Does Merger Simulation Work? A "Natural Experiment" in the Swedish Analgesics Market

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March 2012

Abstract

We exploit a natural experiment associated with a large merger in the Swedish market for analysics (painkillers). We confront the predictions from a merger simulation study, as conducted during the investigation, with the actual merger effects over a two-year comparison window. The merger simulation model is based on a constant expenditures specification for the aggregate nested logit model (as an alternative to the typical unit demand specification). The model predicts a large price increase of 34% by the merging firms, because there is strong market segmentation and the merging firms are the only competitors in the largest segment. The actual price increase after the merger is of a similar order of magnitude, but in fact even larger: +42% in absolute terms and +35% relative to the "control group" of non-merging rivals. These findings at first sight suggest strong support for merger simulation and structural models more generally. But a closer look at a wider range of merger predictions leads to more nuanced conclusions. First, both merging firms raised their prices by a similar percentage, while the simulation model predicted a larger price increase for the smaller firm. Second, the merging firms' market shares dropped, as predicted by the model, but some of the outsider firms' market shares also dropped (in favor of other firms). We discuss the possible reasons for the divergence between the predicted and actual

Keywords: merger simulation, ex post merger evaluation, constant expenditures nested logit, analgesics or painkillers

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1 Introduction

There is an ongoing debate on the usefulness of structural econometric models to predict counterfactual outcomes. Angrist and Pischke (2010) document the recent successes of "design-based" or "treatment effects" approaches in various fields, such as labor and development economics. They suggest that industrial organization would also greatly benefit from a more intense focus on "natural experiments", taking empirical merger analysis as a test case example. At a minimum, they write, empirical evidence should be provided that structural econometric models can deliver reasonably accurate predictions. In a response, Nevo and Whinston (2010) acknowledge that the treatment effects approach may be useful to estimate the effects from mergers. But they also point out limitations, and discuss several circumstances where a structural model and merger simulation can be more useful. The most obvious instance arises when a competition authority has to evaluate the likely price effects of a proposed merger, and does not have information from closely comparable past mergers in the same or related markets. Both Angrist-Pischke and Nevo-Whinston agree that more retrospective merger analysis is clearly needed.

In this paper we provide such an analysis based on a large recent merger between AstraZeneca Tica (AZT) and GlaxoSmithKline (GSK) in the Swedish market for over-thecounter analysics (painkillers). The merger raised competition concerns, since AZT and GSK were the only companies in the largest market segment, which is based on the active substance paracetamol (called acetaminophen in the U.S.). During the investigation, we conducted a merger simulation study for the Swedish competition authority. We allowed for various possible scenario's. Our preferred model was a constant expenditures specification for the nested logit model; this is a new variant where price enters logarithmically instead of linearly as in the typical unit demand specification. The model predicted a substantial price increase in the paracetamol segment in the absence of efficiencies and new entry: +34% under Bertrand competition and +28% under partial coordination (before and after the merger). The competition authority nevertheless decided to clear the merger in April 2009. First, it still expected sufficient competition from the other two main segments (and it referred to our predictions which did not rule out negligible price effects in the absence of sufficiently large efficiencies). Second, and probably more importantly, it was optimistic that the coming deregulation of the pharmacy monopoly would encourage new entry and competition.

A few years after the merger we are able to perform an ex post merger analysis. We confront the predicted price effects, using the simulation methodology as developed during the investigation, with the actual price effects under a two-year comparison window. We obtain striking findings. The merging firms' actual price increase is of a similar order of

magnitude, but in fact even larger than the price increase predicted by the model: +42% in absolute terms, or +35% in a difference-in-difference interpretation where the other firms are the control group. This price increase materialized almost immediately, just one month after the merger, and remained for the entire two-year window after the merger.

At first sight, these results provide strong support for the merger simulation approach in competition policy, and for the usefulness of structural models more generally. However, more nuanced conclusions are warranted after examining a wider range of merger predictions (which we had not yet presented during the investigation although they follow straightforwardly from the merger simulation model). First, our model predicts that the smaller firm in the merger, GSK, would raise its prices by much more than the larger firm, AZT, while in reality the two companies raised their prices by approximately the same percentage. Second, although our model predicts the market share drop of the merging firms fairly well, it did not predict a market share drop by one of the outsiders (in favor of other outsiders). We discuss possible reasons for the divergence between the predicted and actual effects, i.e. the possibility that other things did not remain constant after the merger or that the model specification can be improved. It was possible to test these richer merger predictions, thanks to the unusually large size of the considered merger (where the two merging firms are the only competitors in a segment with limited substitution from other segments).

Our paper contributes to three related strands in the literature: merger simulation, ex post merger evaluation and ex post evaluation of merger simulation.

Merger simulation Merger simulation as a tool for competition policy was introduced by Hausman, Leonard and Zona (1994) and Werden and Froeb (1994). Subsequent research has looked at a variety of issues, such as alternative demand models, e.g. Nevo (2000), Epstein and Rubinfeld (2001) or Ivaldi and Verboven (2005). Some of this work has explicitly compared different demand models and showed how different functional forms may result in rather different price predictions, see Crooke, Froeb, Tschantz and Werden (2003), Huang, Rojas and Bass (2008) and Slade (2009). While these comparisons are informative, it is difficult to disentangle the sources of the differences since the compared models differ in many respects. In contrast, we compare different specifications in a unified demand framework, the nested logit model. As an alternative to the typical unit demand model, we propose the constant expenditures demand model. This enables us to concentrate on the role of the functional form of the price variable, while abstracting from other sources of specification differences (such as more flexible substitution patters for the cross-price elasticities).

Quite surprisingly, the constant expenditures nested logit model has not been used before in empirical work, although it is equally tractable as the unit demand model. We show that only three modifications of the typical estimating equation are required: (i) price enters logarithmically instead of linearly, (ii) market shares are expressed in values instead of units, and (iii) the potential market size refers to the potential aggregate expenditures (in values) instead of the potential number of consumers or households. Apart from the additional flexibility from a new functional form for the price variable, the constant expenditures specification had a particular feature that also be relevant in other applications: the pattern of price elasticities across models is quasi-independent of price, instead of quasi-linearly increase in price as in logit, nested logit and random coefficients logit models with unit demand.

Our simulation model also provides greater flexibility on the supply side. We do not only allow for a standard multi-product Bertrand Nash model. We also allow for the possibility that firms partially coordinate, already before the merger. We introduce a partial coordination parameter, the weight that firms give on their competitors' profits when setting prices. This enables one to better calibrate the premerger marginal costs if reliable outside information on cost is available.

Ex post merger evaluation Ex post merger analysis moved in parallel with merger simulation, and mainly aimed to evaluate the relevance or effectiveness of competition policy towards mergers. Early work focused on mergers in major industries, such as airline markets (Borenstein, 1990; Kim and Singal, 1993), banking (Facacelli and Panetta, 2003) and petroleum (Hastings, 2004; Gilbert and Hastings, 2005; Hosken, Silvia and Taylor, 2011). Most recently, Ashenfelter and Hosken (2008) take advantage of scanner data to assess mergers in five different branded goods industries. They find moderate but significant price effects in the range of 3–7%. Among other things, they argue that their estimates may be viewed as a lower bound on price increases that would have occurred for other mergers that were blocked.

Ex post evaluation of merger simulation There is only a small recent literature that combines both traditions to compare the predictions from mergers simulations with the actual merger effects. Peters (2006) looks at the simulated and actual price increases by the merging firms' in several airline mergers. Weinberg and Hosken (2009) and Weinberg (2011) look at the price increases of both the merging firms and their competing rivals. These papers find mixed support on the performance of merger simulation: the qualitative predictions are in line with the data, but the quantitative predictions show some divergence. Relative to this interesting early work, we make three related contributions. First, we evaluate the performance of merger simulations based on a merger simulation framework that had already been specified during the investigation, i.e. entirely before the merger had been

consummated. Second, we consider a large merger in a concentrated market. This results in large price predictions, which enables us to make quite sharp comparisons, even if other things have changed after the merger. Third, we consider more demanding tests for the merger simulation methodology, since we assess a broader set of merger predictions: we do not only consider the price predictions for each of the merging firms and their competitors, but also the predictions regarding the firms' market shares. More broadly speaking, this large "natural experiment" is therefore not only of interest to evaluate the performance of merger simulations, but also to draw lessons for the relevance of policy counterfactuals in a variety of other oligopoly settings with differentiated products (such as environmental policies, trade policies, taxation, etc).

The paper is organized as follows. Section 2 discusses the industry background, including the merger decision and the dataset. Section 3 develops the framework for merger simulation, as developed during the investigation. Section 4 discusses the empirical results for the demand model and merger simulations, as presented during the case. Section 5 provides the expost analysis. We first present additional predictions from the merger simulations, not presented during the case but based on the same methodology. Next we confront these predictions with what actually happened in terms of prices and market shares of the merging firms and their competitors.

