x])g(x)] = \mathbb{E}[(x-\mathbb{E}[x])(g(x) - g(\mathbb{E}(x))]$. Since $g(\cdot)$ is strictly increasing, $(x-\mathbb{E}[x])(g(x)-g(\mathbb{E}(x)) > 0$ for all $x \neq \mathbb{E}[x]$. Therefore, $\mathrm{cov}(x, g(x)) > 0$.

if we allow for elastic labor supply and sector-specific factors (say, capital). We then show that sectoral and aggregate payments to this fixed factor also share the same characteristics as sectoral and aggregate output. Therefore, to the extent that stock returns move with payments to capital, the predictions in Section 4 also apply to stock returns.

A.2.1 Model with elastic and fixed factors

We consider an economy such that the production function of firms in sector i is given by

$$y_i = z_i \zeta k_i^{1-\alpha-\beta} \ell_i^{\beta} \left(\sum_{j=1}^{n} a_j^{1/\xi} x_{ij,t}^{(\xi-1)/\xi} \right)^{\alpha\xi/(1-\xi)},$$

where ℓ_i is the labor input (which, as in the baseline model, can be flexibly allocated across sectors), k_i is a fixed factor of production that is specific to firms in sector i (which we refer to as capital), and ζ is a normalization constant. Throughout, we normalize the installed value of sector-specific capital to $k_i = 1$ for all i. In the above specification, α and β parametrize share of labor and intermediate inputs and are such that $\alpha + \beta \leq 1$. Finally, we assume that the consumption bundle is given by (2) and the representative household supplies labor elastically at some exogenously-specified real wage w.[2] To simplify the expressions, we choose the normalization constant $\zeta = \alpha^{\beta-1}\beta^{-\beta}w^{\beta}$.

To characterize the equilibrium of this economy, note that since the supply of all sector-specific capital is normalized to one, the economy is equivalent an alternative economy in which firms in sector i have a decreasing returns production function given by

$$y_i = z_i \zeta \ell_i^{\beta} s_i^{\alpha},$$

where s_i denotes the intermediate input bundle of sector i. In this economy, the profit of firms in sector i is given by $\pi_i = p_i y_i - w l_i - s_i$, where p_i denotes the price of good i. The first-order conditions corresponding to sector i's problem imply that sector i's demand for the intermediate input bundle and labor are given by

$$\ell_i = \beta p_i y_i / w \qquad \text{and} \qquad s_i = \alpha p_i y_i, \tag{A.9}$$

respectively. From the above it is immediate that the firm i's expenditure on labor and

[2] A simple microfoundation for this assumption is to assume that household preferences are given by $u(C, L) = C - \chi \frac{1}{1+1/\eta} L^{1+1/\eta}$ for some parameter $\chi > 0$ and consider the limit that the Frisch elasticity of labor supply $\eta \to \infty$. In such an economy, the real wage is given by $w = \chi$.

profits are given by $wl_i = \beta s_i/\alpha$ and $\pi_i = (1-\alpha-\beta)s_i/\alpha$, respectively. Consequently, aggregate output in this economy is equal to

$$\text{GDP} = \sum_{i=1}^{n} \pi_i + \sum_{i=1}^{n} w\ell_i = (1/\alpha - 1)\sum_{i=1}^{n} s_i. \tag{A.10}$$

Next, note that the market-clearing condition for good j in equation (3) can be written as

$$\alpha^{-1} z_j s_j^{\alpha+\beta} = a_j p_j^{-\xi}\,\text{GDP} + a_j p_j^{-\xi}\sum_{i=1}^{n} s_i,$$

where we are using the fact that the household's and sector i's demand for good j are given by $c_j = a_j p_j^{-\xi}\,\text{GDP}$ and $x_{ij} = a_j p_j^{-\xi} s_i$, respectively. Thus, by (A.10),

$$z_j s_j^{\alpha+\beta} = a_j p_j^{-\xi}\sum_{i=1}^{n} s_i. \tag{A.11}$$

Next note that the first-order conditions in (A.9) imply that sector j's demand for the intermediate input bundle is given by

$$s_j = (p_j z_j)^{1/(1-\alpha-\beta)}. \tag{A.12}$$

Therefore, solving for p_j from the above equation and plugging in into (A.11) implies that

$$s_j = \left(a_j z_j^{\xi-1}\sum_{i=1}^{n} s_i\right)^{\frac{1}{1+(\xi-1)(1-\alpha-\beta)}}. \tag{A.13}$$

Summing both sides of the above equation over all j, solving for $\sum_{i=1}^{n} s_i$ and plugging the result into equation (A.10) implies that

$$\log \text{GDP} = \delta + \frac{1+(\xi-1)(1-\alpha-\beta)}{(\xi-1)(1-\alpha-\beta)} \log \sum_{j=1}^{n} \hat{a}_j z_j^{\frac{\xi-1}{1+(\xi-1)(1-\alpha-\beta)}}, \tag{A.14}$$

where δ is some constant and $\hat{a}_j = a_j^{\frac{1}{1+(\xi-1)(1-\alpha-\beta)}}$ for all j. The above equation is the counterpart to equation (5) with flexible and inelastic labor. In particular, when $\xi < 1$, log aggregate output is a concave function of log sectoral shocks.

