Using Voluntary Disclosure Intensity to Infer the Precision of Management's Vision

Miles B. Gietzmann
Faculty of Finance
Cass Business School

Adam J. Ostaszewski
Department of Mathematics
London School of Economics

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Abstract

Shin (2006) has argued that in order to understand the equilibrium patterns of corporate disclosure, it is necessary for researchers to work within an asset pricing model framework in which corporate disclosures are endogenously determined. Furthermore, he argues that without such a framework optimal disclosure strategies may seem counterintuitive. With this in mind, we generalize the Dye (1985) and Penno (1997) upper-tailed disclosure models, so that management's strategic disclosure behaviour can be shown to result in an optimal observable disclosure intensity. We show why a higher equilibrium disclosure intensity may need to be interpreted as implying management has less precise forecasts of future firm value (hence the precision of management's vision of the title). The derived results call into question the specification of empirical studies which test whether firms with higher disclosure intensity will face a lower cost of capital. Working within a generalized Dye-Penno framework this research shows why in equilibrium the converse case applies.

1 Introduction

Companies recognize that implementation of a news-disclosure strategy will affect market value. Simultaneously investors infer that observed disclosure patterns are driven by company type: that is, investor responses (in terms of trading behaviour, and therefore stock price) are guided by beliefs as to the company's type. In this respect some theoretical disclosure models are not readily amenable to empirical investigation, because the key parameters upon which equilibrium beliefs are based are not empirically observable. This research offers an equilibrium model of *market* response to voluntary news-disclosure in a form readily amenable to empirical research design. Specifically, this research establishes why in equilibrium investors may be assumed to act as if they base beliefs upon the observed disclosure intensity of a company. As the starting point for the theoretical modelling, we draw upon Dye's disclosure model and his theorem (see below for details) that in equilibrium, when managers are ex-ante informationally partially-endowed, they will only voluntarily disclose news that has been perfectly revealed to them if it is sufficiently good – above an optimal (equilibrium) cutoff. This is succinctly described as the adoption of an upper-tailed disclosure strategy.

Until now the Dye framework has not been readily amenable to empirical study. One of the difficulties is that an underlying parameter (probability of receiving information, which is referred to here as the *information uncertainty*) is a latent variable that may vary between companies. Dye's model posits a distribution of company value dependent in part upon an exogenously given information endowment faced by management. This research generalizes the setting and specifies *endogenously* how management will optimally choose their information uncertainty; Section 2 develops this theme. Working within such an equilibrium framework, it then becomes possible (see Section 3) to show how the optimal disclosure strategy of a company implies an observable disclosure intensity, and how that in turn can be used to form inferences about the underlying parameters of the company which determine the actual (rational) equilibrium market valuation.

Section 4 provides the principal mathematical formulas which permit tractability of the proposed approach; Section 5 sketches possible empirical research design. Concluding commentary is in Section 6. The bulk of the mathematical analysis is split into appropriate appendices.

2 The Dye Disclosure Calculus with Endogenous Information Endowment

The text below reviews the Dye disclosure calculus and in addition briefly reviews two important issues: in (2.1) the no-arbitrage foundation of the Dye approach (and the associated 'risk-neutral' valuation measure) and in (2.2) some established, alternative ways in which risk may be measured, and utility modelled in portfolio theory – these constructs are central to the model presented here. Next endogenous selection of information endowment by management is introduced in subsection (2.3). It is shown how the choice model of information endowment is consistent with established utility theory approaches in portfolio theory, in particular those where downside and upside risk are not viewed as having symmetric influences on preferences. This is key to further developing the Dye calculus, which is essentially driven by a lower partial moment computed over the range up to the Dye cutoff[1]. Subsection (2.4) links all the above subsections together to show how the model of disclosure strategy leads to a well defined and tractable definition of disclosure intensity.

2.1 No-Arbitrage Risk-Neutral Valuation with Dye's disclosure calculus

In the Dye model there is a rational (equilibrium) reason why management might not disclose information voluntarily (a relaxation of the unravelling paradigm). This necessitates a procedure (due to Dye, further developed below) enabling investors to value the company at other than the Grossman-and-Hart (1980) unconditional minimum (in which 'bad news' is assumed), when investors observe non-disclosure. We point out that Dye's disclosure-cutoff should be viewed as yielding a valuation based on the methodology of arbitrage-free pricing. That is, it is determined by a *no-arbitrage condition*, which has implications for the valuation of an embedded 'disclosure' option, to be discussed below.

When analyzing information flows, the Dye disclosure model assumes three distinctive time $\theta = 0, 1, 2$: ex-ante, interim and terminal times. In the model a random variable X, relating to company valuation (see below for a comprehensive framework for this), has density $f(x)$, an associated

[1] Note that the equilibrium cutoff is below the opening expected value.

distribution function $F_X(x)$ and an ex-ante (i.e. at time $\theta = 0$) expected value m_X. A realization of the random variable is observed by management at the interim time with a probability $1 - p$ (drawn independently of X). Management's decision whether or not to disclose an observed realization of company value x is a voluntary (strategic) decision. Dye (1985) establishes that under continuity and positivity of f there exists a unique threshold value $t = \gamma$ at which management will be indifferent between disclosure or non-disclosure. Here γ will be called the *Dye cutoff*. The indifference point is characterized by equality between a credibly disclosed value γ and the valuation formed by investors when they face non-disclosure (ND); the latter is formally $E[X|ND(\gamma)]$, the computed expected value of the company, conditioned on the absence of information (non-disclosure) of value below γ. That is, the indifference is described by what is termed the *Dye indifference equilibrium* equation:

$$\gamma = E[X|ND(\gamma)]. \tag{1}$$

Under the assumptions above, this implicit definition of a cutoff value γ in fact determines it uniquely; whenever context demands, we emphasize the cutoff's dependence on the underlying information uncertainty p of the manager and the specific model X of firm value by using the notation $\gamma(p)$ or $\gamma_X(p)$. Later, when referring to a family of distributions F parametrized by σ a more convenient variant of this notation will suggest itself.

