Agency Conflicts, Prudential Regulation, and Marking to Market*

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Abstract

We develop a theory of how agency conflicts between the shareholders and debt holders of a financial institution, accounting measurement rules, and prudential capital regulation interact to affect the institution's capital structure and project choices. We show that, relative to a benchmark historical cost regime in which assets and liabilities on the institution's balance sheet are measured at their origination values, fair value or mark-to-market accounting could mitigate asset substitution, but exacerbate under-investment arising from debt overhang. The optimal choices of the accounting regime and the prudential solvency constraint balance the inefficiencies due to asset substitution and under-investment that move in opposing directions. Under fair value accounting, the optimal (value-maximizing) solvency constraint declines with the institution's marginal cost of investment in project quality, and with the excess cost of equity capital relative to debt capital. Fair value accounting dominates historical cost accounting *provided* the solvency constraints in the respective regimes take their optimal values. If the solvency constraint in the fair value regime is sub-optimally set to be too tight, however, historical cost accounting dominates fair value accounting.

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1 Introduction

The role that fair value or "mark–to–market" accounting may have played in the recent financial crisis is the subject of an ongoing debate among practitioners, regulators, and academics (e.g., see Laux and Leuz (2009)). Proponents of fair value accounting argue that a balance sheet based on market prices leads to better insights into the current risk profiles of financial institutions. Regulators can intervene in a more timely and effective manner to influence managerial decisions. Tools such as regulatory capital requirements could be used to prevent the inefficient choices or continuation of bad projects. Opponents counter that market prices can only provide useful signals to outsiders if the assets and liabilities of financial institutions trade in frictionless competitive markets. However, such markets do not exist for several important claims held by institutions. Further, fair value accounting along with regulatory capital requirements that are based on market values could increase the risks faced by institutions, and induce myopic behavior by preventing the selection of efficient, long-term projects. To the best of our knowledge, however, a trade-off that is central to this debate—fair value accounting could mitigate inefficient choices of bad projects, but *simultaneously* hamper the choices of good ones—has not been theoretically formalized and its consequences examined.

We develop a theory of how agency conflicts between the shareholders and debt holders of a financial institution, accounting measurement rules, and prudential capital regulation interact to affect the institution's project choices and capital structure. We show that, relative to the benchmark historical cost regime in which all claims are measured at their origination values, fair value accounting could mitigate the inefficiency arising from *asset substitution* or risk-shifting (the choice of risky, negative NPV projects), but exacerbate under-investment due to *debt overhang* (the avoidance of risky, positive NPV projects). The conflicting effects of fair value accounting hold even in the scenario in which the institution's claims are traded in frictionless, competitive markets. The inefficiencies due to under-investment and asset substitution work in opposing directions in that an increase in one mitigates the other. The optimal choices of accounting measurement regime and prudential capital regulation balance the effects of under-investment and asset substitution. Under fair value accounting, we show that the optimal (value-maximizing) solvency constraint declines with the institution's marginal cost of investment in project quality, and with the excess cost of equity relative to debt financing. If the respective solvency constraints in the two regimes take their optimal values, fair value accounting dominates historical cost accounting. If the solvency constraint in the fair value regime is sub-optimally tighter than a threshold, however, historical cost accounting dominates fair value accounting.

We develop a two-period model in which a financial institution finances a long-term project at date 0 through a combination of debt and equity. The term "project," which we adopt for expositional convenience, could also refer to a "pool" of projects. Our theory is broadly applicable to general financial institutions such as insurance firms and commercial banks that are subject to prudential regulation. Prudential regulation takes the form of a solvency constraint that is imposed by a regulator to ensure that the institution's leverage ratio is below a threshold (Dewatripont and Tirole (1994)). As in studies such as Giammarino *et al.* (1993) and Heaton *et al.* (2010), there are deadweight costs of equity relative to debt financing that are represented by equityholders demanding an additional (risk-adjusted) expected return or premium relative to debt holders.[1] The excess cost of equity creates an incentive for the institution to issue debt. The trade–off between the excess cost of equity and agency costs of debt results in a nontrivial optimal capital structure for the institution.

The project's cash flows are realized at the end of period 2. The cash flows increase stochastically (in the sense of first-order stochastic dominance) in the project's *quality* that is only observable by shareholders. The cost incurred by shareholders in choosing the project's quality is a nonnegative random variable. The mean cost increases with the project's quality. At the end of period 1, there is a publicly observable signal about the performance of the project. Given the public signal, if the financial institution meets the solvency constraint, its shareholders may act opportunistically by engaging in asset substitution or risk-shifting in period 2. Asset substitution effectively results in a transfer of wealth from debt holders to shareholders, but is inefficient in that it lowers the institution's value. If the institution violates the solvency constraint at the end of the first period, control transfers to the regulator who closely monitors its operations and ensures that the ex post efficient continuation strategy—no asset substitution—is chosen in period 2.[2] Consequently, the

[1] As in the aforementioned studies, the precise specification of the particular frictions that give rise to the excess cost is not pertinent to our analysis. Alternate formulations of the deadweight costs of equity financing lead to similar results.

[2] Our analysis does not change if we instead assume that the regulator liquidates the institution's assets where the liquidation payoff equals the financial institution's value under no asset substitution. In other words, our results

project's terminal cash flows are affected by its quality that is chosen in the first period, transfer of control to the regulator if the solvency constraint is violated, and potential asset substitution by shareholders in the second period if the solvency constraint is not violated.

We analyze two accounting regimes: a fair value (FV) regime in which the balance sheet of the institution—and, therefore, the solvency constraint—is marked to market every period, and a "pure" historical cost (HC) regime in which all claims are measured at their origination values. Given a solvency constraint, we first examine the institution's optimal choices of its capital structure, its project quality, and its asset substitution strategy in each regime. We then derive the optimal (value-maximizing) choice of the prudential constraint by the regulator. Finally, we compare the two accounting regimes.

The upshot of our analysis is that, regardless of the accounting measurement regime, there are two key inefficiencies that arise from agency conflicts between shareholders and debt holders. First, because shareholders effectively hold a call option on the terminal payoffs, the larger the face value of debt, the larger are shareholders' incentives to increase risk by engaging in *asset substitution.* Second, the larger the face value of debt, the lower are shareholders' incentives to make a costly investment to increase project quality. In other words, the *debt overhang* problem (Myers, 1977), where a larger proportion of the increased total payoffs from the higher quality project accrues to debt holders, could cause shareholders to *under-invest* in project quality in the first period.

The solvency constraint plays an important role in mediating these two inefficiencies but we show that its effectiveness depends on the prevailing accounting regime. Because the balance sheet is not remeasured in the HC regime, the institution automatically meets the solvency constraint at date 1 if it meets it at date 0. Because there is no possibility of a transfer of control at date 1, the solvency constraint has little bite so that the HC regime is plagued with a high incidence of asset substitution. We show, however, that the high incidence of asset substitution alleviates under-investment in the first period. The potential for *ex post* rents from asset substitution increases shareholders' *ex ante* incentives to invest in higher project quality.

Consistent with the intuition expressed by proponents of fair value accounting that market prices play a disciplining role, we show that the FV regime does indeed alleviate the asset substitution

hold even if the institution's claims are traded in frictionless, competitive markets so that prices reflect fundamental values.

inefficiency pervasive in the HC regime. Because claims are marked to market in the FV regime, the solvency constraint has bite at the intermediate date 1 so that transfer of control to the regulator occurs if it is violated. In mitigating asset substitution, however, we show that FV accounting may exacerbate the debt overhang problem by inducing the shareholders to under-invest relative to the HC regime.

In a second best world, the optimal choices of the accounting regime and the solvency constraint trade off the expected inefficiencies arising from asset substitution against those arising from under-investment. In the HC regime, because the solvency constraint plays no role once the capital structure decision has been made, we show that it should be set as high as possible. In other words, it is sub-optimal for the regulator to constrain the institution's own optimal choice of capital structure by imposing a solvency constraint that does not have bite at the interim date. In the FV regime, however, the regulator faces a dilemma in choosing the optimal solvency constraint. A loose solvency constraint aggravates the asset substitution problem because the constraint has less bite. A tight solvency constraint, however, aggravates under-investment by increasing the likelihood of the transfer of control, thereby curbing potential rents from *ex post* asset substitution. In choosing the optimal solvency constraint, the regulator minimizes the expected inefficiencies arising from asset substitution and the expected inefficiencies arising from under-investment. We show that the optimal solvency constraint does not eliminate either inefficiency, that is, both under-investment and asset substitution are possible at the optimum. Further, the optimal solvency constraint becomes tighter when the marginal cost of investment in project quality or the excess cost of equity capital increase. As the excess cost of equity capital increases, the institution's incentives to use debt financing increase so that asset substitution and under-investment *both* become more likely. Nevertheless, it turns out that the asset substitution problem is relatively more pernicious than the debt overhang problem. Consequently, the solvency constraint becomes tighter to mitigate asset substitution at the expense of potentially increasing under-investment. If the marginal cost of investment in project quality increases, the debt overhang problem becomes less severe because shareholders have less incentives to raise project quality. The optimal solvency constraint, therefore, again becomes tighter to mitigate asset substitution.

Finally, we show that, if the prudential constraints in the two regimes are set at their respective value-maximizing levels, the FV regime always dominates the HC regime. The intuition is that the

FV regime can always replicate the HC regime by setting a loose enough prudential constraint such that the constraint has no bite at the interim date. If the solvency constraints in the two regimes are not set at their optimal values, however, the HC regime could dominate the FV regime. More precisely, we show that, if the solvency constraint in the FV regime is below a threshold, the HC regime dominates the FV regime.

Our results show that the optimal solvency constraint in the FV regime is *institution-specific* in that it depends on parameters that determine the payoff distribution of the institution's projects. These parameters are likely to vary across institutions even if they belong to a particular category such as commercial banks or insurance firms. Consequently, a prudential solvency constraint that is uniform across institutions as in the Basel II and proposed Basel III accords, could be sub-optimal. Moreover, as discussed above, fair value accounting with a sub-optimal solvency constraint could be dominated by HC accounting. Our analysis, therefore, highlights the importance of not only choosing an appropriate accounting regime, but tailoring the solvency constraint to the characteristics of the institution.

In Section 2, we discuss related literature. In Section 3, we present our model. Section 4 analyzes the first-best scenario. Sections 5 and 6 analyze the HC and FV regimes, respectively. Section 7 compares the distortions in the HC and FV regimes and generates some welfare implications. Section 8 concludes. The Appendix contains the proofs of all the main results.

2 Related Literature

We contribute to the growing stream of literature that theoretically analyzes the economic trade–offs of mark–to–market versus historical cost measurement policies. O'Hara (1993) investigates the effect of market value accounting on project maturity and finds that mark-to-market results in a preference for short-term projects over long-term projects. Allen and Carletti (2008) (hereafter, AC) and Plantin, Sapra, and Shin (2008) (hereafter, PSS) are two recent studies that show how fair value accounting may have detrimental consequences for financial stability. In both studies, markets are illiquid and incomplete and therefore a reliance on price signals may lead to inefficiencies.[3] In PSS,

[3] A more recent paper by Bleck and Gao (2010) also models an illiquid asset market in an environment in which liquidity is endogenously determined. Gorton, He, and Huang (2010) study the effect of compensation schemes for traders in principal-agent relationships. They show that marked-to-market compensation contracts introduce an externality. Traders may rationally herd, trading on irrelevant information causing asset prices to be less informative

an illiquid market implies that the price of an asset is sensitive to the decisions of other financial institutions. They show that such illiquidity leads to strategic complementarities that destabilize prices by creating an endogenous risk. AC show that marking–to–market may lead to contagion between the banking sector and the insurance sector. We complement these studies in a number of ways. First, in contrast to the above studies, we analyze the effects of accounting measurement on the capital regulation of financial institutions. Because solvency constraints depend on how the values of assets and liabilities are measured, accounting measurement rules naturally have *real* effects. PSS, instead, assume that managers maximize expected accounting earnings so that accounting has real effects. AC focus solely on fair value accounting. Second, because the issues we examine are different, there are important distinctions in the tensions identified. In our setup, markets are frictionless and competitive so that price signals perfectly impound information about future cash flows. We focus on the effects of agency conflicts between a financial institution's shareholders and its debt holders. We show that, even in the absence of liquidity risk, fair value accounting alleviates asset substitution, but exacerbates under-investment.

Burkhart and Strausz (2009) (hereafter, BS) and Heaton, Lucas, and McDonald (2010) (hereafter, HLM) model the effects of fair value accounting on financial institutions and also assume frictionless and competitive markets so that prices fully reflect fundamentals. BS show that, unlike historical cost accounting, fair value accounting increases the liquidity of a financial institution's assets, which, in turn, increases the institution's asset substitution incentives. Our analysis identifies different frictions, and therefore generates very different conclusions. BS focus on the information asymmetry between the institution's current shareholders and prospective shareholders, while we examine conflicts between debt holders and shareholders. In their environment, fair value accounting reduces information asymmetry that induces asset substitution. In our environment, fair value accounting curbs asset substitution through the intervention of the regulator but unfortunately, the under-investment problem is exacerbated. HLM build a general equilibrium model of an institution and study how accounting interacts with an institution's capital requirements to affect the social costs of regulating an institution. In their model, financial institutions invest in firms whose technologies are *exogenous* and *fixed*. In contrast, our analysis centers on how the optimal choices of the accounting regime and the solvency constraint anticipate the financial institution's *endogenous*

than they would be without the marking-to-market.

project choices.

Because our analysis focuses on how prudential capital regulation interacts with accounting measurement rules, our study is also related to the literature on the capital regulation of banks and, more generally, financial institutions (see Dewatripont and Tirole (1995) and Santos (2001) for surveys). We adopt the perspective in Dewatripont and Tirole (1995) who argue that the main concern of prudential regulation is the solvency of financial institutions that, in turn, is related to their capital structure. Capital structure is relevant because it implies an allocation of control rights (e.g., see Aghion and Bolton (1992)) between shareholders and debt holders. Further, the importance of regulation stems from the fact that small, uninformed debt holders of institutions need a representative to protect their interests. In early studies, Merton (1978) and Bhattacharya (1982) show that capital requirements curb inefficient risk-shifting. However, studies such as Koehn and Santomero (1980), Kim and Santomero (1988), Gennotte and Pyle (1991), and Rochet (1991) argue that capital requirements could alter the equilibrium scale of operations of an institution and, therefore, its optimal asset composition in ambiguous ways. Besanko and Kanatas (1996) show that conflicts of interest between a bank's management and its shareholders could lower, and sometimes even reverse, the beneficial effects of capital regulation in curbing asset substitution. We contribute to this literature by showing how solvency constraints optimally balance the inefficiencies arising from asset substitution and under-investment. More importantly, our study demonstrates how the trade–off between these inefficiencies is affected by the accounting measurement regime.

