Accountability and Control in Buyer-Supplier Relationships

Richard Saouma∗
Stanford Graduate School of Business

January 11, 2006

Abstract

A recent trend in supply chain management is to purchase fully assembled systems from suppliers rather than ordering components. When purchasing assembled systems, a manufacturer can test the assembled unit and hold the supplier accountable for system failures; however she cannot control the assembly effort. On the other hand, when a manufacturer purchases components, she controls the assembly process, yet she may find it difficult to hold the suppliers accountable for the performance of the finished product. In this paper, I explore the manufacturer’s delegation decision when her supplier has a liability constraint. My results are specific to the type of effort used to assemble systems. When systems can be assembled with only binary effort, the manufacturer prefers to assemble the system herself whenever feasible. However, if assembly effort is continuous, then delegation is optimal when the supplier’s cost of producing high quality components is large and his liability constraint is lax. Surprisingly, I find that if the manufacturer cannot penalize the supplier for systems which pass testing but fail in the field, then delegation becomes even more attractive. Finally, I demonstrate that a manufacturer will always perform more testing when the supplier builds the system relative to when the manufacturer assembles the system.

∗I am thankful for helpful comments from Stan Baiman, Erica Plambeck, Madhav Rajan, John Roberts and Stefan Reichelstein. This paper was made possible with the generous funding of the KPMG Foundation. Any errors are my own.
1 Introduction

This paper addresses the task delegation decision facing manufacturers vis-a-vis their suppliers. Over the past two decades, manufacturers have begun ordering complex assemblies or systems, rather than traditional, simple components from their suppliers \cite{Baldwin1997, Sherefkin2003}. The trend is most apparent in the computer industry where PC manufacturers often build entire computers with off-the-shelf components \cite{Economist2001}, though the trend is pervasive in many other industries as well \cite{Kisiel2005}.

However, delegating design or assembly tasks to suppliers is not necessarily always optimal. Some Japanese auto manufacturers have been reluctant to outsource any assembling tasks to their suppliers due to quality concerns \cite{Treece1998}. Inherent in any act of outsourcing is the control loss a manufacturer suffers from no longer directly overseeing the delegated process. On the other hand, when a manufacturer purchases components and builds systems in-house, it may be unclear who is at fault when a system fails, and assigning blame between the manufacturer and supplier could be difficult at best. Assigning blame in the event of a failure is further complicated by the fact that manufacturers are frequently unable to test components until they are built into a system. For example, auto manufacturers are unable to test anti-lock brake computers until the computers are plugged into a finished vehicle\footnote{As noted by Claas Bracklo who works for the electronics integration for vehicle development at DaimlerChrysler: “Despite simplifications in the assumptions I made, I arrived at $10^{180}$ potential test conditions for a single vehicle model. If you wanted to test all these as a simulation, you’d have to book several decades on a Cray supercomputer.” DaimlerChrysler (2004)}. Thus, manufacturers are frequently forced to contract with suppliers on the functionality of finished products, as opposed to contracting on the parts delivered by the supplier. Therefore, contracts between the two parties often include warranty clauses whereby the supplier pays the manufacturer in the event of an end-product failure\footnote{See Smith (1997) and Wernle (2005).} Warranty contracts can be costly to the manufacturer, because suppliers may be too small to afford large warranty clauses preferred by manufacturers \cite{Sherefkin2003}. In order for a small supplier to accept a large warranty contract, manufacturers must often provide additional incentives such as increased upfront payments or a long-term contract. I capture suppliers’ inability to pay large warranties by imposing a liability constraint which restricts the supplier’s payoff in all states. I compare the manufacturer’s payoffs when she assembles the system herself with the payoffs achieved when she outsources assembly to her supplier.

My analysis reveals that the optimality of outsourcing assembly to a supplier is intimately related to the type of effort involved in assembling the system. I assume that a system is either successful or defective, where the probability of a system failing is a function of the assembly efforts. When assembling a system can only be completed one of two ways (binary), I find that the manufacturer always prefers to assemble the system herself rather than outsource assembly to her supplier. In this setting, however, insourcing may be infeasible, and if the manufacturer wants to build a system, she may be forced to outsource assembly. Outsourcing is necessary whenever the manufacturer is unable to write a contract which motivates the supplier to build a high quality component and simultaneously
motivates the manufacturer to assemble the system carefully. In these cases, the dual-moral hazard problem effectively prevents the manufacturer from committing to carefully assembling the system. In particular, when the supplier pays the manufacturer a large warranty should the final product fail, the manufacturer may prefer shirking on the assembly task and collecting the warranty to working hard, thereby increasing the probability of receiving a warranty payment. When assembly is outsourced to the supplier, warranties have the reverse effect on assembly effort, because when faced with a large penalty for failure, suppliers will prefer to work hard assembling the systems to avoid paying the warranty. The downside of outsourcing is that the supplier’s rents may increase because of the additional tasks asked of him.

My results change significantly when I assume that assembly effort is a continuous choice variable. The difference between the two settings is that the binary case imposes a “corner solution” level of assembly effort, whereas making the decision variable continuous allows for “interior” solutions. I find that when the supplier’s cost of producing high quality components is high, the manufacturer will prefer delegation as long as the supplier can shoulder moderate liability. When the assembly effort is a continuous choice variable, a manufacturer who assembles the systems has countless opportunities to shirk. Akin to the binary case, the larger the warranty facing the supplier, the stronger the manufacturer’s incentive to shirk on assembly. In order to motivate a supplier with high component costs, the manufacturer must impose a large warranty. Yet, when the warranty is large, the manufacturer is tempted to shirk when assembling the systems, which in turn mandates an even larger warranty to keep the supplier motivated. This cycle is absent when the manufacturer delegates assembly to the supplier; in this case, a large warranty motivates the supplier both to produce high quality components and to work hard at assembling the systems. Similar to the binary setting, large warranties are costly if the supplier can only accept moderate liability, in which case the gains from outsourcing could potentially be erased by having to pay large economic rents to the supplier.

In my model, the manufacturer’s desire to delegate assembly to a supplier is entirely driven by the manufacturer’s inability to commit to high levels of assembly effort. The dual moral hazard problem facing a manufacturer who assembles the systems disappears if the manufacturer’s assembly effort is a contractible action. I find that if the manufacturer’s actions are contractible, she can always feasibly assemble the system, and moreover, the system quality in this setting exceeds the first-best level of quality.

After comparing the merits of each regime, I analyze the system testing environment and decision. Surprisingly, I find that when the manufacture is unable to contract on system-failures which occur after testing, her preference for outsourcing increases. I then endogenize the testing decision, and analyze the manufacturer’s testing preferences with and without delegation. I find that the manufacturer will always perform more testing when the supplier assembles the systems as opposed to when the manufacturer assembles the systems, regardless of which regime is optimal. Improving the testing procedures decreases the manufacturer’s expected loss of a defective system in both regimes. The reduced expected loss, in turn, dampens the manufacturer’s incentive to accurately assemble the systems. Therefore, when the manufacturer assembles the system, she prefers poor testing to accurate testing, because it increases her ability to commit to building high quality systems. If the manufacturer delegates assembly to her supplier, the commitment problem is absent, as is the
incentive for inefficient testing. Finally, I demonstrate that all my results continue to hold if the manufacturer can imperfectly test the supplier-provided components before assembling the system.

In the following section, I introduce the related literature. Section 3 establishes the model, and Section 4 compares the two regimes when assembly effort is a binary choice variable. Section 5 relaxes the binary restriction and assumes that the assembly effort is a continuous choice variable. Section 6 explores different testing and contracting scenarios and section 7 concludes. All proofs are in the appendix.

2 Literature Review

There is an emerging literature in accounting on the benefits and costs of warranty contract ing. In Baiman et al. (2004), the authors assume that multiple suppliers produce components that piece together to form a single unit manufactured by the buyer. The authors examine the performance of two commonly observed contracts. In the first, each component is individually tested before being used to assemble the end product (Acceptable Quality Level), and in the second, all suppliers are compensated on the basis of the end-product yield rate (Group Warranty). The research suggests that the number of components used to develop the final product is a key determinant as to which type of contract is optimal. In my model, I assume that the supplier’s component cannot be tested prior to completion of the assembly.

A separate line of research has posited the question: under which contracting circumstances can a buyer and seller attain the first-best benchmark in which a single firm controls all facets of production? In Baiman et al. (2000), the supplier produces a good, and the buyer performs a costly appraisal of the incoming goods. The authors find that if either the supplier’s actions are contractible, both internal and external failures are contractible or the buyer’s testing is contractible as are either internal or external failures, then the first-best benchmark can be attained. The paper then analyzes the second-best cases and examines the value of installing an information system which renders the buyer’s appraisal activities contractible. Although my model also involves double moral hazard, I compare these costs with an alternative mode whereby the buyer simply outsources production of the entire system to the supplier.

The benefit to outsourcing or bundling tasks has been examined by the economics literature. In Riordian and Sappington (1987), two regimes are compared: one in which the agent performs both an initial and follow up task, and another where the agent is only asked to perform the initial task and the principal completes the project. In a model of adverse selection where the agent privately knows the cost of completing the first task, the principal may be better off delegating the second task to the agent if the cost of the second task is known to be slightly negatively correlated with that of the first. In their model, the outcome is perfectly contractible, and hence all contracting is complete. In my model, the two parties cannot contract upon the quality of the first task, and are thus limited to contracting on the outcome of the finished product. Again in an adverse selection setting, Gilbert and Riordan (1995) compare the agency costs of procuring two components from two separate firms and

\[3\text{In Section 6 I detail the differences in the analysis were the manufacturer able to test incoming components.}\]
compare them with the costs of procuring both components from a single firm. When two separate firms provide the different components, the authors find that the optimal scheme can be implemented by contracting with only one firm, which sub-contracts with the other firm. Bundling the two components and asking a single firm to produce the bundle is always optimal, because when one firm subcontracts with the other, a double marginalization problem arises. In my model, if the manufacturer outsources all tasks to her supplier, the moral hazard problem between the two parties is aggravated. However, outsourcing allows the manufacturer to sidestep her commitment problem (dual moral hazard), and hence I find that outsourcing is not necessarily always optimal.

In both Riordian and Sappington (1987) and Gilbert and Riordan (1995), outsourcing is used to quell informational asymmetries. On the other hand, both my model and Melumad and Mookherjee (1989) use delegation as a solution to commitment problems. In Melumad and Mookherjee (1989) the government uses strategic audits to induce individuals to truthfully report their incomes. However, after incomes are reported, costly audits are inefficient, and the government would prefer not to pursue the audits. Melumad and Mookherjee show that the government can attain the full-commitment level of welfare by delegating the auditing duties to a third party. Moreover, the authors demonstrate that the government can costlessly contract with the third party in such a way that the welfare-maximizing audit strategy is the third party’s unique equilibrium action. In my model, the manufacturer uses warranties to motivate her supplier to build high quality components. Unlike Melumad and Mookherjee (1989), I allow for moral hazard between the government and the third party, and subsequently find that delegation does not attain the level of welfare possible when the manufacturer can commit to her assembly efforts. Delegation is costly in my model, precisely because the supplier is now charged with an additional unobservable action.

My paper also relates to the task assignment literature, as the manufacturer decides between finishing the system herself and delegating assembly to her supplier. Balachandran and Sridhar (1997) examine a two-step production process, and find that if the manufacturer can handle only the upstream process or the downstream process, then the upstream process should always be outsourced. In my model, the buyer can only perform the final task (assembly), though she may find it optimal to outsource both the initial and final task to the supplier.

In a recent paper about warranty contracts, Balachandran and Radhakrishnan (2005) operationalize what they call the “fairness criterion” by limiting the warranty between a buyer and a seller to the buyer’s incurred damages. In a model where both the buyer and supplier must take an action, the authors find that the first-best outcome can be obtained with the fairness criterion in place as long as the first-best level of testing is sufficiently effective at pinpointing supplier errors. By limiting the warranty itself, the authors do not allow for contractual remedies to the warranty limit. Rather than limiting warranties, I limit the supplier’s payoff in the event of product failure. My model is different, because the manufacturer can pay the supplier rents up-front in exchange for accepting a large warranty which he would otherwise reject.

