Core and 'Crust': Consumer Prices and the Term Structure of Interest Rates*

Online Appendix

Andrea Ajello,[†] Luca Benzoni, [‡] and Olena Chyruk [§]

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†Board of Governors of the Federal Reserve System, 20th Street and Constitution Avenue N.W., Washington D.C. 20551, and Northwestern University, andrea.ajello@u.northwestern.edu.

‡Corresponding author. Federal Reserve Bank of Chicago, 230 S. LaSalle St., Chicago, Il 60604, 312-322-8499, lbenzoni@frbchi.org.

§Federal Reserve Bank of Chicago, 230 S. LaSalle St., Chicago, Il 60604, ochyruk@frbchi.org

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A.1 Computing Zero-coupon Yields

For model estimation, we use two panel data sets of U.S. Treasury yields (see Section 3 of the paper). The first comprises quarterly *zero-coupon* yield observations with maturities of 1, 4, 12, and 20 quarters, computed by the Center for Research in Security Prices (CRSP). The second data set consists of daily constant-maturity *par* yields computed by the U.S. Treasury and distributed by the Board of Governors in the H.15 data release.[1] Prior to analysis we interpolate the daily par yields into zero-coupon rates using all yield maturities available on that day, as we explain below. We then aggregate the daily zero-coupon yields into a panel of quarterly yield series with maturities of 1, 3, 5, 10, and 20 years.

There are many methods to extract zero-coupon yields from a cross-section of par yields and a vast literature has studied the relative advantages of different approaches. The key feature underlying the different methods is the choice of a specific interpolation to fit the discount rate (or forward rate) curve (e.g., Dai, Singleton, and Yang (2007)). On one extreme is the unsmoothed Fama-Bliss approach, which iteratively extracts forward rates from coupon bond prices by building a piecewise linear discount rate function. With this approach, the implied discount rates exhibit kinks at the maturities of the coupon bonds used. At the opposite extreme the Nelson-Siegel-Bliss and smoothed Fama-Bliss methods approximate the discount rates with exponential functions of time to maturity, which yields a smooth term structure interpolation. Here, we follow an approach that falls in between these two extremes and use a 'smoothed' spline interpolation for the discount rates curve. The method is described in Andersen and Benzoni (2010), Appendix A, pages 645-647. For the benefit of the reader we reproduce the details below.[2]

We fit the curve of zero-coupon yields with a cubic spline interpolation $\hat{y}(\tau, \Psi)$, where Ψ is the vector of spline coefficients, τ is time-to-maturity, and $\tau_{min} \le \tau \le \tau_{max}$, with τ_{min} and τ_{max} denoting the nearest and farthest maturities, respectively. We define P_{τ_k}, $k = 1, \ldots, N$, to be the time-t market price of a bond with maturity τ_k and define $\hat{P}_{\tau_k}$ to be the price of the same bond computed by discounting its coupon and principal payments at the discount rate $\hat{y}$. Next, we choose Ψ to solve the problem

$$\min_{\Psi} \left(\sum_{k=1}^{N} (P_{\tau_k} - \hat{P}_{\tau_k})^2 + \int_{\tau_{min}}^{\tau_{max}} \lambda(\tau)\, \hat{y}''(\tau, \Psi)^2 d\tau \right) . \tag{A.1}$$

This is the approach preferred by Waggoner (1997) and used in, for example, Dai, Singleton, and Yang (2007), except that we fit the smoothed cubic spline directly on the zero-coupon yields curve (similar to McCulloch (1975)), while Waggoner (1997) and Dai, Singleton, and Yang (2007) fit the smoothed spline on the forward rates curve. We follow Fisher, Nychka, and Zervos (1995) and impose a penalty for the "roughness" of the spline interpolation, that is, we fit a "smoothed" spline. As in Waggoner (1997) and Dai, Singleton, and

[1] The data are available at http://www.federalreserve.gov/releases/h15/data.htm

[2] This Online Appendix is not for publication.

