Reasonably Certain Estimates, Recognition, and Communication of Uncertainty

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Abstract

This paper presents a model in which asset/liability recognition is a means for a risk-neutral entrepreneur to communicate with risk-averse investors about the riskiness of investments or the uncertainty of future obligations. The model shows that more conservative accounting may produce less conservative investing and lower expected payoffs for the entrepreneur. While the desired uncertainty-based recognition hurdle is increasing in an asset's expected payoff, it is decreasing in a liability's expected magnitude. The impact of further subcategorization of recognized assets is also considered. While finer classification of assets produces no direct benefit, it may provide a cost-effective means to reduce the entrepreneur's incentives to engage in costly voluntary disclosure.

1 Introduction

The requirement for a "reasonably certain estimate" plays a significant role in the accounting for operations. On the asset side, it affects the accounting for tangible and intangible assets (and expenses). Companies make many expenditures that are intended to provide future benefits. But if it is not possible to estimate those future benefits with reasonable certainty, then the expenditure is expensed, rather than recognized as an asset.[1] At first blush, it may appear that this practice should be considered conservative, because it limits the recognition of assets, and therefore shareholders' equity. However, the same requirement applies to liabilities that require estimation.

Rather than focus on the reported results of the recognition process (book values, income, etc.), this paper considers asset/liability recognition as a "classification" system that communicates about uncertainty. In particular, this practice may provide a means for a firm's manager to communicate with investors about the riskiness of the investment which the firm makes or the uncertainty associated with future obligations.

In the model below, a risk-neutral entrepreneur chooses whether to invest in a project (of unknown variance) that must subsequently be sold to a risk-averse investor. The principal results from the model are (1) that increasing the "strictness" of uncertainty-based asset recognition hurdles may have the effect of increasing the risk range of projects that are attractive to the entrepreneur, (2) that the desired uncertainty-based recognition hurdle is increasing in the expected outcome for assets, but decreasing in the expected outcome for liabilities, and (3) further sub-categorization of recognized assets produces no ex ante benefit for the entrepreneur in the price paid by the investor, but it may reduce the deadweight costs of voluntary disclosure.

[1] The same condition applies for the recognition of revenue and receivables, but this analysis will not focus on these aspects of accounting practice. See Antle and Demski (1989) and Dutta and Zhang (2002).

2 Background and Previous Research

The notion that "reasonably certain estimates" of future events are required for recognition is well-integrated into the fabric of accounting. For example, Stickney and Weil (2006, p.40) note that assets are recognized "only if (1) the firm has acquired rights to its use in the future as a result of a past transaction or exchange, and (2) the firm can measure or quantify the furure benefits with a reasonable degree of precision" and liabilities are recognized when "a firm receives benefits or services and in exchange promises to pay the provider of those goods or services a reasonably definite amount at a reasonably definite future time." (Stickney and Weil 2006, p. 46.) Statement of Financial Accounting Concepts No. 5 (paragraph 60) states that "Some events that affect assets, liabilities, or equity are not recognized in financial statements at the time they occur. Some events that result in future benefits, for example, creation of product awareness by advertising and promotion, may perhaps never be recognized as separate assets. Other events, for example, a disaster loss of unkown dimension, are recognized only when sufficient information about the effects of the event has become available at a justifiable cost to reduce uncertainty to an acceptable level."

A prominent example of this condition is the accounting for research and development costs in the United States. In its background and basis for conclusions, Statement of Financial Accounting Standards No. 2 (paragraph 17) notes that research and development "constitute a significant element of the United States economy and are vital for its growth." However, "[t]here is normally a high degree of uncertainty about the future benefits of individual research and development projects," (paragraph 39) and a causal relationship between expenditures and benefits is difficult to establish. "The criterion of measurability would require that a resource not be recognized as an asset for accounting purposes unless at the time it is acquired or developed its future economic benefits can be identified and objectively measured...Although future benefits from a particular reserach and development project may be foreseen, they generally cannot be measured with a reasonably degree of certainty." (SFAS No. 2, paragraphs 44 and 45) As a result, the FASB required that research and development costs be expensed, with the amount to be disclosed in the financial statements.

On the liability/loss side, Statement of Financial Accounting Standards No. 5 requires that an estimated liability be recognized when it is probable that an asset has been impaired or a liability has been incurred and when the amount of the loss can be "reasonably estimated." For instance, if a firm does not have a reasonably certain estimate of warranty costs, it does not recognize a liability (or expense) at the time of sale. However, the firm is required to disclose the existence of the loss contingency.

Two streams of previous literature bear on the questions addressed below: prior research on the communication of uncertainty and prior research on recognition/classification. Jorgensen and Kirschenheiter (2003) consider a multi-firm, portfolio setting, where each manager has private information about his firm-specific risk. They find that a voluntary disclosure equilibrium exists for some levels of disclosure costs and that the Capital Asset Pricing Model holds in this equilibrium. In this equilibrium, managers make a voluntary disclosure of their firm-specific risk when that risk falls below some threshhold level.

Hughes and Pae (2004) consider a setting in which an entrepreneur must disclose a value signal to investors. In addition, the entrepreneur may choose to acquire information about the precision of that value signal and – if acquired – whether to disclose that precision information to investors. Hughes and Pae find that the equilibrium involves the entrepreneur choosing to disclose high precision outcomes when the value signal is favorable (relative to prior expectations) and low precision outcomes when the value signal is unfavorable. They also find that the entrepreneur's discretion over this precision disclosure leads to an over-investment in precision information.

Examples of the second stream of literature include Antle and Demski (1989) and Dutta and Zhang (2002). Both of these papers involve the timing of revenue recognition, with particular attention to agency issues. Antle and Demski find that the preferred timing of revenue recognition is sensitive to nonaccounting information and to the introduction of incentive compatibility constraints and to truth-telling constraints. Dutta and Zhang's analysis of a multi-period agency problem finds that deferring revenue recognition until it is realized is superior to recognizing revenue when one acquires the resources that will produce the revenue in the future. In both cases, the accounting question is not whether one should recognize revenue (and the associated asset), but rather when one should do so.

Dye (2002) examines a model in which an accounting standard produces a reported classification based on the favorability of an investment's prospects. The owner of the investment has private information about the investment's quality and also has the ability to engage in (costly) manipulation of the investment's reported quality. Dye compares the "official" standard to the "shadow" standard and examines how the classification standards and the resulting behavior may evolve over time and how the standards are affected by "the standard setter's knowledge of the economic environment."

The focus of this paper is not on voluntary disclosure of risk information, but rather on a common practice that could be construed as conveying uncertainty information whenever an expenditure is made. Voluntary disclosures will be considered, but principally to see how they are affected by the recognition of assets and by subclassifications of recognized assets.

The model examined below differs from the revenue recognition/classification models by looking at the accounting for assets (and to a lesser extent, liabilities) where recognition is not a "when" question. For instance, the accounting for self-developed intangibles (R&D) in the United States requires that assets never be recognized. Once the cost is expensed, it is never brought back and recognized as an asset once the risk has reduced and it becomes possible to construct a "reasonably certain estimate" of future benefits. And, the focus is less on reporting behavior than on the effect of accounting classifications on investment behavior. The entrepreneur (who owns the investment opportunity) has no reporting discretion, except – in a subsequent section – for the opportunity to engage in a costly voluntary disclosure. How does a recognition hurdle based on uncertainty affect investor assessments and, in turn, the entrepreneur's behavior?

The remainder of the paper proceeds as follows. The model of an entrepreneur and an investor and essential assumptions are described in section 3. Section 4 introduces a recognition partition and considers its impact on the investment behavior of the entrepreneur and the price which the investor is willing to pay. In addition, section 4 applies the model to a liability setting. Section 5 considers information beyond simple recognition, including subclassifications of recognized assets either by required asset category or by (costly) voluntary disclosure. Besides uncertainty, a further difference between tangible assets (e.g., production equipment) and intangible assets (e.g., self-developed intellectual assets) is the speed with which investment results can be observed. This issue is considered in Section 6, along with consideration of two model modifications. Section 7 presents a summary of conclusions.

3 Model and Assumptions

A risk-neutral entrepreneur owns an investment opportunity that will produce benefits for two periods.[2] The opportunity requires an investment of C and will produce benefits x_i in periods $i = 1, 2$. The x_i are independently and identically distributed and have a normal distribution with known mean μ.[3] However, to

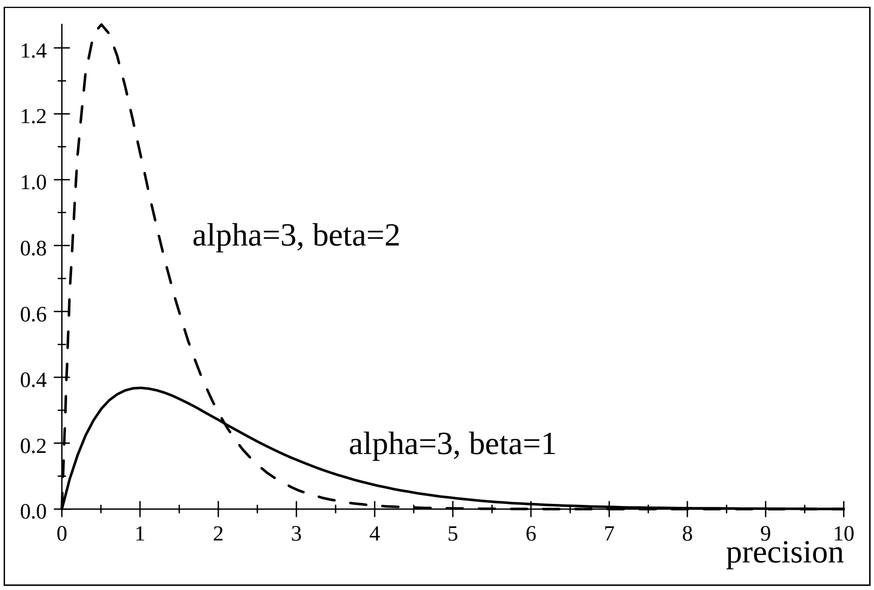
[2] A liability version of the model will be presented in Section 4.4.

[3] The assumption that μ is known can be relaxed without creating substantive changes in the model. See Section 6.3 below.

explore the use of recognition to communicate about uncertainty, the variance of the outcome distribution is not known. A commonly used assumption is that the precision (inverse of variance) is distributed Gamma, with shape parameter $\alpha > 1$, scale parameter $\beta > 0$ and location parameter equal to zero.