The economics literature has used similar assumptions to limit payments in principal-agent models. My limited liability restriction is closest to that found in the seminal Sappington (1983) paper. Sappington characterizes the optimal contract between a risk neutral principal and agent when the agent is asymmetrically informed. In my model, both parties
are risk-neutral, although I assume that there are no informational asymmetries between the two parties. A recent paper that examines moral hazard and limited liability is Kadan and Swinkles (2005), which demonstrates the existence of optimal contracts without having to invoke exogenous assumptions on the optimal contract. My model is simpler in the sense that the only contractible outcomes are binary, though my analysis examines both standard and dual moral hazard problems, unlike the previous limited liability literature.

The delegation literature has also covered a broad number of topics involving multiple parties working for a common goal. The traditional models employed in managerial accounting and economics are nicely captured in the survey piece by Mookherjee (2003). The traditional models covered therein typically ask whether a principal can outsource contracting to one of her agents without incurring any economic losses (Melumad et al. (1995)). However in those models, the inefficiency is asymmetric information, and thus the revelation principle insures that the principal is always at least as well off contracting with agents directly as she would be outsourcing all contracting responsibilities to a single agent. Another branch in the literature has identified circumstances in which the revelation principle no longer applies, and then establishes that in these circumstances, outsourcing contracting responsibilities can potentially be the preferred organizational mode (Melumad et al. (1992)). Although my model does not include asymmetric information but instead moral hazard, the control loss associated with outsourcing is identical in both settings. However, the two settings have different costs associated with insourcing. In the adverse selection literature, insourcing is only costly if the revelation principal is violated; whereas in my moral hazard setting, insourcing is costly because of dual moral hazard.

The hold-up literature has also examined trading problems between two parties, each of which perform a task. Lulfesmann (Forthcoming) examines the hold-up problem when the seller faces a liquidity constraint. In a model with observable actions, Lulfesmann demonstrates that option contracts combined with loans are sufficient to obtain first-best levels of investment. My model is more complicated, because I assume that each party’s actions are privately observed and hence the optimal contract solves a moral hazard problem.

Empirical documentation of outsourcing decisions are fairly limited, because insourcing is rarely observed as an “event.” The few studies that do exist tend to rely on industry survey data or individual case studies. Covering the automotive industry, Anderson et al. (2000) find that when parts must adhere to strict tolerances or are complex, then auto manufacturers are more likely to outsource such parts to a supplier. My model predicts that when either components or assembly labor is expensive, manufacturers are more likely to outsource assembly. However, my findings also rely on the supplier’s ability to accept liability. Using homebuilder surveys, Costantino and Pietroforte (2002) find that general contractors cite transferring liability as the most important reason to subcontract work. Even when the supplier is unable to accept high liability contracts, my model predicts that outsourcing is preferred if the manufacturer has large incentives to behave opportunistically. Sako and Helper (1998) survey over 1,000 automotive suppliers and document the determinants of supplier trust. The authors find that suppliers’ distrust of manufacturers increases concomitant with uncertainty regarding the technology used to build the end-system. In my model, when assembly costs are high, neither the manufacturer nor the supplier would assemble the system with high levels of effort, creating additional uncertainty as to whether the system will be successful or not. In these circumstances, I find that given the choice, the manufac-
turer will behave opportunistically and in equilibrium the supplier correctly anticipates the behavior.

Finally, recent research in the Operations literature has also begun to explore delegation as a solution to agency problems. In [Plambeck and Taylor (2005)], the authors ask whether a number of manufacturers should pool their production amongst themselves, or whether they are better off hiring a third party to satisfy all of their manufacturing needs. Similar to my model, the authors assume that several tasks cannot be measured or perfectly defined and hence are noncontractible. Plambeck and Taylor find that manufacturers prefer to outsource production if they believe they have strong bargaining power vis-a-vis the third party manufacturer; otherwise, they prefer to pool their production capabilities. Although my model does not explicitly include a bargaining stage, the supplier’s liability limit can be interpreted as bargaining power; in particular, a supplier with bargaining power will refuse to accept a contract which involves paying heavy penalties to the manufacturer in the absence of other incentives. In my single-manufacturer single-supplier setup, I find that the manufacturer will prefer to delegate assembly to her supplier when the supplier can accept large warranty contracts and his cost of producing high quality components is high.

3 Model

I consider a single-period setting with a risk-neutral manufacturer and supplier. The manufacturer can sell completed systems for $R$ dollars to the external market. I normalize the manufacturer’s desired quantity to 1 without loss of generality. The system itself is produced in a two-stage process. In the first stage, the supplier must produce a component. In the second stage, the component is assembled into a system, either by the manufacturer or the supplier. When the supplier produces a component, I assume that he selects an effort $x \in \{0, 1\}$. Without loss of generality, I assume that the cost to the supplier of selecting $x = 0$ is zero, whereas the cost of selecting $x = 1$ is given by $c_1$. The probability of the end-system failing is directly tied to the effort $x$. In particular, I assume that if $x = 0$, then the system will fail with certainty, regardless of the processing effort. On the other hand, if $x = 1$, the success of the system is dependent on the processing effort in the second stage. In particular, I assume that assembling the component into a system requires effort $y \in [0, 1]$ and the system is successful with probability $x \cdot y$. Note that the system exhibits what the supply-chain literature labels the “weakest link property,” because if the processing or the component is flawed, then the system fails. Because I am chiefly interested in studying delegation, I solve for the optimal contract when the supplier only produces the component ($x$) and the manufacturer assembles the component into a system ($y$), and then compare the contract to the case where the supplier is asked to build both the component and the system ($x$ and $y$). I will refer to the case where the supplier produces only the component as the insourced regime, and will refer to the case where the supplier is asked to produce the component and the system as the outsourced or delegated regime. Throughout, I assume that the supplier and the manufacturer have the same cost function $c(y)$ for assembly effort $y$.

Because the quality of a component is a deterministic function of the supplier’s effort, after selecting effort $x$ the supplier knows whether the component is of high quality or
However, I assume that the supplier cannot credibly communicate this information to the manufacturer and furthermore, the manufacturer cannot test the component nor can she observe the supplier’s efforts. This assumption is meant to capture scenarios where either the component cannot be entirely specified ex-ante, or when the additional processing used to complete the system is coupled with the component. If the component cannot be specified ex-ante, then the manufacturer will not have a standard against which to compare the delivered component. When the processing and the component are coupled, then testing the component alone does not provide any useful information.

Because testing the component is assumed to be impossible any contract between the manufacturer and the supplier must be based on the success or failure of the finished system. I assume that the completed systems can be perfectly and verifiably tested at no cost. Faulty systems are assumed to be repairable, such that the value to the manufacturer of a faulty good is given by $F < R$; in particular, the cost to repairing a faulty unit is given by $R - F$. I assume that $F$ is sufficiently small that the manufacturer would never choose to produce faulty systems ex-ante.

Any contract between the supplier and manufacturer takes the form of a fixed payment $\hat{\alpha} \geq 0$ plus a warranty $\beta \geq 0$ which the supplier pays the manufacturer in the event of a product failure. I restrict the set of feasible contracts between the manufacturer and supplier with a limited liability constraint which requires that the supplier earns at least $L \leq 0$ dollars in every state of the world. The limited liability constraint facing the supplier can thus be written as:

$$\hat{\alpha} - C_s - \beta \geq L$$

where $C_s$ is the supplier’s equilibrium costs. Whereas Balachandran and Radhakrishnan (2005) limit the size of the warranty, condition (1) allows for any size warranty, as long as the manufacturer provides the supplier with a sufficient upfront payment of $\hat{\alpha}$. Paying large upfront payments and requesting money back in the event of product failure or chargebacks, is an increasingly common practice in manufacturing. Similar to Sappington (1983), the supplier’s limited liability constraint can force the manufacturer to have to pay the supplier rents which are absent in the simpler moral hazard setting without limited liability constraints. Given the contracting environment, I can now outline the time-line of actions and payments.

---

4 My results are unchanged if the component’s functionality is stochastically determined by $x$.

5 The Design literature labels a system as coupled, when each piece of the system and the procedure used to assemble the system cannot be evaluated on its own. For examples see Eppinger and Ulrich (2003).

6 In Section 6, I discuss the implications of making the component contractible.

7 In Section 6, I relax this assumption and examine the consequences of costly and imperfect testing.

8 A point of recent tension between Ford and its suppliers is a provision in Ford’s new supplier contracts which allows the manufacturer to withhold payment to the suppliers to cover potential defects (Armstrong 2004).
Again, the party taking action \( y \) (in period 2) can be either the manufacturer or the supplier, depending on the regime being analyzed. Between period 0 and period 1, the manufacturer’s expected payoff can be expressed as:

\[
Ryx + (1 - yx)(\beta + F) - \hat{\alpha} - C_m
\]  

(2)

The first term in (2) is simply the probability of the system functioning and the subsequent payoff to the manufacturer. The second term in (2) is the expected warranty payoff from the supplier plus the expected payoff of a faulty system and \( C_m \) is the manufacturer’s equilibrium costs associated with producing the system. For instance, in the outsourced regime \( C_m \equiv 0 \), whereas in the insourced regime, the manufacturer bears the cost of assembly effort \( c(y) \). To simplify the notation, denote the marginal value of a successful system \( R - F \) by \( \Gamma \), so that the manufacturer’s objective function can be rewritten as:

\[
\Gamma yx + (1 - yx)\beta + F - \hat{\alpha} - C_m.
\]  

(3)

On the other hand, the supplier’s expected payoff can be expressed as the sum of his upfront payment, his expected warranty cost and his equilibrium cost of effort:

\[
\hat{\alpha} - \beta(1 - yx) - C_s.
\]

In both the insourced and outsourced regimes, the supplier is assumed to have an outside option yielding a payoff of \( u \). I assume \( u = 0 \), which allows us to write the supplier’s individual rationality constraint as:

\[
\hat{\alpha} - \beta(1 - yx) - C_s \geq 0.
\]  

(4)

Because the supplier’s outside opportunity \( u \) is at least as large as his liability constraint \( L \), the supplier’s individual rationality constraint is always binding. In the perverse case where the supplier’s liability constraint \( L \) is larger than his outside opportunity, his individual rationality constraint may no longer be binding.\[9\] In the next section, the insourced and outsourced regimes are compared when the assembly effort \( y \) is binary.

**Binary Second Effort**

The simplest setting in which to examine limited liability and moral hazard is one where the secondary effort \( y \) is a binary choice variable. I begin with the binary setting as a pedagogical exercise building up to the more interesting setting where assembly effort is a continuous choice variable. The significance of limiting the second action to be binary is that only a single shirking option exists which in turn limits the dual moral hazard problem facing the manufacturer. I assume that the assembling task can be completed with two distinct levels

\[9\text{For a general solution to the limited liability problem, see Kadan and Swinkles (2005).}\]
of effort: \( y \in \{0, \mu\} \) with \( \mu > 0 \). Without loss of generality, I assume \( c(0) = 0 \) and \( c(\mu) > 0 \), where the cost of assembling the system, \( c(y) \) is borne by whoever takes the action \( y \).

I begin by considering the insourced regime. Supposing that the manufacturer will select assembly effort \( y = \mu \), the supplier will only exert effort \( x = 1 \) to make a high quality component if any of the following equivalent inequalities hold:

\[
\hat{\alpha} - (1 - \mu)\beta - c_1 \geq \hat{\alpha} - \beta \quad (5) \\
\beta \geq \frac{c_1}{\mu}. \quad (6)
\]

Note that \((5)\) ensures that the supplier is willing to exert effort \( x = 1 \) and incur a cost of \( c_1 \), as opposed to shirking and simply providing a defective component. Constraint \((5)\) assumes the manufacturer takes action \( y = \mu \), but from her objective function \((3)\), she only chooses the high level of effort if either of the equivalent inequalities is satisfied:

\[
\Gamma \mu + (1 - \mu)\beta - \hat{\alpha} + F - c(\mu) \geq F + \beta - \hat{\alpha} \quad (7) \\
\frac{\Gamma \mu - c(\mu)}{\mu} \geq \beta. \quad (8)
\]

The inequality \((7)\) ensures that the manufacturer is willing to put forth positive processing effort \( y = \mu \) and shoulder the cost \( c(\mu) \) rather than shirking. From \((8)\), once the warranty payment \( \beta \) becomes sufficiently large, the manufacturer will prefer to shirk \( (y = 0) \) and collect \( \beta \) with certainty rather than working and collecting \( \beta \) with probability \( 1 - \mu \). Comparing the constraints \((6)\) and \((8)\), production is only feasible if \( \frac{\Gamma \mu - c(\mu)}{\mu} \geq \frac{c_1}{\mu} \).