Based on the assumption of a rational expectations equilibrium (in respect of a conjectural threshold value for the manager's cutoff), Jung and Kwon (1988) derive (their equation (7)) the equation satisfied by γ to be

$$\frac{p}{1-p}(m_X - \gamma) = H_X(\gamma), \tag{2}$$

where

$$H_X(t) := \int_{x \le t} F_X(x)dx.$$

$H_X(t)$ is the lower first partial moment below t, well-known in risk management[2]. As this function is central to the Dye calculus, in our analysis we explicitly name it the *hemi-mean function*. That is, to summarise the Dye valuation equation is made up of just three essential components:

[2]See for example McNeil, Frey and Embrechts (2005), Section 2.2.4.

- $\lambda := \frac{p}{1-p}$ = the management's information uncertainty expressed as *odds,*

- $z := m_X - \gamma$ = the market downgrade resulting from non-disclosure,

- $y := H_X(\gamma)$ = the hemi-mean measure[3] of risk that non-disclosure will occur.

The appeal of this form lies in the separation of the two independent factors of the model: the information odds, i.e the ratio $p/(1-p)$, which characterizes management information technology on one side, and on the other a convex function H_X containing all the information[4] on the distribution of X. The equation (2), when rewritten as

$$p(m_X - \gamma) = qH_X(\gamma), \tag{3}$$

shows the expected downgrade $p(m_X - \gamma)$, i.e. downgrade conditional on the manager receiving no information, balancing what must be some form of 'expected upgrade', conditional on the manager receiving information. Indeed some integration by parts shows why $H_X(\gamma)$ is the essential term in the expected upgrade given by the upper partial first moment above γ.[5]

A feature of the Dye equation, critical to later analysis, is its positive homogeneity[6], in the sense that $H_{\alpha X}(\alpha t) = \alpha H_X(t)$ for $\alpha > 0$. This simplifies

[3] $H_X(t)$ is strictly convex, positive and asymptotic to $t - m_X$ (by l'Hôpital's Rule); clearly $H_X(\underline{X}) = 0$ and $H'_X(\underline{X}) = F_X(\underline{X}) = 0$, where $\underline{X}$ is the lower boundary of the support of F_X (possibly $-\infty$, as for the normal, if that is admissible).

$$\lim_{t\to+\infty} \frac{\int_{x\le\gamma}(t-x)dF(x)}{t-m_X} = \lim_{t\to+\infty} \frac{\int_{x\le t} dF(x)}{1} = 1.$$

[4] See Ostaszewski and Gietzmann (2008). **Proposition.** *Let $H(t)$ be any twice differentiable, strictly convex function on $[\underline{X}, \overline{X}]$ satisfying $H(\underline{X}) = 0$, $H'(\underline{X}) = 0$ and $H'(\underline{X}) = m$. Then $H(t)$ is the hemi-mean function of a continuous distribution with mean given by*

$$m = \overline{X} - H(\overline{X}).$$

This need reinterpretation when either limit of the support interval is infinite. For instance, when $\overline{X} = +\infty$,

$$m = \lim_{t\to\infty} (t - H(t)).$$

[5] $\int_{u\ge\gamma}(u - m_X)dF(u) = \int_{u\le\gamma}(m_X - u)dF(u) = (m_X - \gamma)F(\gamma) + \int_{u\le\gamma} F(u)du$

[6] $E[\alpha X|\alpha X < \alpha\gamma] = \alpha E[X|X < \gamma]$ for $\alpha > 0$; or note that

$$H_{\alpha X}(\alpha\gamma) = \int_{\alpha x\le\alpha\gamma} \Pr(\alpha X \le \alpha x)d(\alpha x) = \alpha \int_{x\le\gamma} \Pr(X \le x)dx = \alpha H_X(t).$$

intra-firm comparisons based on the Dye calculus; writing m_j for firm j's mean $E[X_j]$ and $\bar{\gamma}_j$ for γ_j/m_j, one may cancel by m_j on both sides of the Jung and Kwon equation, and so replace firms with different means by equivalent 're-scaled' firms with equal means.

The framework above can embrace an alternative interpretation: X may validly be replaced by a noisy signal of the true value X , say by $T = T(X, Y)$, where Y models noise. Then one may deduce (see Appendix 1) the existence of a cutoff γ_T above which the noisy signal T would in equilibrium be voluntarily disclosed. Given a disclosure, investors would then form expectations conditioning on the reported noisy signal, and the market values the firm at $E[X|T]$ rather than at T. That is, referring to the *regression function* $\mu_X(t) := E[X|T = t]$, the value is $\mu_X(T)$. If, however, no disclosure occurs, then the market valuation is $\mu_X(\gamma_T)$. The Dye disclosure calculus remains valid in a noisy setting, provided the X in Dye's model is interpreted, not as the true firm value, but as $E[X|T]$, the estimated firm value given T. This is subject to $\mu_X(t)$ being an increasing function. All that needs doing in the Jung and Kwon equation is to replace H_X by another, related, hemi-mean function; the cutoff $\gamma_{E[X|T]}$ defined implicitly in the amended Jung and Kwon equation then is $\mu_X(\gamma_T)$. (Here γ_T is, as above, the disclosure cutoff for the actual signal T.) For details see Appendix 1 (Isotonic transformations).

A particular example we pursue is $T = XY$ with $X = m_X e^{U - \frac{1}{2}\sigma_U^2}$ and $Y = e^{V - \frac{1}{2}\sigma_V^2}$ with U, V independent, normal zero-mean random variables with variances σ_U^2 and σ_V^2. Thus T is log-normally distributed, in accord with the financial benchmark model, as is the signal X. Here $T = m_X e^{W - \frac{1}{2}\sigma_W^2}$ with $W = U + V$, a mean-zero normal with variance $\sigma_W^2 = \sigma_U^2 + \sigma_V^2$. The explicit formulas for $\mu_X(t)$ in this case are given in Section 4 below[7].