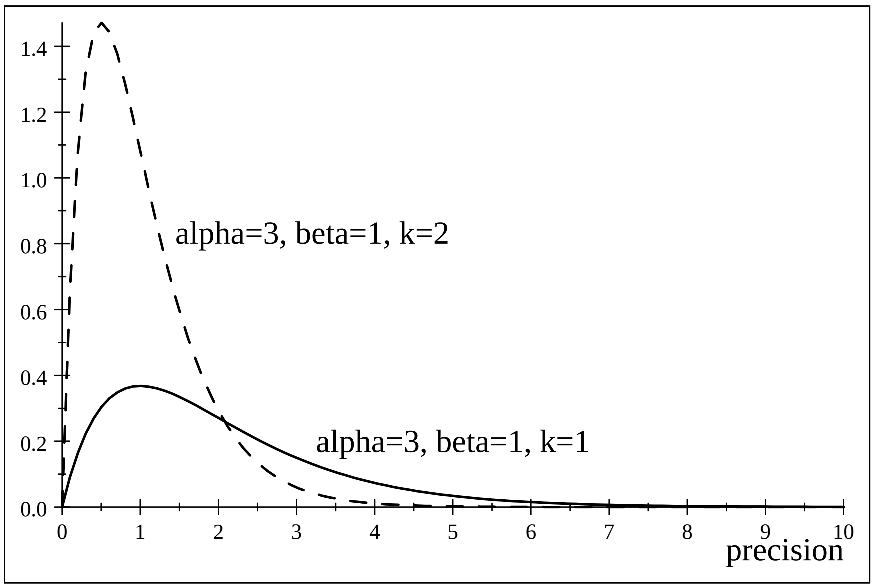
$$g(\tau) = \frac{\tau^{\alpha-1} \cdot e^{-\beta\tau}}{\beta^{-\alpha} \cdot \Gamma(\alpha)}$$

Alternative values of these parameters provide different probability distributions, as exemplified in the figure below. Higher values of β shift the precision distribution toward lower values, thereby making it more likely that the outcomes will have a high variance.



A convenient way to incorporate differential information about the outcome variance is to fix the values of α and β, but to assume that the outcome variance is equal to $\frac{\widetilde{k}}{\tau}$, where $\widetilde{k}$ has density $h(k)$ over the interval $\left[\underline{k}, \overline{k}\right]$ where $\underline{k} > 0$. $h(k)$ is assumed to be continuously differentiable over the interval $\left[\underline{k}, \overline{k}\right]$. The realization of $\widetilde{k}$ is observed by the entrepreneur prior to making the investment.

The precision parameter $\widetilde{\tau}$ is distributed Gamma (with known parameters α and β), and variables $\widetilde{k}$ and $\widetilde{\tau}$ are assumed to be independently distributed. Under these assumptions and conditional on a realization k, the outcome precision $\frac{\tau}{k}$ is distributed Gamma, with shape parameter α and scale parameter $k\beta$. Therefore, the figure above can be reinterpreted with a fixed value of β, but varying values of k.



If no investment is made, the entrepreneur bears no cost and there are no outcomes at the end of periods 1 and 2. If the entrepreneur makes the investment, she bears the cost C, receives the realized value x_1 and then - for reasons exogenous to the model - she must sell the second period payoff to a risk-averse investor. The price paid by the investor is assumed to be the following, where $E\left[\cdot\right]$ is the expectation operator, $\sigma^2\left[\cdot\right]$ is variance, Ω represents the information available to the investor at the time the price is set and ρ is the coefficient representing the investor's level of risk aversion.

$$P[\Omega] = E[\widetilde{x}_2 \mid \Omega] - \rho \cdot \sigma^2[\widetilde{x}_2 \mid \Omega]$$

Conditional on an investment being made, the investor's information Ω is assumed to include C, μ and x_1 at the time the price is set. In effect, this assumes that the investor can observe C whether it is capitalized or expensed[4] and that the investor has a good idea of the expected return to the entrepreneur's investment set. The first period outcome x_1 is publicly observable, and the entrepreneur is assumed to have no reporting latitude. The investor also knows the distributional information concerning $\widetilde{x}_i$, $\widetilde{k}$, and $\widetilde{\tau}$. For parameters that the investor cannot observe, he makes conjectures that - in equilibrium - are correct.

Accounting practice commonly employs binary classifications, and the same practice will be applied to recognition in this model. Specifically, recognition is a binary partition of $\left[\underline{k}, \overline{k}\right]$. For lower values of k (i.e., when future benefit uncertainty is low), the expenditure of C results in an asset being recognized. For higher values of k (i.e., when future benefit uncertainty is high), the expenditure of C is expensed. Therefore, when the entrepreneur expends C, recognition/expensing provides the investor with a partition of $\left[\underline{k}, \overline{k}\right]$ and with information about the uncertainty of the future benefits.

3.1 Belief Revision following x_1

After observing the first period outcome, the investor updates his beliefs about the probability distribution of $\widetilde{x}_2$. The first period outcome $\widetilde{x}_1$ is distributed $N(\mu, \frac{k}{\tau})$, so $g(\tau \mid x_1, k)$ is distributed Gamma, with shape parameter $\widehat{\alpha} = \alpha + \frac{1}{2}$ and scale parameter $\widehat{k\beta} = k\beta + \frac{1}{2}(x_1 - \mu)^2$ (DeGroot 1979, p. 168). It follows that

$$E\left[\frac{\widetilde{k}}{\widetilde{\tau}} \mid x_1, k\right] = \frac{\widehat{k\beta}}{\widehat{\alpha} - 1} = \frac{k\beta + \frac{1}{2}(x_1 - \mu)^2}{\alpha - \frac{1}{2}} = \frac{2k\beta + (x_1 - \mu)^2}{2\alpha - 1}.$$

Let K be the investor's conjectured subset of the support of k.[5] If the

[4] For example, expenditures for research and development (if material) must be disclosed, even though they are not capitalized.

[5] Observing x_1 does not affect the investor's conjectured support K. For any value of k,

investor observes x_1, then

$$E\left[\widetilde{x}_2 \mid x_1, \widetilde{k} \in K\right] = \mu$$

$$\begin{aligned}\sigma^2\left[\widetilde{x}_2 \mid x_1, \widetilde{k} \in K\right] &= E\left[\frac{\widetilde{k}\beta + \frac{1}{2}(x_1-\mu)^2}{\alpha - \frac{1}{2}} \mid x_1, \widetilde{k} \in K\right] \\ &= \frac{2\beta \cdot E\left[\widetilde{k} \mid x_1, \widetilde{k} \in K\right] + (x_1-\mu)^2}{2\alpha - 1}\end{aligned}$$

and the price that the entrepreneur will receive is

$$P = \mu - \rho \cdot \frac{2\beta \cdot E\left[\widetilde{k} \mid x_1, \widetilde{k} \in K\right] + (x_1-\mu)^2}{2\alpha - 1} \tag{1}$$

3.2 The Investment Decision

Prior to making an investment, the entrepreneur observes the realization k. Knowing this value and with a conjecture that the investor believes the support of k to be K, the entrepreneur expects to receive

$$E\left[P \mid k, K\right] = \mu - \rho \cdot \frac{2\beta \cdot E\left[E\left[\widetilde{k} \mid x_1, \widetilde{k} \in K\right] \mid k\right]}{2\alpha - 1} - \rho \cdot \frac{1}{2\alpha - 1} \cdot E\left[(x_1-\mu)^2 \mid k\right]$$

$$= \mu - \frac{2\rho\beta}{2\alpha - 1} \cdot E\left[E\left[\widetilde{k} \mid x_1, \widetilde{k} \in K\right] \mid k\right] - \frac{\rho}{2\alpha - 1} \cdot \frac{k\beta}{\alpha - 1} \tag{2}$$

The risk-neutral entrepreneur will invest as long as the expected payoff is greater than or equal to zero. That is, the investment will be made as long as $-C + \mu + E\left[P \mid k, K\right] \geq 0$. In equilibrium, conjectures and behaviors must coincide. For example, the investor's conjecture about the set of k values leading to investment must be the same as the actual values for which the entrepreneur will invest.

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the support of x_1 is the entire real line. Therefore, any k that has positive probability of occurring prior to observing x_1 will have positive probability after observing x_1.

3.3 k observed by all prior to sale

If k were observed by the investor prior to the sale of the firm by the entrepreneur, no conjectures are required. The first period outcome informs the investor about $\widetilde{\tau}$, but not about $\widetilde{k}$. At the time of investment, the entrepreneur's expectation for the price would be

$$E\left[P \mid k\right] = \mu - \frac{2\rho\beta}{2\alpha-1} \cdot k - \frac{\rho}{2\alpha-1} \cdot \frac{k\beta}{\alpha-1} = \mu - \frac{\rho\beta}{\alpha-1} \cdot k \qquad (3)$$

The entrepreneur's expected price is decreasing in k, so the investment opportunity's attractiveness also decreases in k. If k_I^* is the value of k that sets $-C + \mu + E\left[P \mid k\right]$ to zero ($k_I^* = \frac{2\mu - C}{\rho \cdot \frac{\beta}{\alpha-1}}$), then the entrepreneur's investment region is $[\underline{k}, k_I^*]$. The entrepreneur would invest more often if the expected payoff ($2\mu - C$) were higher and less often if the investor's risk aversion (ρ) were higher.

3.4 No information on k made public

If the investor observes an investment of C, but receives no direct information about k at the time the price is set, then the price must be based upon K, the conjectured support of k (conditional on observing an investment) and on the observed value of x_1, as shown in equation (1) above.

Lemma 1: The expression $E\left[E\left[\widetilde{k} \mid x_1, \widetilde{k} \in [k_L, k_H]\right] \mid k\right]$ is increasing in k_L, k_H and k.

Proof: All proofs are provided in the Appendix.

Lemma 1 implies that the entrepreneur's expected price (and therefore, the expected investment payoff) is decreasing in the observed value of k, so the investment region will consist of a region $\left[\underline{k}, k_I^N\right]$, where the upper limit for acceptable investments (k_I^N) is determined by the following equilibrium relationship.

$$-C + 2\mu - \frac{2\rho\beta}{2\alpha-1} \cdot E\left[E\left[\widetilde{k} \mid x_1, \widetilde{k} \in \left[\underline{k}, k_I^N\right]\right] \mid k_I^N\right] - \frac{\rho}{2\alpha-1} \cdot \frac{\beta}{\alpha-1} \cdot k_I^N = 0 \quad (4)$$

Lemma 2: $k_I^N > k_I^*$. A lack of public information about k causes the entrepreneur to accept a broader range of investments.

Lemma 2 implies that the entrepreneur, upon privately observing the outcome k, is willing to invest in riskier opportunities than she would under full disclosure, as depicted in Figure 1. This situation would fit two extreme accounting practices – one in which these investments were always expensed and one in which these investments were always capitalized. In either case, the reporting would provide no information about k and would lead to the investment behavior in Lemma 2.