3 The Model

3.1 Environment

We model a financial institution that finances a long-term project through a combination of debt and equity. The term "project" could refer to a "pool" of projects. Because our theory is broadly applicable to institutions that are subject to prudential regulation such as insurance firms, banks, and securities firms, we deliberately do not model a specific type of institution.[4] Since our focus is on the conflicts of interest between the institution's shareholders and debt holders, we

[4] If the institution is an insurance firm, its "creditors" include insurance policyholders. With the pooling of insurance risks, the insurance firm's liabilities arising from insurance claims are similar to a debt obligation. If the institution is a bank, its creditors include depositors and other debt holders.

assume that the management behaves in the interests of shareholders and takes all decisions to maximize shareholder value.

The project's payoff increases stochastically (in the sense of first-order stochastic dominance) in the project's *quality*. The institution chooses the quality of the project through careful analysis and selection. The cost incurred by shareholders in choosing the quality of the project is a nonnegative random variable. The mean cost increases with project quality.

At some interim date before the project's payoffs are realized, there is a publicly observable signal about the performance of the project. At this date, the institution's shareholders may act opportunistically by engaging in asset substitution or risk-shifting that results in the transfer of wealth from debt holders to shareholders, but lowers the value of the overall project. Asset substitution could be achieved by engaging in off balance sheet derivative transactions, altering the characteristics of the existing project, etc.

The institution operates in a regulated environment. There is a prudential regulator who protects the interests of small and uninformed debt holders by ensuring that, at any point of time, the institution's leverage ratio is not too high. Consistent with the Basel II and the proposed Basel III regulations, the regulator imposes a prudential or solvency constraint to ensure that the value of the institution's assets are sufficiently high relative to its liabilities. If the prudential constraint is violated at the interim date, control transfers to the regulator who closely monitors the institution and ensures that it chooses the efficient continuation strategy—no asset substitution—in the second period.

Our analysis does not change in any way if we, instead, assume that the regulator sells/liquidates the institution's assets where the total payoff is the market value of the assets assuming the efficient continuation strategy—no asset substitution—is chosen in the second period. In other words, our results hold even if the institution's claims are traded in frictionless, competitive markets, that is, there are no deadweight costs arising from the early sale/liquidation of the institution's assets.

We study two accounting regimes: a "pure" historical cost (HC) regime in which the balance sheet of the institution—and therefore the prudential constraint—is measured using the original prices of the claims; and a fair value (FV) regime in which the balance sheet of the institution—and therefore the prudential constraint—is marked to market every period using the current market prices of the claims. We view the pure HC regime as a benchmark against which we examine the effects of mark-to-market or fair value accounting. We carry out both positive and normative analyses. For each accounting regime, we first examine the effects of a given solvency constraint on the institution's capital structure, its project quality, and its asset substitution strategy. We then derive the optimal (value-maximizing) choice of the prudential constraint by the regulator. Finally, we analyze the optimal choice of accounting regime. We next describe the ingredients of the model in more detail.

3.2 The Long-Term Project and the Institution's Capital Structure

We consider a two-period model with three dates $0, 1, 2$. All agents are risk-neutral, but, as we discuss below, could have differing discount rates.

At the beginning of period 1, i.e., at $t = 0$, the institution makes a fixed investment A_0 in a long-term project. The institution finances the investment through a combination of debt and equity. As in studies such as Giammarino et al. (1993) and Heaton et al. (2010), equity capital is costlier than debt capital. For concreteness, we model the deadweight costs of equity capital by assuming that equityholders demand a higher (risk-adjusted) expected return on their investment than debt holders.[5] For example, if the institution is an insurance firm, the lower cost of debt capital could arise from the fact that agents have a demand for insurance. The insurance firm's core business is the provision of insurance so that it has a comparative advantage in supplying insurance that it does not possess in raising equity capital. If the institution is a bank, the lower cost of debt could arise from the fact that the bank has a comparative advantage in raising capital from depositors. Alternately, the wedge between the cost of equity and the cost of debt could arise because debt has tax advantages and/or equity is associated with higher costs due to asymmetric information, etc. Because the particular frictions that give rise to the excess cost of equity are not relevant to our analysis, we follow studies such as Giammarino et al. (1993) and Heaton et al. (2010) by not modeling them explicitly.

We normalize the cost of debt to 1 and the cost of equity to $1+\lambda$ where λ denotes the excess cost of equity. For simplicity, we restrict consideration to zero coupon debt that pays off at date $t = 2$. The amount of debt the institution chooses to issue is determined by its payoff M at maturity. Let the market value of the debt at date $t = 0$, which is endogenously determined in the model, be D_0.

[5] Other choices of modeling the excess cost of equity capital lead to similar results.

The institution therefore finances the remaining amount $E_0 = A_0 - D_0$ through equity. We assume that capital markets are competitive.

If the institution is a bank, its depositors are protected by deposit insurance in practice. We do not incorporate the presence of deposit insurance in our analysis because, as mentioned earlier, we intend our theory to be applicable to a general financial intermediary whose liabilities need not be secured. Even in the case of banks, a substantial portion of their debt is long-term and unsecured.

It turns out that, even if we restrict ourselves to the specific case in which the institution is a bank and all its debt comprises of demand deposits, our implications are unaffected as long as deposit insurance is fairly priced. The reason is that fairly priced deposit insurance—that is, the deposit insurance premium rationally incorporates the institution's optimal choices of capital structure, project quality, and asset substitution—is merely a transfer of funds from shareholders to debt holders. Shareholders pay the deposit insurance premium to the deposit insurer who, in turn, compensates debt holders if the institution defaults. Consequently, although debt is risk-free due to deposit insurance, the deposit insurance premium lowers the value of equity so that the value of the institution—the size of the total pie—is unchanged. Furthermore, the deposit insurance premium is a sunk cost that is incurred *ex ante*. Consequently, the *ex post* value of equity—that is, after deposit insurance and capital structure are in place—is identical to its value in the scenario in which there is no deposit insurance. The upshot of these implications is that none of the institution's decisions—capital structure, project quality, and asset substitution—is affected by the presence of deposit insurance. Because the size of the total pie is unchanged by deposit insurance, the regulator's objective function is also unaltered. The only result that changes is the magnitude of the optimal solvency constraint which increases with deposit insurance because the value of insured debt is higher than that of uninsured debt.[6]

3.3 Project Quality Choice

The terminal cash flows of the project are realized at date $t = 2$. The terminal cash flows, which we describe shortly, are affected by both the *quality* of the project chosen in period 1 and potential *asset substitution* chosen in period 2. We denote the quality of the institution's project by $q_i \in \{q_L, q_H\}$ where $0 \leq q_L < q_H \leq 1$. The project quality is only observable by shareholders.

[6] An analysis of the model with deposit insurance is available upon request.

The institution can always invest in a default long-term project, i.e., in a project with a low quality level q_L. By carefully analyzing and screening the type of project that it finances, the institution can raise the quality of its project from q_L to q_H. The resources invested by the shareholders in choosing a project of quality q_i is a nonnegative random variable C_i with support $(0, \infty)$ and mean κ_i, where $i = L$ or H. The expected cost of choosing a project is increasing in its quality: $\kappa_H - \kappa_L = k(q_H - q_L)$, where $k > 0$ is the marginal expected cost of quality.

To simplify notation, we normalize q_L to 0. Therefore enhancing the project quality from q_L to q_H requires the institution's shareholders to incur an incremental expected cost of $k(q_H - q_L) = kq_H$. We, henceforth, alternately refer to the additional expected cost kq_H as the *additional investment* to increase project quality.

3.4 Intermediate Signal and Prudential Constraint

At the interim date $t = 1$, there is a publicly observable signal of the final payoff of the project. The signal $y \in \{X_L, X_H\}$ where $X_H > X_L > 0$. If the quality of the project is $q_i \in \{q_L, q_H\}$, then

$$\Pr(X_H) = q_i \text{ and } \Pr(X_L) = 1 - q_i. \tag{1}$$

By (1), the high quality project first-order stochastically dominates the low quality project, that is, the probability of receiving a high intermediate signal is greater with the higher quality project.

At any date t, the institution faces a solvency constraint imposed by a regulator to protect the interests of the institution's creditors. The solvency constraint requires that the value of the institution's assets be high enough relative to the value of its liabilities. In a fair value accounting regime, where all assets and liabilities are marked to market, the constraint takes the form

$$\frac{D_t}{F_t} \leq c \text{ where } t \in \{0, 1\}, \tag{2}$$

where D_t is the *market value* of the institution's debt and F_t is the *market value* of the institution's total assets at date t. In (2), the interval $0 \leq c \leq 1$ implies that the institution's leverage ratio must be below a threshold c.

If the prudential constraint is satisfied at date 1—that is, $\frac{D_1}{F_1} \leq c$—the institution's shareholders maintain control for the second period. However, if it is not satisfied—that is, $\frac{D_1}{F_1} > c$—control transfers to the regulator who closely monitors the institution and ensures that it does not engage in asset substitution. We later describe a benchmark accounting regime that we refer to as the *historical cost regime* in which the institution's assets and liabilities are not marked to market.

3.5 Terminal Payoffs

At the beginning of period 2, the shareholders decide whether to engage in asset substitution. In particular, given the signal $y = X_i$, where $i \in \{L, H\}$, the shareholders take a hidden action that is represented by the ordered pair $(r_j, z_j) \in \{(r_L, z_L), (r_H, z_H)\}$ that alters the terminal payoffs of the institution. Given $y = X_i$, the terminal payoff of the institution, X_{ij}^T, takes two possible values, either $(1 + z_j)X_i$ or $(1 - z_j)X_i$, where

$$\begin{aligned} \Pr(X_{ij}^T &= (1 + z_j)X_i | y = X_i, r_j) = \frac{1}{2} - r_j \\ \Pr(X_{ij}^T &= (1 - z_j)X_i | y = X_i, r_j) = \frac{1}{2} + r_j, \end{aligned} \tag{3}$$

where $0 \leq r_L < r_H \leq \frac{1}{2}$. The parameter $z_j \in \{z_L, z_H\}$ where $0 \leq z_L < z_H \leq 1$ measures the degree to which action (r_j, z_j) alters the terminal payoffs.

Given r_j and $y = X_i$, the expected value of the terminal cash flows X_{ij}^T of the institution is

$$E(X_{ij}^T | y = X_i, r_j) = (1 - 2r_j z_j)X_i \text{ for } i \in \{L, H\} \text{ and for } j \in \{L, H\}.$$

Note that for any $i \in \{L, H\}$, action (r_j, z_j) captures asset substitution because it injects uncertainty in the terminal payoffs in order to benefit the shareholders while simultaneously reducing the expected terminal cash flows of the institution from X_i to $(1 - 2r_j z_j)X_i$.

To simplify the algebra, we make two assumptions about the asset substitution technology. First, we normalize $r_L = 0$ and $z_L = 0$ so that action (r_L, z_L) is associated with no asset substitution, while action (r_H, z_H) represents asset substitution. Second, we assume a "recombining" binomial tree. That is, the best possible terminal payoff from asset substitution when the intermediate signal is low, i.e., when $y = X_L$, equals the worst possible terminal payoff from asset substitution when the intermediate signal is high, i.e., when $y = X_H$, so that

$$(1 + z_H)X_L = (1 - z_H)X_H. \tag{4}$$

We also make the following standing assumption on project parameters:

$$\frac{1}{1+\lambda}(1 + z_H)X_L < A_0 < \frac{1}{1+\lambda}X_H - kq_H. \tag{5}$$

The first inequality implies that, conditional on a low intermediate signal at date 1, even the best possible outcome under asset substitution is not sufficient to recover the initial investment A_0. The second inequality ensures that, conditional on a high intermediate signal at date 1, engaging in no asset substitution has a positive net payoff in the sense that the corresponding terminal payoff X_H is greater than the sum of the initial investment A_0 and the expected incremental cost kq_H of choosing high project quality. Assumption (5) ensures that the inefficiencies arising from asset substitution problem are severe enough for prudential regulation to have significant bite.

By (1) and (3), the distribution of terminal cash flows X_{ij}^T depends on both the unobservable project quality $q_i \in \{q_L = 0, q_H\}$ chosen in period 1 and on the unobservable asset substitution strategy $(r_j, z_j) \in \{(r_L = 0, z_L = 0), (r_H, z_H)\}$ chosen in period 2. We refer to period 1 as the *investment stage* and to period 2 as the *asset substitution* stage.

Figure 1 summarizes the sequence of events for periods 1 and 2. Figure 2 illustrates how the distribution of terminal cash flows X^Tdepends on the institution's investment technology q in period 1 and its asset substitution technology r in period 2.