2 Industry background

In April 2009, the Swedish competition authority cleared the acquisition of AstraZeneca Tika (AZT) by GlaxoSmithKline (GSK). In this section we provide the relevant industry background to introduce the simulation study we conducted for the competition authority during the investigation, and to motivate our ex post analysis carried out several years later. First, we review the market for over-the-counter analgesics or painkillers. Next, we describe the merger between the two companies, GSK and AZT. Finally, we elaborate on the datasets used for the investigation and post-merger analysis.

2.1 The market for OTC painkillers

Over-the-counter analysics or painkillers are non-prescription drugs to treat pain and fever. Painkillers come in three main active substances: paracetamol (called acetaminophen in the U.S.), ibuprofen and acetylsalicylic acid (ASA or aspirin). There are also two less important active substances: diclofenak and naproxen. The active substances may differ in the types of pains they relieve and in their side effects. Paracetamol treats most pains and fevers, and

is known for having little side effects (except that it may damage the liver). Ibuprofen also treats most pains and fevers and is often used to reduce inflammations, but it may have side effects on the stomach. The ASA substance also has a blood-diluting effect, which has both advantages and disadvantages. While each active substance may therefore relieve pain and reduce fever in different ways and with different side effects, consumer perceptions on the companies' brands may also be important. This is evident from the large amount of advertising in the sector. So it is ultimately an empirical question to which extent brands with different active substances are substitutes.

Painkillers also come in various administrative forms. Tablets are the most important form, followed by fizzy tablets. There are also some other forms (such as liquid, suppository and powder), but these are much less important. Table 1 shows the market shares of the three main substances and the two main administrative forms, according to the total value of sales in 2008.¹ With a market share of 42%, paracetamol is by far the most important substance. Ibuprofen and ASA each have a comparable market share of 29%. Paracetamol and Ibuprofen are mainly sold as tablets, whereas ASA is dominantly sold as fizzy tablets.

Table 1: Market shares in 2008, by form and active substance

Form	Paracetamol	Ibuprofen	ASA	Total
Tablet	36.1	29.0	2.6	67.7
Fizzy tablet	6.0		26.3	32.3
Total	42.1	29.0	28.9	100

Note: This table shows the market shares of the main administrative forms and active substances, according to the total value of sales in 2008. Paracetamol is known as acetaminophen in the U.S.

All companies specialize in one or at most two active substances. They typically sell one main brand per segment, and sometimes an additional smaller brand. Table 2 shows the 2008 market shares of the companies and their brands, broken down by active substance. This shows that the two merging companies AZT and GSK are the only companies in the paracetamol segment: AZT sells Alvedon as its main brand and Reliv as a smaller brand, whereas GSK sells the popular brand Panodil . McNeil (selling Ipren) and Nycomed (selling Ibumetin) are the main companies in the Ibuprofen segment. McNeil (selling Treo) is by far

¹Taken together, these three substances and two forms account for 90% of the market.

the largest company in the ASA segment. There are two other companies with much smaller market shares: Meda and Bayer.

Until the deregulation of 2009, the companies distributed all their drugs through the state-owned pharmacy monopoly, Apoteket AB. In 2008 Apoteket operated 850 community pharmacies, 76 hospital pharmacies and 30 shops for over-the-counter and health care services. The pharmaceutical companies determined the wholesale prices, but indirectly also the retail prices, since Apoteket applied a fixed percentage markup on the wholesale prices. After a market investigation, the Swedish government decided to deregulate the distribution of pharmaceutical products in 2009. Several state pharmacies were sold to private companies, and non-pharmacy retail outlets became entitled to sell non-prescription drugs. The reforms also gave more freedom to the pharmacies in various respects. For example, there were no longer obligations to sell all available products in a non-discriminatory fashion, and it became possible to set different retail prices across the country. The government expected that the deregulation of the distribution system would increase competition and encourage entry of new products.

Table 2: Market shares in 2008, by brand and active substance

Firm	Brand	Paracet.	Ibupr.	ASA	Total
AZT	Alvedon	29.3			31.5
	Reliv	2.2			
GSK	Panodil	10.6			10.6
McNeil	Ipren		19.1		44.7
	Treo			22.5	
	Magnecyl			3.1	
Nycomed	Ibumetin		9.2		9.2
Meda (Ellem)	Alindrin		0.7		3.4
	Bamyl			2.7	
Bayer	Aspirin			0.4	0.6
	Alka-selzer			0.0	
	Albyl			0.2	
Total		42.1	29.0	28.9	100

Note: This table shows the market shares of the main firms and brands and active substances, according to the total value of sales in 2008. Paracetamol is known as acetaminophen in the U.S.

2.2 The merger

GSK notified its planned acquisition of AZT on December 22, 2008. Although the merging firms were the only competitors in the paracetamol segment, the Swedish competition authority formally cleared the merger on April 3, 2009.² The competition authority justified its Decision on the grounds that consumers base their decisions more on the brand than on the active substance. Furthermore, and probably more importantly, the competition authority stated that it expected increased competition because of the coming deregulation of the state-owned pharmacy monopoly. This view is well summarized in the competition authority's 2009 Annual Report:³

"GSK and AZT were the only companies providing over-the-counter (OTC) pharmaceuticals on the Swedish market that included the active substance "paracetamol", i.e. Alvedon, Reliv and Panodil. Much of the work associated with the investigation involved assessing the potential effects of the pending deregulation of the pharmacy market. Deregulation would mean that players other than Apoteket would be able to provide OTC pharmaceuticals and at the same time pharmaceutical companies would no longer be able to determine prices for customers. Deregulation would also enable new pharmaceutical stakeholders to enter the Swedish self-care market with their brands; for example including the paracetamol substance. In this way, the buying power of pharmacies and retailers would improve, which could possibly result in improved price competition between the different products available in the self-care market. After conducting a special investigation, the Swedish Competition Authority found that GSK's acquisition of AZT would not manifestly impede effective competition and no action was taken regarding this concentration."

In its Decision, the competition authority described that it based its analysis on a large number of contacts in the industry. It also made a brief reference to the merger simulation study we had conducted for the competition authority during the investigation. It wrote that the simulation study showed that mergers would not lead to significant price increases. As we discuss in detail below, our simulation study covered a wide range of scenario's, with and without efficiencies, and with and without partial coordination between companies. Our simulations only predicted insignificant price increases in one scenario with large efficiencies.

The justification of the Decision was very short, see p. 5-6 on http://www.kkv.se/upload/Filer/Konkurrens/2009/Beslut/beslut_08_0706_2008.pdf (in Swedish).

³See http://www.kkv.se/t/Page____5925.aspx.

Hence, the competition authority's reference to our simulation results may suggest it implicitly had in mind large efficiencies. An alternative possibility is that the competition authority put a large weight on the coming deregulation of the pharmacy monopoly and considered this sufficiently promising to create new competition and compensate for the increase in market power without deregulation.

2.3 Datasets for merger simulation and ex post analysis

During its investigation, the competition authority obtained a rich dataset on the painkiller market from Apoteket AB. The dataset contains monthly aggregate sales information for Sweden during the period 1995-2008 at the level of the product. A product is defined as a brand, form, packsize and dose. For example, one of AZT's products is Alvedon tablet, 30 pieces, 500 mg/piece. An observation for product j in month t contains information on the price, p_{jt} , the total sales volume, q_{jt} , and the total sales value or revenue, $r_{jt} = p_{jt}q_{jt}$. The dataset was combined with two other datasets, one on marketing expenditures by brand and month (collected by Sifo RM), and one on macro-economic variables (from statistics Sweden), such as nominal and real GDP, the number of sick men and sick women (all monthly) and total population of men and women (yearly).

The competition authority collected the dataset for a general descriptive analysis, but in particular to enable us to conduct the simulation study during the investigation. The total number of observations (products/months) during 1995–2008 is 11,185, which amounts to an average of 67 available products per month. The number of observations for the three main active substances (paracetamol, ibuprofen and ASA) and the two main administrative forms (tablets and fizzy tablets) is 7,240. This amounts to an average of 43 products per month. We conducted our analysis on both the full dataset and on the reduced dataset, and obtained robust conclusions. In the remainder of this paper, we focus the discussion on the reduced dataset, which covers about 90% of the total value of sales.

Table 3 presents summary statistics of the main variables over the period 1995-2008. Total sales value r_{jt} refers to Apoteket's total sales per product/month across all its pharmacies in Sweden, expressed in 1 million Swedish Krone (SEK), including VAT. Total sales volume q_{jt} refers to the number of units sold per product/month across the country. Price p_{jt} is the average selling price, including VAT (i.e. r_{jt}/q_{jt}). This coincides with the transaction price paid by every consumer, since Apoteket is required to set uniform prices across all its pharmacies in Sweden.

There is no unambiguous measure for the unit of consumption in the market for painkillers, and hence no obvious measure for the sales volume q_{jt} and the price p_{jt} of each product.

