Testing Incentives in a Buyer-Seller Relationship

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Abstract

One of the most celebrated features of modern manufacturing is the newfound capacity for information sharing between firms. Manufacturing guides suggest that buyer-seller communication can help firms achieve more efficient production and trading decisions (Schonberger (1986)), as long as the trading firms truthfully report to one another. In this paper, we study the incentives for firms to strategically *misreport* private information and solve for the least costly mechanism soliciting truthful communication. When firms can communicate with one another strategically, we find that the cost of motivating truthful reporting may outweigh the benefit associated with the information being shared. By analyzing a buyer and seller's product testing decision, we characterize which of the firms ought to collect private information, and how the choice of tester affects individual testing incentives, end-product quality and industry surplus.

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1 Introduction and Literature Review

One of the most celebrated features of modern manufacturing is the newfound capacity for information sharing between firms. Manufacturing guides suggest that buyer-seller communication can help firms achieve more efficient production and trading decisions (Schonberger (1986)), as long as the trading firms report truthfully to one another. In this paper, we study the incentives for firms to *misreport* private information, and solve for the least costly mechanism soliciting truthful communication. Contrary to conventional wisdom, we find that when firms can strategically misreport to one another, the cost of inducing truthful communication may outweigh the benefits associated with the information being shared–depending largely on which of the firms is reporting to the other.

By analyzing a buyer and seller's product testing decision, we identify the relative merits of buyer- and seller-testing, allowing us the characterize the preferred mode. For example, in the personal computer industry, chip manufacturers typically test and rate the functionality of their processors (seller testing), though software developers rely on computer manufacturers to test and integrate operating systems (buyer testing). In a benchmark setting without hidden information, we find that both firms prefer more to less testing, regardless of who tests the product. However, when the test results are unverifiable, the buyer and seller's informational preferences diverge. Each firm's preferences are driven by the testing firm's incentive to misreport: greater misreporting incentives always benefit the buyer at the seller's expense. When the buyer tests the product, he earns informational rents in line with his incentive to misreport his private information. Whereas when the seller tests, she assuages her temptation to announce exclusively favorable test outcomes by paying the buyer rents in her preferred states. Our primary contribution is to characterize which of the two modes proves least costly.

We find that the seller will outsource testing whenever the tests are sufficiently informative; i.e., when the results strongly sway the expected value of the intermediary product. For instance, testing whether an operating system can function on both 32- and 64-bit processors largely determines the software's value to a computer manufacturer who sells both types of machines, and is thus informative. When testing is relatively informative, the seller finds it overly costly to signal favorable news, and therefore avoids that prospect by having the buyer verify the product himself.

The choice of tester also carries regulatory implications. Our model predicts that buyer testing will result in greater industry surplus, though seller testing may result in higher quality end-products. Surprisingly, we find that increased testing protocols may result in *lower* quality. The link between testing protocol and quality is largely determined by who tests the product. When the buyer tests, the optimal contract induces under-processing of low-quality intermediary products, whereas seller testing results in under-processing of high-quality intermediary products. By discouraging efficient processing, the optimal contract limits the testing firm's payoffs in their preferred state, and therefore dissuades misreporting.

As an extension, we consider an alternative setting where the testing firm is entirely unrestricted in their choice of testing protocol and thus, the informativeness of testing. This choice allows firms to limit contracting inefficiencies by voluntarily restricting their information gathering. When given the choice, the seller prefer's either minimal or maximal testing, whereas the buyer is found to favor a more intermediate level of testing. Unlike our earlier results, we find that with unrestricted testing, the seller will never outsource testing to the buyer. However, we show that should the seller pre-certify the buyer by guaranteeing that he observe a minimum testing protocol (as mandated by various ISO certifications), then the seller will again outsource testing as long as the mandated protocol is sufficiently informative.

Our model relates to earlier work on supply-chain test incentives. For example, Baiman et al. (2000) examine both a buyer's incentive to test incoming products and a seller's incentive to manufacture high-quality products, while Baiman et al. (2001) study an induced signaling problem where the seller's reported product quality is verified by buyer testing. In Cachon and Lariviere (2001), the authors examine how much flexibility to afford the uninformed firm in light of inter-firm communication. We build on these earlier works by endogenizing the choice of testing firm while continuing to assume that the firms contract optimally.

In identifying a preference for less informative testing, our paper relates to the extant agency literature where improved information can be harmful. In Arya et al. (1997) increased information lessens a firm's ability to commit while, in Baiman (1975), additional information induces competitive effects which lower profits. In our model, eschewing information not only results in a loss of production efficiency, but also affects the incentives facing the informed firm to misreport and the subsequent rents paid. Similarly, Arya et al. (2000) find that delaying and coarsening information systems can cause privately informed agents to earn additional rents, providing them an incentive to collect costly information in the first place. Though our buyer also earns informational rents when he tests the product, because we assume that testing protocols are observable, the rents do not assuage any associated moral hazard problem.

Our information structure is similar to that developed in Rajan and Saouma (2006) who characterize the buyer's optimal level of private information. We complement their analysis by asking if the seller would prefer to be privately informed herself. Although the qualitative preferences in each testing mode are similar, the seller prefers to be privately informed unless testing is sufficiently informative, in which case she chooses to outsource testing and thereby endows the buyer with private information.

In the following section we outline the model and benchmark setting. Section III details the second-best setting while Section IV examines the choice of testing party. Section V considers unrestricted testing, and Section VI concludes.

2 Model

We model a seller (she) who sells a pre-manufactured (all production costs are sunk) batch of intermediary product to a buyer (he). The seller's product is of high-, θ_H, or low-, θ_L, quality with $\theta_H > \theta_L$ and mean $\overline{\theta}$. We measure intermediary product heterogeneity, j, as θ_H/θ_L, and assume the seller's production technology generates high-quality product with commonly known probability, $0 < p < 1$. Holding all the bargaining power, the seller outlines the terms of trade with the buyer, who purchases and processes the products with effort, e, at personal cost $T\frac{e^2}{2}$ into an end-product[1] of quality θe. After processing, the buyer sells the good to quality-conscious consumers who value the end-product at: $\theta e + \varepsilon$; their imperfect quality estimate where ε is a finite variance, mean-zero random variable. The consumer's quality signal is unspecified to the contracting firms, as it is meant to capture the consumer's

[1]Hereafter, we refer to the intermediary good transferred between the buyer and seller as the "product" and the processed product sold by the buyer as the "end-product."

idiosyncratic ability to discern quality from both their own observations and their inferences of (potentially biased) third-party reviews.

Testing technology is assumed to be identical across the two firms, allowing either the buyer or seller to costlessly observe a private signal, $\sigma \in \{\sigma_L, \sigma_H\}$.[2] The test realization, σ, carries the same unconditional distribution as product quality; in particular, $\Pr[\sigma = \sigma_H] = p$ and $\Pr[\sigma = \sigma_L] = 1 - p$. We limit contracting between the two firms to menus of revenue-based linear contracts. Specifically, a menu of contracts, $\{(\alpha_i, \beta_i)\}$, is indexed on quality, i, *as reported by the testing firm*,[3] with α_i denoting the fixed price paid to the seller, and β_i the share of the buyer's revenues returned to the seller.[4]

The test realization, σ, relates to the realized quality, θ, according to:

$$Pr[\theta = \theta_i | \sigma_i] = a + (1 - a) \cdot f(\theta_i) \quad i = H, L \quad a \in [0, 1]$$

$$Pr[\theta = \theta_{-i} | \sigma_i] = (1 - a) \cdot f(\theta_{-i}) \quad i = H, L \quad a \in [0, 1].$$

The parameter a measures the correlation between the test outcome and realized product quality. Accordingly, we label a the testing protocol, or equivalently, the level of testing. We note that one can interpret a as the accuracy of a batch-wide test, or the fraction of products (perfectly) tested: our results only require that a relate the test realization to the realized

[2]Including an explicit testing cost induces both firms to test less without changing the marginal incentives identified in this paper. Our study identifies preferences over private testing borne exclusively from the cost and benefit of credibly communicating the results, both of which persist in a more general setting with costly testing (see Grossman and Hart (1983)).

[3]Though the Revelation Principle allows us to assume that the reporting firm truthfully reports its realization, they will only do so when it maximizes their payoffs.

[4]Cachon and Lariviere (2005) report that revenue sharing contracts are ineffective at coordinating supply-chains when firms take hidden actions. Although we use revenue sharing contracts, ours are selected from a menu of potential contracts which allows the choice of contract to signal private information. For a discussion on the efficacy of alternate contract forms in supply-chains, see Corbett et al. (2004).

batch quality. To simplify notation, we label the expected quality, given the test-outcome, σ_i, as $\hat{\theta}_i = \mathrm{E}[\theta|\sigma = \sigma_i] = a\theta_i + (1-a)\bar{\theta}$ for $i = H, L$. The conditional expectations $\hat{\theta}_H$ and $\hat{\theta}_L$ diverge away from the unconditional expectation, $\overline{\theta}$, towards θ_H and θ_L, respectively, with increased testing (Figure 1).

Whereas the level of testing, a, measures the fraction of uncertainty resolved, it does not capture the *informativeness* of testing. To measure test informativeness, a metric must consider both the fraction of uncertainty resolved and the level of uncertainty at the time of testing. Prior to testing, the product's expected quality is $\bar{\theta}$, whereas the posterior expectation is given by either $\hat{\theta}_H$ or $\hat{\theta}_L$. We define test informativeness, k, as $\hat{\theta}_H/\hat{\theta}_L$, which measures the expected quality revision following testing. In particular, when testing is said to be informative, the test outcome causes expected quality to change substantially from the pre-test prior, regardless of whether the test result was favorable or not. Test informativeness is an ex-ante measure, is independent of the test outcome, and is comprised exclusively of commonly known parameters: the efficacy of the test at resolving uncertainty (a), the strength of the priors (p), and the underlying product heterogeneity (j). Figure 1 shows that test informativeness, k, is increasing in both the level of testing, a, and product heterogeneity, j, as one would expect.

To further validate our measure of informativeness, we first examine how buyer and seller profits vary with test informativeness in a benchmark setting where test outcomes are contractible.

Lemma 1. *When the test generates a public signal σ_i, the seller will price the unit at $\alpha_i = \frac{\hat{\theta}_i^2}{2}T$ dollars, and the buyer will exert effort $e_i = \frac{\hat{\theta}_i}{T}$ processing the unit. The seller's expected*

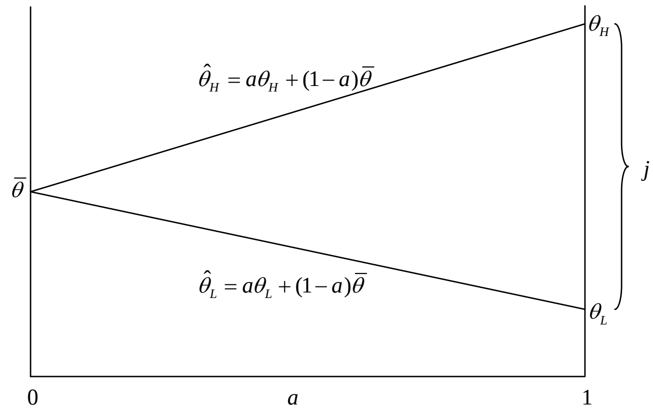

Figure 1: Following the receipt of a signal, σ_i, $\hat{\theta}_i$ is the conditional expected quality as a function of the level of testing, a. The marginal effect of additional testing on informativeness (slope over a) is driven by overall product heterogeneity, $j = \theta_H/\theta_L$, since larger heterogeneity implies a greater marginal informativeness of testing.

profits are convex and increasing in both the level of testing, a, and the informativeness of testing, k, whereas in expectation, the buyer earns his reservation payoff for all levels of a and k.