To simplify the ensuing analysis, I will distinguish the portion of the fixed payment used to cover the supplier’s limited liability, which I denote as \( r \), from fixed payment used to cover the supplier’s cost, which I denote by \( \alpha \), such that \( \hat{\alpha} = \alpha + r \). In this setting, if production is to take place \( (y = \mu \) and \( x = 1 \)) the manufacturer solves:

\[
\max_{\alpha,\beta,r} -\alpha - r + \Gamma \mu + F + \beta(1 - \mu) - c(\mu)
\]

such that

\[
\frac{\Gamma \mu - c(\mu)}{\mu} \geq \beta \geq \frac{c_1}{\mu} \quad (9) \\
\alpha + r - \beta(1 - \mu) - c_1 \geq 0 \quad (10) \\
\alpha + r - \beta - c_1 \geq L \quad (11)
\]

Constraint \((9)\) embodies the manufacturer and supplier’s incentive compatibility constraints, whereas \((10)\) is the supplier’s individual rationality constraint and \((11)\) the supplier’s liability constraint. The following proposition illustrates the optimal contract in this setting.

**Proposition 1** In the insourced regime with \( y \in \{0, \mu\} \), the optimal contract calls for the smallest warranty satisfying the supplier’s incentive compatibility constraint \((9)\). Moreover, \( \alpha = c_1 + \beta(1 - \mu) \) and when positive, the rent term is given by \( r = c_1 + L \).
The proposition demonstrates that the optimal contract minimizes the warranty while still satisfying the supplier’s incentive compatibility constraint (6). To simplify notation, let \((a)^+ \) denote \(\max\{0,a\}\). The supplier’s rents can thus be written as \(r = (c_1 + L)^+\) and are positive whenever \(c_1\), the supplier’s cost of building a high quality component, is larger than his liability limit \(-L\). The rent is increasing in the supplier’s cost \(c_1\), because for large values of \(c_1\), the supplier must be given large incentives to build a high quality good. Since the manufacturer can use only the warranty \(\beta\) to motivate the supplier to work hard, and the supplier’s limited liability constraint mandates that large warranties \(\beta\) be offset by large upfront payments \(r\), whenever the supplier’s cost \(c_1\) is large, he earns economic rents. Clearly, the manufacturer’s incentive compatibility constraint (8) also limits the warranty \(r\), because for a sufficiently large warranty \(\beta\), the manufacturer prefers to shirk on assembling the system and collect the large warranty payment \(\beta\) with certainty. As such, when \(\Gamma_{\mu - c(\mu)} \) is less than \(\frac{c_1}{\mu}\), the largest warranty satisfying the manufacturer’s constraint, is less than \(\frac{c_1}{\mu}\), the smallest warranty satisfying the supplier’s incentive compatibility constraint, the insourced regime is infeasible.

One may question whether allowing for renegotiation would permit the insourced regime to always be feasible. This is in fact the case. However, the only equilibrium in the insourced regime with renegotiation is having both the supplier and manufacturer put forth zero effort. To see this, suppose that after the supplier delivers the component, the manufacturer believes the supplier took the correct actions and proposes to cancel the warranty in exchange for a fixed payment. Being risk neutral, the supplier will pay the manufacturer his expected warranty costs in exchange for abolishing the warranty. Without a warranty, the manufacturer now has no incentives to shirk, and will carry out the first-best level of assembly effort. However, the supplier will never take the correct action if he anticipates the renegotiation, because unlike in the renegotiation literature, the supplier’s actions are unobservable, and thus the manufacturer cannot be sure that the supplier took the correct action. Moreover, there is no way for the manufacturer to make renegotiation unattractive, i.e., there are no mixed strategy equilibria whereby the manufacturer proposes a renegotiation only a fraction of the time. As such, given the opportunity, the manufacturer will always try to renegotiate the contract in the insourced regime, and foreseeing this, the supplier will never build a high quality component if renegotiation is permitted. Thus, in order for the insourced regime to be feasible, any contract between the manufacturer and the supplier must specify that renegotiation is prohibited.\(^{10}\)

In the outsourced regime, the manufacturer no longer assembles the system; instead, all actions are outsourced to the supplier. When the supplier completes the system his actions are assumed to be non-observable to the manufacturer. Thus, the supplier would never shirk on one task but not the other. In particular, note that if the supplier shirks on the component by choosing \(x = 0\), then his expected payoff is no longer a function of his processing effort \(y\), hence he will set \(y = 0\) to avoid the disutility \(c(y)\). On the other hand, if the supplier builds a high quality component \((x = 1)\), then the system will be successful with positive probability only if he selects positive assembly effort \(y = \mu\). Therefore, the supplier must choose between shirking on both tasks, or putting positive effort into both tasks. The supplier prefers to work on both tasks when either of the equivalent inequalities hold:

\[^{10}\text{This is true of all agency models with dual moral-hazard.}\]
\[ \alpha + r - \beta (1 - \mu) - c_1 - c(\mu) \geq \alpha + r - \beta \]  
(12)

\[ \beta \geq \frac{c_1 + c(\mu)}{\mu}. \]  
(13)

The left hand side of (12) is the supplier’s expected payoff if he puts forth effort \( y = \mu \) and \( x = 1 \). The right hand side of (12) is the supplier’s expected payoff when he shirks on both the component (\( x \)) and the processing of the component into a system (\( y \)). Because the supplier is now asked to put forth two types of effort, the supplier’s incentive compatibility constraint requires a larger warranty than that required in the insourced setting. If the manufacturer wishes for the supplier to put forth positive effort on both tasks, the optimal contract will set \( \beta \) as small as possible while still adhering to (13). To see this, recall that the supplier earns positive rents \( r = (\beta \mu + L)^+ \) when the system is successful, and his rents are increasing in the warranty \( \beta \). Because the supplier’s IC constraint (13) mandates a warranty at least as large as \( \frac{c_1 + c(\mu)}{\mu} \), the optimal contract sets \( \beta = \frac{c_1 + c(\mu)}{\mu} \), and pays the smallest possible rents \( r = (c_1 + c(\mu) + L)^+ \). Note that there is no scope for renegotiation in the outsourced regime, because only one party (the supplier) carries out all of the actions. With the outcomes of each regime specified, the following proposition compares the manufacturer’s payoffs in each regime.

**Proposition 2** When both regimes are feasible, the manufacturer always prefers the insourced regime.

Because the product is successful with the same probability in both settings, whenever both setting are feasible, the insourced regime is optimal because the rents paid are less than in the delegated regime. Recall that the rents are greater under the delegated regime because the warranty \( \beta \) must provide sufficient incentives for the supplier to produce a high quality component and process the component into a system, thus necessitating a larger rent payment \( r \). However, the insourced regime is not always feasible. If the marginal payoff to a successful system, \( \Gamma \) times the probability of success is less than the total cost of building a high quality system \( (\Gamma \mu \leq c(\mu) + c_1) \), then the manufacturer cannot commit to assembling the system, and must delegate construction of the system to the supplier.

When the processing effort variable \( y \) is binary, the optimal level of assembly effort is a “corner solution” which is invariant to changes in the warranty \( \beta \). As such, the cost of the dual moral hazard problem is that the insourced regime is sometimes infeasible. However, when \( y \) is a continuous choice variable, the manufacturer can respond to large warranties by shirking a little. Hence, when the processing effort \( y \) is a continuous choice variable, the dual moral hazard problem can be costly even when the insourced regime is feasible. I analyze this setting in detail in the following section.

4 Continuous Second-Effort

In this section, I remove the binary restriction on the processing effort variable \( y \). One would expect the processing effort to be continuous when the second task can be completed with
varying levels of accuracy, whereas the binary setting is better suited to circumstances where the processing task can either be completed correctly, or not. Because the processing effort \( y \) is also the probability of the system succeeding when the component is of high quality, I restrict \( y \) to belong to the interval \([0, 1]\) where the cost of effort \( c(y) \) is again borne by whoever assembles the system. For reasons of tractability, I set \( c(y) = \frac{1}{2} \gamma y \), yet I expect the general results to hold for a much broader class of convex cost functions. Before examining the contracts between the manufacturer and the supplier, I first solve for the first-best benchmark where I assume the manufacturer selects both the component fabrication effort \( x \) and the processing effort \( y \). In the first-best setting the manufacturer solves:

\[
\max_{x, y} \Gamma y x - c(y) - x \cdot c_1 + F.
\]

When the payoff \( \Gamma \) is sufficiently large to merit production, the first-best solution calls for \( x = 1 \) and \( y = \Gamma / \gamma \). In order for the solution to be valid, I must have \( y \leq 1 \), which imposes the restriction \( \Gamma \leq \gamma \). In order for positive production to be profitable, I must additionally assume that \( \frac{\Gamma^2}{2\gamma} \geq c_1 \). I will assume that these two inequalities hold for the remainder of the section.

I again assume that the supplier is unable to accept a contract that leaves him with a payoff less than \( L \) in any state of the world. As in the previous section, I begin by analyzing the insourced setting. The time-line with a continuous assembly effort \( y \) is identical to the case of binary effort \( y \) of the previous section. Once the manufacturer and supplier have agreed on a contract with parameters \( \alpha, \beta \) and \( r \) the manufacturer maximizes the following expression in selecting the optimal level of assembly effort \( y \):

\[
\max_y \Gamma y + \beta (1 - y) - \alpha - r - c(y) + F. \tag{14}
\]

To simplify notation, I will assign the subscript \( m \) to the manufacturer’s optimal processing effort \( y \) in the insourced regime. Solving (14), the manufacturer optimally selects effort \( y_m(\beta) = \frac{1}{\gamma} (\Gamma - \beta) \). Clearly, the effort \( y_m \) is decreasing in the warranty \( \beta \) and increasing in the payoff \( \Gamma \). The optimal level of processing effort is decreasing in the warranty, because the larger \( \beta \) is, the higher the manufacturer’s payoff to the system failing. A similar trade-off is present in Baiman et al. (2000), where the buyer must exert costly effort to inspect incoming deliveries from her suppliers. In both models, the greater the payoff in the event of failure, the more the buyer’s actions seek to identify or cause failures. In a sub-game perfect equilibrium, the supplier will correctly anticipate the manufacturer’s optimal effort level \( y_m \). Correctly anticipating \( y_m \), if the supplier makes a high quality component \((x = 1)\), he expects to earn:

\[
\alpha + r - \beta (1 - y_m(\beta)) - c_1.
\]

On the other hand, if the supplier shirks and selects \( x = 0 \), then the system will fail with certainty and the supplier will have to pay \( \beta \) with certainty, yielding an expected payoff of \( \alpha + r - \beta \). Comparing the two payoffs, the supplier’s incentive compatibility (IC) constraint can be written as:
\[ \alpha + r - \beta (1 - y_m) - c_1 \geq \alpha + r - \beta. \]  \tag{15} \]

The supplier’s individual rationality and liability constraints are identical to those of the previous section. In particular, the supplier will only accept a contract that pays an upfront payment \( \alpha \) at least as large as the supplier’s anticipated cost \( c_1 \), and ensures that his utility never drops below \( L \). Combining the supplier and manufacturer’s constraints, the manufacturer’s contracting problem can be expressed as:

\[
\max_{\alpha, \beta} \Gamma(y_m(\beta)) + \beta (1 - y_m(\beta)) + F - \alpha - c(y_m(\beta))
\]

such that

\[
y_m(\beta) = \frac{1}{\gamma} (\Gamma - \beta) \tag{16}
\]

\[ \alpha + r - \beta (1 - y_m(\beta)) - c_1 \geq \alpha + r - \beta \tag{17} \]

\[ \alpha + r - \beta (1 - y_m(\beta)) - c_1 \geq 0 \tag{18} \]

\[ \alpha + r - \beta - c_1 \geq L. \tag{19} \]

The first constraint is simply the manufacturer’s incentive compatibility constraint, whereas the second constraint is the supplier’s incentive compatibility constraint. The third inequality above is the supplier’s individual rationality constraint followed by the supplier’s limited liability constraint. The following proposition characterizes the solution to the program.