Yang (2007), we calibrate the penalty function $\lambda(\tau)$ to be more severe at long maturities, which stiffens the spline interpolation and reduces its oscillations at the long end of the term structure. In contrast, we impose a small penalty for roughness at short maturities, where more flexibility is necessary to fit the prices of short-term securities, which are available in larger number than long-term bonds. Waggoner (1997) recommends using a number of knots approximately equal to one-third the number of bonds used in the construction of the spline. Dai, Singleton, and Yang (2007) instead use a larger number of knots (as many as 50 to 60). In our application, we take an approach that falls in between these two by adopting a number of knots approximately equal to the number of bonds used in the estimation.

Andersen and Benzoni (2010) validate this approach using several diagnostics. For instance, they show that when applied to a panel of Treasury bills and notes, the method produces short-term interest rates that correlate almost perfectly with zero-coupon rates computed directly from Treasury bills of similar maturity. Treasury bills do not pay coupons; thus, up to a small maturity mismatch due to the Department of Treasury's issuing calendar, yields extracted directly from bills are effectively zero-coupon rates. The fact that interpolated zero-coupon rates match the Treasury bills yields lends support to the quality of the interpolation. Moreover, Andersen and Benzoni (2010) document that realized volatilities computed from interpolated zero-coupon rates match those obtained from raw Treasury yields.

We employ the same interpolation and find results consistent with Andersen and Benzoni (2010). For instance, we find that constant-maturity par yields correlate highly with our estimates of zero-coupon rates. The correlation is nearly perfect for short-maturity rates (Table A.1). This supports the accuracy of the interpolation, as at short maturities there is only a small difference between par and zero-coupon yields. At longer maturities correlations exhibit a mild declining pattern. Andersen and Benzoni (2010) find the same results when applying the interpolation to the same constant-maturity par yields over the sample period from 1991 to 2001 (Table AI, page 646).

As a final check, we consider a linear term-structure interpolation (similar to the unsmoothed Fama-Bliss method) as a substitute for the smoothed spline. We estimate the model with these estimates of the zero-coupon rates and find the results to be similar to those we report in the paper.

Overall, this evidence suggests that the conclusions of our analysis are not biased by the noise due to the procedure that we use to extract the series of zero-coupon yields.

A.2 Simulation Scheme for Yields and Inflation Series

We use the following scheme to simulate 10,000 samples of quarterly yields with maturities of one quarter, one, three, five, and ten years, as well as core, food, and energy inflation series:

1. We simulate 10,000 samples of the state vector X from our preferred DTSM$_{3,3}$ starting from the steady state value $X_0 = (I - \Phi)^{-1}\mu$. This produces simulated paths for the latent factors ℓ^k, $k = 1$, 2, and 3, and the core, food, and energy inflation series π^c, π^f, and π^e.

2. We simulate 10,000 samples of weights ω^c, ω^f, and ω^e using the sample mean of the weights series as starting values:

   (a) Using the simulated inflation series π^c and π^f we recover the level of the core and food price deflators Q^c and Q^f:

$$\begin{aligned} Q^c_{t+1} &= Q^c_t \exp\{\pi^c_t\} \\ Q^f_{t+1} &= Q^f_t \exp\{\pi^f_t\}\,. \end{aligned} \tag{A.2}$$

   (b) Given a draw for the weights ω^c_t, ω^f_t, and ω^e_t, and the level of the total price deflator Q_t, we compute $Q_{t+1} = Q_t \exp\{\omega^c_t\pi^c_t + \omega^f_t\pi^f_t + \omega^e_t\pi^e_t\}$.

   (c) We update the ω^c_t, ω^f_t, and ω^e_t weights according to[3]:

$$\begin{aligned} \omega^c_{t+1} &= \frac{Q^c_{t+1}\omega^c_t/Q^c_t}{Q_{t+1}/Q_t} \\ \omega^f_{t+1} &= \frac{Q^f_{t+1}\omega^f_t/Q^f_t}{Q_{t+1}/Q_t}\,, \end{aligned} \tag{A.3}$$

   and determine the residual weight on energy inflation, $\omega^e_{t+1} = 1 - \omega^f_{t+1} - \omega^e_{t+1}$.