Lemma 1 confirms the value of product testing, and more generally, test informativeness. Given the choice, the seller will always prefer the maximum level of testing, $a = 1$, which leads to the most effective processing decisions, and therefore the greatest generated surplus. However, because the seller captures all of the surplus from trade, the buyer earns his reservation payoff regardless of the information collected. Though additional testing is always valuable, the link between profits and information is subtle. As testing becomes more informative, it raises the precision of *all* test realizations, meaning that the expected profits following a positive test realization rise, and those following a negative realization fall. To see why total expected profits increase, note that the expected *marginal* profitability of the buyer's processing effort $(\hat{\theta}_i)$ is increasing in the expected quality; because the buyer exerts more effort when the test realization is positive, any change to the informativeness of testing will have a greater impact when the test realization is favorable relative to when it is unfavorable. Profits are convex to changes in k, because the resultant increase (decrease) in expected quality causes the buyer to accordingly increase (decrease) his processing level, generating a positive second-order effect.

In the benchmark setting of Lemma 1, the seller prices her products such that only a buyer engaging in the optimal processing will agree to trade. Thus, the seller effectively "sells the firm" to the buyer, who then has no incentive not to process the product efficiently. In the following sections, when the test results are no longer contractible the testing firm will be tempted to misreport their findings, rendering the contract in Lemma 1 infeasible.

3 Non-Contractible Testing

In this section we relax the contractibility of test realizations and instead assume that the test provides either the buyer or seller with a private, non-verifiable signal. Under either buyer or seller testing, the choice of contract communicates private test realizations. We restrict our attention to equilibria where:

i. The testing firm maximizes their payoffs by truthfully communicating the signal.

ii. The non-testing firm correctly infers the signal based on other's communication.

iii. The buyer always selects his payoff maximizing processing effort.

iv. The buyer's expected payoffs from procuring the product are non-negative.

v. If the testing firm fails to communicate, or provides an out-of-equilibrium message, the other firm refrains from trade.

While (i) implies that we only consider separating contracts, when $a = 0$, the contracts are effectively pooled since the test conveys no information. Conditions (ii)-(iv) assume that both firms are individually rational, and condition (v) specifies the off-equilibrium beliefs required to support any equilibrium with communication. Given the conditions above, the Revelation Principle allows us to narrow the analysis to truthful reporting without loss of generality.

Seller Testing

We first consider the case when the seller tests the product and proposes a sales contract to the buyer based on her observed test outcome, as in Figure 2.

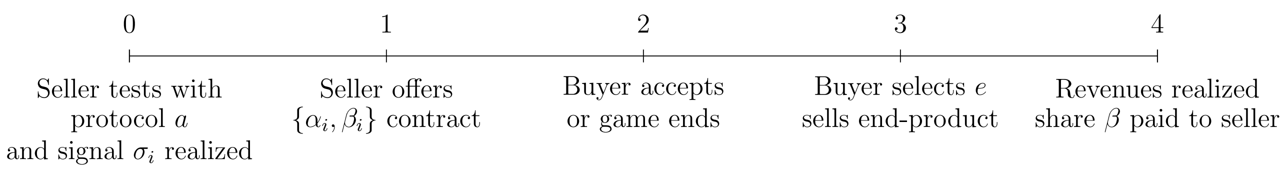

Figure 2: Time-line in the Seller Informed Model

In the first period, the seller offers the buyer a contract thereby revealing her test realization, σ_i from the prior period. The buyer, having learned the seller's signal, decides whether to accept the contract in the second period and, conditional on acceptance, selects effort, e, in the third period. In the final period, revenues are realized and appropriate settlements are made to the seller. The seller therefore solves the following program:

$$\max_{\alpha_i,\beta_i} \quad \mathrm{E}[\beta_i e_i \theta_i + \alpha_i]$$

$$\text{s.t.} \quad \beta_H e_H \hat{\theta}_H + \alpha_H \geq \beta_L e_L \hat{\theta}_H + \alpha_L \tag{1}$$

$$\beta_L e_L \hat{\theta}_L + \alpha_L \geq \beta_H e_H \hat{\theta}_L + \alpha_H \tag{2}$$

$$\beta_i e_i \hat{\theta}_i + \alpha_i \geq 0 \qquad i = H, L \tag{3}$$

$$e_i \in \arg\max_e \quad (1-\beta_i)\mathrm{E}[\theta|\sigma = \sigma_i]e - \alpha_i - \frac{e^2}{2}T \quad i = H, L \tag{4}$$

$$(1-\beta_i)e_i\hat{\theta}_i - \frac{1}{2}Te_i^2 - \alpha_i \geq 0. \qquad i = H, L \tag{5}$$

Constraints (1) and (2) ensure that the seller truthfully reveals her observed test outcome to the buyer. The individual rationality constraints (3) and (5) guarantee that both the buyer and seller are willing to trade, whereas (4) characterizes the buyer's optimal processing effort. After accepting the contract, the buyer maximizes his payoff by solving:

$$\max_e \quad (1-\beta_i)\mathrm{E}[\theta|\sigma = \sigma_i]e - \alpha_i - \frac{e^2}{2}T.$$

The buyer optimally chooses effort $e_i = \frac{(1-\beta_i)}{T}\hat{\theta}_i$; thus the larger the revenue sharing parameter, β_i, the more the buyer distorts his effort away from the efficient benchmark level of Lemma 1: $\frac{\hat{\theta}_i}{T}$. Similarly, the buyer raises his processing effort concomitantly with his expected quality beliefs–regardless of the *realized* test outcome, σ. The buyer's processing effort, and therefore both the buyer and seller's expected revenues, always increase with the seller's *reported* test outcome, providing her with an incentive to misreport unfavorable test outcomes. To overcome the incentive to misreport, the seller must reduce her payoffs when she reports favorable test realizations.

Proposition 1. *When the seller tests, the optimal contract is given by:*

$$\alpha_L = \frac{\hat{\theta}_L^2}{2T} \qquad \beta_L = 0$$

$$\alpha_H = \begin{cases} \frac{\hat{\theta}_L(2\hat{\theta}_L - \hat{\theta}_H)}{4T} & \text{if } k \geq \sqrt{5} - 1 \\ \frac{(1-\beta_H)^2 \hat{\theta}_H^2}{2T} & \text{if } k \in (1, \sqrt{5} - 1) \end{cases} \qquad \beta_H = \begin{cases} \frac{1}{2} & \text{if } k \geq \sqrt{5} - 1 \\ \frac{\hat{\theta}_H^2 - \hat{\theta}_H\hat{\theta}_L - \sqrt{2\hat{\theta}_H\hat{\theta}_L^2(\hat{\theta}_H - \hat{\theta}_L)}}{\hat{\theta}_H^2 - 2\hat{\theta}_H\hat{\theta}_L} & \text{if } k \in (1, \sqrt{5} - 1) \end{cases}.$$

In order to reduce her incentive to misrepresent, the seller decreases her payoff following a favorable signal via a large revenue share, β_H, and a small upfront price, α_H, relative to those following an unfavorable outcome: β_L and α_L. Because end-product revenues relate to the *realized quality* θ, revenue shares provide an effective check against misreporting when the seller announces a favorable test realization, though the same instrument induces the buyer to distort his future processing effort. Since there is no reason to doubt a seller's unfavorable announcement, the same logic implies that effort distortions following unfavorable announcements are unnecessary, thus $\beta_L = 0$. In order for $\beta_H > 0$ to effectively keep the seller honest, the revenue share should maximize the seller's exposure to the realized quality. This maximum is attained when $\beta_H = 1/2$, as evidenced by studying her binding incentive compatibility constraint, (2) (after simplification):

$$\frac{\hat{\theta}_L^2}{2T} - \frac{\beta_H(1-\beta_H)\hat{\theta}_L\hat{\theta}_H}{T} - \alpha_H = 0. \tag{6}$$

The seller's truth-telling constraint, (6), defines a necessary relationship between α_H and β_H which determines how the seller can exchange upfront pricing for revenue shares. When

$\beta_H \neq 1/2$, the seller can profitably perturb the contract by making just such an exchange, implying that the most profitable (from the seller's perspective) contract splits the buyer's end-product revenues evenly between the two firms. However, as Proposition 1 suggests, $\beta_H = 1/2$ contracts are unfeasible when testing is sufficiently uninformative ($k < \sqrt{5} - 1$); i.e., an equal split of end-product revenue would require the buyer to pay a fixed price, α_H so large, that trade would become unprofitable. Intuitively, an uninformative test causes only a negligible revision to the product's expected quality. Asking the buyer to significantly ($\beta_H = 1/2$) under-process a "high" quality product causes him to allocate substantially less processing effort on the more profitable of the two products, making even revenue shares unfeasible.

As the informativeness of testing increases (above $k > 2$), the seller's payoff to misreporting becomes so large that she must pay the buyer to accept the product when she claims to have observes a favorable test outcome. These negative prices, which we term incentive payments, decrease the seller's payoff to misreporting favorable test outcomes by transferring a portion of the seller's profit to the buyer. By revealed preference, the seller is always worse off if she cannot make incentive payments to the buyer, less obvious, however, is how the seller can effectively signal favorable test realizations with the additional contracting restriction.

Corollary 1. *If incentive payments are not allowed ($\alpha_H \geq 0$), then the optimal contract is the same as in Proposition 1 except when $k > 2$, where:*

$$\alpha_H = 0 \qquad\qquad \beta_H = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{\hat{\theta}_L}{2\hat{\theta}_H}}.$$

Corollary 1 identifies the seller's least costly signaling contract when she cannot use incentive payments. Signaling quality is more costly here because large revenue shares ($\beta_H > 1/2$) cause the buyer to grossly under-process the product, whereas smaller revenue shares ($\beta_H < 1/2$) prevent the seller from capturing the gains to informed processing decisions. Regulation preventing incentive payments abound in corporate law; for instance, several states enforce divorcement laws preventing petroleum refineries from acting as anything but "retail" partners with gas stations (Copulos (1980)).

Even when incentive payments are permitted, it would seem the seller may prefer having the buyer test the product. Asking the buyer to test the product overcomes the seller's need to costly signal her private information, though we will show below that screening the information from the buyer proves costly as well.

Buyer Testing

When the buyer tests the product, we assume the seller is uninformed of her product's realized quality at the time of the sale. After purchasing the product, but prior to processing it, the buyer tests the product and chooses whether to communicate his findings to the seller as in Figure 3.

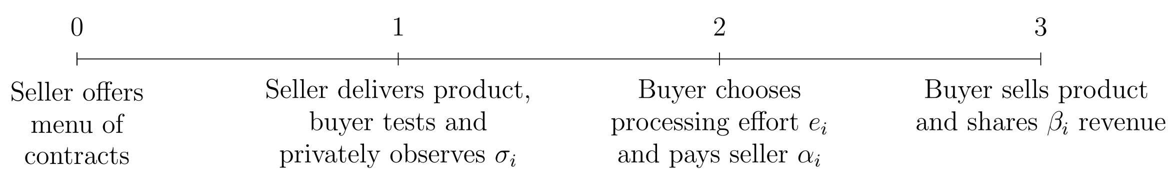

Figure 3: Time-line of the Buyer Informed Model

At date 0, the seller offers a menu of contracts to the buyer based on the testing protocol already in place at his firm. In both testing modes, our predetermined testing protocol assumption is demonstrative of circumstances where testing is comprised exclusively of fixed costs, such as buying equipment, calibrating machinery or obtaining certification, each of which trivialize the marginal cost of testing to be negligible.[5] The seller's proposed menu offers the buyer an untested product and the option to later select one of two individual contracts in line with his test observation. After receiving the product in the first period, the buyer tests the product and observes the test outcome, σ_i. In the second period, the buyer selects one of the two proposed contracts,[6] paying the seller an upfront price of α_i, and agreeing to share a fraction β_i of his end-product revenues.[7] The buyer's choice of processing effort takes place after he locks in a contract with the seller, after which he sells

[5]Models that study investments followed by adverse selection (contracting before test protocols are established) include: Arya et al. (2000), Sappington and Lewis (1997) and Cremer and Khalil (1992).