It remains to be seen whether that equilibrium behavior results in an increased ex ante payoff to the entrepreneur. One way to address that question is to suppose that the entrepreneur could commit to an investment hurdle, k_I^c, where no investments will be made with risk in excess of this amount. Lemma 3 provides the optimum investment hurdle with commitment.

Lemma 3: Suppose the entrepreneur can commit – ex ante – to an upper limit of k that will be accepted for investment. The optimum or "desired" value for the entrepreneur is $k_I^c = k_I^* < k_I^N$.

The implication of Lemma 3 is that the investor's "price protection" takes into account the entrepreneur's willingness to accept investments of greater risk when no information about k is available.[6] Therefore, the entrepreneur is worse off (ex ante) with no information about k being disclosed. This leads to the consideration of a recognition/expensing classification that might provide benefits to the entrepreneur.

[6] The investor is assumed to "price protect" by taking into account the available information and an accurate equilibrium conjecture about the entrepreneur's investment behavior. Therefore, I presume that the investor "breaks even" – at least in expectation – in every situation.

4 A Recognition Partition

It is conventional in accounting to employ binary partitions when establishing recognition criteria. A minority investor has significant influence or not. Revenue is recognized or deferred. Operating expenditures result in an asset or an expense.

The asset recognition process conveys information about the uncertainy surrounding future benefits, and this information affects the entrepreneur's anticipated price from the investor and, ultimately, the set of desirable investment opportunities. In the context of the model presented above, a recognition partition accords different treatment for the entrepreneur's investment depending on the value of k. As would be customary in accounting practice, the asset recognition decision must be made at the time of the expenditure and prior to any observation of x_1. Define $\widehat{k}$ ($\underline{k} \leq \widehat{k} \leq \overline{k}$) as a "recognition hurdle" that divides values of k into "lower risk" and "higher risk." The recognition region will be $\widehat{K}_L = \left[\underline{k}, \widehat{k}\right]$ and the expensing region will be $\widehat{K}_H = \left(\widehat{k}, \overline{k}\right]$. It should be noted that the recognition treatment has no effect on the investor's ability to know the cost of investment C or to observe whether an investment has been made, or to see the first period outcome x_1.[7] Essentially, the first two of these assumptions presume a setting where expenditures that must be expensed must also be disclosed. The third assumption will be discussed in a later section. There are three regions that determine the effect of $\widehat{k}$ on the entrepreneur's behavior. See Figure 2.

4.1 Case 1: $k_I^N < \widehat{k} \leq \overline{k}$.

First consider an equilibrium where information about k (from a partition) is absent, and the equilibrium in Section 3.4 obtains. The investor's equilibrium

[7] Since recognition is simply a classification based on uncertainty, there is nothing particular about a system that recognizes low risk investments and expenses high risk investments. The information conveyed would be the same if low risk investments were expensed and high risk investments were recognized.

conjecture is that the entrepreneur would never invest when $k > k_I^N$ and always invest when $k \leq k_I^N$. This is still an equilibrium when the investor is given information as to whether k is higher or lower than $\widehat{k}$ in this region. Therefore, the choice of $\widehat{k}$ in this region has no effect on the investor's pricing or on the entrepreneur's investment behavior.

4.2 Case 2: $k_I^* \leq \widehat{k} \leq k_I^N$.

If the investor knew that $k = \widehat{k}$, the expected price would make the entrepreneur's return on investment non-positive. In fact, it would be negative nor any $\widehat{k} > k_I^*$. Lemma 1 implies that the expected price will produce a negative investment return for any region bounded below by $\widehat{k}$. Therefore, the entrepreneur makes no investments that don't get recognized when $\widehat{k}$ is set in this region. For investments that would be recognized (i.e., $k \leq \widehat{k}$), Lemma 1 and the definition of k_I^N in equation (4) imply that the expected price would produce a positive investment return for the entrepreneur. Therefore, the equilibrium investment hurdle equals the recognition hurdle, $\widehat{k}_I = \widehat{k}$. An implication of Lemma 3 is that the entrepreneur prefers "stricter" (i.e., lower) recognition hurdles in this region, because the recognition hurdle provides a device by which the entrepreneur can credibly not invest in riskier opportunities.

4.3 Case 3: $\underline{k} \leq \widehat{k} < k_I^*$.

When the recognition hurdle is set between these limits, the acceptable set of investments falls into the recognition and the expensing regions. Any investment opportunity qualifying for recognition will be accepted, and some not qualifying for recognition will be accepted. The entrepreneur's investment hurdle $\widehat{k}_I$ is defined as follows.

$$-C + 2\mu - \frac{2\rho\beta}{2\alpha - 1} \cdot E\left[E\left[\widetilde{k} \mid x_1, \widetilde{k} \in \left[\widehat{k}, \widehat{k}_I\right]\right] \mid \widehat{k}_I\right] - \frac{\rho}{2\alpha - 1} \cdot \frac{\beta}{\alpha - 1} \cdot \widehat{k}_I = 0 \quad (5)$$

The left hand side of this expression decreases in both $\widehat{k}$ and $\widehat{k}_I$. Therefore, a decrease in $\widehat{k}$, say to $\widehat{\widehat{k}} = \widehat{k} - \gamma$ ($\gamma > 0$) would make the left hand side positive. An increase in $\widehat{k}_I$ to $\widehat{\widehat{k}}_I = \widehat{k}_I + \lambda$ ($\lambda > 0$) is necessary to re-establish equilibrium.

> **Proposition 1**: Assume a regime in which the recognition hurdle is set so that desirable investment opportunities (to the entrepreneur) fall into both the recognition and the expensing regions. An incremental reduction in the recognition hurdle (i.e., a stricter hurdle) will increase the riskiness of investments that could be undertaken by the entrepreneur.

The interesting implication of Proposition 1 is that setting the recognition hurdle at a point where the entrepreneur might invest in unrecognized assets means that the entrepreneur has an incentive to implement investments that would not be undertaken under full information. And, this "over-investment" increases as the recognition hurdle becomes more stringent, as depicted in Figure 2. The exact shape of the investment limit function between $\underline{k}$ and k_I^* depends on the density $h(k)$, but the investment limit function starts at k_I^N when $\widehat{k} = \underline{k}$ and declines monotonically to k_I^* when $\widehat{k} = k_I^*$.

For instance, during the 1990s internet bubble, efforts to restrict the types of "less-tangible" assets that firms might recognize (e.g., America Online's deferred subcriber acquisition costs) could actually increase the range of investment risk that was desirable to the entrepreneur. In fact, as $\widehat{k}$ approaches $\underline{k}$ from above, the entrepreneur's investment hurdle approaches k_I^N because when $\widehat{k}$ equals $\underline{k}$, there is no information provided about the risk of investments undertaken by the entrepreneur.[8] Perhaps paradoxically, fewer risky investments might have been undertaken if there were standards for recognizing self-developed intangible assets that were below an uncertainty hurdle.

[8] If recognition is a method for conveying information about uncertainty, then a system in which all expenditures are capitalized is equivalent to a system in which all expenditures are expensed. Neither system provides the investor with any information about the uncertainty of future outcomes.

Or, suppose there are two accounting jurisdictions that are identical except that jurisdiction 1 (J1) allows the capitalization of development costs, while the other (J2) does not. In J2, expenditures for research or development are disclosed, but there is no disclosed division of the two. Assume the entrepreneur may have opportunities for expenditures on tangible resources, on product development activities and on research activities. Further, assume that each opportunity has the same expected payoff, but unknown variance. The expenditure types can be ordered by their values of k, with tangible resource purchases having the lowest values, development activities having higher values, and research activities having the highest k values. How would investment behavior differ between the two jurisdictions?

If the investment set under full information – k_I^* – includes some research projects, then entrepreneurs in both jurisdictions will be willing to make investments in research projects that would not be undertaken under full information. However, the entrepreneur in J2 will be willing to invest in riskier research projects than the entrepreneur in J1. The fact that the investor cannot distinguish between spending on development and spending on research in J2 provides "cover" for the entrepreneur in that jurisdiction to accept riskier projects.

Figure 3 depicts the entrepreneur's expected price for each region, conditional on the realization of k. The valuation differs between the two regions, not from any "transperancy" effect (Ahmed, Kilic and Lobo, 2006) but from the economic differences between the two regions. And, expenditures that must be expensed still create value, as was confirmed in Lev and Sougiannis (1996).

Figure 2 shows that each value of $\widehat{k}$ in the Case 3 region produces an investment region that is identical to that produced by a value of $\widehat{k}$ in the Case 2 region. In effect, the entrepreneur/investor behavior under any "conservative" standard (in which some investments are not recognized as assets) is equivalent to their behavior under a "non-conservative" standard (in which all investments are recognized as assets). In either instance, the recognition standard provides an "opportunity" for the entrepreneur to invest in a project that would be declined under full information. (The investor, it should be remembered, is price-protected against this activity. The entrepreneur bears the expected cost of this over-investment.) Proposition 2 considers the entrepreneur's desired recognition hurdle.

> **Proposition 2:** The recognition hurdle $\widehat{k}$ preferred by the entrepreneur is k_I^*. This recognition hurdle increases in the expected payoff of the investment $(2\mu - C)$ and decreases in the risk aversion (ρ) of the investor.

Within the model, the benefits and costs of different recognition hurdles are born by the entrepreneur, and Proposition 2 shows that the hurdle that produces the most value is the minimum level for which the entrepreneur will invest only in recognized assets. If, under a given recognition regime, the entrepreneur would accept some projects that would not be recognized as assets, then there exists another regime with lower recognition requirements that would produce a higher expected payoff to the entrepreneur. In addition, the desired recognition hurdle increases (decreases) in the investment's expected payoff (the investor's risk aversion). So, comparing two situations in which the expected investment payoff differs, the desired recognition hurdle would be higher in the situation with the higher value of $2\mu - C$.

One interesting observation is that the desired recognition hurdle does not depend on the distribution $h(k)$ or on the support $[\underline{k}, \overline{k}]$. As long as both the entrepreneur and the investor know the distribution, the desired value of $\widehat{k}$ depends on the investment hurdle under full information, and this value does not depend on the distribution of $\widetilde{k}$.

Returning to the earlier comparison of Jurisdiction 1 and Jurisdiction 2, the expected price for recognized assets will be higher under J2 than under J1. Or, since all projects have the same cost C, a firm under J2 will have a higher expected "market to book." However, this higher value for recognized assets does not mean that the entrepreneur is better off. In fact, the entrepreneur's expected payoff increases as $\widehat{k}$ increases up to k_I^*, and decreases thereafter.

