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Information Sale and Competition

Kostas Bimpikis,[a] Davide Crapis,[b] Alireza Tahbaz-Salehi[c]

[a] Graduate School of Business, Stanford University, Stanford, California 94305; [b] Lyft, San Francisco, California 94107; [c] Kellogg School of Management, Northwestern University, Evanston, Illinois 60208

Contact: kostasb@stanford.edu, http://orcid.org/0000-0002-5289-0397 (KB); davide.crapis@gmail.com (DC); alirezat@kellogg.northwestern.edu (AT-S)

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Abstract. This paper studies the strategic interaction between a monopolistic seller of an information product and a set of potential buyers that compete in a downstream market. The setting is motivated by information markets in which (i) sellers have the ability to offer information products of different qualities and (ii) the information product provides potential buyers not only with more precise information about the fundamentals, but also with a coordination device that can be used in their strategic interactions with their competitors. Our results illustrate that the nature and intensity of competition among the information provider's customers play first-order roles in determining the information provider's optimal strategy. We show that when the customers view their actions as strategic complements, the provider finds it optimal to offer the most accurate information at the provider's disposal to all potential customers. In contrast, when buyers view their actions as strategic substitutes, the provider maximizes the provider's profits by either (i) restricting the overall supply of the information product or (ii) distorting its content by offering a product of inferior quality. We also establish that the provider's incentive to restrict the supply or quality of information provided to the downstream market intensifies in the presence of information leakage.

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1. Introduction

Recent advances in information technology have streamlined the process of mining, aggregating, and processing high-volume data about economic activity. Arguably, it is widely believed that the availability of more accurate information about the business environment and market conditions can be hugely beneficial to firms across a wide variety of industries. Such a realization has, in turn, led to a sizable demand for business-to-business information services. Several firms, ranging from Nielsen to Thomson Reuters and IRI, have built their business models around collecting, customizing, and selling information products to other market participants.

Motivated by the growing interest in the markets for information, this paper studies the problem of the optimal design and sale of information by a monopolistic provider of information products. We show that the nature and intensity of competition among the information provider's potential customers have a first-order impact on the information provider's optimal selling strategy and profits. More specifically, our analysis illustrates that the value the provider can extract from the provider's customers is largely determined by the trade-off between (i) the direct (positive) effect of more precise information on the customers' profits by enabling them to make more informed decisions and (ii) the strategic effects that arise because of the fact that the provider's customers may interact with one another in other markets.

Of particular relevance to our study are information markets in which (i) information providers have the ability to sell signals of different precisions (at potentially different prices) to their customers and (ii) information can play a role in how different market participants interact with one another strategically. Potential examples include the multitude of consumer, shopper, and retail market analyses of varying precision offered by firms such as IRI and Nielsen to the consumer packaged goods industry[1] as well as the expansive menus of information products that financial data providers (such as Bloomberg and Thomson Reuters) make available to their customers.[2]

We present our main findings in the context of an environment that involves a monopolistic information provider who can sell potentially informative signals to a collection of firms that compete with one another in a downstream market. We assume that the customer firms face demand uncertainty and that the provider is endowed with a private signal that is (partially) informative about the actual demand realization, thus creating potential gains from trade. Crucially for our argument—and in line with the observation that many real-world information providers offer a variety of information products of varying qualities—we allow for a setting in which the provider can offer information products that are potentially less precise than the provider's private information. In other words, the provider can potentially distort the informativeness of the signal at the provider's disposal by reducing its accuracy.

As our main result, we show that the optimal selling strategy of the provider is largely dependent on the nature and intensity of competition among the provider's potential customers in the downstream market. More specifically, we first show that when firms engage in price competition (Bertrand), the provider finds it optimal to sell the provider's signal with no distortion to the entire set of firms. This is because, in a Bertrand market, firms' actions are strategic complements and, hence, each firm's marginal benefit of procuring a more accurate signal is increasing in the fraction of its competitors that purchase the provider's information product. Therefore, the provider would obtain maximal profits by flooding the market with highly precise signals.

The situation, however, can be dramatically different if the information provider's customers compete with one another in quantities (Cournot). For such a downstream market, we show that the provider may no longer find it optimal to sell an undistorted version of the provider's signal to all firms. Rather, the provider may find it optimal to either (i) reduce the quality of the provider's information product by selling a signal of a lower precision than the one the provider possesses, (ii) strategically limit the provider's market share by excluding a subset of the provider's customers from the sale, or (iii) employ both strategies simultaneously by reducing the quality *and* quantity of the products offered. The optimality of these "information-distorting strategies" is because, in a Cournot market, firms' actions are strategic substitutes, which leads to the emergence of two opposing effects. On the one hand, obtaining additional information about demand directly benefits firms as they can better align their production decisions with underlying market conditions. On the other hand, however, the provider's signal can also serve as a correlating device among the provider's customers' equilibrium actions. In particular, providing the information product to an extra firm can only increase the correlation in the firms' production decisions, an outcome that reduces each firm's profits and, hence, can adversely affect the provider's bottom line. Therefore, when downstream competition is intense enough (for example, when firms' products are sufficiently substitutable), this latter, strategic channel would dominate the positive effect of reducing demand uncertainty, implying that the information provider would be better off by restricting the quantity and/or quality of the provider's information products. Interestingly, unlike in Bertrand competition, the provider's profits in a Cournot market are decreasing in the intensity of competition and may end up being significantly lower than in the absence of any competition.

To further clarify the forces that underpin our results, we also discuss a number of extensions to our benchmark setup. First, we let the provider offer a menu of information products with potentially different precisions and at different prices. We provide an explicit characterization of the optimal selling strategy as a function of the nature and intensity of competition and show that when firms compete in quantities and offer substitutable products, there is a continuum of strategies that lead to the same equilibrium profits for the provider. This characterization thus formalizes the trade-off in the provider's incentives for reducing the quantity or quality of the provider's information product. Second, we extend our benchmark framework by allowing for the possibility of information leakage among the provider's customers. In particular, we assume that, by observing the decisions of their competitors, firms can partially infer the information content of the signal purchased by other firms, thus altering their own willingness to pay for the information provider's signal. We establish that the provider's incentive for reducing the quality and/or quantity of the provider's information product increases as the extent of information leakage among the provider's customers is intensified. Third, we explore the implications of firm heterogeneity for the provider's selling strategy by considering a setting in which firms differ in their production costs. We show that it is optimal for the provider to sell higher precision information products (at higher prices) to the more efficient firms, that is, the firms that have lower production costs. Finally, we establish that our main qualitative insights carry over to a market consisting of finitely many firms.

Taken together, these findings provide a step toward understanding the intricacies involved in markets for information. Unlike traditional markets for physical goods, it is relatively inexpensive to offer a diverse menu of information products that differ in their precision and pricing. Our results highlight that the value that a given buyer can extract from procuring such products depends not only on the product's characteristics (such as its price and precision), but also on the environment in which the information provider's customers interact with one another.

Our paper is related to the extensive literature that studies firms' strategic considerations in sharing information with one another in oligopolistic markets. For example, Vives (1984), Gal-Or (1985), Li (1985), and Raith (1996) provide conditions under which firms find it optimal to share their private information about market conditions with their competitors. A more recent collection of papers, such as Shin and Tunca (2010), Shamir (2012), Shamir and Shin (2016), Ha and Tong (2008), and Ha et al. (2011), studies information-sharing incentives in vertical supply chains. For instance, Shamir and Shin (2016) determine conditions under which firms can credibly share their demand forecasts with one another whereas Cui et al. (2015) provide a theoretical and empirical assessment of the value of information sharing in two-stage supply chains. In contrast to this literature, which, for the most part, focuses on firms' incentives to fully share the information at their disposal with one another, we consider a setting in which a *third party* decides not only the price, but also the *accuracy* of the information product(s) the provider makes available to a set of competing firms. This allows for richer equilibrium outcomes that highlight the interplay between the nature of competition, the optimal selling strategy, and the information provider's profits.

Our paper is also related to the literature, such as Li and Zhang (2008), Anand and Goyal (2009), and Kong et al. (2013), that studies the implications of indirect leakage of information in supply chains via firms' actions. Similar considerations have also been studied in the context of financial markets (Admati and Pfleiderer 1990). Building on the framework of Vives (2011), we show how the intensity of information leakage in the market impacts firms' valuation of information and, hence, alters the provider's incentives in designing the provider's information products.

Our work is also related to the growing theoretical literature on the social and equilibrium value of public information. Morris and Shin (2002) illustrate that public disclosure of information regarding a payoff-relevant parameter may adversely affect social welfare as it may crowd out agents' reliance on their private information. Angeletos and Pavan (2007) extend this framework and provide a complete taxonomy of conditions under which private and public signals are efficiently utilized in equilibrium.[3] Relatedly, Bergemann and Morris (2013) study games of incomplete information with the goal of providing equilibrium predictions that are robust to all possible information structures. Their analysis illustrates that information disclosure policies that involve a partial sharing of a firm's private information may lead to higher equilibrium payoffs.

Also related is the recent work of Myatt and Wallace (2015), who consider a setting in which a set of firms compete in a Cournot market by selling differentiated products to a representative consumer. They characterize the weights firms assign to the private and public signals at their disposal as functions of the signals' precisions, the intensity of the competition, and the extent of product differentiation. They also establish that when signals are costly, firms acquire too much information relative to the socially efficient benchmark. In contrast to their paper, our main focus is on the provider's incentives to reshape the quantity and quality of information sold to the firms.

Finally, our work is related to the more recent work of Bergemann and Bonatti (2015), who explore selling information in the form of cookies in the context of online advertising, as well as Xiang and Sarvary (2013), who consider a market for information with competition on both the demand and supply sides of the market. In a similar context, Babaioff et al. (2012) study the design of optimal mechanisms for a data provider to sell information to a single buyer.

2. Model

Firms. Consider an economy consisting of a unit mass of firms indexed by $i \in [0, 1]$ that compete with one another in a downstream market. Each firm i takes an action $a_i \in \mathbb{R}$ to maximize its profit, which is given by the following expression:

$$\pi(a_i, A, \theta) = \gamma_0 a_i \theta + \gamma_1 a_i A - \frac{\gamma_2}{2} a_i^2, \tag{1}$$

where $A = \int_0^1 a_i \, di$ denotes the aggregate action taken by the firms; $\theta \in \mathbb{R}$ is an unknown payoff-relevant parameter; and $\{\gamma_0, \gamma_1, \gamma_2\}$ are some exogenously given constants. Depending on the context, action a_i may represent the quantity sold or the price set by firm i. As we show in Section 2.2, this framework nests Cournot and Bertrand competition as special cases. For the time being, however, we find it more convenient to work with the general setup without taking a specific position on the mode of competition.

The unknown parameter θ is randomly drawn by nature before firms choose their actions. As we discuss in the following sections, this parameter can represent the intercept of the (inverse) demand curve in the downstream market. All firms hold a common prior belief on θ, which, for simplicity, we assume to be the (improper) uniform distribution over the real line.[4] Even though firms do not know the realization of θ, each firm i observes a noisy private signal

$$x_i = \theta + \epsilon_i, \quad \epsilon_i \sim N(0, 1/\kappa_x)$$

with κ_x capturing the precision of the private signal observed by each firm. The noise terms ϵ_i are independently distributed across firms. Given firm i's profit function in (1), we let

$$\beta = -\frac{\partial^2 \pi}{\partial a \partial A} \Bigg/ \frac{\partial^2 \pi}{\partial a^2} = \frac{\gamma_1}{\gamma_2} \tag{2}$$

denote the degree of strategic complementarity in firms' actions. Note that $\beta > 0$ corresponds to an economy in which firms' actions are strategic complements: the benefit of taking a higher action to firm i

increases the higher the actions of other firms are. In contrast, when $\beta < 0$, firms face a game of strategic substitutes, where i's incentives for taking a higher action decrease with the aggregate action A. Finally, $\beta = 0$ corresponds to a market in which firms face no strategic interactions.

Throughout the paper, we assume that $\gamma_2 > \max\{2\gamma_1, 0\}$. This assumption, which implies that $\beta \in (-\infty, 1/2)$, is made to guarantee that firm i's profits are strictly concave in a_i and that i's marginal profit is more sensitive to its own action a_i than to the aggregate action A.

Information Provider. In addition to the competing firms, the economy contains a monopolist who possesses some private information about the realization of the unknown parameter θ that it can potentially sell to the firms before they take their actions. The provider has access to a private signal z with precision κ_z given by

$$z = \theta + \zeta, \quad \zeta \sim N(0, 1/\kappa_z),$$

where the noise term ζ is independent of ϵ_i's. Given that our main focus is on the market for information, we assume that this signal has no intrinsic value to the provider and that the provider can only benefit from the signal by selling it to the firms.

