Discussion of “Bonds, Currencies, and Expectational Errors”
Granziera and Sihvonen (2022)

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Vienna Symposium on Foreign Exchange Markets
August 2022
Empirical Motivation

- Borrowing in low interest rate currencies and investing in high interest rate currencies delivers large excess returns (Forward premium puzzle).
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• Same does not apply if the trade is implemented using long-term bonds: as the maturity of the bonds increases, the average excess return declines to zero (Lustig, Stathopoulos, and Verdelhan, 2019)

downward-sloping term structure
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  downward-sloping term structure

• Hard to rationalize using standard structural models or no-arbitrage models.
This Paper

• A unified theory of bond and currency returns based on departure from full-information rational expectations (FIRE) benchmark

• **Sticky-information model** a la Mankiw and Reis (2002): at each period, only a fraction $k \in [0, 1]$ of agents adopt the correct expectation

\[
\mathbb{E}_t[x_{t+h}] = k \sum_{\tau=0}^{\infty} (1 - k)^\tau \mathbb{E}_{t-\tau}^* [x_{t+h}]
\]
This Paper

• A unified theory of bond and currency returns based on departure from full-information rational expectations (FIRE) benchmark

• **Sticky-information model** a la Mankiw and Reis (2002): at each period, only a fraction \( k \in [0, 1] \) of agents adopt the correct expectation

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\]

• Main analytical result: if the true process is AR(1), then the sticky-information model can simultaneously generate

1. the forward-premium puzzle
2. the downward-sloping term structure
3. under-reaction of exchange rate forecasts
Currency and Bond Returns

currency excess return: \( r_{x_{t+1}} = x_t + \Delta s_{t+1} \)
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bond excess return differential: \[
\begin{align*}
  r_{x_{t+1}}^{(k)} &= \left( p^*_t (k - 1) - p^*_{t+1} (k - 1) \right) \\
  &- (p^*_t (k) - p_t (k)) + \Delta s_{t+1}
\end{align*}
\]
Currency and Bond Returns

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bond excess return differential: \[ r_{x_{t+1}^{(k)}} = \left( p_{t+1}^*(k - 1) - p_{t+1}(k - 1) \right) \]
\[ - \left( p_{t}^*(k) - p_{t}(k) \right) + \Delta s_{t+1} \]

relative excess return: \[ \Delta r_{x_{t+1}^{(k)}} = r_{x_{t+1}^{(k)}} - r_{x_{t+1}} \]
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exchange rate forecast error: \[ s_{t+h} - \mathbb{E}_t[s_{t+h}] \]
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- Under rational expectations, all objects are unpredictable given time \( t \) information
- How about under misspecified expectations?
Comment 1: Currency and Bond Returns with Misspecified Expectations

- General model of subjective expectations

- Forecast revisions:

\[ FR_{t+1}^{(\tau)} = E_{t+1}[x_{t+\tau}] - E_t[x_{t+\tau}] \]
Comment 1: Currency and Bond Returns with Misspecified Expectations

• General model of subjective expectations

• Forecast revisions:

\[ FR^{(\tau)}_{t+1} = E_{t+1}[x_{t+\tau}] - E_t[x_{t+\tau}] \]

• Impose two assumptions:

(a) UIP under subjective expectations:

\[ s_t = x_t + E_t[s_{t+1}] \]

(b) subjective expectations satisfy the law of iterated expectations
Subjective Expectations: UIP + LIE

- currency excess return:
  \[ r_{x_{t+1}} = \sum_{\tau=1}^{\infty} FR_{t+1}^{(\tau)} \]

- bond excess return differential:
  \[ r_{x_{t+1}}^{(k)} = \sum_{\tau=k}^{\infty} FR_{t+1}^{(\tau)} \]

- relative excess return:
  \[ \Delta r_{x_{t+1}}^{(k)} = -\sum_{\tau=1}^{k-1} FR_{t+1}^{(\tau)} \]

- Exchange rate forecast error:
  \[ s_{t+h} - \mathbb{E}_{t}[s_{t+h}] = \sum_{\tau=0}^{\infty} \sum_{k=1}^{h} FR_{t+k}^{(h+\tau-k+1)} \]
Subjective Expectations: UIP + LIE

- currency excess return:
  \[
  \text{cov}^*(r_{x_{t+1}}, x_t) = \sum_{\tau=1}^{\infty} \text{cov}^*(FR_{t+1}^{(\tau)}, x_t)
  \]

