Discussion of
“Centrality-based Capital Allocations and Bailout Funds”
Alter, Craig, and Raupach (2014)

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Financial Networks

- Much recent interest in the relationship between systemic risk and network effects, mainly a consequence of the Financial Crisis.

- Large body of recent, theoretical works
  - direct contractual linkages
  - fire-sales and pecuniary externalities
  - etc.

- Sure, the models are theoretically nice, but are they relevant?

- Almost no quantitative/empirical analysis.
This Paper

- Uses a very rich dataset to fill this void
  Extracts the interbank network of the German banking system

- A fairly elaborate model of banks’ balance sheets, exogenous shocks, and the contagion mechanism

- Main Question: whether policies that rely on network statistics can improve stability of the system as a whole.

- Simulate the extent of contagion under different policies
  - Tax interconnectivity through capital charges (*ex ante* policy)
    (what I will focus on)
  - A bailout fund based on network statistics (*ex post* policy)
Theoretical Framework (Simplified)

- $n$ institutions/banks
- Banks have outside and interbank liabilities and assets.
- The financial network captures the extent of interbank liabilities.

- Each bank holds some capital $k_i$.
- Depositors and other creditors are senior to interbank creditors ($v_i$).
- Banks are subject to exogenous shocks to their assets $\epsilon_i^{real}$. 

$y_{ij}$ 

$i \rightarrow j$
Contagion Mechanism

- If a bank defaults, its creditors get paid on a *pro rata* basis.
- The total out-payment of bank $i$ is equal to

\[
x_i = \max \left\{ \min \left\{ \sum_{j=1}^{n} \pi_{ij}x_j + k_i - \nu_i - \epsilon_i^{\text{real}}, \sum_{j=1}^{n} y_{ji} \right\}, 0 \right\}.
\]

where $\pi_{ij} = y_{ij} / \sum_{r=1}^{n} y_{rj}$.

- **Solution Concept**: the vector of interbank payments $x^* = (x_1^*, \ldots, x_n^*)$ that solves the above system of equations.

- Alternatively, if $L_i^{\text{IB}} = \sum_{j=1}^{n} y_{ji} - x_i$:

\[
L_i^{\text{IB}} = \min \left\{ \max \left\{ \sum_{j=1}^{n} \pi_{ij}L_j^{\text{IB}} + \nu_i + \epsilon_i^{\text{real}} - k_i, 0 \right\}, \sum_{j=1}^{n} y_{ji} \right\}
\]
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Contagion Mechanism

- Shocks propagate over the interbank linkages.  
  (a variant of the model of Eisenberg and Noe, 2001)

- If a bank defaults, there is a bankruptcy costs of $C_i$ proportional to the bank’s size.

- Total deadweight social loss:
  \[
  L_{agg} = \sum_{i=1}^{n} C_i \cdot 1\{i \text{ defaults}\}
  \]

- Set capital requirements to minimize $\mathbb{E}[L_{agg}]$.  

Network Centralities

- Set capital requirements based not only on banks’ individual riskiness (VaR), but also some widely used network statistics of the banks.

- **Centrality** of bank $i$: $c_i : \mathbb{R}^{n \times n} \rightarrow \mathbb{R}_+$

- **Examples:**
  - the size of $i$’s total interbank assets
    $$c_i = \sum_{j=1}^{n} y_{ij}$$
  - eigenvector centrality: $c$ is the eigenvector of the liabilities matrix
    $$c_i = \sum_{j=1}^{n} y_{ji}c_j.$$
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\[
c_i = \sum_{j=1}^{n} y_{ji}c_j.
\]
Set capital requirements based on centralities and a no-network benchmark:

\[ k_i = \beta k_i^{VaR} + (1 - \beta) \left( \frac{c_i}{\sum_{j=1}^{n} c_j} \right) \sum_{j=1}^{n} k_j^{VaR} \]

Choose \( \beta \) such that \( L_{agg} \) is minimized.

