Managerial Performance Evaluation Under Uncertain Demand^{*}

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Abstract

In a dynamic setting with demand following a random process, we ask how investment and operating decisions can be delegated to a manager with unknown time preferences. Only the manager observes the demand realization in each period and, therefore, has private information when choosing whether to acquire the productive asset and, subsequently, how to utilize it. We derive accrual accounting-based performance measures under which the manager will make the efficient decisions provided the investment date is exogenously given. We show that, in an environment where demand follows a martingale process, the corresponding accounting rules are more decelerated if the firm has the option to idle capacity in case of negative demand shocks. We then extend our results to a scenario in which the investment date is endogenously determined, i.e., the firm has an option to postpone its investment.

1 Introduction

The canonical net present value (NPV) rule calls for one-off investment decisions at a predetermined point in time to be made if and only if the sum of the expected discounted cash flows is positive. Extending this rule to dynamic environments poses two challenges. First, the future benefits of an investment depend on uncertainty resolved only over time and the firm's reaction to the new information about changing market conditions. Second, many investments are not of the "now or never" kind but instead may be timed endogenously by the firm. The *real options* paradigm (Dixit and Pindyck, 1994) has proved fruitful in terms of generating novel predictions for optimal investment timing as a function of characteristics such as the degree of market uncertainty or discount rates.¹ The question that remains, however, is how real options affect the firm's ability to *delegate* investment (and operating) decisions to its managers who may have divergent objectives. This paper aims to answer this question by developing a model in which market conditions follow a dynamic process. The periodic realizations of the market size are privately observed by a firm manager with unknown time preferences, who makes the initial investment decision and subsequent operating decisions regarding the use of the capacity initially acquired.

For settings with a single "now-or-never" investment decision, prior literature (e.g., Rogerson, 1997; Reichelstein, 1997) has shown that investment decisions can be delegated to managers with unknown time preferences by relying on historical cost-based performance measures.² In our fully dynamic setting, two kinds of optionality arise in connection with the acquisition and operation of productive assets. As described above, a flexible investment date generates an option to postpone the investment. But even with an investment date that is exogenously given, the possibility to fully utilize, or partially idle, the productive capacity at a later stage creates a "usage option."³ The fact that future operating decisions

¹On a conceptual level, however, the NPV rule is robust to the introduction of real options in that the NPV can be defined so as to include all relevant option values. Specifically, the firm's expected cash flows can be calculated taking into account the firm's optimal reaction to the new information, and the option value foregone by investing at any given point can be added to the initial cash outflow. The predictions of the real options literature can then be interpreted in terms of the "overall NPV" terminology. This alternative interpretation is well acknowledged in the real options literature; see, e.g., pp. 6-7 in Dixit and Pindyck (1994).

²This literature started with Rogerson (1997); see also Reichelstein (1997), Baldenius and Ziv (2003), Dutta and Reichelstein (2005). The key insights from the goal congruence literature can often be generalized in a straightforward manner to formal agency models, e.g., Christensen et al. (2002), Dutta and Reichelstein (2002), Baldenius and Reichelstein (2005), Pfeiffer and Schneider (2007).

³See also Pindyck (1988), Dixit and Pindyck (1994), and Reichelstein and Rohlfing-Bastian (2014).

can be adjusted to the then prevailing market conditions has a feedback effect on the exante investment decision. To ask how these two types of optionality affect the efficiency of delegated decision making, we first look at a simplified problem in which the investment date is exogenous—i.e., only the usage option is present—and then allow for both forms of optionality to be present by endogenizing the investment date.

With an exogenous investment date, we find that residual income based on historical cost accounting achieves goal congruence: the manager will use his private information in such a manner so as to invest efficiently and then utilize the asset efficiently in subsequent periods.⁴ Specifically, goal congruence is achieved if the firm uses a depreciation rule (which we label the *Relative Expected Optimized Benefits* or the REOB schedule) that matches the cost of the investment with the periods of its useful life in proportion to the expected and properly discounted relative benefits, factoring in the conditionally-optimal capacity utilization decisions. Taking expectations over future demand scenarios is complicated by the manager's informational advantage regarding future demand, as even the *relative* expected benefits over time will generally be a nontrivial function of the current market size—and the current market size is the manager's private information. The REOB schedule derived in this paper nonetheless successfully aligns incentives by allocating capital costs to the periods in proportion to their expected periodic benefits (given optimal operating decisions) evaluated at the threshold (i.e., zero expected NPV) market size. As a result, the compensation for a manager who invests given a privately observed market size above (below) this threshold will increase (decrease) in expectation for each future period.⁵

Given the more demanding notion of goal congruence arising from the presence of operating decisions, how does this performance measure compare with that identified for models with a single decision date, e.g., Rogerson (1997)? To answer this question, we first verify that if the firm did not have the option to idle capacity when demand is low, our REOB rule would coincide with the depreciation schedule identified in Rogerson (1997).⁶ We find

⁴This is despite the fact that the manager has better information than the shareholders at any point in time. Thus, the information asymmetry in our model is magnified compared with earlier models with only one decision date, e.g., Rogerson (1997) and Reichelstein (1997).

⁵Hence, the manager internalizes the shareholders' objective at the investment date. Furthermore, in future periods, the amount of capacity costs reflected in the manager's performance measure is not affected by the actual operating decisions, avoiding the well-known problems associated with absorption costing, so the manager is incentivized to use the available capacity optimally.

⁶To make our model comparable to that in Rogerson (1997), we impose two additional assumptions that i) asset are equally productive over their finite useful life, and ii) the permanent component of the product market size follows a martingale process, i.e., the expectation of price tomorrow given a certain quantity is

that the usage option makes the REOB rule more decelerated (backloaded) relative to the scenario where the firm cannot adjust its capacity utilization. The key force driving this result, at a technical level, is that the (optimized) contribution margin becomes a convex function of the market size realization: the usage option allows the firm to mitigate the effect of unfavorable shocks to demand by idling capacity and saving on variable costs, hence bending upwards the "left tail" of the expected benefit function. Then, Jensen's inequality implies that, from the perspective of the investment date, the expected *optimized* benefits of investment increase over time for a stationary demand process. We show that this, in turn, makes the depreciation rule that facilitates efficient delegation of the initial investment decision more decelerated relative to the benchmark without the usage option.

We next study the effects of demand growth (drift) and demand uncertainty on the REOB depreciation rule. We show that REOB depreciation becomes more decelerated with increases in the drift of the market size process or increases in its variance.⁷ While the drift effect is rather obvious—e.g., outward expected shifts in the inverse demand function over time increase the relative expected benefits in later periods—the variance effect is more subtle. It again hinges on the fact that operating decisions, taken optimally, effectively mitigate the profit implications of negative demand shocks. This result stands in contrast to the traditional view in financial accounting that calls for more accelerated depreciation in environments where investment benefits are less certain.⁸

We then extend our setting to allow for the investment date to be chosen endogenously: it can be undertaken at any point in time, but only once. As is well known from the real options literature (e.g., Pindyck 1988, Dixit and Pindyck 1994), the threshold market size for the investment to be undertaken now exceeds the threshold with a fixed investment date, because the net present value of expected cash flows has to cover not just the investment cash cost but also the option value. After identifying the first-best market size threshold above which fully informed shareholders would want to invest, we again turn to the issue of delegation. Unknown managerial time preferences now cause additional frictions, as they now affect not

equal to the price today at the same quantity level.

⁷Our formal results in Proposition 3 and Corollary 1 rely on additional assumptions about the asset's productivity profile.

⁸For example, Penman and Zhang (2014) observe that "... if revenue from an investment is particularly uncertain, the investment is expensed immediately—as in the case of R&D and advertising—or subject to rapid amortization. In justifying the immediate expensing of R&D under FASB Statement No. 2, the FASB focused on the 'uncertainty of future benefits'."

just the manager's assessment of the present value of the project to be undertaken but also that of the option value. To illustrate, it is useful to describe (heuristically) the distortions that would arise under plausible candidate performance measures.

Suppose the performance measure is residual income with capital charges imputed based on the shareholders' cost of capital, combined with REOB depreciation—i.e., the goal congruent solution if the investment date is exogenous. If the manager had the same time preferences as the shareholders, then this method would induce goal congruence even with an option to wait.⁹ But if the manager were to discount future cash flows at a higher rate than the shareholders, then he would undervalue the option to wait and overinvest (invest too early). In response to this overinvestment bias, the shareholders may consider raising the capital charge rate used for calculating residual income, say, to the internal rate of the return of the marginal investment project (the one that leaves the shareholders indifferent between investing right away and postponing the decision).¹⁰ This approach indeed can induce firstbest investment decisions, but only for a manager who is myopic in that he cares solely about the present period. Other, more patient, managers would underinvest (invest too late) as they would effectively "double-count" the option value: (i) it is reflected in higher capital charges and (ii) the manager will impute his personal option value, because by waiting a period he may potentially garner higher future compensation. A manager compensated on residual income with a hurdle rate equal to the threshold internal rate of return gets zero expected compensation from investing at the threshold market size. Waiting for a period, on the other hand, carries a positive option value; hence the underinvestment bias.

Formalizing these heuristic arguments, we derive an impossibility result: no linear performance measure can achieve goal congruence when there is an option to wait. Given this negative result, a natural question is whether a weaker form of goal congruence can be established. The last result of our paper derives a family of performance measures that allow for efficient delegation of investment and operating decision if the shareholders were to know

⁹This follows immediately from the fact that residual income is fundamentally consistent with discounted cash flow considerations (e.g., Ohlson 1995, Feltham and Ohlson 1996).

¹⁰Several studies have documented that hurdle rates used by firms for investment decisions often exceed the cost of capital (see, for instance, Poterba and Summers 1995 and Summers 1987). Dixit and Pindyck (1994) suggest that the difference between hurdle rates and the cost of capital can be explained by the option value of waiting (see, for example, p.7). This would be consistent with the perfomance measure that we describe above. Our results demonstrate, however, that, in the presence of a delegation problem, setting the hurdle rate equal to the IRR of the marginal project will result in incentives to *underinvest* for managers who discount payoffs at a rate close to that of shareholders.

the manager's time preferences.¹¹ We show that efficient incentives can be provided if both residual income and the REOB depreciation rule are calculated relative to a judiciously chosen hurdle rate that only partially reflects the net present value of the marginal investment project. The hurdle rate must be chosen such that, at the threshold market size, the sum of the option value reflected in the higher capital charges of the performance measure and the manager's own option value is precisely equal to the NPV of the marginal project. Since the manager's option value intrinsically depends on his discount factor, so must the hurdle rate that is used in calculating the manager's performance measure.

The effect of capacity constraints and excess capacity on a firm's cost structure and its product pricing decisions is a classic topic in accounting research (e.g., Banker and Hughes 1994; Göx 2001, 2002; Kallapur and Eldenburg 2005; Banker et al. 2013; Reichelstein and Rohlfing-Bastian 2014).¹² Our model extends this literature by studying the issues of delegation and performance measurement. To the best of our knowledge, the only paper that embeds the option to postpone investment in such a performance measurement framework is Friedl (2005). One of the main differences between Friedl (2005) and our model is that in his model the value of the option to wait is common knowledge. This leads him to conclude that goal congruence is generally achievable with an endogenous investment date—contrary to our findings. Several recent papers consider models where the firm can continuously make investments to expand its capacity (e.g., Rogerson 2008, Rogerson 2011, Nezlobin et al. 2014). However, these papers assume that the product market is always weakly expanding, thus rendering trivial any capacity utilization and investment timing decisions (i.e., in these models, it is always optimal to utilize capacity fully and new investments are made in essentially all periods). Lastly, our paper provides new insights about the structure of hurdle rates for investment decisions in delegation settings (e.g., Poterba and Summers 1995, Christensen et al. 2002, Dutta and Fan 2009).

The rest of the paper is organized as follows. Section 2 lays out the basic model structure. Section 3 addresses a setting where the investment date is given exogenously and Section 4

¹¹Our definition of weak goal congruence is slightly more general than that in Reichelstein (1997), because in that paper the manager's discount factor was assumed to coincide with the shareholders'.

¹²Banker and Hughes (1994) study product pricing and capacity planning decisions in a model with demand uncertainty. However, they do not consider the problem of incentivizing a better informed manager who is responsible for both investment and operating decisions. Another difference between our model and the one in Banker and Hughes (1994) is that in that paper the capacity constraint is "soft" in the sense that it can be relaxed ex-post at an additional cost. See Göx (2002) for a model that incorporates both "soft" and "hard" capacity constraints.

one in which there is an option to wait. Section 5 extends some of our findings from discrete to scalable investment decisions. Section 6 concludes.

2 Model Setup

2.1 The Firm, Capacity, Product Market, and Information

Consider a firm owned by a risk-neutral owner (the principal), managed by a risk-neutral manager (the agent). The firm can invest in a long-lived capacity asset. Initially, we analyze a scenario with a single binary investment decision: at date 0, the firm can either invest b dollars to build capacity for future periods or irretrievably lose the investment opportunity.¹³ We let the indicator function $I \in \{0, 1\}$ denote whether the investment was made or not. If the investment is undertaken (I = 1), the firm can produce at most x_{τ} units of its single product in period $\tau \in \{1, \ldots, T\}$. Let K_{τ} denote the firm's period- τ capacity:¹⁴

$$K_{\tau} = I \cdot x_{\tau}$$

for $1 \leq \tau \leq T$. The vector $\boldsymbol{x} \equiv (x_1, \ldots, x_T)$ describes the productivity pattern of the firm's asset. For some of the results to follow, we will invoke a *one-hoss shay* scenario: for $\tau \in \{1, \ldots, T\}$, all x_{τ} are equal.¹⁵

Demand in the product market is uncertain. For any quantity q > 0, the inverse demand function in period t is given by $P_t(q) = \mu_t \cdot P(q)$. Specifically, we assume a constant-elasticity inverse demand function:¹⁶

$$P_t(q) = \mu_t \cdot q^{-\frac{1}{\eta}},\tag{1}$$

¹³Later, we consider settings with an investment-timing option and a continuous investment decision.

¹⁴In our model, capacity cannot be extended. In Banker and Hughes (1994), in contrast, capacity can be extended at a penalty cost to satisfy greater than expected demand.