**Proposition 3** The optimal contract between the manufacturer and the supplier calls for \( \beta^* \), the smallest warranty satisfying the supplier’s IC constraint, where:

\[ \beta^* = \frac{\Gamma}{2} - \frac{\sqrt{\Gamma^2 - 4 \gamma c_1}}{2}. \]

In equilibrium, the manufacturer selects processing effort:

\[ y_m = \frac{1}{2 \gamma} \left( \Gamma + \sqrt{\Gamma^2 - 4 \gamma c_1} \right). \]

The supplier earns rents if, and only if, \( c_1 + L > 0 \).

Proposition 3 mirrors Proposition 1 as the manufacturer again wishes to set the warranty \( \beta \) as small as possible while continuing to satisfy the supplier’s incentive compatibility constraint (15). Because the equilibrium effort \( y_m(\beta) \) is now a function of \( \beta \), in selecting \( \beta \), the manufacturer is also committing to her processing effort \( y_m \). Note that the manufacturer will only put forth the first-best level of effort if the supplier’s cost of making a high quality component \( c_1 \), is zero. Because \( y_m(\beta) \) is decreasing in \( \beta \) and the supplier’s rents are decreasing in \( \beta \), the manufacturer clearly wishes to minimize the warranty payment. Setting \( \beta > \beta^* \) does not generate any benefits to the manufacturer, since the effort used to build the component, \( x \), is binary. Thus, raising \( \beta \) beyond \( \beta^* \) does not change the supplier’s actions.
and involves paying additional rents. The supplier’s rent is again (weakly) increasing in $c_1$, because the larger the supplier’s cost of making a high quality component $c_1$, the larger the smallest warranty $\beta$ satisfying the supplier’s incentive compatibility constraint. Because the supplier’s rents are given by $(\beta \cdot y_m + L)^+ = (c_1 + L)^+$ when the system is successful, if the supplier’s cost $c_1$ increases and his maximum liability $L$ remains unchanged, his rents will (weakly) increase.

Having solved for the optimal contracts with continuous processing effort $y$ in the insourced regime, I now turn to analyzing the outsourced regime. Given a contract triplet $\{\alpha, \beta, r\}$ and the supplier’s equilibrium choice of component effort $x$, in choosing his level of assembly effort $y$, the supplier solves:

$$
\max_y \alpha + r - \beta (1 - y \cdot x) - c(y_s).
$$

(20)

To simplify notation, label the supplier’s equilibrium choice of processing effort $y_s$. In solving (20), the supplier will put forth effort $y_s(\beta) = \frac{\beta}{\gamma} \cdot x$. Again, the equilibrium processing effort $y$ is a function of the warranty $\beta$. Recall that in the insourced setting the processing effort $y_m$ was a decreasing function of the warranty, whereas now, $y_s$ is increasing in the warranty $\beta$. The supplier increases processing effort $y_s$ when the warranty increases, because increasing the processing effort decreases the likelihood of the system failing and the supplier having to pay the subsequent warranty $\beta$. Given the second stage action $y_s(\beta)$, I can now solve for the first period action choice $x$. The supplier will elect to make the component of high quality ($x = 1$) as long as any of the equivalent inequalities hold:

$$\alpha + r - \beta (1 - y_s(\beta)) - c_1 - c(y_s(\beta)) \geq \alpha + r - \beta$$

(21)

$$\beta \geq \sqrt{2\gamma c_1}.$$  

(22)

The left hand side of (21) is the supplier’s expected payoff when he builds a high quality component, whereas the right hand side is the payoff when the supplier shirks, and makes the component of low quality. Similar to the binary case; in the outsourced regime the supplier will either build a low quality component and spend no effort assembling it, or build a high quality component and spend positive effort assembling it. The supplier never shirks on one task and not the other, that is $x > 0 \iff y_s > 0$. Because the supplier now builds the component and assembles it into a system, his individual rationality constraint is given by:

$$\alpha - c(y_s(\beta)) - \beta (1 - y_s(\beta) \cdot x) - c_1 \cdot x \geq 0.$$  

As in the previous section, the supplier must earn at least $L$ dollars in every state of the world. Combining the three inequalities, the manufacturer’s optimization program becomes:

$$\max_{\{\alpha, \beta, r\}} \Gamma \frac{\beta}{\gamma} + \beta \left(1 - \frac{\beta}{\gamma}\right) + F - \alpha - r$$

such that:
\[\alpha - \beta \left(1 - \frac{\beta}{\gamma}\right) - \frac{\beta^2}{2\gamma} - c_1 \geq 0 \quad (23)\]
\[\beta \geq \sqrt{2\gamma c_1} \quad (24)\]
\[\alpha + r - \beta - c_1 \geq L \quad (25)\]

Constraint (23) is the supplier’s individual rationality constraint with the equilibrium efforts \(y_s\) and \(x\), whereas (24) is the incentive compatibility constraint, and (25) the supplier’s limited liability constraint. The manufacturer clearly wishes to minimize upfront payments \(\alpha\) and \(r\). Unlike the insourced regime, the manufacturer now has an additional incentive to request a large warranty \(\beta\), as the larger the warranty, the more effort \(y_s\) the supplier will apply to processing the component. The next proposition characterizes the optimal contract in relation to the costs \(c_1\) and \(\gamma\), the liability limit \(L\), and the manufacturer’s expected payoff from a successful system \(\Gamma\).

**Proposition 4** When production is profitable, the optimal warranty is given by:

\[
\beta = \begin{cases} 
\Gamma & r^2 < -L \\
\max\{\sqrt{-\gamma L}, \sqrt{2\gamma c_1}\} & r^2 \leq -L \\
\max\{\Gamma/3, \sqrt{2\gamma c_1}\} & -L \leq r^2 \leq \frac{r^2}{\gamma}\.
\end{cases}
\]

The optimal fixed payment is \(\alpha = c_1 \cdot x + c(y_s) + \beta(1 - y_s \cdot x)\) and the supplier earns positive rents if and only if \(2c_1 + L \geq 0\) or \(L > -\frac{r^2}{\gamma}\).

The proposition demonstrates that in choosing the warranty \(\beta\), the manufacturer balances the limited liability constraint (25) with the incentive compatibility constraint (24). The incentive compatibility constraint requires that the manufacturer set the warranty \(\beta\) at least as large as \(\sqrt{2\gamma c_1}\), whereas the supplier’s limited liability constraint curbs the manufacturer’s incentive to offer large warranties. When the limited liability constraint and the incentive compatibility constraint are both non-binding, the manufacturer elicits first-best effort levels and payoffs by setting the warranty \(\beta = \Gamma\). For intermediate values of \(L\), the supplier’s limited liability constraint begins to bind, and the manufacturer optimally decreases the warranty from \(\Gamma\) to \(\Gamma/3\), using the largest possible warranty without having to pay the supplier rents. Decreasing the warranty elicits less effort \(y_s\) from the supplier. However, any warranty must exceed \(\sqrt{2\gamma c_1}\) to satisfy the supplier’s incentive compatibility constraint, otherwise the supplier will produce low quality inputs. Thus, while the limited liability constraint drives the optimal warranty down for larger values of \(L\), the manufacturer will never allow the warranty to fall below \(\sqrt{2\gamma c_1}\) (see figure 1). For large values of \(L\) (i.e. \(L > -\frac{r^2}{\gamma}\)), the manufacturer stops avoiding rents in favor of a fixed warranty \(\Gamma/3\). The threshold for paying rents is \(\beta = \Gamma/3\), because when the liability constraint is binding, the marginal cost of raising \(\beta\) is an increase in rents of \(2\beta/\gamma\), whereas the marginal benefit to the supplier is given by \(\Gamma/\gamma - \beta/\gamma\), the benefit of a successful module times the increased probability of success minus the marginal cost of the supplier’s effort.

Having identified the optimal contracts for each regime, I can now compare the payoffs to the manufacturer between the two regimes. For the remainder of this section, let the subscript
The optimal warranty is the maximum of the bold and dotted curves. When the dotted curve is above the boldface curve, the supplier’s incentive compatibility constraint is binding, and the supplier earns rents. If the dotted curve is beneath the boldface curve, then the supplier earns rents only for sufficiently large values of $L$ as indicated beneath the $L$-axis.

$o$ denote the outsourced setting, and $i$ denote the insourced setting. From Proposition 4, the manufacturer’s expected profits in the outsourced regime can be written as:

$$\pi_o = \frac{\beta}{\gamma} \left( \frac{\Gamma - \beta}{2} \right) - (\beta + L)^+ + F - c_1.$$  

On the other hand, using Proposition 3, the manufacturer’s profits in the insourced regime become:

$$\pi_i = \frac{1}{2\gamma} \left( \Gamma^2 - \beta^*^2 \right) - (\beta^* + L)^+ + F - c_1. \quad (26)$$

In both regimes, the manufacturer must pay the supplier rents to overcome the limited liability constraint, and both regimes must include a minimum warranty $\beta$ to motivate the supplier to make his component of high quality. Where the two regimes differ is in the effect of the warranty upon the assembly effort $y$. In the insourced regime, a large warranty, as necessitated by the supplier’s IC constraint, will induce the manufacturer to put forth low levels of effort $y_m$. On the other hand, in the outsourced regime, a large warranty $\beta$ will induce the supplier to put forth a high level of effort assembling the system, since his equilibrium processing effort $y_s$ is increasing in the warranty $\beta$. As such, one would believe that when the supplier’s cost of making a high quality component $c_1$ is sufficiently large, necessitating a large warranty $\beta$, then outsourcing is optimal. This intuition is correct whenever the supplier’s limited liability constraint is relatively lax. When the supplier’s minimum payoff level is relatively large, the manufacturer prefers to assemble the system herself (insource). The restriction on the payoff limit $L$ is necessary, because when the manufacturer outsources assembly, she employs a larger warranty $\beta$ than in the insourced regime, and thus may have to pay more rents if $L$ is large. If the supplier’s minimum utility level $L$ is sufficiently large, any gains earned from delegation are erased by the additional
rents the manufacturer must pay for the supplier to accept the high-warranty, outsourced contract. The following proposition characterizes the optimality of each regime.

**Proposition 5** Suppose both regimes are feasible \((c_1 \leq \frac{\Gamma^2}{4\gamma})\) and both regimes induce positive levels of processing effort \(y\). Then, outsourcing is optimal when the supplier’s cost of building components \(c_1\) is sufficiently large and his minimum allowable utility \(L\) is sufficiently small.

The proposition predicts that outsourcing is optimal whenever building a high quality component is costly and the supplier’s payoff minimum \(L\) is relatively small. The proof is non-trivial, because typically the outsourced regime produces a higher quality system and pays more rents than the insourced regime. When the cost of producing a high quality component is large, the manufacturer prefers to outsource assembly of the system, because when \(c_1\) is large, the dual moral hazard problem becomes exceedingly costly. However from Proposition 4, the rents paid to the supplier in the outsourced regime are also increasing in \(c_1\) and grow faster than the rents in the insourced regime. The results in Proposition 5 are limited to the case where the insourced regime is feasible \((c_1 \leq \frac{\Gamma^2}{4\gamma})\). Similar to the binary setting, once the supplier’s cost of building a high quality component \(c_1\) exceeds a threshold, the manufacturer can no longer commit to assembling the system and hence the outsourced regime is the only option available to the manufacturer to produce the system. The binary and continuous settings are distinct however, because in the latter, the manufacturer has room to shirk before she loses her ability to commit to assembling the system. In the binary setting, the manufacturer chooses between assembling the system properly or not at all; without any opportunities to shirk, she can only commit to assembling the system properly under a limited set of parameters.