The rationale being that shocks to more “central” banks would propagate more.
Main Results: Capital Requirements

- Setting capital requirements based on total assets leads to the most improvement.
- Aside from the size of total assets, capital requirements based on Opsahl centrality provide the best performance.
First Observation: capital rules based on size outperform all other centrality measures.

However, size is hardwired into the performance measure:

- Larger banks are assumed to have a higher bankruptcy cost.
- Not surprising that the capital allocations best on size dominate all other metrics.
Comment: What is the “Right” Network Statistic?

- The paper relies on a specific structural model of interbank contagion.

- So why not rely on the “network statistic” that is implied by the structural model?

- In fact, off-the-shelf measures can be misleading in identifying systemically important financial institutions.
A Simple Linear Economy

- Suppose that spillovers are linear:
  \[ L_{iB}^I = \sum_{j=1}^{n} \pi_{ij} L_{jB}^I + \epsilon_i^{real}. \]

- Total losses:
  \[ L_{agg}^{IB} = \sum_{i=1}^{n} L_{iB}^I. \]

- In such an economy:
  \[ \frac{dL_{agg}}{d\epsilon_j} = \text{eigenvector centrality of bank } j \]
Eigenvector centrality is the correct notion for systemic importance of a financial institution if interactions/spillovers are linear. (or at least, when the interactions are smooth, so that can be linearly approximated).

This may not generalize to an economy with non-smooth interactions:
  - debt contracts
  - bankruptcy costs
  - etc.
Non-Smooth Interactions: Debt Contracts

- Consider an economy in which all banks are of equal sizes: (identical interbank assets and liabilities)
  \[
  \sum_{j=1}^{n} y_{ij} = \sum_{j=1}^{n} y_{ji} = y.
  \]

- All banks have identical eigenvalue centralities.

- Also, suppose that banks are linked via standard debt contracts (as in the current paper):
  \[
  L_{i}^{IB} = \min \left\{ y, \max \left\{ 0, \sum_{j=1}^{n} \pi_{ij} L_{j}^{IB} + \epsilon_{i}^{real} - k \right\} \right\}
  \]

- Total losses:
  \[
  L_{agg} = C \cdot \sum_{i=1}^{n} 1\{i \text{ defaults}\}
  \]
Network Statistics

- **Harmonic distance** of bank $i$ from bank $j$

\[ m_{ij} = 1 + \sum_{k \neq j} \pi_{ik} m_{kj}. \]

**Proposition (Acemoglu et al., 2014)**

Suppose bank $j$ is hit with a large enough shock. Then, bank $i$ defaults if and only if

\[ m_{ij} \leq m^* \]

for some $m^*$.

- Despite the fact that all banks have the same eigenvector centrality, weighted out-degree, weighted in-degree, Bonacich centrality, ...
Implication: A bank is systemically more important the shorter the harmonic distances of other banks to it are.

Intuition: with linear interactions, positive and negative shocks propagate symmetrically, but not if the interactions are not smooth.

More importantly: the “correct” notion of centrality should come from the structural model of network interactions.
Comment: Equilibrium Multiplicity

- Introducing bankruptcy costs:
  suppose that there is a drop in the value of a bank’s assets if it defaults:
  \[ L_i^{IB} = \min\left\{ y, \max\left\{ 0, \sum_{j=1}^{n} \pi_{ij} L_j^{IB} + \epsilon_i^{real} + C \cdot 1 \{ i \text{ defaults} \} - k \right\} \right\} \]

- Because of the discontinuity, the economy may have multiple equilibria.
- The paper focuses on the equilibrium with the minimum losses.
The set of systemically important banks may depend on the equilibrium selected.

In the “best” eq. banks on the left are more systemically important.

In the “worst” eq. banks on the right are more systemically important.

Optimal capital requirements may be sensitive to the equilibrium selected.
Summary

- Very important and relevant work:
  - Many theoretical studies, but almost no quantitative analyses of the mechanisms studied in the literature
  - The paper fills an important void.

- Main comment: the proper network statistic/moment should be a consequence of the structural model of interbank spillovers.
- Off-the-shelf network statistics can be misleading.
- Equilibrium multiplicity would make the picture more complicated.