To clarify the no-arbitrage underpinnings of Dye's calculus, the market itself is modelled as *arbitrage-free* and has available a probability distribution function F_X which models events corresponding to the three times $\theta = 0$ and $\theta = 1$ and $\theta = 2$ in such a way that this distribution fully reflects the market price of risk at any of these points in time. That is, any contingent contract traded on the market is priced by computing an expectation of the claim under this distribution. This presumes the so-called complete market

[7] For the special case of X, Y Gaussian and $Z = X + Y$ the regression functiom, $\mu_X(z)$ takes the familiar linear format $m_X + \kappa(z - m_X)$; this was used by Penno (1997).

hypothesis and asserts that the distribution itself is an observable, i.e. there is a sufficient range of traded instruments to select a distribution from a proposed parametrized family and so to identify the density of the risk-neutral measure f. Thus the variance at $\theta = 0$ is known, as is $m := E_{\theta=0}[X]$, the market share price of the firm at time $\theta = 0$. This is implicitly part of the Dye framework. Dye assumes that p is known and so any investor can obtain the market value of the firm, denoted $V(t)$, in the circumstance when the manager is known to use an (arbitrary) cutoff t. That is, $V(t) = E_{\theta=1}[X|N(t)]$, where $N(t)$ is the event at $\theta = 1$ of a non-disclosure occurring through the application of a disclosure cutoff of t. That is $m = E_{\theta=0}[X \cdot 1_{D(t)} + V(t) \cdot 1_{N(t)}]$. There is, however, a unique value $t = \gamma$ for which $V(t)$ assumes a minimum, and that value is characterized by the (unique) solution of the equation $t = V(t)$. That solution yields the minimum valuation consistent with the information then available, and coincides with the value at which the manager is indifferent between disclosing and not disclosing. It is this indifference pricing approach that characterizes Dye's own justification for equilibrium. So this is also the unique value of t for which $m = E_{\theta=0}[X \cdot 1_{D(t)} + t \cdot 1_{N(t)}]$. It is well-known that γ as a function of p is strictly monotone.

The market-model approach implies that, in the event that there is no disclosure at $\theta = 1$, the new market value, whatever it is, reveals the value of p through the equation $\gamma_{\text{revealed}} = \gamma(p)$. That is, there is an 'implied p'. Dye assumes that the market has a belief about p, but is mute in regard to how the market has discovered p.

2.2 Preferences and Expected Utility When it is not Appropriate to Measure Risk by (symmetric) Variance

As the above discussion of the Dye calculus makes clear the lower first partial moment, $H_X(t)$, is critical to the analysis. Put simply, investors are concerned about the probabilities of disclosure and non-disclosure, and non-disclosure results in equilibrium revaluation of the firm below its previous mean value m_X to γ. That is, investors focus on the possibility of non disclosure "occurring in the lower tail" of the $F_X(x)$ distribution. In Dye, key parameters such as the information uncertainty p are exogenously assumed. In contrast this research develops a model of rational choice over p and hence explicit modelling assumptions need to be made concerning management preferences. The issue here is that the investor preferences critically relate to concern with lower tail events (non-disclosure), and hence traditional symmetric mean - variance preferences are not appropriate.

Modelling preferences that reflect differential concern with lower (versus upper) tail events, has long been a concern of the portfolio management literature, recognizing the need to steer away from the mean-variance paradigm. This was already acknowledged by Markowitz (1959) explicitly in the seminal work, as he also proposed that semivariance be used to measure the risk of a portfolio, but did not exploit this[8]. Subsequently, a more general risk-measure (for below-target t risk) was studied by Fishburn (1977), which he called the (α, t)-model, namely

$$F_\alpha(t) := \int_{\leq t} (t-x)^\alpha dF(x), \qquad (\alpha > 0).$$

(with t an exogenous target), and showed it to be tractable. See immediately that the hemi mean-function, $H_X(t)$, appearing in the Dye equation is just F_1 for $F = F_X$, i.e. a lower partial moment of order $\alpha = 1$, cf. McNeil-Frey-Embrechts (2005); indeed, an integration by parts yields

$$H_X(t) = \int_{\leq t} (t-x) dF_X(x).$$

Fishburn studies preferences over distributions F representable by a utility $U(\mu(F), \rho(F))$ over two parameters associated with F: the mean $\mu(F)$ and a risk-measure $\rho(F)$ of the general form $\int_{\leq t} \varphi(t-x)dF(x)$ for φ non-negative, non-decreasing with $\varphi(0) = 0$. The latter captures notions of 'riskiness' for outcomes x below the target t. Both parameters are expectations under F and so Fishburn's preference is a 'utility of expectations' rather than an expected utility in the von Neumann-Morgenstern sense.

In this connection we recall Fishburn's result, when specialized to the case $\varphi(t-x) = t-x$, that for such a dominance to be consistent with an expected utility, specifically taking the form $E_F[v(X,t)]$ for some $v(x,t)$ increasing in y, with $v(t,t) = t$ and $v(t+1,t) = t+1$, it is necessary and sufficient for the existence of a constant $k = k(t) > 0$ such that

$$v(x,t;k) := \begin{cases} x, & x \geq t, \\ x - [k(t)(t-x)], & x \leq t. \end{cases}$$

[8] Possibly the reason he did not develop this is, because the analysis of semivariances was not known to be tractable at that time.

Moreover,

$$E_F[v(X,t)] = \mu(F) - k(t)\rho(F).$$

Call the utility v here the Fishburn *kinked* utility, to distinguish it from the utility $U(\mu(F), \rho(F))$ above. The kinked utility has left-sided slope[9] at t greater than the right-sided slope, which recognizes the greater aversion to performance below target.

To summarise: in general, one can still analyse preferences with traditional expected utility analysis when risk is measured by a more general Fisburn risk measure $\rho(F)$, rather than symmetric variance σ^2, provided one identifies the appropriate kink location t in the utility function.

2.3 Endogenising the information endowment in the Dye model

The original Dye theory was an existence of equilibrium non disclosure result and was not designed to explain differences in disclosure cutoff between firms. Thus, in order to be able to apply the Dye paradigm within an empirical setting in which firm cutoffs vary, one needs to be able to develop a rational equilibrium model, in which different managers choose different Dye cutoff's leading to different observable disclosure practices. The fundamental modelling assumption to be introduced here is that managers of firms are represented by a one-parameter family of distributions; this is justified by the positive homogeneity (in relation to the firm's initial value) of the Jung and Kwon equation as discussed in subsection 2.1. Thus manager types are represented by a single parameter called σ. In the case of lognormal models, σ^2 is interpreted as the variance of the underlying normal variable responsible for adding noise to the true firm value X in the received signal T. Thus given differences between managers, there may exist incentives for them to act (disclose) differently in equilibrium. At issue is how to introduce incentives for managers with differing σ.