Table 3: Summary statistics for the Swedish market for analgesics, 1995-2008

Variable	Mean	St. Dev.	Min.	Max.
revenue $(r_{jt} = p_{jt}q_{jt})$	1.24	2.56	.00	22.95
price per tablet (p_{jt})	1.06	.46	.27	2.55
price per defined daily dose (p_{jt})	6.02	2.21	1.74	15.50
price per normal $dose(p_{jt})$	1.61	.60	.43	3.88
number of tablets (q_{jt})	1.11	2.19	.00	16.61
number of defined daily doses (q_{jt})	.21	.43	.00	3.07
number of normal doses (q_{jt})	.77	1.57	.00	11.08
marketing	564.1	1445.7	0	13536
sickwomen	822.9	197.0	391	1204
sickmen	524.5	108.0	254	763
GDPnom (in billions)	621.6	107.4	443.2	859.7
popwomen (in thousands)	4524.2	54.8	4471.4	4652.6
popmen (in thousands)	4437.4	72.5	4366.1	4603.7
	•	•		•

Note: 7240 observations (products, years, months). Sales value or revenue (r_{jt}) is in 1 million SEK, price per unit (p_{jt}) is in SEK, sales volume (q_{jt}) is in 1 million. $1 \in [10.8]$ SEK, 1 = [10.8] SEK in December 2008.

In particular, it is not appropriate to measure q_{jt} as the number of sold packages and p_{jt} as the price per sold package, since the products are sold in different packsizes (number of tablets) and in different doses (mg per tablet). We use three different measures for the unit of consumption. The first measure is the "tablet" (or fizzy tablet). The second measure is the defined daily dose, or "ddd", as defined by the World Health organization. The third measure is the "normal dose", i.e. the number of doses used on a normal single consumption occasion. We thus have three measures of price – price per tablet, price per ddd, and price per normal dose – and three corresponding measures of sales volume. Table 3 shows that the average price per tablet is 1.1SEK, the average price per normal dose is slightly higher, 1.6 SEK, and the average price per ddd is 6.0. More importantly, these measures do not just differ through a scale factor: for example, the ratio of the means to the standard deviations suggest there is more variation in price per normal dose than in price per tablet. During the investigation, we performed a sensitivity analysis using each of the three price and volume measures. We will focus the discussion here on the results from the first measure (price per

tablet and number of sold tablets), but we will point out possible differences when using the other measures.

Two years after the competition authority's investigation, we collected an update of the main dataset from Apoteket AB (now maintained by XXX because of the deregulation). The updated dataset again contains monthly sales information (price, sales volume, and sales value), now for the period 2008 up to May 2011. We thus again collected the information for the year 2008: although we already had this information, this enabled us to verify whether the assembled data were consistent with our previously obtained data, and we verified this was the case. We also updated some of the macro-economic variables, i.e. nominal and real GDP. We no longer collected information on the other variables, since they were only used for estimating the demand model, and we did not aim to update this in our expost analysis.

3 Framework for merger simulation

We now present the framework for the merger simulation, as we developed it during the competition authority's investigation. We first motivate and discuss our adopted demand model, used to estimate the substitution patterns across products. We then present the model of oligopolistic price-setting behavior, used to uncover premerger marginal costs and to predict postmerger prices.

3.1 Unit demand versus constant expenditures nested logit

To conduct the merger simulation, we developed a two-level nested logit model for the demand for painkillers. The model incorporates consumer heterogeneity along two discrete product dimensions: the products' active substance (paracetamol, ibuprofen and ASA) and their administrative form (tablet and fizzy tablet). The nested logit model thus accounts for the possibility of market segmentation, by allowing cross-price elasticities to be greater between products that have the same active substance and/or administrative form. Segmentation according to the active substance may be particularly relevant for the proposed merger, since the merging companies are the only ones active in the paracetamol segment. Segmentation according to the administrative form may however also be relevant. Our market share tables in section 2 showed that two substances (paracetamol and ibuprofen) are mainly sold as tablets, while the other substance (ASA) is mainly sold as fizzy tablets. Hence, if form turns out to be a relevant source of segmentation, this will reveal that consumers mainly substitute from paracetamol to ibuprofen products (both dominantly sold as tablets) and less so to ASA products (sold mainly as fizzy tablets).

As shown by Berry (1994), the aggregate nested logit model can be transformed into a linear estimating equation and is therefore simple to estimate. But this comes at a cost, since the model only allows for consumer heterogeneity along discrete product dimensions and not for heterogeneity in the valuation of continuous characteristics. Accounting for such heterogeneity would require estimating Berry's (1994) and Berry, Levinsohn and Pakes' (1995) aggregate random coefficients logit model. Even though there have been many applications (in particular since Nevo's (2000) practitioner's guide), estimating a full random coefficient model remains considerably more complicated because of practical numerical difficulties, as recently documented by for example Knittel and Metaxoglou (2008). Since in our application the two discrete product dimensions, substance and form, appear important aspects behind consumer heterogeneity, we felt reasonably confident that the more tractable nested logit model would capture the pattern of cross-price elasticities fairly adequately.

We were however more concerned with the pattern of elasticities as induced by the typically adopted functional form for the price variable. Quite surprisingly, the aggregate discrete choice literature since Berry (1994) and Berry, Levinsohn and Pakes (1995) has adopted a utility specification where price enters linearly (or, more generally, enters additively with income). This specification has the property that consumers buy one unit of their preferred product. While this may be an appealing property for some commodities such as automobiles, it may be less realistic for many frequently purchased consumer items. More importantly, the linear price specification implies that the price elasticities of different products are quasi-linearly increasing in prices: if product A is twice as expensive as product B, it also tends to have a price elasticity that is twice as high. This property does not only hold in the logit and nested logit model, but also to some extent in the random coefficients logit model.

For example, in an interesting paper on the same industry, Chintagunta (2002) estimates a random coefficients logit model for five main (U.S.) painkiller brands.⁴ Although he finds significant consumer heterogeneity in the valuation of price, the estimated own-price elasticities show an increasing relationship with prices across products.⁵ This pattern is not unrealistic per se, but it does follow from the linear price specification, as also documented in other random coefficients logit models (e.g. Grigolon and Verboven, 2011). In our application, we were particularly concerned with the linear price specification because, unlike Chintagunta (2002), we have many brands and a large price variation across brands: as

⁴To our knowledge, there are no other papers estimating discrete choice models for painkillers at the brand level. Chevalier, Kashyap and Rossi (2001) estimate a log-log demand model at the category level, and obtain an estimated price elasticity for the painkiller category equal to -1.87.

⁵Tables 2 and 5 in Chintagunta (2002) show the following elasticities (average prices): Advil -2.996 (7.41), Tylenol, -2.69 (6.16), Motrin -2.66 (5.95), Bayer -2.25 (4.95), Store -1.81 (3.55). This pattern is present in other applications with random coefficients, e.g. in Berry, Levinsohn and Pakes (1995).

shown in Table 3, the highest and lowest price differ by a factor of five (only a factor of two in Chintagunta, 2002).

We therefore propose an alternative possible utility specification, where price (as well as income) enters logarithmically instead of linearly. In this specification consumers do not buy one unit of their preferred product (perfectly inelastic conditional demand), but rather a constant expenditure (unit elastic conditional demand). We will obtain an estimating equation that is equally simple as Berry's aggregate logit model, with three differences: price enters logarithmically instead of linearly, market shares are measured in values instead of volumes, and the potential market refers to the potential aggregate budget instead of the potential number of consumers. The implied own- and cross-price elasticities are quasiconstant in price, instead of quasi-linearly increasing in price as in the unit demand model. To our knowledge, no other work has departed from the unit demand model in discrete choice models with aggregate purchasing data. Hendel (1999) and Dubé (2004) used micro-level data to estimate multiple-discrete choice models, where purchasers can buy multiple units as well as multiple products.

Individual utility There are I consumers, $i = 1 \cdots I$. Each consumer chooses one out of J+1 differentiated products, $j = 0 \cdots J$; good 0 is the outside good or no-purchase alternative. Suppose consumer i has the following conditional indirect utility for good $j = 0 \cdots J$:

$$u_{ij} = x_j \beta + \xi_j + \alpha f(y_i, p_j) + \varepsilon_{ij}, \tag{1}$$

where x_j is a vector of observed product characteristics of product j, p_j is price, ξ_j captures unobserved product characteristics, y_i is income of individual i and ε_{ij} is a random utility term or an individual-specific taste parameter for good j. In a typical specification, $f(y_i, p_j) = f(y_i - p_j)$, so that consumers buy one unit of their preferred product. Specification (1) allows income and price to enter non-additively, so that consumers may buy multiple units of their preferred product.

We first consider an individual consumer's decision how many units to buy from her preferred product. Next, we derive her decision which product to buy, based on random utility maximization. Finally, we obtain the aggregate and inverted aggregate demand system for all products.

Conditional individual demand Conditional on buying product j, a consumer i's demand for product j, d_{ij} , follows from Roy's identity

$$d_{ij} = -\frac{\partial f(y_i, p_j)}{\partial p_j} / \frac{\partial f(y_i, p_j)}{\partial y_i}$$
.

Consider the following two specifications for $f(y_i, p_j)$:

Unit demand
$$f(y_i, p_j) = y_i - p_j \Rightarrow d_{ij} = 1$$
Constant expenditures $f(y_i, p_j) = \gamma \ln y_i - \ln p_j \Rightarrow d_{ij} = \gamma \frac{y_i}{p_j}$
(2)

Conditional on choosing j, an individual buys a fixed unit in the first specification and spends a constant fraction of her budget, γ , in the second specification.