The key feature of our model is that the provider has control over both the "quantity" and "quality" of information sold to the firms: the information provider not only chooses the set of firms $I \subseteq [0,1]$ that the provider decides to trade with, but the provider can also choose the precision of the signal offered to the firms. More specifically, the provider offers a signal

$$s_i = z + \xi_i, \quad \xi_i \sim N(0, 1/\kappa_\xi)$$

to firm $i \in I$ at price p_i, where ξ_i is independent from z and $1/\kappa_\xi$ captures the variance of the noise introduced by the provider into s_i. This specification thus captures the idea that the provider can control the quality of the information sold to the firms: by choosing a smaller κ_ξ, the provider can "damage" the signals offered to the firms.[5] Throughout the paper, we refer to s_i as the *market signal* sold to firm i.

In general, the noise added to different firms' signals by the provider may be correlated with one another. To capture this idea formally, we assume that in addition to their precision κ_ξ, the provider can also determine the correlation between different firms' market signals by setting $\rho_\xi = \mathrm{corr}(\xi_i, \xi_j) \in [0,1]$. Our specification thus accommodates situations in which the provider offers identical or conditionally independent signals to any subset of the firms as special cases.

Thus, the market signal s_i offered to firm $i \in I$ can be rewritten as

$$s_i = \theta + \eta_i, \quad \eta_i \sim N(0, 1/\kappa_s) \quad \text{and} \quad \mathrm{corr}(\eta_i, \eta_j) = \rho,$$

where $\kappa_s = (1/\kappa_z + 1/\kappa_\xi)^{-1}$ is the signal's precision and $\rho = (\kappa_\xi + \rho_\xi \kappa_z)/(\kappa_\xi + \kappa_z)$. By construction, signals sold by the provider cannot be more precise than the information the provider possesses, that is, $\kappa_s \le \kappa_z$.

We remark that given firms' ex ante symmetry, we can assume, without loss of generality, that $I = [0, \lambda]$, where $\lambda \in [0,1]$ captures the fraction of firms that the information provider decides to trade with. Also note that, even though we assume that the seller chooses the fraction of firms the seller wants to trade with before offering them the seller's information products, as we show in Section 4, our setting is isomorphic to an environment in which the provider announces the features of the provider's product(s)—that is, price and precision—and firms subsequently decide whether to purchase them.

Finally, with some abuse of terminology, we refer to the firms who purchase the market signal s_i as *informed firms* whereas firms that were denied the signal or decided not to purchase it from the information provider are simply referred to as being *uninformed*.

2.1. Contracts and Equilibrium

Once the seller's and the firms' private signals are realized, the former has the option to sell potentially informative signals about θ to the latter. To capture this idea formally, we assume that the information provider makes a take-it-or-leave-it offer $(\kappa_\xi, \rho_\xi, p_i)$ to a fraction λ of the firms, where κ_ξ captures the quality of the market signal offered to firm i and p_i is the corresponding firm-specific price.

Following the seller's offer, each firm $i \in [0, \lambda]$ then decides whether to accept ($b_i = 1$) or reject ($b_i = 0$) its corresponding offer. This stage is then followed by the *competition subgame* between the firms in which they choose their actions a_i. Note that whereas the strategy of an uninformed firm i is a mapping from its private signal x_i to an action, the strategy of an informed firm maps the pair (x_i, s_i) to an action. We have the following standard solution concept:

Definition 1. A perfect Bayesian equilibrium consists of a strategy $(\lambda, \kappa_\xi, \rho_\xi, \{p_i\}_{i\in[0,\lambda]})$ for the information provider, acceptance/rejection decisions $b_i \in \{0,1\}$ for each firm i, a posterior belief μ_i for each firm i, firm-specific strategies a_i, and an aggregate action A such that

(i) the information provider chooses $(\lambda, \kappa_\xi, \rho_\xi, \{p_i\}_{i\in[0,\lambda]})$ to maximize the information provider's expected profit;

(ii) firm $i \in [0, \lambda]$ accepts the information provider's offer only if doing so maximizes its profit;

(iii) each firm's posterior belief on θ is obtained via Bayes rule, conditional on its information set;

(iv) given its posterior belief, each firm i maximizes its expected payoffs in the competition subgame, taking the strategies of all other firms as given;

(v) the aggregate action A is consistent with individual firm-level actions.

2.2. Examples

As already mentioned, Cournot and Bertrand competition can be derived as special cases of the general framework described in the beginning of Section 2. This feature of the model enables us to provide a comparison of the optimal information selling strategies in markets with different modes and intensities of competition. The following simple examples illustrate how, in the presence of linear demand functions, various forms of competition can induce quadratic profit functions in the form of Equation (1). We use these examples in the subsequent sections to discuss the implications of our results for the optimal trading strategies of the information provider.

Example 1 (Cournot Competition). Consider a market in which firms sell a possibly differentiated product to a downstream market and compete by setting quantities. Firm i faces an inverse demand function given by

$$r_i = \gamma_0\theta - (1-\delta)Q - \delta q_i, \tag{3}$$

where q_i is the quantity sold by firm i, $Q = \int_0^1 q_i\, di$ is the aggregate quantity sold to the downstream market, and θ is a "demand shifter" that captures the intercept of the (inverse) demand curve. In this setting, $\delta \in [0,1]$ represents the degree of product differentiation among firms as a smaller δ corresponds to a more homogenous set of products.[6] Assuming that firms' marginal cost of production is zero, it is then immediate that their profit function $\pi_i = r_i q_i$ is simply a special case of our framework in (1), with action a_i representing the quantity sold by firm i.

Note that, in this environment, the degree of strategic complementarity defined in (2) is equal to $\beta = (\delta-1)/2\delta < 0$, thus implying that firms face a game of strategic substitutes. Parameter β also captures the intensity of competition between the firms. In particular, given that β is increasing in δ, a larger β corresponds to a market in which products are more differentiated. In the extreme case that $\beta \to 0$, the products are no longer substitutes, and each firm essentially becomes a monopolist in its own market. At the other extreme, as $\beta \to -\infty$, the products become perfect substitutes, and the oligopoly converges to a perfectly competitive market.

Example 2 (Bertrand Competition). Next, consider a market in which firms compete in prices and face a linear demand function given by $q_i = \gamma_0\theta + (\phi-1)R - \phi r_i$, where r_i is the price set by firm i and $R = \int_0^1 r_i\, di$ is the average price in the market. Note that this demand system can be obtained by inverting (3) and setting $\phi = 1/\delta > 1$. Once again, it is immediate that firm i's profit function $\pi_i = r_i q_i$ would coincide with (1), where action a_i now represents the price set by firm i. Furthermore, it is straightforward to verify that, in this environment, $\beta = (\phi-1)/2\phi > 0$, thus implying that the competition game between the firms exhibits strategic complementarities, the degree of which is increasing in ϕ.

Example 3. Once again consider the Cournot competition setting described in Example 1 but, instead, suppose that firms produce homogeneous products, that is, $\delta = 0$, and have quadratic production costs given by $c(q_i) = q_i^2/2$. The profit of firm i is then given by $\pi(q_i, Q, \theta) = \gamma_0 q_i\theta - q_i Q - q_i^2/2$, which again fits within our general framework.

We conclude this section by remarking that even though, for the sake of tractability and expositional simplicity, we focus on an environment consisting of a continuum of firms, as we show in Section 7, all our results and insights carry over to a setting consisting of finitely many firms. We also note that, when dealing with a continuum of firms, we assume that a variant of the "exact law of large numbers" guarantees that the cross-sectional average of firm-level variables (such as firms' quantity or price decisions) coincide with the corresponding variables' expectations almost surely.[7]

3. Optimal Sale of Information

In this section, we present our main results and characterize the information provider's optimal information selling strategy. Our results show that the seller's strategy is highly sensitive to the mode and intensity of competition in the downstream market as expressed by β.

3.1. Competition Subgame

We start our analysis by studying the equilibrium in the competition subgame between the firms once the contracts offered by the information provider are accepted or rejected. Without loss of generality, let $[0, l]$ denote the set of firms who accept the seller's offer, where, clearly, $l \le \lambda$.

Proposition 1. *The competition subgame between the firms has a unique Bayes–Nash equilibrium in linear strategies. Furthermore, the equilibrium strategies of the firms are given by*

$$a_i = \begin{cases} \alpha[(1-\omega)x_i + \omega s_i] & \text{if } i \in [0,l] \\ \alpha x_i & \text{if } i \in [l,1], \end{cases}$$

where

$$\omega = \frac{\kappa_s}{(1-\beta l\rho)\kappa_x + \kappa_s} \quad \textit{and} \quad \alpha = \gamma_0/(\gamma_2 - \gamma_1).$$

Proposition 1, which is in line with Angeletos and Pavan (2007) and Myatt and Wallace (2015), provides a characterization of the firms' equilibrium strategies in the competition subgame and serves as a preliminary result for the rest of the results in the paper. It states that the equilibrium action of an informed firm is a weighted sum of its original private signal and the signal it obtains from the information provider. More importantly, however, it shows that the weights firm i assigns to its two signals not only depend on their relative precisions, but also on the fraction of informed firms, l, as well as correlation ρ in the market signals. Furthermore, the equilibrium weight that each informed firm assigns to the market signal s_i is increasing in the degree of strategic complementarities β regardless of the values of ρ and l. This is because, in the presence of stronger strategic complementarities, firms have stronger incentives to coordinate with one another and, as a result, rely more heavily on their market signals, which can function as (imperfect) coordination devices. On the other hand, in the absence of strategic considerations (i.e., when $\beta = 0$), the optimal strategy of all firms would be independent of l and ρ, making the weight assigned to each signal proportional to its relative precision.

Relatedly, Proposition 1 also establishes that, for a given positive (negative) β, the equilibrium weight that informed firms assign to their market signals is increasing (decreasing) in l and ρ. To see the intuition underlying this, suppose that $\beta > 0$ (the argument for $\beta < 0$ is identical). In such an environment, firms face a game of strategic complements as, for example, would be the case if they compete à la Bertrand. Given that firms value coordinating their actions, an informed firm i assigns a higher weight to its market signal—above and beyond what its relative precision would justify—the more other firms base their own decisions on the signal sold by the provider (i.e., higher l) and the more informative s_i is about the signals of other firms (i.e., higher ρ).

With Proposition 1 in hand, in the remainder of this section, we turn to the seller's problem and characterize the seller's optimal information selling strategy as a function of the mode and intensity of competition in the downstream market. To present our results in the most transparent manner, we study Bertrand and Cournot competition separately.

3.2. Bertrand Competition

First, consider the case in which firms compete with one another à la Bertrand. As already mentioned in Example 2, such a market corresponds to a special case of our general framework with $\beta > 0$. Also, recall that the information provider needs to choose the fraction of firms with whom the information provider trades (λ), the precision of the signal offered to the firms (κ_s), and the correlation induced in the noise terms (ρ_ξ). We have the following result:

Proposition 2. *If $\beta > 0$, the information provider sells the information provider's signal without any distortions to all firms; that is, $\kappa_s^* = \kappa_z$ and $\lambda^* = 1$. Furthermore, the provider's expected profit is given by*

$$\Pi^* = \alpha^2\left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_z}{\kappa_x}\right)\frac{\kappa_z + \kappa_x}{[(1-\beta)\kappa_x + \kappa_z]^2}. \tag{4}$$

Thus, Proposition 2 establishes that, under Bertrand competition, it is always optimal for the provider to sell the provider's signal z to the entire set of firms without any additional noise. To understand the intuition underlying this result, recall that, in a Bertrand market, the firms' actions are strategic complements: setting a lower price becomes more attractive the lower the prices of other competing firms are. Such strategic complementarities induce a strong coordination motive among the firms. Therefore, providing the market signal to an additional marginal firm not only increases the profits of the seller directly (via sales to that new marginal firm), but also increases the surplus of all other firms who have already acquired the signal. This extra surplus can thus be appropriated by the seller via higher prices, leading to even higher profits. Consequently, the information provider always finds it optimal to sell to the entire market of firms. An identical argument then shows that the provider would not distort the signal either: sharing a more precise signal with a new firm increases the value of the market signal to the rest of the informed firms.

Proposition 2 also characterizes the expected profit of the seller. From (4), it is easy to verify that Π^* is increasing in the quality of information available to the monopolist (κ_z) but is decreasing in the precision of the firms' private signals (κ_x). The intuition underlying these observations is simple. Given that the information provider always has the option to reduce the precision of the signals it offers to the firms, the provider's profits can never decrease by having access to a more precise signal. On the other hand, however, the extra benefit of the market signal to the firms is lower the more informed they are to begin with, thus reducing the provider's expected profits.