3. For each simulated sample, we discard the first 100 draws and retain the remaining 108 observations. This matches the length of the data sample from 1985Q1 to 2011Q4.

4. Given the simulated weights and the estimated coefficients for the preferred DTSM$_{3,3}$ model, we compute the nominal and real factor loadings (A, B) and (A^*, B^*) that solve the ODEs in equations (9) and (14) of the paper.

5. Using equation equation (15) and the factor loadings (A, B), we construct nominal yields with maturities of one quarter, one, three, five, and ten years. To each series, we add a Gaussian measurement error with mean zero and standard deviation equal to 7 basis points.

6. Using equation (10) and the factor loadings (A^*, B^*), we construct nominal yields with maturities of one quarter, one, three, five, and ten years.

[3] See, e.g., the Bureau of Labor Statistics at `http://www.bls.gov/cpi/cpi_riar.htm`.

A.3 The Components of Core and 'Crust'

Here we provide additional information on the core, food, and energy price indices. First, we describe the main constituents of the consumer price indices (CPI) released by the Bureau of Labor Statistics and the personal consumption expenditures (PCE) price indices computed by the Bureau of Economic Analysis. Second, we outline the main differences in these two sets of indices.

A.3.1 Consumer Price Indices

The CPI is a measure of the average change over time in the prices paid by urban consumers for a market basket of consumer goods and services.[4] It is typically used as an economic indicator, as a deflator of other economic series, and as a means of adjusting (1) dollar values of consumers' income payments; (2) income eligibility levels for government programs; (3) cost-of-living wage adjustments; and (4) the Federal income tax structure.

- *The Core Index:* The market basket for this index consists of:
  - Commodities less food and energy: new vehicles; used cars and trucks; apparel; alcoholic beverages; tobacco and smoking products; and medical care commodities.
  - Services less energy services: shelter (rent of primary residence and owners' equivalent rent of residences); transportation services (motor vehicle maintenance and repair, motor vehicle insurance, and airline fare); and medical care services (physicians' and hospital services).
- *The Food Index:* This index includes food at home (cereals and bakery products; meats, poultry, fish, and eggs; dairy and related products; fruits and vegetables; nonalcoholic beverages and beverage materials; other food at home) and food away from home.
- *The Energy Index:* This index includes energy commodities (gasoline and fuel oil) and energy services (electricity and utility (piped) gas service).

A.3.2 Personal Consumption Expenditures Price Indices

The PCE price index measures the average change in prices paid for goods and services by the personal sector in the U.S. national income and product accounts (NIPA); it is primarily used for macroeconomic and policy analysis and forecasting.

- *The Core Index:* This index includes:

[4] See the Bureau of Labor Statistics Internet site at `http://www.bls.gov/cpi/cpifaq.htm`.

- Goods: durable goods (motor vehicles and parts, furnishings and durable household equipment, recreational goods and vehicles, and other durable goods) and nondurable goods (clothing and footwear and other nondurable goods such as pharmaceutical and other medical products, household supplies, personal care products, tobacco, magazines, newspapers, and net expenditures abroad by U.S. residents).
- Services: housing and utilities, health care, transportation services, recreation services, food services and accommodations, financial services and insurance, communication, education, professional services, personal care services, social services, household maintenance, and net foreign travel.
- Final consumption expenditures of nonprofit institutions serving households.

• *The Food Index:* This index includes food and beverages purchased for off-premises consumption. In particular, the food category includes cereals and bakery products, meats and poultry, fish and seafood, milk, dairy products and eggs, fats and oils, fresh and processed fruits and vegetables, sugar and sweets, other food products, not elsewhere classified. The beverages include nonalcoholic (coffee, tea, mineral waters, soft drinks, vegetable juices, and other beverages) and alcoholic beverages (spirits, wine, and beer). The food produced and consumed on farms is also included in the computation of this index.