To model endogenous uncertainty a reason needs to be introduced why

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[9] Since $v(t-1,t) = t-1-k$, one has

$$k(t) + 1 := \frac{v(t,t) - v(t-1,t)}{v(t+1,t) - v(t,t)},$$

and so $k(t)$ is independent of any scaling or shifting in the utility space.

a manager may have a preference over specific p values[10]. The following approach is adopted. Critical to the Dye analysis is that managers cannot verifiability communicate that they have not observed any news, and so the market downgrades the firm from μ_X to γ when non-disclosure occurs. It is assumed, however, that while the manager cannot communicate in a verifiable manner they can nevertheless trade[11]. Specifically, we assume that managers can extract a benefit from the informational asymmetry with investors that already exist in the Dye model[12]. At the interim time $\theta = 1$, the manager potentially has more information than the investor, either because (i) the manager has observed and chosen not to disclose a value, or (ii) no observation of value has occurred. In case (ii) the Dye model assumes that the there is no verifiable way of communicating that the manager has received no information. In both circumstances the firm has been downgraded. If under these circumstances the manager can benefit from trading in some way, then the manager can be modelled as having preferences over p. The trading in case (i) is excluded as it is illegal (insider dealing, given that the observation is verifiable after the fact). Instead attention is directed to case (ii).

Recalling the Jung and Qwon equation (2), and referring to:

$$z = z(\gamma) := m_X - \gamma, \text{ and } y = y(\gamma) = H_X(\gamma). \tag{4}$$

equation (2) becomes:

$$\frac{p}{1-p} z(\gamma) = y(\gamma),$$

which relate to the downgrade loss z and risk shielding gain y. Now preferences may be defined over the two constructs z and y in terms of a utility function $U(z, y)$ with domain the positive quadrant and the choice of p is reduced to determining the solution of the optimization problem:

$$\max_p U(m_X - \gamma(p), H_X(\gamma(p))) = \max_p U(z(\gamma(p)), y(\gamma(p))).$$

[10] Our fundamental modelling variable is p because it is the (chosen) information endowment of the manager that introduces uncertainty. For instance in the extreme case $p = 0$ management always observe the realisation of the random variable x and in the other polar extreme case $p = 1$ they never observe the realisation.

[11] Or alternatively simply that they benefit from the downgrading eg via the fact that ESOP may now be set at a lower (strike) price.

[12] We make the traditional Kyle noisey trading assumption so investors can't infer with probability one that managers knew they did not know.

Equivalently, the optimal $p = p^*$ may be determined via $z^* = m_X - \gamma(p)$, where

$$(y^*, z^*) := \arg\max\{U(z, y) : \; y = H_X(m_X - z)\}.$$

This uses the fact that: (i) $\gamma(p)$ is strictly monotone in p, for which see Jung and Kwon (1988), and (ii) the function $H_X(t)$ is strictly convex for a positive density f_X, so the opportunity set $\{(y, z) : y = H_X(m_X - z)\}$ is strictly convex (compare footnote 5 in Section 2.1). In Appendix 3, we show that for $U(y, z)$ concave and homothetic, there will be a unique pair (y^*, z^*).solving the maximization problem. When $U(y, z)$ is homothetic or homogeneous of degree 0 that unique pair is characterized by the additional optimality conditions[13]:

$$u(\lambda^*) = F(\gamma^*) \text{ and } \lambda^* = y^*/z^*, \tag{5}$$

where $u(\lambda) = U_z/U_y$ is the marginal rate of substitution.

For U one of the standard homothetic utility functions, the mrs (marginal rate of substitution) function $u(\lambda)$ is increasing in λ. As the cumulative distribution function $F(t)$ is also an increasing function, the *utility effect* of selecting higher odds in equilibrium is to increase the disclosure cutoff: thus increasing the uncertainty parameter p leads to increasing the chances of non-disclosure occurring (when the manager is informed). This parallels and preserves the *gain-to-loss effect* of the Dye model: since the gain-to-loss ratio represented by $\Lambda(\gamma) := H(\gamma)/(m - \gamma)$ is increasing in γ, the Dye equation $\lambda = \Lambda(\gamma)$ implies that increasing the odds λ (exogenously, as is the case in Dye's model), or equivalently increasing p, leads to an increased cutoff.

The combined effect of the two equilibrium conditions in (5) can best understood in the special case of a Cobb-Douglas utility $U_{\text{C-D}}(y, z) = y^\alpha z^\beta$ for which one has $u(\lambda) = \beta\lambda/\alpha$ and so

$$\frac{\beta}{\alpha} = \frac{H(\gamma)}{(m - \gamma)F(\gamma)}. \tag{6}$$

That is, the 'growth rate' of $H(\gamma)$, as represented by the right-hand-side (for a technical clarification see Bingham et al. 1987) balances the substitution coefficient β/α. (For the general situation see Appendix 3).

Note that the above optimization should be read as maximizing a ranking $U(\mu(F, p), \rho(F, p))$ over the parameters $\mu(F, p) = m_X - \gamma(p)$, i.e. an adjusted

[13] The equilibrium condition places constraints on the size of λ since $0 < F(\gamma) \leq F(m) < 1$. Note that $\lambda < 1$ iff $p < 1/2$.

mean, and $\rho(F, p) = H_X(\gamma(p))$, a lower partial moment, as in the Fishburn analysis of Section 2.2. It is thus capable of being interpreted as an expected utility (rather than a utility of the two quantities μ, ρ which are expectations under the model (F,p)).

For a range of explicit trading mechanisms it is possible to derive the implied preferences in the form of a utility function and such a derived function is called the manager's *implied utility.* In this connection it is interesting to recall related results of Fishburn (1977); there the optimization criteria are formulated using targets and lower partial moments of arbitrary order (with target analogous to Dye thresholds) and corresponding implied (expected) utility functions are derived[14].

2.4 Endogenous optimal disclosure intensity

Having established in the previous section that managers can be viewed as optimising of p we return now to the fundamental question of what does this mean about the observed disclosure strategy. It turns out that this is now relatively easy to answer if attention is focused upon the disclosure intensity of such a strategy.