More importantly, however, (4) also shows that the monopolist's expected profit increases in the degree of strategic complementarities β. Recall from Example 2 that $\beta = (\phi - 1)/2\phi$, where $1/\phi = \delta$ is the degree of product differentiation among the firms. Therefore, increasing β is essentially equivalent to a lower degree of product differentiation and, hence, more intense competition. Thus, as β increases, coordination becomes more important to the firms, increasing the value of the seller's signal, which, in turn, leads to higher expected profits.

As a final remark, note that since it is never optimal for the information provider to add noise to the signals, the correlation $\rho_\xi = \text{corr}(\xi_i, \xi_j)$ is immaterial for the provider's profits.

3.3. Cournot Competition

We next focus on the case in which firms compete with one another à la Cournot. Recall from Example 1 that such a market is a special case of our general setup with $\beta < 0$. In this case, firms choose quantities, and their actions are strategic substitutes. Note that, unlike the case of Bertrand competition, firms no longer value coordination per se. The following two propositions provide a characterization of the optimal information selling strategy of the monopolist as a function of the degree of strategic substitutability among the actions of downstream firms.

Proposition 3. *If $-(1 + \kappa_z/\kappa_x) \le \beta < 0$, the information provider sells the provider's signal without any distortions to all firms; that is, $\kappa_s^* = \kappa_z$ and $\lambda^* = 1$. Furthermore, the provider's expected profit is*

$$\Pi^* = \alpha^2\left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_z}{\kappa_x}\right)\frac{\kappa_z + \kappa_x}{[(1-\beta)\kappa_x + \kappa_z]^2}. \tag{5}$$

Thus, in a Cournot market with a weak enough intensity of competition, the seller finds it optimal to follow the same strategy as in a Bertrand market: sell an undistorted version of the seller's signal to the entire set of firms. The intuition underlying this result is straightforward: acquiring information about the demand intercept (θ) allows each firm i to better match its supply decision to the underlying demand and, as a consequence, to increase its profit. The monopolist can then appropriate the increase in i's sales by demanding a higher price for the provider's signal. Therefore, the provider is always better off by making the most precise version of the provider's signal available to all firms.

Even though the seller follows the same strategy as in the Bertrand market, comparing expressions (4) and (5) implies that the provider's expected profit is lower under Cournot competition ($\beta < 0$). This is because, unlike Bertrand competition, firms do not have an incentive to coordinate their actions, undermining the role of the market signal as a coordination device.

Interestingly, the predictions of Propositions 2 and 3 no longer hold if the intensity at which downstream firms compete in a Cournot market is high. We have the following result:

Proposition 4. *If $\beta < -(1 + \kappa_z/\kappa_x)$, the information provider maximizes the provider's expected profit by following any information selling strategy that is a solution to the following equation*:

$$(\kappa_z + \beta\lambda^*\kappa_s^*)\kappa_x + \kappa_z\kappa_s^* = 0. \tag{6}$$

Furthermore, the provider's expected profit is given by

$$\Pi^* = -\alpha^2\left(\frac{\gamma_2}{2}\right)\frac{\kappa_z}{4\beta\kappa_x^2}. \tag{7}$$

The key observation here is that the pair $\kappa_s^* = \kappa_z$ and $\lambda^* = 1$ does not satisfy (6), leading to the following corollary:

Corollary 1. *Suppose that $\beta < -(1 + \kappa_z/\kappa_x)$. Then, either $\kappa_s^* < \kappa_z$ or $\lambda^* < 1$.*

Therefore, when firms compete with one another à la Cournot and offer goods that are strong substitutes—corresponding to a large enough negative β—it is optimal for the seller to distort the information ($\kappa_s^* < \kappa_z$) and/or exclude a fraction of the firms from the sale ($\lambda^* < 1$).

To see the intuition underlying Corollary 1, recall that, in a Cournot market, firms' actions are strategic substitutes; that is, increasing a firm's supply leads to higher marginal profit the lower the supply decisions of its competitors are. Therefore, providing the market signal to an additional firm i affects its profit through two distinct channels. On the one hand, a more precise market signal enables i to better match its supply to the realized demand. On the other hand, however, making such a signal available to i increases the correlation in the firms' actions as now i's action would be more correlated with the market parameter θ. The presence of this second effect implies that the strategic value of the seller's signal to firm i and, consequently, i's willingness to pay for it are decreasing in the fraction of firms that accept the provider's offer. In the presence of sufficiently intense competition (i.e., when the firms offer sufficiently substitutable products), this strategic effect dominates the first effect, thus making it profitable for the information provider to restrict the provider's offer to a strict subset of the firms ($\lambda^* < 1$).

By Proposition 4, an alternative optimal strategy for the monopolist would be to distort the information the monopolist sells to the market. In fact, as Equation (6) suggests, the fraction λ of the firms that the monopolist trades with and the precision κ_s of the signal offered to the firms are substitutes: as the monopolist increases the monopolist's market share, the monopolist finds it optimal to increasingly distort the signals.

Note that Equation (7) in Proposition 4 indicates that the information provider's expected profit decreases in the degree of strategic substitutability ($|\beta|$) of the firms' actions. This is a consequence of the fact that the strategic value of the seller's signal and, hence, a firm's willingness to pay decrease as the market becomes more competitive. This is in contrast with the case of Bertrand competition where the seller's expected profit increases with the intensity of competition as the seller's customers have a stronger incentive to purchase the market signal and coordinate their actions.

We also remark that, regardless of the value of β and the strategy adopted by the information provider, the provider never has an incentive to introduce correlation into market signals; that is, it is always optimal to set $\rho_\xi^* = 0$. Increasing the correlation in the signals provided to downstream firms would invariably increase the correlation among their actions and lead to lower profits for the seller.

Finally, note that the threshold $-(1+\kappa_z/\kappa_x)$ at which the seller finds it optimal to limit the seller's market share and/or strategically distort the market signal is decreasing in the ratio κ_z/κ_x, implying that the more informed the information provider is relative to the provider's customers, the more likely it is that the provider will be able to fully exploit the provider's informational advantage by selling it to the entire market of firms without distortion.

Figure 1 illustrates the optimal selling strategy and the equilibrium profit of the information provider for the following set of parameters: $\alpha = \gamma_2 = 1$, $\kappa_x = 1$, and $\kappa_z = 2$. For these parameters, it is immediate to verify that the threshold at which the seller finds it optimal to strategically distort the market signal is equal to $-(1+\kappa_z/\kappa_x) = -3$. Indeed, as the left panel of Figure 1 illustrates, for values of β greater than this threshold, the provider sets the precision of the market signal to $\kappa_s^* = \kappa_z = 2$; that is, the provider does not distort the information the provider has at the provider's disposal and does not exclude any firms from the sale ($\lambda^* = 1$). On the other hand, for $\beta < -3$, the seller finds it optimal to distort the information the seller sells and limit the seller's market share. The right panel of Figure 1 illustrates how the provider's profit varies with the intensity of competition. Note that the seller is better off when firms view their actions as strategic complements ($\beta > 0$) as opposed to strategic substitutes.

Table 1. Profits Under the Optimal Information Selling Strategy Over Selling the Signal Undistorted to the Market

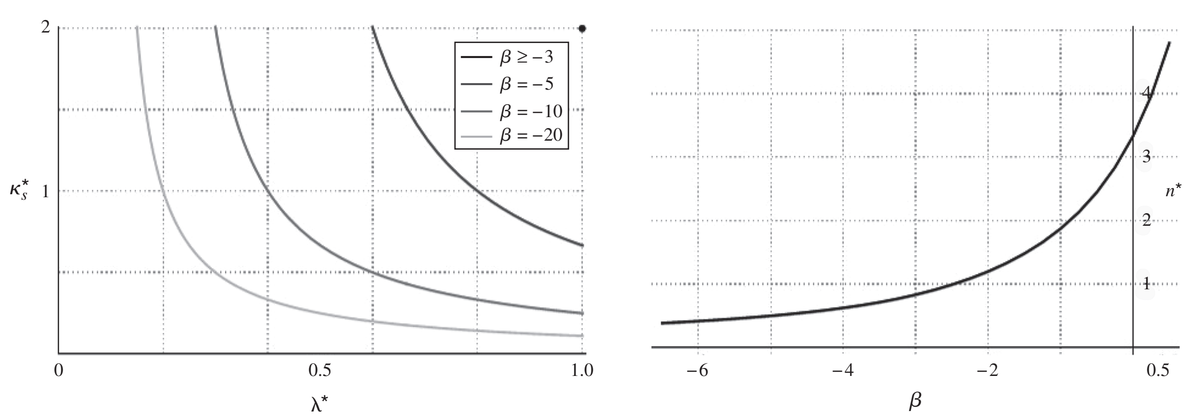
	$\beta=0$	$\beta=-3$	$\beta=-5$	$\beta=-10$	$\beta=-20$
Π_β^*/Π_0^*	1	0.250	0.150	0.075	0.038
$\Pi_\beta^{\text{no-dist}}/\Pi_0^*$	1	0.250	0.141	0.053	0.017
Increase in profits (%)	0	0	6.67	40.83	120.42

We conclude this section by exploring the extent to which an information provider can increase the provider's profits by strategically distorting the information the provider provides to the provider's downstream customers and/or limiting the provider's market share. Table 1 provides a comparison of the provider's profit under the optimal selling strategy (Π_β^*) to the profits of a provider who sells the provider's signal to the entire market with no distortion ($\Pi_\beta^{\text{no-dist}}$). We benchmark Π_β^* and $\Pi_\beta^{\text{no-dist}}$ against the profits for a provider who follows the provider's optimal strategy in the absence of competition, that is, when $\beta = 0$. The first two rows of the table highlight the effect of competition intensity on the providers's profits. More importantly, however, as the bottom row of the table indicates, the provider earns significantly higher profits under competition when the provider distorts the provider's market signal and/or limits the provider's market share: the increase in the provider's profits by following the strategy characterized in Proposition 4 ranges from 6.67% to 120.42% as the extent to which firms view their actions as strategic substitutes increases.

4. Information Quality Discrimination

In our baseline model presented in Section 2 and analyzed in Section 3, we assumed that the information provider can only offer a single product to the entire market in the sense that the provider offers a market signal of the same precision to all firms. In this

Figure 1. Optimal Selling Strategy for Different Levels of β (Left); Equilibrium Profit as a Function of β (Right)



Note. We use the following set of parameters for this example: $\alpha = \gamma_2 = 1$ and $\kappa_x = 1$, $\kappa_z = 2$.

section, we relax this assumption by allowing the seller to offer signals that potentially differ in both price and precision.

Formally, we assume that the information provider offers (κ_{si}, p_i) to each firm $i \in [0,1]$, specifying the signal precision κ_{si} and price p_i. The seller cannot offer a signal of a higher precision than the seller's own private signal, that is, $\kappa_{si} \leq \kappa_z$ for all i. The following result, which generalizes Propositions 2–4, shows that all our earlier insights remain valid under this more general specification.

Proposition 5. *The information provider's optimal strategy is as follows:*

(a) *If $\beta \geq -(1+\kappa_z/\kappa_x)$, the provider offers the provider's signal undistorted to all firms at price*

$$p^* = \alpha^2\left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_z}{\kappa_x}\right)\frac{\kappa_z+\kappa_x}{[(1-\beta)\kappa_x+\kappa_z]^2}.$$

(b) *If $\beta < -(1+\kappa_z/\kappa_x)$, the provider offers a signal of precision κ^*_{si} to firm i, where $\{\kappa^*_{si}\}_{i\in[0,1]}$ solve*

$$\int_0^1 \frac{\kappa^*_{si}}{\kappa_x+\kappa^*_{si}}\,di = -\frac{\kappa_z}{\beta\kappa_x} \tag{8}$$

at price $p^*_i = \alpha^2(\gamma_2/2)(\kappa^*_{si}/(4\kappa_x(\kappa_x+\kappa^*_{si})))$.

Proposition 5(a) establishes that the information provider offers an undistorted version of the provider's signal to all firms in the downstream market if either they compete à la Bertrand or, alternatively, if the intensity of the Cournot competition is not strong enough. In this sense, this result generalizes Propositions 2 and 3, establishing that the seller has no incentive to discriminate among the firms in either price or information quality.