¹⁵The one-hoss shay productivity pattern is frequently seen in the regulation literature (e.g., see Laffont and Tirole, 2000; Nezlobin, Rajan, and Reichelstein, 2012; and Rogerson, 2008, 2011). Alternatively, the finance literature often considers a geometric-decline scenario, (e.g., see Dixit and Pindyck, 1994; Biglaiser and Riordan, 2000), in which $T = \infty$ and $x_{\tau} = (1 - \alpha)^{\tau-1}$, with $\tau \in \{1, \ldots, \infty\}$ and $\alpha \in (0, 1)$. Both of these commonly considered patterns reflect the idea that the asset's capacity is weakly declining with age due to the physical depreciation of the asset or increasing maintenance requirements.

¹⁶We use subscript t to denote periods of time relative to date 0, and subscript τ to denote periods of the asset's useful life relative to the time the asset was acquired. In a scenario with a single investment made at date 0, these two subscripts have essentially the same meaning; the difference becomes important later, when we consider settings where the firm can postpone the investment.

with $\eta > 1.^{17}$ Here, μ_t is the realization of a stochastic *price shift parameter* describing the evolution of the size of the product market over time: for a given level of quantity q, the price between periods t and t+1 changes at a rate $\frac{\mu_{t+1}}{\mu_t}$. If the firm makes and sells $q_t \leq K_t$ units in period $t \in \{1, \ldots, T\}$, cash flows equal

$$R\left(\mu_t, q_t\right) - v \cdot q_t,$$

where $R(\mu_t, q_t) \equiv P_t(q_t) \cdot q_t$ is the firm's revenue, and v > 0 is the unit variable cost. Given the assumed constant elasticity demand formulation in (1), revenues are strictly increasing and concave in quantity. Note that we allow for the product market to contract over time, i.e., μ_t may decrease in t. Therefore, if the firm invests in period 0, it may not find it profitable to utilize its capacity fully in some of the future periods.

Product-market uncertainty and asymmetric information are key ingredients of our analysis. The stochastic price shift parameter μ_t represents uncertain market size; this parameter evolves over time with both a permanent shock and a transitory shock. Specifically,

$$\mu_t = (1 + \epsilon_t) m_{t-1}, \tag{2}$$

with the permanent shock captured by the random variable m_{t-1} , drawn from a probability distribution with support on $(0, \infty)$ at the beginning of period t (i.e., date t - 1), and the transitory shock captured by the random variable ϵ_t , realized at the end of period t (i.e., date t). Transitory shocks $(1 + \epsilon_t)$, are independently and continuously distributed over $(0, \infty)$, with $\mathbb{E}[1 + \epsilon_t] = 1$ and $Var[1 + \epsilon_t] = \sigma_{\epsilon}^2$.

The permanent component of the price shift parameter evolves as follows: at date 0, the manager privately learns the realization of m_0 that determines the permanent demand component in period 1. Then, at the beginning of every period t:

$$m_{t-1} = (1 + g_{t-1}) \cdot m_{t-2}, \tag{3}$$

with the realization of g_{t-1} observed at date t-1 by the manager, but not by the owner.¹⁸

¹⁷It can be shown that some of our results (Proposition 1, Proposition 2 and Corollary 1) hold for (inverse) demand functions more general than those in (1). For those results, we could drop the constant elasticity demand formulation and instead simply assume that $P_t(q) = \mu_t \cdot P(q)$, for $P(\cdot)$ twice differentiable and strictly increasing, and revenues $P(q) \cdot q$ strictly increasing and concave in q.

¹⁸We therefore assume that the manager perfectly anticipates one-period ahead permanent shocks to the

The growth factors, $(1 + g_{t-1})$, are independently log-normally distributed, with $\mathbb{E}[1+g_{t-1}] = 1 + g^*$ and $Var[1+g_t] = \sigma_g^2$. The permanent component of the price shift parameter is a martingale if $g^* = 0$.¹⁹ We assume that the parameters $(g^*, \sigma_g^2, \sigma_\epsilon^2)$ are commonly known. Once period-t cash flows are realized, μ_t can be inferred from revenues and quantities, since both the manager and the owner know the functional form of the demand curves and observe capacity utilization in every period.

With the structure of price shift parameters described above, the manager always enjoys an informational advantage over the firm's owners: when selecting the period-0 investment, the manager privately knows m_0 ; and when choosing period-t quantities, the manager privately knows m_{t-1} . The exact timeline is:

- At date 0, the manager learns the realization of m_0 . The manager then decides whether to invest b dollars to make capacity available in future periods.
- At the beginning of each period $t \in \{1, \ldots, T\}$, the manager learns the realization of m_{t-1} . The manager then selects the period-t quantity level $q_t(m_{t-1}) \leq K_t$. The period's revenues $R(\mu_t, q_t(m_{t-1}))$ and costs $v \cdot q_t(m_{t-1})$ are realized and observed by the manager and the owner at the end of the period (date t), allowing both to infer the realization of μ_t .²⁰

The firm's cost of capital is r; the corresponding discount factor is $\gamma = \frac{1}{1+r}$. If the firm invests in period 0 and sells $q_t(m_{t-1}) \leq K_t$ units in periods $t \in \{1, \ldots, T\}$, the firm's period-0 value is

$$-b + \sum_{t=1}^{T} \gamma^{t} \cdot \mathbb{E}_{0} \left[R\left(\mu_{t}, q_{t}\left(m_{t-1}\right)\right) - v \cdot q_{t}\left(m_{t-1}\right) \right].$$

$$\tag{4}$$

Let $I^*(m_0) \in \{0, 1\}$ denote the optimal investment policy. The owner's problem is to provide incentives for the manager to choose the initial investment $I^*(m_0)$ and quantities $q_t^*(m_{t-1})$

²⁰In our model, the principal cannot offer a "forcing" contract to the agent, under which the manager is compensated only if the quantity produced in a period is equal to the ex-post efficient level, because the manager has to make investment and production decisions based on imperfect information about demand.

inverse demand function, but does not have any advance information about the transitory shocks. Our results are unchanged if the manager's information about permanent shocks is imperfect or if the manager gets signals about transitory shocks as well.

¹⁹To summarize, we assume that demand shocks affect the stochastic price shift parameter in a multiplicative fashion. This specification is consistent with much of the dynamic real options literature (e.g., Dixit and Pindyck 1994, Chapters 5 and 6). In contrast, many studies on product costing and pricing assume that shocks to demand are additive (see, e.g., Banker and Hughes 1994, Göx 2001, 2002). One advantage of the multiplicative assumption for our model is that it ensures that the product price is non-negative after any sequence of unfavorable shocks to the price shift parameter.

that maximize (4) subject to the capacity constraint: for any t,

$$q_t^*(m_{t-1}) \le K_t = I^*(m_0) \cdot x_t.$$
(5)

2.2 Performance Measures, The Manager's Preferences, and Goal Congruence

To guide the manager's investment and operating decisions, the owner uses performance measures $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_T)$. We consider performance measures that, in each period $t \in \{1, \ldots, T\}$, are based on that period's observed accounting information: the firm's realized contribution margin, depreciation, and the asset's book value. We use d_{τ} to represent the period- τ depreciation charge per dollar of upfront investment. The investment is fully capitalized at date 0, and the date- τ book value per dollar of upfront investment is given by

$$bv_{\tau} = 1 - \sum_{i=1}^{\tau} d_i$$

for $0 \le \tau \le T$, with $bv_T = 0$ (i.e., $\sum_{\tau=1}^T d_\tau = 1$). The total period-t depreciation charge is

$$D_{\tau} = d_{\tau} \cdot b,$$

and the date- τ book value of assets is $BV_{\tau} = bv_{\tau} \cdot b$. Let $\boldsymbol{d} = (d_1, \ldots, d_T)$ represent the depreciation schedule. For any $q_t \leq K_t$, period-t income is then equal to

$$Inc_t = R\left(\mu_t, q_t\right) - v \cdot q_t - D_t.$$

We will say that depreciation rule d is more accelerated than d' if book values per dollar of upfront investment are lower under rule d:

$$bv_{\tau} \leq bv'_{\tau}$$

for all τ . Likewise, we will refer to a depreciation rule d as more decelerated than d' if $bv_{\tau} \geq bv'_{\tau}$ for all τ .

Realizations of accounting variables depend on the manager's investment and operating

decisions. The owner seeks to design the performance measure to guide the manager to make optimal decisions. The manager is risk neutral and attaches arbitrary utility weights β_i to future payoffs in period t. These weights may reflect both the manager's discount rate and the bonus parameters attached to performance measures. Formally, at any date $t \in \{0, \ldots, T-1\}$, the manager's utility function is given by:

$$U_t = \mathbb{E}_t \left[\sum_{i=t}^T \beta_i \cdot \pi_i \mid m_t \right].$$
(6)

For example, a recurring theme in the incentives literature is that managers may be less patient than owners. To accommodate this in our framework, let $\beta_t = \hat{\gamma}^t \cdot u$ for some time-invariant bonus coefficient, u > 0, and the manager's personal discount factor, $\hat{\gamma} \equiv \frac{1}{1+\hat{r}}$, with $\hat{r} > r$.

The manager selects each period's quantity q_t to maximize the objective in (6) subject to the capacity constraint that

$$q_t \le K_t = I \cdot x_t. \tag{7}$$

Note that, as long as the depreciation schedule is determined in advance, each period's depreciation is independent of that period's quantity, and the manager's quantity choice will not affect future performance measures.

At the investment date, the manager observes m_0 , anticipates making future quantity choices that maximize (6) based on the available information in each period, and invests if and only if

$$U_0 = \mathbb{E}_0\left[\sum_{i=0}^T \beta_i \cdot \pi_i \mid m_0\right] > 0.$$

The principal does not observe the manager's time preferences and seeks to design a performance measure that will provide incentives for optimal investment and operating decisions for all possible time preferences of the manager. Formally, a performance measure $\boldsymbol{\pi}$ attains goal congruence for operating and investment decisions if and only if, for any $\boldsymbol{\beta} = (\beta_0, \beta_1, ..., \beta_T)$: (i) each period $t \in \{1, \ldots, T\}$, the optimal owner's quantity $q_t^*(m_{t-1})$ maximizes the manager's objective in (6) subject to the capacity constraint (7); (ii) the optimal owner's investment choice $I^*(m_0)$ maximizes the manager's objective at the investment date 0; and (iii) if $\beta_t > 0$ for at least one $t, 1 \leq t \leq T$, the manager must strictly prefer to implement a project if it makes the principal strictly better off.²¹

To summarize, the model entails a truly dynamic goal congruence setting in which information arrives over time and, after the initial asset acquisition decision, the manager continues to make a sequence of operating decisions regarding the utilization of the asset.

3 Single Investment Opportunity

Earlier studies on performance measurement for investment decisions have shown that goal congruence can be attained under certain accrual accounting rules if residual income is used as the performance measure (e.g., Rogerson 1997, 2008). Residual income is defined as the difference between the firm's net income and an imputed interest charge on the beginning-of-period book value of assets:

$$RI_t = Inc_t - r \cdot BV_{t-1}.$$

We will refer to the sum of depreciation expense and the interest charge on the book value of assets as the historical cost of capacity in period t:

$$z_t \cdot b \equiv (d_t + r \cdot bv_{t-1}) \cdot b = D_t + r \cdot BV_{t-1}.$$

Let $\boldsymbol{z} \equiv (z_1, ..., z_T)$. Earlier literature has shown that there is a one-to-one mapping between the vectors of per-dollar historical cost charges, \boldsymbol{z} , satisfying

$$\sum_{\tau=1}^{T} \gamma^{\tau} \cdot z_{\tau} = 1,$$

²¹We impose condition (iii) to avoid trivial solutions, where the manager is always indifferent between investing and not investing in the project. Note that if only $\beta_0 > 0$ and all other $\beta_i = 0$, then satisfying condition (*iii*) is impossible, since π_0 is calculated before any cash flows from the project are realized. Since our focus is on inducing the privately informed manager to make first-best decisions in a robust manner (i.e., for any possible time preferences and any realizations of the product market size), the principal's probability distribution over the dynamic demand process is irrelevant for the solution. Also, the solution will be simplified by the fact that, once the investment is made, the operating decisions in each period are intertemporally separable. The solution to the overall problem, however, still requires applying dynamic programming on the part of the decision maker, because the ex-ante investment decision will depend on the anticipated future demand and the anticipated future operating decisions.

and depreciation schedules, d, satisfying the (nominal) clean surplus requirement²²

$$\sum_{\tau=1}^{T} d_{\tau} = 1$$

Therefore, in order to define a certain depreciation rule, it suffices to specify the corresponding vector of historical cost charges.

We now turn to characterizing a depreciation rule based on which residual income is a goal congruent performance measure. First, observe that if the firm invests, the expectation at date 0 of the firm's period- τ optimized cash flow is increasing in the (permanent component of the) price shift parameter, m_0 . This is explained by the fact that higher realizations of m_0 correspond to greater expected values of m_{τ} , for any $\tau \geq 1$, which, in turn, imply a greater expected price for any given quantity of product supplied in that period.²³ As a consequence, if the firm finds it optimal to invest for a certain realization of m_0 , then the investment should also be made for all greater realizations of m_0 . Therefore, the first-best investment policy is characterized by a threshold market size, \overline{m}_0 , such that the investment is optimal if and only if $m_0 \geq \overline{m}_0$. Since the principal observes all parameters of the model, except for the actual process realization, $\{m_{\tau}\}$, the threshold \overline{m}_0 is known to the principal.

At the threshold market size, the principal must be indifferent between investing and not investing in the capital asset, which implies that the net present value of the investment opportunity is zero. Therefore, \overline{m}_0 is such that

$$b = \sum_{\tau=1}^{T} \gamma^{\tau} \cdot \mathbb{E}_0 \left[\mathbb{E}_{\tau-1} \left\{ R \left(\mu_{\tau}, q_{\tau}^* \left(m_{\tau-1} \right) \right) - v \cdot q_{\tau}^* \left(m_{\tau-1} \right) \right\} \mid m_0 = \overline{m}_0 \right].$$
(8)

It will be convenient to denote the expected value of the *optimized constrained contribution* margin in period t by $CM_t^*(K_t, m_{t-1})$:

$$CM_{t}^{*}(K_{t}, m_{t-1}) = \mathbb{E}_{t-1} \left\{ R\left(\mu_{t}, q_{t}^{*}(m_{t-1})\right) - v \cdot q_{t}^{*}(m_{t-1}) \right\}.$$

At the threshold market size, \overline{m}_0 , the expected discounted value of constrained contribution margins is equal to the initial investment.