Random utility maximization Each consumer i chooses the product j that maximizes random utility u_{ij} . Using (2), we can write utility (1) as follows

$$u_{ij} = K_i + \delta_j + \varepsilon_{ij}, \tag{3}$$

where $K_i = \alpha y_i$ in the unit demand specification and $K_i = \alpha \gamma \ln y_i$ in the constant expenditures specification), and δ_j is the mean utility component of product j:

Unit demand
$$\delta_j \equiv x_j \beta - \alpha p_j + \xi_j$$

Constant expenditures $\delta_j \equiv x_j \beta - \alpha \ln p_j + \xi_j$. (4)

Normalize the mean utility of the outside good to zero, $\delta_0 = 0$.

The random utility terms ε_{ij} follow the extreme value distributional assumptions of a two-level nested logit model. Partition the set of products into G groups, $g=0,\ldots,G$, where group 0 is degenerate and only consists of the outside good 0. Further partition each group g into H_g subgroups, $h=1,\ldots,H_g$. Each subgroup h of group g contains J_{hg} products, so that $\sum_{g=1}^{G}\sum_{h=1}^{H_g}J_{hg}=J$. Given random utility maximization, the probability that a consumer i chooses product $j=1,\ldots,J$ takes the following well-known form:

$$s_j = s_j(\boldsymbol{\delta}, \sigma) \equiv \frac{\exp((\delta_j)/(1 - \sigma_1))}{\exp(I_{hg}/(1 - \sigma_1))} \frac{\exp(I_{hg}/(1 - \sigma_2))}{\exp(I_g/(1 - \sigma_2))} \frac{\exp(I_g)}{\exp(I)},$$
(5)

where I_{hg} , I_g , and I, are "inclusive values", defined by:

$$I_{hg} \equiv (1 - \sigma_1) \ln \sum_{k=1}^{J_{hg}} \exp((\delta_k)/(1 - \sigma_1))$$

$$I_g \equiv (1 - \sigma_2) \ln \sum_{h=1}^{H_g} \exp(I_{hg}/(1 - \sigma_2))$$

$$I \equiv \ln \left(1 + \sum_{g=1}^{G} \exp(I_g)\right),$$
(6)

 $\boldsymbol{\delta}$ is a $J \times 1$ vector containing the mean utilities δ_j , and $\sigma = (\sigma_1, \sigma_2)$ are the nesting parameters associated with the nested logit distribution, measuring the preference correlation across products of the same subgroup (σ_1) or group (σ_2) . Note that the separable terms K_i cancel out from the choice probabilities (5).

As shown by McFadden (1978), the model is consistent with random utility maximization if $1 \ge \sigma_1 \ge \sigma_2 \ge 0$. When σ_1 is high, consumer preferences are strongly correlated across products of the same subgroup, and when σ_2 is high, consumer preferences show additional correlation across products of the same group. Further intuition obtains from considering a few special cases. If $\sigma_1 = \sigma_2$, the model reduces to a one-level nested logit model, where groups are the nests: preferences only show correlation across products of the same group; there is no additional correlation across products of different subgroups within a group. If $\sigma_1 > \sigma_2 = 0$, the model also reduces to a one-level nested logit model, where the subgroups are the nests. Finally, if $\sigma_1 = \sigma_2 = 0$, the model reduces to a simple logit model, so consumer preferences do not show correlation across products from the same subgroups or groups.

Aggregate and inverted aggregate demand We can now derive the aggregate demands q_j for products j = 1, ..., J. Aggregate demand for product j is the probability that a consumer buys product j multiplied by the quantity purchased, d_{ij} , summed over all consumers. Under the two utility specifications (2) we obtain the following aggregate demand system for j = 1, ..., J:

Unit demand
$$q_{j} = \sum_{i=1}^{I} s_{j}(\delta, \sigma) d_{ij} = s_{j}(\delta, \sigma) I$$
Constant expenditures
$$q_{j} = \sum_{i=1}^{I} s_{j}(\delta, \sigma) d_{ij} = s_{j}(\delta, \sigma) \frac{B}{p_{j}}$$
(7)

where $s_j(\delta, \sigma)$ is given by (5), $B = \gamma Y$ and $Y = \sum_{i=1}^{I} y_i$. Hence, B is the total potential budget allocated to the differentiated products in the economy, a constant fraction γ of total income of all consumers Y.

The goal is to estimate the parameters (α, β, σ) entering the demand system (7). The econometric error term ξ_j enters non-linearly through the mean utility terms (4). To obtain a tractable model with a linear error term ξ_j , we proceed in two steps. In the first step, we follow Berry (1994) and invert the system of choice probabilities $s_j = s_j(\boldsymbol{\delta}, \sigma)$, $j = 1, \ldots, J$, to solve for the mean utilities $\delta_j = \delta_j(s, \sigma)$ as a function of the choice probability vector s. Following Berry (1994) for the one-level nested logit and Verboven (1996) for the two-level nested logit, we obtain the inverted choice probability system

$$\delta_j = \ln(s_j/s_0) - \sigma_1 \ln(s_{j|hg}) - \sigma_2 \ln(s_{h|g}),$$
 (8)

where $s_{j|hg} = s_j / \sum_{k=1}^{J_{hg}} s_k$ is the probability of choosing j given that an alternative from subgroup h of g is chosen (the first factor in (5)), and where $s_{h|g} = \left(\sum_{k=1}^{J_{hg}} s_k\right) / \left(\sum_{h=1}^{H_{hg}} \sum_{k=1}^{J_{hg}} s_k\right)$ is the probability of choosing subgroup h of g given that group g is chosen (the second factor in (5)).

In the second step, we use the aggregate demand expressions (7) to write the unobserved choice probabilities s_j , $s_{j|g}$ and s_0 in terms of observables. Using (7), the choice probabilities are equal to the market shares in volume terms for the familiar unit demand model

$$s_j = \frac{q_j}{I}, \quad s_{j|hg} = \frac{q_j}{\sum_{j \in H_{hg}} q_j}, \quad s_{h|g} = \frac{\sum_{j \in H_{hg}} q_j}{\sum_{h=1}^{H_{hg}} \sum_{j \in H_{hg}} q_j},$$

and they are equal to market shares in value terms for the constant expenditures model

$$s_j = \frac{p_j q_j}{B}, \quad s_{j|hg} = \frac{p_j q_j}{\sum_{j \in H_{hg}} p_j q_j}, \quad s_{h|g} = \frac{\sum_{j \in H_{hg}} p_j q_j}{\sum_{h=1}^{H_{hg}} \sum_{j \in H_{hg}} p_j q_j}.$$

We can insert these expressions, together with the specification (4) for δ_j , into the inverted choice probability system (8). This results in the following estimating equations for the unit demand model

$$\ln \frac{q_j}{I - \sum_{j=1}^{J} q_j} = x_j \beta - \alpha p_j + \sigma_1 \ln \frac{q_j}{\sum_{j \in H_{hg}} q_j} + \sigma_2 \ln \frac{\sum_{j \in H_{hg}} q_j}{\sum_{h=1}^{H_{hg}} \sum_{j \in H_{hg}} q_j} + \xi_j$$
 (9)

and for the constant expenditures model

$$\ln \frac{p_j q_j}{B - \sum_{j=1}^J q_j p_j} = x_j \beta - \alpha \ln p_j + \sigma_1 \ln \frac{p_j q_j}{\sum_{j \in H_{hg}} p_j q_j} + \sigma_2 \ln \frac{\sum_{j \in H_{hg}} p_j q_j}{\sum_{h=1}^{H_{hg}} \sum_{j \in H_{hg}} p_j q_j} + \xi_j. \quad (10)$$

The unit demand estimating equation (9) is the familiar one for the two-level nested logit. The constant expenditures estimating equation (10) has not been considered before, although it is equally simple. It differs from the unit demand specification in three respects. First, price enters logarithmically instead of linearly. Second, market shares are measured in value terms instead of volume terms. Third, the potential market is the total budget as a fixed fraction of GDP, B, instead of the total number of consumers, I. Note that

⁶Some other papers have used a logarithmic price term, for example Peters (2006) or Gowrisankaran and Rysman (2009). Verboven (1996) uses a Box-Cox transformation of the price term, $(p_j^{\mu} - 1)/\mu$ to nest both the linear and logarithmic specifications. While these approaches are useful to obtain a more flexible functional form for price, they are not consistent with utility maximization. As we show here, the logarithmic specification can be made consistent after some simple adjustments regarding the computation of market shares and the potential market (and it is straightforward to generalize this to the Box-Cox transformation, but the model is then no longer linear in the parameters).

the unit demand specification can immediately be interpreted as an inverse demand system (by writing price on the left hand side). This is not the case for the constant expenditures specification.

Both models can be estimated using an instrumental variable regression of volume or value market shares (relative to the outside good market share) on product characteristics, price and subgroup and group market shares, where the endogeneous variables are price and the (sub)group market shares.