[6]The Revelation Principle ensures that there is no loss of generality in linking the buyer's report to his choice of contract, though, as in the seller testing regime, the buyer will have to be induced to report his observation truthfully.

[7]The timing of the fixed payment is irrelevant in what follows. In particular, the buyer can pay the seller after testing the product, or can instead pay the minimum of the two possible fixed prices and pay the remainder after communicating his test results.

the completed end-product at date 3. At the outset of the game the seller solves:

$$
\begin{aligned}
\max_{\alpha_i,\beta_i} \quad & \mathrm{E}[\alpha_i + \beta_i e_i \theta_i | \sigma] \\
s.t. \quad & (1-\beta_H) e_H \hat{\theta}_H - \frac{e_H^2}{2} T - \alpha_H \geq 0 && (7) \\
& (1-\beta_L) e_L \hat{\theta}_L - \frac{e_L^2}{2} T - \alpha_L \geq 0 && (8) \\
& (1-\beta_H) e_H \hat{\theta}_H - \frac{e_H^2}{2} T - \alpha_H \geq (1-\beta_L) e_{HL} \hat{\theta}_H - \frac{e_{HL}^2}{2} T - \alpha_L && (9) \\
& (1-\beta_L) e_L \hat{\theta}_L - \frac{e_L^2}{2} T - \alpha_L \geq (1-\beta_H) e_{LH} \hat{\theta}_L - \frac{e_{LH}^2}{2} T - \alpha_H && (10) \\
& e_i \in \arg\max_e \quad \mathrm{E}[(1-\beta_i) e\theta - \frac{e^2}{2} T - \alpha_i | \sigma = \sigma_i] && (11) \\
& e_{ij} \in \arg\max_e \quad \mathrm{E}[-\alpha_j + (1-\beta_j) \hat{\theta}_i e_{ij} - \frac{e_{ij}^2}{2} T - \alpha_j | \sigma = \sigma_i]. && (12)
\end{aligned}
$$

Constraints (7) and (8) ensure that the buyer can profitably process the product after testing and will therefore accept a product of unknown quality in the first period. The incentive compatibility constraints, (9) and (10), require that the buyer truthfully reveal his test realization to the seller. If the buyer misreports the test realization, then in accordance with equilibrium condition (iii), his choice of processing effort maximizes his resulting payoffs as determined by (12). The buyer's incentive compatibility constraint reflects the fact that the seller only observes revenues which are an imperfect proxy for quality,[8] not buyer's actual effort choice. As expected, the buyer's truth telling constraint, (9), always binds, causing the contract to leave him indifferent between honestly disclosing and misrepresenting the receipt of a favorable test result. Though the buyer always benefits from reporting an unfavorable test outcome, the seller can rationally anticipate such behavior and encourage truth telling

[8] If end-product revenues were a deterministic function of end-product quality, the seller's problem becomes degenerate as she can perfectly back out the buyer's test outcome and processing effort based on the shifting revenue support.

by paying informational rents.

Proposition 2. *When the buyer reports a test realization σ_i, the optimal contract is comprised of:*

$$\alpha_H = \frac{2(1-2p)p\hat{\theta}_H^4\hat{\theta}_L^2 + p(3p-2)\hat{\theta}_H^2\hat{\theta}_L^4 + p2\hat{\theta}_H^6 + (p-1)^2\hat{\theta}_L^6}{2T\left(p\left(\hat{\theta}_H^2 - 2\hat{\theta}_L^2\right) + \hat{\theta}_L^2\right)^2}$$

$$\alpha_L = \frac{(p-1)^2\hat{\theta}_L^6}{2T\left(p\left(\hat{\theta}_H^2 - 2\hat{\theta}_L^2\right) + \hat{\theta}_L^2\right)^2}$$

$$\beta_H = 0$$

$$\beta_L = \frac{p\left(\hat{\theta}_H - \hat{\theta}_L\right)\left(\hat{\theta}_H + \hat{\theta}_L\right)}{p\left(\hat{\theta}_H^2 - 2\hat{\theta}_L^2\right) + \hat{\theta}_L^2}.$$

Independent of who tests the product, each firms' informational preferences are entirely determined by the informed firm's incentive to misreport. Because she has all the bargaining power, the seller retains all gains from trade net of rents paid to encourage truthful reporting. In particular, when the seller attempts to lower the testing firm's incentive to misrepresent–be it herself or the buyer–the buyer's payoff increases. Therefore the seller always prefers test protocols where the testing firm's incentive to misreport is minimal, whereas the buyer prefers protocols netting the opposite incentives. Surprisingly, the following proposition shows that testing protocol, and consequently, the informativeness of testing, do not move in tandem with the reporting firm's incentive to misreport.

Proposition 3. *Independent of who tests, the buyer's (seller's) payoffs are single peaked ("U" shaped) over the level of testing, a. The seller prefers either full testing* $(a = 1)$*, or no testing* $(a = 0)$*, while the buyer prefers a non-zero level of testing. If the seller is charged* *with testing and incentive payments are prohibited, then the seller's payoff always decreases with testing.*

If the seller were to observe a negative test outcome and misreport it as being favorable, she would effectively fool the buyer into over-processing the intermediary product and earn herself an additional $e_H - e_L$ in end-product revenues. Such a roue would be costly, however, as the seller would have to simultaneously decrease the buyer's upfront purchase price by $\frac{e_H^2}{2}T - \frac{e_L^2}{2}T$ in order for her announcement to be credible. Whereas the benefit to misrepresentation is linear in the level of additional processing, the cost is convex. When the induced effort levels, e_H and e_L' i.e., when testing is relatively uninformative, then the payoff to misrepresentation is both positive and increasing. However, as the test becomes more informative, the induced effort levels grow further apart and the convex cost eventually overwhelms the linear benefit, decreasing the seller's incentive to misreport. Put simply, the seller's incentive to misreport is single peaked in the informativeness of testing, and consequently, in the level of testing.

The proposition also shows that when incentive payments are prohibited, as in Corollary 1, credibly sharing information proves too costly to merit any testing whatsoever, as the required (inefficient) revenue sharing therein erases all the benefit associated with more informed processing. The finding provides a rationale for firms to invest in downstream, arm's length partners, as absent such investments or incentive payments, the former may be unable to credibly communicate helpful information. For example, McDonalds has traditionally purchased the land on which its franchises operate (Gross and Staff (1997)). In return, the franchisee pays McDonalds revenue based royalties in addition to a small annual franchise fee. Our model suggests that this arrangement encourages franchise owners to trust their franchisor's communications; e.g., "high-quality" product offerings, suggested pricing, recommended inventory strategies, and promotions, because in the absence of such investments, the franchisee could infer that the franchisor has no incentive to share meaningful information.

On the other hand, when the buyer tests, his sole objective is to maximizing his informational rents. The buyer's binding truth-telling constraint (9), sets his rents equal to his incentive to misreport products which test favorably. The seller can always reduce the buyer's rents by raising the revenue share, β_L, though doing so will induce the buyer to later distort his processing effort, e_L. The seller's choice of revenue share thus balances the cost of these distortions with the buyer's rents. Because the expected marginal productivity of the buyer's low effort–and therefore the loss associated with processing distortions–is decreasing in the informativeness of testing, the seller raises the revenue share β_L as testing becomes more informative. As testing becomes even more informative, the difference in expected qualities increases, causing the buyer's payoff to misreporting a high-quality product (higher end-product revenues) as a low-quality product (lower payment to the seller) to rise as well. Combined, the varying revenue payoff to misreporting and the varying revenue share cause the buyer's rents to peak once over test informativeness.

While Proposition 3 shows that the seller's profits are qualitatively similar regardless of whether she or the buyer tests the product, the following section ranks her outcomes across the two testing regimes, and characterizes the preferred setting.

4 Choice of Tester

The prior section exogenously assigned testing to either the buyer or seller, and therein characterized each firms' informational preferences. We now consider the broader question of who should be responsible for product testing. For example, the United Stated Department of Agriculture (USDA) requires that slaughterhouses test outgoing meat before supplying it to meat packers. Specifically, slaughterhouses propose and follow pre-specified test protocols which limit testing variability. Similar adhesion to protocols are required for firms to qualify for ISO certification. Motivated by relatively fixed testing choices, we begin by studying the consequences of buyer and seller testing, when the testing protocol is fixed.

Proposition 4. *The seller will outsource testing if the testing protocol results in informative testing. Thus, the seller favors outsourcing if testing is sufficiently rigorous; i.e.,* $a > \hat{a}(j) > 0$*, where* $\hat{a}(j)$ *is decreasing in product heterogeneity,* j*.*

To understand how testing intensity, a, influences the seller's preferences, recall that when the seller (buyer) tests, the buyer will only processes the intermediary product efficiently following an unfavorable (favorable) test report. When testing is relatively uninformative, the seller prefers to test the product herself, as the processing losses associated with under-processing high-quality products are negligible, and she avoids paying rents. However, as testing becomes more informative, either via increased testing, greater product heterogeneity, j, or a smaller ex-ante probability of producing a high-quality product, (p), the losses associated with under-processing high-quality products under seller testing outweigh the informational rents and low-quality processing distortions associated with buyer testing, causing the seller to favor buyer-testing instead.

When the products can vary substantially (large heterogeneity j) the end-product prices facing consumers will vary significantly as well, driving a large premium for high- as opposed to low-quality intermediary products. Regardless of the testing intensity, additional heterogeneity will always result in more informative testing, causing the seller to again favor outsourcing. Proposition 4 also speaks to the seller's manufacturing choices. While we have thus far assumed that the seller's production facility dictates both the probability of manufacturing a high-quality product, p, and the different qualities of product produced (j), the previous analysis may influence these longer-term decisions, albeit the relation is intricate. If the seller were to invest in her plant and raise the yield of high-quality product, p, then she: (1) decreases her opportunities for misreporting unfavorable test realizations and, (2) raises the expected product quality conditional on it testing favorably. The conflicting effects do not admit intuitive thresholds allowing us to sign how changes in production facilities affect the seller's test sourcing decision, however we do find that the seller's investment incentive is maximized concomitantly with her expected profits. In particular, the seller's marginal return to facility upgrades is maximized when she is free to choose who ought to test the product once the improvements are in place, without requiring any additional commitment.

The choice of testing firm also carries welfare implications which may limit the seller's test sourcing options. We consider two welfare measures, total expected surplus and total expected quality. Regulators pay close attention to total industry surplus whenever one firm is capable of exercising monopoly power over the over, as evidenced by antitrust rulings surrounding computer operating systems (Jackson (1999)). Meanwhile, in consumer markets, regulation frequently seeks to ensure minimum end-product quality, especially as it pertains to safety.