- bond excess return differential:
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  \text{cov}^*(r_{x_{t+1}}^{(k)}, x_t) = \sum_{\tau=k}^{\infty} \text{cov}^*(FR_{t+1}^{(\tau)}, x_t)
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- relative excess return:
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  \text{cov}^*(\Delta r_{x_{t+1}}^{(k)}, x_t) = -\sum_{\tau=1}^{k-1} \text{cov}^*(FR_{t+1}^{(\tau)}, x_t)
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- Exchange rate forecast error:
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  \text{cov}^*(s_{t+h} - \mathbb{E}_t[s_{t+h}], x_t) = \sum_{\tau=0}^{\infty} \sum_{k=1}^{h} \text{cov}^*(FR_{t+k}^{(h+\tau-k+1)}, x_t)
  \]
Assumption: $\text{cov}^*(\text{FR}_{t+h}^{(\tau)}, x_t) > 0$

- currency excess return:

$$\text{cov}^*(r_{x_{t+1}}, x_t) = \sum_{\tau=1}^{\infty} \text{cov}^*(\text{FR}_{t+1}^{(\tau)}, x_t) > 0$$
Assumption: $\text{cov}^*(FR_{t+h}^{(\tau)}, x_t) > 0$

- currency excess return:

$$\text{cov}^*(rx_{t+1}, x_t) = \sum_{\tau=1}^{\infty} \text{cov}^*(FR_{t+1}^{(\tau)}, x_t) > 0$$

- bond excess return differential:

$$\text{cov}^*(rx_{t+1}^{(k)}, x_t) = \sum_{\tau=k}^{\infty} \text{cov}^*(FR_{t+1}^{(\tau)}, x_t) > 0 \text{ and decays with } k$$
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- relative excess return:
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Assumption: $\text{cov}^*(FR_{t+h}^{(\tau)}, x_t) > 0$

- **Currency excess return:**
  
  $\text{cov}^*(rx_{t+1}, x_t) = \sum_{\tau=1}^{\infty} \text{cov}^*(FR_{t+1}^{(\tau)}, x_t) > 0$

- **Bond excess return differential:**
  
  $\text{cov}^*(rx_{t+1}^{(k)}, x_t) = \sum_{\tau=k}^{\infty} \text{cov}^*(FR_{t+1}^{(\tau)}, x_t) > 0$ and decays with $k$

- **Relative excess return:**
  
  $\text{cov}^*(\Delta rx_{t+1}^{(k)}, x_t) = -\sum_{\tau=1}^{k-1} \text{cov}^*(FR_{t+1}^{(\tau)}, x_t) < 0$

- **Exchange rate forecast error:**
  
  $\text{cov}^*(s_{t+h} - \mathbb{E}_t[s_{t+h}], x_t) = \sum_{\tau=0}^{\infty} \sum_{k=1}^{h} \text{cov}^*(FR_{t+k}^{(h+\tau-k+1)}, x_t) > 0$
Comment 1: Driving Force Behind the Results

- One can reproduce all the paper’s predictions as long as subjective expectations and the true data-generating process jointly satisfy:

$$\text{cov}^\ast(FR_{t+h}^{(\tau)}, x_t) > 0$$
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- One can reproduce all the paper’s predictions as long as subjective expectations and the true data-generating process jointly satisfy:

\[
\text{cov}^*\left(\text{FR}_{t+h}^{(\tau)}, x_t\right) > 0
\]

- In the model with sticky information and AR(1) data-generating process:

\[
\text{cov}^*\left(\text{FR}_{t+h}^{(\tau)}, x_t\right) = \frac{1 - \lambda^2}{1 - \lambda^2(1 - k)} \lambda^{\tau+h-1}(1 - k)^h \left(k1_{\{\tau>1\}} + 1_{\{\tau=1\}}\right).
\]

- Positive as long as \( k < 1 \) (information is sticky) and \( \lambda \neq 0 \) (serial correlation)
Comment 2: Predictability Reversal Puzzle

\[ r x_{t+h} = \alpha_h + \beta_h x_t + \epsilon_{t,t+h} \]

- **Forward Discount Puzzle**
  - Fama (1984)

- **Predictability Reversals Puzzle**
  - Bacchetta and van Wincoop (2010)
Comment 2: Predictability Reversal Puzzle

\[ \text{cov}^*(r_{x_{t+h}}, x_t) = \sum_{\tau=1}^{\infty} \text{cov}^*(FR_{t+h}^{(\tau)}, x_t) > 0 \]

• Higher interest rates today predict positive excess returns at all horizons \( h \)
• Inconsistent with the predictability reversal puzzle

• In general, too much to ask a simple model to explain all puzzles simultaneously
• But the key force that drives all the results in the model is exactly what violates the reversal puzzle.
Taking Stock

• One needs $\text{cov}^*(\text{FR}_{t+h}^{(\tau)}, x_t)$ to change signs to simultaneously explain
  
  (1) the forward premium puzzle
  (2) the predictability reversal puzzle
  (3) downward-sloping terms structure of term premia

• But how exactly?