While reporting discretion (defined as an ability of the entrepreneur to deviate from the formal recognition hurdle) is not a focus of the paper, one can see that discretion in the recognition of assets could be either beneficial or detrimental to the entrepreneur. If $\widehat{k}$ is set below k_I^*, then a modest amount of (known) reporting discretion can increase the entrepreneur's expected payoff (remembering that the investor price protects against the expected use of discretion) by creating a "shadow standard" similar to that in Dye (2002). However, if $\widehat{k}$ is set at or above k_I^*, then reporting discretion increases the effective recognition hurdle, and that is detrimental to the entrepreneur.

4.4 Application to Liabilities

As noted at the outset, the "reasonably certain estimate" condition applies to liabilities as well as to assets, and the model can be modified to consider whether the findings above change for liabilities. Suppose again that the entrepreneur will receive a transaction opportunity. If the opportunity is accepted, then at the start of each of the two periods the customer pays R and the entrepreneur (or investor) spends V.[9] At the end of each period, the entreprenuer's/investor's firm must provide a service costing an additional amount $\widetilde{s}_i, i = 1, 2$. It is assumed that the $\widetilde{s}_i$ are i.i.d., with known mean and unknown variance, and it is this cost that is the focus of the accounting choice.

To fit the accounting model a little more closely, suppose that the first period is divided into two reporting periods – 1A and 1B. Assume that incurring the cost V allows the entrepreneur to say that the revenue has been earned when it is received from the customer, but the recognition of a liability for the end-

[9] If the transaction opportunity is declined, there are no cash flows and nothing is sold to the investor at the end of the first period.

of-period service depends on its level of uncertainty. If the expected service cost is recognized as a liability, then the income in 1A is $R - V - E[\widetilde{s}_1]$ and the income in 1B is $E[\widetilde{s}_1] - s_1$. If the expected service cost is not recognized as a liability, the 1A income is $R - V$ and the 1B income is $-s_1$.

The previous sections' model can be applied, with $\widetilde{x}_i = R - V - \widetilde{s}_i$ and $\mu = R - V - E[\widetilde{s}_i]$. The previous assumptions about the uncertainty of $\widetilde{x}_i$ can be attributed to $\widetilde{s}_i$, with the entrepreneur observing the realization of $\widetilde{k}$ prior to choosing whether to engage in the transaction, and the structure of the transaction opportunity and the distributional assumptions being common knowledge between the entrepreneur and the investor. As in the previous section, a recognition hurdle for the liability provides the investor with information about the riskiness of the transaction that the entrepreneur has implemented. However, Corollary 2.1 shows an interesting difference from the previous findings.

> **Corollary 2.1:** As the expected value of the liability increases (*ceteris paribus*), the desired recognition hurdle decreases, i.e., it becomes more stringent.

This result runs counter to the usual thinking that liabilities with a large expected value should be recognized even if there is significant uncertainty about their ultimate amount (e.g., health care costs for retirees). However, the difference may be attributed to the model's assumption that the investor already knows the expected value of the obligation, and recognition communicates only about uncertainty. In addition, the *desired* recognition hurdle allows the entrepreneur a method of credibly accepting only those transactions that qualify for recognition. So, when $E[\widetilde{s}_i]$ is high, the entrepreneur would like to restrict the accepted transactions to those with very low expected variance, and the low recognition hurdle allows her to do so.

In practice, the classification of liabilities in such cases is not bimodal, but rather trimodal. When post-sale service costs are relatively certain, the entrepreneur would recognize the revenue and the estimated liability. As the post-sale service uncertainty increases, the entrepreneur would not recognize the estimated liability and, as uncertainty increases yet further, the entrepreneur would recognize *neithe*r the revenue *no*r the estimated liability. It is difficult to make sense of this practice from an income level point of view: increases in uncertainty first *increase* income and then *decrease* income.

However, as an uncertainty classification system, the practice makes more sense. One region $\left[\underline{k}, k^{\ell}\right]$ allows for recognition of revenue and the estimated liability, one more region $\left(k^{\ell}, k^{R}\right]$ allows for recognition of the revenue only, and the final region $\left(k^{R}, \overline{k}\right]$ defers the revenue and recognizes no estimated liability. Within the context of the model, there is no informational difference between the regions, except for the uncertainty classification. (For instance, the investor would be able to observe R whether it was recognized as revenue or appeared as a deferred revenue liability.) For some levels of k^{ℓ}, the entrepreneur could benefit from an additional cutoff (k^{R}) to communicate about uncertainty and to reduce the range of uncertainty that she will accept in a transaction.

5 Information Beyond Recognition

5.1 Subclassifications of Recognized Assets

Common accounting practice involves a variety of classifications. The model above considers a classification based on the recognition or non-recognition of an asset following an entrepreneur's investment in a project that is expected to produce (uncertain) future benefits.[10] But even assets that are recognized are subject to further classification, as balance sheets commonly disclose levels of cash, receivables, inventory, and so on. These classifications are usually described in terms of their "closeness to cash," or the length of time or number of transactions that must transpire before the asset's ultimate benefits are received.

[10]In fact, the model above would be informationally equivalent to an accounting system that recognized all investments as assets, with an asset category classification based on $\widehat{k}$.

One might think of these asset classifications as being related to the uncertainty of assets' benefits – perhaps because projects that require additional time or many transactions before receiving a payoff would have less certain payoffs.

In the model above, does the entrepreneur benefit from further subclassifications of recognized assets? That is, if the recognition hurdle $\widehat{k}$ is set in the region $\underline{k} < \widehat{k} \leq k_I^*$, does the entrepreneur benefit from setting an additional "classification" hurdle $\widehat{\widehat{k}} \in \left(\underline{k}, \widehat{k}\right)$? This provides a richer reporting space for the entrepreneur, who can now classify an investment as "recognized and below $\widehat{\widehat{k}}$" or "recognized and between $\widehat{\widehat{k}}$ and $\widehat{k}$" or "unrecognized," and thereby provide additional information to the risk-averse investor.

> **Proposition 3:** Assume a regime in which the recognition hurdle $\widehat{k}$ is set so that desirable investment opportunities (to the entrepreneur) fall into both the recognition and the expensing regions. The entrepreneur receives no ex ante benefit from a finer subclassification of recognized assets by the value of k.

This result is driven by the linear effect of k on the "price protection" of the investor. That is, the expected price protection in region $\left[\underline{k}, \widehat{k}\right]$ is simply the weighted average of the expected price protection in the two regions $\left[\underline{k}, \widehat{\widehat{k}}\right]$ and $\left(\widehat{\widehat{k}}, \widehat{k}\right]$. This phenomenon is caused by the assumption that the risk-averse investor's pricing is linear in mean and variance, and by the linear effect of k on variance. However, no particular assumption is needed for the probability distribution of k.[11] In fact, there is no incremental benefit (ex ante) to public disclosure of k exactly within the recognition region, as long as $\widehat{k} \leq k_I^*$.

The entrepreneur could benefit from a further classification of recognized assets if $\widehat{k} > k_I^*$. With the recognition hurdle set in this region, the entrepreneur

[11] Suppose that k's influence on the outcome variance were $\frac{\theta(k)}{\tau}$ instead of $\frac{k}{\tau}$, with k's density being $h(k)$ and $\theta(\cdot)$ being a continuously increasing function. It would be straightforward to reconfigure the model by making the variance $\frac{\kappa}{\tau}$ with $\kappa = \theta(k)$, which would then have a linear structure like the model.

is willing to invest in projects that exceed the risk hurdle when k is observed by all, k_I^*. Further, the entrepreneur would make no investments that were not recognized as assets. (See section 4.2 above.) Therefore, a subclassification might enable the entrepreneur to communicate that she was not investing in riskier assets, thereby reducing the investor's price protection and increasing the entrepreneur's expected payoff.

5.2 Voluntary Truthful Disclosures

Of course, the lack of benefit from further disclosures about assets is based on an ex ante evaluation. What would happen if the entrepreneur had the opportunity to make a costly, truthful voluntary disclosure after observing the realization of k (but before observing the outcome x_1)? If the entrepreneur can expend D to disclose the realization of k, then the investor must make an equilibrium conjecture about the disclosure region and the investment region. When the value of k is disclosed and outcome x_1 is observed, the price paid by the investor is the following:

$$P(k, x_1) = \mu - \frac{2\rho\beta}{2\alpha - 1} \cdot k - \frac{\rho}{2\alpha - 1} \cdot (x_1 - \mu)^2$$

If k is not disclosed, the investor must make a conjecture about the disclosure region and the investment region. The expected price above is decreasing in k, so if the entrepreneur discloses for one value of k she discloses for all values less than that value. So, the investor's conjectured region of investments without disclosure can be described by an interval $\left[\widehat{\kappa}^d, \kappa_I^d\right]$. In this region, the price paid by the investor following observation of x_1 is the following:

$$P(\widehat{\kappa}^d, \kappa_I^d, x_1) = \mu - \frac{2\rho\beta}{2\alpha - 1} \cdot E\left[\widetilde{k} \mid \widetilde{k} \in \left[\widehat{\kappa}^d, \kappa_I^d\right], x_1\right] - \frac{\rho}{2\alpha - 1} \cdot (x_1 - \mu)^2$$

The following conditions (6) and (7) hold in equilibrium, where $\widehat{k}^d$ is the highest value of k for which the entrepreneur voluntarily discloses its value and k_I^d is the highest value of k for which the entrepreneur will make an investment.

The first equation describes the entrepreneur's choice of disclosure region based on the investor's conjectured disclosure and investment regions. The second equation requires that the conjectured investment region be the correct one.

$$2\mu - C - \rho \cdot \frac{\widehat{k}^d \beta}{\alpha - 1} - D = \mu - C + E\left[E\left[P \mid \widetilde{k} \in \left(\widehat{k}^d, k_I^d\right)\right] \mid k = \widehat{k}^d\right] \quad (6)$$

and

$$2\mu - C - \frac{2\rho\beta}{2\alpha - 1} \cdot E\left[E\left[\widetilde{k} \mid x_1, \widetilde{k} \in \left(\widehat{k}^d, k_I^d\right)\right] \mid k_I^d\right] - \frac{\rho}{2\alpha - 1} \cdot \frac{\beta}{\alpha - 1} \cdot k_I^d = 0 \quad (7)$$

While the result that favorable outcomes are voluntarity disclosed is a familiar one in the disclosure literature, one interesting feature of this disclosure situation is that it has real economic effects (besides the disclosure costs). Recall from section 3.4 that the entrepreneur will "over-invest" when there is no information about k. With voluntary disclosures, if the investor believes that k will be revealed whenever it is less than $\widehat{k}^d$, then a lack of disclosure requires the investor to price protect. It can easily be seen that $k_I^d < k_I^N$, and the entrepeneur will not invest in a range of risky assets that would have been acceptable with no voluntary disclosure.[12]

> **Proposition 4A:** Assume a regime in which asset recognition provides no incremental information to the investor. (I.e., all accepted projects are recognized as assets or no accepted projects are recognized as assets.) In addition, assume that the entrepreneur may voluntarily disclose the value of k. For sufficiently small disclosure cost D, there exists a disclosure region $\left[\underline{k}, \widehat{k}^d\right]$ for the less risky investments and a non-disclosure region for the more risky investments.