Proposition 5. *Expected total surplus is always greater when the buyer tests. Expected end-product quality is also greatest when the buyer tests the product, unless the informativeness of testing, k, is sufficiently large and the high-quality product yield, p, is sufficiently small. More informative testing will only lead to greater total surplus and expected quality if testing was sufficiently informative to begin with.*

Amidst recent outbreaks in food-borne illnesses, the U.S. Food and Drug Administration has recently issued guidelines moving the burden of meat testing further downstream to buyers (Moss (2009)), in line with Proposition 5. Our finding also highlights the subtle fact that end-product quality is not monotonically driven by testing intensity. Increased testing always results in more information with which to make processing decisions, however, if the marginal cost of communicating the newfound information requires substantial under-processing, then the benefits of improved decision making can be outweighed by the burden of communicating the newfound information. We find this to be the case, regardless of who is testing, when testing is sufficiently uninformative. The complementarities between the choice of testing firm and the actual testing protocol require that policy makers consider both decisions to avoid writing regressive regulation.

5 Endogenous Testing

Thus far our analysis has fixed the testing protocol, a, and studied the consequences of buyer and seller testing. In this section, we expand the model to allow the testing firm the unrestricted choice of testing protocol ($a \in [0, 1]$). Specifically, we allow the buyer and seller the opportunity to choose their own testing protocol once the seller has chosen who should test the product in anticipation of the subsequent protocols. For example, the a seller may choose not to outsource testing if she knows that the buyer will rationally under-test the product. To simplify our endogenous testing analysis, we set $\theta_L = 1$ and assume that the seller's product quality is uniformly distributed ($p = 1/2$).

In choosing their testing protocols, the seller seeks to minimize the testing firm's incentive to misreport whereas the buyer's orthogonal payoff induce him to maximize the very same incentive. The opposing preferences cause the seller to abandon test outsourcing altogether. To understand why the seller's position on outsourcing has changed, recall that when a was fixed, the seller only outsourced testing when the results were sufficiently informative. If the the seller were to outsource testing in the present setting, the buyer would opt for a lesser testing protocol, making testing less informative and thereby eliminating the seller's incentive to outsource testing in the first place. As a solution to the sellers' reluctance to outsource testing, a number of industries have begun requiring that buyers be pre-certified (Hwang et al. (2006)). Pre-certification can come in the form of ISO-certification, factory visits, or any other mechanism which places a lower bound on the buyer's testing protocol. By pre-certifying the buyer, the seller regains control over some of the buyer's testing choices, and when the certification guarantees sufficiently informative testing, the seller will again favor buyer testing.

Proposition 6. *With endogenous product testing, the seller will outsource testing only if she can guarantee a minimum level of testing from the buyer* $a > \underline{a} > 0$ *and product heterogeneity is sufficiently large,* $j > j(a) > 1$.

Product heterogeneity, j, enters the seller's outsourcing decisions, since testing becomes more informative as product diversity increases. Therefore requiring a minimum heterogeneity threshold guarantees that the buyer's testing will be informative even when the minimum protocol, $\underline{a}$, is relatively small. Although minimum protocols can be helpful at the firm level, they have a more nebulous effect once imposed on the industry level.

In our specialized setting, a minimum testing level of $a = 1/2$ is sufficient to motivate the seller to outsource testing as long as product heterogeneity, j, is sufficiently large ($j > 1.5$). In Table 1, we report the buyer and seller's preferred testing level when industry standards mandate that $a \geq 1/2$. The bold face values denotes the seller's preferred tester, while the two right columns indicate each firm's preferred protocol assuming they each choose their own. Table 1 shows that as product heterogeneity, j, increases, the seller will outsource testing to the buyer, who will eventually choose the minimum level of testing.

Table 1: Seller and buyer testing preferences as a function of product heterogeneity, j, in the face of minimum testing standards $a \geq 1/2$. The equilibrium testing intensities are marked in boldface when the seller is free to choose who should test the product.

Heterogeneity (j)	Seller's Pref.	Buyer's Pref.
$\left(1, \frac{1}{4}(1+\sqrt{17}\right)$	$\mathbf{a = 1/2}$	$a = 1$
$\left[\frac{1}{4}(1+\sqrt{17}, 1.5\right)$	$\mathbf{a = 1/2}$	$1/2 < a < 1$
$\left[1.5, \frac{1}{13}(5+4\sqrt{17})\right)$	$a = 1/2$	$\mathbf{1/2 < a < 1}$
$\left[\frac{1}{13}(5+4\sqrt{17}), 5\right)$	$a = 1/2$	$\mathbf{a = 1/2}$
$\left[5, \infty\right)$	$a = 1$	$\mathbf{a = 1/2}$

Although omitted from the table, we find that both expected surplus and quality can also decrease with industry-imposed minimum testing protocols. To see why, recall that the seller eschews more informative testing whenever the cost of communicating the results outweigh the benefits: if the seller is forced to undertake additional testing, then she will also have to additionally distort the buyer's processing incentives to credibly communicate her findings. When product heterogeneity is small, the additional distortions erase the efficiency gains associated with increased testing, and in doing so decrease both total quality and surplus.

6 Conclusion

In a setting where testing yields unverifiable results, our model finds that buyers and sellers differ in their incentives to collect and share private product information. In particular, we showed that a seller favors testing so long as it reduces the testing firm's incentive to misreport the results, whereas the buyer favors testing only when it increases the same incentive. The firms' opposing preferences highlight the importance of who tests traded products, as the choice is shown to affect end-product quality, industry surplus and agency costs.

Throughout, we normalized the volume of trade to a single production batch, preventing the buyer or seller from communicating their information via the ordered or offered quantities. Our setting is analogous to one where the buyer no longer processes the product, but instead uses the test result to inform his order quantity. In such a setting, the two sides could use the volume of trade to signal and screen private information between one another. Unfortunately, the results of such a study would be largely influenced by the assumed end-product demand function, and our limited experience with such models has generated perverse equilibria where the seller signals high-quality products by limiting the volume offered for sale. To avoid such outcomes, future research would have to endow the buyer and seller with multiple signaling tools, such as volume *and* processing effort. It would be interesting to see how the choice of communication tool depends on who tests the product and the outstanding level of uncertainty.

Another possible extension would allow for joint testing where both the buyer and seller test the product. Characterizing the information and testing protocols in this case would require delicate modeling choices. In our model, if the buyer can test the product after the seller, then the seller's test is redundant unless the buyer's test reveals supplemental information. If the buyer's test did reveal more information than the seller's test, then the model reduces to our setting with an additional noisy auditing game. Though an additional round of auditing will not qualitatively change our results, additional research could consider how the results would change if multiple rounds of testing revealed different types of information.

A final extension could consider the choice of testing intensity when the outcome is verifiable, but the testing protocol is not. Though we believe that such circumstances are rare, it would be interesting to see how the results compare to our setting where only the testing protocol is verifiable.

Appendix

Table 2: Parameters

Parameter Name	Definition
α_i	Fixed payment to seller
β_i	Revenue share to seller
e_i	Buyer's effort
θ_i	Product quality
T	Cost function parameter
a	Testing intensity
j	θ_H/θ_L
k	$\hat{\theta}_H/\hat{\theta}_L$
σ	Signal

Proof of Lemma 1

The seller maximizes the following problem in the first best setting:

$$\max_{e_H, e_L} \quad p \cdot E_\theta \left[\theta_H e_H - T \frac{e_H^2}{2} \middle| \sigma = H \right] + (1-p) \cdot E_\theta \left[\theta_L e_L - T \frac{e_L^2}{2} \middle| \sigma = L \right].$$

The optimal efforts are found via the first order condition:

$$\begin{aligned} e_H^{FB} &= \frac{\hat{\theta}_H}{T} \\ e_L^{FB} &= \frac{\hat{\theta}_L}{T}. \end{aligned}$$

Benchmark, or so-called first best expected profits are thus:

$$\begin{aligned} \Pi^{FB} &= \frac{p\left(\hat{\theta}_H^2 - \hat{\theta}_L^2\right) + \hat{\theta}_L^2}{2T} \\ &= \frac{(p(\theta_H - \theta_L) + \theta_L)^2 + a^2(1-p)p(\theta_H - \theta_L)^2}{2T}. \end{aligned}$$

By inspection, Π^{FB} is increasing and convex in a, implying that as the testing increases, profits are monotonically increasing (strictly increasing except at $a = 0$). Since the informativeness of testing, k, and the level of testing a, strictly move together, this completes the proof.

Proof of Proposition 1

The seller solves:

$$
\begin{aligned}
\max_{\alpha_i,\beta_i} \quad & \mathrm{E}[\beta_i e_i \theta_i + \alpha_i] \\
\text{s.t.} \quad & \beta_H e_H \hat{\theta}_H + \alpha_H \geq \beta_L e_L \hat{\theta}_H + \alpha_L && (\mathrm{ICS}_H) \\
& \beta_L e_L \hat{\theta}_L + \alpha_L \geq \beta_H e_H \hat{\theta}_L + \alpha_H && (\mathrm{ICS}_L) \\
& \beta_i e_i \hat{\theta}_i + \alpha_i \geq 0 \qquad i = H, L && (\mathrm{IRS}_i) \\
& e_i \in \arg\max_e \quad (1-\beta_i)\mathrm{E}[\theta|\sigma = \sigma_i]e - \alpha_i - \frac{e^2}{2}T \quad i = H, L && (\mathrm{ICB}_i) \\
& (1-\beta_i)e_i\hat{\theta}_i - \frac{1}{2}Te_i^2 - \alpha_i \geq 0. \qquad i = H, L && (\mathrm{IRB}_i)
\end{aligned}
$$

Differentiating the buyer's IC constraints yield:

$$
e_i^{SB} = \frac{(1-\beta_i)\hat{\theta}_i}{T} \quad i = H, L. \tag{13}
$$

As is standard, we ignore the ICS_H, IRS_H and IRS_L constraints for the time being and later verify that the solution to the relaxed problem satisfies those constraints. IRB_L will always bind, because, if it did not then α_L could be increased which would raise profits, create slack in ICS_L and leave the other constraints unchanged. Simplifying IRB_L using (13) yields:

$$
\begin{aligned}
\alpha_L &= (1-\beta_L)e_L\hat{\theta}_L - T\frac{e_L^2}{2} \\
&= \frac{(1-\beta_L)^2\hat{\theta}_L^2}{2T}.
\end{aligned} \tag{14}
$$

After substituting in (14) and (13), the seller's program becomes:

$$
\begin{aligned}
\max_{\alpha_H,\beta_H,\beta_L} \quad & p \cdot \left(\alpha_H + \frac{\beta_H(1-\beta_H)\hat{\theta}_H^2}{T}\right) + (1-p)\left(\frac{(1-\beta_L^2)\hat{\theta}_L^2}{2T}\right) \\
s.t. \quad & \frac{(1-\beta_H)^2\hat{\theta}_H^2}{2T} \geq \alpha_H && (\mathrm{IRB}_H) \\
& \frac{(1-\beta_L^2)\hat{\theta}_L^2}{2T} - \frac{\beta_H(1-\beta_H)\hat{\theta}_H\hat{\theta}_L}{T} \geq \alpha_H. && (\mathrm{ICS}_L)
\end{aligned}
$$

Both profits and the left hand side (LHS) of the ICS_L constraint are decreasing in $|\beta_L|$ while the rest of the program is independent of β_L. To maximize profits, β_L is set to 0, implying that $\alpha_L = \frac{\hat{\theta}_L^2}{2T}$. The program thus reduces to:

$$\begin{aligned}
\max_{\alpha_h,\beta_h} \quad & p\left(\alpha_H + \frac{\beta_H(1-\beta_H)\hat{\theta}_H^2}{T}\right) + (1-p)\left(\frac{\hat{\theta}_L^2}{2T}\right) \\
s.t. \quad & \frac{(1-\beta_H)^2\hat{\theta}_H^2}{2T} \geq \alpha_H && (\text{IRB}_H) \\
& \frac{\hat{\theta}_L^2}{2T} - \frac{\beta_H(1-\beta_H)\hat{\theta}_H\hat{\theta}_L}{T} \geq \alpha_H. && (\text{ICS}_L)
\end{aligned}$$

