Discussion of “The Macroeconomic Impact of Microeconomic Shocks”
David Baqaee and Emmanuel Farhi

Alireza Tahbaz-Salehi
Columbia Business School

LSE Workshop on Networks in Macro & Finance
June 2017
What is Hulten’s Theorem?

- In an efficient economy, the macro impact of a shock to industry $i$ depends on $i$’s sales as a share of aggregate output, up to a first-order approximation.

- **Corollary:** Firm size distribution is a sufficient statistic for how micro shocks shape macroeconomic outcomes.

- As long as one is concerned with macro outcomes, one can ignore
  - details of firm-to-firm linkages
  - complementarities in production
  - reallocation of primary factors across industries
What is Hulten’s Theorem?

- Though mathematically true, the result sounds somewhat unintuitive:
  - Shutting down electricity or the transportation system can have impacts above and beyond each industry's sales as a share of GDP.

- Turns out the theorem’s quantifiers actually matter!
- In an efficient economy, the macro impact of shocks to $i$ depends on $i$'s sales as a share of output, up to a first-order approximation.
What is Hulten’s Theorem?

- Though mathematically true, the result sounds somewhat unintuitive:
  - Shutting down electricity or the transportation system can have impacts above and beyond each industry's sales as a share of GDP.

- Turns out the theorem’s quantifiers actually matter!
- In an efficient economy, the macro impact of shocks to $i$ depends on $i$'s sales as a share of output, up to a first-order approximation.
Where Does Hulten’s Theorem Come from?

- Consider an economy in which the FWT holds:

\[ C(A_1, \ldots, A_n) = \max C(c_1, \ldots, c_n) \]

subject to

\[ y_i = A_i f_i(x_{i1}, \ldots, x_{in}, l_i, L_i) \]

\[ y_i = c_i + \sum_{j=1}^{n} x_{ji}, \quad \sum_{j=1}^{n} l_j = \bar{l}, \quad L_i = \bar{L}_i. \]

- By the envelope theorem:

\[ \frac{\partial C}{\partial A_i} = p_i f_i(x_{i1}, \ldots, x_{in}, l_i, L_i). \]

- Which leads to Hulten’s:

\[ \frac{\partial \log C}{\partial \log A_i} = \frac{p_i y_i}{C} := \lambda_i \quad \text{Domar weight of industry } i \]

- Natural (but very much ignored) question: how good is this approximation?
Where Does Hulten’s Theorem Come from?

- Consider an economy in which the FWT holds:

\[
C(A_1, \ldots, A_n) = \max \ C(c_1, \ldots, c_n)
\]

s.t. \( y_i = A_i f_i(x_{i1}, \ldots, x_{in}, l_i, L_i) \)

\[
y_i = c_i + \sum_{j=1}^{n} x_{ji}, \quad \sum_{j=1}^{n} l_j = \bar{l}, \quad L_i = \bar{L}_i.
\]

- By the envelope theorem: \( \frac{\partial C}{\partial A_i} = p_i f_i(x_{i1}, \ldots, x_{in}, l_i, L_i) \).

- Which leads to Hulten’s:

\[
\frac{\partial \log C}{\partial \log A_i} = \frac{p_i y_i}{C} := \lambda_i \quad \text{Domar weight of industry } i
\]

- Natural (but very much ignored) question: how good is this approximation?
Where Does Hulten’s Theorem Come from?

- Consider an economy in which the FWT holds:

\[ C(A_1, \ldots, A_n) = \max C(c_1, \ldots, c_n) \]

\[ \text{subject to } \]

\[ y_i = A_i f_i(x_{i1}, \ldots, x_{in}, l_i, L_i) \]

\[ y_i = c_i + \sum_{j=1}^{n} x_{ji}, \quad \sum_{j=1}^{n} l_j = \bar{l}, \quad L_i = \bar{L}_i. \]

- By the envelope theorem:

\[ \frac{\partial C}{\partial A_i} = p_i f_i(x_{i1}, \ldots, x_{in}, l_i, L_i). \]

- Which leads to Hulten’s:

\[ \frac{\partial \log C}{\partial \log A_i} = \frac{p_i y_i}{C} := \lambda_i \quad \text{Domar weight of industry } i \]

- Natural (but very much ignored) question: how good is this approximation?
A Differential Identity