 $^{^{22}\}mathrm{See}$ Rogerson (1997) and Reichelstein (1997).

²³Technically speaking, for any m_0^o and $m_0^{oo} > m_0^o$, the distribution over m_t , t > 0, conditional on m_0^{oo} first-order stochastically dominates that conditional on m_0^o

Let us consider a depreciation rule, d^* , corresponding to the following vector of historical cost charges, \boldsymbol{z}^* , with

$$z_{\tau}^{*} = \frac{\mathbb{E}_{0}\left[CM_{\tau}^{*}\left(K_{\tau}, m_{\tau-1}\right) \mid \overline{m}_{0}\right]}{b}.$$
(9)

It follows from equation (8) that the discounted value of all z_{τ}^* is equal to one, therefore, the corresponding depreciation rule will satisfy the clean surplus condition, $\sum_{\tau=1}^{T} d_{\tau}^* = 1$. We will refer to this depreciation rule as *Relative Expected Optimized Benefit (REOB) depreciation*.²⁴

The REOB rule allocates the cost of investment to different periods in proportion to the expected optimized contribution margin the investment will generate in those periods at the threshold market size. (Henceforth we will drop the "expected" and just say "optimized contribution margin" when there is no potential for confusion.) This rule is related to the Relative Benefit Rule identified in Rogerson (1997), with the main difference that the expected benefits in each period are calculated assuming optimal future capacity utilization decisions. This stems from the fact that, as described above, the manager in our setting makes both investment and operating (quantity choice) decisions.

Let us now consider a manager who is compensated based on residual income calculated using the REOB rule. First, if the firm invests at date 0, then at date t - 1, the manager will choose a production level that maximizes the expected residual income in the following period:

$$\mathbb{E}_{t-1}\left[R\left(\mu_t, q_t\right) - v \cdot q_t \mid m_{t-1}\right] - z_t^* \cdot b.$$

Since the historical cost charge does not depend on the actual asset utilization, the manager will choose the quantity that maximizes the expected contribution margin, which is the optimal quantity for the firm.

Second, note that from the perspective of date t - 1, the expected residual income in period t, based on the REOB rule, can be rewritten as:

$$CM_t^*(K_t, m_{t-1}) - \mathbb{E}_0[CM_t^*(K_t, m_{t-1}) \mid \overline{m}_0].$$

At date 0, if, after observing the realization of m_0 , the manager proceeds with the investment,

²⁴This name is not precise: if m_0 exceeds \overline{m}_0 , then the d^* depreciation rule will not allocate the cost of investment to the periods of useful life in proportion to expected benefits. The cost is always allocated in proportion to expected optimized benefits at the *threshold* market size.

the expected value of period-t residual income, viewed from that initial date, is:

$$\mathbb{E}_0\left[CM_t^*\left(K_t, m_{t-1}\right) \mid m_0\right] - \mathbb{E}_0\left[CM_t^*\left(K_t, m_{t-1}\right) \mid \overline{m}_0\right].$$

Since the expected value of constrained contribution margins increases in the initial value m_0 , the quantity above will exceed zero if and only if $m_0 > \overline{m}_0$. Therefore, from the perspective of date 0, the manager's performance measure in each future period will have the same sign as the net present value of the firm's expected cash flows. As a consequence, the manager will make the optimal investment decision at date 0, regardless of his discount factor. We can now formulate the following Proposition.

Proposition 1. In a model with a single investment opportunity, residual income based on the REOB rule is a goal congruent performance measure for investment and operating decisions.

While Proposition 1 does not speak directly to the question of uniqueness of a goal congruent performance measure, we note that residual income is indeed the only measure with this property if certain additional conditions are imposed. Specifically, if the set of projects available to the firm is sufficiently rich, the owners may have to restrict attention to performance measures that allow for aggregation across assets. A performance measure can be used to aggregate across assets with different productivity profiles if the coefficients that are attached to the accounting numbers are independent of the \boldsymbol{x} vector. For this set of performance measures, it can be verified that residual income based on the REOB rule is indeed the only goal congruent performance evaluation system for investment and operating decisions.²⁵

We now turn to characterizing how uncertainty about future market conditions and the manager's ability to react to new information by idling capacity affect the performance evaluation system described in the Proposition above. To that end, we consider a special case of assets with one-hoss shay productivity pattern,

$$x_1 = x_2 = \dots = x_T \iff K_t = \overline{K}, \text{ for any } t,$$
 (10)

 $^{^{25}}$ This uniqueness result is a variant of the one found in Reichelstein (1997), derived for a setting without operating decisions. Nezlobin, Reichelstein and Wang (2014) provide further uniqueness results for investments in assets with usage-driven capacity decline and in a model with overlapping capacity investments.

and assume that the permanent component of the price shift parameter is a martingale, $g^* = 0$. To isolate the effect of the manager's option to idle capacity, we first consider a benchmark where the firm does not have this option, i.e., the firm is artificially constrained to make and sell \overline{K} units in every period regardless of the market conditions. In this scenario, the structure of the firm's cash flows is effectively equivalent to those in Rogerson (1997) and Reichelstein (1997).²⁶ Consistent with those papers, goal congruence in the benchmark case is achieved by the so-called *r*-annuity rule under which depreciation charges increase at a rate *r* over time,

$$d_{\tau+1}^r = (1+r) \, d_{\tau}^r$$

and the historical cost charges, denoted \boldsymbol{z}^r , are constant over time:²⁷

$$z_{\tau}^r = z_{\tau+1}^r.$$

The *r*-annuity depreciation rule is commonly recommended for assets with one-hoss shay productivity in the practitioner and academic literature.²⁸

We now ask the question: relative to the benchmark described above, how does the presence of operating decisions, i.e., the option to idle capacity, affect the REOB depreciation rule? We start by identifying conditions under which the ability of the firm to adjust its capacity utilization does not affect the optimal performance evaluation system.

$$z_{ au}^* = rac{R\left(\overline{m}_0, \overline{K}
ight) - v \cdot \overline{K}}{b},$$

²⁶One remaining difference between Rogerson's (1997) model with one-hoss shay assets and our benchmark scenario is that in Rogerson's model the investment opportunity is scalable (the firm can decide how much to invest at date 0), while in our model it is binary (the firm can only decide whether or not to make a fixed investment in a project). We demonstrate in Section 5 of this paper that our results in Propositions 1 and 2 can be extended to the model with a scalable investment opportunity.

²⁷If the firm has to operate at capacity at all times, assets have one-hoss shap productivity and m_t is a martingale, then, from the perspective of date 0, the expected value of the contribution margin in period τ is $R(m_0, \overline{K}) - v \cdot \overline{K}$. Therefore, goal congruence is achieved if

i.e., all z_{τ}^* are equal. The corresponding depreciation schedule is the *r*-annuity rule, see Rogerson (1997) and Reichelstein (1997).

²⁸See, for instance, Ehrbar (1998) and Young and O'Byrne (2000). Rogerson (2008) shows that goal congruence can be attained with the *r*-annuity rule if assets have one-hoss shay productivity and the product market is always expanding. The assumption of an expanding product market is important in Rogerson's (2008) model to make sure that the firm never finds itself in an excess capacity situation. In contrast, the focus of our paper is on providing efficient investment incentives to the manager taking into account the possibility of such situations. The *r*-annuity rule also corresponds closely to the treatment for right-of-use assets in Type B leases suggested by the IASB and FASB in their Exposure Draft #ED/2013/6.

Observation 1. Assume that assets have one-hoss shay productivity and the permanent component of the price shift parameter is a martingale $(g^* = 0)$. Then, the REOB rule coincides with the r-annuity rule if:

- 1. All price shocks are transitory ($\sigma_g^2 = 0$), or
- 2. variable production costs are zero (v = 0).

With productivity being one-hoss shay and demand stationary $(g^* = \sigma_g^2 = 0)$, the capacity utilization and optimized contribution margins, in expectation at date 0, will be the same for all periods. Therefore, the historical cost charges should also be the same every period. On the other hand, with zero marginal costs (v = 0), if the investment is made, the firm's manager will (efficiently) choose to operate at capacity in every period, since the firm's contribution margin collapses to revenues, and revenues are increasing in quantity. Therefore, the REOB rule coincides with the benchmark solution where the manager is constrained to operate at capacity, i.e., the *r*-annuity rule.

The conditions in Observation 1 are rather strong. We therefore now assess how the possibility of permanent price shocks in conjunction with nontrivial marginal production cost affect the REOB depreciation rule.

Proposition 2. Assume that assets have one-hoss shay productivity and the permanent component of the price shift parameter is a martingale ($g^* = 0$, $\sigma_g^2 > 0$). Then, the REOB rule is more decelerated than the r-annuity rule.

The intuition for this result is as follows. With the permanent component of the price process being martingale ($g^* = 0$), the expected market price is constant, but its variance (conditional on date 0 information) is greater for later periods. At the heart of Proposition 2 lies the observation that the optimized contribution margin in each period, $CM_t^*(K_t, m_{t-1})$, is a (weakly) convex function of the permanent component of the price shift parameter in that period, m_{t-1} . To see why, note that given the demand formulation in (1), if the firm always were to sell its entire capacity, K_t , revenues would be linear in m_{t-1} , and thus so would be the optimized contribution margin. For favorable realizations of m_{t-1} , such that the capacity constraint (5) is binding, this will indeed be optimal. For unfavorable realizations, however, the firm will adjust the selling quantity accordingly, which mitigates the shortfall in contribution margin. The optimized contribution margin, $CM_t^*(K_t, m_{t-1})$, hence, is strictly convex in m_{t-1} in the left tail, and linear in the right tail.

Having established the convexity of optimized contribution margin in the permanent component of the price shift parameter, Jensen's inequality then implies that the expectation at date 0 of the optimized contribution margin in period τ increases in τ , if $g^* = 0$.²⁹ The REOB depreciation rule is then characterized by monotonically increasing historical cost charges, z_{τ}^* . We show in the proof of Proposition 2 that the fact that historical cost charges are increasing of the asset's useful life implies that the REOB rule is more decelerated than the *r*-annuity rule. Under the more decelerated depreciation rule, the manager faces lower historical cost charges upfront and, therefore, is more willing to invest at date 0. This is precisely what the principal wants in the presence of permanent shocks to market prices (as compared to the scenario with only transitory shocks), since greater uncertainty about prices in later periods increases the expected value of optimized contribution margins in those periods.

We will now explore the behavior of the REOB rule as a function of the product market parameters: the drift and variance of the expected price process. Proposition 2 has assumed a martingale price process and relied on a variance argument to make the case for depreciation decelerated relative to the annuity rule. Deviating from martingale processes introduces a drift effect: if the drift is positive $(g^* > 0)$, it further raises far-future expected optimized contribution margins relative to early ones, thereby making the REOB rule even more decelerated. On the other hand, one would expect the REOB rule to be accelerated relative to the annuity rule if the drift in the permanent component of price shifts, g^* , is sufficiently negative, i.e., if prices are expected to decline for given quantities. With constant elasticity demand, there is a straightforward mapping between expected growth in prices (holding quantity fixed) and expected growth in quantities (holding price fixed). Specifically, if prices are described by a martingale process $(g^* = 0)$, then the corresponding dynamic process describing quantities will be such that expected quantity in t + 1 will be greater than realized quantity in t, holding price constant. Conversely, for quantities to be a martingale, holding prices fixed, it would have to be the case that $q^* < 0.^{30}$

²⁹Banker et al (2013) show in a static, stochastic model with exogenously given congestion costs that a firm is more willing to invest in fixed assets, the more uncertainty it faces. Numerous papers in the adjustment-cost literature show that, with symmetric convex adjustment costs, investment levels increase in price uncertainty, see Hartman (1972), Abel (1983, 1984), Caballero (1991), and Abel and Eberly (1994).

³⁰More generally, if prices are submartingale, then quantities, *a fortiori*, will be submartingale, too. Con-

Proposition 3. Assume assets have one-hoss shay productivity, $\sigma_g^2 > 0$, and the permanent component of the price shift parameter declines sufficiently quickly in expectation, $g^* < 0$, so that the expected quantity demanded at a given price is a martingale (i.e., quantities are expected to remain constant over time, holding price constant). Then, the REOB is more accelerated than the r-annuity rule.

Because negative price drift parameters, $g^* < 0$, push down the expected investment benefits in later periods relative to earlier ones, it is not surprising that for g^* sufficiently negative the REOB depreciation rule becomes more accelerated than the *r*-annuity rule. Proposition 3 derives an (implicit) upper bound for g^* that is sufficient for this to happen: if g^* is such that quantities are expected to remain constant over time, holding price constant, the drift effect dominates the variance effect. Note that Proposition 3 also holds, *a fortiori*, for even lower values of g^* , such that quantities are expected to decline over time for given price (i.e., if quantities evolve as a supermartingale).³¹

Characterizing the behavior of the REOB rule in g^* and σ_g^2 beyond the case of undiminished (one-hoss shay) productivity is difficult, because our notion of accelerated/decelerated depreciation is a demanding one in that it requires a uniform ranking of book values across different depreciation schedules period-by-period. To consider time-variant patterns of asset productivity in a tractable fashion, we turn to assets with a two-period lifetime, i.e., T = 2:

Corollary 1. Suppose T = 2. Then, the REOB depreciation schedule becomes more decelversely, if quantities are supermartingale, then prices, a fortiori, will be supermartingale, too. To see this formally, note that rewriting (1) yields

$$Q_t(p) = \left(\frac{p}{(1+\epsilon_t)\cdot m_{t-1}}\right)^{-\eta}, \ \eta>1.$$

Thus, $Q_t(\cdot)$ is proportional to m_{t-1}^{η} and thereby a convex function of the price shift parameter, m_{t-1} . This implies that if prices evolve as a submartingale, then quantities evolve as a submartingale, too. Conversely, if quantities evolve as a supermartingale, then so do prices.