Disclosure occurs when management are informed (which occurs with probability $1-p$) and that informed value, be it the value X or $T(X,Y)$, is above the (respective) cutoff γ which occurs with probability $1-F(\gamma)$. Here F denotes the probability distribution function F_X if X is observed above γ_X, or F_T if T is observed above $\gamma = \gamma_T$ (cf. Appendix 1). Thus it is natural to define a firm's disclosure intensity τ as:

$$\tau = (1-p)(1-F(\gamma)).$$

So at issue next is whether the firm's disclosure intensities τ varies in a systematic way (e.g. monotonically) with the underlying uncertainty (noise) faced by management represented by the relevant model parameter σ, be it measured by σ_X when X is observed or by σ_Y when $T = T(X,Y)$ is observed with σ_X fixed. (These are the appropriate parameters for the model $T = XY$ of Section 2.1 above.) As we shall comment later in Appendix 6, Penno (1997) produced an existence result which suggested that one should not assume that the relationship between τ and σ may be a simple monotonic function, and

[14] See also the more recent Pedersen and Satchell (2002) who also look at targets and lower partial moments.

hence that inference of the relative underlying uncertainty faced by different management from disclosure intensity would be problematic[15]. Actually this suggestion turns out to have been over-pessimistic, as the following example shows.

The structurally minimal model. We illustrate the theory developed so far in a tractable way by concentrating on details of the pay-offs that may arise, rather than on abstract utilities above. We concentrate on the expression pz, i.e. $p(m_X - \gamma(p))$, which arises in (3), the notation here stressing the dependence of γ on p. With probability p a payoff $m_X - \gamma(p)$ may arise to the manager in the following circumstance: the manager knows that no new information is available on the company's future value, but investors have nevertheless downgraded the value of the firm (because of non-disclosure). Here $\gamma(p)$ may be interpreted as γ_X (corresponding to the case when X is observed), or as $\mu_X(\gamma_T)$ (when T is observed). Conditional on this absence of information, the manager could, if permitted, buy the stock at the interim market price γ, and then liquidate the stock at the terminal time. The expected terminal value is m_X given the absence of information. Thus ex-ante the manager holds an option with expected value (under the measure F_X, which is here the risk-neutral valuation measure, or, in a noisy context, its correction F_T^L – see Appendix 1) equal to

$$p(m_X - \gamma). \tag{7}$$

More generally, if the manager can receive a share of this value in remuneration, then the expression above becomes the manager's objective function. For instance, this does not necessarily have to involve explicit trading, instead simply assume the manager's share options are set at the money immediately after investors, not seeing a disclosure, downgrade the firm when the manager knows there is no new company information[16].

See Ostaszewski and Gietzmann (2008) for an explanation that this expected value here (under F_X) is also a risk-neutral valuation realizable through

[15] The work by Penno is a timely reminder of the care that needs to be taken when trying to extend theory to model real world practice. However, the following section shows that in fact the Penno result on non consistency (monotonicity) arises because of the soemwhat restrictive functional form he used. We generalise his result and show what class of probability functions admit consistency.

[16] See Aboody and Kaznik (2000) and Gao and Shrieves (2002) for a discussion of strategic granting of options.

a trading strategy. The trading mechanism does assume that the manager's trade remains *unobserved* by the investors, as would be the case in the traditional Kyle (1985) one-shot market model. We refer to this as the *unobserved trading mechanism*. However, it is possible to relax this assumption (without changing the qualitative features of the results) in a setting with a sequence of observed trades[17], but at the price of less tractability.

We continue, parsimoniously, with this structurally minimalist assumption where a manger's trades or option grants are not observed in a timely fashion (before the final date). With the manager's objective set at $p(m_X - \gamma(p))$; this turns out to be a very tractable model. Indeed, we find that under these current assumptions a particular CES utility function is uniquely determined, namely $U(y,z) = (y^{-1} + z^{-1})^{-1}$, and we refer to it as the *implied utility* in order to stress that it is not imposed, but derived from the managerial payoff structure (7). See Appendix 2, for details.

The principal feature of the structurally minimal model is summarized as:

Optimal Intensity Theorem for the structurally minimal model. *The odds-ratio $\lambda = \widehat{p}/(1-\widehat{p})$ and the intensity of disclosure τ sum to unity, i.e.*

$$\tau + \lambda = 1,$$

iff the value of p is selected optimally as in Section 2.3 above, i.e. $p = \hat{p}$, or, equivalently $\tau = \hat{\tau}$. In this case the corresponding Dye cutoff, denoted $\hat{\gamma}$, and the odds ratio $\hat{\lambda}$ are related according to the rule

$$\hat{\lambda}^2 = F(\hat{\gamma}(\sigma), \sigma). \tag{8}$$

The result here is driven by the condition (8) which corresponds to the utility function $U(z,y) = (y^{-1}+z^{-1})^{-1}$ derived in Appendix 2. The definition

[17] In such a sequential market model (allowing trades at dates in between the interim and terminal dates) the manager's trading, having become observable, is subject to inferential analysis; the revised managerial opportunity set necessitates that the optimal managerial behaviour (given the manager's incentive) employs a mixed strategy of buying and selling – to preserve optimally the manager's informational advantage. On game-theoretic grounds, one expects that the revised valuation of the manager's option to trade is a convex function of p, say of the form $V(p)x(\gamma(p))$, with $V(p)$ taking zero value at the endpoints $p = 0$ and $p = 1$. (In this respect that is similar to the case with $p(m_X - \gamma(p))$). See De Meyer and Moussa Saley (2002) for a sequential auction model yielding just such a result. Our theory applies also to such general valuations – see below.

of τ and some simple arithmetic yields[18]:

$$\tau = 1 - \lambda \text{ iff } F(\gamma, \sigma) = \lambda^2,$$

from which the sum-to-unity formula follows.

According to this simple rule, the intensity of a firm selecting its optimal odds at λ is negatively linear in λ. As the optimal choice of p increases (across different firms) the intensity falls.

We stress that the simplicity of this formula is evidence of the tractability of the valuation (7).

The more general situation is given by the following result (see Appendix 3 for proof), which includes the structurally minimal case given by $u(\lambda) = \lambda^2$, where $u(\lambda)$ is the marginal rate of substitution of the utility function $U(z, y)$.