Outsourcing is only optimal in this section because of the dual moral hazard problem. If the manufacturer could make his actions contractible, then the dual moral hazard problem disappears and the manufacturer can always feasibly assemble the systems and optimally elicit the first-best effort levels.

**Corollary 1** If the manufacturer’s action \(y\) is contractible, then the insourced regime is always feasible and preferred by the manufacturer.

If the manufacturer’s actions are contractible, then she can commit to any level of assembly effort \(y\). The warranty \(\beta\) no longer influences the manufacturer’s choice of \(y\), allowing her to set \(\beta\) as large as necessary to motivate the supplier in the insourced regime. Of course, the supplier’s limited liability constraint continues to mandate that the supplier be paid rents for large values of \(L\) and \(c_1\), however these rents are always less than those of the outsourced regime. Hence, when the manufacturer’s actions are contractible, she always prefers the insourced regime.

One would anticipate the manufacturer’s effort \(y\) to be contractible in cases where an information linkage exists between the supplier and the manufacturer. Another case where the action may be contractible is when the processing effort is not an action, but an investment. If \(y\) is an investment, then \(y\) being contractible would imply that the accounting

---

11I have assumed that the cost of processing effort is given by \(c(y)\); if I treat \(y\) as an investment, then \(c(y)\) would denote the cost of capital to whoever makes the expenditure \(y\).
information for the investment can be credibly shared between the manufacturer and the supplier. Several other papers have also identified the benefits of making the manufacturer’s (or buyer’s) effort verifiable, and demonstrated that the first-best can frequently be obtained under such circumstances (Baiman et al. (2000), Balachandran and Radhakrishnan (2005)). In the next section, I analyze the manufacturer’s testing decision.

5 Testing

Thus far, it was assumed the manufacturer could perfectly test the finished system at no cost. Yet testing limitations have a real impact on manufacturers’ decisions. Following the literature on quality testing (Baiman et al. (2000), Baiman et al. (2001)), in this section I assume that the manufacturer has a technology which correctly identifies a functional system, but with probability δ mislabels a defective system as being functional. As before, the manufacturer values a system found to be faulty via testing (internal failures), at F dollars. On the other hand, I assume that the manufacturer values defective units which pass testing and are subsequently sold to consumers (external failures) at E dollars, with F > E. The benefit E is meant to capture both the cost of repairing the defective system (R – F) plus any reputational damage suffered by the manufacturer (E can be less than zero). Studies have shown that intangible costs such as loss of reputation and goodwill can be several magnitudes larger than the tangible costs involved with a product failure (Nagar and Rajan (2001)).

When testing is imperfect, a faulty system is accidently sold to the public with probability δ, in which case an external failure occurs with certainty. Contracting on external failures, however, may be infeasible (Joseph (1999)), in which case the manufacturer can no longer penalize the supplier for system failures that go undetected by the manufacturer’s testing process. The following proposition compares the optimal warranty when external failures are contractible to when they are not.

Proposition 6 If external failures are non-contractible, then the manufacturer increases the warranties in both regimes.

When external failures are non-contractible, the manufacturer must impose higher warranties than otherwise, because now the supplier can shirk and not have to pay the warranty with positive probability, which makes the supplier’s IC constraint bind more often. Recall that increasing warranties has a different effect in the outsourced and insourced regimes. In the insourced regime, a larger warranty exacerbates the dual moral hazard problem since the manufacturer’s ability to commit to working hard decreases. However in the outsourced setting, the larger warranty motivates the supplier to increase his assembly effort. In both regimes, the larger warranty requires that larger rents be paid to the supplier, yet the following corollary demonstrates that the larger warranties have a distinct effect on the manufacturer’s outsourcing preferences.

12 Citing quality concerns, for several years Toyota refused to follow the rest of the automotive industry in outsourcing systems or modules instead of simple components (Treece (1998)).

13 For instance, producers of manufacturing robots do not guarantee that their robots will coordinate with the buyer’s existing assembly lines. (Fine and Whitney (1999)).
Corollary 2 When external failures are non-contractible, the manufacturer prefers outsourcing assembly to the supplier more frequently than if external failures were contractible.

The proof of Proposition 6 shows that when external failures are non-contractible, the manufacturer faces the identical decision problem as if external failures were contractible and the supplier’s cost of building a high quality component was given by \( \frac{c_1}{1-\delta} > c_1 \). From Propositions 3 and 4, outsourcing is preferred precisely when the supplier’s cost is high and can accept contracts with large warranties. Because all the results are robust to external failures being contractible or not, assume for the remainder of this section that external failures are contractible.

Although the model has thus far taken the level of testing \( \delta \) to be exogenous, manufacturer’s frequently have a choice of testing intensity\(^{14}\). To solve for the manufacturer’s testing preferences, continue to assume that the probability of a type II error is given by \( \delta \) and additionally, let \( t(\delta) \) denote the manufacturer’s cost of testing. With costly testing, the manufacturer’s objective function becomes:

\[
R(y \cdot x) + (\delta E + (1-\delta)F)(1-y \cdot x) - t(\delta) - \alpha - r - \beta(1-y \cdot x).
\]

Because defective systems are assumed to be costlier to the manufacturer when they fail “in the field” \( F > E \), reducing the number of type II errors \( \delta \) decreases the manufacturer’s expected loss given a faulty system. The manufacturer thus treats testing quality \( (1-\delta) \) and system quality \( (x \text{ and } y) \) as substitutes.

The losses associated with faulty systems are identical across the outsourced and insourced regimes. Hence, if each regime produces a system with the same level of quality, the manufacturer would test the finished system with the same level of effort across the two regimes. In the binary setting of Section 3, the optimal level of induced assembly effort \( y \) and component quality \( x \) were the same in both regimes; thus, the manufacturer would choose the same intensity of testing \( 1-\delta \) in each regime. On the other hand, when assembly effort is a continuous choice variable, as in Section 4, the equilibrium assembly efforts diverge as does the manufacturer’s choice of testing intensity. The following proposition shows that the manufacturer will always test more rigorously in the outsourced regime.

**Proposition 7** Suppose there exists a unique, interior optimal level of testing in both the insourced and outsourced regime. Then the manufacturer will conduct at least as much testing in the outsourced regime as in the insourced regime.

Improving the testing procedures in both regimes helps the manufacturer avoid costly external failures by repairing a greater portion of the faulty systems before they are sold to the public. Because testing and system quality are substitutes, when the manufacturer ameliorates her tests, she becomes more willing to accept low quality systems. In the outsourced regime, the manufacturer will lower the warranty facing the supplier when testing is improved, which in turn lowers the expected quality of the systems. In the insourced regime, improving system tests exacerbates the dual moral hazard problem: the manufacturer can

\(^{14}\)For example, auto manufacturers select acceptable tolerances for their motors and transmissions. The smaller the acceptable tolerance, the more rigorous the testing.
commit to even less effort than before which mandates that she increase the supplier’s warranty. Yet with a larger warranty, the manufacturer’s commitment strength drops even further, and the cycle continues. The indirect effect of testing on the manufacturer’s commitment strength is why the manufacturer always tests at least as much in the outsourced regime as she does in the insourced regime.

Throughout the paper, component testing has been ruled impossible. The supply chain literature has labelled systems as “coupled” or “non-separable” when a component’s quality cannot be tested without testing the entire system (Baiman et al. (2001) and Eppinger and Ulrich (2003)). On the other hand, when a system is “decoupled” or exhibits “separable architecture” then a manufacturer may be able to test components before they are assembled into a system. Allowing for component testing in the present model can resolve some of the costs associated with the insourced regime. In the outsourced regime, nothing is gained when the component can be tested, as the supplier already knows whether the component is of high quality or not based on his production effort $x$. On the other hand, when the manufacturer assembles the system, she does not know what efforts were taken by the supplier, and hence the test is beneficial.

To illustrate the impact of component testing, suppose that the manufacturer can test components in such a way that all functional components pass the test, yet with probability $\tau$, a defective component also passes the test, where $\tau \in [0, 1]$. Furthermore, assuming the component test is contractible, let $\beta_c$ denote the penalty the manufacturer imposes on the supplier when a faulty component is identified. I continue to assume the finished system can be tested, and that the manufacturer charges the supplier $\beta$ dollars once a faulty system is identified. In order for the supplier to build a functional component in the insourced regime, any of the following inequalities must hold:

$$- (1 - y_m) \beta - c_1 + \alpha + r \geq -\tau \beta - (1 - \tau) \beta_c + \alpha + r$$

$$-(1 - \tau - y_m) \beta + (1 - \tau) \beta_c \geq c_1.$$ 

The expression on the left hand side of (27) is the supplier’s expected utility when he builds a high quality component, whereas the expression on the right hand side depicts his expected utility should he build a low quality component. The first expression on the right hand side of (27) is the penalty for a failed system times the probability of a low quality component passing the manufacturer’s test ($\tau$) and then the system failing (which happens with certainty). The second expression on the right hand side of (27) is the probability of a low quality component failing the manufacturer’s test times the penalty for such a failure. Because the manufacturer can now spread the warranties across two tests, she can satisfy the supplier’s IC constraint (28) with a pair of warranties, each smaller than the warranty used when component testing was unavailable. Since the supplier will never have to pay both $\beta$ and $\beta_c$, the smaller warranties allow the manufacturer to pay less rents. To see this, note that the supplier’s limited liability constraint (19) now becomes $\min\{r - \beta y_m, r - \beta_c y_m\} \geq L$. Given a choice, the manufacturer always prefers a small component warranty $\beta_c$ over a small system warranty $\beta$, because the component warranty has no effect on the manufacturer’s assembly incentives whereas Section 3 and 4 showed that a system warranty does. Hence, if $\beta_c > 0$ and $\beta = 0$ satisfy the supplier’s IC constraint (28) and his limited liability constraint without
paying rents, then the manufacturer can attain the first-best outcome and payoff. However, it may be impossible to satisfy the supplier’s IC constraint using only a component warranty $\beta_c$ and without paying rents. In particular, if the smallest component warranty satisfying the supplier’s IC constraint $\beta_c = \frac{c_1}{1 - \tau}$ is sufficiently large, i.e., $\beta_c y_m = \frac{c_1}{1 - \tau} > -L$, then the manufacturer must pay the supplier rents if she wishes to use only a component warranty. In these cases, the manufacturer decides between paying the supplier rents, or imposing an additional warranty $\beta$ penalizing the supplier when the system fails. If the manufacturer employs a system warranty $\beta$, then she again faces the commitment issues outlined in Sections 3 and 4. The following proposition shows that the manufacturer optimally imposes a system warranty $\beta > 0$ when she cannot costlessly satisfy the supplier’s IC constraint using only a component warranty and component testing is inefficient.

**Proposition 8** Suppose the manufacturer and supplier can contract on the outcome of both component and system warranties. If the manufacturer cannot attain the first-best payoff using only a component warranty $\beta_c$ and $\Gamma > 2(1 - \tau)$, then she will use a positive system warranty $\beta$.

The proposition demonstrates that allowing for component testing does not eliminate the dual moral hazard problem, that is, the manufacturer will still employ a warranty $\beta > 0$ when component testing is sufficiently ineffective. If component testing is effective ($\Gamma < 2(1 - \tau)$), then imposing a positive system warranty $\beta > 0$ is counterproductive, as it decreases the supplier’s incentive to build a high quality input. To see this, note that if $\Gamma < 2(1 - \tau)$, the supplier’s expected warranty costs are greater when he builds a high quality component than when he builds a low quality component, because the probability of a system failing with a high quality component $(1 - y_m \leq 1 - \Gamma)$ is greater than the probability of a low quality component passing testing ($\tau$) and being produced into a unsuccessful system (probability 1). Hence, when component testing is effective, the manufacturer will abandon system warranties, but if component testing is ineffective, the tradeoffs considered in Sections 3 and 4 continue to hold.