> **Proposition 4B:** Assume a regime with recognition hurdle $\widehat{k} \in [\underline{k}, k_I^*)$, so accepted projects with $k \leq \widehat{k}$ are recognized as

[12] For any positive value of disclosure cost, D, there will exist a non-disclosure region $\left(\widehat{k}_d, k_I^d\right]$, and there is always a positive probability of non-disclosure. Therefore, there is no need to assume any off-equilibrium beliefs when the investor receives no information from the entrepreneur.

assets and all other accepted projects are expensed. In addition, assume that the entrepreneur may voluntarily disclose the value of k. For sufficiently small disclosure cost D, there exists a disclosure region with two distinct intervals. One disclosure interval will encompass the less risky investments that are recognized as assets $\left[\underline{k}, \widehat{k}^r\right]$, while the other interval will cover the less risky investments that are not recognized as assets $\left(\widehat{k}, \widehat{k}^{nr}\right]$. In addition, there exists a non-disclosure region with two distinct invervals encompassing more risky recognized assets and more risky investments that are not recognized as assets.

So, even though the entrepreneur receives no ex ante benefit from a further classification of recognized assets, the ex post desire to inform the investor of positive (i.e., low uncertainty) outcomes will cause the entrepreneur to voluntarily incur costs to do so. In effect, the entrepreneur would be better off ex ante if such voluntary disclosures were proscribed. This finding raises the possibility that the entrepreneur might benefit from a lost-cost, standardized asset classification system that diminishes the ex post voluntary disclosure of k at a cost of D. Suppose that the recognized assets could be further subdivided into two classifications at a cost D'.

Proposition 5: Assume a regime with a recognition hurdle $\widehat{k} \in [\underline{k}, k_I^*)$, and assume that the entrepreneur may voluntarily disclose the value of k at a cost D. Further, assume that a required subclassification of recognized assets can be accomplished at a cost $D' < D$. There exist values of D and D' such that the subclassification of recognized assets may provide an ex ante benefit to the entrepreneur.

This result relates to Jorgensen and Kirschenheiter's finding that required disclosure of firm risk would lower share prices ex ante, due to the costs of disclosure. The benefit in Proposition 5 would arise if it were cheaper to set up a classification/reporting system ex ante, where the classification system did not have to verify the actual value of k, but rather only whether k fell above or below the classification cutoff.

The entrepreneur may choose to make voluntary disclosures about investments that are not recognized as assets, but there does not seem to be a natural way for the accounting system to provide additional information other than to describe the nature of the expenditure (research, development, employee training, etc.).[13] As noted above, these voluntary disclosures should reduce the investment risk hurdle in equilibrium, providing some ex ante benefit to the entrepreneur.

6 The Role of Outcome Observability and Other Issues

6.1 Outcome Observability

The investor's observation of the first period outcome is very important to the model because it provides information for the investor to update his estimates of k and τ prior to setting the price. The impact of these updates is that the entrepreneur's expected price (at the time of investment) is decreasing in the value of k that the entrepreneur has observed. If no outcomes were observable prior to setting the price, the investor would only be able to use prior distribution information to set the price. If $E[k]$ were low enough, the entrepreneur would find all projects acceptable and the project would be sold to the investor. If $E[k]$ were high enough, the investor's break-even price is negative and the entrepreneur would invest in no projects. The observation of x_1 allows for an interior investment hurdle.

This issue raises another common distinction between investments that are

[13] See pages 24-27 of the 2005 10-K of Prepaid Legal Services, Inc. for an example of more extensive voluntary disclosure on expenditures that management considers to have future benefits but are not permitted to be recognized as assets.

recognized as assets and those that are not – the speed with which information about results becomes available. For instance, when the entrepreneur spends to acquire inventory, the outcome of that expenditure becomes observable within an operating cycle. When the entrepreneur spends to acquire productive capacity, outcomes begin to be observed as soon as the capacity comes on line. On the other hand, expenditures for research may take years before there is any observation of the outcomes from those activities.

Consider the following modification of the above model. The entrepreneur's investment opportunities last N years ($N > 2$), and any investments made are sold to an investor at the end of year $N - 1$. At the start of period 1, the entrepreneur can invest C to create a string of N random cash flows. The entrepreneur will consume the first $N - 1$ of these cash flows, and the N^{th} will be sold to the investor. As before, the cash flows have a known mean μ and an unknown variance $\frac{\widetilde{k}}{\widetilde{\tau}}$. The substantive difference with this model modification is that the investor will have $N - 1$ outcome observations to learn about the realizations of $\widetilde{k}$ and $\widetilde{\tau}$, rather than just a single observation.

This modification affects the substance of the results very little. The principal difference is that the entrepreneur's expected price (when the investor has imperfect information about $\widetilde{k}$) slopes downward more steeply in the entrepreneur's known value of the outcome, k. The over-investment in Lemma 2 and Proposition 1 will have a lower magnitude, but it will still exist. The effects of a recognition hurdle will remain substantively the same, and the entrepreneur will still receive no ex ante benefit from further subclassification of recognized assets, as long as the recognition hurdle is set so that desirable investments – under full information – may fall in the non-recognition region.

6.2 Multiple Investment Opportunities

Another characteristic of the model is that it considers a single investment project that the entrepreneur receives. The model's implications would not change substantively if the entrepreneur were to consider M independent, identically-distributed investment projects with no budget constraints. In fact, the projects might have differing distributions $h(k)$, and the same recognition hurdle would be desired for all projects.

Two modifications might provide interesting avenues for future research. First, if the entrepreneur faced M investment projects with a budget constraint, then the fact that the entrepreneur invested in a particular project might convey additional information about the project's riskiness, even though the entrepreneur herself has no aversion to risk.[14] A further extension might consider portfolios of projects and their implications for expected return and uncertainty. For instance, an investment in productive capacity without an accompanying investment in intellectual property may leave the entrepreneur producing a commodity that is subject to greater uncertainties than would be the case if intellectual property allowed production of a differentiated product.

6.3 Unknown Mean and Variance

For ease of exposition, the model in Section 3 assumes that both the entrepreneur and the investor know μ, the mean of the operating outcome. However, that model can be modified to allow x_1 to provide information about both the mean and variance of the operating outcomes. Assume that the x_i are independent, identically and normally distributed outcomes with unknown mean M and unknown precision $\Phi = \frac{\tilde{\tau}}{\tilde{k}}$. Let k be defined as above, with marginal distribution $h(k)$. Given k, the conditional distribution of the precision Φ is assumed to be Gamma with shape parameter $\alpha > 1$ and scale parameter $k\beta > 0$. Given the realized precision $\varphi = \frac{\tau}{k}$, the conditional distribution of M is assumed to be normal with mean μ and precision $\lambda\varphi$.

After observing x_1, the investor has the following updated beliefs (DeGroot

[14] The model presented above provides some allowance for this effect. For example, in section 3.4, the investor's observation that an investment took place conveys information that $k \leq k_I^N$.

1979, p. 168-169). The revised mean of the operating outcome is $\widehat{\mu} = \frac{\lambda\mu + x_1}{\lambda+1}$. The updated outcome precision, conditional on k, is distributed Gamma with shape parameter $\widehat{\alpha} = \alpha + \frac{1}{2}$ and scale parameter $\widehat{k\beta} = k\beta + \frac{\lambda(x_1-\mu)^2}{2(\lambda+1)}$. And, as above, the distribution of k is updated based on information Ω, which includes x_1 and any other information the investor might receive or infer. Note that as $\lambda \to \infty$ (that is, the priors on μ become more certain), these expressions converge to those presented in the model above.

These modifications would create modest changes in the pricing formula (equation 1): μ would be replaced by $\widehat{\mu}$, etc. Because the entrepreneur has superior knowledge of outcome variance, but not mean, there are even fewer changes in the expected price calculation (equation 2). The expected value of $\widehat{\mu}$ is μ, the second term in equation (2) would be unaffected, and the last term in equation (2) would have an additional multiplier, $\frac{\lambda}{\lambda+1}$. Therefore, the basic forces affecting the entrepreneur's investment decision would remain unchanged, and the principal results of the simpler model would continue to hold.

7 Conclusions

This paper looks at asset (and liability) recognition practices as a means of communicating to investors about uncertainty. When there exists a "reasonably certain estimate" of an asset's benefits (or a liability's costs), the accounting model recognizes an asset (or a liability). So, when an expenditure intended to create future benefits is expensed, the (risk-averse) investor can make an inference about the variance of those future benefits. And, when a firm discloses a future obligation for which it has accrued no liability, the investor can make an inference about the variance of those future costs. This recognition practice conveys information about risk and affects the price that the entrepreneur expects to receive from investors.

The principal effect of a recognition hurdle in the model is on the types of investment opportunities that a risk-neutral entrepreneur will find to be attractive. One interesting prediction of the model is that "tightening" the requirements for recognition as an asset may increase the investment risk levels that the entrepreneur is willing to accept. So, more conservative accounting can produce less conservative investment behavior.

The model assumes that the investor "price protects" effectively, so the economic consequences of the recognition hurdle and the ensuing investment behavior fall on the entrepreneur. The entrepreneur's preferred recognition hurdle is equal to the investment hurdle under full information – the lowest recognition hurdle for which every accepted project would be recognized as an asset. This hurdle is increasing in the expected payoff from the investment, so the entrepreneur would prefer that the recognition hurdle be relaxed for projects that are expected to produce higher payoff.

When the model is modified to consider liability recognition, this relationship reverses. *Ceterus paribus*, as the expected value of the liability increases, the entrepreneur prefers a "tighter" recognition hurdle, but not because she wishes to hide something from the investor. The model assumes that the investor knows whether a liability has been incurred and the expected value of that liability, regardless of the liability recognition. So, liability recognition conveys information only about the uncertainty of those future costs. In fact, at the entrepreneur's preferred recognition hurdle, she would never incur a liability that would not be recognized. In effect, the recognition hurdle is a mechanism that allows the entrepreneur to communicate to the investor that she is not taking on risky obligations when the overall profitability of the project is modest.

Firms usually provide more information than just totals for recognized assets and totals for expenditures that produce future benefits but don't qualify for asset recognition. Recognized assets might be further subclassified by risk and reported to the investor, or the entrepreneur might – at some cost – voluntarily disclose the risk of an accepted project. For recognized assets, neither of these produce a direct ex ante benefit to the entrepreneur, though voluntary disclosure may be desirable ex post. If it is less costly, a reported subclassification may have an indirect benefit of reducing the amount of resources that the entrepreneur expends on voluntary ex post disclosure.