We claim that ICS_L must bind. To see this, note that the seller's profits are increasing in α_H, though α_H is bounded above by either IRB_H or ICS_L. If the LHS of ICS_L is less than that of IRB_H, then ICS_L binds. Otherwise, IRB_H binds and $\alpha_H = \frac{(1-\beta_H)^2\hat{\theta}_H^2}{2T}$. However, if $\alpha_H = \frac{(1-\beta_H)^2\hat{\theta}_H^2}{2T}$, the program becomes:

$$\begin{aligned}
\max_{\beta_H} \quad & p \cdot \left(\frac{(1-\beta_H^2)\hat{\theta}_H^2}{T}\right) + (1-p)\left(\frac{\hat{\theta}_L^2}{2T}\right) \\
s.t. \quad & \frac{\hat{\theta}_L^2}{2T} \geq \frac{\beta_H(1-\beta_H)\hat{\theta}_H\hat{\theta}_L}{T} + \frac{(\beta_H-1)^2\,\hat{\theta}_H^2}{2T}. && (\text{ICS}_L)
\end{aligned}$$

The right hand side (RHS) of ICS_L is decreasing with respect to β_H, as is objective function. Thus, the seller could lower β_H until ICS_L binds and simultaneously increase profits. Therefore, ICS_L will always bind, allowing us to relate α_H and β_H with the following equations:

$$\beta_H(\alpha_H) = \frac{\hat{\theta}_H\hat{\theta}_L \pm \sqrt{\hat{\theta}_H\hat{\theta}_L\left(\hat{\theta}_L\left(\hat{\theta}_H - 2\hat{\theta}_L\right) + 4\alpha_H T\right)}}{2\hat{\theta}_H\hat{\theta}_L} \tag{15}$$

$$\alpha_H(\beta_H) = \frac{\hat{\theta}_L\left(2\left(\beta_H-1\right)\beta_H\hat{\theta}_H + \hat{\theta}_L\right)}{2T}. \tag{16}$$

The principal's objective function is maximized when β_H is equal to 1/2, since substituting (16) into the objective function and differentiating with respect to β_H yields:

$$\frac{\partial\Pi(\beta_H,\alpha_H(\beta_H))}{\partial\beta_H} = \frac{p\left(1-2\beta_H\right)\hat{\theta}_H\left(\hat{\theta}_H - \hat{\theta}_L\right)}{T}.$$

Since the objective function is concave in β_H, $\beta_H = 1/2$ maximizes profits. Using $\beta_H = 1/2$ and α_H as defined by (16), the ICS_H constraint is satisfied since:

$$\text{ICS}_H : \frac{\hat{\theta}_H\left(\hat{\theta}_H - \hat{\theta}_L\right)}{4T} \geq 0.$$

Simplification also yields that IRS_H and IRS_L are satisfied. However, the candidate solution sets IRB_H to $\frac{(k(k+2)-4)\hat{\theta}_L^2}{8T}$ which is non-negative when $k \geq \sqrt{5}-1$. For values $k < \sqrt{5}-1$, the seller will set β_H as large as possible without violating IRB_H. The contract will therefore have a second form when $k < \sqrt{5}-1$; i.e., when IRB_H binds:

$$\alpha_H = \frac{(1-\beta_H)^2\hat{\theta}_H^2}{2T}, \tag{17}$$

which combined with (15) implies:

$$\beta_H = \frac{\hat{\theta}_H^2 - \hat{\theta}_H\hat{\theta}_L \pm \sqrt{2}\sqrt{\hat{\theta}_H\hat{\theta}_L^2\left(\hat{\theta}_H - \hat{\theta}_L\right)}}{\hat{\theta}_H^2 - 2\hat{\theta}_H\hat{\theta}_L}.$$

One of the above roots can be rejected since it falls outside of the relative range, $[0,1]$. Therefore, when $k < \sqrt{5}-1$:

$$\beta_H = \frac{\hat{\theta}_H^2 - \hat{\theta}_H\hat{\theta}_L - \sqrt{2}\sqrt{\hat{\theta}_H\hat{\theta}_L^2\left(\hat{\theta}_H - \hat{\theta}_L\right)}}{\hat{\theta}_H^2 - 2\hat{\theta}_H\hat{\theta}_L}$$

and α_H is defined by (17). Using these values, ICS_H, yields:

$$\frac{(k-1)\hat{\theta}_L^2}{(k-2)^2T}\left(2-k-k^2+\sqrt{2}\sqrt{(k-1)k}\right),$$

which is positive for $k \in (1, \sqrt{5}-1)$. Simplification also implies that IRS_H and IRS_L are satisfied by this solution.

Proof of Corollary 1

If incentive payments are disallowed ($\alpha_H \geq 0$), then when $k > 2$ the contract from Proposition 1 is no longer valid. To obtain the optimal contract on this range, we first verify that α_H should be zero by substituting (13), (16), $\beta_L = 0$, and (14) into the profit function:

$$\Pi = \frac{2pT\alpha_H\left(\hat{\theta}_L - \hat{\theta}_H\right) + \hat{\theta}_L^2\left((1-p)\hat{\theta}_L + p\hat{\theta}_H\right)}{2T\hat{\theta}_L},$$

which implies that profits are decreasing in α_H since:

$$\frac{\partial \Pi}{\partial \alpha_H} = p\left(1 - \frac{\hat{\theta}_H}{\hat{\theta}_L}\right) < 0.$$

The seller therefore sets α_H to zero and, using (15), solves for β_H:

$$\beta_H = \frac{1}{2}\left(1 \pm \sqrt{1 - \frac{2\hat{\theta}_L}{\hat{\theta}_H}}\right).$$

Both roots of β_H are in (0,1) and yield the same profit to the seller:

$$\Pi = \frac{\hat{\theta}_L\left(p\left(\hat{\theta}_H - \hat{\theta}_L\right) + \hat{\theta}_L\right)}{2T}.$$

We verify the constraints:

$$\begin{aligned}
IRB_H: &\quad \frac{\left(\hat{\theta}_H\hat{\theta}_L + \sqrt{\hat{\theta}_H\hat{\theta}_L^2\left(\hat{\theta}_H - 2\hat{\theta}_L\right)}\right)^2}{8T\hat{\theta}_L^2} > 0 \\
ICS_H: &\quad \frac{\hat{\theta}_L\left(\hat{\theta}_H - \hat{\theta}_L\right)}{2T} \geq 0 \\
IRS_H: &\quad \frac{\hat{\theta}_H\hat{\theta}_L}{2T} \geq 0 \\
IRS_L: &\quad \frac{\hat{\theta}_L(\hat{\theta}_H + \hat{\theta}_L)}{2T} \geq 0.
\end{aligned}$$

Proof of Proposition 2

The seller's maximization program is given by:

$$
\begin{aligned}
\max_{\alpha_H,\alpha_L,\beta_H,\beta_L} \quad & E_\theta[\alpha_i + \beta_i \cdot e_i \cdot \theta_i] \\
s.t. \quad & e_H \in \arg\max_e E[(1-\beta_H)\theta e - \alpha_H - T\frac{1}{2}e^2|\sigma = H] && \text{(ICB1}_H\text{)} \\
& e_L \in \arg\max_e E[(1-\beta_L)\theta e - \alpha_L - T\frac{1}{2}e^2|\sigma = L] && \text{(ICB1}_L\text{)} \\
& e_{HL} \in \arg\max_e E[(1-\beta_L)\theta e - \alpha_L - T\frac{1}{2}e^2|\sigma = H] && \text{(ICB2}_H\text{)} \\
& e_{LH} \in \arg\max_e E[(1-\beta_H)\theta e - \alpha_H - T\frac{1}{2}e^2|\sigma = L] && \text{(ICB2}_L\text{)} \\
& E[(1-\beta_H)e_H\theta - \alpha_H - T\frac{1}{2}e_H^2|\sigma = H] \geq 0 && \text{(IRB}_H\text{)} \\
& E[(1-\beta_L)e_L\theta - \alpha_L - T\frac{1}{2}e_L^2|\sigma = L] \geq 0 && \text{(IRB}_L\text{)} \\
& E[(1-\beta_H)e_H\theta - \alpha_H - T\frac{1}{2}e_H^2|\sigma = H] \geq E[(1-\beta_L)e_{HL}\theta - \alpha_L - T\frac{1}{2}e_{HL}^2|\sigma = H] && \text{(ICB}_H\text{)} \\
& E[(1-\beta_L)e_L\theta - \alpha_L - T\frac{1}{2}e_L^2|\sigma = L] \geq E[(1-\beta_H)e_{LH}\theta - \alpha_H - T\frac{1}{2}e_{LH}^2|\sigma = L] && \text{(ICB}_L\text{)} \\
& E[\beta_H e_H \theta_H + \alpha_H|\sigma = H] \geq 0 && \text{(IRS}_H\text{)} \\
& E[\beta_L e_L \theta_L + \alpha_L|\sigma = H] \geq 0. && \text{(IRS}_H\text{)}
\end{aligned}
$$

Solving ICB1 and ICB2 yields:

$$
\begin{aligned}
e_H &= \frac{(1-\beta_H)\hat{\theta}_H}{T} & \qquad e_L &= \frac{(1-\beta_L)\hat{\theta}_L}{T} \\
e_{LH} &= \frac{(1-\beta_H)\hat{\theta}_L}{T} & \qquad e_{LH} &= \frac{(1-\beta_L)\hat{\theta}_H}{T}.
\end{aligned}
$$

As is standard, we ignore IRB_H, ICB_L, IRS_H and IRS_L and verify that the solution to the relaxed problem satisfies the constraints, so the seller's program becomes:

$$
\begin{aligned}
\max_{\alpha_H,\alpha_L,\beta_H,\beta_L} \quad & p\left(\frac{(1-\beta_H)\,\beta_H\hat{\theta}_H^2}{T} + \alpha_H\right) + (1-p)\left(\frac{(1-\beta_L)\,\beta_L\hat{\theta}_L^2}{T} + \alpha_L\right) \\
s.t. \quad & \frac{(1-\beta_L)^2\hat{\theta}_L^2}{2T} \geq \alpha_L && \text{(IRB}_L\text{)} \\
& \frac{(1-\beta_H)^2\hat{\theta}_H^2}{2T} - \frac{(1-\beta_L)^2\hat{\theta}_H^2}{2T} + \alpha_L \geq \alpha_H. && \text{(ICB}_H\text{)}
\end{aligned}
$$

Increasing α_H until ICB$_H$ binds will increase profits without changing the other constraint, so ICB$_H$ binds and consequently determines α_H. This yields:

$$\max_{\alpha_L,\beta_H,\beta_L} \quad \frac{-p\hat{\theta}_H^2\left(\beta_H^2+(\beta_L-2)\beta_L\right)+2(p-1)(\beta_L-1)\beta_L\hat{\theta}_L^2+2T\alpha_L}{2T}$$

$$s.t. \quad \frac{(1-\beta_L)^2\hat{\theta}_L^2}{2T} \geq \alpha_L. \tag{IRB$_L$}$$

Profits are increasing in α_L, so the seller will set it equal to the maximum allowable level while maintaining IRB$_L$ and the program becomes:

$$\max_{\beta_H,\beta_L} \quad \frac{(\beta_L-1)\hat{\theta}_L^2\left((2p-1)\beta_L-1\right)-p\hat{\theta}_H^2\left(\beta_H^2+(\beta_L-2)\beta_L\right)}{2T}. \tag{18}$$

The derivative of (18) is decreasing in β_H, so β_H is set to 0, while β_L is found via the first order condition. The optimal contract values are:

$$\alpha_H = \frac{2(1-2p)p\hat{\theta}_H^4\hat{\theta}_L^2+p(3p-2)\hat{\theta}_H^2\hat{\theta}_L^4+p^2\hat{\theta}_H^6+(p-1)^2\hat{\theta}_L^6}{2T\left(p\left(\hat{\theta}_H^2-2\hat{\theta}_L^2\right)+\hat{\theta}_L^2\right)^2}$$

$$\alpha_L = \frac{(p-1)^2\hat{\theta}_L^6}{2T\left(p\left(\hat{\theta}_H^2-2\hat{\theta}_L^2\right)+\hat{\theta}_L^2\right)^2}$$

$$\beta_H = 0$$

$$\beta_L = \frac{p\left(\hat{\theta}_H-\hat{\theta}_L\right)\left(\hat{\theta}_H+\hat{\theta}_L\right)}{p\left(\hat{\theta}_H^2-2\hat{\theta}_L^2\right)+\hat{\theta}_L^2}.$$

Substituting these expressions into IRB$_H$, ICB$_L$, IRS$_H$ and IRS$_L$ yields expressions which are always positive and therefore the final constraints are verified.