³¹Proposition 3 may be (mis)read as suggesting that the manager's ability to adjust capacity utilization makes the REOB rule more accelerated in environments where g^* sufficiently less than zero—a message seemingly at odds with that of Proposition 2. We note that Proposition 3 does not in fact identify the effect of the manager's option to idle capacity on the REOB rule, since if $g^* < 0$, the r-annuity rule ceases to be the proper benchmark for the case in which the usage option were absent (i.e., if the firm were constrained to operate at capacity). If $g^* < 0$ and the firm is constrained to operate at capacity, then, consistent with Rogerson (1997), the corresponding benchmark depreciation rule will be characterized by declining historical cost charges and will be itself more accelerated than the r-annuity rule. The goal of Proposition 3 is to illustrate the effect of product market growth on the REOB rule: specifically, we derive an upper bound for g^* below which the REOB rule is more accelerated than the annuity rule and, consequently, more accelerated than the REOB rule for $g^* \geq 0$. erated as g^* increases or as σ_g^2 increases, all else equal.

As we show in the proof, for increases in either the variance or the drift of the process describing the evolution of prices over time, the project becomes more profitable in expectation, for any m_0 . As a result, the investment threshold \overline{m}_0 decreases. The period-1 cost charge according to REOB, as in (9), depends on g^* and σ_g^2 only through this threshold—and is inversely related to the latter. Hence, the period-1 cost charge declines as either g^* and σ_g^2 increase, and the depreciation becomes more decelerated.

The central takeaway from this section is that firms may employ depreciation schedules that are more decelerated as a result of demand uncertainty. A key concern in managerial accounting is how to account for the cost of idle capacity. With demand functions being subject to a sequence of shocks with permanent components, one might expect the expected cost of idle capacity to increase over time—hence the conventional wisdom associating greater uncertainty with more accelerated depreciation (e.g., Penman and Zhang, 2014). As we have shown, this conventional wisdom omits the option to idle capacity. Specifically, given proper incentives, the manager will adjust his operating decisions to the state of the world in a manner that mitigates unfavorable demand shocks. As a result, we have established convexity of periodic benefits in the market size. Uncertainty about future market size therefore translates into more decelerated depreciation rules, unless offset by a sufficiently negative drift in expected market sizes over time, i.e., an expected contraction in demand.

4 Option to Wait

In the preceding analysis, the firm was endowed with the option to fully utilize or partially leave idle the productive capacity, but the investment point in time was assumed exogenously given. We now consider a scenario in which the firm can choose *when* to make its investment. By adding a classic investment timing (real) option to the picture, we now have two different kinds of options entering the firm's decision making process.

As before, we assume that at date t the manager learns m_t , the permanent component of the price shift in period t+1. The manager can then decide whether to invest in the capital asset immediately or postpone the investment decision to a future date. If the investment is made at date t, the firm will have capacity to produce x_{τ} units of the output in period $t + \tau$ for $1 \leq \tau \leq T$. The firm can invest only once, and the firm's capacity cannot be expanded after the investment is made. We use the indicator variable $I_t \in \{0, 1\}$ to denote the investment decision in period t, so that $\sum_{t=0}^{\infty} I_t \leq 1$. If the investment is made at date t, the firm's capacity is $K_{t+\tau} = x_{\tau}$, for $1 \leq \tau \leq T$ and $K_{t-i} = 0$ for $0 \leq i \leq t - 1$. We assume that

$$\mathbb{E}[(1+g_t)^{\eta}] < 1+r \tag{11}$$

to ensure that the expected quantity of the product demanded by the market at a given price does not grow faster than at a rate equal to the firm's cost of capital.³²

We show in the proof of Proposition 4 that the first-best investment policy is to invest as soon as the market size exceeds a certain critical level \overline{m} . The critical market size \overline{m} will be greater than the one identified in the scenario without the option to wait. At any given date t, if the firm invests in the project, the present value of the cash inflows, given optimal operating decisions, equals

$$f(m_t) \equiv \sum_{\tau=1}^{T} \gamma^{\tau} \cdot \mathbb{E}_t \left[CM_{t+\tau}^* \left(K_{t+\tau}, m_{t+\tau-1} \right) \mid m_t \right].$$

On the other hand, we denote the present value of postponing the investment decision at that date, for a given realization of m_t , by $\Theta_w^{\overline{m}}(m_t)$, which we refer to as the *option value*, calculated using the shareholders' cost of capital.³³ Formally,

$$\Theta_{w}^{\overline{m}}\left(m_{t}\right) = \mathbb{E}_{t}\left[\gamma^{t(\overline{m})-t}\cdot\left(f\left(m_{t(\overline{m})}\right)-b\right) \mid m_{t}\right],$$

where $t(\overline{m})$ is the random stopping time given by $t(\overline{m}) = \min\{t + \tau \mid m_{t+\tau} \geq \overline{m}\}$. Intuitively, if the firm invests at date t, it incurs an immediate cash outflow of b dollars and it forgoes loses the option value. Therefore, at the threshold market size \overline{m} , the expected value of future optimized benefits exceed the direct cost of investment by the option value:

$$f(\overline{m}) \equiv b + \Theta_w^{\overline{m}}(\overline{m}).$$
⁽¹²⁾

³²Otherwise, the firm's value can be made arbitrarily large by postponing the investment indefinitely.

³³The superscript \overline{m} indicates that the option value is calculated under the rule that the investment will be made as soon as the market size exceeds \overline{m} . The option value also depends on the shareholders' cost of capital, r, and the cost of investment (the "exercise" price of the option), b. We suppress the dependence of option values on these variables whenever the relevant discount rate is r and the cost of investment is b, and explicitly state it otherwise. That is, more generally, we write $\Theta_w^{\overline{m}}\left(m_t, \tilde{r}, \tilde{b}\right)$ for some discount rate \tilde{r} and investment amount \tilde{b} , and adopt the convention that $\Theta_w^{\overline{m}}\left(m_t\right) \equiv \Theta_w^{\overline{m}}\left(m_t, r, b\right)$.

In most earlier real option models, single threshold investment policies usually exist. Those models typically do not consider operating decisions once the investment has been undertaken—or if they do, they model those as stark binary decisions (e.g., shut down operations altogether in a period of negative shocks as in Dixit and Pindyck, 1994, Ch.6). As a result, the investment problem was modeled for the most part as a perpetual American call option with a payoff that is linear in market size, once the latter exceeds the threshold for exercising the option. Because our model incorporates sequentially optimal operating decisions, the payoff is no longer linear. As the following result demonstrates, a unique investment threshold nonetheless exists:

Proposition 4. Assume the growth rate of the permanent component of the price shift parameter, g^* , is greater than some threshold $g^*_{min} < 0$, such that the expected quantity for given price is a submartingale (i.e., quantities are expected to weakly increase over time, holding price constant). Then, it is optimal for the firm to invest as soon as the permanent component of the price-shift parameter reaches a certain (uniquely-defined) threshold, \overline{m} .

We now turn again to delegated decision making and the issue of goal congruence. To accommodate the fact that the investment, at any date t, can be delayed by the manager, we need to modify the manager's expected utility function at date t as

$$U_t = \mathbb{E}_t \left[\sum_{i=t}^{\infty} \beta_i \cdot \pi_i \mid m_t \right].$$

As before, the utility weights β_i may reflect the manager's discount rate as well as bonus coefficients attached to the performance measure in each period. For example, if the manager's bonus coefficient, u > 0, is time-invariant and the manager's personal discount factor is \hat{r} , then $\beta_i = \hat{\gamma}^i \cdot u$ for all i, where $\hat{\gamma}^i = \frac{1}{1+\hat{r}}$. If, in addition, the manager plans on leaving the firm after period \bar{i} , then $\beta_i = \hat{\gamma}^i \cdot u$ for $i \leq \bar{i}$ and $\beta_i = 0$ for $i \geq \bar{i}$.

With an option to wait, a performance measure π attains goal congruence for operating and investment decisions if and only if, for any $\boldsymbol{\beta} = (\beta_0, \beta_1, ...)$: (i) once the investment is made at some date t, the optimal owner's quantity $q_{t+\tau}^*(m_{t+\tau-1})$ maximizes the manager's objective each period $t+\tau$, $1 \leq \tau \leq T$; (ii) if the shareholders strictly prefer to invest at some date t, then the manager should also have strict incentives to invest at date t if $\beta_{t+\tau} > 0$ for at least one $1 \leq \tau \leq T$. The delegation problem is now compounded by the fact that the manager's (unknown) time preferences affect not just how he assesses the present value of compensation associated with investing today, but also his tradeoff between investing now and waiting for another period. To illustrate how this complicates the search for goal congruent metrics, we proceed heuristically by proposing two manager "archetypes" and studying the distortions that would arise for those under plausible candidate performance measures. We call a manager *perfectly aligned* if his time preferences can be represented as $\beta_i = \gamma^i \cdot u$, for some u > 0. That is, the manager receives a time-invariant bonus coefficient, u, and discounts the future at the shareholders' discount rate. We call the manager *myopic* at date t if he intends to leave the firm at the end of period t+1. In other words, $\beta_t, \beta_{t+1} > 0$, but all other $\beta_i = 0$ for i > t+1.

How will these two managers act if the firm were to offer them the same performance measure identified in Section 3, i.e., residual income with capital charges imputed using r and REOB depreciation?

Observation 2. Assume that residual income based on the shareholders' discount rate, r, and REOB depreciation is used as the performance measure. Then:

- 1. A perfectly aligned manager will invest at the optimal time and make optimal operating decisions.
- 2. A myopic manager will overinvest at date t, i.e., he will invest for some values of $m_t < \overline{m}$ when waiting is preferred by the principal.

Part 2 of the observation follows directly from the conservation property of residual income (e.g., Preinreich 1937),³⁴ combined with the fact that an aligned manager also values the option to wait equally as the shareholders. A myopic manager, on the other hand, effectively assigns zero value to the option to wait. Hence, the latter will invest at date t as long as $f(m_{t-1}) \geq b$, which by comparison with (12) implies overinvestment.

One way to make an impatient manager internalize the option value is by incorporating the shareholders' value of the option to wait into the performance measure itself, specifically by raising the capital charge rate used for computing residual income.³⁵ Define *residual income based on the REOB rule relative to rate* \tilde{r} as a performance measure where both

 $^{^{34}}$ See also Ohlson (1995) and Feltham and Ohlson (1996). In fact, for the aligned manager, the choice of depreciation method becomes arbitrary as long as the asset is fully depreciated over its life time.

³⁵Hurdle rates employed in practice tend to exceed the firm's cost of capital significantly, e.g., Poterba and Summers (1995).

residual income and the REOB rule are calculated relative to an arbitrary discount rate \tilde{r} (we will also call it the "hurdle" rate). Let $d^*(\tilde{r})$, $bv^*(\tilde{r})$, and $z^*(\tilde{r})$ denote the vectors defining the depreciation charges, book values, and historical cost charges under the REOB rule relative to \tilde{r} . The three vectors must satisfy the following condition:

$$z_{\tau}^{*}\left(\widetilde{r}\right) = d_{\tau}^{*}\left(\widetilde{r}\right) + \widetilde{r} \cdot bv_{\tau-1}^{*}\left(\widetilde{r}\right)$$

reflecting the idea that residual income is now calculated relative to some rate \tilde{r} . Formally, we define the REOB rule relative to \tilde{r} as the one corresponding to the following historical cost charges:

$$z_{\tau}^{*}(\widetilde{r}) = \frac{CM_{\tau}^{*}}{\sum_{i=1}^{T} \widetilde{\gamma}^{i} \cdot CM_{i}^{*}},\tag{13}$$

where $CM_{\tau}^* = \mathbb{E}_t \left[CM_{t+\tau}^*(K_{\tau}, m_{t+\tau-1}) \mid m_t = \overline{m} \right]$ and $\widetilde{\gamma} = (1+\widetilde{r})^{-1}$. Note that, for any value of \widetilde{r} , the historical cost charges defined above are proportional to the optimized expected contribution margins of a project implemented at the threshold market size. The numerator in (13) is chosen so as to ensure that the corresponding depreciation rule satisfies the clean surplus condition. Further, let r^* denote the internal rate of return of the project evaluated at the first-best threshold \overline{m} , as given by:

$$\sum_{\tau=1}^{T} \frac{1}{(1+r^*)^{\tau}} \cdot \mathbb{E}_t \left[CM_{t+\tau}^* \left(K_{t+\tau}, m_{t+\tau-1} \right) \mid m_t = \overline{m} \right] \equiv b.$$

Then:

Observation 3. Assume that residual income based on the REOB rule relative to r^* is used as the performance measure. Then:

- A perfectly aligned manager will underinvest, i.e., will not invest for some values of m_t > m
 when investment is preferred by the shareholders.
- A myopic manager will make the optimal investment decision at date t.

A perfectly aligned manager, when charged the higher hurdle rate of r^* , will internalize the option value twice.³⁶ While even this manager will eventually invest, he will do so later

³⁶The logic is reminiscent of that in Christensen et al (2002) regarding the double counting of risk premia if risk-averse agents get charged hurdle rates for capital that are "inflated" so as to incorporate the shareholders' risk premium.

than the shareholders would prefer. On the other hand, incorporating the option value into the hurdle rates forces the myopic manager to internalize the option value and thereby invest optimally.

Observations 2 and 3 together suggest that goal congruence may be elusive if the shareholders don't know the manager's time preferences. We now confirm this intuition formally. In our search for goal congruent performance measures, we restrict attention to linear performance measures of the form:

$$\pi_t = \alpha_R \cdot R_t + \alpha_v \cdot v \cdot q_t + \alpha_d \cdot D_t + \alpha_B \cdot BV_{t-1} + \alpha_I \cdot b \cdot I_t, \quad \alpha_i \in \mathbb{R}.$$
(14)

Proposition 5. In the model with the option to wait, there does not exist a linear performance measure that attains goal congruence.

No linear performance can ensure that a manager with unknown time preferences balances the tradeoff between the value of a project available today (including the prospect of making efficient operating decisions throughout the project's lifetime) and the value of the option to wait in the shareholders' best interest.³⁷

Given the proven impossibility of achieving goal congruence in the sense of costless delegation to a manager with unknown time preferences, the natural question is what the firm could achieve if it had more information. To that end, we now consider a weaker notion of goal congruence. We restrict attention to (potentially impatient) managers with known constant discount factors, i.e., $\hat{r} \geq r$, and time invariant bonus coefficients. Overall,

$$\beta_i = \widehat{\gamma}^i \cdot u.$$

A performance evaluation system (which can now depend on \hat{r}) is *weakly goal-congruent* if the manager whose discount factor is \hat{r} is incentivized to time the investment optimally and make optimal operating decisions.³⁸

³⁷In Friedl (2005), a manager can invest today or one period later. Unlike here, in his model, the value of investing tomorrow is assumed to be common knowledge. Therefore, goal congruence becomes attainable, either by raising the hurdle rate or by capitalizing the option value in the asset base. In our model, because of the presence of permanent demand shocks, the option value at any given point in time depends on the current market size m_{t-1} and hence is the manager's private information.