The payoffs of the shareholders and debt holders depend on whether the solvency constraint (2) is violated at the end of period 1 and, therefore, on whether the regulator takes control. If the regulator takes control at $t = 1$, it ensures that the institution chooses the ex post efficient strategy of no asset substitution, that is, it chooses $(r_L, z_L) = (0, 0)$ in the second period. The debt holders' payoffs equal the lower of the face value M of the debt or the terminal cash flows X^T of the institution. Because shareholders are the residual claimants, they receive cash flows net of payments to debt holders minus the cost of investment in project quality. Table 1 summarizes the payoffs of the shareholders, the debt holders, and their combined payoffs:

Figure 1: Sequence of Events

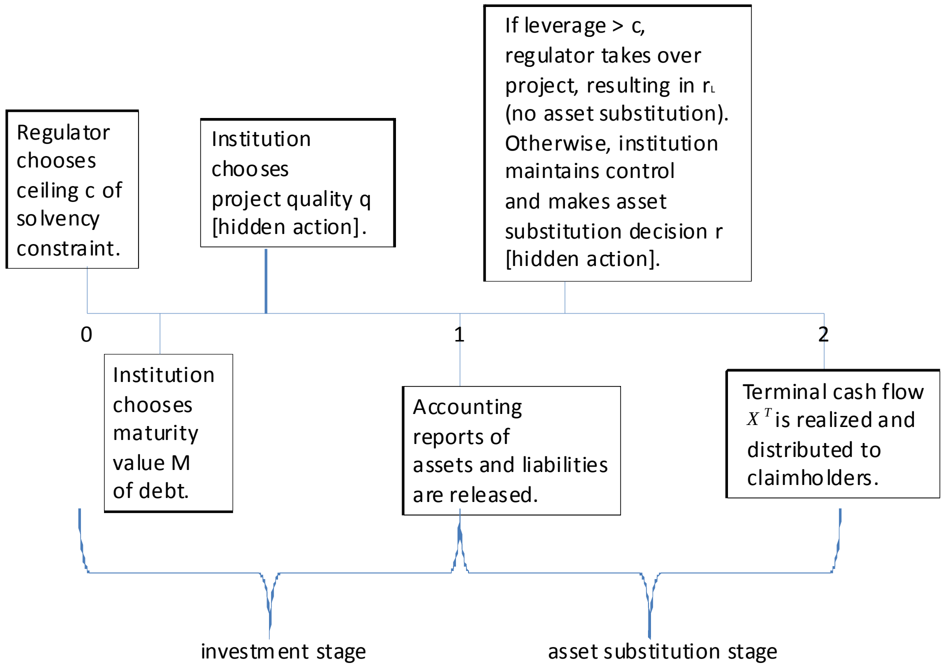

Figure 2: Technology

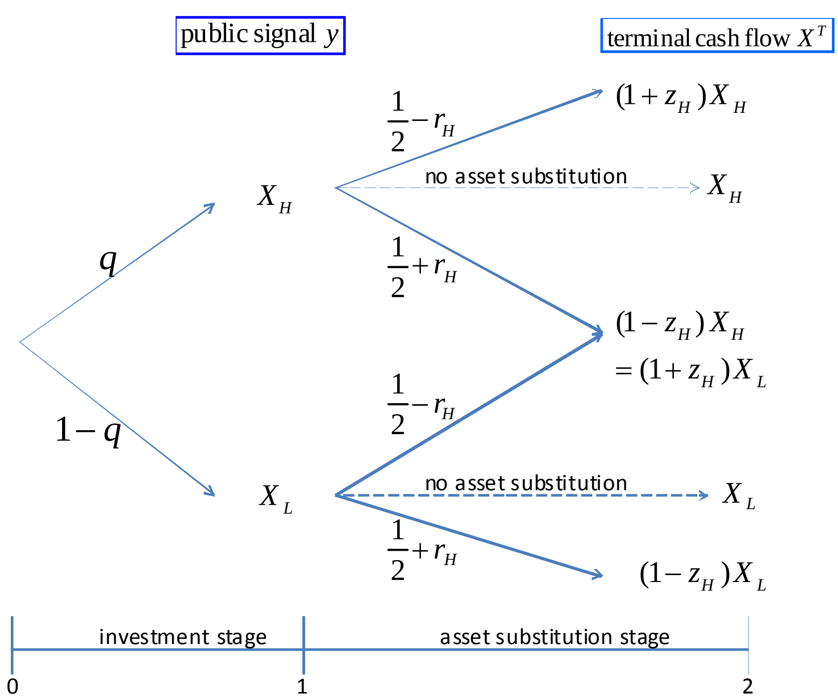

Table 1: Payoffs of Debt Holders and Shareholders		
	Institution Maintains Control	Regulator Takes Control
Debt holders' Payoff	$\min\{M, X_{ij}^T\}$	$\min\{M, X_{iL}^T\}$
Shareholders' Payoff	$-C_i + \max\{X_{ij}^T - M, 0\}$	$-C_i + \max\{X_{iL}^T - M, 0\}$
Total Payoff	$-C_i + X_{ij}^T$	$-C_i + X_{iL}^T$

The payoffs in the scenario where the regulator takes control reflect the fact that the regulator ensures that the institution chooses the efficient continuation strategy—no asset substitution—in the second period, that is, $(r_j, z_j) = (r_L, z_L) = (0, 0)$.

3.6 Accounting Regimes

We study two accounting measurement regimes. The first regime, which should be viewed as a benchmark regime, is a pure *historical cost* regime (HC) in which the institution's assets and liabilities are measured at their initial date 0 "origination" values. More precisely, in the context of our model, the prudential constraint is given by

$$\frac{D_0}{A_0} \leq c^{HC} \tag{6}$$

at the initial date $t = 0$ *and* the intermediate date $t = 1$. In the above, D_0 is the initial value of the institution's debt and A_0 is the acquisition cost of its assets.

The second regime is the *fair value* regime (FV) in which the institution's balance sheet is marked to market every period so that the solvency constraint is given by (2), that is,

$$\begin{aligned} \frac{D_0}{F_0} &\leq c^{FV} \text{ at } t = 0, \\ \text{and } \frac{D_1}{F_1} &\leq c^{FV} \text{ at } t = 1. \end{aligned} \tag{7}$$

The superscripts on the solvency constraints in the two regimes reflect the fact that they could differ across the regimes.

4 First-Best Regime

We start by briefly discussing the first-best (FB) scenario in which all decisions are made to maximize the total value of the institution (rather than just shareholder value), and the excess cost of equity λ is zero.

Given that asset substitution reduces the total value of the institution, it is optimal to choose no asset substitution in the first best regime, that is, $(r^{FB}, z^{FB}) = (0, 0)$. No asset substitution in period 2, in turn, implies that, given the public signal y,

$$E(X_{ij}^T | y, (r^{FB}, z^{FB}) = (0, 0)) = y.$$

At $t = 0$, the institution chooses the project quality q_j, $j \in \{L, H\}$, to maximize $-\kappa_j + E(y|q_j)$. Because $-\kappa_j + E(y|q_j)$ equals $q_H X_H + (1 - q_H)X_L - \kappa_H$ if q_H is chosen and $X_L - \kappa_L$ if q_L is chosen, q_H is chosen if and only if

$$q_H X_H + (1 - q_H)X_L - \kappa_H > X_L - \kappa_L$$
$$\Leftrightarrow \kappa_H - \kappa_L < q_H(X_H - X_L) \Leftrightarrow k < X_H - X_L,$$

which is true from (5). The last implication above follows from the fact that $\kappa_H - \kappa_L = k(q_H - q_L) = kq_H$ (recall that we have normalized q_L to zero).

We, therefore, have the following proposition that describes the institution's project quality and asset substitution choices in the first best regime.

Proposition 1 (First Best Regime) *In the first best regime, the institution always chooses the high quality project and does not engage in asset substitution. Because the incentives of shareholders are aligned with those of creditors, the institution's capital structure and therefore its leverage play no role. Therefore, accounting measurement issues are moot.*

We now proceed to analyze the second-best world in which maximizing shareholder value is not necessarily equivalent to maximizing the total value of the institution. Our analysis highlights two inefficiencies that reduce the institution's value. First, shareholders may engage in *asset substitution* that increases shareholder value to the detriment of institution value. Second, there is also a *debt* *overhang* problem (Myers, 1977) in the sense that the shareholders may under-invest in project quality by choosing the low quality project, q_L. Under-investment also reduces the institution's value.

We analyze each accounting regime using backward induction. We start at the beginning of period 2 when the public signal y has been released. For a given face value M of debt and a given public signal y, we first derive the transfer of control and asset substitution decision r in period 2, and then we derive the project quality decision q in period 1. Next, we determine the capital structure decision via the choice of M, the face value of debt. Finally, given the institution's optimal capital structure, investment, and asset substitution decisions, we derive the optimal/value-maximizing solvency constraint.

5 Historical Cost Regime

Under the HC regime, because the institution's assets and liabilities are measured at their date 0 origination values until terminal payoffs are realized, the solvency constraint $\frac{D_0}{A_0} \leq c$ is used at *both* $t = 0$ and $t = 1$. In the HC regime, therefore, if the solvency constraint is satisfied at date 0, it is automatically satisfied at date 1. Consequently, there is no transfer of control at date 1.

5.1 Asset Substitution

At date $t = 1$, given the public signal y and the debt face value M, shareholders decide whether to engage in asset substitution by choosing the hidden action $r_j \in \{r_L = 0, r_H\}$ to maximize

$$\begin{aligned} & E[\max\{X^T - M, 0\}|y, M] \\ = \ & (\frac{1}{2} - r_j)\max\{(1+z_j)y - M, 0\} + (\frac{1}{2} + r_j)\max\{(1-z_j)y - M, 0\}. \end{aligned} \tag{8}$$

If the shareholders choose (r_H, z_H), it follows from (8) that their value is

$$(\frac{1}{2} - r_H)\max\{(1+z_H)y - M, 0\} + (\frac{1}{2} + r_H)\max\{(1-z_H)y - M, 0\}. \tag{9}$$

However, if they choose $(r_L, z_L) = (0, 0)$, their value is

$$\max\{y - M, 0\}. \tag{10}$$

Proposition 2 (Asset Substitution in HC Regime) *For a given face value M of debt and intermediate signal y, shareholders choose asset substitution if and only if the face value M of debt is sufficiently high relative to the intermediate signal y:*

$$(r^{HC}, z^{HC}) = (r_H, z_H) \Leftrightarrow M > \left(1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H\right) y. \tag{11}$$

The underlying intuition for Proposition 2 is well known. Because shareholders effectively hold a call option on the terminal payoff with strike price equal to the face value of debt, it is optimal for them to increase risk by choosing asset substitution when the intermediate signal is sufficiently low relative to the face value of debt. We also note that as $\frac{1}{2} - r_H$ (the probability of a good outcome given asset substitution) and/or z_H (the spread of outcomes resulting from asset substitution) increases, thereby making asset substitution more attractive to shareholders in period 2, the threshold value of M above which asset substitution takes place decreases, i.e., $\left(1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H\right) y$ decreases, thereby increasing the incidence of asset substitution.

5.2 Project Quality

At date 0, given the face value M of debt, shareholders choose the project quality $q \in \{q_L, q_H\}$ anticipating the asset substitution decision in period 2 given by (11). For example, suppose the face value M is such that $M < (1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H) X_L$. Then, from (11), regardless of the value of y at date 1, shareholders will choose $(r_L, z_L) = (0, 0)$. For a given M, their date 2 payoffs would be

$$\max\{y - M, 0\} = y - M \text{ because } M < (1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H) X_L < X_L.$$

From the perspective of date $t = 0$, therefore, shareholders choose project quality $q \in \{q_L = 0, q_H\}$ to maximize

$$\frac{1}{1+\lambda} \left[q(X_H - M) + (1 - q)(X_L - M)\right] - \kappa(q),$$

where $\kappa(q) = \kappa_H$ if $q = q_H$, and $\kappa(q) = \kappa_L$ if $q = q_L$. Using similar arguments to analyze all the possible face values of debt, M, we derive the next proposition that describes the optimal choice of project quality by the shareholders.

Proposition 3 (Project Quality in HC Regime) *If the marginal cost of investment, k, is below a threshold, i.e., $k \leq k^* \equiv \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H X_H/(1+\lambda)$, shareholders choose q_H if and only if $M \leq (1+z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}$. If the marginal cost of investment, k, is above the threshold, i.e., $k \geq k^*$, shareholders choose q_H if and only if $M < X_H - k(1+\lambda)$. To summarize,*

if $k \leq k^*$	*if* $M < (1+z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}$ $q^{HC} = q_H$	*if* $M > (1+z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}$ $q^{HC} = q_L$
if $k > k^*$	*if* $M < X_H - k(1+\lambda)$ $q^{HC} = q_H$	*if* $M > X_H - k(1+\lambda)$ $q^{HC} = q_L$

Proposition 3 states that shareholders choose the low quality project if the face value of debt is sufficiently high. This is essentially a consequence of the well known "debt overhang" problem (for example, Myers (1977)). If the amount of debt in the institution's capital structure is sufficiently high, shareholders' incentives to make a costly investment in the higher quality project are curbed because a larger proportion of the increased total payoff from such an investment will accrue to debt holders.

However, in contrast with previous literature, Proposition 3 also shows that the debt overhang problem in period 1 is *alleviated* by the possibility of asset substitution in period 2. To make this result more transparent, suppose asset substitution were infeasible. In that hypothetical environment, it is straightforward to show that shareholders would choose q_H if and only if

$$M < X_H - k(1+\lambda).$$

In words, shareholders would choose the high quality project q_H if and only if the face value of the debt is lower than the maximum payoff to the shareholders from choosing q_H.

When $k \leq k^*$, it can easily be verified that

$$X_H - k(1+\lambda) < (1+z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}.$$

It follows from the above and Proposition 3 that the range of values of M for which under-investment occurs shrinks with the possibility of asset substitution. Furthermore, as $\frac{1}{2} - r_H$ and/or z_H increases, the threshold k^* increases so that the region $k \leq k^*$ expands while the region $k > k^*$ shrinks. Therefore, not only does the range of values of M that induce under-investment shrink in the presence of asset substitution, but as asset substitution becomes more attractive, the latter effect also persists for even large values of k.

The intuition for these results follows from the fact that the anticipation of enhanced *ex post* rents from asset substitution mitigates shareholders' incentives to under-invest in project quality due to the debt overhang problem. Specifically, the anticipation of asset substitution in period 2 induces the shareholders to choose the high quality project q_H in period 1 provided the marginal cost of investment is below a threshold. Furthermore, when asset substitution becomes more profitable in period 2, i.e., when $\frac{1}{2} - r_H$ and/or z_H increases, q_H is attractive even for large values of k.

5.3 Capital Structure

In the preceding analysis, we first derived the optimal asset substitution strategy in period 2 taking the institution's capital structure represented by the face value M of debt as given. Given the optimal asset substitution strategy, we then derived the optimal project quality choice in period 1. The bank optimally finances the project by issuing debt and equity rationally anticipating the *ex post* project quality and asset substitution choices. In particular, at $t = 0$, the bank's original shareholders anticipate these optimal q and r decisions and choose the institution's capital structure to maximize their value subject to the date $t = 0$ solvency constraint (6).

Because the cost of debt is normalized to 1 and that of equity to $1 + \lambda$, from the perspective of $t = 0$, shareholder value equals the present value of their expected future cash flows discounted at the cost of equity $1 + \lambda$ net of the cost of investment $\kappa(q)$ and net of their equity investment E_0, i.e., the shareholders maximize

$$\underbrace{\frac{E(\max\{X^T_{HC} - M, 0\})}{1 + \lambda} - \kappa(q^{HC}) - E_0}_{\text{net present value of expected future cash flows to shareholders}}, \tag{12}$$

where $X^T_{HC} \equiv X^T(q^{HC}, r^{HC})$ denotes the terminal cash flows of the institution given the optimal choices of investment q^{HC} and asset substitution decision r^{HC}.