Proof of Proposition 3

To complete this proof we break the analysis into four separate pieces, conditioned on the testing party and the party in question.

Seller's Profits Under Seller Testing

To show that the seller's profit function, when she tests, is "U" shaped in a, we first show that the function is decreasing in a for $k < \sqrt{5} - 1$. We will then show that as k increases, profits are initially decreasing, but carry a positive second derivative. We begin by differentiating the seller's profits over testing intensity, a:

$$\frac{d\Pi(\beta_H(a), a)}{da} = \left.\frac{\partial \Pi(\beta_H, a)}{\partial \beta_H}\right|_{\beta_H = \beta_H(a)} \cdot \frac{\partial \beta_H(a)}{\partial a} + \left.\frac{\partial \Pi(\beta_H, a)}{\partial a}\right|_{\beta_H = \beta_H(a)}. \tag{19}$$

There are three parts to (19), which we sign individually. We first show that $\frac{\partial \Pi(\beta_H, a)}{\partial \beta_H} < 0$ by writing the profit function in terms of β_H and a, and setting $\theta_H = j \cdot \theta_L$:

$$\Pi(\beta_H, a) = \frac{\theta_L^2 \left(((a-1)(j-1)p-1)^2 - p\left(\left(\beta_H^2 - 1\right)\left(a(j-1)(p-1) - jp + p - 1\right)^2 + ((a-1)(j-1)p-1)^2\right)\right)}{2T}.$$

Differentiating the profits over β_H yields:

$$\begin{aligned} \frac{\partial \Pi(\beta_H, a)}{\partial \beta_H} &= -\frac{p \beta_H \theta_L^2 (a(j-1)(p-1) - jp + p - 1)^2}{T} \\ &< 0. \end{aligned} \tag{20}$$

Inequality (20) will hold for any positive function $\beta_H(a)$. To sign $\frac{\partial \beta_H}{\partial a}$, the second term in (19), we first compute:

$$\frac{\partial \beta_H}{\partial a} = \frac{\partial \beta_H}{\partial \hat{\theta}_H} \frac{\partial \hat{\theta}_H}{\partial a} + \frac{\partial \beta_H}{\partial \hat{\theta}_L} \frac{\partial \hat{\theta}_L}{\partial a}. \tag{21}$$

Now:

$$\frac{\partial \hat{\theta}_H}{\partial a} = (1-p)(\theta_H - \theta_L) > 0 \tag{22}$$

$$\frac{\partial \hat{\theta}_L}{\partial a} = -p(\theta_H - \theta_L) < 0. \tag{23}$$

Differentiating β_H with respect to $\hat{\theta}_H$ and yields:

$$\frac{\partial \beta_H}{\partial \hat{\theta}_H} = \frac{-1}{k}\left(\frac{2\sqrt{(k-1)k}k + \sqrt{2}((3-2k)k-2)}{2(k-2)2\sqrt{(k-1)k}\hat{\theta}_L}\right)$$
$$\frac{\partial \beta_H}{\partial \hat{\theta}_L} = \left(\frac{2\sqrt{(k-1)k}k + \sqrt{2}((3-2k)k-2)}{2(k-2)2\sqrt{(k-1)k}\hat{\theta}_L}\right).$$

Since the expressions within the parenthesis above are the same, the derivatives have opposite signs. When $k \leq \sqrt{5}-1$ we find that, over the entire interval:

$$\frac{\partial \beta_H}{\partial \hat{\theta}_H} > 0 \tag{24}$$
$$\frac{\partial \beta_H}{\partial \hat{\theta}_L} < 0. \tag{25}$$

From (21), (22), (23), (24), (25) we have, when $k < \sqrt{5}-1$:

$$\frac{\partial \beta_H}{\partial a} > 0 \tag{26}$$

which signs the second part of (19). To sign the final term in (19), we substitute $j = \frac{k-1}{(a-1)(k-1)p+a} + 1$ and $\beta_H = \beta_H(a)$ to yield the following expression (un-ambiguously positive pre-multiplier has been removed):

$$\left.\frac{\partial \Pi(\beta_H, a)}{\partial a}\right|_{\beta_H=\beta_H(a)} = -3k + 2\sqrt{2}\sqrt{k-1}\sqrt{k} + 2.$$

On the interval $k \in (1, \sqrt{5}-1)$, the above expression is negative, therefore:

$$\left.\frac{\partial \Pi(\beta_H, a)}{\partial a}\right|_{\beta_H=\beta_H(a)} < 0. \tag{27}$$

From (27), (26) and (20) we have that (19) is negative and therefore profits are decreasing in a when $k < \sqrt{5}-1$.

Next, we will show that the seller's profits are "U" shaped when $k \geq \sqrt{5}-1$ by demonstrating that the derivative is initially negative and the second derivative is positive over the entire region. When $k \geq \sqrt{5}-1$, the profit function takes the form:

$$\Pi = \frac{p\hat{\theta}_H\left(\hat{\theta}_H - \hat{\theta}_L\right) + 2\hat{\theta}_L^2}{4T}$$

which implies:

$$\frac{\partial \Pi}{\partial a} = \frac{1}{4T}\bigg(2ap(p+1)\left(\theta_H-\theta_L\right)^2 + p\left(\theta_H-\theta_L\right)\left(-3p\left(\theta_H-\theta_L\right)-3\theta_L\right)\bigg). \tag{28}$$

At the end-point, $k=\sqrt{5}-1$, (28) becomes:

$$-\frac{(j-1)p\left(\left(\sqrt{5}-2\right)p-2\sqrt{5}+7\right)\left((j-1)p+1\right)\theta_L^2}{4\left(\left(\sqrt{5}-2\right)pT+T\right)}<0.$$

The second derivative is positive over the entire interval, as:

$$\begin{aligned}\frac{\partial^2 \Pi}{\partial a^2} &= \frac{p(p+1)\left(\theta_H-\theta_L\right)^2}{2T}\\ &> 0.\end{aligned}$$

To determine the seller's preference for information, we only need to evaluate the interval end-points, since the function is "U" shaped. Note that we compare the second contract profit when $a=1$ and the first contract profit when $a=0$ since we have previously shown that the seller's profits are decreasing over the entire first contract region. Therefore, it is not possible for the first contract region, with $a=1$, to achieve a maximum value. Substituting $a=1$ and $a=0$ into the relevant profit functions yields:

$$\begin{aligned}\Pi\bigg|_{a=0} &= \frac{\left(p\left(\theta_H-\theta_L\right)+\theta_L\right)^2}{2T}\\ \Pi\bigg|_{a=1} &= \frac{p\theta_H\left(\theta_H-\theta_L\right)+2\theta_L^2}{4T}.\end{aligned}$$

Comparing these two expressions implies that the seller prefers to be fully informed when:

$$\begin{aligned}0<p<\frac{1}{2}\\ j>\frac{2p-4}{2p-1}.\end{aligned}$$

To conclude the proof, we need to exclude the possibility of a jump discontinuity where the contract changes form. To do so, note that:

$$\lim_{k\to(\sqrt{5}-1)^+}\Pi = \lim_{k\to(\sqrt{5}-1)^-}\Pi.$$

We next consider the case when incentive payments are not allowed. Note that the contract with and with out incentive payments is the same when $k<\sqrt{5}-1$ and we have shown that profits are decreasing in a on this range above. When incentive payments are not allowed and $k>2$ differentiating the profit function yields:

$$\begin{aligned}\frac{\partial}{\partial a}\left\{\frac{\hat{\theta}_L\left(p\left(\hat{\theta}_H-\hat{\theta}_L\right)+\hat{\theta}_L\right)}{2T}\right\} &= \frac{\partial}{\partial a}\left\{\frac{\hat{\theta}_L E[\theta]}{2T}\right\}\\ &= \frac{\partial\hat{\theta}_L}{\partial a}\cdot\frac{E[\theta]}{2T}\\ &< 0.\end{aligned}$$

Therefore, when incentives payments are disallowed, profits are decreasing when $k > 2$ and when $k < \sqrt{5}-1$. However, in the range $k \in [\sqrt{5}-1, 2]$, it may be possible for the seller's profits to increase. As we've shown above, the second derivative of the profit function is negative on this interval, so we need only check the sign of the first derivative at the end point to see if profits are ever increasing on this region. If $j > 2$ then the relevant end-point to check is $k = 2$, while if $j < 2$ then the correct end-point is given by $a = 1$ or equivalently, $k = j$. Under this later scenario, at $a = 1$, we obtain:

$$\begin{aligned}\left.\frac{\partial\Pi}{\partial a}\right|_{a=1} &= \frac{(j-1)p\theta_L^2(j(2-p)+p-5)}{4T}\\ &= \frac{p\theta_L^2}{4T}(j-1)\left(2j+p(1-j)-5\right)\\ &\leq \frac{p\theta_L^2}{4T}(\sqrt{5}-2)\left(3\sqrt{5}-7\right)\\ &\leq 0.\end{aligned}$$

When $j > 2$, to determine if profits are ever increasing, we check the end-point generated by the restriction $k \leq 2$:

$$\begin{aligned}\frac{(a(p-1)-p)\left(\theta_H-\theta_L\right)-\theta_L}{(a-1)p\left(\theta_H-\theta_L\right)-\theta_L} &\leq 2\\ \Leftrightarrow \frac{p\theta_H - p\theta_L+\theta_L}{p\theta_H+\theta_H-p\theta_L-\theta_L} &\geq a. \end{aligned}\tag{29}$$

Imposing (29) onto the profit function, (28), we find that profits are decreasing in a:

$$\frac{-p\left(\theta_H-\theta_L\right)\left(p\left(\theta_H-\theta_L\right)+\theta_L\right)}{4T} \leq 0.$$

Therefore, the seller's profits are everywhere decreasing if incentive payments are disallowed.

