Discussion of
“Product Differentiation and Oligopoly: a Network Approach”
Bruno Pellegrino (2023)

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A Network Model of Oligopoly

- Generalized Hedonic-Linear (GHL) Demand: consumers have additively separable preferences over attributes:

\[ u(x_1, \ldots, x_m) = \sum_{k=1}^{m} \left( b_k x_k - \frac{1}{2} x_k^2 \right) - L \]

- examples: antibodies, organisms, purification, yeast, enzymes, …
A Network Model of Oligopoly

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• \( n \) firms producing differentiated products, which can be represented on the attribute space:

good \( i \)'s representation:

\[ a_i = [a_{i1} \ a_{i2} \ \ldots \ \ a_{im}]' \]

- representation of the product characteristics space: \( A \)
A Network Model of Oligopoly

- Firms compete à la Cournot
- Cosine similarity as a natural measure of how similar two products are
  \[ \cos ij = a'_i a_j \in [0, 1] \]

• Main force: firms that produce more similar products compete more intensely
• Implication: firms with high product market centralities...
  ▶ set lower markups
  ▶ have a smaller (weighted) market share

• Empirical Finding: a significant portion (90%) of the rise in markups can be attributed to changes in product market centrality.
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• **Empirical Finding**: a significant portion (90%) of the rise in markups can be attributed to changes in product market centrality.
• Truly impressive paper

• Lots and lots of generalizations:
  ▶ multiproduct firms
  ▶ input-output linkages
  ▶ competitive fringe of firms
  ▶ etc.
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- Empirical implementation using the Hoberg and Phillips product similarity data
  - Model maps beautifully to the cosine similarity constructed by HP
- Model is used to think about important counterfactuals:
  - welfare costs of oligopoly, implications of collusion, M&A’s
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- This discussion: narrow focus on the theory
Comment 1: What Is $\chi$?

- **Product market centrality** of firm $i$ as

$$\chi_i = 1 - 2 \sum_{j=1}^{n} (I + A'A)^{-1} \left( \frac{(A'b - c)_j}{(A'b - c)_i} \right)$$

A measure of how intensely a firm competes with others

- Characterize equilibrium quantities, markups, consumer surplus, profits, market share, etc. in terms of product market centrality

$$q = \frac{1}{2} \text{diag}(A'b - c)(1 - \chi)$$

$$\mu = 1 + \frac{1}{2} \text{diag}^{-1}(c)\text{diag}(A'b - c)(1 - \chi).$$
Comment 1: What Is $\chi$?

- A solid case that the **product market centrality** $\chi_i$ is economically relevant

  - markups:
    \[ \mu_i = \chi_i + (1 - \chi_i) \bar{\mu}_i \]

  - weighted market share:
    \[ M_i = \frac{q_i}{q_i + \sum_{j \neq i} \sigma_{ij} q_j} = \frac{1 - \chi_i}{1 + \chi_i} \]
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- key properties:
  - firm is a monopolist: $\chi_i = 0$
  - all products identical: $\chi_i = 1 - \frac{2}{n + 1}$
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- key properties:
    
    - firm is a monopolist: $\chi_i = 0$
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- But beyond these, the paper doesn’t explore what $\chi_i$ is or how it behaves, even though it is the central statistic in the model.
Comment 1: What Is $\chi$?

- No matter the environment and the market structure, I can always find a $\chi_i$ as follows and call it “centrality”:

\[ \mu_i = \chi_i + (1 - \chi_i) \bar{\mu}_i \]

- But this would only be useful as a measure if one understands how this object depends on product characteristics.
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- There is an expression in the paper in terms of model primitives, but understanding what the object really captures requires **comparative statics** analysis.
  \[ \chi = 1 - 2 \text{diag}^{-1}(A'b - c)(I + A'A)^{-1}(A'b - c). \]
Comment 1: Comparative Statics

- Consider the following change in the product space.
- Intuitively: goods are more become more similar as $\gamma$ grows

$$B \propto (1 - \gamma)A + \gamma \mathbf{1}\mathbf{1}'/\sqrt{n}, \quad A = \begin{bmatrix} 0.0641 & 0.7271 & 0.2212 \\ 0.9365 & 0.3822 & 0.9015 \\ 0.3448 & 0.5703 & 0.3719 \end{bmatrix}$$
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• For $\gamma = 0.1$:

\[ \cos(b_i, b_j) > \cos(a_i, a_j) \text{ for all } i \neq j \]
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- Product market centrality has the flavor of “how far firm $i$ is from every other rival $j$ in the space of product characteristics,” but it’s not exactly that.
Comment 1: What Is $\chi$?