To obtain the counterpart to equation (4) for sectoral output, note that equations (A.9) and (A.12) imply that $y_j = z_j s_j^{\alpha+\beta}/\alpha$. Therefore, equation (A.13) implies that

$$\log y_j = \hat{\delta}_j + \frac{\xi}{1+(\xi-1)(1-\alpha-\beta)} \log z_j + \frac{1}{1+(\xi-1)(1-\alpha-\beta)} \log \text{GDP}, \qquad \text{(A.15)}$$

for some constant $\hat{\delta}_j$. Therefore, as in equation (4) for our baseline model, sectoral output has a factor structure consisting of a common factor that is proportional to log aggregate output and an idiosyncratic component that is proportional to log productivity shock to that sector.

Taken together, the similarity between equation pairs (A.14)–(A.15) and (4)–(5) implies that Propositions 1–3 also hold for the economy with elastic and immobile factors of production.

A.2.2 Payments to capital

We next argue that Propositions 1–3 also apply to sectoral and aggregate payments to capital. To this end, we show that (i) log sectoral profits have a factor structure similar to equation (4) and (ii) log aggregate profits is a concave CES aggregate of log productivity shocks.

To see the first claim above, recall that in Section A.2.1, we established that the profits of sector j (which is equal to payments on its fixed capital stock) is given by $\pi_j = (1-\alpha-\beta)s_j/\alpha$, where s_j denotes sector j's demand for the intermediate input bundle. We also established that $y_j = z_j s_j^{\alpha+\beta}/\alpha$. Therefore, π_j is proportional to $(y_j/z_j)^{1/(\alpha+\beta)}$, which given (A.15), implies that (log) sectoral profits also have a factor structure similar to that of (log) sectoral output.

Similarly, $\pi_j = (1-\alpha-\beta)s_j/\alpha$ coupled with the fact that $\sum_{j=1}^{n} s_j$ is proportional to aggregate output (equation (A.10)) also implies that aggregate profits is proportional to aggregate output. Therefore, in view of (A.14), (log) aggregate profit is also a CES aggregate of sectoral shocks and is concave in those shocks when intermediate inputs are gross complements.

A.2.3 Skewness of growth rates

This section gives conditions under which the growth rates of output in the model are skewed left. We study a continuous time specification. In particular, a case where productivity follows a mean-reverting version of a finite-activity Lévy process (finite activity means that in any given time period, the number of jumps is almost surely finite). A Lévy process is a general class in which the increments are independent and stationary. There are well known results on representations for such processes. We impose one restriction, which is that the jump component of the process (from the Lévy–Itô representation) has finite activity (i.e. the Lévy measure is finite). This is a restriction to allow only certain types of jumps, which we impose because it allows the jump process to be expressed as a simple compound Poisson process, keeping the analysis relatively simple (see Cont and Tankov (2004) sections 3.4 and 4.1.1). Jump diffusions are widely studied in economics, particularly within finance.

Formally, we assume that for all i, $\epsilon_{i,t}$ follows

$$d\epsilon_{i,t} = -m\left(\epsilon_{i,t}\right)dt + \sigma dW_{i,t} + \Delta\epsilon_{i,t} \tag{A.16}$$

where $W_{i,t}$ is a standard Wiener process and $\Delta\epsilon_{i,t}$ is a compound poisson process, equal to a random number $k_{i,t}$ with probability λdt and zero otherwise (for a more formal definition, see Cont and Tankov (2004) section 8.3). $k_{i,t}$ is a symmetrically distributed random variable. We assume that the function m is such that ϵ has a well-defined unconditional distribution with finite moments. Intuitively, that requires that $m(\cdot)$ induces mean reversion in ϵ, for example as in an Ornstein–Uhlenbeck process.

The solution of the model is such that aggregate output is a function f of the sector productivities with the characteristics that $f_i > 0$ and $f_{ii} < 0$ $\forall i$ and $f_{ij} > 0$ $\forall i \neq j$. The question here is under what circumstances df_t is skewed left. That is, when do our results on skewness in levels also apply to growth rates?

Itô's lemma in the case of a jump diffusion (see Cont and Tankov (2004) section 8.3.2) yields

$$\begin{aligned} df_t &= \sum_i \left(-m\left(\epsilon_{i,t}\right)f_{i,t} + \frac{\sigma^2}{2}f_{ii,t}\right)dt \\ &\quad + \sum_i f_i \sigma dW_{i,t} + \sum_i f\left(..., \epsilon_{i,t-} + \Delta\epsilon_i, ...\right) - f\left(..., \epsilon_i, ...\right) \end{aligned} \tag{A.17}$$

Now first assume that there are no jumps, so that the final summation above is equal to zero. Then we have

$$\mathbb{E}\left[df_t^3\right] = O\left(dt^3\right) \tag{A.18}$$

$$\mathbb{E}\left[df_t^2\right] = O\left(dt\right) \tag{A.19}$$

and hence skewness is $O\left(dt^{3/2}\right) \to 0$.

Alternatively, suppose there are jumps. Then

$$\mathbb{E}\left[df_t^3\right] = \sum_i \lambda dt \mathbb{E}\left[\left(f\left(..., \epsilon_{i,t-} + k_t, ...\right) - f\left(..., \epsilon_i, ...\right)\right)^3\right] + o\left(dt\right) \tag{A.20}$$

The fact that f is globally concave implies that the expectation on the right-hand side is negative. Furthermore, $\mathbb{E}\left[df_t^2\right] = O\left(dt\right)$, so that skewness is negative and $O\left(dt^{-1/2}\right)$.