³⁸Our definition of weak goal congruence is slightly more general than that in Reichelstein (1997), as we allow for the manager's discount rate, while common knowledge, to diverge from that of the shareholders.

Proposition 6. Assume the growth rate of the permanent component of the price shift parameter, g^* , is greater than some threshold $g^*_{min} < 0$, such that the expected quantity for given price is a submartingale (i.e., quantities are expected to weakly increase over time, holding price constant). Then, for any \hat{r} , there exists an $\tilde{r} < r^*$ such that residual income based on the REOB rule relative to \tilde{r} is a weakly goal-congruent performance measure for a manager whose discount rate is \hat{r} .

The manager will have incentives to optimally time the investment if and only if, at the threshold market size, he internalizes precisely the owners' direct cost of investment as well as the owners' option value of waiting. From the manager's perspective, the cash flows from the marginal project must exceed the capital charges associated with the investment, so as to result in a positive performance measure. In addition, the expected bonus payments to the manager must cover the manager's option value of future compensation that is forgone by investing immediately. The rate \tilde{r} must be chosen such that the sum of the manager's option value and the value of historical cost charges reflected in the performance measure is equal to the owners' total cost of investment. This rate depends on the manager's discount rate, \hat{r} , since the latter determines the manager's option value of forgone future compensation. Therefore, the performance evaluation system described in Proposition 6 achieves weak goal congruence, but, as demonstrated in Proposition 5, will not attain strong goal congruent solution entails hurdle rates that are below the internal rate of return of the marginal project.

5 Scalable Investments

Up to this point, we have assumed that the firm's investment opportunity is of fixed scale: the firm can spend b dollars to start a project with fixed capacity levels $(x_1, ..., x_T)$. While such an assumption is plausible in certain environments and has been widely used in the earlier literature (e.g., Dixit and Pindyck 1994, Chapter 6), in many cases, firms can choose the scale of their projects.³⁹ In this section, we extend our results to a model where the firm's investment decision is continuous: the firm can decide *how much* capacity to purchase

 $^{^{39}}$ A firm could be facing an investment opportunity with a fixed scale if, for example, it considers buying an existing asset or a widget factory. Expanding the capacity of an existing asset or a factory may be economically infeasible. In contrast, firms can often decide how large their *new* plants should be.

at the investment date.⁴⁰

We start by assuming that the investment opportunity is only available at date 0. A unit of the capital asset purchased at date 0 generates x_{τ} units of capacity in period τ . Without loss of generality, the cost of one unit of the capital asset is normalized to one. In contrast to our setting in Section 2, we now assume that the firm can decide how many units of the asset to purchase. Specifically, if the firm invests *b* dollars at date 0, then the capacity available for production in period τ is:

$$K_{\tau} = b \cdot x_{\tau}.$$

As before, let $CM_{\tau}^*(K_{\tau}, m_{\tau-1})$ denote the optimized contribution margin in period τ if the permanent component of the price-shift parameter for that period is $m_{\tau-1}$ and the firm's available production capacity is K_{τ} . The principal seeks to design a performance evaluation system that incentivizes the manager to: (i) choose the optimal scale of the project at date 0 (i.e., the optimal b) and (ii) implement the optimal capacity utilization decisions in future periods. We note that the two problems are interrelated: the optimal capacity utilization decisions depend on the amount of capacity installed at date 0, since $K_{\tau} = b \cdot x_{\tau}$.

Formally, the optimal investment level solves the following problem:

$$\max_{b} -b + \sum_{\tau=1}^{T} \gamma^{\tau} \cdot \mathbb{E}_{0} \left[CM_{\tau}^{*} \left(K_{\tau}, m_{\tau-1} \right) \mid m_{0} \right].$$
(15)

Let $b^*(m_0)$ denote the optimal investment policy. Taking the first-order condition yields:

$$\sum_{\tau=1}^{T} \gamma^{\tau} \cdot \mathbb{E}_{0} \left[\frac{\partial CM_{\tau}^{*} \left(K_{\tau}, m_{\tau-1} \right)}{\partial K_{\tau}} \Big|_{K_{\tau} = b^{*}(m_{0}) \cdot x_{\tau}} \cdot x_{\tau} \mid m_{0} \right] = 1.$$
(16)

Intuitively, the discounted sum of the expected marginal net benefits over the project's lifetime has to be equal to one (which is the marginal cost of investment at date 0).

We now turn again to the issue of goal congruence under delegated decision-making. Recall that with binary investment decisions, as in Section 3, the first-best investment was described by a hurdle policy such that the investment was to be made if and only if the date-0 market size exceeded a certain threshold. The REOB rule achieved goal congruence while simply relying on information about the threshold market size, as the actual one

 $^{^{40}\}mathrm{We}$ thank an anonymous referee for suggesting this extension of our model.

was known only to the manager. Given that the REOB rule resulted in zero expected incremental residual income each period as a result of investing at the threshold market size, the manager would expect a positive (negative) performance measure in each period for market size greater (smaller) than this threshold, by stochastic dominance arguments. With scalable investments, no such threshold policy exists; instead, the optimal investment rule, $b^*(m_0)$, is a monotonic function of the date-0 market size. The principal, however, has to design the accounting system without knowledge of m_0 . Nevertheless, as we show in this section, proper adjustments to the accounting rules identified in Section 3 ensure goal congruence.

Specifically consider the following variant of the depreciation rule discussed in Section 3, which we label the *marginal* REOB rule:

$$z_{\tau}^{*} = \mathbb{E}_{0} \left[\frac{\partial CM_{\tau}^{*}(K_{\tau}, m_{\tau-1})}{\partial K_{\tau}} \Big|_{K_{\tau} = b^{*}(m_{0}) \cdot x_{\tau}} \cdot x_{\tau} \mid m_{0} \right].$$
(17)

We verify in the proof of Proposition 1' that the expected marginal net benefit in period τ (the expression on the right-hand side of equation (17)) does not depend on m_0 . Therefore, the vector of z_{τ}^* for $\tau = 1, ..., T$, can be calculated by the principal at date 0. Equations (16) and (17) together imply that

$$\sum_{\tau=1}^{T} \gamma^{\tau} \cdot z_{\tau}^* = 1;$$

hence, there exists a depreciation rule $d^* = (d_1^*, ..., d_T^*)$ that corresponds to historical cost charges given by z_{τ}^* in every period. The main difference between this rule and the one described in Section 3 is that instead of setting the historical cost charges equal to the expected optimized benefits at the *threshold* market size, now they are set equal to the *marginal* expected optimized benefits calculated at the current market size. While the principal does not know the current market size and the absolute value of future benefits, the *marginal* expected benefits in future periods do not, in fact, depend on m_0 .

Consider a manager who is compensated based on residual income calculated under the marginal REOB rule. First, it is clear that once the investment is made, the manager will optimize the expected contribution margin in every period. Therefore, it is sufficient to check the manager's investment incentives at date 0. From the perspective of date 0, the expected

value of residual income in period τ is given by:

$$\mathbb{E}_0\left[CM^*_{\tau}\left(b\cdot x_{\tau}, m_{\tau-1}\right) \mid m_0\right] - z^*_{\tau} \cdot b,$$

where b is the investment level chosen by the manager. The definition of z_{τ}^* in (17) implies that the expression above is maximized at $b^*(m_0)$, because

$$\mathbb{E}_0\left[\left.\frac{\partial CM^*_{\tau}\left(K_{\tau},m_{\tau-1}\right)}{\partial K_{\tau}}\right|_{K_{\tau}=b^*(m_0)\cdot x_{\tau}}\cdot x_{\tau}\mid m_0\right]-z^*_{\tau}=0.$$

Therefore, the optimal investment level maximizes the manager's objective in each and every period, and the manager has incentives to invest optimally regardless of his time preferences.

Proposition 1'. In a model with a single scalable investment opportunity, residual income based on the marginal REOB rule is a goal congruent performance measure for investment and operating decisions.

Our result in Proposition 2 also extends to the setting with a scalable investment opportunity. However, to apply the same intuition as in our discussion following Proposition 2, we need to show that the *marginal* expected optimized cash flow in period τ is a convex function of the permanent component of the price shift parameter, $m_{\tau-1}$. This indeed turns out to be the case, as we demonstrate in the proof of the following result.

Proposition 2'. Assume that assets have one-hoss shay productivity and the permanent component of the price shift parameter is a martingale $(g^* = 0, \sigma_g^2 > 0)$. Then, the marginal *REOB* rule is more decelerated than the r-annuity rule.

To conclude this section, we ask how scalable investments should be timed optimally if the firm has the option to postpone its investment. It follows from the proof of Proposition 1' that the optimal investment decision becomes degenerate in the sense that the firm either invests immediately or postpones the investment ad infinitum. Specifically, one can show that the net present value of the firm's expected cash flows is proportional to m_{t-1}^{η} with the proportionality factor being common knowledge. Depending on the sign of this proportionality factor (which indeed can take on either sign even with limit on expected demand growth specified in (11) in place), the firm will invest immediately or always postpones by another period.⁴¹

6 Conclusion

This paper has developed a dynamic goal congruence model in which information arrives over time and investment and operating decisions are delegated to a manager who always has a leg up, informationally, over the firm's owners and whose time preferences are unknown to the principal. For the case of an exogenously given investment date, we have identified an asset valuation rule (the REOB depreciation schedule) that achieves goal congruence, if combined with residual income as the periodic performance measure. Two features of the REOB rule are worth emphasizing. First, we have shown that if demand demand follows a martingale process, the REOB rule is decelerated relative to the scenario where the firm cannot adjust its capacity utilization. The reason is that the manager's option to idle capacity if the firm is hit by a sequence of unfavorable demand shocks mitigates the negative profit consequences of those shocks. Second, in terms of informational requirements, the REOB rule can be implemented by a corporate controller without exact knowledge of the current market conditions. With demand following a dynamic process with permanent shocks, a manager who is informed about the market size at the outset thereby also has better information about all future expected demand scenarios. Nonetheless, goal congruence is attainable if the investment date is exogenous. These findings apply equally to binary and scalable investments with only minor adjustments to the respective depreciation schedules.

We have also considered a scenario where the manager can time the investment. From the shareholders' point of view, the investment should be undertaken only if the attendant NPV exceeds the value of the option to wait, which comprises the expected cash flows associated with future possible investment dates, properly weighted and discounted. The manager follows similar logic when deciding when to invest, by comparing the present value of his

⁴¹More technically, the net present value of expected cash flows (after deducting the investment cost) is given by $A \cdot m_{t-1}^{\eta}$, with the constant A being common knowledge. The firm, then, faces an optimal stopping problem with the payoff $A \cdot m_{t-1}^{\eta}$. The process that determines the net present value of the project (if undertaken at date t-1), discounted to date 0 is given by $A \cdot m_{t-1}^{\eta} \cdot \gamma^{t-1}$. This process is a geometric Brownian motion with a drift equal to $\mathbb{E}[(1+g_t)^{\eta}]\gamma < 1$. Therefore, if A is positive, the firm should start the project immediately, and if A is negative, the project is never implemented. A corner solution obtains in this model because instead of postponing its investment when the market conditions are relatively unfavorable, the firm can reduce the investment amount optimally. In contrast, most earlier models that study a single investment decision with endogenous timing assume that the project in question has a fixed scale, i.e., the investment option has a fixed exercise price (see, for instance, Dixit and Pindyck 1994, Chapters 5 and 6).

compensation if investing today versus investing later. The fact that now the manager's opportunity cost of investing is non-zero but instead depends on his time preferences, which only he knows, makes goal congruence impossible to achieve. Only a weak form of goal congruence is attainable in that case, in the sense that the principal would need to know the manager's time preferences.

Our model does not include formal agency problems.⁴² Earlier literature has shown that many key findings obtained from goal congruence models carry over qualitatively to models that include elements of moral hazard and/or adverse selection. Performing a similar robustness check on our model would be complicated by the fact that permanent negative demand shocks may lead agents to quit the firm, given plausible commitment assumptions. Another possible next step in this research agenda is to look at overlapping capacity investments. Earlier studies on such overlapping investments have assumed that the markets are sufficiently fast-growing (at least in expectation) to avoid any idle capacity issues (e.g., Rogerson 2008, Dutta and Reichelstein 2010, Nezlobin et al 2014). Our setting with temporary and permanent price shocks could be fruitfully employed to study the consequences for performance measurement if that continuing growth assumption were dropped.

⁴²Several other studies have incorporated other types of options into agency models. For instance, in Pfeiffer and Schneider (2007) the firm can abandon its multistage project prior to completion. Arya and Glover (2001) study a model with two uncorrelated projects that become available at different points in time. They present a control problem that can make the option to wait valuable. In our model, the option to wait is valuable because the manager learns new information about the changing product market conditions over time. Johnson et al. (2013) focus on externalities across divisions when they share a common resource.