Statement (b) of Proposition 5 considers the setting in which firms' actions are strong strategic substitutes, for example, when they compete à la Cournot and produce goods that are highly substitutable. Consistent with the discussion in Section 3.3, this result shows that the information provider finds it optimal to either distort the signals sold to the downstream firms or strategically restrict the provider's market share. In particular, it is easy to verify that $\kappa^*_{si} = \kappa_z$ for all i does not satisfy the optimality condition (8). The intuition underlying this result parallels those behind Proposition 4 and Corollary 1: providing high-quality signals to all firms increases the induced correlation in their actions, which, in turn, reduces their profit when their actions are strong strategic substitutes. Thus, the monopolist would be better off by limiting the monopolist's market share or reducing the quality of the signals sold to the firms. Note, however, that the optimal strategy of the information provider is not unique. Rather, any signal precision profile $\{\kappa^*_{si}\}$ that satisfies (8) would lead to the same expected profit. Nevertheless, irrespective of the strategy chosen by the monopolist, the monopolist's incentive to lower the precision of the market signals increases as firms' actions become stronger strategic substitutes. In particular, as $\beta \to -\infty$, the downside of coordination among firms that trade with the monopolist is so strong that essentially no trade takes place in equilibrium: the information provider offers a completely uninformative signal $\kappa^*_{si} \to 0$ to all firms at price $p^*_i \to 0$.

Example 4 (Selling Two Products)**.** Consider a Cournot market in which $\beta < -(1+\kappa_z/\kappa_x)$, and suppose that the information provider can offer two information products: a premium product of precision $\bar{\kappa}_s$ at price $\bar{p}$ and an inferior one of precision $\underline{\kappa}_s < \bar{\kappa}_s$ at price $\underline{p}$. Let $\bar{\lambda}$ and $\underline{\lambda}$ denote the fraction of firms offered the premium and inferior products, respectively, where by construction $\bar{\lambda}+\underline{\lambda} \leq 1$. Condition (8) implies that it is optimal for the seller to design the seller's information products such that $\bar{\lambda}(\bar{\kappa}_s/(\kappa_x+\bar{\kappa}_s)) + \underline{\lambda}(\underline{\kappa}_s/(\kappa_x+\underline{\kappa}_s)) = -\kappa_z/(\beta\kappa_x)$. This equation highlights the trade-off between information quality and quantity faced by the information provider in designing the provider's menu of products. In particular, increasing the precision $\bar{\kappa}_s$ of the premium product requires either a reduction in its supply $\bar{\lambda}$ or, alternatively, a reduction in the precision or the supply of the inferior product.

We end by remarking that the ability to discriminate on quality does not offer the seller any advantage compared with our benchmark model of Sections 2 and 3. In particular, Equation (8) always has a solution such that $\kappa_{si} = \kappa_s$ for a fraction λ of the firms and $\kappa_{si} = 0$ for the rest. In other words, offering two products, one with nonzero precision at a strictly positive price and another with zero precision at zero price, is sufficient for the seller to maximize the seller's expected profit.

5. Information Leakage

Thus far, we assumed that purchasing a signal from the information provider is the only channel available to the firms for acquiring information about the unknown parameter θ. Firms, however, can also infer potentially valuable information by observing their competitors' actions. For instance, a firm's price or quantity decisions can (partially or fully) reveal the information it has at its disposal to other firms. In this section, we extend our baseline model to allow for the possibility of such indirect "information leakage" and study the information provider's optimal selling strategy when the provider's customers can potentially free ride on the information purchased by other firms.

We capture the possibility of information leakage by allowing firms to condition their actions on an extra piece of information that is informative about their competitors' actions. More specifically, we assume that, in addition to its signal x_i and the market signal s_i

(if purchased from the information provider), firm i can also condition its action on a *leakage signal*:

$$S_i = A + \nu_i, \quad \nu_i \sim N(0, 1/\kappa_\nu), \tag{9}$$

where $A = \int_0^1 a_i\, di$ denotes the aggregate action and the noise terms ν_i are independently distributed across the firms. The key observation is that, as long as firms' actions are based (even in part) on the information at their disposal, signal S_i would be informative about such information. As such, the precision κ_ν can serve as a proxy for the extent of information leakage in the market: S_i is perfectly informative about the aggregate action A when $\kappa_\nu = \infty$ whereas as κ_ν decreases, the information content of the leakage signal is reduced. In the extreme case that $\kappa_\nu = 0$, signal S_i does not convey any payoff-relevant information. It is immediate to see that this latter case reduces to the no-leakage setting in our benchmark model.[8]

To formally model firms' ability to incorporate any information leaked through the market into their decisions, we follow Vives (2011) and extend the firms' strategy space by assuming that firm i's strategy is a contingent schedule $a_i(\cdot, S_i)$ that maps its private and market signals, (x_i, s_i), to an action depending on the realization of the leakage signal S_i.[9] Thus, the equilibrium of the subgame between firms requires (i) each firm i to choose $a_i(x_i, s_i, S_i)$ to maximize its expected profit conditional on its information set (that is, $\mathbb{E}[\pi_i \mid x_i, s_i, S_i]$), taking the strategies of all other firms as given, and (ii) the aggregate action to be consistent with the realization of the firms' individual actions; that is, $A = \int_0^1 a_i(x_i, s_i, S_i)\, di$.

We remark that, despite the slightly more complex nature of the firms' strategies, this modeling approach enables us to directly incorporate information leakage into our benchmark model without resorting to a multi-period, dynamic model of interaction between firms. Crucially, it also enables us to study how the provider's optimal strategy and profits vary as a function of the intensity of information leakage in the market. We have the following result:

Proposition 6. *For sufficiently small $\kappa_\nu > 0$,*

(a) *the provider's profit decreases in the extent of information leakage (i.e., $\partial\Pi^*/\partial\kappa_\nu < 0$);*

(b) *there exists $-(1 + \kappa_z/\kappa_x) < \bar{\beta} < 0$ such that $\kappa_s^* < \kappa_z$ for all $\beta \in (-(1 + \kappa_z/\kappa_x), \bar{\beta})$.*

Therefore, Proposition 6 establishes that, regardless of whether actions are strategic substitutes or complements (and, hence, regardless of the mode of competition), the information provider's profits decrease as the extent of information leakage is intensified. This is because firms' willingness to pay for an extra piece of information reduces whenever they can free ride on the information purchased by their competitors. Given that more information leakage would only intensify this free-riding incentive, the information provider is forced to charge lower prices for the provider's signal, thus making less profit.

More importantly, however, Proposition 6 establishes that the range of β's for which the information provider finds it optimal to distort the market signal offered to the provider's customers widens in the presence of information leakage. Recall from Corollary 1 and Proposition 5 that, with no information leakage, the information provider would reduce the quality of the market signal if and only if $\beta < -(1 + \kappa_z/\kappa_x)$. In contrast, part (b) of Proposition 6 shows that, no matter how small the extent of leakage, the provider would offer distorted signals for some $\beta > -(1 + \kappa_z/\kappa_x)$. This is because the provider's ability to extract surplus from the firms by increasing the precision of s_i is hindered in the presence of leakage. That said, the fact that $\bar{\beta} < 0$ means that, regardless of the presence or absence of information leakage, distorting the signal sold to the firms is never optimal when firms' actions are strategic complements (for example, as in Bertrand competition).

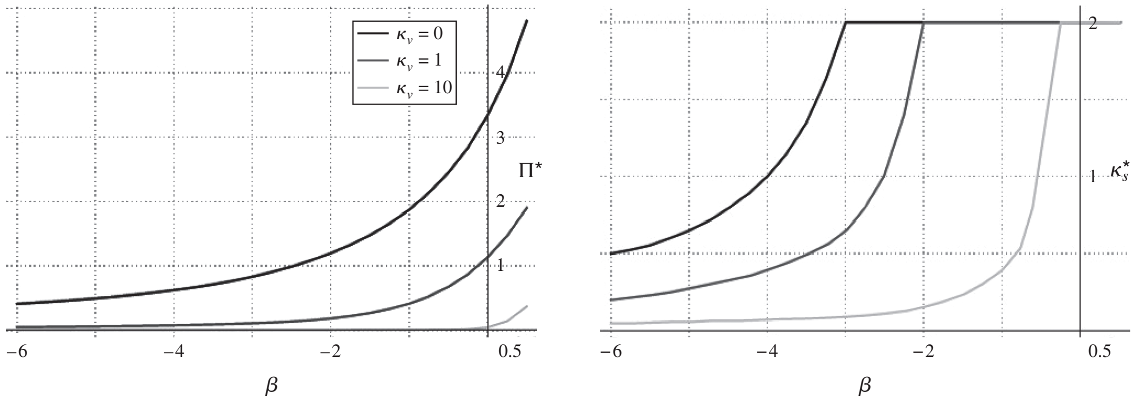
Figure 2 illustrates the provider's equilibrium profits (left panel) and the provider's optimal distortion strategy (right panel) for different levels of information leakage. As the left panel indicates, the provider's profits are decreasing in the leakage intensity irrespective of the value of β. As for the precision of the signal offered to the firms, the right panel clearly illustrates the two key observations we already mentioned: (i) as leakage is intensified (higher κ_ν), the provider finds it optimal to sell signals of lower quality for a wider range of β's; and (ii) distortion is never optimal in the presence of strategic complementarities ($\beta > 0$) irrespective of the value of κ_ν.

Table 2 presents the results of a numerical simulation for the effect of information leakage on the provider's optimal strategy and equilibrium profits for different values of β with $\kappa_\nu = 0$, $\kappa_\nu = 1$, and $\kappa_\nu = 10$ corresponding, respectively, to a scenario with no, low, and high levels of leakage intensity. The last column of the table indicates that, at $\beta = 1/3$, the equilibrium profit in the high leakage regime is only 5% of the corresponding profit in the benchmark case with no information leakage. Finally, the lower panel of Table 2 indicates that for certain values of β (say, $\beta = -2$), the optimal strategy may entail selling a signal with maximal precision when leakage is absent or low whereas the seller finds it optimal to dramatically decreases the signal precision to only 8% of the seller's best signal precision in the high-leakage regime.

6. Heterogeneous Firms

In this section, we discuss how our results are affected by introducing heterogeneity among the firms (in terms of their production costs).

Figure 2. The Provider's Equilibrium Profits (Left) and the Provider's Optimal Selling Strategy (Right) as Functions of β for Different Levels of Information Leakage



Notes. We use the following set of parameters for this example: $\alpha = \gamma_2 = 1$ for the firms' payoff functions and $\kappa_x = 1, \kappa_z = 2$ for the signal precisions of the firms' private signals and the provider's information, respectively. We plot the provider's profits and the precision of the signal the provider sells to the downstream market (κ_s^*) as a function of β for three levels of information leakage $\kappa_v = 0$ (no leakage), $\kappa_v = 1$, and $\kappa_v = 10$.

We generalize the setting described in Section 2 along two dimensions. First, we allow for heterogeneity in firms' production costs, and second, we introduce a transaction cost borne by the information provider whenever the provider trades with a downstream firm. We mostly focus on the case that firms view their actions as strategic substitutes and thus interpret our results in the context of Cournot competition with quadratic production costs (Example 3 from Section 2.2). Specifically, we assume that downstream firms are heterogeneous with respect to their costs of production: firm i faces a quadratic production cost of $C_i(q_i) = c_i q_i^2/2$, where q_i is the quantity produced by i and $c_i > 0$. The firm's profit is thus given by

$$\pi_i(q_i, Q, \theta) = \gamma_0 q_i \theta + \gamma_1 q_i Q - \tfrac{1}{2} c_i q_i^2, \tag{10}$$

where Q denotes the aggregate quantity in the market and $\gamma_1 < 0$ is some constant. Note that, even though Expression (10) is similar to (1), the extent of strategic complementarities can no longer be captured by a single parameter β as now firms face different production costs.

As for transaction costs, we assume that the seller incurs a cost equal to $v\kappa_{si}$ whenever the seller sells a signal of precision κ_{si} to firm i, where $v > 0$. This cost can, for example, capture the idea that the firm cannot provide verifiable and/or credible information to the seller's customers at no cost. Rather, it needs to spend resources to assure the seller's customer that the market signal is indeed as informative as claimed. Alternatively, it can be thought of as the cost associated with customizing the provider's information to meet the customer's informational needs. As in Section 4, we allow the seller to discriminate along both signal precision and price. We have the following result:

Proposition 7. *There exist $\bar{v} > \underline{v}$ such that*

(a) *if $v > \bar{v}$, the information provider does not transact with any of the firms (i.e., $\kappa_{si}^* = 0$ for all i);*

(b) *if $v < \underline{v}$, the information provider sells the provider's signal with no distortion to all firms;*

(c) *for any $v \in (\underline{v}, \bar{v})$, then there exist c^* such that*

$$\kappa_{si}^* = \begin{cases} 0 & \text{if } c_i > c^* \\ \kappa_z & \text{if } c_i < \dfrac{\kappa_x^2}{(\kappa_x + \kappa_z)^2} c^* \\ \kappa_x(\sqrt{c^*/c_i} - 1) & \text{otherwise.} \end{cases}$$

Table 2. The Provider's Equilibrium Profits and Optimal Precision for Different Levels of Leakage at Different Levels of β

	Leakage level	$\beta = -4$	$\beta = -3$	$\beta = -2$	$\beta = -1$	$\beta = 0$	$\beta = 1/3$
Equilibrium profits	$\kappa_v = 0$	0.0625	0.0833	0.1200	0.1875	0.3333	0.4218
	$\kappa_v = 1$	0.0082	0.0014	0.0189	0.0420	0.1134	0.1606
	$\kappa_v = 10$	0.0002	0.0003	0.0005	0.0008	0.0050	0.0209
Optimal precision	$\kappa_v = 0$	1	2	2	2	2	2
	$\kappa_v = 1$	0.40	0.65	2	2	2	2
	$\kappa_v = 10$	0.08	0.12	0.16	0.36	2	2

Note. The other parameter values are the same as in the example of Figure 2.