8 Appendix

8.1 Proof of Lemma 1

First, show that $E\left[E\left[\widetilde{k} \mid \widetilde{k} \in [k_L, k_H], x_1\right] \mid k\right]$ is increasing in k_H. First, consider the interior expectation of an investor who observes a particular realization of x_1 and knows that $\widetilde{k} \in \left[\underline{k}, \overline{k}\right]$. The investor's posterior density for k could be written as

$$\widehat{h}(k \mid x_1) = \frac{f(x_1 \mid k) \cdot h(k)}{\int_{\ell \in \left[\underline{k}, \overline{k}\right]} f(x_1 \mid \ell) \cdot h(\ell) d\ell} \text{ for all } k \in \left[\underline{k}, \overline{k}\right]$$

Now define a cumulative posterior probability that $\widetilde{k} \in [k_L, k_H]$, $\widehat{H}(K_{LH} \mid x_1) \equiv \int_{k_L}^{k_H} \widehat{h}(\ell \mid x_1) d\ell$.

$$E\left[\widetilde{k} \mid \widetilde{k} \in [k_L, k_H], x_1\right] = \int_{k_L}^{k_H} \ell \cdot \frac{\widehat{h}(\ell \mid x_1)}{\widehat{H}(K_{LH} \mid x_1)} d\ell$$

The derivative of this expression with respect to k_H is the following:·

$$\begin{aligned}
& k_H \cdot \frac{\widehat{h}(k_H \mid x_1)}{\widehat{H}(K_{LH} \mid x_1)} + \int_{k_L}^{k_H} \ell \cdot \left\{ -\frac{\widehat{h}(\ell \mid x_1) \cdot \widehat{h}(k_H \mid x_1)}{\widehat{H}(K_{LH} \mid x_1)^2} \right\} d\ell \\
= \; & k_H \cdot \frac{\widehat{h}(k_H \mid x_1)}{\widehat{H}(K_{LH} \mid x_1)} - \frac{\widehat{h}(k_H \mid x_1)}{\widehat{H}(K_{LH} \mid x_1)} \cdot \int_{k_L}^{k_H} \ell \cdot \frac{\widehat{h}(\ell \mid x_1)}{\widehat{H}(K_{LH} \mid x_1)} d\ell \\
= \; & \frac{\widehat{h}(k_H \mid x_1)}{\widehat{H}(K_{LH} \mid x_1)} \cdot \left\{ k_H - E\left[\widetilde{k} \mid \widetilde{k} \in [k_L, k_H], x_1\right] \right\} > 0
\end{aligned}$$

If the expectation is increasing for every realization of x_1, then it is increasing for the expectation over all x_1 given k. The proof that $E\left[E\left[\widetilde{k} \mid \widetilde{k} \in [k_L, k_H], x_1\right] \mid k\right]$ is increasing in k_L proceeds in a similar manner. The final derivative expression (comparable to the one just above is

$$\frac{\widehat{h}(k_L \mid x_1)}{\widehat{H}(K_{LH} \mid x_1)} \cdot \left\{ E\left[\widetilde{k} \mid \widetilde{k} \in [k_L, k_H], x_1\right] - k_L \right\} > 0$$

The proof that $E\left[E\left[\widetilde{k} \mid \widetilde{k} \in [k_L, k_H], x_1\right] \mid k\right]$ is increasing in k requires two steps that – together – show that the posterior distribution for $\widetilde{k}$ is first-order stochastic dominant in k. Fix τ for the time being and, to simplify the notation that follows, fix $\tau = 1$. Define $H(K_{LH}) \equiv \int_{k_L}^{k_H} h(\ell)d\ell$. The posterior density of $\widetilde{k}$ conditional on x_1 and $\widetilde{k} \in [k_L, k_H]$ is the following:

$$\begin{aligned} h(k \quad \mid \quad \widetilde{k} \in [k_L, k_H], x_1) &= \frac{f(x_1 \mid k) \cdot \frac{h(k)}{H(K_{LH})}}{\int_{k_L}^{k_H} f(x_1 \mid \ell) \cdot \frac{h(\ell)}{H(K_{LH})} d\ell} \\ &= \frac{f(x_1 \mid k) \cdot h(k)}{\int_{k_L}^{k_H} f(x_1 \mid \ell) \cdot h(\ell) d\ell} \end{aligned}$$

Recall that x_1 is normally distributed, so $f(x_1 \mid k)$ can be be expressed in terms of $f((x_1 - \mu)^2 \mid k)$ and that implies that the posterior density of $\widetilde{k}$ can be determined in terms of $(x_1 - \mu)^2$ rather than x_1. Let $z \equiv (x_1 - \mu)^2$. The posterior culumative probability distribution can be espressed as

$$\begin{aligned} \widehat{H}(k_0 \quad \mid \quad \widetilde{k} \in [k_L, k_H], z) &= \frac{\int_{k_L}^{k_0} f(z \mid \ell) \cdot h(\ell) d\ell}{\int_{k_L}^{k_H} f(z \mid \ell) \cdot h(\ell) d\ell} \\ &= \frac{\int_{k_L}^{k_0} f(z \mid \ell) \cdot h(\ell) d\ell}{\int_{k_L}^{k_0} f(z \mid \ell) \cdot h(\ell) d\ell + \int_{k_0}^{k_H} f(z \mid \ell) \cdot h(\ell) d\ell} \end{aligned}$$

As $z \equiv (x_1 - \mu)^2$ increases, does the posterior probability of $k < k_0$ decrease? The expression just above is in the form $\frac{A(z)}{A(z)+B(z)}$. Both $A(z)$ and $B(z)$ decrease in z, the question is which decreases faster. $\frac{A(z)}{A(z)+B(z)}$ decreases in z iff $\frac{A'(z)}{A(z)} < \frac{B'(z)}{B(z)}$. If $f(z \mid \ell) = \frac{2}{\sqrt{2\pi\ell}} e^{-\frac{z}{2\ell}}$, then $\frac{\partial}{\partial z} f(z \mid \ell) = -\left(\frac{1}{2\ell}\right) \cdot f(z \mid \ell)$.

$$\begin{aligned} -\frac{1}{2k_0} &= -\frac{1}{2k_0} \cdot \frac{\int_{k_0}^{k_H} f(z \mid \ell) \cdot h(\ell) d\ell}{\int_{k_0}^{k_H} f(z \mid \ell) \cdot h(\ell) d\ell} \\ &= \frac{\int_{k_0}^{k_H} \left(-\frac{1}{2k_0}\right) f(z \mid \ell) \cdot h(\ell) d\ell}{\int_{k_0}^{k_H} f(z \mid \ell) \cdot h(\ell) d\ell} \\ &< \frac{\int_{k_0}^{k_H} \left(-\frac{1}{2\ell}\right) f(z \mid \ell) \cdot h(\ell) d\ell}{\int_{k_0}^{k_H} f(z \mid \ell) \cdot h(\ell) d\ell} = \frac{B'(z)}{B(z)} \end{aligned}$$

By the same process, it can be shown that $\frac{A'(z)}{A(z)} < -\frac{1}{2k_0} < \frac{B'(z)}{B(z)}$. Therefore, the posterior distribution of $\widetilde{k}$ is first-order stochastic dominant in $z \equiv (x_1 -$

$\mu)^2$. It remains to be shown that the distribution of $\widetilde{z}$ is first-order stochastic dominant in k.

Given k, the cumulative distribution of z can be calculated as $2 \cdot N\left[\frac{\sqrt{z}}{k}\right] - 1$, where $N[\cdot]$ is the standard normal cumulative. Given z, this is decreasing in k, so $\widetilde{z}$ is first-order stochastic dominant in k. So, the posterior distribution of k is first-order stochastic dominant in $z \equiv (x_1 - \mu)^2$, and $z \equiv (x_1 - \mu)^2$ is first-order stochastic dominant in k. Therefore, $E\left[E\left[\widetilde{k} \mid \widetilde{k} \in [k_L, k_H], x_1\right] \mid k\right]$ is increasing in k. (Note that this is true for each value of τ, so it will be true when integrating across the possible values of τ.)

8.2 Proof of Lemma 2

The lemma can be shown by considering the value of the left-hand side of equation (4) in the text at k_I^*, and showing it to be positive. A version of the full information limit can be written using the notation in equation (4).

$$-C + 2\mu - \frac{2\rho\beta}{2\alpha - 1} \cdot E\left[E\left[\widetilde{k} \mid \widetilde{k} \in [k_I^*, k_I^*], x_1\right] \mid k_I^*\right] - \frac{\rho}{2\alpha - 1} \cdot \frac{\beta}{\alpha - 1} \cdot k_I^* = 0$$

But Lemma 1 shows that reducing the lower limit of the range of $\widetilde{k}$ will reduce the value of the expectation of k in the above expression. Therefore,

$$-C + 2\mu - \frac{2\rho\beta}{2\alpha - 1} \cdot E\left[E\left[\widetilde{k} \mid \widetilde{k} \in [\underline{k}, k_I^*], x_1\right] \mid k_I^*\right] - \frac{\rho}{2\alpha - 1} \cdot \frac{\beta}{\alpha - 1} \cdot k_I^* > 0$$

Since equation (4) is decreasing in k_I^N, it must be that $k_I^N > k_I^*$, and the range of investment opportunities that are attractive to the entrepreneur increases when there is no information about k available to the investor.

