This appendix contains the proofs and derivations omitted from the main body of the paper. Section A derives Equation (11) in the paper. Section B provides the proof of Theorem 4.

A CES Production Technologies

In what follows, we derive the expression in Equation (11) in the paper. Suppose that the production technology of firms in industry $i$ is given by Equation (10) in the paper. The first-order conditions of firms in industry $i$ are therefore given by

$$l_i = \alpha_i p_i y_i / w$$

$$x_{ij} = (1 - \alpha_i) a_{ij} p_i y_i p_j^{-\sigma_i} \left( \sum_{k=1}^n a_{ik} p_k^{1 - \sigma_i} \right)^{-1},$$

where we are using the fact that $\alpha_i + \sum_{j=1}^n a_{ij} = 1$ for all $i$. Plugging the above expressions back into the production function of firms in industry $i$ implies that

$$p_i z_i = w^{\alpha_i} \left( \frac{1}{1 - \alpha_i} \sum_{k=1}^n a_{ik} p_k^{1 - \sigma_i} \right)^{(1-\alpha_i)/(1-\sigma_i)}.$$

Taking logarithms from both sides of the above equation leads to the following system of equations

$$\log(p_i / w) = -\epsilon_i + \frac{1 - \alpha_i}{\sigma_i - 1} \log \left( \frac{1}{1 - \alpha_i} \sum_{k=1}^n a_{ik} (p_k / w)^{1 - \sigma_i} \right).$$

We make two observations. First, the above system of equations immediately implies that when $\epsilon_i = 0$ for all industries $i$, then all relative prices coincide with another, that is, $p_i = w$ for all $i$. Second, differentiating both sides of the above equation with respect to $\epsilon_j$ and evaluating it at $\epsilon = 0$ leads to

$$d\hat{p}_i / d\epsilon_j = -\mathbb{I}_{\{i=j\}} + \sum_{k=1}^n a_{ik} d\hat{p}_k / \epsilon_j,$$

where recall that $\hat{p}_i = \log(p_i / w)$ is the log relative price of good $i$ and $\mathbb{I}$ denotes the indicator function. Rewriting the previous equation in matrix form, we obtain

$$d\hat{p} / d\epsilon_j = -e_j + A d\hat{p} / d\epsilon_j,$$

where $e_j$ is the $j$-th unit vector. Consequently, $d\hat{p} / d\epsilon_j = -(I - A)^{-1} e_j$, which in turn can be rewritten as

$$d\hat{p}_i \bigg|_{\epsilon=0} = -\ell_{ij}.$$
The above equation therefore illustrates how shocks to industry \( j \) change the relative prices of all other industries up to a first-order approximation.

Next, recall that the market-clearing condition for good \( i \) is given by \( y_i = c_i + \sum_{j=1}^{n} x_{ji} \). Multiplying both sides by \( p_i \) and dividing by GDP implies that
\[
\lambda_i = \beta_i + \sum_{k=1}^{n} \omega_{ki} \lambda_k,
\]
where \( \lambda_i = p_i y_i / \text{GDP} \) is the Domar weight of industry \( i \) and \( \omega_{ki} = p_i x_{ki} / p_k y_k \). Note that in deriving the above equation, we are using the fact that the household’s first-order condition requires that \( p_i c_i = \beta_i \text{GDP} \). Differentiating both sides of the above equation with respect to \( \epsilon \)
\[
\frac{d\lambda_i}{d\epsilon_j} = \sum_{k=1}^{n} \omega_{ki} \frac{d\lambda_k}{d\epsilon_j} + \sum_{k=1}^{n} \lambda_k \frac{d\omega_{ki}}{d\epsilon_j}. \tag{A.4}
\]
On the other hand, Equation (A.2) implies that \( \omega_{ki} = (1 - \alpha_k) a_{ki} p_i^{1-\sigma_k} / \left( \sum_{r=1}^{n} a_{kr} p_r^{1-\sigma_k} \right) \). Hence, differentiating both sides of this expression, evaluating them at \( \epsilon = 0 \), and plugging the resulting expression back into Equation (A.4) implies that
\[
\frac{d\lambda_i}{d\epsilon_j} = \sum_{k=1}^{n} a_{ki} \frac{d\lambda_k}{d\epsilon_j} + \sum_{k=1}^{n} (1 - \sigma_k) a_{ki} \lambda_k \left( \frac{d\hat{p}_i}{d\epsilon_j} - \frac{1}{1 - \alpha_k} \sum_{r=1}^{n} a_{kr} \frac{d\hat{p}_r}{d\epsilon_j} \right).
\]
Hence, using Equation (A.3), we obtain
\[
\frac{d\lambda_i}{d\epsilon_j} - \sum_{k=1}^{n} a_{ki} \frac{d\lambda_k}{d\epsilon_j} = \sum_{k=1}^{n} (\sigma_k - 1) a_{ki} \lambda_k \left( \ell_{ij} - \frac{1}{1 - \alpha_k} \sum_{r=1}^{n} a_{kr} \ell_{rj} \right).
\]
Multiplying both sides of the above equation by \( \ell_{is} \), summing over all \( i \), and noting that \( L = (I - A)^{-1} \) leads to
\[
\frac{d\lambda_i}{d\epsilon_j} = \sum_{k=1}^{n} (\sigma_k - 1) \lambda_k \left( \sum_{s=1}^{n} a_{ks} \ell_{si} \ell_{sj} - \frac{1}{1 - \alpha_k} \sum_{r=1}^{n} a_{kr} \ell_{rj} \sum_{s=1}^{n} a_{ks} \ell_{si} \right). \tag{A.5}
\]
On the other hand, the fact that \( \lambda_i = p_i y_i / \text{GDP} \) implies that
\[
\frac{d \log y_i}{d\epsilon_j} = \frac{d\hat{p}_i}{d\epsilon_j} + \frac{1}{\lambda_i} \frac{d\lambda_i}{d\epsilon_j} = \ell_{ij} + \frac{1}{\lambda_i} \frac{d\lambda_i}{d\epsilon_j},
\]
where the second equality is a consequence of Equation (A.3). Plugging for \( d\lambda_i/d\epsilon_j \) from Equation (A.5) into the above equality leads to Equation (11).

