Discussion of “Collateralized Debt Networks with Lender Default”
by Jin-Wook Chang

Alireza Tahbaz-Salehi
Northwestern University

Midwest Finance Association
August 2020
Financial Networks

- Growing literature on how financial linkages...
  1. function as a mechanism for propagation and amplification of shocks
  2. generate systemic risk from micro shocks

- For the most part, the literature makes two simplifying assumptions
  - unsecured lending
  - exogenously-specified network of relationships

- But in reality....
  - interbank lending is mostly collateralized
  - banks choose their partners and terms of contracts
Financial Networks

• Growing literature on how financial linkages...
  (i) function as a mechanism for propagation and amplification of shocks
  (ii) generate systemic risk from micro shocks

• For the most part, the literature makes two simplifying assumptions
  ▶ unsecured lending
  ▶ exogenously-specified network of relationships

• But in reality....
  ▶ interbank lending is mostly collateralized
  ▶ banks choose their partners and terms of contracts
This Paper: A Model of Collateralized Debt Markets

• Allows for collateralized lending and borrowing

• Quantities and prices (including that of the collateral) are endogenously determined

• Two methodological contributions:
  ▶ a framework for propagation of shocks in a collateralized lending network
  ▶ a model of financial network formation
    (though sidesteps some of the intricate issues of network formation by assuming lenders/borrowers are competitive)

• Main results:
  ▶ trade-off between counterparty risk and leverage
  ▶ under-diversification in equilibrium
  ▶ an application to loss coverage by a CCP
This Paper: A Model of Collateralized Debt Markets

• Allows for collateralized lending and borrowing
• Quantities and prices (including that of the collateral) are endogenously determined

• Two methodological contributions:
  ▶ a framework for propagation of shocks in a collateralized lending network
  ▶ a model of financial network formation
    (though sidesteps some of the intricate issues of network formation by assuming lenders/borrowers are competitive)

• Main results:
  ▶ trade-off between counterparty risk and leverage
  ▶ under-diversification in equilibrium
  ▶ an application to loss coverage by a CCP
Propagation of Shocks over Collateralized Debt Networks

• An exogenous network of pairwise collateralized debt contracts

\[ y_{ij} : \text{ borrowed cash per unit of collateral} \]
\[ c_{ij} : \text{ amount of posted collateral} \]

• Financial network represented by a pair of matrices \((Y, C)\)

• Payment from \(j\) to \(i\), per unit of posted collateral:

\[ x_{ij} = \min\{y_{ij}, p\}, \]

where \(p\) is the equilibrium price of collateral (non-recourse)
Propagation of Shocks over Collateralized Debt Networks

- Nominal wealth of agent $j$:
  \[
  m_j = e_j - \epsilon_j + h_j p + \sum_k c_{jk} \min\{p, y_{jk}\} - \sum_k c_{kj} \min\{p, y_{kj}\} - \sum_{k : m_k < 0} \xi(c_{kj})[p - y_{kj}]^+
  \]

- Failure of lender makes the borrower incur a cost to recover her collateral
  - counterparty risk channel

- Nominal wealth depends on the equilibrium price of the asset
  - collateral price channel of contagion
Payment Equilibrium

- Collection of nominal wealth \((m_1^*, \ldots, m_n^*)\) and asset price \(p^*\) such that:

  (i) nominal wealths are mutually consistent with pairwise contracts:

  \[
  m_j^* = e_j - \epsilon_j + h_j p^* + \sum_k c_{jk} \min\{p^*, y_{jk}\} - \sum_k c_{kj} \min\{p^*, y_{kj}\} - \sum_{k : m_k^* < 0} \zeta(c_{kj})[p^* - y_{kj}]^+
  \]

  (ii) asset market clears:

  \[
  \begin{cases}
  \sum_{i=1}^n \max\{m_i^*, 0\} = p^* \sum_{i=1}^n h_i & \text{if } p^* < s \\
  \sum_{i=1}^n \max\{m_i^*, 0\} \geq p^* \sum_{i=1}^n h_i & \text{if } p^* = s,
  \end{cases}
  \]

  where \(s\) is the asset’s payoff.
Comment 1: Monotone Comparative Statics?

- Paper discusses the interaction of fire sale and counterparty risk channels:

  - But this is an argument about “best responses” and not equilibrium.
  - Should not be hard to prove a formal result for equilibrium using monotone comparative statics arguments:

Proposition

\( p^*, (m_1^*, \ldots, m_n^*), \text{ and aggregate welfare, are decreasing in the shock } \epsilon_j. \)
Comment 1: Monotone Comparative Statics?

- Paper discusses the interaction of fire sale and counterparty risk channels:

  - But this is an argument about “best responses” and not equilibrium.
  - Should not be hard to prove a formal result for equilibrium using monotone comparative statics arguments:

**Proposition**

\[ p^*, (m_1^*, \ldots, m_n^*), \text{ and aggregate welfare, are decreasing in the shock } \epsilon_j. \]
Comment 1: Monotone Comparative Statics?

- Paper discusses the interaction of fire sale and counterparty risk channels:

- But this is an argument about “best responses” and not equilibrium.
- Should not be hard to prove a formal result for equilibrium using monotone comparative statics arguments:

Proposition

\[ p^*, (m_1^*, \ldots, m_n^*), \text{ and aggregate welfare, are decreasing in the shock } \epsilon_j. \]
Comment 2: Network Comparative Statics?

- There is a literature that studies the role of network structure for systemic risk, but mostly focused on unsecured lending.
- Current framework can be used to investigate the robustness of those results.

- Can one say anything about how changes in $C$ and $Y$ impact equilibrium objects?

- For example, suppose $y_{ij} \geq \tilde{y}_{ij}$ for all $i \neq j$. What can one say about $p^*$ and $\tilde{p}^*$?
- How about uniformly higher haircuts? More diversified patterns of lending?
Network Formation Stage

• Where do $C$ and $Y$ come from?
  Assume agents disagree about asset payoff $s \rightarrow$ gains from trade.

• Each agent takes pairwise interest rates and the price of the asset (today and tomorrow) as given and chooses the contracts:

$$\max \{ c_{jk}, c_{kj}, y_{jk}, y_{kj} \} \quad \max \left\{ m_j \frac{s}{p_1}, 0 \right\}$$

subject to

$$\sum_k c_{kj} = \sum_k c_{jk} + h_j.$$

• Interest rates and prices determined such that asset market and contract markets clear.
Network Equilibrium Results

(1) If there is a cost to recovering collateral, borrowers have an incentive to borrow from more than one lender
   → counterpart risk vs. leverage trade-off

(2) Yet, they do not internalize the full benefit of diversification on others
    → under-diversification externality.
Comment 3: Framing?

• Both results are novel, and yet, they have counterparts in the earlier literature and are well known (for non-secured lending).

• Would be great to use the powerful framework (and characterization) to obtain novel comparative statics and answer policy-relevant questions.

• Already some of the ingredients are there:
  ▶ impact of distress on lending volume, velocity of collateral
  ▶ CCP

• Would be great to push this further