Theorem 1 (First Theorem of Monotonicity in Equilibrium). *The intensity of disclosure as a function of the optimal odds is decreasing in the following two circumstances:*

(i) *If $u(\lambda)$ is increasing, then $\tau(\lambda)$ is decreasing for all $\lambda > 0$.*

(ii) *If $u(\lambda)$ is convex and $\tau'(0) < 0$, then $\tau'(\lambda) < 0$ for all $\lambda > 0$.*

(ii) *If $u(\lambda)$ is concave and $\tau'(\bar{\lambda}) < 0$, then $\tau'(\lambda) < 0$ for all $0 < \lambda < \bar{\lambda}$.*

3 Monotonicity between disclosure intensity τ and signal noise σ

We have just traced the dependence of τ on $\lambda = \hat{\lambda}$. In the noisy signal model $T = T(X, Y)$ there are two sources of uncertainty:
(i) in X, i.e. in the sector return variabiablity, captured by the variance σ_X^2, and
(ii) in Y, i.e. in the variability of the noise which models the managerial 'vision', captured by the variance σ_Y^2 (or the precision $1/\sigma_Y^2$).

Since, as discussed in Section 2.1, the noisy signal model's Dye equation replaces X by $\mu = \mu_X(T)$ so we will be concerned with the variability σ_μ^2 in the estimated firm-value $E[X|T]$. We refer to σ_μ^2 as the *aggregate variability*, because it is a function of σ_X^2 and σ_Y^2. As we show later and to guide the current intuition, note that, if T is highly uninformative (in the limit as $\sigma_Y^2 \to$

[18] $1 - F(\gamma) = 1 - \lambda^2 = (1 - \lambda)(1 + \lambda)$ iff $\tau = (1 - p)(1 - F(\gamma)) = 1 - \lambda$, as $1 - p = 1/(1 + \lambda)$.

∞), then $E[X|T] \to E[X] = m_X$. In such circumstances the distribution of $E[X|T]$ is tightly bounded around m_X and so σ_μ^2 is small. Although this comment concerns the scenario of extremely poor managerial vision, it correctly brings out the inversion: larger σ_Y^2 results in smaller σ_μ^2.

We now consider the dependence of $\hat{\lambda}$ on the aggregate variability. For this work we will need to exploit stochastic dominance. This should comes as no surprise in view of the seminal work by Vijay Bawa (1975) on lower partial moments, where he mapped out the relationship between lower partial moment and stochastic dominance. Indeed Bawa (1975) was the first to define lower partial moment (LPM) as a general family of below-target risk measures, one of which is the below-target semivariance, and studied them in regard to risk tolerance. See Levy (1992) for a survey, or Nawrocki (1999) for a more recent review of the issues.

We begin by establishing some results which refine the first-order notion of stochastic dominance and the associated comparative statics concerned with variations in a distribution parameter of a single random variable. We use this to analyse the comparative statics with respect to σ of the uncertainty parameter p (see the end of the last section), equivalently of the endogenously defined equilibrium value $\hat{\lambda}$ defined by (**??**) and of the corresponding cutoff. We will work with a family of distributions parametrized by $\sigma = \sigma_\mu$, so we henceforth identify the corresponding Dye cutoff as $\gamma(\lambda, \sigma)$. We continue to use hatted notation to refer to situations where the uncertainty parameter p, or λ, takes its optimal value, thus we write $\hat{\lambda}(\sigma)$ and $\hat{\gamma}(\sigma) := \gamma(\hat{\lambda}(\sigma), \sigma)$.

We show first that $\hat{\gamma}$ decreases with σ (subject to dominance assumptions). We then formulate an assumption about the preferences of investors facing increased risk; from this and the comparative statics of $\hat{\gamma}$ follows the sensitivity to changes in σ of $\hat{\lambda}$, or equivalently of $\hat{p}$. This may be reduced to a modelling condition on the distribution F to be used in relation to the log-normal models in the next section (Section 4). This section ends with the corollary that under the circumstances $\hat{\tau}$, the equilibrium value of the disclosure intensity, increases with σ.

The entire analysis of these statics is necessitated by the fact that the statics conducted by Jung and Kwon (1988) are inappropriate here because theirs is a 'partial statics analysis' with one parameter held fixed namely p; we need to relax that assumption and to allow variation in p as well as in the distributions (which in our case are parameterized by σ). As commented before, in Section 2.1, we hold the expected firm value m_X fixed.

3.1 Stochastic dominance

Recall that F_1 dominates F_2 in the sense of *first degree stochastic dominance* (FDSD) if $F_1 \neq F_2$ and $F_1(t) \leq F_2(t)$ for all t, i.e. $\bar{F}_1(t) \geq \bar{F}_2(t)$ for all t, so that the event $X_1 \geq t$ is more likely than $X_2 \geq t$, so that the former is preferred over the latter.

Thus a family of distributions $F(t, \sigma)$ parametrized by variance σ exhibits FDSD if for all t

$$F(t, \sigma_1) \leq F(t, \sigma_2) \text{ provided } 0 < \sigma_1 < \sigma_2.$$

Likewise F_1 dominates F_2 in the sense of *second degree stochastic dominance* (SDSD) if $F_1 \neq F_2$ and $H_1(t) \leq H_2(t)$ for all t. (This implies in particular that $m_2 \leq m_1$ since $m_i = 1 - H_i(1)$). Note that in this case the reciprocal function $(H_1(t))^{-1}$ majorizes $(H_2(t))^{-1}$ for $t > 0$ in the usual function sense.

Recall two relevant result from Jung and Kwon (1988). By their Prop. 2 one has[19] for fixed σ

$$\gamma(\lambda_1, \sigma) < \gamma(\lambda_2, \sigma) \text{ provided } 0 < \lambda_1 < \lambda_2. \tag{JK1}$$

Also by their Prop. 3, for fixed p, one has[20]

$$\gamma(\lambda, \sigma_2) \leq \gamma(\lambda, \sigma_1) \text{ provided } 0 < \sigma_1 < \sigma_2, \tag{JK2}$$

when the family $F(t, \sigma)$ parametrized by variance σ exhibits FDSD or SDSD.