Thus, Proposition 7 establishes that the information provider finds it optimal to follow an information selling strategy that involves offering a signal to firm i with a precision that is decreasing in the firm's cost c_i; that is, the provider sells higher quality signals to more efficient firms. Formally, κ^*_{si} is always nonincreasing in c_i. However, note that this does not mean that the monopolist sells the monopolist's best available information to all firms even when transactions are costless. Rather, because of the presence of strategic interactions between downstream firms (and in line with our earlier results), the provider may either sell distorted signals to some firms or simply even exclude them by offering noninformative signals $\kappa^*_{si} = 0$ altogether. Thus, Proposition 7 generalizes Propositions 4 and 5 to the case in which firms face heterogeneous production costs.

Finally, note that, depending on the parameter values, the threshold $\underline{v}$ in Proposition 7 may be negative, thus ruling out the case in which the information provider sells an undistorted signal to all firms. In fact, as the proof of the proposition highlights, $\underline{v} < 0$ whenever

$$\int_0^1 \frac{1}{c_i}\,di < -\frac{1}{\gamma_1}(1 + \kappa_z/\kappa_x),$$

which reduces to the condition of Proposition 4 when firms face identical production costs.

Cost dispersion and optimal information selling strategy. To further clarify the impact of firm heterogeneity on the provider's equilibrium strategy and profits, consider a special case consisting of two types of firms $i \in \{1, 2\}$ with production costs $C_i(q_i) = c_i q_i^2/2$, where

$$\frac{1}{c_1} = \frac{1}{c} + \delta, \quad \text{and} \quad \frac{1}{c_2} = \frac{1}{c} - \delta$$

for some $\delta > 0$. Also, assume that both types have mass equal to $1/2$. It is immediate to see that in such a setting δ measures the cost dispersion in the market. We have the following corollary:

Corollary 2. *Let κ^*_{s1} and κ^*_{s2} be the optimal signal precisions offered to firms of type* 1 *and type* 2, *respectively. Then, for any $\delta < 1/(c\sqrt{2})$,*

$$\frac{\partial \kappa^*_{s1}}{\partial \delta} \geq 0, \quad \textit{and} \quad \frac{\partial \kappa^*_{s2}}{\partial \delta} \leq 0.$$

Corollary 2 thus establishes that, as the cost dispersion among the downstream firms increases, the provider finds it optimal to sell increasingly more accurate signals to the efficient type while the provider decreases the accuracy of the signals the provider sells to the type that has high production costs. This change in selling strategy occurs despite the fact that the average cost in the downstream market remains constant. Table 3 reports the results of a numerical simulation that quantifies the effect of cost dispersion on the provider's optimal information selling strategy and equilibrium profits. As the table indicates, when the dispersion between firms' production costs is sufficiently high ($\delta = 4$ in this case), the optimal strategy of the information provider requires excluding the less efficient firms from trade altogether. It is also immediate to see that the provider's profits are increasing in the cost dispersion parameter δ with the provider's profits roughly 20% higher when $\delta = 4$ compared with the benchmark case with no dispersion.

Table 3. Optimal Information Selling Strategy and Equilibrium Profits as a Function of the Cost Dispersion Between the Two Types of Firms

	$\delta = 0.5$	$\delta = 1$	$\delta = 2$	$\delta = 4$
κ^*_{s1}	1.334	1.505	1.819	2
κ^*_{s2}	0.976	0.789	0.409	0
Profits	1.067	1.076	1.114	1.251

Note. For this example, we use $\kappa_x = 1$, $\kappa_z = 2$, $c = 1/6$, $\gamma_1 = 3/5$, and $\gamma_0 = 10$.

7. Finite Markets

To simplify the exposition and allow for a tractable analysis, most of the paper focused on an environment with a continuum of firms. In this section, we show that our qualitative insights regarding the monopolist's optimal information selling strategy carry over to a market consisting of finitely many firms. In particular, we focus on a Cournot market, in which n firms compete with one another in quantities with the inverse demand function given by $r = \gamma_0\theta + \gamma_1 Q$, where $Q = (1/n)\sum_{i=1}^n q_i$ is the average quantity in the market, q_i is the quantity produced by firm i, r denotes the market price, and $\gamma_1 < 0$ is some constant. Assuming that firm i faces quadratic production costs $c(q_i) = \gamma_2 q_i^2/2$, its profits can be expressed as

$$\pi_i = \gamma_0 q_i \theta + \frac{n-1}{n}\gamma_1 q_i Q_{-i} - \left(\frac{\gamma_2}{2} - \frac{\gamma_1}{n}\right) q_i^2,$$

where $Q_{-i} = (1/(n-1))\sum_{j\neq i} q_j$ is the average quantity of i's competitors. It is immediate to see that this expression is similar to the firms' profit function (1) for a market consisting of a continuum of firms. The degree of strategic complementarity among firms' actions can also be defined as

$$\beta_n = -\frac{\partial^2 \pi_i}{\partial q_i \partial Q_{-i}} \Bigg/ \frac{\partial^2 \pi_i}{\partial q_i^2} = \left(\frac{n-1}{n}\right)\frac{\gamma_1}{\gamma_2 - 2\gamma_1/n}.$$

As in the environment with a continuum of firms, we assume that each firm i observes a noisy private signal x_i about the realization of θ and that the information provider can offer a market signal s_i to firm i. Let K denote the set of firms that the information provider trades with, where $|K| = k \leq n$. Lemma EC.1 in the online appendix provides a complete characterization of the equilibrium of the competition subgame for any k, which can be viewed as the discrete analog of Proposition 1 in Section 3. However, as we argued in Section 3, there always exist an equilibrium in which the provider offers the market signal to all firms. Thus, without loss of generality, we can restrict our attention to the case of $k = n$. We have the following result:

Proposition 8. *The optimal information selling strategy is given as follows*:

(a) *If $\beta_n \geq -(1+\kappa_z/\kappa_x)$, the provider offers an undistorted version of the provider's signal to all firms.*

(b) *If $\beta_n < -(1+\kappa_z/\kappa_x)$, the provider offers a signal of precision $\kappa_s^* = -\kappa_z/(\beta_n + \kappa_z/\kappa_x) < \kappa_z$.*

Furthermore, the seller's expected profit is given by

$$\Pi^* = \begin{cases} n\alpha_n^2\left(\dfrac{\gamma_2}{2} - \dfrac{\gamma_1}{n}\right)\left(\dfrac{\kappa_z}{\kappa_x}\right)\dfrac{\kappa_z + \kappa_x}{[(1-\beta_n)\kappa_x + \kappa_z]^2} \\ \quad \text{if } \beta_n \geq -(1+\kappa_z/\kappa_x) \\ n\alpha_n^2\left(\dfrac{\gamma_2}{2} - \dfrac{\gamma_1}{n}\right)\dfrac{\kappa_z}{-4\beta_n\kappa_x^2} \quad \text{otherwise,} \end{cases}$$

where $\alpha_n = \gamma_0/(\gamma_2 - ((n+1)/n)\gamma_1)$.

Proposition 8 thus illustrates that the insights underlying our main results remain unchanged when the downstream market comprises a finite number of firms. Additionally, it is straightforward to verify that, as n grows to infinity, the expressions characterizing the provider's optimal strategy and the provider's expected profits (normalized by the total number of firms n) reduce to those we obtained in Section 3.3 for a market consisting of a continuum of firms. Finally, the fact that β_n is decreasing in n implies that the range of parameters over which the monopolist finds it optimal to distort the information the monopolist sells to the market grows with the number of firms n.

Proposition 8 also illustrates that, when $\beta_n < -(1+\kappa_z/\kappa_x)$, the gain to the provider from optimally distorting the information can be obtained by comparing

$$\Pi_n^{\text{no-dist}} = \alpha_n^2\left(\frac{\gamma_2}{2} - \frac{\gamma_1}{n}\right)\frac{\kappa_z}{\kappa_x}\frac{\kappa_z + \kappa_x}{[(1-\beta_n)\kappa_x + \kappa_z]^2}$$

$$\Pi_n^* = \alpha_n^2\left(\frac{\gamma_2}{2} - \frac{\gamma_1}{n}\right)\frac{\kappa_z}{-4\beta_n\kappa_x^2},$$

where $\Pi_n^{\text{no-dist}}$ is the provider's expected profit per customer under no information distortion and Π_n^* is the normalized expected profit under optimal distortion. Thus, the gain from distorting the information, normalized by the number of firms n, is given by

$$\Delta_n = \frac{(n-1)|\gamma_1|\alpha_n^2}{8n\beta_n^2}\frac{\kappa_z[(1+\beta_n)\kappa_x + \kappa_z]^2}{\kappa_x^2[(1-\beta_n)\kappa_x + \kappa_z]^2}, \tag{11}$$

where $\alpha_n = \gamma_0/(\gamma_2 - ((n+1)/n)\gamma_1)$ and $\beta_n = ((n-1)/n)\cdot(\gamma_1/(\gamma_2 - 2\gamma_1/n))$, and recall that γ_1, γ_2, κ_x, and κ_z are model primitives that do not depend on n. Expression (11) thus leads to two key observations. First, it illustrates that, consistent with the findings of Proposition 8, the gain from distorting the information is positive for all values of $\beta_n < -(1+\kappa_z/\kappa_x)$. Second, taking the market size n and intensity of competition β_n (which only depends on parameters γ_1 and γ_2) as given, the gain Δ_n can be made arbitrarily large by increasing the value of γ_0 (and, hence, α_n). This observation thus illustrates that, even though the *percentage change* in profit gain $(\Pi_n^* - \Pi_n^{\text{no-dist}})/\Pi_n^* = [(1+\beta_n)\kappa_x + \kappa_z]^2/[(1-\beta_n)\kappa_x + \kappa_z]^2$ is maximized when the intensity of competition is maximized ($\beta_n \to -\infty$), the *level* of gain Δ_n can be large even away from the competitive limit.

8. Conclusions

This paper considers the problem of selling information to a set of firms that compete in a downstream market. We establish that both the information provider's optimal selling strategy as well as the provider's profits depend critically on the environment in which the provider's customers operate. In particular, our results highlight that the extent of strategic substitutability and complementarity in the latter's actions has a first-order impact on the former's optimal strategy: when the firms' actions are strategic complements, the provider finds it optimal to sell an undistorted version of the provider's information to the entire market whereas, if their actions are strategic substitutes, the optimal strategy involves offering an inferior information product and/or limiting the supply of information.

Our results are largely driven by the following tradeoff: On the one hand, information about market conditions, for example, demand realization, always has a direct positive effect on firms' profits as they can better align their actions with the underlying environment. On the other hand, however, in the presence of strategic substitutability among the firms, the provider's signal may have an additional (adverse) effect by increasing the correlation between the firms' actions. It turns out that this latter effect may dominate the former when firms view their actions as strong strategic substitutes, in which case the provider finds it optimal to degrade the quality of the provider's information products and/or exclude a subset of the firms from the sale.

We showcase the implications of our results in the context of Bertrand and Cournot competition, thus complementing the extensive prior literature in operations management that explores vertical and horizontal information sharing in a supply chain. In addition, we discuss how the extent of information leakage in the market can affect the provider's selling strategy and profits. Finally, we extend our findings to the case when firms differ in their production costs and establish that the optimal selling strategy involves offering several information products with varying precisions and at different prices. We also show that in equilibrium, the information provider offers more precise signals to the more efficient firms at higher prices to maximize the provider's profit.

Taken together, our findings illustrate that the optimal provision and pricing of information products cannot be decoupled from the market structure in which the firm's potential customers operate. They also uncover a potential rationale for why information markets typically feature several versions of essentially the same information product but of varying qualities and price tags. Identifying the prevalence of the said mechanism and its relative importance compared with other potential explanations (such as price discrimination driven, for example, by the heterogeneity in the willingness to pay among potential buyers) in various contexts is an important question with both positive and normative insights for the pricing of information products.