Price elasticities Following Berry (1994), both demand specifications generate simple analytic expressions for the aggregate price elasticities of demand. To illustrate, consider the own-price elasticities. The derivatives of the choice probability (5) with respect to δ_j can be shown to be

$$\frac{\partial s_j}{\partial \delta_i} = s_j \left(\frac{1}{1 - \sigma_1} - \left(\frac{1}{1 - \sigma_1} - \frac{1}{1 - \sigma_2} \right) s_{j|hg} - \frac{\sigma_2}{1 - \sigma_2} s_{j|g} - s_j \right). \tag{11}$$

Using (7), the aggregate demand derivatives are

Unit demand
$$\frac{\partial q_{j}}{\partial p_{j}} = -\alpha \frac{\partial s_{j}}{\partial \delta_{j}} I$$
Constant expenditures
$$\frac{\partial q_{j}}{\partial p_{j}} = -\alpha \frac{\partial s_{j}}{\partial \delta_{j}} \frac{B}{p_{j}^{2}} - s_{j} \frac{B}{p_{j}^{2}},$$
(12)

Substituting (11) into (12), we obtain the aggregate own-price elasticity in the unit demand specification

$$\frac{\partial q_j}{\partial p_j} \frac{p_j}{q_j} = -\alpha \left(\frac{1}{1 - \sigma_1} - \left(\frac{1}{1 - \sigma_1} - \frac{1}{1 - \sigma_2} \right) s_{j|hg} - \frac{\sigma_2}{1 - \sigma_2} s_{j|g} - s_j \right) p_j, \tag{13}$$

and in the constant expenditures specification

$$\frac{\partial q_j}{\partial p_j} \frac{p_j}{q_j} = -\alpha \left(\frac{1}{1 - \sigma_1} - \left(\frac{1}{1 - \sigma_1} - \frac{1}{1 - \sigma_2} \right) s_{j|hg} - \frac{\sigma_2}{1 - \sigma_2} s_{j|g} - s_j \right) - 1. \tag{14}$$

These expressions reveal the well-known role of subgroup, group and overall market shares in the measurement of the price elasticities (in interaction with the nesting parameters σ_1 and σ_2). Furthermore, they show that in the typical unit demand specification the own-price elasticities are increasing quasi-linearly in prices across products, whereas they are quasi-independent of prices in the constant expenditures demand specification.⁷ The Appendix provides similar expressions for the cross-price elasticities.

⁷In both cases, we write "quasi", since there is indirect dependence on the prices through the market shares.

3.2 Oligopoly model

The oligopoly model serves two purposes. First, in combination with the demand parameters it enables one to uncover the premerger marginal costs. Second, based on the demand parameters and uncovered marginal costs, it can be used to predict the price effects of the merger. We begin with the basic model where multi-product firms set prices non-cooperatively. We then extend it to allow firms to partially coordinate (to the same extent before and after the merger). We found the introduction of a partial coordination parameter a useful approach to calibrate the premerger marginal cost such that they are close to outside estimates for the marginal costs.

Each firm f owns a portfolio of products F_f . Its total variable profits are given by the sum of the profits for each product $k \in F_f$:

$$\Pi_f(\mathbf{p}) = \sum_{k \in F_f} (p_k - c_k) \, q_k(\mathbf{p}) \tag{15}$$

where c_k is the constant marginal cost for product k and $q_k(\mathbf{p})$ is demand, as given by (7), now written as a function of the $J \times 1$ price vector \mathbf{p} . The profit-maximizing price of each product $j = 1, \ldots, J$ should satisfy the following first-order condition:

$$q_j(\mathbf{p}) + \sum_{k \in F_f} (p_k - c_k) \frac{\partial q_k(\mathbf{p})}{\partial p_j} = 0.$$
 (16)

A price increase affects profits through three channels. First, it directly raises profits, proportional to current demand $q_j(\mathbf{p})$. Second, it lowers the product's own demand, which lowers profits proportional to the current markup. Third, it raises the demand of the other products in the firm's portfolio, which partially compensates for the reduced demand of the own product. If the first-order conditions (16) hold for all products $j = 1 \cdots J$, a multiproduct Bertrand-Nash equilibrium obtains.

To write this system of J first-order conditions in vector notation, define the $J \times J$ matrix $\boldsymbol{\theta}^F$ as the firms' product ownership matrix, a block-diagonal matrix with a typical element $\boldsymbol{\theta}^F(j,k)$ equal to 1 if products j and k are produced by the same firm and 0 otherwise. Let $\mathbf{q}(\mathbf{p})$ be the $J \times 1$ demand vector, and $\boldsymbol{\Delta}(\mathbf{p}) \equiv \partial \mathbf{q}(\mathbf{p})/\partial \mathbf{p}'$ be the corresponding $J \times J$ Jacobian matrix of first derivatives. Let \mathbf{c} be the $J \times 1$ marginal cost vector. Using the operator \odot to denote element-by-element multiplication of two matrices of the same dimension, we have

$$\mathbf{q}(\mathbf{p}) + (\boldsymbol{\theta}^F \odot \boldsymbol{\Delta}(\mathbf{p})) (\mathbf{p} - \mathbf{c}) = 0.$$

This can be inverted to give the following expression:

$$\mathbf{p} = \mathbf{c} - \left(\boldsymbol{\theta}^F \odot \boldsymbol{\Delta}(\mathbf{p})\right)^{-1} \mathbf{q}(\mathbf{p}). \tag{17}$$

It is straightforward to generalize this expression to allow for (partial) coordinated behavior. Suppose that firms put a weight $\phi \in (0,1)$ on the profits of their competitors and modify the objective function (15) accordingly. The same expression (17) then obtains, where the zeros in the matrix $\boldsymbol{\theta}^F$ are replaced by the parameter ϕ .⁸

Intuitively, (17) decomposes the price into two terms: marginal cost and a markup, which depends on the own- and cross-price elasticities of demand. The lower the own-price elasticities and the greater the cross-price elasticities, the greater will be the markup over marginal cost.

Equation (17) serves two purposes. First, it can be rewritten to uncover the pre-merger marginal cost vector \mathbf{c} based on the pre-merger prices and estimated price elasticities of demand, i.e.

$$\mathbf{c}^{pre} = \mathbf{p}^{pre} + \left(oldsymbol{ heta}^{F,pre} \odot oldsymbol{\Delta}(\mathbf{p}^{pre})
ight)^{-1} \mathbf{q}(\mathbf{p}^{pre}).$$

Second, (17) can be used to predict the post-merger equilibrium. The merger involves two possible changes: a change in the product ownership matrix from $\boldsymbol{\theta}^{F,pre}$ to $\boldsymbol{\theta}^{F,post}$ and, if there are efficiencies, a change in the marginal cost vector from \mathbf{c}^{pre} to \mathbf{c}^{post} . To simulate the new price equilibrium, we used fixed point iteration on (17), and, if this fails to converge, we revert to the Newton method.

4 The merger simulation

We now present the results from the merger simulation. We deliberately chose to maintain the estimation and simulation methodology as developed during the merger investigation. We first present the estimation and specification choices and then discuss the estimated demand parameters and the implied price elasticities. Finally, we present the simulated price effects of the merging firms, under a variety of scenario's considered during the investigation. While the model also makes other predictions (on competitor price increases, on market share changes), we do not present these here yet. We defer this to the next section, where we confront all predictions from the preferred demand model with what actually happened after the merger.

4.1 Estimation and specification

Estimation We estimate the two versions of the nested logit model: the unit demand specification (9) and the constant expenditures specification (10). Several econometric is-

⁸It would be possible to allow for more general patterns of coordinated behavior, allowing ϕ to vary across products, but since there is little information about the possibility and the extent of coordination we keep a simple specification.

sues need to be addressed. First, it is necessary to specify the potential market, i.e. the total number of potential consumers I in the unit demand model and the total potential budget B in the constant expenditures model. For both specifications, we assume that the potential market is twice the average amount spent over the entire period, in units for the first specification and in values for the second specification. We performed a sensitivity analysis with alternative factors: 1.5, 2 (base), 4 and 6 and obtained similar results.

Second, we do not observe a single cross-section of products $j = 1 \cdots J$, but rather a panel of multiple periods (months and years during 1995-2008). We therefore estimate the model including fixed effects per product j to account for time-invariant unobserved product characteristics.

Third, the price variable and the group share variables $\ln s_{j|hg}$ and $\ln s_{h|g}$ are endogenous variables that may be correlated with the error term, even conditional on the fixed effects. Berry, Levinsohn and Pakes (1995) proposed to use sums of the characteristics of the other products, and counts of the number of products, over all products and over all competing firms' products. For the nested logit model, Verboven (1996) suggested to also take the sums and counts by subgroups and groups. We follow the same approach here. Intuitively, identification comes from the fact that the choice sets (number of products per subgroup, group, firm and market) show variation over time.