Substituting for $E_0 = A_0 - D_0$ in (12), shareholder value at $t = 0$ is

$$\frac{E(\max\{X_{HC}^T - M, 0\})}{1+\lambda} + \ D_0 - \kappa(q^{HC}) - A_0.$$

The debt face value, M, determines the institution's capital structure. At $t = 0$, the optimal choice of the debt face value in the HC regime solves

$$M^{HC} = \arg\max_M \frac{E(\max\{X_{HC}^T - M, 0\})}{1+\lambda} + D_0 - \kappa(q^{HC}) - A_0 \tag{13}$$

subject to the $t = 0$ solvency constraint

$$\frac{D_0}{A_0} \leq c, \tag{14}$$

where D_0, the date $t = 0$ market value of debt, is given by

$$D_0 = E(\min\{M, X_{HC}^T\}). \tag{15}$$

Note that, by (15), the market value of debt, D_0, depends on the debt face value M.

By (13), M^{HC} balances the trade–off between the excess cost of equity represented by λ and the inefficiencies arising from under-investment and asset substitution due to the presence of debt in the institution's capital structure. In Lemma 1 in the Appendix, we analyze the above optimization program. The optimal face value of debt, M^{HC}, depends on the underlying parameters of our environment. In particular, as one would expect, M^{HC} increases in λ, that is, the optimal amount of debt financing increases with the excess cost of equity.

5.4 Prudential Constraint

The regulator chooses the optimal solvency constraint, c^{HC}, to maximize the total value of the institution. Given that the debt market is competitive, the institution's original shareholders extract all the surplus from its operations. Therefore, in choosing the optimal solvency constraint, c^{HC}, the regulator's problem of maximizing the total value of the institution is equivalent to maximizing shareholder value subject to the solvency constraint (6). Of course, in the presence of agency costs, the institution's total value in our second-best environment is strictly lower than the first-best benchmark.

In the HC regime, the solvency constraint does not have any bite at date $t = 1$ and thus transfer of control never occurs. The prudential constraint is only relevant at $t = 0$ by constraining the shareholders' optimal choice of M. But in the HC regime, shareholders optimally choose M in order to maximize their value that, as we have argued above, is equivalent to maximizing the value of the institution. Constraining the choice of M via the prudential constraint only reduces the institution's date 0 value below the unconstrained maximum. Therefore, c^{HC} should be set as high as possible, that is, it should be set to 1.

Proposition 4 (Optimal Prudential Constraint in HC Regime) *The optimal prudential constraint in the historical cost regime, c^{HC}, is 1.*

To summarize, in the HC regime, because transfer of control does not occur at the end of period 1, there is a prevalence of asset substitution in period 2. However, the potential for expected rents from asset substitution in period 2 alleviates the under-investment problem in period 1 by increasing shareholders' incentives to invest in the high quality project. Furthermore, the more severe the asset substitution problem is in the second period, the less severe the under-investment problem is in the first period. Finally, in the HC regime, because the prudential constraint only restricts the institution's choice of capital structure and plays no role once the capital structure is in place, it should be set as high as possible.

The absence of transfer of control implies that the pure HC regime is essentially equivalent to a regime with no prudential regulation, that is, we could also interpret the benchmark HC regime as a "no regulation" regime. However, if we modify the model to incorporate the possibility of interim cash flows at date 1, transfer of control becomes feasible in the HC regime at date 1. Nevertheless, it can be shown that, for reasonable parameterization, transfer of control would still be significantly less likely in the HC regime compared with the FV regime. Our main results hinge on the trade-off between the higher likelihood of transfer of control in the FV regime against the lower incidence of asset substitution. Consequently, the key implications of our study are unlikely to be altered even if we were to complicate the model by incorporating interim cash flows so that transfer of control is possible in the HC regime.

6 Fair Value Regime

In the fair value regime (FV), the institution's balance sheet is marked to market every period so that the prudential constraint is

$$\frac{D_0}{F_0} \leq c \text{ at } t = 0 \text{ and } \frac{D_1}{F_1} \leq c \text{ at } t = 1, \tag{16}$$

where D_t and F_t, respectively, denote the market values of the institution's debt and assets at t. If $\frac{D_1}{F_1} > c$, the regulator takes control and closely monitors the institution to ensure that there is no asset substitution in period 2.

6.1 Asset Substitution

We again proceed by backward induction. At $t = 1$, the institution's asset substitution decision (r, z) is unobservable. Consequently, in order to value the institution's debt, the capital market forms a conjecture $(\widehat{r}, \widehat{z})$ about (r, z). Given the date $t = 1$ signal y, the capital market values the institution's debt at

$$\begin{aligned} D_1(y, (\widehat{r}, \widehat{z})) &= E[\min\{M, X^T\}|y, (\widehat{r}, \widehat{z})] \\ &= (\frac{1}{2} - \widehat{r}) \min\{M, (1 + \widehat{z})y\} + (\frac{1}{2} + \widehat{r}) \min\{M, (1 - \widehat{z})y\}. \end{aligned} \tag{17}$$

Similarly, at date $t = 1$, the market values the institution's assets at

$$F_1(y, (\widehat{r}, \widehat{z})) = E[X^T|y, (\widehat{r}, \widehat{z})] = (1 - 2\widehat{r}\widehat{z})y. \tag{18}$$

Suppose the market conjectures that $(\widehat{r}, \widehat{z}) = (r_H, z_H)$, then

$$D_1(y, (\widehat{r}, \widehat{z}) = (r_H, z_H)) = (\frac{1}{2} - r_H) \min\{M, (1 + z_H)y\} + (\frac{1}{2} + r_H) \min\{M, (1 - z_H)y\} \tag{19}$$

and

$$F_1(y, (\widehat{r}, \widehat{z}) = (r_H, z_H)) = (1 - 2r_H z_H)y. \tag{20}$$

Suppose, instead, the market conjectures that $(\widehat{r}, \widehat{z}) = (r_L, z_L) = (0, 0)$, then

$$D_1(y, (\widehat{r}, \widehat{z}) = (0, 0)) = \min\{M, y\} \tag{21}$$

and

$$F_1(y, (\widehat{r}, \widehat{z}) = (0, 0)) = y. \tag{22}$$

These date $t = 1$ market prices along with the prudential constraint determine whether transfer of control occurs. Given the continuation or control transfer outcome, the institution chooses $r \in \{r_L, r_H\}$ in period 2.

If transfer of control occurs at date $t = 1$, the regulator ensures that the efficient continuation decision $(\widehat{r}, \widehat{z}) = (0, 0)$ in period 2 is chosen so that the payoff at date 2 is y. If transfer of control does not occur at date $t = 1$, then shareholders could either choose or not choose asset substitution. In equilibrium, the market's conjecture must be confirmed in the sense that given $D_1(y, (\widehat{r}, \widehat{z}))$ and $F_1(y, (\widehat{r}, \widehat{z}))$ and the prudential constraint c, the institution's optimal asset substitution strategy is indeed $(\widehat{r}, \widehat{z})$.

The following proposition characterizes the optimal continuation/transfer of control and asset substitution decisions given the debt face value M and prudential constraint c.

Proposition 5 (Asset Substitution in FV Regime) *Given the face value M of the debt, the prudential constraint c, and the signal y, the optimal transfer of control/continuation and asset substitution decisions in the FV regime are summarized below:*

$c \leq c^* \equiv 1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H$	$M < cy$ *continue and* (r_L, z_L)		$M > cy$ *transfer and* (r_L, z_L)
$c > c^* \equiv 1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H$	$M < c^* y$ *continue and* (r_L, z_L)	$M \in [c^* y, cy]$ *continue and* (r_H, z_H)	$M > cy$ *transfer and* (r_L, z_L)

Note that unlike the HC regime, transfer of control occurs in the FV regime if M is sufficiently large relative to y. In fact, the preceding proposition shows that, regardless of the value of c, as long as $M > cy$, transfer of control occurs. This is a direct consequence of violating the date $t = 1$ solvency constraint. Transfer of control prevents the possibility of asset substitution in period 2.

Furthermore, in the FV regime, for values of c below a threshold, i.e., $c < c^* \equiv 1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H$, either transfer of control occurs or shareholders retain control and *voluntarily* do not choose asset substitution in period 2. Consequently, regulators may eliminate the inefficiencies created by asset substitution by choosing a low value of the ceiling c. In fact, as $\frac{1}{2} - r_H$ and/or z_H increases, c^* shrinks. Therefore as shareholders find asset substitution more enticing in period 2, an even tighter solvency constraint is necessary to eliminate asset substitution.

For relatively high values of c, i.e., $c > c^* \equiv 1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H$, asset substitution only occurs for intermediate values of M relative to y. For large values of M relative to y, the institution violates the prudential solvency constraint so that transfer of control occurs, and the regulator ensures that no asset substitution is chosen.

To summarize, compared to the HC regime in which transfer of control does not occur at $t = 1$, the incidence of asset substitution is lower in the FV regime because transfer of control occurs for large values of M relative to y. A low enough value of c may completely rule out asset substitution. Conversely, a high value of c exacerbates asset substitution. In fact, as c increases so that the ceiling is very high, the FV regime becomes effectively equivalent to the HC regime.

6.2 Project Quality

At date 0, given the face value M of the debt, shareholders choose the project quality q anticipating the transfer of control/continuation and asset substitution rules described in Proposition 5. The following result describes shareholders' optimal choice of project quality at date $t = 0$.

Proposition 6 (Project Quality in FV Regime) *Let* $c_1 \equiv 1 - \frac{k(1+\lambda)}{X_H}$, $c_2 \equiv (1+z_H) - \frac{k(1+\lambda)}{(\frac{1}{2}-r_H)X_H}$, *and* $k^* \equiv \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H X_H/(1+\lambda)$.

For $k \leq k^*$	
values of c:	*shareholders choose* q_H *if and only if*
$c > c_2$	$M < (1 + z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}$
$c \in [c_1, c_2]$	$M < cX_H$
$c < c_1$	$M < X_H - k(1 + \lambda)$

For $k > k^*$
shareholders choose q_H *if and only if*
$M < X_H - k(1+\lambda)$

Note that, unlike the HC regime in which the solvency constraint plays no direct role in affecting the choice of project quality, Proposition 6 illustrates the crucial role that it plays in the FV regime in determining the project quality choice q when the marginal cost of investment is below a threshold ($k \leq k^*$).

Recall from Proposition 5 that, the smaller c is, the higher the likelihood of transfer of control. Proposition 6 implies that, the smaller c is, the higher the likelihood of under-investment (q_L). Taken together, these two propositions imply a positive relationship between transfer of control and under-investment.

For high values of c ($c > c_2$), transfer of control is highly unlikely so that the FV regime becomes equivalent to the HC regime. In fact, for high values of c, we recover the same result obtained in the HC regime: shareholders under-invest if and only if $M > (1+z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}$.

For intermediate values of c ($c \in [c_1, c_2]$), the threshold (cX_H) of the face value of debt triggering under-investment becomes lower. Thus, as the likelihood of transfer of control increases, the under-investment problem worsens.

For low values of c ($c < c_1$), the prudential constraint is so low that the institution is very likely to exceed it. A high likelihood of transfer of control implies that the incidence of asset substitution is very low. Not surprisingly, the FV regime becomes equivalent to a world of no asset substitution, so that shareholders under-invest (q_L) if and only if $M > X_H - k(1+\lambda)$.

Furthermore, as shareholders find asset substitution more attractive in period 2, i.e., as $\frac{1}{2} - r_H$ and/or z_H increases, both c_2 and k^* increase. In other words, as the *ex post* rents from asset substitution increase, the positive relationship between transfer of control and under-investment becomes more pervasive as it applies to a larger set of values of the ceiling c and the marginal cost k.

The discussion above along with the intuition for Proposition 5 suggests that, while transfer of control at date $t = 1$ *mitigates* inefficiencies created by asset substitution in period 2, it *exacerbates* inefficiencies created by under-investment in period 1. As we discuss shortly, the optimal choice of the solvency constraint balances the trade-off between these two sources of inefficiencies.

6.3 Prudential Constraint

As in the HC regime, the optimal capital structure of the institution maximizes shareholder value subject to (14) and (15). However, unlike the HC regime in which there is no transfer of control at $t = 1$, in the FV regime, the shareholders' payoffs are affected by the potential transfer of control at $t = 1$. In Lemma 2 in the Appendix, we analyze the optimization program that determines the institution's optimal capital structure.

We now turn to the derivation of the optimal solvency constraint in the FV regime. In other words, anticipating the institution's capital structure, quality investment, and asset substitution decisions, how should a regulator set the optimal value of the solvency constraint?

In choosing the optimal solvency constraint in the FV regime, the regulator faces a dilemma. Choosing a high value of c aggravates the asset substitution problem in period 2 while choosing a low value of c aggravates under-investment in period 1. In choosing the optimal constraint, the regulator therefore minimizes the expected inefficiencies arising from asset substitution and the expected inefficiencies arising from under-investment. The next result characterizes the optimal solvency constraint in the FV regime.

Proposition 7 (Optimal Prudential Constraint in FV Regime) *In the FV regime, the optimal solvency constraint,* c^{FV}*, is* $1 - \frac{k(1+\lambda)}{X_H}$*.*

By setting $c^{FV} = 1 - \frac{k(1+\lambda)}{X_H}$, it follows from Proposition 5 and from Proposition 6 that the regulator maximizes the expected surplus by reducing the incidence of asset substitution while tolerating the under-investment problem arising from excessive transfer of control. With this constraint, however, neither inefficiency is completely eliminated, that is, under-investment and asset substitution *both* occur at the optimum.

Proposition 7 also shows that the optimal solvency constraint becomes tighter as the excess cost of equity λ or the marginal cost k of investment in project quality increase. As λ increases, the institution's incentives to use debt financing increase so that asset substitution and under-investment *both* become more likely. Nevertheless, it turns out that the asset substitution problem is relatively more pernicious than the debt overhang problem. Consequently, the solvency constraint becomes tighter to mitigate asset substitution at the expense of potentially increasing under-investment. As k increases, the debt overhang problem becomes less severe because shareholders have less incentives to raise project quality. The optimal solvency constraint, therefore, again becomes tighter to mitigate asset substitution.

To summarize, in the FV regime, because the balance sheet of the institution is marked to market every period, the solvency constraint reflects current market values. The institution therefore faces a threat of transfer of control at the end of period 1. If the institution violates the solvency constraint, transfer of control eliminates the possibility of asset substitution in period 2. The tighter (looser) the prudential constraint, the higher (lower) the likelihood of transfer of control. Therefore, to reduce the incidence of asset substitution, the solvency constraint must be tightened. Unfortunately, in doing so, the under-investment problem is aggravated. The regulator trades off the asset substitution problem against the under-investment problem.