8.3 Proof of Lemma 3

Suppose the entrepreneur could make an ex ante commitment not to invest in any project that had a value of k greater than k_I^c. The expected payoff to the entrepreneur would be the following:

$$\begin{aligned}
& \int_{\underline{k}}^{k_I^c}\left(-C+2\mu-\frac{2\beta\rho}{2\alpha-1}\cdot E\left[E\left[\widetilde{k}\mid\widetilde{k}\in[\underline{k},k_I^c]\right]\mid k\right]-\frac{\beta\rho}{(2\alpha-1)(\alpha-1)}\cdot k\right)h(k)dk \\
= & \int_{\underline{k}}^{k_I^c}(-C+2\mu)\,h(k)dk-\frac{2\beta\rho}{2\alpha-1}\cdot H(k_I^c)\cdot\int_{\underline{k}}^{k_I^c}E\left[E\left[\widetilde{k}\mid\widetilde{k}\in[\underline{k},k_I^c]\right]\mid k\right]\frac{h(k)}{H(k_I)}dk \\
& -\frac{\beta\rho}{(2\alpha-1)(\alpha-1)}\cdot H(k_I^c)\cdot\int_{\underline{k}}^{k_I^c}k\frac{h(k)}{H(k_I)}dk \\
= & H(k_I^c)\cdot\left\{-C+2\mu-\frac{2\beta\rho}{2\alpha-1}\cdot E\left[\widetilde{k}\mid\widetilde{k}\in[\underline{k},k_I^c]\right]-\frac{\beta\rho}{(2\alpha-1)(\alpha-1)}\cdot E\left[\widetilde{k}\mid\widetilde{k}\in[\underline{k},k_I^c]\right]\right\} \\
= & H(k_I^c)\cdot\left\{-C+2\mu-\frac{\beta\rho}{\alpha-1}\cdot E\left[\widetilde{k}\mid\widetilde{k}\in[\underline{k},k_I^c]\right]\right\}
\end{aligned}$$

Differentiating this expression with respect to k_I yields the following:

$$\begin{aligned}
& h(k_I^c)\cdot\left\{-C+2\mu-\frac{\beta\rho}{\alpha-1}\cdot E\left[\widetilde{k}\mid\widetilde{k}\in[\underline{k},k_I^c]\right]\right\} \\
& -H(k_I^c)\cdot\frac{\beta\rho}{\alpha-1}\left\{k_I\cdot\frac{h(k_I^c)}{H(k_I^c)}-\int_{\underline{k}}^{k_I}\frac{k\cdot h(k)\cdot h(k_I^c)}{H(k_I^c)^2}dk\right\} \\
= & h(k_I^c)\cdot\left\{-C+2\mu-\frac{\beta\rho}{\alpha-1}\cdot E\left[\widetilde{k}\mid\widetilde{k}\in[\underline{k},k_I^c]\right]\right\}-\frac{\beta\rho}{\alpha-1}\cdot k_I^c\cdot h(k_I^c) \\
& +\frac{\beta\rho}{\alpha-1}\cdot h(k_I^c)\cdot\int_{\underline{k}}^{k_I^c}\frac{k\cdot h(k)}{H(k_I^c)}dk \\
= & h(k_I^c)\cdot\left\{-C+2\mu-\frac{\beta\rho}{\alpha-1}\cdot E\left[\widetilde{k}\mid\widetilde{k}\in[\underline{k},k_I^c]\right]\right\}-\frac{\beta\rho}{\alpha-1}\cdot k_I^c\cdot h(k_I^c) \\
& +\frac{\beta\rho}{\alpha-1}\cdot h(k_I^c)\cdot E\left[\widetilde{k}\mid\widetilde{k}\in[\underline{k},k_I^c]\right] \\
= & h(k_I^c)\cdot\left\{-C+2\mu-\frac{\beta\rho}{\alpha-1}\cdot k_I^c\right\}
\end{aligned}$$

The expression inside the curly brackets is the entrepreneur's payoff when k is public. It equals zero at k_I^*, so if the entrepreneur could commit ex ante to a maximum risk investment, she would set $k_I^c = k_I^*$.

8.4 Proof of Proposition 1

The proof follows the same lines as Lemma 2. The left-hand side of equation (5) is decreasing in both $\widehat{k}$ and $\widehat{k}_I$. A reduction in $\widehat{k}$ would make the left-hand side of equation (5) positive, requiring an increase in $\widehat{k}_I$ to restore the left-hand side back to zero.

8.5 Proof of Proposition 2

Given a value of $\widehat{k}$, the expected payoff to the entrepreneur can be written as the following:

$$
\begin{aligned}
& \int_{\underline{k}}^{\widehat{k}} \left(-C+2\mu-\frac{2\beta\rho}{2\alpha-1}\cdot E\left[E\left[\widetilde{k}\mid\widetilde{k}\in\left[\underline{k},\widehat{k}\right],x\right]\mid k\right]-\frac{\beta\rho}{(2\alpha-1)(\alpha-1)}\cdot k\right)h(k)dk \\
& +\int_{\widehat{k}}^{\widehat{k}_I}\left(-C+2\mu-\frac{2\beta\rho}{2\alpha-1}\cdot E\left[E\left[\widetilde{k}\mid\widetilde{k}\in\left[\widehat{k},\widehat{k}_I\right],x\right]\mid k\right]-\frac{\beta\rho}{(2\alpha-1)(\alpha-1)}\cdot k\right) \\
= & \int_{\underline{k}}^{\widehat{k}_I}\left(-C+2\mu-\frac{\beta\rho}{(2\alpha-1)(\alpha-1)}\cdot k\right)h(k)dk \\
& -\frac{2\beta\rho}{2\alpha-1}\cdot\int_{\underline{k}}^{\widehat{k}} E\left[E\left[\widetilde{k}\mid\widetilde{k}\in\left[\underline{k},\widehat{k}\right],x\right]\mid k\right]h(k)dk \\
& -\frac{2\beta\rho}{2\alpha-1}\cdot\int_{\widehat{k}}^{\widehat{k}_I} E\left[E\left[\widetilde{k}\mid\widetilde{k}\in\left[\widehat{k},\widehat{k}_I\right],x\right]\mid k\right]h(k)dk \\
= & \int_{\underline{k}}^{\widehat{k}_I}\left(-C+2\mu-\frac{\beta\rho}{(2\alpha-1)(\alpha-1)}\cdot k\right)h(k)dk \\
& -\frac{2\beta\rho}{2\alpha-1}\cdot H(\widehat{k})\cdot\int_{\underline{k}}^{\widehat{k}} E\left[E\left[\widetilde{k}\mid\widetilde{k}\in\left[\underline{k},\widehat{k}\right],x\right]\mid k\right]\frac{h(k)}{H(\widehat{k})}dk \\
& -\frac{2\beta\rho}{2\alpha-1}\cdot\left(H(\widehat{k}_I)-H(\widehat{k})\right)\cdot\int_{\widehat{k}}^{\widehat{k}_I} E\left[E\left[\widetilde{k}\mid\widetilde{k}\in\left[\widehat{k},\widehat{k}_I\right],x\right]\mid k\right]\frac{h(k)}{\left(H(\widehat{k}_I)-H(\widehat{k})\right)}dk \\
= & \int_{\underline{k}}^{\widehat{k}_I}\left(-C+2\mu-\frac{\beta\rho}{(2\alpha-1)(\alpha-1)}\cdot k\right)h(k)dk-\frac{2\beta\rho}{2\alpha-1}\cdot H(\widehat{k})\cdot E\left[\widetilde{k}\mid\widetilde{k}\in\left[\underline{k},\widehat{k}\right]\right] \\
& -\frac{2\beta\rho}{2\alpha-1}\cdot\left(H(\widehat{k}_I)-H(\widehat{k})\right)\cdot E\left[\widetilde{k}\mid\widetilde{k}\in\left[\widehat{k},\widehat{k}_I\right]\right] \\
= & \int_{\underline{k}}^{\widehat{k}_I}\left(-C+2\mu-\frac{\beta\rho}{(2\alpha-1)(\alpha-1)}\cdot k\right)h(k)dk-\frac{2\beta\rho}{2\alpha-1}\cdot H(\widehat{k}_I)\cdot E\left[\widetilde{k}\mid\widetilde{k}\in\left[\underline{k},\widehat{k}_I\right]\right] \\
= & \int_{\underline{k}}^{\widehat{k}_I}\left(-C+2\mu-\frac{\beta\rho}{(2\alpha-1)(\alpha-1)}\cdot k\right)h(k)dk-\frac{2\beta\rho}{2\alpha-1}\cdot H(\widehat{k}_I)\cdot\int_{\underline{k}}^{\widehat{k}_I} k\cdot\frac{h(k)}{H(\widehat{k}_I)}dk \\
= & \int_{\underline{k}}^{\widehat{k}_I}\left(-C+2\mu-\frac{\beta\rho}{(2\alpha-1)(\alpha-1)}\cdot k\right)h(k)dk-\frac{2\beta\rho}{2\alpha-1}\cdot\int_{\underline{k}}^{\widehat{k}_I} k\cdot h(k)dk
\end{aligned}
$$

Differentiating this last expression with respect to $\widehat{k}$ yields the following:

$$\begin{aligned} & \left(-C+2\mu-\frac{\beta\rho}{(2\alpha-1)(\alpha-1)}\cdot\widehat{k}_I\right)\cdot h(\widehat{k}_I)\cdot\frac{d\widehat{k}_I}{d\widehat{k}}-\frac{2\beta\rho}{2\alpha-1}\cdot\widehat{k}_I\cdot h(\widehat{k}_I)\cdot\frac{d\widehat{k}_I}{d\widehat{k}} \\ = & \left(-C+2\mu-\frac{\beta\rho}{\alpha-1}\cdot\widehat{k}_I\right)\cdot h(\widehat{k}_I)\cdot\frac{d\widehat{k}_I}{d\widehat{k}} \end{aligned}$$

For all values of $\widehat{k}\neq k_I^*$, $\widehat{k}_I>k_I^*$ making the expression inside the parentheses negative. For $\widehat{k}<k_I^*$, $\frac{d\widehat{k}_I}{d\widehat{k}}<0$ which makes the derivative above positive. For $\widehat{k}>k_I^*$, $\frac{d\widehat{k}_I}{d\widehat{k}}>0$ which makes the derivative above negative. The expression inside the parentheses equals zero when $\widehat{k}_I=k_I^*$, but that can only happen when $\widehat{k}=k_I^*$. Therefore, the entrepreneur's desired recognition hurdle is equal to k_I^*.