To facilitate our analysis, we focused on an environment with a monopolistic provider of information interacting with a market of competing firms. Extending our framework to incorporate competition among information providers is an interesting direction for future research. Even though the basic mechanism we identify—that is, that the extent of strategic complementarities/substitutabilities in the downstream market matters for products firms should offer—will be present regardless of market conditions, departures from our baseline framework can potentially impact the optimal degree of distortion in the quality or quantity of information.

Appendix. Proofs

With the exception of our results in Section 6, firms in our model are assumed to be ex ante symmetric. Therefore, unless otherwise noted, we assume without loss of generality that the price offered by the provider to the firms is nondecreasing in the firms' index; that is, $p_i \geq p_j$ for $i > j$. Given that excluding a firm i from trade is equivalent to offering a price $p_i = \infty$, this assumption also implies that the set of firms that are offered a contract by the provider is of the form $[0, \lambda]$ for some $\lambda \in [0, 1]$. Let l denote the fraction of firms who accept the provider's offer. It is immediate that $l = \sup\{i \in [0, \lambda]: b_i = 1\}$ and that $b_i = 1$ for all $i \leq l$.

Proof of Proposition 1

The first-order optimality condition for firm i's problem with respect to action a_i is given by

$$\mathbb{E}\left[\frac{\partial}{\partial a_i}\pi(a_i, A, \theta)\,\middle|\, \mathcal{I}_i\right] = 0,$$

where $\mathcal{I}_i = \{x_i\}$ if $i \in [l, 1]$, that is, the firm is uninformed, and $\mathcal{I}_i = \{x_i, s_i\}$ if $i \in [0, l]$, that is, the firm is informed. Consequently, $a_i = \mathbb{E}[\beta A + (1-\beta)\alpha\theta \mid \mathcal{I}_i]$, where $\beta = \gamma_1/\gamma_2$ is the degree of strategic complementarity in the downstream market as defined in (2) and $\alpha = \gamma_0/(\gamma_2 - \gamma_1)$. Thus, the firms' equilibrium actions are given by

$$a_i = \begin{cases} \mathbb{E}[\beta A + (1-\beta)\alpha\theta \mid x_i] & \forall\, i \in [l, 1], \\ \mathbb{E}[\beta A + (1-\beta)\alpha\theta \mid x_i, s_i] & \forall\, i \in [0, l]. \end{cases}$$

Noticing that $\mathbb{E}[\theta \mid x_i]$ is linear in x_i and $\mathbb{E}[\theta \mid x_i, s_i]$ is linear in x_i and s_i, we conjecture that equilibrium strategies are linear functions of x_i and s_i and then verify our hypothesis. In particular, we conjecture that

$$a_i = \begin{cases} c_0 x_i & \forall\, i \in [l, 1] \\ c_1 x_i + c_2 s_i & \forall\, i \in [0, l] \end{cases}$$

for some constants $c_0, c_1, c_2 \in \mathbb{R}$. Replacing the candidate equilibrium strategy of an uninformed firm $i \in (l, 1]$ in its first-order optimality condition yields

$$\begin{aligned} c_0 x_i &= \mathbb{E}\left[\beta\left(\int_0^l c_1 x_j + c_2 s_j\, dj + \int_l^1 c_0 x_j\, dj\right) + (1-\beta)\alpha\theta \mid x_i\right] \\ &= [\beta l(c_1 + c_2) + \beta(1-l)c_0 + (1-\beta)\alpha]x_i, \end{aligned} \tag{A.1}$$

where we are using the fact that $\mathbb{E}[\theta \mid x_i] = \mathbb{E}[x_j \mid x_i] = \mathbb{E}[s_j \mid x_i] = x_i$. Similarly, the first-order optimality condition for the optimization problem of an informed firm $i \in [0, l]$ yields

$$\begin{aligned} c_1 x_i + c_2 s_i = \mathbb{E}\Bigg[&\beta\left(\int_0^l (c_1 x_j + c_2 s_j)\, dj + \int_l^1 c_0 x_j\, dj\right) \\ &+ (1-\beta)\alpha\theta \mid x_i, s_i\Bigg]. \end{aligned} \tag{A.2}$$

Note that

$$\mathbb{E}[\theta \mid x_i, s_i] = \mathbb{E}[x_j \mid x_i, s_i] = \delta_1 x_i + (1-\delta_1)s_i \quad \text{and}$$
$$\mathbb{E}[s_j \mid x_i, s_i] = \delta_1(1-\rho)x_i + [1 - \delta_1(1-\rho)]s_i,$$

where $\delta_1 = \kappa_x/(\kappa_x + \kappa_s)$. Consequently, we can rewrite (A.2) as

$$\begin{aligned} &c_1 x_i + c_2 s_i \\ &\quad = [\beta l c_1 \delta_1 + \beta l c_2 \delta_1(1-\rho) + \beta(1-l)c_0\delta_1 + (1-\beta)\alpha\delta_1]x_i \\ &\qquad + [\beta l c_1(1-\delta_1) + \beta l c_2(1 - \delta_1(1-\rho)) \\ &\qquad\quad + \beta(1-l)c_0(1-\delta_1) + (1-\beta)\alpha(1-\delta_1)]s_i. \end{aligned} \tag{A.3}$$

From Equation (A.1) we have for the equilibrium strategy coefficients $[c_0 = \beta l(c_1 + c_2) + \beta(1-l)c_0 + (1-\beta)\alpha$, for any admissible $l \in [0, 1]$. In turn, this implies that $c_0 = \alpha$ and $c_1 + c_2 = \alpha$. Replacing $c_0 = \alpha$ and $c_2 = \alpha - c_1$ in Equation (A.3) implies that equilibrium coefficient c_1 must satisfy $c_1 = \beta l c_1 \delta_1 \rho + \alpha\delta_1(1 - \beta l \rho)$. Solving for c_1 yields $c_1 = \alpha((1-\beta l\rho)\kappa_x/((1-\beta l\rho)\kappa_x + \kappa_s))$, and hence, $c_2 = \alpha - c_1 = \alpha(\kappa_s/((1-\beta l\rho)\cdot\kappa_x + \kappa_s))$. Thus, we conclude that firms' actions at equilibrium are given by

$$a_i = \begin{cases} \alpha x_i & \forall\, i \in [l, 1] \\ \alpha \dfrac{(1-\beta l\rho)\kappa_x}{(1-\beta l\rho)\kappa_x + \kappa_s} x_i + \alpha \dfrac{\kappa_s}{(1-\beta l\rho)\kappa_x + \kappa_s} s_i & \forall\, i \in [0, l], \end{cases}$$

completing the proof. Q.E.D.

Two Auxiliary Lemmas

We state and prove two lemmas that we use in the remainder of the appendix. The first lemma characterizes the expected surplus of an informed firm whereas the second lemma shows that, for any given λ, the provider always finds it optimal to charge a constant price to all firms $i \in [0, \lambda]$.

Lemma 1. *The expected surplus of each firm from buying the market signal is given by*

$$\Delta(l, \kappa_s, \rho, \kappa_x) = \alpha^2 \left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_s}{\kappa_x}\right)\frac{\kappa_s + \kappa_x}{[(1-\beta l\rho)\kappa_x + \kappa_s]^2}, \quad \text{(A.4)}$$

where l denotes the fraction of informed firms.

Proof. Let $a_i^1 := \alpha((\kappa_s s_i + (1-\beta l\rho)\kappa_x x_i)/(\kappa_s + (1-\beta l\rho)\kappa_x))$ denote the equilibrium action of an informed firm, and let $a_i^0 := \alpha x_i$ denote the equilibrium action of an uninformed firm. Recall that l denotes the fraction of informed firms, and thus, the aggregate equilibrium action is $A = \int_0^l a_i^1\, di + \int_l^1 a_i^0\, di$. By replacing the equilibrium actions in the expressions for the firms' payoffs and then taking the expectations conditional on θ, we get

$$\mathbb{E}[\pi(a_i^1, A, \theta) \mid \theta] = \alpha^2\left(\frac{\gamma_2}{2}\right)\left[\theta^2 + \frac{2\beta l\rho\kappa_s}{[(1-\beta l\rho)\kappa_x + \kappa_s]^2} - \frac{(1-\beta l\rho)^2\kappa_x + \kappa_s}{[(1-\beta l\rho)\kappa_x + \kappa_s]^2}\right], \quad \text{(A.5)}$$

and

$$\mathbb{E}[\pi(a_i^0, A, \theta) \mid \theta] = \alpha^2\left(\frac{\gamma_2}{2}\right)\left[\theta^2 - \frac{1}{\kappa_x}\right]. \quad \text{(A.6)}$$

Next note that we can use the two conditional expectations (A.5) and (A.6) to compute the (unconditional) expectation for a firm's surplus given by

$$\Delta := \mathbb{E}[\pi(a_i^1, A, \theta)] - \mathbb{E}[\pi(a_i^0, A, \theta)].$$

Applying the law of total expectation yields

$$\Delta = \alpha^2\left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_s}{\kappa_x}\right)\frac{\kappa_s + \kappa_x}{[(1-\beta l\rho)\kappa_x + \kappa_s]^2},$$

which completes the proof of the lemma. Q.E.D.

Lemma 2. *The provider sets $p_i = p^*(\lambda)$ for all $i \in [0, \lambda]$, where $p^*(\lambda)$ is equal to the expected equilibrium surplus of an informed firm when the fraction of informed firms is λ. Furthermore, $p^*(\lambda)$ is such that all firms that receive the provider's offer accept in equilibrium; thus, $l = \lambda$.*

Proof. Consider the simultaneous game of accepting/rejecting the provider's offer. Recall that, in such a game, each firm $i \in [0, \lambda]$ accepts the offer if the firm's expected surplus is bigger than the firm's individual price p_i while taking the decisions of the rest of the firms as given.

We suppose that a fraction $l \in [0, \lambda]$ of firms has accepted the provider's offer, and we write the optimal decision of each firm $i \in [0, \lambda]$ as a function of firm's i individual price. We have

$$b_i(p_i) = \begin{cases} 1 & \text{if } \Delta(l) > p_i \\ 0 & \text{if } \Delta(l) < p_i \\ \in \{0,1\} & \text{if } \Delta(l) = p_i, \end{cases}$$

where $\Delta(l)$ is given by Equation (A.4) and denotes the expected surplus of an informed firm when a fraction l is informed. We can write the provider's optimization problem as follows

$$\max_{\{p_i\}_{i\in[0,\lambda]}} \int_0^\lambda p_i b_i(p_i)\, di$$
$$\text{s.t. } b_i(p_i) = \begin{cases} 1 & \text{if } \Delta(l) > p_i \\ 0 & \text{if } \Delta(l) < p_i \\ \in \{0,1\} & \text{if } \Delta(l) = p_i \end{cases}, \quad \forall\, i \in [0, \lambda]. \quad \text{(A.7)}$$

Before solving for the provider's optimal selling strategy, we rewrite the set of constraints (A.7) as

$$\begin{cases} \sup_{i\in[0,\lambda]} p_i \le \Delta(\lambda) & \text{if } l = \lambda, \\ \inf_{i\in[0,\lambda]} p_i \ge \Delta(0) & \text{if } l = 0, \\ \int_0^\lambda \mathbb{1}_{\{p_i < \Delta(l)\}}\, di \le l \le \int_0^\lambda \mathbb{1}_{\{p_i \le \Delta(l)\}}\, di & \text{if } l \in (0, \lambda). \end{cases}$$

Recall that without loss of generality the pricing schedule $p\colon [0, \lambda] \to \mathbb{R}_+$ is nondecreasing; thus, we can further simplify the set of constraints as

$$\begin{cases} p_\lambda \le \Delta(\lambda) & \text{if } l = \lambda & \text{(A.8a)} \\ p_0 \ge \Delta(0) & \text{if } l = 0 & \text{(A.8b)} \\ p_l \le \Delta(l) \text{ and } p_{l^+} \ge \Delta(l) & \text{if } l \in (0, \lambda). & \text{(A.8c)} \end{cases}$$

The proof proceeds by showing that for any equilibrium of the subgame that results from a fraction l of the firms accepting the provider's offer, there exists an optimal pricing schedule such that $p_i = \Delta(l)$ for all $i \le l$ and $p_i = \infty$ for all $i > l$. There are the following three cases to consider:

(i) For case (A.8a), the problem simplifies to

$$\max_{\{p_i\}_{i\in[0,\lambda]}} \int_0^\lambda p_i\, di$$
$$\text{s.t. } p_\lambda \le \Delta(\lambda).$$

In this case, a fraction $l = \lambda$ of firms accepts, and as we show in what follows, it is optimal for the provider to set $p_i = \Delta(\lambda)$ for all $i \in [0, \lambda]$. Suppose, for the sake of contradiction, that p is optimal but $u := \sup\{i \in [0, \lambda]: p_i < \Delta(\lambda)\} \ge 0$. If $u = 0$, then we have $p_i = \Delta(\lambda)$ except for a set of measure 0, so this case is immaterial. If $u > 0$,, the maintained assumption that p is nondecreasing implies that

$$p_i < \Delta(\lambda), \quad \forall\, i < u \quad \text{and} \quad p_i = \Delta(\lambda), \quad \forall\, i \ge u.$$

This implies that we can construct pricing schedule p' such that

$$p_i < p'_i \le \Delta(\lambda), \quad \forall\, i < u \quad \text{and} \quad p'_i = p_i, \quad \forall\, i \ge u,$$

which is feasible and achieves a higher objective value. Thus, it must be that $p_i = \Delta(\lambda)$ for all $i \le \lambda$.