- Clear from the analysis that low centrality firms have higher markups
- But what do we learn about their product characteristics?
  - is it really because of they have more differentiated products?
  - maybe! maybe not!
  - would be great if the paper can pin this down.
Comment 1: What Is $\chi$?
Comment 2: Markup Growth Decomposition

- Markups in the model can be expressed in terms of product market centrality and hedonic-adjust productivity

\[ \mu_i = \chi_i + \frac{1}{2} (1 - \chi_i)(1 + \omega_i) \]

- Use this result to decompose the rise of markups to either increased productivity or reduction in centrality
Comment 2: Markup Growth Decomposition

- But one cannot move these two objects independently:

\[ \chi_i = 1 - 2 \sum_{j=1}^{n} (I + A'A)_i^j \left( \frac{b_j - c_j}{b_i - c_i} \right) \]

\[ \omega_i = \frac{b_i}{c_i} \]
Comment 2: Markup Growth Decomposition

• But one cannot move these two objects independently:

\[
\chi_i = 1 - 2 \sum_{j=1}^{n} (I + A'A)_{ij}^{-1} \left( \frac{b_j - c_j}{b_i - c_i} \right)
\]

\[
\omega_i = \frac{b_i}{c_i}
\]

• For example, when all firms have identical marginal costs:

\[
\chi_i = 1 - 2 \sum_{j=1}^{n} (I + A'A)_{ij}^{-1} \left( \frac{\omega_j - 1}{\omega_i - 1} \right)
\]

• So, an increase in the productivity of firm \(i\) also increases its centrality.

• Having a hard time thinking about this decomposition
The paper argues that the model can handle goods that are gross complements, even though the utility function is submodular:

\[
\frac{\partial^2 u}{\partial q_i \partial q_j} = -(A' A)_{ij} \leq 0 \quad \text{for all } i \neq j
\]

\[
\frac{\partial q_i}{\partial p_j} = -(A' A)^{-1}_{ij} \leq 0
\]

In fact, the paper finds evidence for gross complementarities in the data: “General Motors’s output is gross complement vis-a-vis energy and consumer finance companies.”
Comment 3: Complementarities?

\[ A' A = \begin{bmatrix} 1 & 1/\sqrt{3} & 0 \\ 1/\sqrt{3} & 1 & 1/\sqrt{3} \\ 0 & 1/\sqrt{3} & 1 \end{bmatrix}, \quad (A' A)^{-1} = \begin{bmatrix} 2 & -\sqrt{3} & 1 \\ -\sqrt{3} & 3 & -\sqrt{3} \\ 1 & -\sqrt{3} & 2 \end{bmatrix} \]

- Goods 1 and 3 are gross complements:

\[ \frac{\partial q_1}{\partial p_3} = -1 \]
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• Goods 1 and 3 are gross complements:

\[
\frac{\partial q_1}{\partial p_3} = -1
\]

• Useful to understand what happens here

• Suppose \( p_3 \) increases
  
  ▶ **direct effect**: increases in demand for 2 (because 2 and 3 are substitutes)
  
  ▶ **indirect effect**: the increase consumption of good 2 reduces demand for 1 (because 1 and 2 are substitutes)
  
  ▶ **total effect**: 1 and 3 are act as complements.
Comment 3: Complementarities?

• Back to the example:

• Automobile and fuel are complements because I have no use for gas if I don’t have a car, independently of the presence of any third good (submodular preferences)

• In the model, automobile and fuel are complements only because when the price of cars go up, I switch to a third good (bicycles?) that is a substitute to both of them.
Summary

• Really impressive and ambitious paper

• It can benefit from exploring in more detail what the objects are really capturing