7 Comparison of Fair Value and Historical Cost Regimes

We have shown that, in a second best world, regardless of the accounting measurement regime, there are two distortions that reduce the total value of the institution. First, there is an asset substitution problem whereby shareholders engage in risk-shifting to transfer wealth from debt holders to the detriment of institution value. Second, there is a debt overhang problem that causes shareholders to under-invest in project quality, which reduces institution value. We have also shown how the solvency constraint mediates these two distortions. In the HC regime, because the balance sheet is not remeasured in the interim date, the solvency constraint has no bite in the interim date. The institution faces no threat of transfer of control so that the incidence of asset substitution is high. In the FV regime, because the balance sheet is marked to market, the solvency constraint serves as a credible threat of transfer of control, thereby alleviating the asset substitution inefficiency pervasive in the HC regime.

The following table summarizes Propositions 2 and 5 and confirms the intuition that the incidence of asset substitution is higher in the HC regime than in the FV regime. In fact, for low values of the solvency constraint c, FV eliminates asset substitution.

HC	if $M < (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)y$	if $M > (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)y$
	No Asset Substitution	Asset Substitution
FV	if $c > 1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H$:	
	if $M < (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)y$ or $M > cy$	if $M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)y, cy]$
	No Asset Substitution	Asset Substitution
	if $c \leq 1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H$:	
	No Asset Substitution	

The following proposition compares the under-investment problem in the two regimes.

Proposition 8 (Under-Investment in the Two Regimes) *(i) If the marginal cost of investment is low, i.e., $k \leq k^* \equiv \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H X_H/(1+\lambda)$, and the solvency constraint is tight, i.e., $c < c_2 \equiv (1+z_H) - \frac{k(1+\lambda)}{(\frac{1}{2}-r_H)X_H}$, then (a) the under-investment problem in the FV regime is worse than in the HC regime and (b) as $\frac{1}{2} - r_H$ and/or z_H increases, the under-investment problem deteriorates in the FV regime relative to the HC regime.*

(ii) If $k \leq k^*$ and $c > c_2$, or $k > k^*$, the extent of under-investment is the same in both regimes.

Note that as the asset substitution problem increases, i.e., $\frac{1}{2} - r_H$ and/or z_H increases, both k^* and c_2 increase so that the under-investment region in the FV regime expands relative to the under-investment region in the HC regime over a larger range of values of parameters. That is, the debt overhang problem in the FV regime worsens.

It follows from the above results that the interplay between asset substitution and under-investment could cause the HC regime to dominate the FV regime if the solvency constraints in the two regimes do not take their optimal values, c^{HC} and c^{FV}, respectively. However, if the two constraints take their optimal values, it easily follows from our analysis that the FV regime unambiguously dominates the HC regime. This is because the regulator operating in the FV regime can always replicate the HC regime by setting a loose enough solvency constraint so that it will not have bite at the interim date and there is no transfer of control as in the HC regime. Consequently, the FV regime can do at least as well as the HC regime. The following corollary states the result.

Corollary 1 (Comparison Between Accounting Regimes) *Suppose that* $c^{HC} = 1$ *and* $c^{FV} = 1 - \frac{k(1+\lambda)}{X_H}$*. The FV regime always dominates the HC regime.*

Note that the optimal solvency constraint, c^{FV}, in the FV regime depends on the marginal cost of investment in project quality, k, and the value of the high signal X_H. These parameters are likely to vary across institutions even if they belong to the same category such as commercial banks or insurance firms. The optimal solvency constraint is, therefore, *institution-specific*. Further, the optimal solvency constraint also depends on the excess cost of equity λ that could vary over time and, in particular, with the business cycle.

The above discussion implies that a uniform solvency constraint across institutions such as in the Basel II and proposed Basel III regulations may not be optimal. Further, the result of Corollary 1 crucially depends on the respective solvency constraints in the HC and FV regimes taking their optimal values. The following proposition shows that, if the solvency constraint in the FV regime is below a threshold, the HC regime dominates the FV regime.

Proposition 9 (HC Versus FV Regime) *Suppose that* $c^{HC} = 1$*. There exists* $c_0 \in (0, c_1 \equiv 1 - \frac{k(1+\lambda)}{X_H})$ *such that for* $c \in [0, c_0)$*, the HC regime dominates the FV regime.*

Proposition 9 shows that, if the solvency constraint in the fair value regime is below a threshold, the historical cost regime would be superior to the fair value regime. Our results, therefore, highlight the importance of not only choosing the appropriate accounting regime, but tailoring the solvency constraint to the characteristics of the institution.

8 Conclusion

In the aftermath of the recent financial crisis, the merits and demerits of fair value accounting are being actively debated by academics, practitioners, and regulators. Our study contributes to the debate by showing how accounting measurement affects the intensity of agency conflicts between a financial institution's shareholders and debt holders in the presence of prudential capital regulation. We demonstrate that, relative to the "historical cost" regime in which assets and liabilities on an institution's balance sheet are measured at their origination values, fair value accounting could alleviate the inefficiencies arising from asset substitution, but exacerbate those arising from under-investment due to debt overhang. Debt overhang and asset substitution work in opposing directions. The optimal choices of accounting regime and prudential solvency constraint balance the conflicts between shareholders and debt holders. Under fair value accounting, the optimal solvency constraint declines with the institution's marginal cost of investment in project quality and the excess cost of equity capital relative to debt capital. Fair value accounting dominates historical cost accounting provided the solvency constraints in the respective regimes take their optimal values. If the solvency constraints are sub-optimally chosen, however, historical cost accounting could dominate fair value accounting.

We have shown that by ignoring current price signals, the HC regime is plagued with asset substitution that reduces social welfare. By relying on current market prices, FV alleviates ex post asset substitution, but exacerbates ex ante underinvestment in project quality that also reduces social welfare. Our analysis therefore suggests that there exists an *optimal* degree of reliance on market prices that minimizes these inefficiencies. In future research, we hope to shed light on the optimal design of accounting measurement rules for financial institutions. Such an extension would inform the current debate about the extent to which a financial institution's prudential regulation and capital requirements should rely on fair value measurements.

To sharpen the analysis, we assumed a standard capital structure for the financial institution consisting of equity and debt. Our results, however, suggest an important role for hybrid securities such as "convertible debt." In the aftermath of the recent financial crisis, there is an ongoing debate about the role that "contingent capital" could play to resolve agency conflicts when financial institutions are insolvent. In our environment, the optimal prudential solvency constraint in the FV regime determines the transfer of control to the regulator to curb asset substitution in the second period. These "real" effects of regulation and the transfer of control could also be achieved by using a hybrid security such as convertible debt that retains the characteristics of debt when the intermediate signal is above a threshold, but converts to equity when it is below the threshold. In future research, it would be important to build on these insights and extend the analysis to study the optimal security design of a financial institution.

Appendix

Proof of Proposition 2

Using expressions (9) and (10), Table 2 summarizes how the asset substitution decision r depends on the relative magnitudes of the face value M of the debt and the signal y:

Table 2			
	payoff from r_H	payoff from r_L	asset substitution decision r
$M < (1 - z_H)y$	$(\frac{1}{2} - r_H)[(1 + z_H)y - M]$ $+(\frac{1}{2} + r_H)[(1 - z_H)y - M]$	$y - M$	r_L
$M \in [(1 - z_H)y, y]$	$(\frac{1}{2} - r_H)[(1 + z_H)y - M]$	$y - M$	r_L if $M < (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)y$ r_H if $M > (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)y$
$M \in [y, (1 + z_H)y]$	$(\frac{1}{2} - r_H)[(1 + z_H)y - M]$	0	r_H
$M > (1 + z_H)y$	0	0	r_H

The statements of the proposition directly follow from the above table. ■

Proof of Proposition 3

The shareholders' expected payoffs from choosing q at date $t = 0$ are summarized in Table 3 below for all the feasible values of M:

Table 3		
Feasible Values of M	asset substitution	shareholders' expected payoffs
$M < (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$	r_L if $y = X_H$ r_L if $y = X_L$	$\frac{1}{1+\lambda}\{q[X_H - M]$ $+(1 - q)[X_L - M]\}$ $-\kappa(q)$
$M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L, (1 + z_H)X_L]$	r_L if $y = X_H$ r_H if $y = X_L$	$\frac{1}{1+\lambda}\{q[X_H - M]$ $+(1 - q)(\frac{1}{2} - r_H)[(1 + z_H)X_L - M]\}$ $-\kappa(q)$
$M \in [(1 + z_H)X_L, (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H]$	r_L if $y = X_H$ r_H if $y = X_L$	$\frac{1}{1+\lambda}q[X_H - M]$ $-\kappa(q)$
$M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H, (1 + z_H)X_H]$	r_H if $y = X_H$ r_H if $y = X_L$	$\frac{1}{1+\lambda}q(\frac{1}{2} - r_H)[(1 + z_H)X_H - M]$ $-\kappa(q)$

Using Table 3, the shareholders' expected payoff from choosing q can be investigated as follows:

Case 1: $M < (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$: Comparing the shareholders' expected payoff from q_H and that from q_L, shareholders choose q_H if and only if $k < \frac{1}{1+\lambda}(X_H - X_L)$, which is true by assumption in (5). Therefore, shareholders will choose q_H.

Case 2: $M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L, (1 + z_H)X_L]$: Comparing the shareholders' expected payoff from q_H and that from q_L, shareholders choose q_H if and only if $M < \frac{X_H - (\frac{1}{2} - r_H)(1+z_H)X_L - k(1+\lambda)}{\frac{1}{2}+r_H}$.

Case 3: $M \in [(1+z_H)X_L, (1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)X_H]$: Comparing the shareholders' expected payoff from q_H and that from q_L, shareholders choose q_H if and only if $M < X_H - k(1+\lambda)$.
Case 4: $M \in [(1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)X_H, (1+z_H)X_H]$: Comparing the shareholders' expected payoff from q_H and that from q_L, shareholders choose q_H if and only if $M < (1+z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}$.
Using the above results, we derive the optimal choice of q for different values of k.
(i) $k < \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H X_H/(1+\lambda)$: In Case 1, shareholders will always choose q_H. In Case 2, shareholders choose q_H if and only if $M < \frac{X_H-(\frac{1}{2}-r_H)(1+z_H)X_L-k(1+\lambda)}{\frac{1}{2}+r_H}$. But even for the highest possible value of M in Case 2, $(1+z_H)X_L$, that inequality always holds as long as $k < X_H-(1+z_H)X_L$. Because of the assumption of $(1+z_H)X_L = (1-z_H)X_H$, $k < X_H-(1+z_H)X_L \Leftrightarrow k < z_H X_H$, which is satisfied because $k < \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H X_H$. So q_H will be chosen. In Case 3, shareholders choose q_H if and only if $M < X_H - k(1+\lambda)$. But even for the highest possible value of M in Case 3, $(1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)X_H$, that inequality always holds as long as $k < \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H X_H/(1+\lambda)$, which is true by the assumption for k in this scenario (i). So q_H will be chosen. In Case 4, shareholders choose q_H if and only if $M < (1+z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}$, which is exactly stated in the statements of this proposition.
(ii) $k \in [\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H X_H/(1+\lambda), (X_H-(1+z_H)X_L)/(1+\lambda)]$: In Case 1, shareholders will always choose q_H. In Case 2, shareholders choose q_H if and only if $M < \frac{X_H-(\frac{1}{2}-r_H)(1+z_H)X_L-k(1+\lambda)}{\frac{1}{2}+r_H}$. But even for the highest possible value of M in Case 2, $(1+z_H)X_L$, that inequality always holds as long as $k < X_H-(1+z_H)X_L$, which is true by the assumption in (5). So q_H will be chosen. In Case 3, shareholders choose q_H if and only if $M < X_H - k(1+\lambda)$, which is exactly stated in the statements of this proposition. In Case 4, shareholders choose q_L if and only if $M > (1+z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}$. But even for the lowest possible value of M in Case 4, $(1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)X_H$, that inequality holds as long as $k > \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H X_H/(1+\lambda)$, which is true by the assumption for k in this scenario (ii). So q_L will be chosen.
(iii) $k > (X_H-(1+z_H)X_L)/(1+\lambda)$: This case is infeasible by the assumption in (5).

For the purpose of future reference, we summarize the shareholders' expected payoff from the optimal choice of q in the following:

$k < \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H X_H/(1+\lambda)$	choice of q	shareholders' expected payoff from the optimal choice of q
$M < (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$	q_H	$\frac{1}{1+\lambda}\{q_H X_H + (1-q_H)X_L - M\}$ $-\kappa_H$
$M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L, (1+z_H)X_L]$	q_H	$\frac{1}{1+\lambda}\{q_H X_H$ $+(1-q_H)(\frac{1}{2}-r_H)(1+z_H)X_L$ $-[q_H + (1-q_H)(\frac{1}{2}-r_H)]M\}$ $-\kappa_H$
$M \in [(1+z_H)X_L, (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H]$	q_H	$\frac{1}{1+\lambda} q_H [X_H - M] - \kappa_H$
$M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H, (1+z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}]$	q_H	$\frac{1}{1+\lambda} q_H (\frac{1}{2}-r_H)[(1+z_H)X_H - M]$ $-\kappa_H$
$M \in [(1+z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}, (1+z_H)X_H]$	q_L	$-\kappa_L$

$k > \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H X_H/(1+\lambda)$	choice of q	shareholders' expected payoff from the optimal choice of q
$M < (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$	q_H	$\frac{1}{1+\lambda}\{q_H X_H + (1-q_H)X_L - M\}$ $-\kappa_H$
$M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L, (1+z_H)X_L]$	q_H	$\frac{1}{1+\lambda}\{q_H X_H$ $+(1-q_H)(\frac{1}{2}-r_H)(1+z_H)X_L$ $-[q_H + (1-q_H)(\frac{1}{2}-r_H)]M\}$ $-\kappa_H$
$M \in [(1+z_H)X_L, X_H - k(1+\lambda)]$	q_H	$\frac{1}{1+\lambda} q_H [X_H - M] - \kappa_H$
$M \in [X_H - k(1+\lambda), (1+z_H)X_H]$	q_L	$-\kappa_L$

■

Lemma 1: Capital Structure in HC Regime

In Section 5.3, we discussed the general guidelines for deriving the shareholders' capital structure decision, M, the face value of debt, in the HC regime. The following table shows the choices of M by regions and the shareholders' Date 0 expected payoffs by regions:

HC regime: $k < k^*$		
	Choice of M	Shareholders' *ex ante* payoffs
$M \leq (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H) X_L$	$(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H) X_L$	$\pi_I^{HC} \equiv -A_0 - \kappa_H$ $+\frac{1}{1+\lambda}[q_H X_H + (1-q_H) X_L]$ $+\frac{\lambda}{1+\lambda}(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H) X_L$
$M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H) X_L,$ $(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H) X_H]$	$(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H) X_H$	$\pi_{II}^{HC} \equiv -A_0 - \kappa_H$ $+\frac{1}{1+\lambda} q_H X_H$ $+(1-q_H)(1-2r_H z_H) X_L$ $+\frac{\lambda}{1+\lambda} q_H (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H) X_H$
$M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H) X_H,$ $(1+z_H) X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}]$	$(1+z_H) X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}$	$\pi_{III}^{HC} \equiv -A_0 - \kappa_H$ $+q_H(1-2r_H z_H) X_H$ $+(1-q_H)(1-2r_H z_H) X_L$ $-\lambda q_H k$
$M \in [(1+z_H) X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H},$ $(1+z_H) X_H]$	$[(1+z_H) X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H},$ $(1+z_H) X_H]$	$\pi_{IV}^{HC} \equiv -A_0 - \kappa_L + (1-2r_H z_H) X_L$

HC regime: $k > k^*$		
	Choice of M	Shareholders' *ex ante* payoffs
$M \leq (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H) X_L$	$(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H) X_L$	$-A_0 - \kappa_H$ $+\frac{1}{1+\lambda}[q_H X_H + (1-q_H) X_L]$ $+\frac{\lambda}{1+\lambda}(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H) X_L$
$M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H) X_L,$ $X_H - k(1+\lambda)]$	$(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H) X_H$	$-A_0 - \kappa_H$ $+\frac{1}{1+\lambda} q_H X_H$ $+(1-q_H)(1-2r_H z_H) X_L$ $+\frac{\lambda}{1+\lambda} q_H [X_H - k(1+\lambda)]$
$M \in [X_H - k(1+\lambda),$ $(1+z_H) X_H]$	$[(1+z_H) X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H},$ $(1+z_H) X_H]$	$-A_0 - \kappa_L + (1-2r_H z_H) X_L$

Proof of Lemma 1

(i) The case where $k < \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H X_H/(1+\lambda)$:

Under the historical cost regime, taking into consideration of the optimal choices of r and q, we first derive D_0 using (15):

HC regime	
$k < \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H X_H/(1+\lambda)$	market value of debt at date 0, D_0
$M \leq (1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$	M
$M \in [(1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L,$ $(1+z_H)X_L]$	$[q_H+(1-q_H)(\frac{1}{2}-r_H)]M+(1-q_H)(\frac{1}{2}+r_H)(1-z_H)X_L$
$M \in [(1+z_H)X_L,$ $(1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H]$	$q_H M+(1-q_H)(1-2r_H z_H)X_L$
$M \in [(1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H,$ $(1+z_H)X_H-\frac{k(1+\lambda)}{\frac{1}{2}-r_H}]$	$q_H(\frac{1}{2}-r_H)M+q_H(\frac{1}{2}+r_H)(1-z_H)X_H+(1-q_H)(1-2r_H z_H)X_L$
$M \in [(1+z_H)X_H-\frac{k(1+\lambda)}{\frac{1}{2}-r_H},$ $(1+z_H)X_H]$	$(1-2r_H z_H)X_L$

Under the historical cost regime, taking into consideration of the optimal choices of r and q, we derive the expected payoffs for those regions above using (13):

HC regime	
$k < \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H X_H/(1+\lambda)$	Shareholders' *ex ante* payoffs
$M \leq (1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$	$-A_0-\kappa_H$ $+\frac{1}{1+\lambda}[q_H X_H+(1-q_H)X_L]$ $+\frac{\lambda}{1+\lambda}M$
$M \in [(1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L,$ $(1+z_H)X_L]$	$-A_0-\kappa_H$ $+\frac{1}{1+\lambda}[q_H X_H+(1-q_H)(\frac{1}{2}-r_H)(1+z_H)X_L]$ $+(1-q_H)(\frac{1}{2}+r_H)(1-z_H)X_L$ $+\frac{\lambda}{1+\lambda}[q_H+(1-q_H)(\frac{1}{2}-r_H)]M$
$M \in [(1+z_H)X_L,$ $(1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H]$	$-A_0-\kappa_H$ $+\frac{1}{1+\lambda}q_H X_H$ $+(1-q_H)(1-2r_H z_H)X_L$ $+\frac{\lambda}{1+\lambda}q_H M$
$M \in [(1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H,$ $(1+z_H)X_H-\frac{k(1+\lambda)}{\frac{1}{2}-r_H}]$	$-A_0-\kappa_H$ $+\frac{1}{1+\lambda}q_H(\frac{1}{2}-r_H)(1+z_H)X_H+q_H(\frac{1}{2}+r_H)(1-z_H)X_H$ $+(1-q_H)(1-2r_H z_H)X_L$ $+\frac{\lambda}{1+\lambda}q_H(\frac{1}{2}-r_H)M$
$M \in [(1+z_H)X_H-\frac{k(1+\lambda)}{\frac{1}{2}-r_H},$ $(1+z_H)X_H]$	$-A_0-\kappa_L+(1-2r_H z_H)X_L$

Because regional payoffs are increasing in M, the optimal value of M for a given region is the upper bound of that region, as shown in the statement of this lemma.

Substituting the regional optimal M into the regional payoff function yields the regional expected payoffs for shareholders at Date 0. For example, for the region of $M \leq (1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$, the shareholders' expected payoff is $-A_0-\kappa_H+\frac{1}{1+\lambda}[q_H X_H+(1-q_H)X_L]+\frac{\lambda}{1+\lambda}M$ and the optimal value of M for this particular region is therefore $(1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)X_L$. Inserting $M=(1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)X_L$ into the payoff function yields $-A_0-\kappa_H+\frac{1}{1+\lambda}[q_H X_H+(1-q_H)X_L]+\frac{\lambda}{1+\lambda}(1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)X_L$. Similar analyses apply to other regions. Note that the payoff in the third region is always larger than that in the second region, and therefore we combined those two regions.

(ii) The case where $k>\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H X_H/(1+\lambda)$: The analysis of this case is analogous to that of the preceding case. The only difference is that the fourth region in the preceding case disappears and that the optimal M in the third region is now $X_H-k(1+\lambda)$.

HC regime	
$k>\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H X_H/(1+\lambda)$	market value of debt at date 0, D_0
$M\leq(1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)X_L$	M
$M\in[(1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)X_L,$ $(1+z_H)X_L]$	$[q_H+(1-q_H)(\frac{1}{2}-r_H)]M+(1-q_H)(\frac{1}{2}+r_H)(1-z_H)X_L$
$M\in[(1+z_H)X_L,$ $X_H-k(1+\lambda)]$	$q_H M+(1-q_H)(1-2r_H z_H)X_L$
$M\in[X_H-k(1+\lambda),$ $(1+z_H)X_H]$	$(1-2r_H z_H)X_L$

The subsequent steps are to derive the optimal M and the shareholder value at the optimal M. The analysis follows the exact logic in the preceding case and so is omitted. ■

Proof of Proposition 5

The transfer of control/continuation outcome depends on the prudential constraint c. For the feasible range of values of M, using equations (19) through (22), Table 4 shows the corresponding market values D_1 and F_1:

Table 4		
Values of M	$\widehat{r} = r_H$	$\widehat{r} = r_L$
$M < (1 - z_H)y$	$D_1 = M$ $F_1 = (1 - 2r_H z_H)y$ $\Longrightarrow \frac{D_1}{F_1} < c \Longleftrightarrow M < c(1 - 2r_H z_H)y$	$D_1 = M$ $F_1 = y$ $\Longrightarrow \frac{D_1}{F_1} < c \Longleftrightarrow M < cy$
$M \in [(1 - z_H)y, y]$	$D_1 = (\frac{1}{2} - r_H)M + (\frac{1}{2} + r_H)(1 - z_H)y$ $F_1 = (1 - 2r_H z_H)y$ $\Longrightarrow \frac{D_1}{F_1} < c \Longleftrightarrow M < \frac{c(1-2r_H z_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y$	$D_1 = M$ $F_1 = y$ $\Longrightarrow \frac{D_1}{F_1} < c \Longleftrightarrow M < cy$
$M \in [y, (1 + z_H)y]$	$D_1 = (\frac{1}{2} - r_H)M + (\frac{1}{2} + r_H)(1 - z_H)y$ $F_1 = (1 - 2r_H z_H)y$ $\Longrightarrow \frac{D_1}{F_1} < c \Longleftrightarrow M < \frac{c(1-2r_H z_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y$	$D_1 = y$ $F_1 = y$ $\Longrightarrow \frac{D_1}{F_1} = 1 \geq c$
$M > (1 + z_H)y$	$D_1 = (1 - 2r_H z_H)y$ $F_1 = (1 - 2r_H z_H)y$ $\Longrightarrow \frac{D_1}{F_1} = 1 \geq c$	$D_1 = y$ $F_1 = y$ $\Longrightarrow \frac{D_1}{F_1} = 1 \geq c$

Note that Table 4 implies the following three general facts:
(i) For $M > (1 + z_H)y$, transfer of control will occur for sure.
(ii) For $M \in [(1-z_H)y, (1+z_H)y]$, transfer of control will occur if and only if $M > \frac{c(1-2r_H z_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y$ given $\widehat{r} = r_H$ and $M > cy$ given $\widehat{r} = r_L$.
(iii) For $M < (1 - z_H)y$, transfer of control will occur if and only if $M > c(1 - 2r_H z_H)y$ given $\widehat{r} = r_H$ and $M > cy$ given $\widehat{r} = r_L$.

We now proceed by discussing the cases of various values of c. Recall that from Proposition 2 that, given continuation, shareholders will choose r_L if and only if $M < (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)y$.

(a) $c < 1-z_H$: In this case, $\frac{c(1-2r_H z_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y < c(1-2r_H z_H)y < cy < (1-z_H)y$. Therefore, transfer of control occurs for $M > cy$ and continuation occurs for $M < c(1 - 2r_H z_H)y$, regardless of what the investors' conjecture is, $\widehat{r} = r_H$ or $\widehat{r} = r_L$. What happens if $M \in [c(1 - 2r_H z_H)y, cy]$? If the market's conjecture is $\widehat{r} = r_L$, continuation will occur because $M < cy$, and the firm will indeed choose r_L because $M < (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)y$; therefore, the market's conjecture is confirmed. If the market's conjecture is $\widehat{r} = r_H$, transfer of control will occur because $M > c(1 - 2r_H z_H)y$, and the firm will choose r_L because of the transfer of control; therefore, the market's conjecture is incorrect. Taken together, for $M \in [c(1 - 2r_H z_H)y, cy]$, continuation will occur. To summarize, continuation and r_L will occur for $M < cy$ and transfer of control occurs for $M > cy$.
(b) $c \in [1 - z_H, \frac{1-z_H}{1-2r_H z_H}]$: In this case, $\frac{c(1-2r_H z_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y < c(1 - 2r_H z_H)y < (1 - z_H)y < cy < (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)y$. Therefore, transfer of control occurs for $M > cy$ and continuation occurs for $M < c(1 - 2r_H z_H)y$, regardless of what the investors' conjecture is, $\widehat{r} = r_H$ or $\widehat{r} = r_L$. What happens if $M \in [c(1 - 2r_H z_H)y, cy]$? If the market's conjecture is $\widehat{r} = r_L$, continuation will occur because $M < cy$, and the firm will indeed choose r_L because $M < (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)y$; therefore, the

market's conjecture is confirmed. If the market's conjecture is $\widehat{r} = r_H$, transfer of control will occur because $M > c(1-2r_Hz_H)y$ and $M > \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y$, and the firm will choose r_L because of the transfer of control; therefore, the market's conjecture is incorrect. Taken together, for $M \in [c(1-2r_Hz_H)y, cy]$, continuation will occur. To summarize, continuation and r_L occur for $M < cy$ and transfer of control occurs for $M > cy$.

(c) $c \in [\frac{1-z_H}{1-2r_Hz_H}, \frac{(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}+r_H-2r_Hz_H}]$: In this case, $(1-z_H)y < c(1-2r_Hz_H)y < \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y < cy < (1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)y$. Therefore, transfer of control occurs for $M > cy$ and continuation occurs for $M < \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y$, regardless of what the investors' conjecture is, $\widehat{r} = r_H$ or $\widehat{r} = r_L$. What happens if $M \in [\frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y, cy]$? If the market's conjecture is $\widehat{r} = r_L$, continuation will occur because $M < cy$, and the firm will indeed choose r_L because $M < (1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)y$; therefore, the market's conjecture is confirmed. If the market's conjecture is $\widehat{r} = r_H$, transfer of control will occur because $M > \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y$, and the firm will choose r_L because of the transfer of control; therefore, the market's conjecture is incorrect. Taken together, for $M \in [\frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y, cy]$, continuation will occur. To summarize, continuation and r_L occur for $M < cy$ and transfer of control occurs for $M > cy$.

(d) $c \in [\frac{(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}+r_H-2r_Hz_H}, \frac{(\frac{1}{2}-r_H)(1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)+(\frac{1}{2}+r_H)(1-z_H)}{1-2r_Hz_H}]$: In this case, $(1-z_H)y < c(1-2r_Hz_H)y < cy < \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y < (1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)y$. Therefore, transfer of control occurs for $M > \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y$ and continuation occurs for $M < cy$, regardless of what the investors' conjecture is, $\widehat{r} = r_H$ or $\widehat{r} = r_L$. What happens if $M \in [cy, \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y]$? If the market's conjecture is $\widehat{r} = r_L$, transfer of control will occur because $M > cy$, and the firm will indeed choose r_L because of the transfer of control; therefore, the market's conjecture is confirmed. If the market's conjecture is $\widehat{r} = r_H$, continuation will occur because $M < \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y$, but the firm will choose r_L because $M < (1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)y$; therefore, the market's conjecture is incorrect. Taken together, for $M \in [cy, \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y]$, transfer of control will occur. To summarize, continuation and r_L occur for $M < cy$ and transfer of control occurs for $M > cy$.