Specification Our maintained specification defines the upper nesting level by the administrative form and the lower nesting level by the active substance. Under this nesting structure, consumers are most likely to substitute to another product of the same form and substance, and would substitute more to another substance than to another form. We also estimated a model with the reverse nesting order (where consumers would substitute more to another form than to another substance), but this led to estimates of the nesting parameters $\sigma_1 < \sigma_2$, inconsistent with random utility theory. Following common practice (e.g. Goldberg, 1995), we therefore limit attention to the model that gave parameters consistent with random utility theory $(1 \ge \sigma_1 \ge \sigma_2 \ge 0)$. We also estimated a constrained one-level nested logit models $(\sigma_1 = \sigma_2)$, where segmentation is either according to substance or form.

We estimated the model on both the full sample, and on the reduced sample with only the three main substances and two main forms. Since we obtained robust conclusions, we present only the results for the reduced sample: this was our preferred sample during the investigation since it already captures more than 90% of sales and reduces heterogeneity across (smaller) brands. For both samples, we estimated the model using the three different measures for the consumption unit (tablet, defined daily dose, and normal dose at a single occasion). Our base specifications are based on the tablet measure, but we also comment on

the results for the other measures.

Both the unit demand and constant expenditures nested logit model include the following variables as determinants of mean utility (relative to the outside good): price (unit demand) or log of price (constant expenditures), marketing expenditures, the fraction of sick women and sick men in the total population, a time trend and monthly dummy variables capturing seasonal effects. One of these explanatory variables, marketing expenditures, is potentially endogeneous. As in Chintagunta (2002), we treat it as exogenous, uncorrelated with the error term. This assumption may be justified to the extent that the full set of product fixed effects takes away the main source of correlation with the error term.

4.2 Demand parameters and price elasticities

Table 4 presents the estimated demand parameters for the base specifications of the unit demand and constant expenditures specifications. In both specifications most parameters have the expected sign and are estimated significantly different from zero. As in Chintagunta (2002), marketing expenditures have a positive effect on the products' demands. There is a positive and significant time trend, and monthly dummy variables indicate that demand for painkillers is especially strong during the winter months December and March. Demand grows with the number of sick men but, surprisingly, in the unit demand specification it decreases with the number of sick women. This may be because this variable picks up some other effects, or because women use other drugs (perhaps prescription drugs) when they report sickness.

In both specifications the price coefficient α has the expected sign. The subgroup and group nesting parameters are fairly comparable ($\sigma_1 = 0.93$ and $\sigma_2 = 0.79$ in the linear specification, and $\sigma_1 = 0.84$ and $\sigma_2 = 0.67$ in the constant expenditures specification). These estimates satisfy the requirements for the model to be consistent with random utility theory, $1 \ge \sigma_1 \ge \sigma_2 \ge 0$. In both specifications, the inequalities are strict, which implies that consumers perceive products of the same form and substance as the closest substitutes, products of a different substance but the same form as weaker substitutes, and products with both different substance and different form as the weakest substitutes.

The bottom part of Table 4 shows what the estimates of α , σ_1 and σ_2 imply for the ownand cross-price elasticities. The numbers refer to averages across products during December 2008, the last month of our dataset during the investigation. In the constant expenditures specification, the average own-price elasticity is -2.7, while the cross-price elasticity is much larger for products of the same substance and form (0.17) than for products of a different substance but the same form (0.04), which in turn is larger than for products of different

	Constant expenditures		Unit demand		
	Parameter	St. Error	Parameter	St. Error	
price $(-\alpha)$	304	.097	-2.042	.147	
subgroup (σ_1)	.835	.019	.928	.012	
group (σ_2)	.667	.018	.792	.010	
marketing expenditures	15.50	2.66	8.85	1.75	
sickwomen	.357	.123	699	.081	
sickmen	1.145	.235	.809	.155	
time trend	.0013	.0005	.0007	.0002	
constant	-2.513	.211	669	.030	
February	059	.012	047	.008	
March	.115	.012	.094	.008	
April	122	.012	087	.008	
May	127	.013	103	.008	
June	109	.013	095	.008	
July	.044	.013	164	.009	
August	.127	.013	085	.008	
September	.177	.012	001	.008	
October	095	.013	.010	.008	
November	206	.012	085	.008	
December	.124	.012	.259	.008	
R^2	0.983		0.972		
	Implied price elasticit		ties (December 2008)		
	Average	Range	Average	Range	
Own	-2.68	-2.84; -1.91	-12.4	-24.1; -3.9	
Cross: same subgroup	.16	.00; .93	1.5	.00; 8.3	
Cross: different subgroup	.04	.00; .29	.25	.00; 1.76	
Cross: different group	.01	.00; .06	.02	.00; .16	

Note: 7,240 observations for 1995–2008, 56 product fixed effects included.

substance and form (0.01). There is a similar pattern in the unit demand specification, but the level of elasticities is considerably higher.⁹

4.3 Predicted price effects under alternative scenario's

During the merger investigation by the Swedish competition authority, we reported the predicted price effects from the merger under both the unit demand and the constant expenditures specifications. For each specification, we considered four scenario's: no cost savings versus 25% cost savings, and multiproduct Bertrand competition versus partial coordination. The partial coordination parameter was calibrated to $\phi = 0.75$, i.e. both before and after the merger all firms take into account their competitors' profits by 75% when setting their own prices. Calibrating $\phi = 0.75$ leads to premerger marginal costs in line with outside information available to the competition authority, so it has some intuitive appeal as an alternative to Bertrand competition.

Table 5 shows the pre-merger markups and predicted price increases, under the four scenario's and the two demand specifications. The predicted price increases are average percentage price increases in the paracetamol segment, where the merging firms (and no other firms) are active. Table 5 is essentially what we reported during the competition investigation.¹⁰ We defer a richer and more systematic set of predictions from the merger simulations to our expost analysis below.

According to the constant expenditures specification, the merger between AZT and GSK would lead to rather substantial price increases in the absence of efficiencies: +34.1% under Bertrand competition, and +28.4% under partial coordination. The predicted price effects only become small or negligible if the merger involves at least 25% marginal cost savings (price increase of +4.7% under Bertrand competition and -0.1% under partial coordination). These results therefore imply large efficiency requirements for the merger to benefit consumers. Nevertheless, during the investigation we stressed that caution was warranted, because such large price increases may not materialize if they trigger entry, a possibility that

⁹It is of interest to compare these estimates with the ones from a unit demand random coefficients logit, as obtained by Chintagunta (2002). The estimates for the five analgesics brands he considered vary between -1.8 and -3.0. This is of a similar range as in the constant expenditure specification. However, the variation in elasticities in Chintagunta's model is largely driven by variation in prices, whereas in our model it is mainly driven by variation in market shares. Also, while the own-price elasticities are comparable, the cross-price elasticities obtained by Chintagunta are considerably lower despite the fact that he considered only five products.

¹⁰In the report to the Swedish competition authority we also presented the results from a constant expenditure specification based on the full dataset instead of the reduced dataset. This gave very similar results.

Table 5: Predicted price effects of the merger during the investigation

	percentage pric	pre-merger markup			
	no cost saving	25% cost saving			
	Constant exper	ons			
Bertrand	+34.0%	+4.7%	.49		
partial coordination	+28.4%	-0.1%	.76		
	Unit demand specification				
Bertrand	+12.9%	+1.6%	.16		
partial coordination	+16.1%	+9.0%	.54		

Note: This table shows the pre-merger markups and the predicted price effects of the merging firms under alternative scenario's, exactly as reported in the merger investigation.

became more likely in light of the then coming deregulation of the distribution system.

According to the unit demand model, the predicted price effects from the merger are considerably smaller, but they remain quite substantial. In the absence of efficiencies, the model predicts that the merging firms would raise prices by +12.9% under Bertrand competition and by +16.1% under partial coordination. The lower predicted price effects are due to the larger estimated price elasticities in the unit demand model. If we account for 25% cost savings, the predicted price effects become negligible under Bertrand competition, but they remain significant under partial coordination. In the unit demand model, the cost savings are passed on to a lesser extent than in the constant expenditures specification. This clearly follows from the functional form: in the unit demand model consumers tend to become more price elastic as price increases, whereas they remain more or less equally price elastic in the constant expenditures specification.

Despite the rather large predicted price increase in the constant expenditures specification, we favoured these results for two reasons. First, as discussed above, we considered the constant expenditures as our preferred base specification, because it entailed a more plausible relationship between price elasticities and prices, and in particular because the pattern of price elasticities does not depend on the chosen unit of consumption (tablet, defined daily dose, or normal dose on a single occasion). Second, we found the computed premerger markups to be more plausible. As shown in the last column of Table 5, in the constant expenditures model the average premerger markups are 49% under Bertrand competition 76% under partial coordination. These numbers were broadly in line with the variable cost information provided by the parties during the investigation (cost of purchasing the active substance, production cost and packaging cost). In contrast, in the unit demand specifica-

tion, the average premerger markups are much smaller (16% under Bertrand competition and 54% under partial coordination) and in fact well below the markups from the parties' information.

In sum, the merger requires substantial cost savings, in the order of at least 25%, for the price effects to become small. In the absence of cost savings, the preferred constant expenditures specification predicts a very large price increase: +34% under Bertrand competition and +28.4% under partial coordination before the merger. The unit demand specification predicts lower price increases, but still well above 10%. If one were to apply a SSNIP test for market definition, the conclusion would clearly be that the merging firms constitute a monopoly by themselves.