Component testing also allows the manufacturer to always assemble the system feasibly herself. Recall that in both the binary setting of Section 3 and the continuous setting of Section 4, when the supplier’s cost of building a high quality component was sufficiently large relative to the manufacturer’s value for a functional system, the insourced regime was infeasible. In these circumstances, the smallest warranty which motivated the supplier to build a high quality component was too large for the manufacturer to credibly assemble the system. From the supplier’s IC constraint with component testing (28), the manufacturer can always impose a large component warranty $\beta_c$ to motivate the supplier to build a high quality component. Although insourcing is now always feasible, it remains non-optimal whenever the supplier’s cost $c_1$ is sufficiently large, and his minimum acceptable utility $L$ is sufficiently negative for the same reasons outlined in the previous sections.

Finally, in the presence of component testing, the manufacturer may be better off charging the supplier for component failures, and compensating him for system failures. A negative system warranty, whereby the manufacturer pays the supplier in the event of a system failure, allows the manufacturer to commit to high levels of assembly effort and sidestep the commitment issues that arise with positive warranties. This opportunity is only feasible
when component testing is effective at spotting low quality components (small $\tau$). To see this, note that for small levels of $\tau$ the term in brackets on the left hand side of the supplier’s IC constraint (28) will be positive, and hence the supplier will accept to build a high quality component in exchange for the remote possibility that the manufacture assembles a faulty system, earning the supplier a payment of $-\beta$ dollars. If component testing is either ineffective (large $\tau$) or infeasible ($\tau = 1$), then the supplier will prefer to build a low quality component, because with probability $\tau$ the low quality component will go unnoticed by the manufacturer who will then assemble it into a system which fails with certainty, earning the supplier $-\beta$ dollars again. Because negative warranties are rarely observed, and may be difficult to uphold in a court of law, I have assumed throughout that any transfer from the supplier to the manufacturer which occurs after a failure (either component or system) must be positive.

6 Conclusion

In this paper, I have analyzed a supplier’s decision to outsource assembly of a system to a supplier who already provides the manufacturer with a component. My analysis assumed that the supplier can only accept contracts that leave him with at least a minimum level of utility in each state of the world. The liability limit restricted the types of warranties the manufacturer could pen with the supplier, and necessitated paying the supplier rents when his minimum acceptable utility level was high. I showed that when assembling the system was a binary process, given the choice, the manufacturer would never delegate assembly to the supplier. However, the manufacturer is not always able to assemble the system herself, because given a sufficiently large warranty or a small relative payoff to owning a successful system, she would rather shirk and complete a faulty system than assemble it carefully. In these cases, delegation is the only option for the manufacturer if she wishes to procure a system. Several more subtle effects arose when I lifted the binary restriction and assumed that the second task could be assembled with countless levels of effort. When the component is costly to build, the manufacturer must motivate the supplier with a large warranty. Similar to the binary setting, when the manufacturer employs a large warranty, she is again tempted to shirk on assembly, so much so that delegating assembly to the supplier may be preferred even when both options are feasible. Delegation is preferred in these cases, when the supplier accepts large warranty contracts without necessitating large upfront payments. I found that the manufacturer’s preference for delegation disappears if her actions are contractible or the component can be effectively tested before being built into a system, indicating that the incentive to delegate assembly to a supplier is entirely driven by the manufacturer’s dual-moral hazard problem. Skeptics may find it unlikely that manufacturers would shirk on assembling their own products to collect warranty payments. However, Sako and Helper (1998) has clearly documented suppliers’ distrust of their customers. The same study shows that a suppliers’ distrust is unaffected by previous dealings with the same customer, suggesting that the problem persists even between established trade partners.

Lastly, I examined the manufacturer’s testing preferences and found that when external failures are non-contractible, the manufacturer develops a stronger preference for outsourcing. If the manufacturer selects the level of intensity with which to test outgoing systems,
she will always test more when the supplier assembles the system as opposed to when the manufacturer herself assembles the systems. This result is independent of whether delegation is optimal or not. I then examined what happens when the manufacturer can directly test the components in addition to the finished systems. Testing components is not always feasible. As has been described in the design literature, when systems have an “integrated” design, testing individual components is useless, because the outcome of such tests is non-correlated with the probability of the system failing. When systems are “non-integrated,” component testing is fruitful, and I found that if component testing was sufficiently effective, the manufacture may be able to eliminate the dual-moral hazard problem and obtain the first-best outcome. When component testing is not particularly precise, the manufacturer must still penalize the supplier for system failures and incur the agency costs associated with the dual moral hazard problem.

Comparing the binary- and continuous-action settings of Section 3 and 4, it becomes clear that system design is an important variable when deciding whether to outsource or not. In particular, as the manufacturer’s opportunities to shirk decrease she can commit to ever higher levels of assembly effort. Research on the effect of design on contracting costs has only recently begun (Baiman et al. (2001)). Empirical studies have shown that the modular or non-integrated systems are more likely to be outsourced than integrated systems (Veloso and Fixson (2001)). Although this paper has not explicitly examined the effect of design choices on outsourcing, a more comprehensive model could potentially analyze these effects.

I have also assumed that finished systems only require a single component from a single supplier. If the manufacturer dealt with multiple suppliers for multiple components, then the analysis remains largely unchanged. Outsourcing assembly to a supplier always eliminates the manufacturer’s commitment problem at the cost of aggravating the moral hazard problem with the supplier asked to assemble the system. With multiple suppliers, the commitment problem is significantly greater because multiple suppliers rely on the manufacturer to assemble the system carefully, yet the costs of delegating assembly to a supplier remain unchanged, because the moral hazard problem is constant. As such, the model predicts more outsourcing in the presence of multiple suppliers.

Lastly, in my model both the buyer and seller were aware of one another’s technology, and neither party was privy to any asymmetric information. To expand the present analysis, it would be interesting to compare the delegation decision in the presence of asymmetric information. In particular, if the supplier’s ability to build components is privately known, then the informational rents paid to the supplier may vary dependent on whether or not the supplier assembles the systems. Unlike Riordan and Sappington (1987), such a study would continue to assume that the supplier’s components are non-contractible. Based on recent studies on the cost of asymmetric information (Rajan and Saouma (Forthcoming)), I conjecture that the level of asymmetric information will determine whether or not delegation is optimal in such settings.

References

Sharon Anderson, David Glenn, and Karen Sedatole. Sourcing parts of complex products: evidence on transactions costs, high-powered incentives and ex-post opportunism. Ac-


6.1 Appendix

Proof of Proposition 1 The manufacturer solves:

\[
\max_{\alpha, \beta, r} -\alpha - r + \Gamma \mu + F + \beta (1 - \mu) - c(\mu)
\]

such that

\[
\frac{\Gamma \mu - c(\mu)}{\mu} \geq \beta \geq \frac{c_1}{\mu} \tag{29}
\]

\[
\alpha - \beta (1 - \mu) - c_1 \geq 0 \\
\alpha + r - \beta - c_1 \geq L. \tag{30}
\]

Because the fixed payment \(\alpha\) enters the manufacturer’s objective function negatively, the manufacturer wishes to minimize \(\alpha\), and clearly, the smallest \(\alpha\) satisfying the supplier’s IR constraint is given by \(\alpha = \beta (1 - \mu) + c_1\). Plugging in the optimal \(\alpha\), the manufacturer’s problem can be rewritten as:

\[
\max_{\beta, r} -r + \Gamma \mu + F - c(\mu) - c_1
\]

such that (29) and (30) hold. The liability limit (30) implies that the larger the warranty \(\beta\), the larger the necessary upfront payment \(r\), which again enters the manufacturer’s objective function negatively. As such, the manufacturer wishes to minimize \(\beta\) and satisfy (29); the smallest such warranty is given by \(\beta = \frac{c_1}{\mu}\). Because the manufacturer wishes to minimize the upfront payment \(r\), she clearly sets \(r = (\beta y + L)^+ = (c_1 + L)^+\).

Proof of Proposition 2: The two regimes yield the same quality system (identical \(x\) and \(y\)). The only difference between the two regimes in the warranty \(\beta\). Because the liability limit imposes a rent payment increasing in \(\beta\), the regime with the smallest warranty is preferred, i.e., the insourced regime.

Proof of Proposition 3: The proof technique is identical to that of Proposition 1 and is hence omitted.

Proof of Proposition 4: The optimal contract is detailed in the following table:

<table>
<thead>
<tr>
<th>(\beta)</th>
<th>(r)</th>
<th>when</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Gamma)</td>
<td>0</td>
<td>(2c_1 \leq \frac{r}{\gamma} \leq -L)</td>
</tr>
<tr>
<td>(\sqrt{2\gamma c_1})</td>
<td>(2c_1 + L)</td>
<td>(\frac{r^2}{\gamma} &gt; -L \geq \frac{r^2}{\gamma_L} \text{ and } 2c_1 \geq -L)</td>
</tr>
<tr>
<td>(\sqrt{-\gamma L})</td>
<td>0</td>
<td>(\frac{r^2}{\gamma} &gt; -L \geq \frac{r^2}{\gamma_L} \text{ and } 2c_1 \leq -L)</td>
</tr>
<tr>
<td>(\frac{\gamma}{3})</td>
<td>(\frac{r^2}{\gamma_L} + L)</td>
<td>(\frac{r^2}{\gamma} &gt; -L \text{ and } 2c_1 \leq \frac{r^2}{\gamma_L})</td>
</tr>
<tr>
<td>(\sqrt{2\gamma c_1})</td>
<td>(2c_1 + L)</td>
<td>(\frac{r^2}{\gamma} &gt; -L \text{ and } 2c_1 \geq \frac{r^2}{\gamma_L})</td>
</tr>
</tbody>
</table>
The supplier’s sequentially rational choice of assembly effort is given by $y = x \cdot \frac{\beta}{\gamma}$. In order to satisfy the supplier’s individual rationality constraint (assuming no rents), the manufacturer sets:

$$\alpha = \beta \left(1 - \frac{\beta}{\gamma}\right) + \frac{\beta^2}{2\gamma} + c_1$$

Plugging the payment $\alpha$, the manufacturer’s Kuhn-Tucker problem becomes:

$$\max_{\{r, \beta\}} -r + \Gamma \frac{\beta}{\gamma} - \frac{\beta^2}{2\gamma} + \lambda \left(r - \frac{\beta^2}{\gamma} - L\right) + \mu \left(\beta - \sqrt{2\gamma c_1}\right)$$

With conditions:

$$-1 + \frac{\lambda}{\Gamma} \leq 0 \quad r \geq 0 \quad r(\lambda - 1) = 0 \quad (31)$$

$$\beta = \frac{\Gamma + \mu \gamma}{1 + 2\lambda} \quad \beta \geq 0 \quad \beta \left(\beta - \frac{\Gamma + \mu \gamma}{1 + 2\lambda}\right) = 0 \quad (32)$$

$$r - \frac{\beta^2}{\gamma} \geq L \quad \lambda \geq 0 \quad \lambda(r - \frac{\beta^2}{\gamma} - L) = 0 \quad (33)$$

$$\beta \geq \sqrt{2\gamma c_1} \quad \mu \geq 0 \quad \mu \left(\beta - \sqrt{2\gamma c_1}\right) = 0. \quad (34)$$

Suppose $\frac{\Gamma^2}{\gamma} \leq -L$ and $\frac{\Gamma^2}{\gamma} \geq 2c_1$, I claim that the optimal warranty is $\beta = \Gamma$. To this end, note that $\frac{\Gamma^2}{\gamma} \leq -L$ implies $r = 0$ is feasible. Now, if $\frac{\Gamma^2}{\gamma} < -L$, then $\beta = 0$, whereas if $\frac{\Gamma^2}{\gamma} = -L$, then $\beta = 0$, but I will show that $\lambda = 0$ is optimal. Because $\frac{\Gamma^2}{\gamma} \geq 2c_1$, the candidate solution $\beta = \Gamma \geq \sqrt{2\gamma c_1}$, where a strict inequality implies $\mu = 0$. However, if $\Gamma = \sqrt{2\gamma c_1}$, then $\mu$ need not be zero, although setting $\lambda = \mu = 0$ yields the first-best solution, i.e. $\lambda = \mu = 0$ must be optimal. Finally, note that $\beta = 0$, and then the system is no longer profitable, as the first-best profits are given by $\frac{\Gamma^2}{\gamma} - c_1$.