• *The Energy Index:* This index consists of gasoline and other energy goods, electricity, and gas services.

A.3.3 Comparing the CPI and the PCE Indices

McCully, Moyer, and Stewart (2007) attribute the differences in the PCE and CPI price indices to four main sources:

• *Formula Effect:* The CPI is computed using a modified Laspeyres index formula, while the PCE is based on a Fisher-Ideal index formula (see, e.g., McCully, Moyer, and Stewart (2007) for more details).

• *Weight Effect:* The relative weights of each item price in the CPI and PCE indices are based on different data sources. The weights in the CPI are formed using household surveys, while the weights in the PCE are mainly based on business surveys.

• *Scope Effect:* This effect is due to conceptual differences in the composition of the two indices. PCE measures spending by and on behalf of the personal sector, which includes both households and nonprofit institutions serving households; the CPI measures out-of-pocket spending by households. Thus, some items included in one of the two indices are out-of-scope for the other.

- *Other Effects:* This group includes differences in the computation of seasonal adjustments and the valuation of the items included in the baskets.

Among these effects, McCully, Moyer, and Stewart (2007) find the formula and weight effects to be the most important to reconcile differences in the two indices during the period from 2002Q1 to 2007Q2.

A.4 Nominal Yields Forecasts

Table A.2 shows out-of-sample RMSEs for the forecasts of nominal yields with maturity of one quarter, five and ten years. We first focus on the $\text{DTSM}_{3,3}$ estimated on post-1984 CPI inflation data and CMT yields with maturity up to ten years. The model produces RMSEs that are similar to those of the ARMA estimated on the univariate spot rate series; it outperforms the ARMA on 5- and 10-year maturity yields (the p-values show a significant improvement). The $\text{DTSM}_{3,3}$ outperforms the SPF forecasts of one-quarter and ten-year yields.[5] It beats the random walk on the one-quarter yield, and does as well on the other maturities. Consistent with Calvet, Fisher, and Wu (2010), increasing the number of latent factors to four improves the forecasts slightly. Estimation on the yields gives the same results as estimation on the first four yields' principal components. Similar conclusions characterize the results obtained when estimating the model on PCE data.

Table A.3 reports results for models estimated over long sample periods starting from 1962Q1. For completeness, the table considers the same cases we discussed in Section 4.3 of the paper. Panel A focuses on the 1985Q4-2008Q4 out-of-sample window, while Panel B shows results for the 1998Q4-2008Q4 period. Overall, the findings are similar to those obtained when estimating the model with data starting from 1985Q1 (Table A.2).

Taken together, these results indicate that our preferred $\text{DTSM}_{3,3}$ estimated on nominal yields and inflation data does quite well at forecasting Treasury yields. It is plausible that extending our $\text{DTSM}_{3,3}$ to include other factors (e.g., a measure of real activity or the Cochrane and Piazzesi (2005, 2008) tent-shaped linear combination of forward rates) would further improve these forecasts (e.g., Ang and Piazzesi (2003), Joslin, Priebsch, and Singleton (2010)). Since this is not the focus of our analysis, we point the reader to those studies for more details.

A.5 References

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[5]The SPF defines the one-quarter yield as the quarterly average of the monthly averages of the daily three-month Treasury bill secondary market rates, expressed on a discount basis. The ten-year yield is a similar average of the 10-year constant-maturity Treasury bond rate. We rely on these definitions to determine yield realizations used to compute the RMSEs for the SPF forecasts.

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A.6 Extra Tables

Table A.1: **Sample Percentage Correlations Between Zero-Coupon and Par Treasury Yields.** The table shows sample correlations between constant-maturity daily par yields computed by the U.S. Treasury and released by the Federal Reserve Board and our daily zero-coupon yields interpolated from these constant-maturity par yields.