5 Ex post merger analysis

We now confront the predicted merger effects with what actually happened after the merger. We first provide a more systematic overview of a broad range of merger predictions under the preferred constant expenditures model, and then confront these with the actual effects over a two-period window before and after the merger.

5.1 Predicted price and market share effects in the preferred model

As discussed in section 4, during the investigation we focused on the predicted average price increase of the merging firms. We now consider a much broader range of merger predictions: the predicted price increase by each of the merging firms, the price increase by their competitors, and the market share effects. To maintain focus, we now limit attention to our preferred model during the investigation, i.e. the constant expenditures nested logit where administrative form is the upper nest and active substance is the lower nest. We also only consider the scenario without efficiencies, since there was no concrete evidence on the actual realization of efficiencies.

The predicted merger effects are shown in Table 6, under columns 1 and 2 (prices) and columns 3 and 4 (market shares). Consider first the predicted price effects at the level of the active substance (top panel). As already discussed, under Bertrand competition (column 1) the predicted price increase is a substantial 34.1% in the paracetamol segment (where only the merging firms are active). Furthermore, prices in the other segments (where only the competitors are active) only increase by a small amount: by 0.7% for ibuprofen products and by +0.8% for ASA products. If firms partially coordinate, the predicted price increase in paracetamol is lower at 28%, but the price increase in the competing segments of the other

firms becomes higher at 4.1% for Ibuprofen and 3% for ASA.

Table 6: Predicted price and market share effects in the preferred model

	Price effec	ets	Market share effects			
	Bertrand	Partial coord.	Bertrand	Partial coord.		
	Prediction	Predictions at the level of the active substance				
Paracetamol	+34.1%	+28.0%	-7.1%	-5.4%		
Ibuprofen	+0.7%	+4.1%	+3.7%	+2.7%		
ASA	+0.8%	+3.0%	+3.3%	+2.7%		
	Predictions at the level of the firm					
AZT	+21.3%	+19.5%	-3.4%	-2.7%		
GSK	+59.8%	+45.1%	-3.7%	-2.7%		
Nycomed	+0.6%	+4.0%	+1.3%	+0.9%		
Meda (Ellem)	+0.1%	+2.7%	+0.6%	+0.5%		
McNeil	+1.7%	+4.1%	+5.1%	+3.9%		
Bayer	+0.1%	+2.5%	+0.1%	+0.1%		

Note: This table shows predicted price and market share effects, based on the preferred model.

The predicted price effects at the level of the firm give interesting additional insights (bottom panel). Perhaps most interestingly, the model predicts that the merging firm with the lower pre-merger market share, GSK, raises its price by a much larger amount (+60% under Bertrand competition) than its partner with the larger market share, AZT (+21.3%). Intuitively, this follows from the fact that markups of small firms tend to be lower than those of large firms, and they become equalized after a merger (see already Anderson and de Palma, 1992). The model also predicts that most competitors raise their prices by a negligible amount under Bertrand competition, but by a more sizeable amount under partial coordination.

Now consider the predicted market shares effects from the merger, measured in volume terms. Under Bertrand competition the paracetamol market share is predicted to decrease by 7.1% (from 47.9% to 40.8%). This comes to the benefit of both ibuprofen and ASA (+3.8% and +3.3%). The larger firm AZT is predicted to suffer a market share drop of -3.4% (from 36.1% to 32.7%), while the smaller partner GSK will suffer a proportionately more substantial market share drop of -3.7% (from 11.8% to 8.1%). These market share effects are qualitatively similar under partial coordination, though quantitatively less pronounced.

5.2 Actual price and market share effects

We can now confront these various predictions with the actual price and market share effects following the merger. We use a two-year comparison window around the merger event of April 3, 2009, so we compare the periods April 2007–April 2009 and May 2009–May 2011.

Price effects Figure ?? shows the price evolution during both periods for the three main segments: paracetamol, ibuprofen and ASA. The results are striking. In the paracetamol segment, where the merging firms AST and GSK are the only competitors, average prices increase from about 1.5 SEK to 2 SEK, already one month after the merger. The price increase is especially striking since prices only show a small gradual increase two years prior to the merger (from SEK1.4 to SEK 1.5) and remained more or less constant after the sharp increase just after the merger. Only near the end of the period, there is a slight tendency of a price drop, perhaps associated with new entry threats following the deregulation. In sharp contrast, in the ibuprofen segment prices remained stable after the merger, whereas in the ibuprofen segment they appear to increase by a modest amount (from 1.4 SEK to 1.55 SEK). This suggests that the sharp price increase by the merging firms was indeed due to the merger, and not due to a general cost or demand shock unrelated to the merger.

To gain further insights on this, we estimate the following regression, in line with Ashenfelter and Hosken (2008) and other recent work on ex post merger evaluation discussed in the introduction

$$ln p_{it} = \alpha_i + \beta_i PostMerger_t + \varepsilon_{it},$$
(18)

where p_{it} is the average price of "product group" i, and $PostMerger_t$ is a dummy variable equal to 1 after the merger event.¹¹ The literature sometimes assumes that the merger does not have an impact on the competitors' prices. If this assumption is satisfied, one can interpret this regression as a difference-in-difference estimator, where the difference between the merging firms' β_i and the competitors' β_i measures the merger price effect. In practice, it is possible that the merger raises the competitors' prices (under Bertrand competition, but especially if there is some coordination, as the merger simulations also predict). If this is the case, the difference between the merging firms' and the competitors' β_i 's can be viewed as a lower bound for the merger price effect.

We define the product group i in the above regression at three levels: the substance, the firm and the product. Since we obtained similar results, we only presents the results at

¹¹Our specification is slightly more general than Ashenfelter and Hosken (2008) and other work. They typically constrain the same effect for the control group after the merger, whereas we allow different product groups i to have different price changes.

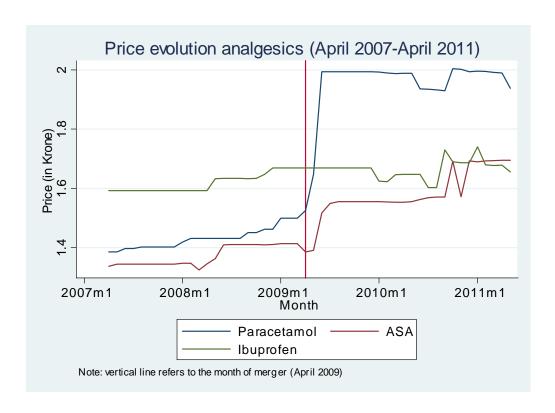


Figure 1: Price evolution analgesics (April 2007 - April 2011)

the level of the substance and firm. Table 7 shows the results. According to column 1 (top panel), the merger led to a log price increase of 0.351 in the paracetamol segment, implying an average price increase of the merged firms' products by 42%. This is of a same order of magnitude, and in fact even larger than the already large predicted price increase of 34% under Bertrand competition.

Why did such a large and sudden price increase not raise a significant amount of controversy in Sweden? In fact, the merged firm AZT-GSK implemented the price increase by reducing their packsizes from 30 to 20 tablets, while reducing prices per package by only a small amount, for example from 41.5 crowns to 38.5 crowns for one of their most selling products. The reduction in packsize had been required by the Swedish medical products agency (Läkemedelsverket), because of concerns with a too wide availability of painkillers. While the firms argued that the price increase was warranted because of the increased costs with the reduced package size, this appears implausible because other companies had also been required to lower packsizes and this did not coincide with large price increases. If one would account for a modest increase in the packaging costs, the predicted price increase from the merger simulation model would become even closer to the actual price increase.

The merger simulation model thus performed quite well in predicting the average price increase of the merging firms. But we can dig deeper, thanks to the particularly large size of the merger. We now consider how well the model predicted other effects: the average price changes in the competing segments and the price increases of the individual firms.

Table 7: Actual price and market share effects, two year window

	Price effects		Market share effects		
	Estimate	Stand. err.	Estimate	Stand. err.	
	Regressions at the level of the active substance				
Constant	.303	.004	.468	.002	
Ibuprofen	.171	.006	199	.003	
ASA	.208	.006	204	.003	
Paracetamol*merger	.351	.006	033	.003	
Ibuprofen*merger	.001	.006	.050	.003	
ASA*merger	.100	.006	016	.003	
\mathbb{R}^2	.969		.986		
	Regressions at the level of the firm				
Constant	.304	.011	.344	.003	
GSK	004	.016	221	.004	
Nycomed	.107	.016	254	.004	
Meda	121	.018	316	.004	
McNeil	.229	.016	.052	.004	
Bayer	149	.016	339	.004	
AZT*merger	.356	.016	056	.004	
$GSK^*merger$.379	.016	003	.004	
Nycomed*merger	.012	.016	.001	.004	
Meda*merger	.029	.018	.011	.004	
McNeil*merger	.084	.016	027	.004	
Bayer*merger	.105	.016	.005	.004	
\mathbb{R}^2	.907		.990		

Note: This table shows actual price and market share effects, based on the regression model (18) for price and analogous model for market share.