Appendix

Proof of Proposition 1

Using constant elasticity of demand in (1), the optimal unconstrained quantity for period t solves

$$\max_{q} \mathbb{E}_{t-1} \left[\mu_t \cdot P(q) \cdot q - v \cdot q \right].$$

This problem can be rewritten as:

$$\max_{q} m_{t-1} q^{\frac{\eta-1}{\eta}} - vq.$$

Therefore, the optimal unconstrained quantity is:

$$\underline{q}_t(m_{t-1}) \equiv \left(\frac{(\eta-1)m_{t-1}}{\eta v}\right)^{\eta}.$$

The firm will operate at capacity for m_{t-1} large enough such that $\underline{q}_t(m_{t-1}) \ge K_t$, and it will sell the unconstrained quantity if $\underline{q}_t(m_{t-1}) < K_t$. Equivalently, $q_t(m_{t-1}) = K_t$ if

$$m_{t-1} \ge m_{t-1}(K_t) \equiv \frac{\eta v}{\eta - 1} K^{\frac{1}{\eta}},$$

and $q_t(m_{t-1}) = \underline{q}_t(m_{t-1})$ if $m_{t-1} < m_{t-1}(K_t)$. By straightforward algebra, the contribution margin in these two cases is given by:

$$CM_{t}^{*}(K_{t}, m_{t-1}) = \begin{cases} \frac{1}{\eta - 1} \left(\frac{\eta - 1}{\eta}\right)^{\eta} v^{1 - \eta} \cdot m_{t-1}^{\eta}, & \text{if } m_{t-1} < m_{t-1}(K_{t}), \\ K_{t}^{\frac{\eta - 1}{\eta}} \cdot m_{t-1} - v \cdot K_{t}, & \text{otherwise.} \end{cases}$$
(18)

Note that

$$\frac{1}{\eta - 1} \left(\frac{\eta - 1}{\eta} \right)^{\eta} v^{1 - \eta} \cdot m_{t-1}^{\eta} \Big|_{m_{t-1}(K_t)} = \left(K_t^{\frac{\eta - 1}{\eta}} \cdot m_{t-1} - v \cdot K_t \right) \Big|_{m_{t-1}(K_t)} = \frac{v}{\eta - 1} K_t$$

Therefore, the function in (18) is continuous in m_{t-1} . Note further that $CM_t^*(K_t, m_{t-1})$ is linear in m_{t-1} to the right of $m_{t-1} = m_{t-1}(K_t)$, and is proportional to m_{t-1}^{η} before that point. Therefore, to verify that $CM_t^*(K_t, m_{t-1})$ is convex in m_{t-1} , it suffices to check that at $m_{t-1} = m_{t-1}(K_t)$ the right and left derivatives match. The right derivative of $CM_t^*(K_t, m_{t-1})$ with respect to m_{t-1} at $m_{t-1}(K_t) = \frac{\eta}{\eta-1}vK_t^{1/\eta}$ is:

$$K_t^{\frac{\eta-1}{\eta}} = m_{t-1}^{\eta-1} \left(\frac{\eta-1}{\eta}\right)^{\eta-1} v^{-(\eta-1)}.$$

The left derivative at that point equals the expression because:

$$\frac{\eta}{\eta-1} \left(\frac{\eta-1}{\eta}\right)^{\eta} v^{1-\eta} \cdot m_{t-1}^{\eta-1} = m_{t-1}^{\eta-1} \left(\frac{\eta-1}{\eta}\right)^{\eta-1} v^{-(\eta-1)}.$$

Therefore, $CM_t^*(K_t, m_{t-1})$ is convex in m_{t-1} .

Let us show that if it is optimal for the firm to invest given a certain realization of m_0 , then it is also optimal to invest for any realization $m'_0 > m_0$. Undertaking the investment is optimal given m_0 if and only if

$$\sum_{t=1}^{T} \gamma^{t} \cdot \mathbb{E}_{0} \left[CM_{t}^{*} \left(K_{t}, m_{t-1} \right) \mid m_{0} \right] \geq b.$$

Observe that the distribution of m_{t-1} given m'_0 first-order stochastically dominates the distribution of $m_{t-1} \mid m_0$:

$$Pr\{m_{t-1} > k \mid m_0\} = Pr\left\{\prod_{\tau=1}^{t-1} (1+g_{\tau}) > \frac{k}{m_0}\right\} < Pr\left\{\prod_{\tau=1}^{t-1} (1+g_{\tau}) > \frac{k}{m'_0}\right\}$$
$$= Pr\{m_t > k \mid m'_0\}.$$

Since $CM_t^*(K_t, m_{t-1})$ is increasing in m_{t-1} , it follows that

$$\sum_{t=1}^{T} \gamma^{t} \cdot \mathbb{E}_{0} \left[CM_{t}^{*} \left(K_{t}, m_{t-1} \right) \mid m_{0}^{\prime} \right] \geq \sum_{t=1}^{T} \gamma^{t} \cdot \mathbb{E}_{0} \left[CM_{t}^{*} \left(K_{t}, m_{t-1} \right) \mid m_{0} \right] \geq b_{t}$$

and undertaking the investment is optimal given m'_0 .

Let us now show that $\mathbb{E}_0[CM_t^*(K_t, m_{t-1}) \mid m_0]$ is continuous in m_0 . It follows from the global convexity of $CM_t^*(K_t, m_{t-1})$ and equation (18) that the derivative of $CM_t^*(K_t, m_{t-1})$

with respect to m_{t-1} is everywhere bounded by $K_t^{\frac{n-1}{\eta}}$. Then, for any m_0, m'_0 , we have:⁴³

$$\begin{aligned} \left\| \mathbb{E}_{0} \left[CM_{t}^{*} \left(K_{t}, m_{t-1} \right) \mid m_{0} \right] - \mathbb{E}_{0} \left[CM_{t}^{*} \left(K_{t}, m_{t-1} \right) \mid m_{0}^{\prime} \right] \right] \\ = \left\| \mathbb{E}_{g_{1}, \dots, g_{t-1}} \left[CM_{t}^{*} \left(K_{t}, m_{0} \cdot \prod_{\tau=1}^{t-1} (1+g_{\tau}) \right) \right] - \mathbb{E}_{g_{1}, \dots, g_{t-1}} \left[CM_{t}^{*} \left(K_{t}, m_{0}^{\prime} \cdot \prod_{\tau=1}^{t-1} (1+g_{\tau}) \right) \right] \right\| \\ = \left\| \mathbb{E}_{g_{1}, \dots, g_{t-1}} \left[CM_{t}^{*} \left(K_{t}, m_{0} \cdot \prod_{\tau=1}^{t-1} (1+g_{\tau}) \right) - CM_{t}^{*} \left(K_{t}, m_{0}^{\prime} \cdot \prod_{\tau=1}^{t-1} (1+g_{\tau}) \right) \right] \right\| \\ \leq K_{t}^{\frac{\eta-1}{\eta}} \cdot \left| m_{0} - m_{0}^{\prime} \right| \cdot \left\| \mathbb{E}_{g_{1}, \dots, g_{t-1}} \left[\prod_{\tau=1}^{t-1} (1+g_{\tau}) \right] \right\|. \end{aligned}$$

The last term of the inequality above approaches zero as m'_0 approaches m_0 , therefore $\mathbb{E}_0[CM_t^*(K_t, m_{t-1}) \mid m_0]$ is continuous in m_0 .

We have shown that the benefits of investment are monotonically increasing and continuous in m_0 . From this it follows that the optimal investment policy is characterized by a threshold realization of m_0 , denoted \overline{m}_0 , at which the present value of future expected optimized cash flows is precisely equal to the initial investment cash outflow of b. The remainder of the proof follows from the argument that precedes the statement of the Proposition in the main text of this paper.

Proof of Proposition 2

We have shown in the proof of Proposition 1 that $CM_{\tau}^*(K_{\tau}, m_{\tau-1})$ is a convex function of $m_{\tau-1}$. Under the conditions stated in Proposition 2, capacity is constant over time, $K_{\tau} = \overline{K}$ for any τ . Now consider the role of the parameters describing the permanent component of the price shift process, g^* and σ_g^2 . If $g^* = 0$, then m_{τ} is a mean-preserving spread of $m_{\tau-1}$. Given that the proof of Proposition 1 has established that $CM_{\tau}^*(K_{\tau}, m_{\tau-1})$ is convex in $m_{\tau-1}$, it follows by Jensen's inequality that

$$CM_{\tau}^{*}\left(\overline{K}, m_{\tau-1}\right) \leq \mathbb{E}_{\tau-1}\left[CM_{\tau+1}^{*}\left(\overline{K}, m_{\tau}\right) \mid m_{\tau-1}\right]$$

⁴³In some of the proofs, we write $\mathbb{E}_{g_1,\ldots,g_{t-1}}$ [·] to emphasize that the expectation is taken over the random variables listed in the subscript - g_1,\ldots,g_{t-1} .

for any $m_{\tau-1}$. Then, by the Law of Iterated Expectations,

$$\mathbb{E}_0\left[CM^*_{\tau}\left(\overline{K}, m_{\tau-1}\right) \mid m_0\right] \le \mathbb{E}_0\left[CM^*_{\tau+1}\left(\overline{K}, m_{\tau}\right) \mid m_0\right],\tag{19}$$

and, therefore,

$$z_{\tau}^* \le z_{\tau+1}^*.$$
 (20)

It remains to show that equation (20) implies that the REOB rule is more decelerated than the r-annuity rule.

First, note that for any vector of historical cost charges, \boldsymbol{z} , the corresponding vector of book values (per dollar of the initial investment) is given by:

$$bv_{\tau} = \gamma \cdot z_{\tau+1} + \dots + \gamma^{T-\tau} \cdot z_T.$$

Then,

$$d_{\tau} + r \cdot bv_{\tau-1} = bv_{\tau-1} - bv_{\tau} + r \cdot bv_{\tau-1} = \gamma^{-1} \cdot bv_{\tau-1} - bv_{\tau}$$
$$= (z_{\tau} + \dots + \gamma^{T-\tau} \cdot z_T) - (\gamma \cdot z_{\tau+1} + \dots + \gamma^{T-\tau} \cdot z_T)$$
$$= z_{\tau}.$$

Recall that for the r-annuity rule, all z_{τ}^{r} are equal, and, therefore,

$$\frac{z_{\tau}^r}{z_{\tau+1}^r} = 1 \ge \frac{z_{\tau}^*}{z_{\tau+1}^*},$$

and, equivalently,

$$\frac{z_{\tau}^{*}}{z_{\tau}^{r}} \le \frac{z_{\tau+1}^{*}}{z_{\tau+1}^{r}}.$$
(21)

We will now show that the inequality above implies that the sequence bv_{τ}^*/bv_{τ}^r increases in τ .

Let us rewrite bv_{τ}^*/bv_{τ}^r as:

$$\begin{aligned} \frac{bv_{\tau}^{*}}{bv_{\tau}^{r}} &= \frac{z_{\tau+1}^{*} + \dots + \gamma^{T-\tau-1} \cdot z_{T}^{*}}{z_{\tau+1}^{r} + \dots + \gamma^{T-\tau-1} \cdot z_{T}^{r}} \\ &= \frac{z_{\tau+1}^{*} + bv_{\tau+1}^{*}}{z_{\tau+1}^{r} + bv_{\tau+1}^{*}}. \end{aligned}$$

It can be easily verified that the equation above implies that

$$\frac{bv_{\tau}^*}{bv_{\tau}^r} \le \frac{bv_{\tau+1}^*}{bv_{\tau+1}^r}$$

whenever

$$\frac{z_{\tau+1}^*}{z_{\tau+1}^r} \le \frac{bv_{\tau+1}^*}{bv_{\tau+1}^r}$$

The latter inequality holds because

$$\begin{split} \frac{bv_{\tau+1}^*}{bv_{\tau+1}^r} &= \frac{\gamma \cdot z_{\tau+2}^* + \dots + \gamma^{T-\tau-1} \cdot z_T^r}{\gamma \cdot z_{\tau+2}^r + \dots + \gamma^{T-\tau-1} \cdot z_T^r} \\ &\geq \frac{\gamma \cdot z_{\tau+2}^r \cdot \frac{z_{\tau+1}^*}{z_{\tau+1}^r} + \dots + \gamma^{T-\tau-1} \cdot z_T^r \cdot \frac{z_{\tau+1}^*}{z_{\tau+1}^r}}{\gamma \cdot z_{\tau+2}^r + \dots + \gamma^{T-\tau-1} \cdot z_T^r} \\ &= \frac{z_{\tau+1}^*}{z_{\tau+1}^r}. \end{split}$$

We have shown that

$$\frac{bv_{\tau}^*}{bv_{\tau}^r} \le \frac{bv_{\tau+1}^*}{bv_{\tau+1}^r}$$

To conclude that proof note that the inequality above implies that

$$bv_{\tau+1}^* \ge \frac{bv_{\tau}^*}{bv_{\tau}^r} \cdot bv_{\tau+1}^r.$$

Therefore, if for some value of τ , bv_{τ}^* is (weakly) greater than bv_{τ}^r , it will also be the case for all greater values of τ . At the beginning, $bv_0^* = bv_0^r = 1$, so the depreciation rule d^* is more decelerated than d^r .

Proof of Proposition 3

Let $\widetilde{m}_{t-1} = m_{t-1}^{\eta}$. We will first verify that if σ_g^2 and g^* are such that $\mathbb{E}[(1+g_t)^{\eta}] = 1$, then g^* is less than zero. For this value of g^* , the process $\{\widetilde{m}_t\}$, and therefore the expected quantity demanded at a given price, is a martingale, while the expected price holding the quantity fixed declines over time.⁴⁴

$$E_{\epsilon_{t}}\left[p^{-\eta}\left(1+\epsilon_{t}\right)^{\eta}m_{t-1}^{\eta}\right] = p^{-\eta}E_{\epsilon_{t}}\left[\left(1+\epsilon_{t}\right)^{\eta}\right]m_{t-1}^{\eta}$$

⁴⁴The expected quantity demanded at price p given m_{t-1} is given by:

Therefore, the expected quantity demanded at a given price is always proportional to $\widetilde{m}_{t-1} \equiv m_{t-1}^{\eta}$.

Recall that $(1 + g_t)$ are independently log-normally distributed and let κ and δ^2 denote the mean and variance of the underlying *normal* distribution. Then, σ_g^2 and $\mathbb{E}\left[(1 + g_t)^{\eta}\right]$ can be expressed in terms of κ and δ^2 :

$$\sigma_g^2 = \left(e^{\delta^2} - 1\right)e^{2\kappa + \delta^2},\tag{22}$$

$$\mathbb{E}\left[(1+g_t)^{\eta}\right] = e^{\kappa\eta + \delta^2 \eta^2/2} = 1.$$
(23)

From (23), it follows that $\kappa = -\delta^2 \eta/2$. Then,

$$1 + g^* = \mathbb{E}\left[(1 + g_t)\right] = e^{\kappa + \delta^2/2} = e^{\delta^2(1 - \eta)/2} < 1.$$
(24)

Therefore, the corresponding value of g^* is less than zero.⁴⁵

The optimized constrained contribution margin in (18) can be rewritten as a function of \widetilde{m}_{t-1} as follows:

$$CM_t^*\left(K_t, m_{t-1}\right) = \widetilde{CM}_t^*\left(K_t, \widetilde{m}_{t-1}\right) = \begin{cases} \frac{1}{\eta-1} \left(\frac{\eta-1}{\eta}\right)^\eta v^{1-\eta} \cdot \widetilde{m}_{t-1}, & \text{if } \widetilde{m}_{t-1} < \left(\frac{\eta}{\eta-1}\right)^\eta v^\eta K_t, \\ K_t^{\frac{\eta-1}{\eta}} \cdot \widetilde{m}_{t-1}^{1/\eta} - v \cdot K_t, & \text{otherwise.} \end{cases}$$

This function is continuous and increasing in \widetilde{m}_{t-1} . Furthermore, this function is linear in \widetilde{m}_{t-1} for $\widetilde{m}_{t-1} < \left(\frac{\eta}{\eta-1}\right)^{\eta} v^{\eta} K_t$, concave in \widetilde{m}_{t-1} otherwise, and it is differentiable everywhere.