Now, suppose $\Gamma^2/\gamma \geq -L \geq \frac{\Gamma^2}{\gamma}$. I claim that $\beta = \sqrt{2\gamma c_1}$ when $2c_1 \geq -L$ and $\beta = \sqrt{-L}$ when $2c_1 < 2L$. To this end, note that $\lambda = 0$ is no longer feasible, because this would imply via (31) that $r = 0$, yet (32) implies $\beta \geq \Gamma$ which violates the first inequality in (31). As such, it must be the case that $\lambda > 0$ hence (33) implies $r = \beta^2/\gamma + L$. Now there are two possibilities, either $r = \beta^2/\gamma + L \leq 0$ or $r > 0$.

**case 1:** $(2c_1 \geq -L)$ Suppose $r > 0$, then (31) implies that $\lambda = 1$, which implies $\beta = \frac{\Gamma + \mu \gamma}{3}$. Next, because $2c_1 \geq -L \geq \frac{\Gamma^2}{\gamma}$, the only possible value for $\mu$ is $\mu > 0$, because if $\mu = 0$, then $\beta = \Gamma/3$ which violates (34) since $2c_1 \geq -L \geq \frac{\Gamma^2}{\gamma}$ or $\frac{\Gamma}{3} < \sqrt{2\gamma c_1}$. Finally, (34) combined with $\mu > 0$ imply $\beta = \sqrt{2\gamma c_1}$. On the other hand, if $r = 0$, then (33) implies $\beta = \sqrt{-L}$, which violates (34), hence $\mu, r > 0$, and the value of $r$ is given by (33).

**case 2:** $(2c_1 < -L)$ I will show that $r = 0$. Suppose $\mu > 0$, then (34) implies that $\beta = \sqrt{2\gamma L}$, if $r > 0$, (33) implies that $-\beta^2/\gamma > L$, but $-\beta^2/\gamma = -2c_1 > L$, a contradiction. Hence,
if $\mu > 0$ then $r = 0$. Now, I claim that $\mu = 0$ is infeasible. To see this note that $r = L + \frac{\beta^2}{\gamma}$, and recall that (32) implies $\beta = \frac{\Gamma + \mu \gamma}{3} = \frac{\Gamma}{3}$, allowing us to write $r = L + \frac{r^2}{9\gamma}$ which is negative, contradicting (31).

Now, suppose $\frac{r^2}{9\gamma} > -L$. I claim that $\beta = \frac{\Gamma}{3}$ when $\frac{r^2}{9\gamma} \geq 2c_1$ and $\beta = \sqrt{2\gamma c_1}$ when $2c_1 > \frac{r^2}{9\gamma}$. If $2c_1 > \frac{r^2}{9\gamma}$, then note that $r > 0$, because if $r = 0$, then (33) implies that $\beta = \sqrt{-\gamma L}$, but then $\beta < \sqrt{2\gamma c_1}$, contradicting (34). Now, if $r > 0$, then (31) implies that $\lambda = 1$, implying $\mu > 0$, because if $\mu = 0$, then (32) implies that $\beta = \frac{\Gamma}{3}$, but then $-\frac{\beta^2}{\gamma} - L \neq 0$, and hence (33) is violated. As such, $\mu > 0$ and this again implies that $\beta = \sqrt{2\gamma c_1}$. Now, if $\frac{r^2}{9\gamma} \geq 2c_1$ then $r > 0$, because if $r = 0$, then $\beta = \sqrt{-\gamma L} > 2c_1$, hence (34) implies $\mu = 0$ and (32) implies $\beta = \frac{\Gamma}{1+2\lambda}$, which in turn violates (33), since $-\beta^2/\gamma \in \left[-\frac{r^2}{9\gamma}, -\frac{r^2}{9\gamma}\right]$ is strictly less than $L$. Thus, if $\frac{r^2}{9\gamma} \geq 2c_1$ then $r > 0$ which combined with (31) implies $\lambda = 1$. If $\mu > 0$, then (34) implies $\beta = \sqrt{2\gamma c_1}$, yet $\lambda = 1$ and (32) imply that $\beta = \frac{\Gamma + \mu \gamma}{3}$, the two solutions do not agree since $\frac{\Gamma + \mu \gamma}{3} \geq \sqrt{2\gamma c_1}$. As such, it must be the case that $\mu = 0$ when $\frac{r^2}{9\gamma} \geq 2c_1$.

Combining the results, the only feasible $\beta$ is given by $\frac{\Gamma}{3}$.

Proof of Proposition 5

Insourced profits, $\pi_i$, are everywhere decreasing in $c_1$. If $c_1 = 0$, then the insourced regime obtains first-best profits. The outsourced regime generates first-best profits as long as $2c_1 \leq \frac{r^2}{9\gamma} \leq -L$. As such, for $c \in (0, \frac{\Gamma^2}{(2\gamma)})$ and $\frac{r^2}{9\gamma} \leq -L$, the outsourced regime is preferred.

Next consider the case where $\frac{r^2}{9\gamma} > -L \geq \frac{r^2}{9\gamma} > 2c_1$. Beginning with $c_1 = 0$, the insourced regime yields first-best profits, whereas the second best regime does not. If one ignores the common $-c_1$ term in both the insourced and outsourced profits, then as $c_1$ increases, the insourced regime becomes less profitable, whereas the outsourced regime’s profits remain unchanged. In this sub-case, neither regime pays rents. Since both regimes induce the supplier to build a high quality component ($x = 1$), the regime which induces the highest effort $y$ is optimal, because the problem without agency is single peaked in $y$ with peak $y = \frac{\Gamma}{\gamma}$ whenever $x = 1$. To this end, note that the insourced effort $y_m = \frac{1}{2\gamma} \left( \Gamma + \sqrt{\Gamma^2 - 4\gamma c_1} \right)$ is decreasing in $c_1$, whereas the outsourced assembly effort $y_s = \frac{\beta}{\gamma} = \sqrt{-\gamma L/\gamma}$ where $y_s$ is decreasing in $L$. The outsourced effort is larger whenever any of the equivalent statements hold:

$$\frac{1}{2\gamma} \left( \Gamma + \sqrt{\Gamma^2 - 4\gamma c_1} \right) \leq \sqrt{-\gamma L/\gamma}$$

$$\left( \Gamma + \sqrt{\Gamma^2 - 4\gamma c_1} \right) \leq 2\sqrt{-\gamma L}$$

$$\frac{\gamma L + \Gamma \sqrt{-\gamma L}}{\gamma} \leq c_1.$$

Hence there exists a threshold $c^* = \frac{\gamma L + \Gamma \sqrt{-\gamma L}}{\gamma}$ such that when $c_1 \geq c^*$, the manufacturer prefers outsourcing assembly to the supplier, and when $c_1 < c^*$, the manufacturer prefers to
assemble the product herself. Note that in order for \( c^* \leq c_1 < -L/2 \), \( L < -\frac{4r^2}{9\gamma} \), otherwise the manufacturer will always assemble the product herself.

Now, for larger values of \( c_1 \) such that \( 2c_1 \geq -L \), the outsourced regime begins to pay a rent of \( 2c_1 + L \), whereas the insourced regime only pays a rent of \( c_1 + L \). Recall that the largest possible value for \( c_1 \) is given by \( \frac{r^2}{4\gamma} \), because only for smaller values of \( c_1 \) is the insourced regime feasible. Similarly, the smallest possible value of \( c_1 \) in this subcase is \( c_1 = -L/2 \geq \frac{r^2}{18\gamma} \). Now, when \( c_1 = \frac{r^2}{18\gamma} \), the insourced regime generates a profit of \( \frac{r^2(10+2\sqrt{7})}{36\gamma} \), whereas the outsourced regime earns \( \frac{4r^2}{18\gamma} \) (neither regime pays rents, as \( 2c_1 = -L \)) which is less, hence the insourced regime is preferred. On the other hand, assuming no rents are paid, when \( c_1 = \frac{r^2}{4\gamma} \), the insourced regime generates a profit of \( \pi_i = \frac{3r^2}{8\gamma} \), whereas the outsourced regime earns \( \pi_o = \frac{(2\sqrt{2}-1)r^2}{4\gamma} \) which is larger. Moreover, even with the largest rent differential of \( c_1 = \frac{r^2}{4\gamma} \), the outsourced regime is still more profitable. The rent differential may be less as \( -L \in [c_1, 2c_1] \). I claim that there exists a threshold \( \hat{c} \) such that for \( c_1 < \hat{c} \) the insourced regime is preferred, and when \( c_1 \geq \hat{c} \) the outsourced regime is preferred. To see this, suppose to the contrary that there exists an interval \([a, b]\) with \( a > \frac{r^2}{18\gamma} \) and \( b < \frac{r^2}{4\gamma} \) such that for \( c_1 \in [a, b] \) the outsourced regime is preferred, yet for \( c_1 \in \{a-\varepsilon, b+\varepsilon\} \) the insourced regime is preferred for sufficiently small \( \varepsilon \). Because the insourced profits lie above the outsourced profits when \( c_1 = \frac{r^2}{18\gamma} \) and below the outsourced profits when \( c_1 = \frac{r^2}{4\gamma} \), the two profits must intersect at least three times. I will show that after the first crossing, which I label \( \hat{c} \), the profit functions never cross again. To prove this, I need the following lemma:

**Lemma 1** Suppose a differentiable concave function \( f(z) \) lies above a differentiable concave function \( g(z) \) at \( z = a \) and \( z = b \), then as long as \( f'(b) < g'(b) \), then \( f > g \) for \( z \in [a, b] \).

**Proof:** Suppose \( g(z) > f(z) \) for some \( z = z^* \). Then in order for \( g(b) < f(b) \), I must have \( \min_{z \in [z^*, b]} g'(z) < \min_{z \in [z^*, b]} f'(z) \). By the concavity of \( f \), I have \( \min_{z \in [z^*, b]} f'(z) = f'(b) \), hence there must exist a \( \tilde{z} \) such that \( g'(< \tilde{z}) < f'(b) \), but by the concavity of \( g \), \( g'(< \tilde{z}) < g'(\tilde{z}) < f'(b) \), a contradiction. \( \blacksquare \)

Therefore it is sufficient to show that both \( p_i^o(c_1) \) and \( \pi_i(c_1) \) are concave, and \( \pi_o' < \pi_i' \) when \( c_1 = \frac{r^2}{4\gamma} \).

Simple algebra yields:

\[
\pi_i''(c_1) = \frac{-\gamma \Gamma}{(\Gamma^2 - 4\gamma c_1)^3} < 0
\]

Hence the insourced profits are concave in \( c_1 \). Note that:

\[
\pi_o'(c_1) = -3 + \frac{\Gamma}{\sqrt{2\gamma c_1}}, \\
\pi_o''(c_1) = -\frac{1}{2\sqrt{2\gamma c_1}}.
\]

Hence, outsourced profits are also concave, and have a finite derivative at \( c_1 = \frac{r^2}{4\gamma} \). Finally, note that:
\[ \pi'_i(c_1) = \begin{cases} \frac{1}{2} - \frac{\Gamma}{2\sqrt{\Gamma^2 - 4yc_1}} < 0 & c_1 < -L \\ -\frac{1}{2} - \frac{\Gamma}{2\sqrt{\Gamma^2 - 4yc_1}} < 0 & c_1 \geq -L \end{cases} \]

And hence \( \pi'_i\left(\frac{\Gamma^2}{4}\right) = -\infty \). I have thus shown that outsourcing is optimal for sufficiently large \( c_1 \). Because the two profits intersect only once, I can approximate their intersection by solving \( \pi_i(c_1) = \pi_o(c_1) \) for \( c_1 < -L \) and for \( c_1 \geq -L \). When \( c_1 < -L \), the two profits are equal when \( c_1 \approx \frac{12\Gamma^2}{7} \), and when \( c_1 \geq -L \) the profits are equal when \( c_1 \approx \frac{19\sqrt{7}\Gamma^2}{4} \), hence I can conclude that the threshold \( \hat{c} \) is weakly decreasing in \( L \).