First, the price regression at the level of the substance shows that ibuprofen prices essentially remained constant (+0.1%), which is consistent with the model predictions. But the ASA prices increased by 0.10 (in logs) or 11%, which is much larger than the model predic-

tions (+0.8% under Bertrand competition and +3.0% under partial coordination). Using the difference-in-difference interpretation of Ashenfelter and Hosken (2008), we would conclude that the price increase caused by the merger is 0.35 (in logs) or 42.0% if ibuprofen is the control group, and 0.251 (in logs) or 29% if ASA is the control group. The merger simulation prediction of 34% falls in between these bounds.¹²

Second, the price regression at the level of the firms (column 1, bottom panel) shows that both of the merging firms raised their prices substantially and more or less proportionately: AZT by 0.356 or 43% and GSK by 0.379 or 46%. The price increase is thus slightly smaller for the bigger firm (AZT) than for the smaller firm (GSK). But the merger simulations predicted a much wider difference between both firms (+21% for the bigger firm versus +60% for the smaller firm). The competitors raised their prices by much lower or negligible amounts (Bayer by +0.105, Pfizer by +0.084, Meda by +0.029 and Nycomed by +0.012): this is again qualitatively consistent with the model, but not quantitatively, since the model predicted negligible price increases (under Bertrand competition).

Market share effects Did the large price increase of the merging firms also affect market shares? Figure 3 shows the market share evolution (expressed in volumes), using the same comparison window as Figure 2. This shows that the market share of the merging firms' paracetamol segment suddenly dropped by a sizeable 5% (down from about 47% to about 42%), whereas the market share of especially ibuprofen increased sharply (from about 27% to 32%). It is less clear from Figure 3 whether these market share changes were permanent, since they show some volatility over the sample. We therefore estimated a regression similar to (18), but with the log of price replaced by the market share as the dependent variable (again, in line with Ashenfelter and Hosken's (2008) ex post study).

The market share of the paracetamol segment, occupied by the merging firms, dropped by a significant 3.3% (95% confidence interval of 2.7%–3.9%). This loss was entirely in favor of the ibuprofen market share, which increased by a substantial 5.0%. The market share of ASA unexpectedly, decreased (by 1.6%). Interesting additional findings obtain for the market shares at the level of the firms. Despite the fact that prices increased slightly more for GSK than for AZT products, AZT experienced the largest market share drop (–5.6%, compared with an insignificant –0.3% for GSK). Pfizer also experienced a market share drop (–2.7%), whereas the other competitors all experienced market share increases.

In sum, several of the estimated market share effects are, at least qualitatively, in line

¹²If we follow other work and estimate a restricted version of (18) where ibuprofen and ASA are a common control group, we find that ibuprofen-ASA prices increase by 0.05 (in logs) and paracetamol prices by 0.35, i.e. an extra increase of 0.30 (in logs) or 35%, which is very close to the prediction from the merger simulation.

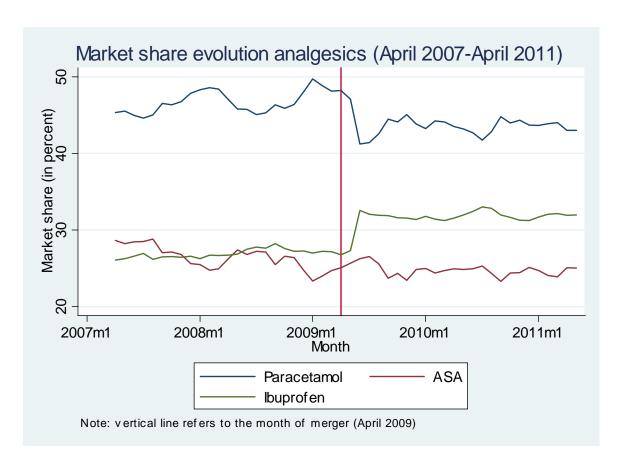


Figure 2: Market share evolution analgescis (April 2007 - April 2011)

with the merger predictions: the market share decrease of paracetamol in favor of ibuprofen. But the market share drop of ASA is inconsistent with the merger predictions (as it was predicted to gain from the merger). Similarly, the insignificant market share drop of one of the merging firms, GSK, is not consistent with the predicted market share drop for GSK.¹³

A possible explanation for the deviations between actual and predicted market share effects is that other things did not remain equal after the merger. For example, the market share of ASA already shows a small gradual decline during the two-year period before the merger. Also, the insignificant drop in GSK's market share (at the expense of the bigger drop in AZT's market share) may be related to the fact that GSK was the acquiring firm. After the merger it may have restructured its operations to favour the GSK brands, which would benefit GSK's market share. An alternative explanation for the deviations is of course that the merger simulation model is not specified correctly in all respects. We already noted the

 $^{^{13}}$ But it is consistent with our earlier observation that GSK did not raise its price as much as the model had predicted.

property of the logit model with Bertrand competition that markups are equalized across products of a multi-product firm. This is driven by functional forms and may not be realistic in practice.

6 Conclusions

We have made use of a unique "natural experiment" to measure a merger's effects, and in particular to evaluate the usefulness of merger simulation as a "structural approach" to predict the effects from mergers. The merger case is unique for several reasons. First, it involves large players who have no other competition in their own segment. This leads to large merger predictions, enabling us to test a broad range of predictions. Second, the merger simulation methodology was entirely implemented during the case, without information on the actual merger effects.

The merger simulation model started from a two-level nested logit demand system, where we proposed a constant expenditures specification as an alternative to the typical unit demand specification. Our empirical results show the following two key points. First, market segmentation according to active substance is a very important differentiation dimension. This implies that the two merging firms form a strong competitive constraint on prices before the merger. Second, the constant expenditures specification entails a more plausible pattern of price elasticities across products. Based on these two findings, the model predicts a large price increase of 34% by the merging firms.

Our ex post analysis shows that the actual price increase by the merging firms is of a similar order of magnitude, but in fact even larger than the price increase predicted by the model: +42% in absolute terms, or +35% in a difference-in-difference interpretation where the other firms are the control group. The average price predictions are thus quite accurate, but a closer look leads to more nuanced conclusions. First, both merging firms raised their prices by a similar percentage, while the simulation model predicted a larger price increase for the smaller firm. Second, although the merging firms' market share dropped, as predicted by the model, some of the outsiders' market shares also dropped (in favor of other outsiders). We discussed possible reasons for the divergence between the predicted and actual effects, i.e. the possibility that other things did not remain constant after the merger or that the model specification can be improved. It was possible to test these richer predictions, thanks to the unusually large size of the considered merger (where the two merging firms are the only competitors in a segment with limited substitution from other segments).

It is interesting to observe that our predictions were obtained from a fairly simple differentiated products oligopoly model without the "elaborate superstructure" to which Angrist and Pischke refer in their discussion. In future research it may nevertheless be interesting to consider various extensions of the model (alternative equilibrium, further sensitivity of functional form of demand) to see whether these can improve the accuracy of the predictions. But in our view more importantly, it would be interesting to see a lot more work that confronts the merger simulations during a case with the actual merger effects.

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7 Appendix

In the text, we reported the own-price elasticities for the unit demand (13) and constant expenditures nested logit, (13). This Appendix also reports the cross price elasticities with respect to products from the same subgroup, from a different subgroup within the same group, and from a different subgroup. The derivatives of the choice probability s_j , as given by (5), with respect to the mean utility δ_k can be shown to be

$$\frac{\partial s_j}{\partial \delta_k} = s_j \left(\frac{1}{1 - \sigma_1} D_{jk}^1 - \left(\frac{1}{1 - \sigma_1} - \frac{1}{1 - \sigma_2} \right) s_{j|hg} D_{jk}^2 - \frac{\sigma_2}{1 - \sigma_2} s_{j|g} D_{jk}^3 - s_j \right) \tag{19}$$

where $D_{jk}^1 = 1$ if j = k, $D_{jk}^2 = 1$ if j and k are in same subgroup, $D_{jk}^3 = 1$ if j and k are in same group. Using (19) and (12), one can obtain the following expressions for the aggregate price elasticities. In the unit demand specification, we have

$$\frac{\partial q_j}{\partial p_k} \frac{p_k}{q_j} = -\alpha \left(\frac{1}{1 - \sigma_1} D_{jk}^1 - \left(\frac{1}{1 - \sigma_1} - \frac{1}{1 - \sigma_2} \right) s_{j|hg} D_{jk}^2 - \frac{\sigma_2}{1 - \sigma_2} s_{j|g} D_{jk}^3 - s_j \right) p_j,$$

while in the constant expenditures specification we have

$$\frac{\partial q_j}{\partial p_k} \frac{p_k}{q_j} = -\alpha \left(\frac{1}{1 - \sigma_1} D_{jk}^1 - \left(\frac{1}{1 - \sigma_1} - \frac{1}{1 - \sigma_2} \right) s_{j|hg} D_{jk}^2 - \frac{\sigma_2}{1 - \sigma_2} s_{j|g} D_{jk}^3 - s_j \right) - D_{jk}^1.$$