The remainder of the proof proceeds along similar steps as that of Proposition 1, combined with the proof of Proposition 2 with all inequalities "flipped". \Box

Proof of Corollary 1

Given T = 2, the REOB cost allocation schedule, \boldsymbol{z}^* as in (9), reduces to:

$$z_1^* = \frac{CM_1^*(K_1, \overline{m}_0)}{b},$$

$$z_2^* = \frac{\mathbb{E}_0 \left[CM_2^*(K_2, m_1) \mid \overline{m}_0\right]}{b}.$$

$$\sigma_g^2 = \left((1+g^*)^{-\frac{2}{\eta-1}} - 1 \right) \left(1+g^* \right)^2.$$

Clearly, for any $-1 < g^* < 0$, there exists a σ_g^2 such that the condition above is satisfied. It can be verified that if $\eta < 2$, then for any σ_g^2 there exists a corresponding $g^* < 0$ such that the condition above is satisfied. If $\eta \ge 2$, then such g^* exists for $\sigma_g^2 \in (0, \overline{\sigma_g}^2)$ for some $\overline{\sigma_g}^2$ that depends on η .

⁴⁵It follows from equations (22), (23), and (24) that Proposition 3 applies to environments characterized by $\{g^*, \sigma_q^2\}$ where

Note in particular that the first-period cost charge depends on the parameters describing the demand dynamics, g^* and σ_g^2 , only via their effect on \overline{m}_0 . The latter is implicitly defined by (8), which for T = 2 reads

$$(1+r) \cdot b \equiv CM_1^* \left(K_1, \overline{m}_0 \right) + \gamma \cdot \mathbb{E}_0 \left[CM_2^* \left(K_2, m_1 \right) \mid \overline{m}_0 \right].$$

At the same time, $z_1^* = d_1^* + r$ and $z_2^* = (1 - d_1^*) \cdot (1 + r)$. In this case, we can say that depreciation becomes more decelerated if and only if z_1^* goes down.

An increase in g^* , all else equal, increases $\mathbb{E}_0[CM_2^*(K_2, m_1) \mid \overline{m}_0]$ and thereby lowers the investment threshold \overline{m}_0 . This in turn reduces z_1^* , i.e., depreciation becomes more decelerated. On the other hand, an increase in σ_g^2 , all else equal, implies a mean-preserving spread over m_1 's. Together with the fact that $CM_2^*(K_2, m_1)$ is convex in m_1 (see proof of Proposition 1), this implies, again, a drop in the investment threshold \overline{m}_0 , and thus a more decelerated depreciation schedule.

Proof of Proposition 4

Using the same notation as in the proof of Proposition 3, let us write the firm's optimized contribution margin in period $t + \tau$ as a function of $\widetilde{m}_{t+\tau-1} \equiv m_{t+\tau-1}^{\eta}$:⁴⁶

$$\widetilde{CM}_{t+\tau}^{*}\left(K_{t+\tau}, \widetilde{m}_{t+\tau-1}\right) = \begin{cases} \frac{1}{\eta-1} \left(\frac{\eta-1}{\eta}\right)^{\eta} v^{1-\eta} \cdot \widetilde{m}_{t+\tau-1}, & \text{if } \widetilde{m}_{t+\tau-1} < \left(\frac{\eta}{\eta-1}\right)^{\eta} v^{\eta} K_{t+\tau}, \\ K_{t+\tau}^{\frac{\eta-1}{\eta}} \cdot \widetilde{m}_{t+\tau-1}^{1/\eta} - v \cdot K_{t+\tau}, & \text{otherwise.} \end{cases}$$

It is straightforward to verify that $\widetilde{CM}_{t+\tau}^{*}(K_{t+\tau}, \widetilde{m}_{t+\tau-1})$ and

$$\frac{\partial \widetilde{CM}^*_{t+\tau} \left(K_{t+\tau}, \widetilde{m}_{t+\tau-1} \right)}{\partial \widetilde{m}_{t+\tau-1}} \cdot \widetilde{m}_{t+\tau-1}$$
(25)

are concave functions of $\widetilde{m}_{t+\tau-1}$.

The total benefits of investing at date t can be written as:

$$\phi\left(\widetilde{m}_{t}\right) \equiv \sum_{\tau=1}^{T} \gamma^{\tau} \cdot \mathbb{E}_{t} \left[\widetilde{CM}_{t+\tau}^{*}\left(K_{t+\tau}, \widetilde{m}_{t+\tau-1}\right) \mid \widetilde{m}_{t}\right].$$

⁴⁶See the proof of Proposition 3 for details.

Let g_{min}^* be such that $E\left[(1+g_t)^{\eta}\right] = 1$ so that \widetilde{m}_t is a martingale. Then, for any $g^* > g_{min}^*$, $\{\widetilde{m}_t\}$ is a submartingale. The firm is presented with an optimal stopping problem with a nonlinear payoff of $\phi(\widetilde{m}_t)-b$. We need to show that there exists a unique optimal stopping policy of the "threshold" type: the firm should invest as soon as \widetilde{m}_t exceeds a certain threshold, \overline{m}^{η} .⁴⁷ For future reference, we note that the concavity of $\widetilde{CM}_{t+\tau}^*(K_{t+\tau}, \widetilde{m}_{t+\tau-1})$ and the concavity of the functions in (25) for all τ imply that $\phi(\widetilde{m}_t)$ and $(\phi'(\widetilde{m}_t) \cdot \widetilde{m}_t)$ are concave.

There exists a unique optimal threshold policy under the following two conditions (see Dixit and Pindyck (1994), pp. 128-129):

1. Writing the difference between the value of waiting for exactly one period and stopping right away as

$$\Delta(\widetilde{m}_{t}) \equiv \gamma \cdot \mathbb{E}_{t} \left[\phi\left(\widetilde{m}_{t+1}\right) - b \mid \widetilde{m}_{t} \right] - \left(\phi\left(\widetilde{m}_{t}\right) - b \right),$$

then this difference is decreasing in \widetilde{m}_t : $\Delta'(\widetilde{m}_t) \leq 0$.

2. The distribution of future values of \widetilde{m}_{t+1} shifts uniformly to the right for higher values of \widetilde{m}_t .

It is straightforward to see that condition 2 holds for the process $\{\widetilde{m}_t\}$.⁴⁸ To verify that condition 1 holds, note that

$$\begin{aligned} \Delta'(\widetilde{m}_{t}) &= \gamma \cdot \mathbb{E}_{g_{t+1}} \left[(1 + g_{t+1})^{\eta} \cdot \phi' \left((1 + g_{t+1})^{\eta} \, \widetilde{m}_{t} \right) \mid \widetilde{m}_{t} \right] - \phi' \left(\widetilde{m}_{t} \right) \\ &\leq \gamma \cdot \mathbb{E}_{g_{t+1}} \left[(1 + g_{t+1})^{\eta} \right] \cdot \phi' \left(\mathbb{E}_{g_{t+1}} \left[(1 + g_{t+1})^{\eta} \right] \widetilde{m}_{t} \right) - \phi' \left(\widetilde{m}_{t} \right), \end{aligned}$$

which follows from Jensen's inequality and the concavity of $(\phi'(\widetilde{m}_t) \cdot \widetilde{m}_t)$ established above. Note, further, that since $\phi'(\cdot)$ is concave and $1 \leq \mathbb{E}_{g_{t+1}}[(1+g_{t+1})^{\eta}] \leq 1+r$, we have

$$\gamma \cdot \mathbb{E}_{g_{t+1}} \left[(1+g_{t+1})^{\eta} \right] \cdot \phi' \left(\mathbb{E}_{g_{t+1}} \left[(1+g_{t+1})^{\eta} \right] \widetilde{m}_t \right) - \phi' \left(\widetilde{m}_t \right) \le \phi' \left(\mathbb{E}_{g_{t+1}} \left[(1+g_{t+1})^{\eta} \right] \widetilde{m}_t \right) - \phi' \left(\widetilde{m}_t \right) \le \phi' \left(\widetilde{m}_t \right) - \phi' \left(\widetilde{m}_t \right) = 0,$$

which concludes the proof of the proposition.

Proof of Proposition 5

⁴⁷The condition $\widetilde{m}_t \geq \overline{m}^{\eta}$ is equivalent to $m_t \geq \overline{m}$.

 $^{^{48}}$ See also the proof of Proposition 1.

We prove impossibility of goal congruence for T = 1. For T = 1, we can drop capacity K_t as an argument in the contribution margin. Then, the present value of cash flows if the firm invests for m_t is:

$$f(m_t) - b = \gamma \cdot CM_{t+1}^*(m_t) - b.$$
(26)

Let \overline{m} denote the first-best investment threshold. Then, the firm is indifferent between investing and waiting further when $m_t = \overline{m}$:

$$f(\overline{m}) - b - \Theta_w^{\overline{m}}(\overline{m}) = 0$$

Expanding, we can write:

$$\gamma \cdot CM_{t+1}^*(\overline{m}) - b - \mathbb{E}_t \left\{ \gamma^{t(\overline{m}) - t + 1} \cdot \left[CM_{t(\overline{m}) + 1}^*(m_{t(\overline{m})}) - b \right] \mid m_t = \overline{m} \right\} = 0.$$
(27)

Now turn to delegation. Suppose the manager's time preferences are given by $\beta_t = \hat{\gamma}^t \cdot u$, where $\hat{\gamma} = (1+\hat{r})^{-1}$ and we normalize u = 1, without loss of generality. That is, the manager discounts his performance measures, given in (14) for general T, at a rate of \hat{r} . Start by normalizing $\alpha_R = 1$. To ensure operating decisions are delegated efficiently, $\alpha_v = -\alpha_R = -1$ must hold. Therefore, for T = 1, if the firm invests at date t, the present value of the performance measures from the manager's point of view simplifies to

$$\widehat{\gamma} \cdot CM_{t+1}^*(m_t) + \alpha_d \cdot \widehat{\gamma} \cdot D_{t+1} + \alpha_B \cdot \widehat{\gamma} \cdot BV_t + \alpha_I \cdot b, \qquad (28)$$

that is, for $\alpha_v = -\alpha_R = -1$, we can take as given that the manager will make optimal operating decisions and focus on showing that the shareholders cannot align the investment incentives with their own if they don't know the manager's discount rate, \hat{r} . ⁴⁹ Since for T = 1 the accounting system fully depreciates the asset in the only year of its useful life, i.e., $d_1 = 1$, $BV_t = b$ and $D_{t+1} = b$, the manager's payoff can be written as:

$$\widehat{\gamma} \cdot CM_{t+1}^*(m_t) + (\alpha_d \cdot \widehat{\gamma} + \alpha_B \cdot \widehat{\gamma} + \alpha_I) b.$$

Goal congruence requires that the manager have incentives to invest as soon as $m_t \geq \overline{m}$.

⁴⁹The first three terms in (31) reflect the manager's (discounted) performance measure in period t + 1, π_{t+1} , and the last term is equal to π_t .

Therefore, at $m_t = \overline{m}$, the manager must be indifferent between investing and waiting further. Formally, the manager's payoff net of the personal option value must be equal to zero:

$$\widehat{\gamma} \cdot CM_{t+1}^*(\overline{m}) + (\alpha_d \cdot \widehat{\gamma} + \alpha_B \cdot \widehat{\gamma} + \alpha_I) \cdot b$$
$$-\mathbb{E}_{t(\overline{m})} \left\{ \widehat{\gamma}^{t(\overline{m})-t+1} \cdot \left[CM_{t(\overline{m})+1}^*(m_{t(\overline{m})}) + (\alpha_d \cdot \widehat{\gamma} + \alpha_B \cdot \widehat{\gamma} + \alpha_I) \cdot b \right] \mid m_t = \overline{m} \right\} = 0.(29)$$

for all \hat{r} . Note that when $\hat{r} \to \infty$, the left-hand side of the equation above approaches $\alpha_I \cdot b$, therefore, α_I must be equal to zero.

Consider two special cases: (i) $\hat{r} = r$ and (ii) $\hat{r} \to \infty$.

Case (i): Here we show that, if $\hat{r} = r$, then $\alpha_d \cdot \gamma + \alpha_B \cdot \gamma = -1$ will have to hold for the manager to internalize the correct investment threshold. If $\hat{r} = r$, then subtracting (27) from (29), we obtain

$$(\alpha_d \cdot \gamma + \alpha_B \cdot \gamma + \alpha_I + 1) \cdot b - (\alpha_d \cdot \gamma + \alpha_B \cdot \gamma + \alpha_I + 1) \cdot b \cdot \mathbb{E}_{t(\overline{m})} \left\{ \gamma^{t(\overline{m}) - t + 1} \mid m_t = \overline{m} \right\} = 0.$$
(30)

Therefore,

$$\alpha_d \cdot \gamma + \alpha_B \cdot \gamma = -1.$$

Case (ii): Now assume $\hat{r} \to \infty$. Since we have shown that $\alpha_I = 0$, equation (29) can be simplified as follows:

$$\begin{aligned} \widehat{\gamma} \cdot CM_{t+1}^*(\overline{m}) + \left(\alpha_d \cdot \widehat{\gamma} + \alpha_B \cdot \widehat{\gamma}\right) b \\ -\mathbb{E}_{t(\overline{m})} \left\{ \widehat{\gamma}^{t(\overline{m})-t+1} \cdot \left[CM_{t(\overline{m})+1}^*(m_{t(\overline{m})}) + \left(\alpha_d \cdot \widehat{\gamma} + \alpha_B \cdot \widehat{\gamma}\right) b \right] | m_t = \overline{m} \right\} = 0 \end{aligned}$$

for all \hat{r} . Dividing by $\hat{\gamma}$ and then taking the limit, as $\hat{r} \to \infty$, this expression tends to

$$CM_{t+1}^*(\overline{m}_t) + (\alpha_d + \alpha_B) \cdot b = \gamma^{-1} \cdot (f(\overline{m}) - b) > 0,$$

where the last inequality holds because the NPV of the threshold project exceeds zero by the option value. Therefore, equation (29) cannot simultaneously hold for $\hat{r} = r$ and for sufficiently high values of \hat{r} .