Finally, I examine the case where \( -L \leq \frac{\Gamma^2}{3\gamma} \). In this setting, the outsourced regime sets a warranty of \( \frac{\Gamma}{3} \) whence \( 2c_1 \leq \frac{\Gamma^2}{9\gamma} \) and \( \sqrt{2}\gamma c_1 \) otherwise. Suppose \( 2c_1 \leq \frac{\Gamma^2}{9\gamma} \), such that the outsourced regime uses a warranty \( \beta = \frac{\Gamma}{3\gamma} \). When \( -2c_1 = L \), neither regime pays any rents. Recall from before, that when neither regime pays any rents, and the outsourced regime uses a warranty of \( \beta = \sqrt{-\gamma L} \), the outsourced regime is preferred for sufficiently large \( c_1 \) as long as \( L < -\frac{4\Gamma^2}{9\gamma} \). When \( L = -\frac{\Gamma^2}{9\gamma} \), I have \( \sqrt{-\gamma L} = -\frac{\Gamma}{3\gamma} \), and since \( L = -\frac{4\Gamma^2}{9\gamma} > -\frac{4\Gamma^2}{9\gamma} \), the insourced regime is clearly preferred when \( 2c_1 = -L = -\frac{\Gamma^2}{9\gamma} \). For larger values of \( L \), the outsourced warranty remains unchanged, in fact the only terms which vary between the insourced and outsourced regime are the rents, which are larger in the outsourced regime (recall that the rents in the outsourced regime are given by \( (2c_1 + L)^+ \) and only \( (c_1 + L)^+ \) in the insourced regime). As such, for \( L \geq -\frac{\Gamma^2}{9\gamma} \) and \( 2c_1 = -L \), the insourced regime is preferred. On the other hand, for smaller values \( c_1 \) below \( -L/2 \), neither regime pays rents, and hence the outsourced regime payoffs remain constant, whereas the insourced regime payoffs increase, thus for \( L \geq -\frac{\Gamma^2}{9\gamma} \) and \( c_1 \leq -L/2 \), the insourced regime is optimal. Now, recall that the upper-bound for \( c_1 \) is \( \frac{\Gamma^2}{4\gamma} \), because for larger values of \( c_1 \), the insourced regime is no longer feasible. The payoff to the insourced regime when \( c_1 = \frac{\Gamma^2}{4\gamma} \) (without the rent payments) is given by \( \frac{\Gamma^2(2\sqrt{2}-1)}{4\gamma} \), which is larger. However, the rents are larger in the outsourced regime \( (2c_1 > c_1) \), the largest rent differential between the two regimes is when \( L = 0 \) and the difference is \( c_1 = \frac{\Gamma^2}{3\gamma} \); the smallest differential in this sub-case is when \( -L = -\frac{\Gamma^2}{9\gamma} \) and the difference is given by \( 2c_1 + L = \frac{\Gamma^2}{9\gamma} \). Clearly, the outsourced payoffs are always less than the insourced payoffs after taking into consideration the difference in rents, hence at \( c_1 = \frac{\Gamma^2}{4\gamma} \), the insourced regime is preferred. I claim that the insourced regime is preferred in the entire interval \([ -L/2, \frac{\Gamma^2}{4\gamma} ] \). To see this, note that both the \( \pi_i(c_1) \) and \( \pi_o(c_1) \) are concave, yet only \( \pi_o(c_1) \) has a finite derivative at \( c_1 = \frac{\Gamma^2}{4\gamma} \), thus the earlier lemma proves that \( \pi_i(c_1) > \pi_o(c_1) \) for \( c_1 \in [ -L/2, \frac{\Gamma^2}{4\gamma} ] \) when \( L \in \left[ -\frac{\Gamma^2}{9\gamma}, 0 \right] \).

**Proof of Proposition 6** In the insourced regime, recall that the optimal warranty \( \beta \) is the smallest warranty satisfying the supplier’s incentive compatibility constraint. When both the test results and external failures are contractible, the supplier is asked to pay \( \beta \) with probability \( (1 - y \cdot x) \). If the manufacturer cannot hold the supplier responsible for external failures, then the supplier only pays the warranty \( \beta \) with probability \( (1 - y \cdot x)(1 - \delta) \), the
probability of the system failing times the probability of detecting a failed system during testing. This makes the supplier’s IC constraint:

$$\alpha - \beta (1 - x \cdot y)(1 - \delta) - c_1 \geq \alpha - \beta (1 - \delta).$$

The IC constraint can be rewritten as $$\beta c \cdot y(1 - \delta) \geq c_1$$. The new IC constraint is identical to the old IC constraint when the cost to the supplier of building a high quality good is $$\frac{c_1}{1 - \delta} > c_1$$. Because all warranties were found to be increasing in $$c_1$$ in the previous propositions, all warranties increase when external failures are non-contractible.

**Proof of Proposition 7**

Note that $$\Gamma$$, the relative benefit of a functional system to the manufacturer, is no longer given by $$R - F$$. To re-derive $$\Gamma$$, recall that the manufacturer’s objective function can be written:

$$R(x \cdot y) + (\delta E + (1 - \delta)F)(1 - x \cdot y) - \alpha - r - \beta - C_m,$$

where $$C_m$$ are the manufacturer’s anticipated costs. For the remainder of this proof, let $$\Gamma = (R - \delta E - (1 - \delta)F)$$, such that the manufacturer’s objective function becomes:

$$\Gamma x \cdot y - \alpha - r + \beta (1 - x \cdot y) - C_m + \delta E + (1 - \delta)F = \Gamma x \cdot y - r - c(y) - c_1 + \delta E + (1 - \delta)F.$$

Let $$\pi_c$$ denote the value function for the manufacturer’s profits under integration and similarly $$\pi_o$$ the value under delegation. Because a unique, interior solution to the testing problem is assumed, if $$\frac{d\pi_c}{d\delta} > \frac{d\pi_o}{d\delta}$$, then the outsourced regime will test more than the insourced regime (recall that test intensity is given by $$1 - \delta$$), which is true if any of the following equivalent inequalities is true:

$$\frac{d\pi_c}{d\delta} > \frac{d\pi_o}{d\delta}$$

The third inequality holds because $$\frac{\partial \pi_c}{\partial \delta} = \frac{\partial \pi_o}{\partial \delta}$$ and $$\frac{\partial \Gamma}{\partial \delta} = F - E > 0$$. Hence, it suffices to show that the $$\frac{\partial \pi_c}{\partial \Gamma} \geq \frac{\partial \pi_o}{\partial \Gamma}$$ to demonstrate that the centralized regime always tests more rigorously. Begin by noting that $$\frac{\partial \pi_c}{\partial \Gamma}$$ is maximized when the optimal warranty $$\beta$$ in the decentralized regime is given by $$\Gamma$$. To see this, first assume that the IC constraint is not binding in the outsourced setting. Thus, $$\beta \in \{\Gamma, \sqrt{-\gamma L}, \Gamma/3\}$$. If $$\beta = \Gamma/3$$, then:

$$\frac{\partial \pi_o}{\partial \Gamma} = y_s + (\Gamma - y_s \gamma) \frac{\partial y_s}{\partial \Gamma} - \frac{\partial r}{\partial \Gamma} = \frac{\Gamma}{3\gamma} + (\Gamma - \Gamma/3) \frac{1}{3\gamma} - \frac{2\Gamma}{9\gamma} = \frac{3\Gamma}{9\gamma}$$

By raising $$\Gamma$$, the manufacturer will elect to increase the warranty and thus yield a higher quality system from the supplier, yet at the same time, the rents paid to the supplier also increase. When $$\beta = \Gamma$$:
\[ \frac{\partial \pi_o}{\partial \Gamma} = y_s + (\Gamma - y_s \gamma) \frac{\partial y_s}{\partial \Gamma} - \frac{\partial r}{\partial \Gamma} = \Gamma / \gamma \]

Finally, when \( \beta = \sqrt{-\Gamma L} \):

\[ \frac{\partial \pi_o}{\partial \Gamma} = y_s + (\Gamma - y_s \gamma) \frac{\partial y_s}{\partial \Gamma} - \frac{\partial r}{\partial \Gamma} = \sqrt{-L \gamma} / \gamma. \]

From Proposition 5, when \( \beta = \sqrt{-\gamma L}, \Gamma / 3 \leq \sqrt{-\gamma L} \leq \Gamma \), hence an upper bound for \( \frac{\partial \pi_o}{\partial \Gamma} \) is \( \frac{\Gamma}{\gamma} \). On the other hand, in the insourced setting:

\[ \frac{\partial \pi_i}{\partial \Gamma} = y_m + (\Gamma - y_s \gamma) \frac{\partial y_m}{\partial \Gamma} - \frac{\partial r}{\partial \Gamma} = \frac{1}{2 \gamma} \left( \Gamma + \sqrt{\Gamma^2 - 4 \gamma c_1} \right) + \frac{1}{4 \gamma} \left( \Gamma - \sqrt{\Gamma^2 - 4 \gamma c_1} \right) \left( 1 + \frac{\Gamma}{\sqrt{\Gamma^2 - 4 \gamma c_1}} \right) \]

\[ = \frac{\Gamma \left( \Gamma + \sqrt{\Gamma^2 - 4 \gamma c_1} \right) - 2 \gamma c_1}{2 \gamma \sqrt{\Gamma^2 - 4 \gamma c_1}} \] (35)

Hence, it is sufficient to show that \( (35) > \Gamma / \gamma \) to establish that when the IC constraint is non-binding in the outsourced regime, the manufacturer will test more in the outsourced regime. To see this, note that the following statements are equivalent:

\[ \frac{\Gamma \left( \Gamma + \sqrt{\Gamma^2 - 4 \gamma c_1} \right) - 2 \gamma c_1}{2 \gamma \sqrt{\Gamma^2 - 4 \gamma c_1}} > \Gamma / \gamma \]

\[ \frac{\Gamma}{2} + \frac{\Gamma^2 - 2 \gamma c_1}{2 \sqrt{\Gamma^2 - 4 \gamma c_1}} > \Gamma \]

\[ \frac{\Gamma^2 - 2 \gamma c_1}{\sqrt{\Gamma^2 - 4 \gamma c_1}} > \Gamma \]

\[ \Gamma^2 - 2 \gamma c_1 > \Gamma \sqrt{\Gamma^2 - 4 \gamma c_1} \]

\[ 4 \gamma c_1 \gamma^2 > 0 \]

To complete the proof, note that if the IC constraint is binding in the outsourced regime, then

\[ \frac{\partial \pi_o}{\partial \Gamma} = \frac{\sqrt{2 \gamma c_1}}{\Gamma} < \Gamma / \gamma \]

where the inequality holds, because Proposition 4 showed if \( \sqrt{2 \gamma c_1} > \Gamma \), then no production occurs, as the manufacturer is better off shutting down.■

**Proof of Proposition** 8 To prove the proposition, it is sufficient to establish that the marginal cost of increasing \( \beta \) is less than that of increasing \( \beta_c \) when \( \beta = 0 \) and the
manufacturer cannot obtain the First-best payoffs using only $\beta_c$. first-best payoffs are attainable as long as the smallest component warranty $\beta_c$ satisfies the supplier’s IC constraint \((28)\) ($\beta_c \leq \frac{\omega_1}{(1-\tau)}$) and the component warranty does not require paying the supplier rents: $\beta_c \leq -L/\Gamma$ (since the first-best payoff requires $y_m = \Gamma$ and that no rents be paid). The marginal cost of raising the component warranty $\beta_c$ when $\frac{\omega_1}{(1-\tau)} \leq \beta_c$ and $\beta_c > L/\Gamma$ is given by $y_m$, the additional rents paid to the supplier caused by raising $\beta_c$ by one dollar. To this end, differentiating the manufacturer’s insourced profits \((26)\) by $\beta$ when $\beta = 0$ obtains:

\[
\left. \frac{\partial \pi_i}{\partial \beta} \right|_{\beta=0} = \left. \frac{1}{2\gamma} (\beta - 2\Gamma) \right|_{\beta=0} = \frac{\Gamma}{\gamma}
\]

Recall that when $\beta = 0$, $y_m = \Gamma > \Gamma/\gamma$, thus the marginal cost to the manufacturer of raising the system warranty is less than the marginal cost of raising the component warranty, and the manufacturer would use $\beta > 0$ in such a setting.