8.6 Proof of Proposition 3

If the recognition region is $\left[\underline{k},\widehat{k}\right]$, the entrepreneur's expected payoff coming from recognized investments can be written as follows:

$$\int_{\underline{k}}^{\widehat{k}}\left(-C+2\mu-\frac{2\beta\rho}{2\alpha-1}\cdot E\left[E\left[\widetilde{k}\mid\widetilde{k}\in\left[\underline{k},\widehat{k}\right],x\right]\mid k\right]-\frac{\beta\rho}{(2\alpha-1)(\alpha-1)}\cdot k\right)h(k)dk$$

If the recognition region is given a further refinement into $[\underline{k},k']$ and $\left(k',\widehat{k}\right]$, the entrepreneur's expected payoffs from these investments can be written as follows:

$$\begin{aligned} & \int_{\underline{k}}^{k'}\left(-C+2\mu-\frac{2\beta\rho}{2\alpha-1}\cdot E\left[E\left[\widetilde{k}\mid\widetilde{k}\in[\underline{k},k'],x\right]\mid k\right]-\frac{\beta\rho}{(2\alpha-1)(\alpha-1)}\cdot k\right)h(k)dk \\ & +\int_{k'}^{\widehat{k}}\left(-C+2\mu-\frac{2\beta\rho}{2\alpha-1}\cdot E\left[E\left[\widetilde{k}\mid\widetilde{k}\in\left[k',\widehat{k}\right],x\right]\mid k\right]-\frac{\beta\rho}{(2\alpha-1)(\alpha-1)}\cdot k\right)h(k)dk \end{aligned}$$

Clearly, the difference between these two expressions hinges on the third term inside the parentheses. Let $H(k)$ be defined as the cumulative probability of k, that is $\int_{\underline{k}}^{k} h(\ell)d\ell$.

$$\begin{aligned}
&\int_{\underline{k}}^{\widehat{k}} E\left[E\left[\widetilde{k} \mid \widetilde{k} \in \left[\underline{k}, \widehat{k}\right], x\right] \mid k\right] h(k)dk \\
= \;& H(\widehat{k}) \cdot \int_{\underline{k}}^{\widehat{k}} E\left[E\left[\widetilde{k} \mid \widetilde{k} \in \left[\underline{k}, \widehat{k}\right], x\right] \mid k\right] \frac{h(k)}{H(\widehat{k})} dk \\
= \;& H(\widehat{k}) \cdot E\left[\widetilde{k} \mid \widetilde{k} \in \left[\underline{k}, \widehat{k}\right]\right]
\end{aligned}$$

Likewise,

$$\int_{\underline{k}}^{k'} E\left[E\left[\widetilde{k} \mid \widetilde{k} \in [\underline{k}, k'], x\right] \mid k\right] h(k)dk = H(k') \cdot E\left[\widetilde{k} \mid \widetilde{k} \in [\underline{k}, k']\right]$$

and

$$\int_{k'}^{\widehat{k}} E\left[E\left[\widetilde{k} \mid \widetilde{k} \in \left[k', \widehat{k}\right], x\right] \mid k\right] h(k)dk = \left(H(\widehat{k}) - H(k')\right) \cdot E\left[\widetilde{k} \mid \widetilde{k} \in \left[k', \widehat{k}\right]\right]$$

Adding these two latter terms together,

$$\begin{aligned}
&H(k') \cdot E\left[\widetilde{k} \mid \widetilde{k} \in [\underline{k}, k']\right] + \left(H(\widehat{k}) - H(k')\right) \cdot E\left[\widetilde{k} \mid \widetilde{k} \in \left[k', \widehat{k}\right]\right] \\
= \;& H(\widehat{k}) \cdot E\left[\widetilde{k} \mid \widetilde{k} \in \left[\underline{k}, \widehat{k}\right]\right]
\end{aligned}$$

So, the division of $\left[\underline{k}, \widehat{k}\right]$ into $[\underline{k}, k']$ and $\left(k', \widehat{k}\right]$ produces no change in the expected payoff to the entrepreneur.

8.7 Proof of Proposition 4

Consider first the recognition region in Proposition 4A and in both regions of Proposition 4B. It can be easily shown by contradiction that if the entrepreneur voluntarily discloses when the realization of $\widetilde{k}$ is k_1, then she discloses for all values less than k_1. Suppose this is not the case, and there exists a $k_2 < k_1$ for which it is better not to disclose. Recall from equation (3) that $E[P \mid k]$ is decreasing in k, and let $E[P \mid \emptyset]$ be the expected price when there is no disclosure. This would imply

$$E[P \mid \emptyset] \;\geq E[P \mid k_2] - D > E[P \mid k_1] - D \geq E[P \mid \emptyset]$$

which is a contradiction. Therefore, the disclosure region is "left-tailed" and if any realizations of k are disclosed, it will be the lower values.

To see that not all realizations will be voluntarily disclosed, first consider the recognition region in Proposition 4B. The expected (gross) benefit from disclosure when $\widetilde{k} = k^d$ can be found by subtracting the expected price with no disclosure from the expected price with disclosure.

$$\begin{aligned}
& \left\{\mu - \frac{\rho\beta}{\alpha - 1} \cdot k^d\right\} \\
& - \left\{\mu - \frac{2\rho\beta}{2\alpha - 1} \cdot E\left[E\left[\widetilde{k} \mid \widetilde{k} \in \left[k^d, \widehat{k}\right]\right] \mid k^d\right] - \frac{\rho\beta}{(2\alpha - 1)(\alpha - 1)} \cdot k^d\right\} \\
= \; & \frac{2\rho\beta}{2\alpha - 1} \cdot \left\{E\left[E\left[\widetilde{k} \mid \widetilde{k} \in \left[k^d, \widehat{k}\right]\right] \mid k^d\right] - k^d\right\}
\end{aligned}$$

The expression inside the curly brackets approaches zero as k^d approaches $\widehat{k}$. Therefore, there will be a nondisclosure region to the left of $\widehat{k}$, as long as $D > 0$. For Proposition 4A and for the non-recognition region of Proposition 4B, the gross benefit approaches zero as the disclosure limit approaches k_I^*, so there will be a nondisclosure region to the left of k_I^*, as long as $D > 0$.

8.8 Proof of Proposition 5

Recall from Proposition 3 that the subdivision will have no effect on the expected price to be received from the investor. Therefore, the effect of the subclassification would be limited to the costs of voluntary disclosure. Without the subclassification, assume that the entrepreneur makes a voluntary disclosure whenever $k \leq k^d < \widehat{k}$. If $H(k^d) = \int_{\underline{k}}^{k^d} h(k)dk$, then the expected disclosure cost for the entrepreneur is $D \cdot H(k^d)$. Suppose the entrepreneur can incur a fixed cost D' and subdivide the recognized assets into two reported classes. Set the subdivision at k^d. There will be no voluntary disclosure for projects which fall into the more risky subclassification of recognized assets: if it were economic to make such disclosures, they would have been made without the subclassification.

Voluntary disclosures may still be made for some of the projects that fall into the less risky subclassification of recognized assets, but not for all of them. A new – smaller – region will result, with voluntary disclosure occurring when $k \leq k^{d\prime} < k^d$ and with expected disclosure cost of $D \cdot H(k^{d\prime})$. As long as $D' < D \cdot \left(H(k^d) - H(k^{d\prime})\right)$, the entrepreneur is better off.

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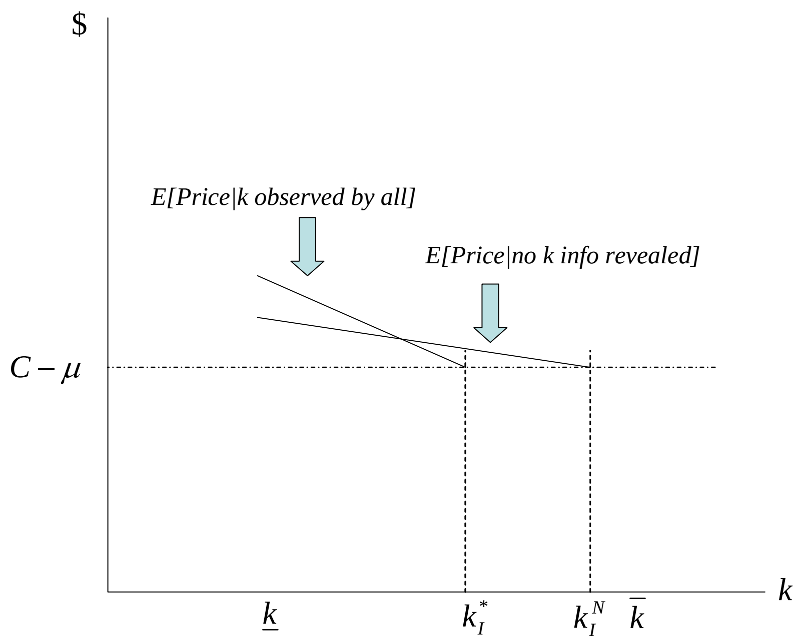

Figure 1: Expected price as a function of k

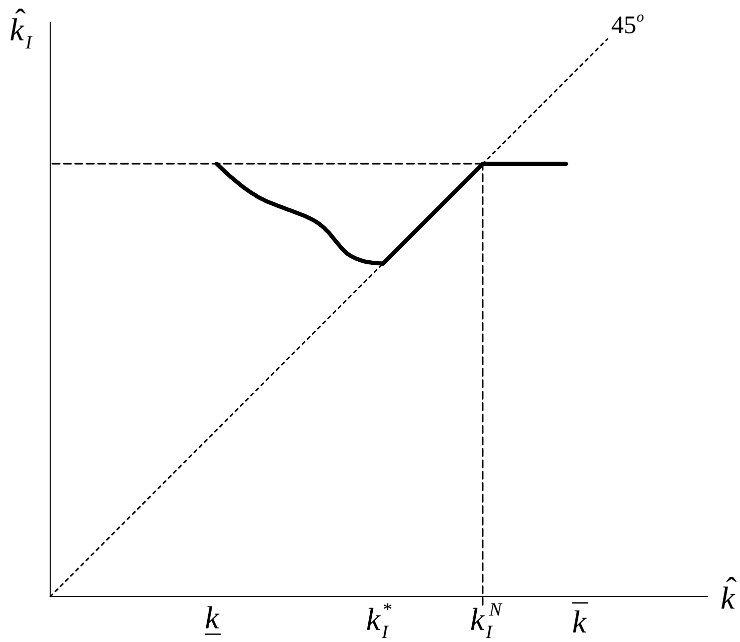

Figure 2: Investment hurdle as a function of the recognition hurdle, $\widehat{k}$

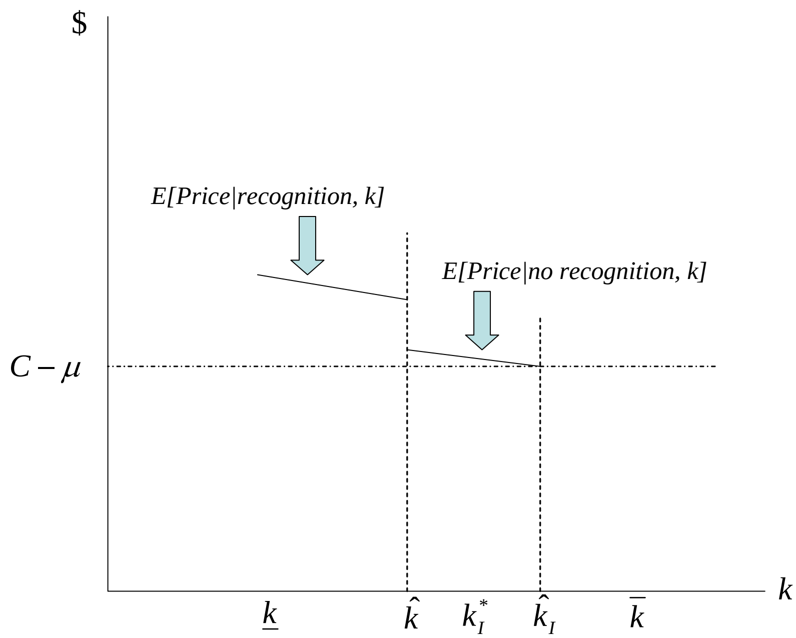

Figure 3: Expected price as a function of recognition and k