(ii) For case (A.8b), $l = 0$ and the objective function is always equal to zero. Thus, p can be chosen such that $p_i = \infty$ for all $i \in [0, \lambda]$.

(iii) Finally, for case (A.8c), the problem simplifies to

$$\max_{\{p_i\}_{i\in[0,\lambda]}} \int_0^l p_i\, di$$
$$\text{s.t. } p_l \le \Delta(l) \quad \text{and} \quad p_{l^+} \ge \Delta(l).$$

First, we show that the provider can always set $p_i = \infty, \forall\, i > l$. Note that the individual price of each firm $i > l$ does not affect the objective function of the provider. This implies that all feasible solutions p that differ only on $(l, \lambda]$ attain the same objective value, so it is without loss of generality to focus on solutions that are such that $p_i = \infty$ for all $i > l$. Next, we show that $p_i = \Delta(l), \forall\, i \le l$. Suppose, for the sake of contradiction, that p is optimal but $u := \sup\{i \in [0, l]\colon p_i < \Delta(l)\} \ge 0$. If $u = 0$, we have $p_i = \Delta(l)$, except for a set of measure 0. If $u > 0$,, the assumption that p is nondecreasing implies that

$$p_i < \Delta(l), \quad \forall\, i < u \quad \text{and} \quad p_i = \Delta(l), \quad \forall\, i \ge u,$$

which, in turn, implies that we can construct a pricing schedule p'' such that

$$p_i < p_i'' \le \Delta(l), \quad \forall\, i < u \quad \text{and} \quad p_i'' = p_i, \quad \forall\, i \ge u,$$

which is feasible and achieves a higher objective value. Thus, it must be that $p_i = \Delta(l)$ for all $i \le l$.
Thus, there exists an optimal pricing schedule such that $p_i = \Delta(l)$ for all $i \le l$ and $p_i = \infty$ for all $i > l$, which implies that only a fraction l of firms accepts the provider's offer, and the latter's optimal profit is $l \cdot \Delta(l)$. Without loss of generality, the provider sets $\lambda = l$ and $p_i = \Delta(\lambda)$ for all $i \in [0, \lambda]$. Thus, all firms accept the provider's offer, and the provider's profit is given by $\lambda \cdot \Delta(\lambda)$. Setting $p^*(\lambda) = \Delta(\lambda)$ completes the proof. Q.E.D.

Proof of Proposition 2

By Lemma 2, the provider's problem simplifies to choosing λ, κ_y and ρ to maximize the expected profit $\Pi := \lambda \cdot p^*(\lambda, \kappa_s, \rho, \kappa_x) = \lambda \cdot \Delta(\lambda, \kappa_s, \rho, \kappa_x)$, subject to the constraints imposed by the information structure. Replacing the expected surplus (A.4) into the objective function yields

$$\Pi(\lambda, \kappa_s, \rho, \kappa_x) = \lambda \alpha^2 \left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_s}{\kappa_x}\right)\frac{\kappa_s + \kappa_x}{[(1-\beta\lambda\rho)\kappa_x + \kappa_s]^2}, \tag{A.9}$$

and thus, the provider's problem can be rewritten as

$$\max_{\rho, \kappa_s, \lambda} \ \Pi(\lambda, \kappa_s, \rho, \kappa_x) \tag{A.10}$$
$$\text{s.t. } \frac{\kappa_s}{\kappa_z} \le \rho \le 1, \quad \text{and} \quad 0 \le \lambda \le 1.$$

Note that the partial derivative of Π with respect to ρ, that is,

$$\frac{\partial \Pi}{\partial \rho} = \lambda\ \alpha^2 \gamma_2 \frac{\beta\lambda\kappa_s(\kappa_x + \kappa_s)}{[(1-\beta\lambda\rho)\kappa_x + \kappa_s]^3}, \tag{A.11}$$

is positive for $\beta \in (0, 1/2)$; thus, $\rho^* = 1$. Replacing this into (A.9) and differentiating with respect to λ yields

$$\frac{\partial \Pi}{\partial \lambda} = \alpha^2 \left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_s}{\kappa_x}\right)\frac{(\kappa_x + \kappa_x)[(1+\beta\lambda)\kappa_x + \kappa_s]}{[(1-\beta\lambda)\kappa_x + \kappa_s]^3}. \tag{A.12}$$

Similarly, the partial derivative with respect to κ_s is given by

$$\frac{\partial \Pi}{\partial \kappa_s} = \lambda\alpha^2\left(\frac{\gamma_2}{2}\right)\frac{(1-\beta\lambda)\kappa_x + (1-2\beta\lambda)\kappa_s}{[(1-\beta\lambda)\kappa_x + \kappa_s]^3}. \tag{A.13}$$

In addition, note that (A.12) and (A.13) are positive for $\beta \in (0, 1/2)$, so the provider finds it optimal to set $\lambda^* = 1$ and $\kappa_s^* = \kappa_z$. Replacing ρ^*, λ^* and κ_s^* into (A.9) yields $\Pi^* = \alpha^2(\gamma_2/2)(\kappa_z/\kappa_x)((\kappa_z + \kappa_x)/([(1-\beta)\kappa_x + \kappa_z]^2)$. Q.E.D.

Proof of Proposition 3

Consider the provider's expected profit (A.9) and the provider's profit-maximization problem (A.10), and let $-(1 + \kappa_z/\kappa_x) \le \beta < 0$. In this case, the partial derivative of Π with respect to ρ given in (A.11) is negative, which implies that the provider finds it optimal to set the level of correlation to its minimum; that is, $\rho_\xi^* = 0$ or $\rho^* = \kappa_s/\kappa_z$. Replacing this into (A.9) and differentiating with respect to κ_s yields

$$\frac{\partial \Pi}{\partial \kappa_s} = \lambda\alpha^2\left(\frac{\gamma_2}{2}\right)\frac{(1+\beta\lambda\kappa_s/\kappa_z)\kappa_x + \kappa_s}{[(1-\beta\lambda\kappa_s/\kappa_z)\kappa_x + \kappa_s]^3} \tag{A.14}$$

while differentiating with respect to λ yields

$$\frac{\partial \Pi}{\partial \lambda} = \alpha^2\left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_s}{\kappa_x}\right)\frac{(\kappa_x + \kappa_s)[(1+\beta\lambda\kappa_s/\kappa_z)\kappa_x + \kappa_s]}{[(1-\beta\lambda\kappa_s/\kappa_z)\kappa_x + \kappa_s]^3}. \tag{A.15}$$

The assumption on β implies that (A.14) and (A.15) are positive, which results in $\lambda^* = 1$ and $\kappa_s^* = \kappa_z$. The proof follows by replacing the optimal values for ρ^*, λ^*, and κ_s^* into expression (A.9). Q.E.D.

Proof of Proposition 4

Consider the provider's expected profit (A.9) and the provider's profit-maximization problem (A.10), and let $\beta < -(1 + \kappa_z/\kappa_x)$. First, note that in this case (A.11) is negative, so the provider finds it optimal to set $\rho^* = \kappa_s/\kappa_z$. Replacing this into (A.9) and differentiating with respect to κ_s and λ, we again obtain (A.14) and (A.15), respectively. Both (A.14) and (A.15) are equal to zero if and only if $(\kappa_z + \beta\lambda\kappa_s)\kappa_x + \kappa_z\kappa_s = 0$. Moreover, Π is unimodal in both κ_s and λ, which implies that the set of optimal allocations (κ_s^*, λ^*) is given by the solutions to $(\kappa_z + \beta\lambda\kappa_s)\kappa_x + \kappa_z\kappa_s = 0$. Finally, replacing $\kappa_s^*, \rho^* = \kappa_s^*/\kappa_z$, and $\lambda^* = (\kappa_x + \kappa_s^*)\kappa_z/(-\beta\kappa_x\kappa_s^*)$ into (A.9) yields $\Pi^* = -\alpha^2(\gamma_2/2)(\kappa_z/(4\beta\kappa_x^2))$. Q.E.D.

Proof of Proposition 5

We solve the game by backward induction; that is, first, we characterize the firms' equilibrium actions in the competition subgame that results from a (subset) of them obtaining the provider's information signal; then, we solve for their acceptance/rejection decisions; and, finally, we turn to the provider's problem and complete the proofs of parts (a) and (b) of the proposition. Recall that the provider possesses a signal $z = \theta + \zeta$, with $\zeta \sim N(0, 1/\kappa_z)$, and offers to firm $i \in [0, 1]$ a signal $s_i = z + \xi_i$ with $\xi_i \sim N(0, \kappa_{\xi i})$. Without loss of generality, we assume that the provider does not add any correlation to the signal the provider sells; that is, $\mathrm{corr}(\xi_i, \xi_j) = 0$. The market signal s_i offered to firm $i \in [0, 1]$ can be rewritten as $s_i = \theta + \eta_i$ with $\eta_i \sim N(0, 1/\kappa_{si})$, where $\kappa_s = (1/\kappa_z + 1/\kappa_{\xi i})^{-1}$ and $\mathrm{Cov}(s_i, s_j) = 1/\kappa_z$. We have the following auxiliary lemma.

Lemma 3. *The competition subgame has a unique Bayes–Nash equilibrium in linear strategies given by* $a(\kappa_{si}, \kappa_{s-i}) = \alpha[(1-\omega_i) \cdot x_i + \omega_i s_i]$ *for all* $i \in [0, 1]$, *where*

$$\omega_i = \left(\frac{\kappa_{si}}{\kappa_x + \kappa_{si}}\right) \bigg/ \left(1 - \beta\frac{\kappa_x}{\kappa_z}\int_0^1 \frac{\kappa_{si}}{\kappa_x + \kappa_{si}}\, di\right)$$
$$\text{and} \quad \alpha = \gamma_0/(\gamma_2 - \gamma_1).$$

Proof. The first-order optimality condition of firm i with respect to action a_i implies that in equilibrium

$$a_i = \mathbb{E}[\beta A + (1-\beta)\alpha\theta \mid x_i, s_i]. \tag{A.16}$$

Assume that each firm $i \in [0,1]$ uses a linear strategy $c_i x_i + h_i s_i$, for constants $c_i, h_i \in \mathbb{R}$. Then, we can rewrite the equilibrium condition (A.16) as $c_i x_i + h_i s_i = \mathbb{E}[\beta \int_0^1 (c_j x_j + h_j s_j)\,dj + (1-\beta)\alpha\theta \mid x_i, s_i]$. Using equations

$$\mathbb{E}[s_j \mid x_i, s_i] = \frac{\kappa_x(1-\kappa_{si}/\kappa_z)}{\kappa_x+\kappa_{si}} x_i + \frac{\kappa_{si}(1+\kappa_x/\kappa_z)}{\kappa_x+\kappa_{si}} s_i,$$

$$\text{and} \quad \mathbb{E}[\theta \mid x_i, s_i] = \mathbb{E}[x_j \mid x_i, s_i] = \frac{\kappa_x}{\kappa_x+\kappa_{si}} x_i + \frac{\kappa_{si}}{\kappa_x+\kappa_{si}} s_i,$$

which are obtained by the conditional expectation of Gaussian random vectors, we have

$$\begin{aligned} c_i x_i + h_i s_i = \beta\Bigg[&\left(\frac{\kappa_x}{\kappa_x+\kappa_{si}} x_i + \frac{\kappa_{si}}{\kappa_x+\kappa_{si}} s_i\right)\int_0^1 c_j\,dj \\ &+ \left(\frac{\kappa_x(1-\kappa_{si}/\kappa_z)}{\kappa_x+\kappa_{si}} x_i + \frac{\kappa_{si}(1+\kappa_x/\kappa_z)}{\kappa_x+\kappa_{si}} s_i\right)\int_0^1 h_j\,dj\Bigg] \\ &+ (1-\beta)\alpha\left(\frac{\kappa_x}{\kappa_x+\kappa_{si}} x_i + \frac{\kappa_{si}}{\kappa_x+\kappa_{si}} s_i\right). \end{aligned}$$