- For any function $C(A_1, \ldots, A_n)$, let,

$$\nabla C = \sum_{i=1}^{n} \frac{\partial \log C}{\partial \log A_i}$$

and define the elasticities

$$1/\rho_{ij} = -\frac{\partial \log (C_i/C_j)}{\partial \log A_i}.$$

- Differential identity:

$$\frac{\partial^2 \log C}{(\partial \log A_i)^2} = \frac{\partial \log C}{\partial \log A_i} \left( \frac{1}{\nabla C} \sum_{j \neq i} (1 - 1/\rho_{ij}) \frac{\partial \log C}{\partial \log A_j} + \frac{\partial \log \nabla C}{\partial \log A_i} \right).$$
Beyond Hulten’s Theorem

• As a result of Hulten’s, these mechanical objects are economically meaningful in an efficient economy.

• Input-output multiplier:

\[ \xi := \nabla C = \sum_{i=1}^{n} \frac{\partial \log C}{\partial \log A_i} = \frac{\text{Gross Output}}{\text{GDP}} \]

• Elasticities:

\[ 1 - \frac{1}{\rho_{ij}} = \frac{\partial \log (\lambda_i / \lambda_j)}{\partial \log A_i}. \]

• Hence,

\[ \frac{\partial^2 \log C}{(\partial \log A_i)^2} = \frac{\lambda_i}{\xi} \sum_{j \neq i} \lambda_j \frac{\partial \log (\lambda_i / \lambda_j)}{\partial \log A_i} + \lambda_i \frac{\partial \log \xi}{\partial \log A_i} \]
Beyond Hulten’s Theorem

- As a result of Hulten’s, these mechanical objects are economically meaningful in an efficient economy.

- Input-output multiplier:

  \[ \xi := \nabla C = \sum_{i=1}^{n} \frac{\partial \log C}{\partial \log A_i} = \frac{\text{Gross Output}}{\text{GDP}} \]

- Elasticities:

  \[ 1 - \frac{1}{\rho_{ij}} = \frac{\partial \log (\lambda_i/\lambda_j)}{\partial \log A_i}. \]

- Hence,

  \[ \frac{\partial^2 \log C}{(\partial \log A_i)^2} = \frac{\lambda_i}{\xi} \sum_{j \neq i} \lambda_j \frac{\partial \log (\lambda_i/\lambda_j)}{\partial \log A_i} + \lambda_i \frac{\partial \log \xi}{\partial \log A_i}. \]
Beyond Hulten’s Theorem

• As a result of Hulten’s, these mechanical objects are economically meaningful in an efficient economy.

• Input-output multiplier:

\[ \xi := \nabla C = \sum_{i=1}^{n} \frac{\partial \log C}{\partial \log A_i} = \text{Gross Output} \frac{\text{GDP}}{\text{GDP}} \]

• Elasticities:

\[ 1 - 1/\rho_{ij} = \frac{\partial \log(\lambda_i/\lambda_j)}{\partial \log A_i} \]

• Hence,

\[ \frac{\partial^2 \log C}{(\partial \log A_i)^2} = \frac{\lambda_i}{\xi} \sum_{j \neq i} \lambda_j \frac{\partial \log(\lambda_i/\lambda_j)}{\partial \log A_i} + \lambda_i \frac{\partial \log \xi}{\partial \log A_i} \]
Second-order approximation:

\[
\frac{\partial^2 \log C}{(\partial \log A_i)^2} = \frac{\lambda_i}{\xi} \sum_{j \neq i} \lambda_j \frac{\partial \log (\lambda_i / \lambda_j)}{\partial \log A_i} + \lambda_i \frac{\partial \log \xi}{\partial \log A_i}
\]

Key observations:

1. When firm-level shocks are not small, the domar weights may no longer be sufficient statistics for measuring the macro impact of the micro shocks.
2. Second-order macro effects depend on first-order “micro effects”.
First-Order Micro Effects in a Structural Model

• Suppose all firms have Cobb-Douglas production technologies, whereas the representative consumer has a CES utility:

\[ u(c_1, \ldots, c_n) = \left( \sum_{j=1}^{n} \beta_j c_j^{\frac{\sigma}{\sigma-1}} \right)^{\frac{\sigma}{\sigma-1}} \]

• Input-output matrix: \( A_{ij} = p_i x_{ij} / p_i y_i \).

• Leontief inverse: \( \mathcal{L} = (I - A)^{-1} \).