Proof of Proposition 6

Without loss of generality, we can assume that u = 1. By Proposition 4, it is optimal for the firm to invest as soon as m_t exceeds \overline{m} . Assume that if the manager invests when the permanent component of the price shift variable is equal to m_t , his payoff is given by

$$f(m_t, \hat{r}) - \hat{b},\tag{31}$$

where $f(m_t, \hat{r})$ is the present value of the expected optimized cash flows from the project, calculated at the manager's discount rate and \hat{b} is some constant.⁵⁰ Let us show that there exists a \hat{b} , such that the manager will invest as soon as m_t exceeds \overline{m} .

Given the manager's objective function in (31), the manager's problem is equivalent to the principal's problem studied in Proposition 4 with a different discount factor and investment cost. From the proof of Proposition 4, it follows that the optimal investment policy for the manager is to invest as soon as the value of m_t exceeds a certain threshold level \hat{m} . At this threshold, the manager is indifferent between investing immediately and waiting further:

$$f\left(\widehat{m},\widehat{r}\right) - \widehat{b} = \Theta_w^{\widehat{m}}\left(\widehat{m},\widehat{r},\widehat{b}\right),\tag{32}$$

where $\Theta_w^{\widehat{m}}(\widehat{m})$ represents the option value of postponing the project until the next date when m_t exceeds \widehat{m} , calculated at the manager's interest rate, \widehat{r} , and assuming that the cost of investment is \widehat{b} . Formally,

$$\Theta_w^{\widehat{m}}\left(\widehat{m},\widehat{r},\widehat{b}\right) = \mathbb{E}_t\left[\gamma^{t(\widehat{m})-t} \cdot \left(f\left(m_{t(\widehat{m})},\widehat{r}\right) - \widehat{b}\right) | m_t = \widehat{m}\right],$$

where $t(\widehat{m})$ is the random stopping time given by $t(\widehat{m}) = \min\{t + \tau | m_{t+\tau} \ge \widehat{m}\}.$

We need to show that there exists a \hat{b} such that equation (32) is satisfied at \overline{m} , the principal's optimal investment threshold:

$$f(\overline{m},\widehat{r}) - \widehat{b} = \Theta_w^{\overline{m}}\left(\overline{m},\widehat{r},\widehat{b}\right).$$
(33)

⁵⁰By the manager's payoff, we mean the manager's utility measured in date t dollars. Formally, we assume that if the manager invests at date t, then $U_t = \hat{\gamma}^t \cdot \left(f(m_t, \hat{r}) - \hat{b}\right)$.

The derivative of the right-hand side of the equation above with respect to \hat{b} is equal to

$$-\widehat{\gamma} \cdot \Pr\left\{t\left(\overline{m}\right) = t+1 \mid m_t = \overline{m}\right\} - \widehat{\gamma}^2 \cdot \Pr\left\{t\left(\overline{m}\right) = t+2 \mid m_t = \overline{m}\right\} - \dots,$$

which is strictly greater than -1, as $t(\overline{m})$ is finite with probability one. The derivative of the right-hand side of (33) with respect to \hat{b} is equal to -1. It is easy to verify that for $\hat{b} = 0$,

$$f(\overline{m}, \widehat{r}) > \Theta_w^{\overline{m}}(\overline{m}, \widehat{r}, 0) ,$$

i.e., if the manager is impatient and does not internalize any cost of investment, then the manager will overinvest. On the other hand, if $\hat{b} = b + \Theta_w^{\overline{m}}(\overline{m})$ (i.e., the manager internalizes the principal's direct cost of investment plus the principal's option value), then

$$f\left(\overline{m},\widehat{r}\right) - \widehat{b} < \Theta_w^{\overline{m}}\left(\overline{m},\widehat{r},\widehat{b}\right),$$

and the manager underinvests. The observations above imply that there exists \hat{b} , $0 \leq \hat{b} \leq b + \Theta_w^{\overline{m}}(\overline{m})$, such that

$$f(\overline{m},\widehat{r}) - \widehat{b} = \Theta_w^{\overline{m}}\left(\overline{m},\widehat{r},\widehat{b}\right).$$

It remains to construct a performance measure with the property that the manager's expected payoff is equal to

$$f\left(m_t, \widehat{r}\right) - \widehat{b}$$

if the project is implemented at date t. Assume that the manager is compensated based on residual income. Then, once the investment is made, the manager will be optimizing the contribution margin in each period. Therefore, the manager's expected compensation is:

$$\sum_{\tau=1}^{T} \widehat{\gamma}^{\tau} \left(\mathbb{E}_{t} \left[CM_{t+\tau}^{*} \left(K_{t+\tau}, m_{t+\tau-1} \right) | m_{t} \right] - z_{\tau}^{*} \left(\widetilde{r} \right) \cdot b \right) = f \left(m_{t}, \widehat{r} \right) - \sum_{\tau=1}^{T} \widehat{\gamma}^{\tau} \cdot z_{\tau}^{*} \left(\widetilde{r} \right) \cdot b,$$

where t is the investment date and $z_{\tau}^*(\tilde{r})$ are the historical cost charges corresponding to the REOB rule relative to \tilde{r} (i.e., $z_{\tau}^*(\tilde{r}) = d_{\tau}^*(\tilde{r}) + \tilde{r} \cdot bv_{\tau-1}^*(\tilde{r})$). We need to show that there exists an \tilde{r} such that

$$\sum_{\tau=1}^{T} \widehat{\gamma}^{\tau} \cdot z_{\tau}^* \left(\widetilde{r} \right) = \frac{\widehat{b}}{\overline{b}}.$$

Recall that

$$z_{\tau}^{*}(\widetilde{r}) = \frac{CM_{\tau}^{*}}{\sum_{i=1}^{T} \widetilde{\gamma}^{i} \cdot CM_{i}^{*}}$$

where $CM_{\tau}^* = \mathbb{E}_t \left[CM_{t+\tau}^*(K_{\tau}, m_{t+\tau-1}) | m_t = \overline{m} \right]$. Therefore, it suffices to choose \widetilde{r} such that

$$\frac{\sum_{\tau=1}^{T} \widehat{\gamma}^{\tau} \cdot CM_{\tau}^{*}}{\sum_{\tau=1}^{T} \widetilde{\gamma}^{\tau} \cdot CM_{\tau}^{*}} = \frac{\widehat{b}}{b}.$$
(34)

Note that $\sum_{i=1}^{T} \hat{\gamma}^{\tau} \cdot CM_{\tau}^{*}$ must be greater than \hat{b} , for otherwise condition (33) cannot be satisfied at \hat{b} . Therefore, equation (34) implies that $\tilde{r} < r^{*}$, since at r^{*} , the denominator in the left-hand side is equal to b, and this denominator is monotonically decreasing in \tilde{r} . \Box

Proof of Proposition 1'

Recall that the optimal investment policy, b^* , is given by:

$$\sum_{\tau=1}^{T} \gamma^{\tau} \cdot \mathbb{E}_0 \left[\left. \frac{\partial CM_{\tau}^* \left(K_{\tau}, m_{\tau-1} \right)}{\partial K_{\tau}} \right|_{K_{\tau} = b^*(m_0) \cdot x_{\tau}} \cdot x_{\tau} \mid m_0 \right] = 1.$$

We need to show that

$$\mathbb{E}_{0}\left[\left.\frac{\partial CM_{\tau}^{*}\left(K_{\tau},m_{\tau-1}\right)}{\partial K_{\tau}}\right|_{K_{\tau}=b^{*}\left(m_{0}\right)\cdot x_{\tau}}\mid m_{0}\right]$$

does not depend on m_0 . Then, the result in Proposition 1' will follow from our discussion that precedes its statement in Section 5.

Recall from the proof of Proposition 1 that

$$CM_{\tau}^{*}(K_{\tau}, m_{\tau-1}) = \begin{cases} \frac{1}{\eta-1} \left(\frac{\eta-1}{\eta}\right)^{\eta} v^{1-\eta} \cdot m_{\tau-1}^{\eta}, & \text{if } m_{\tau-1} < \frac{\eta}{\eta-1} v K_{\tau}^{1/\eta} \\ K_{\tau}^{\frac{\eta-1}{\eta}} \cdot m_{\tau-1} - v \cdot K_{\tau}, & \text{otherwise.} \end{cases}$$

Let $G_{\tau-1} \equiv \frac{m_{\tau-1}}{m_0}$ be the cumulative growth factor of the permanent component of the price shift parameter from date 0 to date $\tau - 1$, and $\chi_{\tau-1}(\cdot)$ be its probability density function.⁵¹

⁵¹Specifically, $G_{\tau-1} = \prod_{i=1}^{\tau-1} (1+g_i)$, and is, therefore, log-normally distributed.

Then, we have:

$$\mathbb{E}_{0} \left[CM_{\tau}^{*} \left(K_{\tau}, m_{\tau-1} \right) \mid m_{0} \right] = \int_{0}^{\underline{G}_{\tau-1}} A_{1} \cdot m_{0}^{\eta} \cdot G_{\tau-1}^{\eta} \chi_{\tau-1} \left(G_{\tau-1} \right) dG_{\tau-1} \qquad (35)$$

$$+ \int_{\underline{G}_{\tau-1}}^{\infty} \left(K_{\tau}^{\frac{\eta-1}{\eta}} \cdot m_{0} \cdot G_{\tau-1} - v \cdot K_{\tau} \right) \chi_{\tau-1} \left(G_{\tau-1} \right) dG_{\tau-1} (36)$$

$$A_{1} \equiv \frac{1}{\eta-1} \left(\frac{\eta-1}{\eta} \right)^{\eta} v^{1-\eta},$$

$$\underline{G}_{\tau-1} \equiv \frac{m_{\tau-1}(K_{\tau})}{m_{0}} = \frac{\frac{\eta}{\eta-1} v K_{\tau}^{1/\eta}}{m_{0}}.$$

We can now differentiate the expression in (36) with respect to K_{τ} by applying Leibniz's rule (note that $\underline{G}_{\tau-1}$ depends on K_{τ} and recall that $CM_{\tau}^*(K_{\tau}, m_{\tau-1})$ is continuous at $m_{t-1}(K_{\tau}) = \frac{\eta}{\eta-1}vK_t^{1/\eta}$):

$$\mathbb{E}_{0}\left[\frac{\partial CM_{\tau}^{*}\left(K_{\tau},m_{\tau-1}\right)}{\partial K_{\tau}}\mid m_{0}\right] = \int_{\underline{G}_{\tau-1}}^{\infty} \left(\frac{\eta-1}{\eta}K_{\tau}^{-\frac{1}{\eta}}\cdot m_{0}\cdot G_{\tau-1}-v\right)\chi_{\tau-1}\left(G_{\tau-1}\right)dG_{\tau-1}.$$
 (37)

The first-order condition for b^* becomes:

$$\sum_{\tau=1}^{T} \left(\gamma^{\tau} \cdot \int_{\frac{\eta}{\eta-1}v(b^*x_{\tau})^{\frac{1}{\eta}}/m_0}^{\infty} x_{\tau} \left(\frac{\eta-1}{\eta} \left(b^*x_{\tau} \right)^{-\frac{1}{\eta}} \cdot m_0 \cdot G_{\tau-1} - v \right) \chi_{\tau-1} \left(G_{\tau-1} \right) dG_{\tau-1} \right) = 1.$$

Note that the expression on the left-hand side can be written as a function of $(b^*)^{-\frac{1}{\eta}} \cdot m_0$. It follows that if we let $b_1 \equiv b^*(1)$ denote the optimal investment level for $m_0 = 1$, then

$$b^*(m_0) = b_1 \cdot m_0^{\eta}. \tag{38}$$

We can now use the condition defining the optimal investment policy (38) and equation

(37) to calculate the marginal expected cash flow in period τ :

$$\mathbb{E}_{0} \left[\frac{\partial CM_{\tau}^{*}(K_{\tau}, m_{\tau-1})}{\partial K_{\tau}} \Big|_{K_{\tau} = b^{*}(m_{0}) \cdot x_{\tau}} \cdot x_{\tau} \mid m_{0} \right]$$

= $x_{\tau} \int_{\frac{\eta}{\eta-1} \cdot v \cdot b_{1}^{1/\eta} \cdot x_{\tau}^{1/\eta}}^{\infty} \left(\frac{\eta-1}{\eta} \cdot b_{1}^{-1/\eta} \cdot x_{\tau}^{-1/\eta} \cdot G_{\tau-1} - v \right) \chi_{\tau-1} \left(G_{\tau-1} \right) dG_{\tau-1}.$

The expression above does not depend on m_0 , which concludes the proof of Proposition 1'.

It is useful to note that one can use equations (35) and (38) to verify that the expected value of the contribution margin in period τ is proportional to m_0^{η} . Since, according to (38), the firm's investment amount is also proportional to m_0^{η} , the net present value of the optimal investment project is proportional to m_0^{η} .

Proof of Proposition 2'

Given our discussion in the main text of Section 5, it is sufficient to verify that

$$\frac{\partial CM_{\tau}^{*}\left(K_{\tau},m_{\tau-1}\right)}{\partial K_{\tau}}\bigg|_{K_{\tau}=b^{*}(m_{0})\cdot x_{\tau}}$$

is a convex function of $m_{\tau-1}$. The rest of the proof follows from the argument in the proof of Proposition 2.

Under the one-hoss shay assumption, all x_{τ} are equal to one, so let $\overline{K} \equiv b^* = K_{\tau}$ denote the capacity available in all periods. Then, we have

$$\frac{\partial CM_{\tau}^{*}\left(\overline{K}, m_{\tau-1}\right)}{\partial \overline{K}} = \begin{cases} 0, & \text{if } m_{\tau-1} < \frac{\eta}{\eta-1} v \overline{K}^{1/\eta}, \\ \frac{\eta-1}{\eta} \cdot \overline{K}^{-1/\eta} \cdot m_{\tau-1} - v, & \text{otherwise.} \end{cases}$$

This function is continuous and convex in $m_{\tau-1}$.

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