1M	3M	6M	1Y	2Y	3Y	5Y	7Y	10Y	20Y	30Y
Sample Period: January 2, 1962 to Dec 31, 2009										
N.A.	N.A.	N.A.	99.99	N.A.	99.99	99.97	N.A.	99.92	99.55	N.A.
Sample Period: January 2, 1985 to Dec 31, 2009										
N.A.	100.00	100.00	100.00	100.00	100.00	99.98	99.96	99.92	99.64	98.30
Sample Period: August 1, 2001 to Dec 31, 2009										
100.00	100.00	100.00	100.00	100.00	99.99	99.96	99.84	99.53	96.47	95.48

Table A.2: **Treasury Yields Forecasts: Post-1984 Estimation**. We use data from 1985Q1 for model estimation and forecast nominal treasury yields out of sample at the one-year horizon over 1998Q4 to 2008Q4. Section 4.1, "Model Specifications and Fit," explains the different model specifications. For each model, the table shows the p-value for a test of equal forecast accuracy (West (1996)) computed under the null that the RMSE for that model equals the RMSE for the ARMA(1,1), when the alternative is that the RMSE for the ARMA(1,1) exceeds the RMSE for that model.

	1Q Yield		5Y Yield		10Y Yield	
	RMSE	p-val.	RMSE	p-val.	RMSE	p-val.
Panel A: Univariate models and survey forecasts						
ARMA	1.69		1.19		0.85	
RW	2.19	1.00	1.18	0.46	0.72	0.03
SPF	1.85	0.87			0.95	0.99
Panel B: DTSMs, estimation on CPI data						
DTSMs, CMT yields $\leq$ 10Y, Kalman Filter estimation on the yields' PCs						
$DTSM_{3,3}$ AR(1-1-1)	1.64	0.22	1.06	0.07	0.75	0.06
$DTSM_{4,3}$ AR(1-1-1)	1.67	0.41	0.98	0.10	0.71	0.14
$DTSM^{*}_{3,3}$ AR(1-1-1)	1.87	0.92	1.23	0.78	0.87	0.64
$DTSM_{3,2}$	1.62	0.17	1.06	0.10	0.77	0.11
$DTSM^{*}_{3,2}$	1.90	0.94	1.31	0.98	0.94	0.95
DTSMs, CMT yields $\leq$ 10Y, Kalman Filter estimation on the yields						
$DTSM_{3,3}$ AR(1-1-1)	1.65	0.25	1.06	0.05	0.75	0.03
Panel C: DTSMs, estimation on PCE data						
DTSMs, CMT yields $\leq$ 10Y, Kalman Filter estimation on the yields' PCs						
$DTSM_{3,3}$ AR(1-1-1)	1.64	0.22	1.06	0.07	0.76	0.05
$DTSM_{4,3}$ AR(1-1-1)	1.70	0.51	0.99	0.10	0.71	0.13
$DTSM^{*}_{3,3}$ AR(1-1-1)	1.88	0.93	1.27	0.93	0.91	0.82
$DTSM_{3,2}$	1.62	0.17	1.05	0.10	0.76	0.10
$DTSM^{*}_{3,2}$	1.89	0.93	1.29	0.97	0.92	0.91
DTSMs, CMT yields $\leq$ 10Y, Kalman Filter estimation on the yields						
$DTSM_{3,3}$ AR(1-1-1)	1.68	0.41	1.12	0.13	0.82	0.25

Table A.3: **Treasury Yields Forecasts**. We estimate each model using yields data from 1962Q1 and forecast yields at the one-year horizon over the 1985Q4-2008Q4 and 1998Q4-2008Q4 out-of-sample periods. Section 4.1, "Model Specifications and Fit," explains the different model specifications. For each model, the table shows the p-value for a test of equal forecast accuracy (West (1996)) computed under the null that the RMSE for that model equals the RMSE for the ARMA(1,1), when the alternative is that the RMSE for the ARMA(1,1) exceeds the RMSE for that model.