We prove the complementary result below in Theorem that, subject to stochastic dominance assumptions,

$$\hat{\gamma}(\sigma_2) := \gamma(\hat{\lambda}(\sigma_2), \sigma_2) < \gamma(\hat{\lambda}(\sigma_1), \sigma_1) := \hat{\gamma}(\sigma_1). \tag{9}$$

This does not follow from their results, because here in fact $\hat{\lambda}(\sigma_2) > \hat{\lambda}(\sigma_1)$, or equivalently, $\hat{p}(\sigma_2) > \hat{p}(\sigma_1)$. For the latter result see the Section 3.2. Our first statics result is as follows: for a proof and technical terms here, see Appendix 7.

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[19] In their notation this result would read read $\gamma(p_1, \sigma) < \gamma(p_2, \sigma)$ provided $0 < p_1 < p_2$, since λ is increasing in p.

[20] As before, in their notation, one has $\gamma(p, \sigma_2) \leq \gamma(p, \sigma_1)$ provided $0 < \sigma_1 < \sigma_2$.

Theorem 2. *Given two distributions, with log-concave hemi-means, with F_1 increasingly dominating F_2 with respect to $\sigma = \sigma_\mu$, the corresponding optimized Dye triggers satisfy*

$$\hat{\gamma}(\sigma_2) < \hat{\gamma}(\sigma_1), \text{ i.e.,} \gamma(\hat{\lambda}(\sigma_2), \sigma_2) < \gamma(\hat{\lambda}(\sigma_1), \sigma_1) \text{ provided } 0 < \sigma_1 < \sigma_2.$$

Thus the cutoff falls if the aggregate variability σ_μ^2 increases.

3.2 Monotonicity and Investor Preferences

Here we argue that in modelling a firm by a family of distributions $F(x, \sigma)$ one should respect the fact that investors require to be rewarded when accepting increased risk. We take the view that investors measure the risk by the gain-to-loss ratio, defined in response to a $\sigma = \sigma_\mu$ and a (freely chosen) cutoff γ by

$$\Lambda = \Lambda(\gamma, \sigma) := \frac{H(\gamma, \sigma)}{m_X - \gamma}.$$

Here H refers either to H_X when the observed signal is that of true value X, or else its correction H_T^L (for which see Appendix 1). The latter corresponds to the observed signal being $T = T(X, Y)$ and the true value is estimated by the manager as $\mu_X(T)$.

In fact, of course, since here γ is fixed only $H(\gamma, \sigma)$ varies with σ.

Assumption MIP (Monotonic Investor Preferences). *We assume that for any cutoff γ an investor facing an increase in $\sigma = \sigma_\mu$ demands a higher value of $\Lambda(\gamma, \sigma)$, equivalently a higher value of $H(\gamma, \sigma)$, to reward the extra risk-exposure.*

The investor's demanding for larger Λ value may be translated to forcing an increase in γ, because H is increasing in γ, for fixed $\sigma = \sigma_\mu$. As Dye's analysis characterizes the equilibrium choice of γ to be such that $\Lambda = p/(1 - p)$, the MIP in turn translates into requiring that p be adjusted upwards.

With σ_X^2 held fixed, note the countervailing directions of higher precision (lower σ_Y) inducing higher p as though sharper vision could be traded-off against higher informational uncertainty p (lower opportunity to use the higher vision). Note also that a disclosed signal T is heavily discounted to $\mu_X(T)$ when σ_Y^2 is large, but nevertheless, since that signal was already sufficiently large to trigger a disclosure, such a disclosure results in an upgrade of the firm value above the mean[21]. Recall that $\mu_X(t)$ is assumed to be increasing (a fact verified below for the log-normal model), and so the larger the observed signal the larger is the estimate of X. That is, good news from a noisy manager is still good news.

Remark. The use of Λ is an equivalent tool to that of return on investment. Indeed, investors can equivalently work with the reciprocal, which is connected to the Omega-function of Shadwick and Keating (2002)

$$\Omega(t) = 1 + \frac{1}{\Lambda(t)},$$

i.e. as

$$\Omega(t) = \frac{m - t + H(t)}{H(t)} = 1 + \frac{m-t}{H(t)}.$$

The theorem below relates MIP to parameters occurring in the derivation of the optimized utility U. In particular we need to refer here to $u(\lambda)$ which represents the marginal rate of substitution for the utility $U(y,z)$ as in Appendix 3 and a function Π which determines that the cutoff is an appropriate λ-driven quantile (refer to the growth-rates discussion in Section 2.3).

For the latter, recall from Section 2.3 the equilibrium condition for utility optimization

$$u(\lambda) = F(\gamma, \sigma),$$

and form the inverse function

$$\Pi(\gamma, \sigma) = u^{-1}(F(\gamma, \sigma)).$$

Theorem 3 (Jacobian condition). *Suppose $H(\gamma, \sigma)$ is log-concave as a function of γ, that $F(x, \sigma)$ exhibits (first and) second order stochastic dominance with respect to (increasing) $\sigma = \sigma_\mu$ and that the system is invertible. The Assumption MIP is equivalent to the following Jacobian (determinant) being positive:*

$$\frac{\partial(\Pi, \Lambda)}{\partial(\gamma, \sigma)} = \frac{1}{u'(\bar{\lambda})(m-\gamma)} \begin{vmatrix} F_\gamma & F_\sigma \\ H_\gamma + H/(m-\gamma) & H_\sigma \end{vmatrix}.$$

Remark. This is the two-variable analogue of the well-known positive slope condition used to characterize monotonic increasing functions of one

[21] This neglects for intuitive argument the event here with very low probability that the signal T falls in the narrow band between the cutoff γ_T and the mean. (See Appendix 1.)

variable. The condition asserts that the transformation Θ defined below preserves orientation, a feature that is equivalent to the Assumption MIP. Here $\Theta : (\gamma, \sigma) \to (\pi, \lambda)$ from $[0, m) \times [0, 1)$ to $[0, m) \times [0, 1)$ with $\Theta(\gamma, \sigma) = (\pi, \lambda)$ where:

$$\begin{aligned}
\pi &= u^{-1}(F(\gamma, \sigma)), \\
\lambda &= \Lambda(\gamma, \sigma) := \frac{H(\gamma, \sigma)}{m - \gamma}.
\end{aligned}$$

As a corollary, we obtain the following monotonicity result. For technical terms here see Appendix 7.