Seller's Profits Under Buyer Testing

The seller's profits under the buyer informed model are given by:

$$\frac{p^2\hat{\theta}_H^4 + (1-2p)p\hat{\theta}_H^2\hat{\theta}_L^2 + (p-1)^2\hat{\theta}_L^4}{2T\left(p\left(\hat{\theta}_H^2 - 2\hat{\theta}_L^2\right) + \hat{\theta}_L^2\right)}. \tag{30}$$

Taking the second derivative with respect to a and simplifying yields a single term, the denominator of which carries the sign of:

$$-\frac{a^2\left(\left(k^2-2\right)p+1\right)}{((a-1)(k-1)p+a)^2}, \tag{31}$$

which is negative. The numerator is the product of two functions:

$$\frac{a^6(k-1)^2(p-1)^2p^2\theta_L^2}{((a-1)(k-1)p+a)^8}\cdot \tag{32}$$

$$\left(\left((k(3k+8)-15)k^2+3\right)p+6k^2+\left(k^2\left(k^4-8k+9\right)-2\right)p^2-1\right). \tag{33}$$

The first expression (32) is always positive while the second (33) can be written as:

$$(-1+(3-2p)p)+k^2(-6+15p-9p^2)+k^3(-8p+8p^2)+k^4(-3p)+k^6(-p^2). \tag{34}$$

To search for possible roots to (34) in $p \in (0,1)$, we apply a Mobius transform and complete a Descartes test[9] and find that the resulting quadratic has all non-positive coefficients, implying that (34) is constant in sign over the entire interval. Evaluation shows that (32) is always negative. The seller's profits are thus "U" shaped, since the second derivative of her profits has both a negative numerator and denominator.

Buyer's Rents Under Buyer Testing

When the buyer tests, his rents $(r(a,j))$ are given by:

$$r(a,j) = \frac{(1-p)^2p(\hat{\theta}_H - \hat{\theta}_L)\hat{\theta}_L^4(\hat{\theta}_H + \hat{\theta}_L)}{2T\left(\hat{\theta}_L^2 + p(\hat{\theta}_H^2 - 2\hat{\theta}_L^2)\right)}.$$

Note that the rents are influenced by the level of testing via the dependence of both $\hat{\theta}_H$ and $\hat{\theta}_L$ on a. To prove that the rents are single peaked over a, we show that $\frac{\partial r(0,j)}{\partial a} > 0$ and the function, $\frac{\partial r(a,j)}{\partial a}$ has, at most, one root over the interval $a \in (0,1)$. To this end, we have $\frac{\partial r(0,j)}{\partial a} = \frac{p\theta_L^2(j-1)(1+(j-1)p)}{T} > 0$.

In order to show that the function $\frac{\partial r(a,j)}{\partial a}$ has at most a single root over $a \in (0,1)$, we begin by examining

[9] See Eigenwillig (2007) for details.

its denominator:

$$2T(-1-(-3+a^2(1-j)^2+2j)p-(1-a)(j-1)(j-3+a(j-1))p^2+(1-a)^2(j-1)^2p^3)^3$$

which carries the same sign as:

$$-1-(-3+a^2(1-j)^2+2j)p-(1-a)(j-1)(j-3+a(j-1))p^2+(1-a)^2(j-1)^2p^3.$$

The denominator has two roots in a, both of which fall out of the range $(0,1)$ when $0<p<1$ and $j>1$; implying that the denominator has a constant (negative) sign throughout the relevant range. We can express the numerator as:

$$f(a,p,j)\Big((-1+j)(-1+p)^2p(1+(-1+a+j-aj)p)^3\theta_L^2\Big)$$

where $f(a,p,j)$ is a fourth order polynomial in a and the rest of the expression is always positive. We search for possible roots between the $(0,1)$ interval by applying the Descartes test to the Mobius transform on $f(a,p,j)$. In particular, we verify the number of roots of $R(a,j)=(a+1)^4f(T(a),j,p)$, where by $T(a)=\frac{1}{1+a}$.[10] The function $R(a,j)$ is a fourth degree polynomial. Each coefficient is negative at $p=0$ and has a unique root over $0<p<1$, which we denote C_i, where i is the order of a:

$$\begin{aligned}
C_0 &= \frac{-j^3+\sqrt{j^6+8j^5-2j^4-14j^3+j^2+6j+1}-j+1}{2\left(2j^4-j^3-3j^2+2\right)}\\
C_1 &= \frac{-j^3-3j^2+\sqrt{j^6+6j^5+99j^4-68j^3-117j^2+54j+41}-3j+3}{2\left(7j^3-9j^2-6j+8\right)}\\
C_2 &= \frac{-j^2+\sqrt{j^4+12j^3-10j^2-12j+13}-2j+1}{2\left(2j^2-5j+3\right)}\\
C_3 &= \frac{2j-\sqrt{3j^2-2j+3}}{j-1}\\
C_4 &= 1
\end{aligned}$$

The roots above are ordered, in that for any j: $C_0<C_1<C_2<C_3<C_4=1$. Therefore, for any value of j and p, the total sign variation amongst the ordered coefficients is, at most, 1. Since the denominator of $\frac{\partial r(a,j)}{\partial a}$ does not vary in sign and the numerator has, at most, a single sign change as a varies, $\frac{\partial r(a,j)}{\partial a}$ has at most a single root in a over the interval $(0,1)$ and is thus single peaked.

[10]See Eigenwillig (2007) for details.

Buyer's Rents Under Seller Testing

When $k \leq \sqrt{5}-1$ both of the buyer's (IR) constraints bind and he collects no rents. When $k \geq \sqrt{5}-1$, the buyer's rents are given by:

$$p\left(\frac{2\hat{\theta}_H\hat{\theta}_L + \hat{\theta}_H^2 - 4\hat{\theta}_L^2}{8T}\right). \tag{35}$$

The buyer's rents are initially increasing in a, as taking the derivative of (35) and substituting in the value of a at which $k(a) = \sqrt{5}-1$ yields:

$$\frac{(j-1)p\left(\sqrt{5}+\left(5-2\sqrt{5}\right)p\right)((j-1)p+1)\theta_L^2}{4\left(\left(\sqrt{5}-2\right)pT+T\right)}$$

which is always positive. The second derivative of (35) is equal to:

$$(1-p(p+4))\frac{(\theta_H-\theta_L)^2}{4T}, \tag{36}$$

and does not vary in sign over the relevant range. If the second derivative is positive then, coupled with the fact that the rents are initially increasing, the buyer's rents are increasing over the entire region and are therefore single peaked. If, on the other hand, the second derivative is negative then the buyer's rents are single peaked and there possibly exists an internal solution. The two conditions below are necessary and sufficient for there to be an internal solution:

$$\begin{aligned} 1-p(p+4) &\leq 0 \\ 1+j+3p-2jp &< 0. \end{aligned}$$

The first condition guarantees that the second derivative is negative while the second condition implies that the first derivative is negative at $a=1$. Simplification yields:

$$\begin{aligned} p &> \frac{1}{2} \\ j &> \frac{1+3p}{-1+2p}. \end{aligned}$$

Proof of Proposition 4

When $k \in (1, \sqrt{5}-1)$, we can write the seller's profit in the buyer informed regime minus those in the seller informed regime as:

$$\left(k^5 p - k^4 p + k^3\left(p - 2\sqrt{2}\sqrt{(k-1)kp}\right) - k^2(p-2) + 2\left(\sqrt{2}\sqrt{(k-1)k} - 1\right)k(2p-1) + 4(p-1)\right)$$
$$\cdot \frac{\theta_L^2 (k-1)p((j-1)p+1)^2}{2(k-2)^2 T((k-1)p+1)^2\left(\left(k^2-2\right)p+1\right)}.$$

The second term is positive, and therefore any sign variation will be the result of the first term:

$$k^5 p - k^4 p + k^3\left(p - 2\sqrt{2}\sqrt{(k-1)kp}\right) - k^2(p-2) + 2\left(\sqrt{2}\sqrt{(k-1)k} - 1\right)k(2p-1) + 4(p-1).$$

The expression above is linear in p with a positive coefficient, so the following condition is both necessary and sufficient for the difference in profits to be positive:

$$p > \frac{2}{k(k+1)\left(k + \sqrt{2}\sqrt{(k-1)k} - 1\right) + 2}. \tag{37}$$

If (37) is satisfied, then the seller's profits are greater when the buyer tests the product. The RHS of (37) is monotonically decreasing in k, demonstrating that for a fixed level of p, the difference in profits between the two modes may be equal to zero at most once.

If we substitute $k = 1$ into the expression, the RHS is equal to 1 and the expression is never true, showing that the seller informed model initially dominates. However, as a increases, k increases, and thus the (RHS) of (37) decreases, causing the buyer informed model to leave the seller with ever increasing profits. Allowing k to increase indefinitely, (37) will eventually fail, and the buyer informed model will dominate. However, k is bounded above for two reasons. First, j could be small enough to bound k below the crossing level, in which case the seller informed model always dominates. Secondly, we assumed k was less than or equal to $\sqrt{5}-1$ in justifying (37), for larger values, the threshold condition is no longer valid.

Finally, note that if we evaluate the expression at the end-point ($k = \sqrt{5}-1$), then we see that if $p < \frac{1}{22}\left(7+\sqrt{5}\right)$, the seller prefers to do testing in house throughout the region, whereas if $p > \frac{1}{22}\left(7+\sqrt{5}\right)$, the seller will prefer to outsource testing for any k above some threshold level of k in $(1, \sqrt{5}-1)$.

In the second contract region, where $k \geq \sqrt{5}-1$, subtracting the seller profits in the seller informed model from those in the buyer informed yields:

$$\frac{(k(k(k+2)p-1)-2p)}{4\left(k^2-2\right)p+4} \frac{(k-1)p\hat{\theta}_L^2}{T}. \tag{38}$$

Now, the second expression and the denominator of the first expression (38) are always positive, so the only possible sign variation will come from:

$$k(k(k+2)p-1)-2p.$$

Simplifying, the following condition holds if and only if (38) is positive:

$$p > \frac{k}{k^3 + 2k^2 - 2}. \tag{39}$$

If p is above this level, then the buyer informed model dominates and if p falls below the threshold $\frac{k}{k^3+2k^2-2}$, then the seller informed model dominates. The derivative of the RHS of (39) with respect to k is $-\frac{2(k^3+k^2+1)}{(k^2(k+2)-2)^2} < 0$, implying that the RHS of (39) is monotonically decreasing with respect to k (and therefore also a), establishing that the profits can only cross once.

For a fixed p value, if we increase a (and therefore k) this condition is more likely to be satisfied, since the p threshold decreases. Assuming that k is unbounded above, then for sufficiently large k, the condition would eventually hold regardless of p and the buyer informed model would always dominate. However, if j is small, then it is possible that the cut-off value of k is never attained, in which case the seller informed model would always dominate.

Note that evaluating this condition at the endpoint $k = \sqrt{5} - 1$ yields the same cut-off from the first contract region: $p > \frac{1}{22}\left(7 + \sqrt{5}\right)$, thereby excluding any jump discontinuities as the contract form changes.

Proof of Proposition 5

We begin this proof by studying the relative levels of welfare under each regime and then discuss the effect of marginally increasing the testing intensity under each regime and welfare measure.