• First-order micro effect:

\[ \frac{\partial \lambda_j}{\partial \log A_i} = (\sigma - 1) \left( \sum_{k=1}^{n} \beta_k \ell_{ki} \ell_{kj} - \left( \sum_{k=1}^{n} \beta_k \ell_{ki} \right) \left( \sum_{k=1}^{n} \beta_k \ell_{kj} \right) \right) \]
First-Order Micro Effects in a Structural Model

• Suppose all firms have Cobb-Douglas production technologies, whereas the representative consumer has a CES utility:

\[
  u(c_1, \ldots, c_n) = \left( \sum_{j=1}^{n} \beta_j c_j^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}
\]

• Input-output matrix: \( A_{ij} = p_i x_{ij} / p_i y_i \).

• Leontief inverse: \( L = (I - A)^{-1} \).

• First-order micro effect:

\[
  \frac{\partial \lambda_j}{\partial \log A_i} = (\sigma - 1) \left( \sum_{k=1}^{n} \beta_k \ell_k i_{\ell k} - \left( \sum_{k=1}^{n} \beta_k \ell_k i \right) \left( \sum_{k=1}^{n} \beta_k \ell_k j \right) \right)
\]
Second-Order Macro Effects

- Second-order macro effects are *identical to* first-order micro effects.

\[
\frac{\partial^2 \log C}{\partial (\log A_i)^2} = \frac{\partial \lambda_i}{\partial \log A_i} = (\sigma - 1) \left( \sum_{k=1}^{n} \beta_k \ell_{ki}^2 - \left( \sum_{k=1}^{n} \beta_k \ell_{ki} \right)^2 \right)
\]

- The second-order effects depend on the dispersion of how various goods rely on firm \(i\) as a (direct or indirect) supplier: a higher dispersion means a larger second-order term.

- Intuition: Substitutability can only matter when there is differential exposure to the shock.
Second-Order Macro Effects

- Second-order macro effects are *identical to* first-order micro effects.

\[
\frac{\partial^2 \log C}{\partial (\log A_i)^2} = \frac{\partial \lambda_i}{\partial \log A_i} = (\sigma - 1) \left( \sum_{k=1}^{n} \beta_k \ell_{ki}^2 - \left( \sum_{k=1}^{n} \beta_k \ell_{ki} \right)^2 \right)
\]

- The second-order effects depend on the dispersion of how various goods rely on firm \(i\) as a (direct or indirect) supplier: a higher dispersion means a larger second-order term.

- Intuition: Substitutability can only matter when there is differential exposure to the shock.
Operationalizing the Characterization?

- Hulten, even though imprecise, provides a result in terms of quantities that can be measured.

- Is there an equivalent for the second-order effects?

- Or does one have to rely on a structural model?
The paper mostly concerned with the limitations of relying on Hulten's and makes a convincing case by focusing on the second-order terms.

But the same criticism applies to the second-order approximation as well, at least quantitatively (even if one thinks higher-order terms are not structurally meaningful).

In the presence of large shocks, no guarantee that second-order terms are what matter.

Two alternative take-aways:

1. Non-linearities are important and one has to rely on the full non-linear model (as is done in the paper’s quantitative section)
2. The second-order approximation ($\zeta$ & $\rho_{ij}$) is in and of itself useful.
• **Possible solution**: Upper bound on the size of the approximation error as a function of the shocks and structural elasticities using Taylor’s Theorem.

• Clearly, the approximation error is highly network and elasticity dependent. But even rough bounds (say, based on the smallest/largest elasticities) would be useful.

• Not a common practice in the literature! But the paper makes a convincing case that it should be.
A User’s Manual?

- **Possible solution**: Upper bound on the size of the approximation error as a function of the shocks and structural elasticities using Taylor’s Theorem.

- Clearly, the approximation error is highly network and elasticity dependent. But even rough bounds (say, based on the smallest/largest elasticities) would be useful.

- Not a common practice in the literature! But the paper makes a convincing case that it should be.
Summary

- Important contribution, clarifying the role of non-linearities, input-output linkages, and reallocation of factors in translating micro shocks to macro outcomes.

- Clarified a disconnect in my understanding: how come first-order micro effects depend on the elasticities but not the macro effects?

- Would be nice to have a thorough discussion of how the characterizations can be operationalized empirically/quantitatively.