\[\Box\]

**B Proof of Theorem 4**

Consider two economies with symmetric circulant input-output matrices \( A \) and \( \tilde{A} \) and suppose the latter is more interconnected than the former, that is, there exists a \( \gamma \in [0, 1] \) such that
\[
\tilde{A} = \gamma A + (1 - \gamma)(1 - \alpha)J,
\]
where \( J = (1/n)I \) is a matrix with all entries equal to \( 1/n \). We first prove statement (b) of the theorem by showing that the above transformation can only decrease the volatility of each industry, i.e., \( \text{var}(\log \tilde{y}_i) \leq \text{var}(\log y_i) \) for all \( i \). We then use this result to establish statement (a).
Proof of statement (b). Recall from Theorem 1 that the output of industry \( i \) satisfies \( \log y_i = \sum_{j=1}^n \ell_{ij} \epsilon_j \). Under our assumption that all microeconomic shocks are i.i.d. with a common variance \( \sigma^2 < \infty \), it is immediate that \( \text{var}(\log y_i) = \sigma^2 \sum_{j=1}^n \ell_{ij}^2 \). Therefore, sectoral log outputs are more volatile in the less interconnected economy (that is, \( \text{var}(\log \hat{y}_i) \leq \text{var}(\log y_i) \) for all \( i \)) if and only if \( \sum_{j=1}^n \hat{\ell}_{ij}^2 \leq \sum_{j=1}^n \ell_{ij}^2 \) for all \( i \). On the other hand, the assumption that input-output matrices \( A \) and \( \hat{A} \) are symmetric and circulant implies that \( \sum_{j=1}^n \hat{\ell}_{ij}^2 = (1/n) \sum_{i,j=1}^n \ell_{ij}^2 = (1/n) \text{trace}(\hat{L}^2\hat{L}) = (1/n) \text{trace}(\hat{L}^2) \).

Hence, it is sufficient to show that

\[
\frac{d}{d\gamma} \text{trace}(\hat{L}^2) \bigg|_{\gamma=1} \geq 0. \tag{B.1}
\]

To this end, first note that, by definition, \( \hat{L} = (I - \hat{A})^{-1} \). Therefore, differentiating \( \hat{L}^2 \) with respect to \( \gamma \) leads to

\[
d\hat{L}^2/d\gamma = \hat{L}^2 (d\hat{A}/d\gamma)\hat{L} + \hat{L} (d\hat{A}/d\gamma)\hat{L}^2.
\]

On the other hand, \( d\hat{A}/d\gamma = A - (1 - \alpha)J \). Consequently,

\[
\frac{d\hat{L}^2}{d\gamma} \bigg|_{\gamma=1} = L^2 AL + LAI^2 - (1 - \alpha)(L^2JL + JL^2)
= 2(L^3 - L^2) - 2(1 - \alpha)\alpha^{-3}J,
\]

where the second equality uses \( LA = AL = L - I \) and the fact that the row and column sums of \( L \) are equal to \( 1/\alpha \), i.e., \( L1 = L1 = (1/\alpha)1 \). Hence,

\[
\frac{d}{d\gamma} \text{trace}(\hat{L}^2) \bigg|_{\gamma=1} = 2 \text{trace}(L^3) - 2 \text{trace}(L^2) - 2(1 - \alpha)/\alpha^3.
\]