Relative Welfare Measures

Total Expected Surplus

When $k \in (1, \sqrt{5} - 1)$, subtracting the expected total surplus under the seller informed-model from that of the buyer informed model yields:

$$g(a, p, k) \cdot \frac{p\hat{\theta}_L^2}{2T(-2+k)^2(1-2p+k^2p)^2}$$

where $g(a, p, k)$ is quadratic in p. The discriminant of this polynomial is:

$$-(k-2)^2(k-1)^4(k+1)^3\left(k\left(k(3k-1) - 8\sqrt{2}\sqrt{(k-1)k} + 8\right) - 4\right). \tag{40}$$

On the relevant range, the sign of (40) is the same as the sign of:

$$4 - k\left(k(3k-1) - 8\sqrt{2}\sqrt{(k-1)k} + 8\right),$$

which has 3 roots: $k = 1$, $k = 2$ and $k = -2.97$, all of which are outside $k \in (1, \sqrt{5} - 1)$. Therefore, (40) has no sign changes over the interval and is always negative (at $k = 1.1$ (40) is equal to -3.45). Since the discriminate is negative, $g(a, p, k)$ has no real roots; evaluation implies that it is always positive. Therefore, when $k \in (1, \sqrt{5} - 1)$ the buyer informed model has higher expected total surplus.

When $k \geq \sqrt{5}-1$, the difference in expected total surplus under the seller informed and the buyer informed models is equal to:

$$\frac{p\hat{\theta}_L^2}{8T\left(\left(k^2-2\right)p+1\right)^2}\left(k^2+\left(k^6-4k^2+4\right)p^2-2\left(k^4-2k^2+2\right)p\right). \tag{41}$$

The first part of (41) is positive, while the second part is quadratic in p and has discriminate equal to:

$$-16(-1+k^2)^3<0,$$

implying that there are no real roots. Evaluation yields that (41) is positive, so the buyer informed model yields larger expected surplus when $k \geq \sqrt{5}-1$.

Total Expected Quality

When $k \leq \sqrt{5}-1$, subtracting the expected total quality when the seller tests from that of when the buyer tests yields:

$$\left((k-1)\left(\left(k\left(k^3-k-1\right)-2\right)p+k+2\right)-\sqrt{2}k\left(\left(k^2-2\right)p+1\right)\sqrt{(k-1)k}\right)\cdot\frac{\hat{\theta}_L^2 p}{(2-k)\left(\left(k^2-2\right)p+1\right)}.$$

The second expression is always positive, so we focus on the first, which is linear in p,

$$\left((k-1)\left(\left(k\left(k^3-k-1\right)-2\right)p+k+2\right)-\sqrt{2}k\left(\left(k^2-2\right)p+1\right)\sqrt{(k-1)k}\right). \tag{42}$$

The constant term of (42) is equal to:

$$2+k(-1-k+\sqrt{2(-1+k)k}).$$

This expression has 3 real roots (± 1 and 2) all of which are outside the relevant range, implying that for any value of k, the constant term of (42) is invariant in sign over k. Evaluation reveals that it is positive. Similarly, when $p=1$, (42) becomes:

$$k(-1+k^2)(k(k-1)+\sqrt{2(-1+k)k}).$$

which has four roots (0 , ± 1 and 2), all of which are outside of the relevant region. Evaluation reveals that this expression is also positive. This implies that expression (42) is positive since it is a line connecting two positive quantities. Therefore, when $k \in (1, \sqrt{5}-1)$ we find that the buyer informed model has higher expected total quality. When $k \geq \sqrt{5}-1$ the difference in expected quality between the two modes is equal to:

$$\left(k^4p-k^2-2p+2\right)\cdot\frac{p\hat{\theta}_L^2}{2T\left(\left(k^2-2\right)p+1\right)}. \tag{43}$$

The second part of (43) is positive while the first part is negative when the following two conditions hold:

$$k > \sqrt{2}$$
$$p \leq \frac{-2 + k^2}{-2 + k^4}.$$

Therefore, unless these two conditions hold, (43) is positive and the buyer informed model has higher expected total quality.

The low signal expected quality is always going to be larger in the seller informed model, because the buyer's process at the first best level. Similarly, the high signal expected quality is larger under the buyer informed model since the buyer's effort is at the first best level following a favorable test result.

Marginal effects of increasing testing intensity

Total Expected Surplus

Total expected surplus, in the seller informed model when $k < \sqrt{5} - 1$, is simply the seller's profits since the buyer collects no rents. As shown in Proposition 3, the seller's profits are decreasing with a in this region and therefore total expected surplus is decreasing. When $k \geq \sqrt{5} - 1$ and the seller tests, the derivative of the total expected surplus with respect to a is equal to:

$$\frac{a(k-1)(1-p)p\theta_L^2}{4T((a-1)(k-1)p+a)^2}(3k-4) \tag{44}$$

The first part of this expression is always positive while the second is positive when $k \geq 4/3$. In other words, under the seller informed model, the total welfare is increasing with a only when $k \geq 4/3$ and decreasing otherwise. In the buyer informed model, the derivative of the total expected surplus with respect to a yields a positive expression multiplied by:

$$1 + (-6 + (-2 + k)k)p + (11 + k(-1 + 3k)(-3 + k^2))p^2 + (-6 + k^2(9 + k(-1 + k(-5 + k + k^2))))p^3. \tag{45}$$

For large enough k, all terms are positive and hence the derivative is increasing in a. For smaller values of k, this expression can be either positive or negative and therefore total expected surplus can be either increasing or decreasing in a.

Total Expected Quality

We first show that when $k < \sqrt{5} - 1$, that total expected quality under the seller informed model is decreasing in a. First, note that since:

$$TQ(\beta_H, a) = \frac{p(1-\beta_H)\hat{\theta}_H^2}{T} + \frac{(1-p)\hat{\theta}_L^2}{T}$$

we have that:

$$\frac{dTQ(\beta_H, a)}{da} = \frac{\partial TQ}{\partial \beta_H}\frac{\partial \beta_H}{\partial a} + \frac{\partial TQ}{\partial a}. \tag{46}$$

$$\frac{\partial TQ}{\partial \beta_H} = -\frac{p\hat{\theta}_H^2}{T} \leq 0 \tag{47}$$

$$\frac{\partial^2 TQ}{\partial a \partial \beta_H} = -\frac{2p\hat{\theta}_H}{T}\frac{\partial \hat{\theta}_H}{\partial a} \leq 0 \tag{48}$$

$$\frac{\partial \beta_H}{\partial a} \geq 0. \tag{49}$$

Where (49) was shown in Proposition 3. From (48) and (47), the first part of (46) is negative. Since the cross partial of expected total quality with respect to a and β_H is negative, if we can find a function β_H^* such that (1) $\beta_H^*(a) \leq \beta_H(a)$, ($\beta_H(a)$ is the second best optimal value), and (2) $\frac{\partial TQ}{\partial a}|_{\beta_H^*(a)} \leq 0$ then we will have shown that the overall derivative, (46), is negative. To find such a function we solve:

$$\frac{\partial TQ}{\partial a} = 0,$$

which yields:

$$\beta_H^*(a) = \frac{a - aj}{a(j-1)(p-1) - jp + p - 1}.$$

We next show that $\beta_H^*(a) < \beta_H(a)$. Taking the difference, $\beta_H^* - \beta_H(a)$, yields:

$$\frac{-\sqrt{2}\sqrt{-a(j-1)((a-1)(j-1)p-1)^2(a(j-1)(p-1)-jp+p-1)} - 2a(j-1)((a-1)(j-1)p-1)}{(a(j-1)(p-1)-jp+p-1)(a(j-1)(p+1)-jp+p-1)}.$$

Note that $(a(j-1)(p-1) - jp + p - 1) < 0$, simplification implies that the difference carries the sign of:

$$\Big((1-a)(j-1)p+1\Big)\frac{\sqrt{2}\sqrt{-a(j-1)(a(j-1)(p-1)-jp+p-1)} - 2a(j-1)}{(a(j-1)(p+1)-jp+p-1)},$$

The first expression is positive, so we ignore it. Further simplification yields:

$$\frac{\sqrt{2}\sqrt{-a(j-1)(a(j-1)(p-1)-jp+p-1)} - 2a(j-1)}{(a(j-1)(p+1)-jp+p-1)}$$
$$= \Big(2a(j-1)\Big)\frac{-1}{\sqrt{2}\sqrt{-a(j-1)(a(j-1)(p-1)-jp+p-1)} + 2a(j-1)}.$$

Ignoring the positive expression, the numerator is negative while the denominator is positive, implying that $\beta_H^* < \beta_H(a)$. Therefore, in the region $k < \sqrt{5} - 1$, the seller informed model has decreasing total expected quality with respect to a. When $k \geq \sqrt{5} - 1$ the derivative of total expected quality is equal to:

$$-\frac{a(k-2)(k-1)(p-1)p\theta_L^2}{T((a-1)(k-1)p+a)^2}$$

After removing the positive terms, this expression is positive when $k \geq 2$. Therefore, under the seller testing model, total expected quality is decreasing when $k < 2$ and increasing afterwards.

When the buyer tests, the denominator of the derivative with respect to a can be written as:

$$\frac{a^4(1+(-2+k^2)p)^2 T\theta_L^4}{(p-kp+a(1+(-1+k)p))^4},$$

which is always positive. After removing positive terms, the denominator of the derivative is equal to:

$$-1+(3+2k(-1+(-1+k)k))p+(-1+k)^3(1+k)(2+k)p^2,$$

which is positive when

$$\frac{2}{\sqrt{4(k-1)^2(k+1)k^3+1}+2k((k-1)k-1)+3} < p < 1.$$

This expression is decreasing in k and also quite small (when $k = 2$ the p cutoff is .118).

Proof of Proposition 6

In this numeric example we have an exogenous minimum standard of $a = 1/2$, $\theta_L = 1$ and $p = 1/2$. To determine when the seller will choose to outsource, we first find her preferred level of testing as a function of j. When the seller tests, her preferences are complicated by the contract dichotomy. But, because of the "U" shaped profit function the seller, we only need to compare her testing preferences at the end-points $a = 1/2$ and $a = 1$. When $j < \sqrt{5} - 1$, both end-points are in the first contract region and the seller prefers $a = 1/2$. When $\sqrt{5} - 1 < j < \frac{4-3\sqrt{5}}{\sqrt{5}-4}$ then $a = 1/2$ is in the first contract. In this region, the seller still prefers $a = 1/2$. When $j > \frac{4-3\sqrt{5}}{\sqrt{5}-4}$ then both $a = 1/2$ and $a = 1$ are in the second contract region and the seller prefers $a = 1$ until $j > 5$.

When the buyer tests, his preferences can be found by studying his rents. Using the first order condition to solve for internal optima, the buyer's rents are maximized when:

$$a = \frac{-4j^2 + \sqrt{17}\sqrt{j^4 - 2j^2 + 1} + 4}{j^2 - 2j + 1}. \tag{50}$$

However, (50) only applies when $\frac{1}{13}(5 + 4\sqrt{17}) \geq j \geq \frac{1}{4}(1 + \sqrt{17})$, since below this range the preferred a is greater than 1 and for values of j above this range the preferred a is less than 1/2. Given each firm's preferences, the results indicate that for small values of j, the seller prefers to keep testing in house, while for large value of j, she chooses outsourcing. All findings are presented in Table 1.

While the analysis above was restricted to the case when $a > 1/2$, if we allow j to become sufficiently large, then comparing the seller's profits between seller and buyer testing, we find that for any $a > .388$ there exists a level of product heterogeneity, j, at which the seller prefers outsourcing. To see this, subtract the seller's profits under seller testing from buyer testing profits to obtain (unambiguously positive terms have been omitted):

$$(3 - 8a - 2a^2 - a^4) + (6 - 6a - 11a^2 + 3a^4)j + (3 + 8a^2 - 3a^4)j^2 + (-2a + 5a^2 + a^4)j^3.$$

If the coefficient on the j^3 term is positive than, eventually, for large enough j, the positive leading coefficient will dominate any other negative coefficients. Solving for the single positive root in the leading coefficient implies that any value of a greater than .388 will suffice for the argument to hold.

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