(e) $c \in [\frac{(\frac{1}{2}-r_H)(1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)+(\frac{1}{2}+r_H)(1-z_H)}{1-2r_Hz_H}, 1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H]$: In this case, $(1-z_H)y < c(1-2r_Hz_H)y < cy < (1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)y < \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y$. Therefore, transfer of control occurs for $M > \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y$ and continuation occurs for $M < cy$, regardless of what the investors' conjecture is, $\widehat{r} = r_H$ or $\widehat{r} = r_L$. What happens if $M \in [cy, \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y]$? Let's split that interval into two subintervals: $M \in [cy, (1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)y]$ and $M \in [(1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)y, \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y]$. First, assume $M \in [cy, (1-\frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)y]$. If the market's conjecture is $\widehat{r} = r_L$, transfer of control will occur because $M > cy$, and the firm will indeed choose r_L because of the transfer of control; therefore, the market's conjecture is confirmed. If the market's conjecture is $\widehat{r} =$

r_H, continuation will occur because $M < \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y$, but the firm will choose r_L because $M < (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)y$; therefore, the market's conjecture is incorrect. Taken together, for $M \in [cy, (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)y]$, transfer of control will occur. Second, assume $M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)y, \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y]$. If the market's conjecture is $\widehat{r} = r_L$, transfer of control will occur because $M > cy$, and the firm will indeed choose r_L because of the transfer of control; therefore, the market's conjecture is confirmed. If the market's conjecture is $\widehat{r} = r_H$, continuation will occur because $M < \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y$, and the firm will choose r_H because $M > (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)y$; therefore, the market's conjecture is confirmed. Because the market prefers r_L to r_H, the market's conjecture of r_L is more reasonable. Taken together, for $M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)y, \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y]$, transfer of control will occur. To summarize, continuation and r_L occur for $M < cy$ and transfer of control occurs for $M > cy$.
(f) $c > 1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H$: In this case, $(1 - z_H)y < c(1 - 2r_Hz_H)y$ and $(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)y < cy < \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y$. Therefore, transfer of control occurs for $M > \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y$ and continuation occurs for $M < cy$, regardless of what the investors' conjecture is, $\widehat{r} = r_H$ or $\widehat{r} = r_L$. What happens if $M \in [cy, \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y]$? If the market's conjecture is $\widehat{r} = r_L$, transfer of control will occur because $M > cy$, and the firm will indeed choose r_L because of the transfer of control; therefore, the market's conjecture is confirmed. If the market's conjecture is $\widehat{r} = r_H$, continuation will occur because $M < \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y$, and the firm will choose r_H because $M > (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)y$; therefore, the market's conjecture is confirmed. Because the market prefers r_L to r_H, the market's conjecture of r_L is more reasonable. Taken together, for $M \in [cy, \frac{c(1-2r_Hz_H)-(\frac{1}{2}+r_H)(1-z_H)}{\frac{1}{2}-r_H}y]$, transfer of control will occur. To summarize, continuation occurs for $M < cy$ and transfer of control occurs for $M > cy$. In addition, given continuation, r_L will occur for $M < (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)y$ and r_H will occur for $M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H)y, cy]$.

The results from the preceding analysis are summarized in the statement of the proposition. ■

Proof of Proposition 6

For relatively low values of $c \leq 1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}z_H$, using Proposition 5, the shareholders' expected payoffs from choosing q at date $t = 0$ are summarized in Table 5 below for all the feasible values of M:

Table 5: $c \leq 1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H$	
Feasible Values of M	shareholders' expected payoffs from choosing q
$M < X_L$:	r_L regardless of y $-\kappa(q) + \frac{1}{1+\lambda}\{q[X_H - M] + (1-q)[X_L - M]\}$
$M \in [X_L, X_H]$:	r_L regardless of y $-\kappa(q) + \frac{1}{1+\lambda} q[X_H - M]$
$M \in [X_H, (1+z_H)X_H]$:	r_L regardless of y $-\kappa(q)$

Similarly, for values of $c \geq 1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H$, using Proposition 5, the shareholders' expected payoffs from choosing q at date $t = 0$ are summarized in Table 6 below for all the feasible values of M:

Table 6 $c \geq 1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H$	
Feasible Values of M	shareholders' expected payoffs from choosing q
$M < (1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H) X_L$:	r_L regardless of y $-\kappa(q) + \frac{1}{1+\lambda}\{q[X_H - M] + (1-q)[X_L - M]\}$
$M \in [(1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H) X_L, cX_L]$	r_L if $y = X_H$ r_H if $y = X_L$ $-\kappa(q) + \frac{1}{1+\lambda}\{q[X_H - M] + (1-q)(\frac{1}{2} - r_H)[(1+z_H)X_L - M]\}$
$M \in [cX_L, X_L]$:	r_L regardless of y $-\kappa(q) + \frac{1}{1+\lambda}\{q[X_H - M] + (1-q)[X_L - M]\}$
$M \in [X_L, (1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H) X_H]$:	r_L regardless of y $-\kappa(q) + \frac{1}{1+\lambda} q[X_H - M]$
$M \in [(1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H) X_H, cX_H]$:	r_H if $y = X_H$ r_L if $y = X_L$ $-\kappa(q) + \frac{1}{1+\lambda} q(\frac{1}{2} - r_H)[(1+z_H)X_H - M]\}$
$M \in [cX_H, X_H]$:	r_L regardless of y $-\kappa(q) + \frac{1}{1+\lambda} q[X_H - M]$
$M \in [X_H, (1+z_H)X_H]$:	r_L regardless of y $-\kappa(q)$

Using Tables 5 and 6, we derive the following optimal investment rules in the FV regime.

We first analyze the case where $c \leq c^* \equiv 1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H$. It is obvious that shareholders always choose q_H in region 1 and q_L in region 3. In region 2, the shareholders choose q_H if and only if $M < X_H - k(1 + \lambda)$.

For the case where $c > c^*$, using Table 6, a similar analysis can be done. It is obvious that shareholders always choose q_H in regions 1, 2, and 3 and q_L in region 7. In regions 4 and 6, the shareholders choose q_H if and only if $M < X_H - k(1+\lambda)$. In region 5, the shareholders choose q_H if and only if $M < (1+z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}$.

(i) $k > k^*$: In regions 1, 2, and 3, shareholders always choose q_H. In region 4, shareholders choose q_H if and only if $M < X_H - k(1+\lambda)$, which is stated in the proposition. Furthermore, $k > k^*$ implies that $(1+z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H} < X_H - k(1+\lambda)$, which implies that shareholders choose q_L in region 5. Because $X_H - k(1+\lambda)$ is in region 4, shareholders choose q_L in region 6. Finally, shareholders always choose q_L in region 7.

(ii) $k < k^*$: In regions 1, 2, and 3, shareholders always choose q_H. Furthermore, $k < k^*$ implies that $(1+z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H} > X_H - k(1+\lambda)$, which implies that shareholders choose q_H in region 4. The choices of q in regions 5 and 6 depend on the values of c. When $c < c_1$, shareholders always choose q_H in region 5 but chooses q_H in region 6 if and only if $M < X_H - k(1+\lambda)$. When $c > c_2$, shareholders always choose q_L in region 6 but chooses q_H in region 5 if and only if $M < (1+z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}$. When $c \in [c_1, c_2]$, shareholders always choose q_H in region 5 and q_L in region 6. Therefore, the cutoff value of M dividing the q_H and q_L regions is the boundary of regions 5 and 6, that is, cX_H. Finally, shareholders always choose q_L in region 7.

For the purpose of future reference, we summarize the shareholders' expected payoff from the optimal choice of q in the following:

$c > c^*$ and $k \le k^*$ and $c < c_1$	choice of q	shareholders' expected payoff from the optimal choice of q
$M < (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$	q_H	$\frac{1}{1+\lambda}\{q_H X_H + (1-q_H)X_L - M\}$ $-\kappa_H$
$M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L, cX_L]$	q_H	$\frac{1}{1+\lambda}\{q_H X_H$ $+(1-q_H)(\frac{1}{2}-r_H)(1+z_H)X_L$ $-[q_H + (1-q_H)(\frac{1}{2}-r_H)]M\}$ $-\kappa_H$
$M \in [cX_L, X_L]$	q_H	$\frac{1}{1+\lambda}\{q_H X_H + (1-q_H)X_L - M\}$ $-\kappa_H$
$M \in [X_L, (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H]$	q_H	$\frac{1}{1+\lambda} q_H [X_H - M] - \kappa_H$
$M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H, cX_H]$	q_H	$\frac{1}{1+\lambda} q_H (\frac{1}{2}-r_H)[(1+z_H)X_H - M] - \kappa_H$
$M \in [cX_H, X_H - k(1+\lambda)]$	q_H	$\frac{1}{1+\lambda} q_H [X_H - M] - \kappa_H$
$M \in [X_H - k(1+\lambda), (1+z_H)X_H]$	q_L	$-\kappa_L$

$c > c^*$ and $k \le k^*$ and $c \in [c_1, c_2]$	choice of q	shareholders' expected payoff from the optimal choice of q
$M < (1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H) X_L$	q_H	$\frac{1}{1+\lambda}\{q_H X_H + (1 - q_H) X_L - M\}$ $-\kappa_H$
$M \in [(1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H) X_L, cX_L]$	q_H	$\frac{1}{1+\lambda}\{q_H X_H$ $+(1 - q_H)(\frac{1}{2} - r_H)(1 + z_H) X_L$ $-[q_H + (1 - q_H)(\frac{1}{2} - r_H)]M\}$ $-\kappa_H$
$M \in [cX_L, X_L]$	q_H	$\frac{1}{1+\lambda}\{q_H X_H + (1 - q_H) X_L - M\}$ $-\kappa_H$
$M \in [X_L, (1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H) X_H]$	q_H	$\frac{1}{1+\lambda} q_H [X_H - M] - \kappa_H$
$M \in [(1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H) X_H, cX_H]$	q_H	$\frac{1}{1+\lambda} q_H (\frac{1}{2} - r_H)[(1 + z_H) X_H - M] - \kappa_H$
$M \in [cX_H, (1 + z_H) X_H]$	q_L	$-\kappa_L$

$c > c^*$ and $k \le k^*$ and $c > c_2$	choice of q	shareholders' expected payoff from the optimal choice of q
$M < (1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H) X_L$	q_H	$\frac{1}{1+\lambda}\{q_H X_H + (1 - q_H) X_L - M\}$ $-\kappa_H$
$M \in [(1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H) X_L, cX_L]$	q_H	$\frac{1}{1+\lambda}\{q_H X_H$ $+(1 - q_H)(\frac{1}{2} - r_H)(1 + z_H) X_L$ $-[q_H + (1 - q_H)(\frac{1}{2} - r_H)]M\}$ $-\kappa_H$
$M \in [cX_L, X_L]$	q_H	$\frac{1}{1+\lambda}\{q_H X_H + (1 - q_H) X_L - M\}$ $-\kappa_H$
$M \in [X_L, (1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H) X_H]$	q_H	$\frac{1}{1+\lambda} q_H [X_H - M] - \kappa_H$
$M \in [(1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H) X_H, (1 + z_H) X_H - \frac{k(1+\lambda)}{\frac{1}{2} - r_H}]$	q_H	$\frac{1}{1+\lambda} q_H (\frac{1}{2} - r_H)[(1 + z_H) X_H - M] - \kappa_H$
$M \in [(1 + z_H) X_H - \frac{k(1+\lambda)}{\frac{1}{2} - r_H}, (1 + z_H) X_H]$	q_L	$-\kappa_L$

$c > c^*$ and $k > k^*$	choice of q	shareholders' expected payoff from the optimal choice of q
$M < (1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H) X_L$	q_H	$\frac{1}{1+\lambda}\{q_H X_H + (1-q_H)X_L - M\}$ $-\kappa_H$
$M \in [(1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H) X_L, cX_L]$	q_H	$\frac{1}{1+\lambda}\{q_H X_H$ $+(1-q_H)(\frac{1}{2} - r_H)(1+z_H)X_L$ $-[q_H + (1-q_H)(\frac{1}{2} - r_H)]M\}$ $-\kappa_H$
$M \in [cX_L, X_L]$	q_H	$\frac{1}{1+\lambda}\{q_H X_H + (1-q_H)X_L - M\}$ $-\kappa_H$
$M \in [X_L, X_H - k(1+\lambda)]$	q_H	$\frac{1}{1+\lambda} q_H [X_H - M] - \kappa_H$
$M \in [X_H - k(1+\lambda), (1+z_H)X_H]$	q_L	$-\kappa_L$

$c \leq c^*$	choice of q	shareholders' expected payoff from the optimal choice of q
$M < X_L$	q_H	$\frac{1}{1+\lambda}\{q_H X_H + (1-q_H)X_L - M\}$ $-\kappa_H$
$M \in [X_L, X_H - k(1+\lambda)]$	q_H	$\frac{1}{1+\lambda} q_H [X_H - M] - \kappa_H$
$M \in [X_H - k(1+\lambda), (1+z_H)X_H]$	q_L	$-\kappa_L$

■

Lemma 2: Capital Structure in FV Regime

In Section 6.3, we discussed the general guidelines for deriving the shareholders' capital structure decision, M, the face value of debt, in the FV regime. In the following claim, we state and prove the specifics of this decision rule. The result in Lemma 2 will be used in the proof of Proposition 7. The following table show the choices of M by regions and the shareholders' Date 0 expected payoffs by regions:

FV regime: Scenario A: $c \leq c^* \equiv 1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H}$		
	Choice of M	Shareholders' *ex ante* payoffs
$M \leq X_L$	X_L	$\pi_{A1} \equiv -A_0 - \kappa_H$ $+\frac{1}{1+\lambda}[q_H X_H + (1-q_H)X_L]$ $+\frac{\lambda}{1+\lambda} X_L$
$M \in [X_L, X_H - k(1+\lambda)]$	$X_H - k(1+\lambda)$	$\pi_{A2} \equiv -A_0 - \kappa_H$ $+q_H X_H$ $+(1-q_H)X_L$ $-\lambda q_H k$
$M \in [X_H - k(1+\lambda), (1+z_H)X_H]$	$[X_H - k(1+\lambda), (1+z_H)X_H]$	$\pi_{A3} \equiv -A_0 - \kappa_L + X_L$

FV regime: Scenario B: $c > c^*$ and $k \leq k^*$ and $c < c_1$		
	Choice of M	Shareholders' *ex ante* payoffs
$M \leq (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$	$(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$	$\pi_{B1} \equiv -A_0 - \kappa_H$ $+\frac{1}{1+\lambda}[q_H X_H + (1-q_H)X_L]$ $+\frac{\lambda}{1+\lambda}(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$
$M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L, cX_L]$	cX_L	$\pi_{B2} \equiv -A_0 - \kappa_H$ $+\frac{1}{1+\lambda}[q_H X_H + (1-q_H)(\frac{1}{2} - r_H)(1+z_H)X_L]$ $+(1-q_H)(\frac{1}{2}+r_H)(1-z_H)X_L$ $+\frac{\lambda}{1+\lambda}[q_H + (1-q_H)(\frac{1}{2} - r_H)]cX_L$
$M \in [cX_L, X_L]$	X_L	$\pi_{B3} \equiv -A_0 - \kappa_H$ $+\frac{1}{1+\lambda}[q_H X_H + (1-q_H)X_L]$ $+\frac{\lambda}{1+\lambda}X_L$
$M \in [X_L,$ $(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H]$	$(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H$	$\pi_{B4} \equiv -A_0 - \kappa_H$ $+\frac{1}{1+\lambda}q_H X_H$ $+(1-q_H)X_L$ $+\frac{\lambda}{1+\lambda}q_H(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H$
$M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H,$ $cX_H]$	cX_H	$\pi_{B5} \equiv -A_0 - \kappa_H$ $+\frac{1}{1+\lambda}q_H(\frac{1}{2} - r_H)(1+z_H)X_H$ $+q_H(\frac{1}{2}+r_H)(1-z_H)X_H$ $+(1-q_H)X_L$ $+\frac{\lambda}{1+\lambda}q_H(\frac{1}{2} - r_H)cX_H$
$M \in [cX_H,$ $X_H - k(1+\lambda)]$	$X_H - k(1+\lambda)$	$\pi_{B6} \equiv -A_0 - \kappa_H$ $+q_H X_H$ $+(1-q_H)X_L$ $-\lambda q_H k$
$M \in [X_H - k(1+\lambda),$ $(1+z_H)X_H]$	$[X_H - k(1+\lambda),$ $(1+z_H)X_H]$	$\pi_{B7} \equiv -A_0 - \kappa_L + X_L$