Note that the trace of a matrix is equal to the sum of its eigenvalues. Furthermore, the fact that \( L = (I - A)^{-1} \) implies that \( \lambda_k(L) = (1 - \lambda_k(A))^{-1} \), where \( \lambda_k(L) \) and \( \lambda_k(A) \) are the \( k \)-th largest eigenvalues of \( L \) and \( A \), respectively. Consequently,

\[
\frac{d}{d\gamma} \text{trace}(\hat{L}^2) \bigg|_{\gamma=1} = 2 \sum_{k=1}^n \frac{1}{(1 - \lambda_k(A))^3} - 2 \sum_{k=1}^n \frac{1}{(1 - \lambda_k(A))^2} - 2(1 - \alpha)/\alpha^3 = 2 \sum_{k=2}^n \frac{\lambda_k(A)}{(1 - \lambda_k(A))^3}.
\]

The second equality above is a consequence of the fact that the row sums of matrix \( A \) are all equal to \( 1 - \alpha \), and hence, by the Perron-Frobenius theorem, its largest eigenvalue is given by \( \lambda_1(A) = 1 - \alpha \).

Multiplying and dividing the right-hand side of the above equation by \( n - 1 \) and using the fact that the function \( g(z) = z/(1 - z)^3 \) is convex over the interval \((-1, 1)\) implies that

\[
\frac{d}{d\gamma} \text{trace}(\hat{L}^2) \bigg|_{\gamma=1} \geq \frac{2 \sum_{k=2}^n \lambda_k(A)}{(1 - \frac{1}{n-1} \sum_{k=2}^n \lambda_k(A))^3}. \tag{B.2}
\]

Next, note that \( \sum_{k=2}^n \lambda_k(A) = \text{trace}(A) - \lambda_1(A) = na_{ii} - (1 - \alpha) \geq 0 \), where we are using the assumption that \( a_{ii} \geq 1/n \) for all \( i \). This implies that the numerator of the fraction on the right-hand side of (B.2) is nonnegative. Furthermore, the fact that \( \lambda_k(A) \leq \lambda_1(A) = 1 - \alpha \) guarantees that the denominator of the fraction on the right-hand side of (B.2) is strictly positive. Taken together, these two observations establish inequality (B.1). □
Proof of statement (a). We now use statement (b) to establish statement (a) of the theorem. Recall from the previous part that the variance-covariance matrix of sectoral log outputs is given by $\sigma^2 \tilde{L}\tilde{L}'$. On the other hand, the assumption that the input-output matrix $A$ is symmetric and circulant guarantees that all row and column sums of $\tilde{L}$ are equal to $1/\alpha$. Therefore,

$$
\sum_{i,j=1}^{n} \text{cov}(\log \tilde{y}_i, \log \tilde{y}_j) = 1'\tilde{L}1 = n/\alpha^2.
$$

Furthermore, the assumption that the economy’s input-output matrix is circulant implies that all industries are equally volatile, that is, $\text{var}(\log \tilde{y}_i) = \text{var}(\log \tilde{y}_1)$ for all $i$. Hence,

$$
\sum_{i\neq j} \text{cov}(\log \tilde{y}_i, \log \tilde{y}_j) = n(1/\alpha^2 - \text{var}(\log \tilde{y}_1)).
$$

Hence, the average pairwise correlation between sectoral log outputs is given by

$$
\hat{\rho} = \frac{1}{n(n-1)} \sum_{i\neq j} \text{corr}(\log \tilde{y}_i, \log \tilde{y}_j) = \frac{1}{(n-1) \text{var}(\log \tilde{y}_1)} (1/\alpha^2 - \text{var}(\log \tilde{y}_1)).
$$

Identical derivations for the less interconnected economy with input-output matrix $A$ imply that

$$
\rho = \frac{1}{(n-1) \text{var}(\log y_1)} (1/\alpha^2 - \text{var}(\log y_1)).
$$

Comparing the right-hand sides of the above two equations completes the proof: by statement (b) of the theorem, $\text{var}(\log y_1) \geq \text{var}(\log \tilde{y}_1)$, which in turn implies that $\rho \leq \hat{\rho}$. \qed