Panel A: 1985Q4 to 2008Q4 out-of-sample period

	1Q Yield		5Y Yield		10Y Yield	
	RMSE	p-val.	RMSE	p-val.	RMSE	p-val.
Univariate models, CRSP yields						
ARMA	1.61		1.11			
RW	1.87	1.00	1.17	0.85		
Univariate models, CMT yields						
ARMA			1.13		0.93	
RW			1.19	0.87	0.95	0.63
Survey Forecasts						
SPF	1.54	0.03				
DTSMs, CRSP yields, Chen-Scott estimation						
$DTSM_{2,1}$ tot	1.48	0.15	1.10	0.30		
$DTSM_{2,1}$ core	1.48	0.15	1.09	0.26		
$DTSM_{2,3}$ AR(1-1-1)	1.45	0.10	1.09	0.26		
$DTSM_{2,3}$ AR(3-1-1)	1.47	0.12	1.10	0.32		
DTSMs, CRSP yields, Kalman Filter estimation on yields and their PCs						
$DTSM_{2,3}$ AR(3-1-1) yields	1.52	0.22	1.13	0.66		
$DTSM_{2,3}$ AR(3-1-1) PCs	1.42	0.09	1.03	0.09		
DTSMs, CMT yields $\leq$ 10Y, Kalman Filter estimation on the yields' PCs						
$DTSM_{3,3}$ AR(1-1-1)	1.48	0.13	1.12	0.44	0.93	0.57
DTSMs, CMT yields $\leq$ 20Y, Kalman Filter estimation on the yields' PCs						
$DTSM_{3,3}$ AR(1-1-1)	1.53	0.26	1.05	0.09	0.93	0.54
$DTSM_{4,3}$ AR(1-1-1)	1.54	0.26	1.18	0.82	1.00	0.98
$DTSM_{5,3}$ AR(1-1-1)	1.47	0.11	1.10	0.37	0.95	0.71

Table A.3, continued

Panel B: 1998Q4 to 2008Q4 out-of-sample period	1Q Yield		5Y Yield		10Y Yield	
	RMSE	p-val.	RMSE	p-val.	RMSE	p-val.
Univariate models, CRSP yields						
ARMA	1.94		1.15			
RW	2.19	0.94	1.18	0.68		
Univariate models, CMT yields						
ARMA			1.16		0.79	
RW			1.18	0.61	0.72	0.06
Survey Forecasts						
SPF	1.85	0.06			0.95	1.00
DTSMs, CRSP yields, Chen-Scott estimation						
$DTSM_{2,1}$ tot	1.65	0.07	1.11	0.20		
$DTSM_{2,1}$ core	1.64	0.07	1.10	0.18		
$DTSM_{2,3}$ AR(1-1-1)	1.63	0.07	1.10	0.20		
$DTSM_{2,3}$ AR(3-1-1)	1.65	0.07	1.11	0.20		
DTSMs, CRSP yields, Kalman Filter estimation on yields and their PCs						
$DTSM_{2,3}$ AR(3-1-1) yields	1.66	0.03	1.10	0.25		
$DTSM_{2,3}$ AR(3-1-1) PCs	1.58	0.06	0.99	0.04		
DTSMs, CMT yields $\leq$ 10Y, Kalman Filter estimation on the yields' PCs						
$DTSM_{3,3}$ AR(1-1-1)	1.64	0.03	1.07	0.10	0.75	0.16
DTSMs, CMT yields $\leq$ 20Y, Kalman Filter estimation on the yields' PCs						
$DTSM_{3,3}$ AR(1-1-1)	1.67	0.05	0.98	0.05	0.73	0.05
$DTSM_{4,3}$ AR(1-1-1)	1.70	0.04	1.11	0.23	0.82	0.70
$DTSM_{5,3}$ AR(1-1-1)	1.64	0.04	1.02	0.13	0.74	0.18