FV regime: Scenario C: $c > c^*$ and $k \leq k^*$ and $c \in [c_1, c_2]$		
	Choice of M	Shareholders' *ex ante* payoffs
$M \leq (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$	$(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$	π_{B1}
$M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L, cX_L]$	cX_L	π_{B2}
$M \in [cX_L, X_L]$	X_L	π_{B3}
$M \in [X_L, (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H]$	$(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H$	π_{B4}
$M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H, cX_H]$	cX_H	π_{B5}
$M \in [cX_H, (1+z_H)X_H]$	$[cX_H, (1+z_H)X_H]$	π_{B7}

FV regime: Scenario D: $c > c^*$ and $k \leq k^*$ and $c > c_2$		
	Choice of M	Shareholders' *ex ante* payoffs
$M \leq (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$	$(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$	π_{B1}
$M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L, cX_L]$	cX_L	π_{B2}
$M \in [cX_L, X_L]$	X_L	π_{B3}
$M \in [X_L, (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H]$	$(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H$	π_{B4}
$M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_H, (1+z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}]$	$(1+z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}$	$-A_0 - \kappa_H$ $+\frac{1}{1+\lambda} q_H(\frac{1}{2} - r_H)(1+z_H)X_H$ $+q_H(\frac{1}{2}+r_H)(1-z_H)X_H$ $+(1-q_H)X_L$ $+\frac{\lambda}{1+\lambda} q_H(\frac{1}{2} - r_H)(1+z_H)X_H - \frac{k(1+\lambda)}{\frac{1}{2}-r_H}$
$M \in [cX_H, (1+z_H)X_H]$	$[cX_H, (1+z_H)X_H]$	π_{B7}

FV regime: Scenario E: $c > c^*$ and $k > k^*$		
	Choice of M	Shareholders' *ex ante* payoffs
$M \leq (1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$	$(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L$	π_{B1}
$M \in [(1 - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H)X_L, cX_L]$	cX_L	π_{B2}
$M \in [cX_L, X_L]$	X_L	π_{B3}
$M \in [X_L, X_H - k(1+\lambda)]$	$X_H - k(1+\lambda)$	$-A_0 - \kappa_H$ $+q_H X_H + (1-q_H)X_L - \lambda\kappa(q_H)$
$M \in [X_H - k(1+\lambda), (1+z_H)X_H]$	$[X_H - k(1+\lambda), (1+z_H)X_H]$	π_{B7}

Proof of Lemma 2

(i) The case where $c \leq c^*$:

Under the fair value regime, taking into consideration of the optimal choices of r and q, we first derive D_0 using (15):

FV regime: $c \leq c^*$	
	market value of debt at date 0, D_0
$M \leq X_L$	M
$M \in [X_L, X_H - k(1+\lambda)]$	$q_H M + (1 - q_H) X_L$
$M \in [X_H - k(1+\lambda), (1+z_H) X_H]$	X_L

Under the fair value regime, taking into consideration of the optimal choices of r and q, we derive the expected payoffs for those regions above using (13):

FV regime: $c \leq c^*$	
	Shareholders' *ex ante* payoffs
$M \leq X_L$	$-A_0 - \kappa_H + \frac{1}{1+\lambda}[q_H X_H + (1-q_H) X_L] + \frac{\lambda}{1+\lambda} M$
$M \in [X_L, X_H - k(1+\lambda)]$	$-A_0 - \kappa_H + \frac{1}{1+\lambda} q_H X_H + (1-q_H) X_L + \frac{\lambda}{1+\lambda} q_H M$
$M \in [X_H - k(1+\lambda), (1+z_H) X_H]$	$-A_0 - \kappa_L + X_L$

Because regional payoffs are increasing in M, the optimal value of M for a given region is the upper bound of that region, as shown in the statement of this lemma.

Substituting the regional optimal M into the regional payoff function yields the regional expected payoffs for shareholders at Date 0. For example, for the region of $M \leq X_L$, the shareholders' expected payoff is $-A_0 - \kappa_H + \frac{1}{1+\lambda}[q_H X_H + (1-q_H) X_L] + \frac{\lambda}{1+\lambda} M$ and the optimal value of M for this particular region is therefore X_L. Inserting $M = X_L$ into the payoff function yields $-A_0 - \kappa_H + \frac{1}{1+\lambda}[q_H X_H + (1-q_H) X_L] + \frac{\lambda}{1+\lambda} X_L$. Similar analyses apply to other regions.

(ii) The case where $c > c^*$: The analysis follows the exact logic in the preceding case and so is omitted to save space. ■

Proof of Proposition 7

Lemma 2 gives the shareholders' ex ante (date 0) expected payoffs under the fair value regime, assuming no solvency constraint exists at date 0. We will take into consideration the effect of c at date 0 at the end of this proof.

We first outline the roadmap of the proof: (i) Show that π_{B6} (in Lemma 2) is the highest payoff in Scenario B. (ii) Show that $\pi_{A2} = \pi_{B6}$ is the highest payoff in Scenario A. Therefore, (i) and (ii) imply that Scenarios A and B are equivalent. (iii) Show that $\pi_{A2} = \pi_{B6}$ exceeds any payoff in Scenario E. (iv) Show that $\pi_{A2} = \pi_{B6}$ exceeds any payoff in Scenario D. (v) Show that $\pi_{A2} = \pi_{B6}$ exceeds any payoff in Scenario C. Therefore, (i) through (v) imply that the highest payoff among all the five scenarios is $\pi_{A2} = \pi_{B6}$. Next, we show that, to induce Scenario A or B, the regulator must set $c \leq c_1$. However, any further reduction of c below c_1 will constrain shareholders' choice of M at date 0 and therefore damage their *ex ante* welfare. This tension gives rise to the socially optimal constraint, $c = c_1$.

(i) It is straightforward to show the following results except the fifth one, which we will furnish the proof at the end of the list:

- $\pi_{B3} > \pi_{B1}$ (the payoff in region 3 exceeds that in region 1 in Scenario B);
- $\pi_{B3} > \pi_{B2}$ (the payoff in region 3 exceeds that in region 2 in Scenario B);
- $\pi_{B6} > \pi_{B3}$ (the payoff in region 6 exceeds that in region 3 in Scenario B);
- $\pi_{B6} > \pi_{B4}$ (the payoff in region 6 exceeds that in region 4 in Scenario B);
- $\pi_{B6} > \pi_{B5}$ (the payoff in region 6 exceeds that in region 5 in Scenario B);
- $\pi_{B6} > \pi_{B7}$ (the payoff in region 6 exceeds that in region 7 in Scenario B).

Therefore, π_{B6} is the highest payoff in Scenario B. Recall that Scenario B will be viable when

$$c \in [c^*, c_1 \equiv 1 - \frac{k(1+\lambda)}{X_H}]. \tag{23}$$

{We prove $\pi_{B6} > \pi_{B5}$ in the following:

$$\begin{aligned} \pi_{B6} &> \pi_{B5} \\ &\Leftrightarrow X_H - \lambda k > \frac{1}{1+\lambda}(\frac{1}{2} - r_H)(1+z_H)X_H + (\frac{1}{2}+r_H)(1-z_H)X_H + \frac{\lambda}{1+\lambda}(\frac{1}{2}-r_H)cX_H. \end{aligned}$$

Because in Scenario B, $c \leq c_1 \equiv 1 - \frac{k(1+\lambda)}{X_H}$, the right-hand side of the preceding inequality takes its maximum value at $c = 1 - \frac{k(1+\lambda)}{X_H}$. Therefore, we can prove that $\pi_{B6} > \pi_{B5}$ by showing that $X_H - \lambda k$ exceeds the right-hand side's maximum value:

$$\begin{aligned} X_H - \lambda k &> \frac{1}{1+\lambda}(\frac{1}{2} - r_H)(1+z_H)X_H + (\frac{1}{2}+r_H)(1-z_H)X_H + \frac{\lambda}{1+\lambda}(\frac{1}{2}-r_H)(1 - \frac{k(1+\lambda)}{X_H})X_H \\ &\Leftrightarrow [1 + \frac{1}{\lambda} - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}\frac{1}{\lambda}]z_H X_H > k(1+\lambda). \end{aligned}$$

Because $k \leq k^* \equiv \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H X_H/(1+\lambda)$, the right-hand side of the preceding inequality takes its maximum value at $k = k^*$. Then the preceding inequality will be established if we show that the left-hand side of it exceeds the maximum value of the right-hand side.

$$\begin{aligned} [1 + \frac{1}{\lambda} - \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}\frac{1}{\lambda}]z_H X_H &> \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H} z_H X_H \\ &\Leftrightarrow 1 > \frac{\frac{1}{2}-r_H}{\frac{1}{2}+r_H}, \end{aligned}$$

which always holds.}

(ii) It is straightforward to show the following results for Scenario A:

- $\pi_{A1} = \pi_{B3}$ (the payoff in region 1 in Scenario A equals that in region 3 in Scenario B);
- $\pi_{A2} = \pi_{B6}$ (the payoff in region 2 in Scenario A equals that in region 6 in Scenario B);
- $\pi_{A3} = \pi_{B7}$ (the payoff in region 3 in Scenario A equals that in region 7 in Scenario B).

Therefore, $\pi_{A2} = \pi_{B6}$ is the highest payoff in Scenerios A and B. Recall that Scenario A will be viable when

$$c \leq c^*. \tag{24}$$

(iii) It is easy to see that $\pi_{A2} = \pi_{B6}$ exceeds any payoff in Scenario E. Specifically, not only region 6 in Scenario B does not exist in Scenario E, but also the payoff in region 4 in B is larger than the payoff in region 4 in E.

(iv) It is easy to see that $\pi_{A2} = \pi_{B6}$ exceeds any payoff in Scenario D. Specifically, not only region 6 in Scenario B does not exist in Scenario D, but also the payoff in region 5 in B is larger than the payoff in region 5 in D.

(v) It is easy to see that $\pi_{A2} = \pi_{B6}$ exceeds any payoff in Scenario C. Specifically, region 6 in Scenario B does not exist in Scenario C.

Therefore, overall, the regulator prefers Scenarios A and B to Scenarios C, D, and E. Furthermore, both Scenario A and Scenario B give rise to the same maximum payoff, $\pi_{A2} = \pi_{B6}$. To induce Scenario A or B, in light of (24) and (23), it suffice for the regulator to set $c \leq c_1 \equiv 1 - \frac{k(1+\lambda)}{X_H}$. However, any further reduction of c below c_1 will constrain shareholders' choice of M at date 0 and therefore damage their *ex ante* welfare. This tension gives rise to the socially optimal constraint, $c = c_1$. ■

Proof of Proposition 8

If $k > k^*$, from Propositions 3 and 6, the threshold value of M in both regimes is $X_H - k(1+\lambda)$.

If $k \leq k^*$, from Proposition 3, the threshold value of M for which under-investment occurs in the HC regime is $(1 + z_H)X_H - \frac{k}{\frac{1}{2}-r_H}$. From Proposition 6 it is $X_H - k(1 + \lambda)$, cX_H, or $(1+z_H)X_H - \frac{k}{\frac{1}{2}-r_H}$ under the FV regime. The under-investment region is larger under FV regime than that under HC regime if and only if

$$X_H - k(1 + \lambda) < (1 + z_H)X_H - \frac{k(1 + \lambda)}{\frac{1}{2} - r_H} \Leftrightarrow k \leq \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H X_H/(1 + \lambda) \equiv k^*,$$

which is true by assumption. Furthermore,

$$cX_H < (1 + z_H)X_H - \frac{k(1 + \lambda)}{\frac{1}{2} - r_H} \Leftrightarrow c < (1 + z_H) - \frac{k(1 + \lambda)}{(\frac{1}{2} - r_H)X_H} \equiv c_2,$$

which is true when cX_H is the threshold value. When $\frac{1}{2} - r_H$ and/or z_H increases, both c_2 and k^* increases, thereby expanding the under-investment (q_L) region. ■

Proof of Proposition 9

We already know from Corollary 1 that at $c = c_1$, the fair value regime dominates the historical cost regime. We show in the following that at $c = 0$, the historical cost regime dominates the fair value regime. Taken together these two facts, by continuity, there must exist $c \in [0, c_0]$ where the historical cost regime dominates the fair value regime.

Under the fair value regime, when $c = 0$, it must be the case that the debt/asset ratio at date 0, $\frac{D_0}{F_0}$, exceeds or equals $c = 0$. For the business to continue beyond date 0, the shareholders must choose the minimal face value of debt in order to satisfy the solvency constraint. Therefore, $M(c = 0) = 0$. This face value of debt implies that the shareholders' *ex ante* payoff, given in the table of payoffs in the proof of Lemma 2, is

$$\begin{aligned} & -A_0 - \kappa_H + \frac{1}{1+\lambda}[q_H X_H + (1-q_H)X_L] + \frac{\lambda}{1+\lambda}M \\ = \quad & -A_0 - \kappa_H + \frac{1}{1+\lambda}[q_H X_H + (1-q_H)X_L] + \frac{\lambda}{1+\lambda} \times 0, \end{aligned}$$

which is less than its counterpart in the historical cost regime, given in Lemma 1,

$$\pi_I^{HC} \equiv -A_0 - \kappa_H + \frac{1}{1+\lambda}[q_H X_H + (1-q_H)X_L] + \frac{\lambda}{1+\lambda}(1 - \frac{\frac{1}{2} - r_H}{\frac{1}{2} + r_H} z_H) X_L.$$

Therefore, at $c = 0$, the historical cost regime dominates the fair value regime. ■

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