Note that the equilibrium coefficients $(c_i, , h_i)$ for $i \in [0,1]$, must solve the following sets of equations:

$$\begin{aligned} c_i = \beta\frac{\kappa_x}{\kappa_x+\kappa_{si}}\int_0^1 c_j\,dj + \beta\frac{\kappa_x(1-\kappa_{si}/\kappa_z)}{\kappa_x+\kappa_{si}} \\ \cdot \int_0^1 h_j\,dj + (1-\beta)\alpha\frac{\kappa_x}{\kappa_x+\kappa_{si}} \quad \forall i \in [0,1], \end{aligned} \tag{A.17}$$

and

$$\begin{aligned} h_i = \beta\frac{\kappa_{si}}{\kappa_x+\kappa_{si}}\int_0^1 c_j\,dj + \beta\frac{\kappa_{si}(1+\kappa_x/\kappa_z)}{\kappa_x+\kappa_{si}} \\ \cdot \int_0^1 h_j\,dj + (1-\beta)\alpha\frac{\kappa_{si}}{\kappa_x+\kappa_{si}} \quad \forall i \in [0,1]. \end{aligned} \tag{A.18}$$

Integrating over $[0,1]$ in (A.17) and (A.18) yields a linear-system of two equations, which implies that

$$\int_0^1 c_i\,di = \alpha\left(1 - \left(1+\beta\frac{\kappa_x}{\kappa_z}\right)\int_0^1 \frac{\kappa_{si}}{\kappa_x+\kappa_{si}}\,di\right) \Bigg/ \left(1 - \beta\frac{\kappa_x}{\kappa_z}\int_0^1 \frac{\kappa_{si}}{\kappa_x+\kappa_{si}}\,di\right)$$

and

$$\int_0^1 h_i\,di = \alpha\left(\int_0^1 \frac{\kappa_{si}}{\kappa_x+\kappa_{si}}\,di\right) \Bigg/ \left(1 - \beta\frac{\kappa_x}{\kappa_z}\int_0^1 \frac{\kappa_{si}}{\kappa_x+\kappa_{si}}\,di\right).$$

Thus, we can rewrite (A.17) and (A.18) as $c_i = \alpha(\kappa_x/(\kappa_x+\kappa_{si})) \cdot (1 - \beta((\kappa_x + \kappa_{si})/\kappa_z)\int_0^1(\kappa_{si}/(\kappa_x+\kappa_{si}))\,di)/(1-\beta(\kappa_x/\kappa_z) \cdot \int_0^1(\kappa_{si}/(\kappa_x+\kappa_{si}))\,di)$ and $h_i = \alpha(\kappa_{si}/(\kappa_x+\kappa_{si}))/(1 - \beta(\kappa_x/\kappa_z)\int_0^1(\kappa_{si}/(\kappa_x+\kappa_{si}))\,di)$. Finally, noting that $c_i + h_i = \alpha$ and setting $h_i = \alpha\omega_i$ completes the proof. Q.E.D.

The next step in our analysis involves studying the firms' acceptance/rejection decisions that precede the competition subgame. We restrict attention to subgame perfect equilibria in which all firms accept the provider's offers. This is without loss of generality since the case in which there is a firm i that rejects the provider's offer is surplus-equivalent to the case in which the provider offers a signal of precision $\kappa_{si} = 0$ at price $p_i = 0$ to firm i and firm i accepts the offer. The equilibrium acceptance/rejection decisions can be characterized as follows. Each firm $i \in [0,1]$ accepts the provider's offer if $\Delta_i = \mathbb{E}[\pi(a(\kappa_{si}, \kappa_{s-i}))] - \mathbb{E}[\pi(a(0, \kappa_{s-i}))] \geq p_i$, that is, if price p_i is lower than the expected surplus of firm i. Thus, it is optimal for the provider to offer $p_i = \Delta_i$ for all $i \in [0,1]$.

Using the equilibrium characterization from Lemma 3, we can compute the expected surplus Δ_i of firm i, which, in turn, is equal to price p_i; that is,

$$p_i = \alpha^2\left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_{si}}{\kappa_x+\kappa_{si}}\right)\left(\frac{1}{\kappa_x}\right)\Bigg/\left[1 - \beta\frac{\kappa_x}{\kappa_z}\int_0^1 \frac{\kappa_{si}}{\kappa_x+\kappa_{si}}\,di\right]^2. \tag{A.19}$$

The provider's expected equilibrium profit is given by

$$\begin{aligned} \Pi(\kappa_s, \beta) = \int_0^1 p_i\,di = \alpha^2\left(\frac{\gamma_2}{2}\right)\left(\frac{1}{\kappa_x}\int_0^1 \frac{\kappa_{si}}{\kappa_x+\kappa_{si}}\,di\right) \\ \Bigg/\left[1 - \beta\frac{\kappa_x}{\kappa_z}\int_0^1 \frac{\kappa_{si}}{\kappa_x+\kappa_{si}}\,di\right]^2, \end{aligned} \tag{A.20}$$

and the provider's problem can now be simply written as

$$\begin{aligned} &\max_{\{\kappa_{si}\}_{i\in[0,1]}} \ \Pi(\kappa_s, \beta) \\ &\text{s.t.} \ \ 0 \leq \kappa_{si} \leq \kappa_z \quad \forall i \in [0,1]. \end{aligned} \tag{A.21}$$

The following lemma allows us to further simplify optimization problem (A.21).

Lemma 4. *The objective function of problem (A.21) depends on* $\{\kappa_{si}\}_{i\in[0,1]}$ *only through a constant*

$$D = \int_0^1 \frac{\kappa_{si}}{\kappa_x+\kappa_{si}}\,di. \tag{A.22}$$

Furthermore, for any optimal solution $\{\kappa^*_{si}\}_{i\in[0,1]}$ *of problem (A.21), there exists a constant solution* $\bar{\kappa}_s$ *that is feasible and achieves the same objective value of* $\{\kappa^*_{si}\}_{i\in[0,1]}$.

Proof. The first statement follows directly from expression (A.20). For the second statement, let $\{\kappa^*_{si}\}_{i\in[0,1]}$ be an optimal solution of problem (A.21) with corresponding $D^* = \int_0^1(\kappa^*_{si}/(\kappa_x+\kappa^*_{si}))\,di$. Define constant $\bar{\kappa}_s$ as $\bar{\kappa}_s := D^*\kappa_x/(1-D^*)$. Note that $\bar{\kappa}_s/(\kappa_x+\bar{\kappa}_s) = D^*$, which implies that $\bar{\kappa}_s$ achieves the same objective value as $\{\kappa^*_{si}\}_{i\in[0,1]}$. Finally, we need to verify that $\bar{\kappa}_s$ is feasible. By the feasibility of $\{\kappa^*_{si}\}_{i\in[0,1]}$, that is, $0 \leq \kappa^*_{si} \leq \kappa_z$ for all $i \in [0,1]$, it follows that $0 \leq D^* \leq \kappa_z/(\kappa_x+\kappa_z)$ and thus $0 \leq \bar{\kappa}_s \leq \kappa_z$. This implies that the constant $\bar{\kappa}_s$ is feasible, and it achieves the maximum objective value, which completes the proof. Q.E.D.

Lemma 4 allows us to solve a simplified problem, in which the provider offers a signal of precision κ_s to all firms $i \in [0,1]$. Furthermore, using the optimal value for κ_s together with Equation (A.22) allows us to characterize the set of optimal solutions for the original problem (A.21). In particular, replacing κ_s for κ_{si} in problem (A.21), the provider's problem simplifies to

$$\begin{aligned} &\max_{\kappa_s} \ \Pi = \alpha^2\left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_s}{\kappa_x}\right)\frac{\kappa_s+\kappa_x}{[(1-\beta\kappa_s/\kappa_z)\kappa_x+\kappa_s]^2} \\ &\text{s.t.} \ \ 0 \leq \kappa_s \leq \kappa_z. \end{aligned} \tag{A.23}$$

Proof of Part (a). Let $\beta \geq -(1+\kappa_z/\kappa_x)$, and consider the simplified problem (A.23). Differentiating the objective with respect to κ_s yields

$$\frac{\partial \Pi}{\partial \kappa_s} = \alpha^2\left(\frac{\gamma_2}{2}\right)\frac{(1+\beta\kappa_s/\kappa_z)\kappa_x + \kappa_s}{[(1-\beta\kappa_s/\kappa_z)\kappa_x + \kappa_s]^3}. \tag{A.24}$$

The assumption on β implies that (A.24) is positive, which means that it is optimal to set $\kappa_s^* = \kappa_z$. By Lemma 4, this implies that any solution $\{\kappa_{si}^*\}_{i\in[0,1]}$ to problem (A.21) that is feasible and such that

$$\int_0^1 \frac{\kappa_{si}^*}{\kappa_x + \kappa_{si}^*}\,di = \frac{\kappa_z}{\kappa_x + \kappa_z}$$

is an optimal solution. Thus, problem (A.21) has a unique optimal solution in this case; that is, $\kappa_{si}^* = \kappa_z, \forall\, i \in [0,1]$. Replacing this solution into (A.19), we obtain

$$p_i^* = \alpha^2\left(\frac{\gamma_2}{2}\right)\left(\frac{\kappa_z}{\kappa_x}\right)\frac{\kappa_z + \kappa_x}{[(1-\beta)\kappa_x + \kappa_z]^2} = p^*.$$

Proof of Part (b). Let $\beta < -(1+\kappa_z/\kappa_x)$, and consider problem (A.23). In this case, the partial derivative given in (A.24) evaluated at $\kappa_s^* = \kappa_z$ is negative, so the provider is better off by offering noisy signals to the firms. Solving for the optimal κ_s using a firm's first-order optimality condition yields

$$\kappa_s^* = \frac{\kappa_x}{-(1+\beta\kappa_x/\kappa_z)} < \kappa_z.$$

By Lemma 4, this implies that any solution $\{\kappa_{si}^*\}_{i\in[0,1]}$ to problem (A.21) that is feasible and such that

$$\int_0^1 \frac{\kappa_{si}^*}{\kappa_x + \kappa_{si}^*}\,di = -\frac{\kappa_z}{\beta\kappa_x} \tag{A.25}$$

is an optimal solution. Finally, replacing (A.25) into (A.19) yields $p_i^* = \alpha^2(\gamma_2/2)(\kappa_{si}^*/(4(\kappa_x+\kappa_{si}^*)\kappa_x)$. Q.E.D.

Endnotes

[1] IRI offers an array of information products at different price points. For example, the basic "Market Advantage Solution" includes a summary of industry sales and a detailed analysis of pricing strategies employed by a firm's competitors. The premium "Market Advantage Solution," on the other hand, provides a more in-depth analysis of sales and competitors' pricing strategies along with more specialized analytic services. The basic product is priced around \$10,000 whereas the price for the premium offering can range between \$100,000 and \$500,000.

[2] Besides the obvious case of differentiating their data based on its granularity (say, its coverage or level of aggregation), financial data providers also use frequency as a dimension to differentiate their information products. For instance, in the context of the U.S. macroeconomic data announcements by various government agencies (such as monetary policy announcements by the Federal Reserve or nonfarm employment numbers released by the Bureau of Labor Statistics), Kurov et al. (2019) argue that some private data providers release information to exclusive groups of subscribers before making it available to others with the documented early releases in the range of seconds. Also see Mullins et al. (2013) for another example. Thus, to the extent that slightly outdated information can be considered as information of lower quality (e.g., because of fast-moving market conditions), such an environment also exhibits the key features of our model.

[3] Also see Chen and Tang (2015), who study the value of market information for farmers in developing economies.

[4] More formally, suppose that θ is distributed according to a Gaussian distribution with mean 0 and variance σ_θ^2. By letting $\sigma_\theta \to \infty$, we obtain a distribution with full support over $(-\infty,\infty)$ that, in the limit, assigns the same probability to all intervals that have the same Lebesgue measure.

[5] Note that, in our baseline setting, the provider offers a signal of the same precision to all firms $i \in I$; that is, κ_ξ is independent of i. We relax this assumption in Section 4 and show that all our insights are robust to this assumption.

[6] See Myatt and Wallace (2015) for micro-foundations for this demand system.

[7] We provide the formalism and the required conditions for such a variant of the law of large numbers in the electronic companion of the paper. For a thorough treatment of the subject, see Sun (2006) and Sun and Zhang (2009).

[8] Recall from the payoff function (1) that each firm i cares about the actions of other firms only insofar as these actions impact the aggregate action A. This observation thus implies that any (symmetric) setting in which firm i observes noisy signals about other firms' individual actions can be mapped into an isomorphic setting in which firm i only observes a signal about the aggregate action as in (9).

[9] It is immediate to see that the setting in which firms' actions cannot be contingent on the realization of S_i reduces to our benchmark model.

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