Theorem 4 (Monotonicity Theorem for Aggregate variability). *Assume MIP and that H is log-concave. For $u(\lambda)$ strictly and regularly increasing, $\hat{\tau}$ is decreasing in $\sigma = \sigma_\mu$.*

We now fix the sector returns variability and focus on the influence of managerial vision on the equilibrium disclosure intensity. Our final aim is to apply Theorem 4 to the log-normal model of a manager receiving a noisy signal of firm value to show that $\hat{\tau}$ is increases as the managerial vision deteriorates.

As an aid to intuition, we briefly recall the Kalman-gain approach to estimation for the linear filtering of normally distributed random variables. Abbreviating to μ the estimator $\mu_X(T)$ of X conditioned on $T = X + Y$, we have

$$\mu = \mu_X(T) = m_X + \kappa(T - m_X),$$

where the Kalman gain coefficient is

$$\kappa = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_Y^2}, \tag{10}$$

and so

$$\sigma_\mu^2 = \kappa^2 \sigma_T^2 = \frac{\sigma_X^4}{\sigma_X^2 + \sigma_Y^2} = \kappa \sigma_X^2. \tag{11}$$

Let us assume that in the passage to log-normality, we may use the formula (10) and (11). Notice that σ_μ decreases as σ_Y increases. (That a similar formula is indeed correct, is confirmed by Section 4.) Recall also (under MIP which rewards investor risk taking) that λ increases with σ_μ (by the MIP assumption), and τ decreases as λ increases (Theorem 1, in Section 2.4). So τ decreases as σ_μ increases. But, as we have just seen, σ_μ decreases as σ_Y increases; so τ increases as σ_Y increases.

To place in perspective these findings, consider an analysis which neglects the effect of σ_μ on λ. So hold λ fixed, and model the reaction of γ to $\sigma = \sigma_\mu$ simplistically, in line with the Jung-and-Kwon comparative statics, by

$$\gamma = m_X - \sigma\xi,$$

for some positive and now 'constant' $\xi = \xi(\lambda)$ (as in the Penno model – see Appendix 6). Then, as the distribution $F(t)$ is an increasing function of t, the disclosure intensity, given by

$$\tau = (1-p)(1-F(\gamma)) = (1-p)(1-F(m-\sigma\xi)),$$

appears under the current circumstances to decrease as σ_Y increases. This naive analysis contradicts the true state of affairs.

The conclusion is, that the effects of σ_μ on λ may not be validly neglected. Indeed, as σ_Y decreases, so too does λ, and so $1-p$ increases. This factor has a countervailing dominant tendency.

This highlights the insights gained from allowing the information uncertainty parameter p to be chosen endogenously.

4 Log-normal models

In this section we note the details of the log-normal model. The explicit formulas quoted below show its tractability, all the way down to an application of Theorem 3 which yields the desired monotonicity result.

Recall that we are concerned with the possibility that at time $\theta = 2$ the manager observes either true value X, or a transform $T = T(X,Y)$ of the random variable X with Y a source of noise. Here we take $T = XY$ with $X = m_X e^{U-\frac{1}{2}\sigma_U^2}$ and $Y = e^{V-\frac{1}{2}\sigma_V^2}$ with U, V independent, normal zero-mean random variables with variances σ_U^2 and σ_V^2. Thus X is log-normally distributed, in accord with the financial benchmark model, as is the signal T. Here $T = m_X e^{W-\frac{1}{2}\sigma_W^2}$ with $W = U+V$ a mean-zero normal with variance $\sigma_W^2 = \sigma_U^2 + \sigma_V^2$.

The clean (Dye) signal cutoff for an observation of true value X is given by

$$\bar{x} = m_X \cdot \bar{\gamma}, \tag{12}$$

where $\bar{\gamma} = \bar{\gamma}(\lambda, \sigma_U)$ is the 'dimensionless' or rescaled solution to the Jung and Kwon equation:

$$\lambda(1 - \bar{\gamma}) = H_{\mathrm{LN}}(\bar{\gamma}, \sigma_U), \tag{13}$$

Up to rescaling against m_X the value of $\bar{\gamma}$ yields the market valuation under non-disclosure. Here H_{LN} the hemi-mean function for the log-normal, is given by

$$H_{\mathrm{LN}}(\gamma, \sigma) = \gamma \cdot \Phi_{\mathrm{N}}\left(\frac{\log(\gamma) + \frac{1}{2}\sigma^2}{\sigma}\right) - \Phi_{\mathrm{N}}\left(\frac{\log(\gamma) - \frac{1}{2}\sigma^2}{\sigma}\right),$$

where Φ_{N} denotes the standard normal probability distribution function. Its similarity to the Black-Scholes equation is not accidental (see Shadwick and Keating (2002) identifying call options in this context).

Since T is log-normal, it is straightforward from this formula to compute the regression function $\mu_X(t)$, i.e. the conditional expectation estimator of Section 2.1. It is given by indirectly in terms of $W = U + V$ by the equation:

$$X^{\mathrm{est}} = E[X|T] = m_X \exp\left(\kappa W - \frac{1}{2}\kappa^2 \sigma_W^2\right) = m_X \exp\left(\kappa W - \frac{1}{2}\kappa \sigma_U^2\right),$$

where

$$\kappa := \frac{\sigma_U^2}{\sigma_U^2 + \sigma_V^2} = \frac{\rho_V}{\rho_U + \rho_V}, \text{ employing the precision } \rho_U = 1/\sigma_U^2, \text{etc.}$$

Here the aggregate variability is given by:

$$\sigma_\mu^2 := \kappa^2 \sigma_W^2 := \frac{\sigma_U^4}{(\sigma_U^2 + \sigma_V^2)^2}(\sigma_U^2 + \sigma_V^2) = \frac{\sigma_U^4}{\sigma_U^2 + \sigma_V^2} = \kappa \sigma_U^2.$$

As a result the cutoff for the estimator X^{est} is given by

$$\hat{x}^{\mathrm{est}} = m_X \cdot \widehat{\gamma}^{\mathrm{est}},$$

where $\widehat{\gamma}^{\mathrm{est}} = \widehat{\gamma}(\lambda, \kappa\sigma_W)$ is the solution to the Dye equation corrected for noise, namely:

$$\lambda(1 - \gamma) = H_{\mathrm{LN}}(\gamma, \kappa\sigma_W).$$

and so this gives the market valuation in the noisy model in the event of non-disclosure at time $\theta = 1$. We note the form of the (increasing) regression function which is:

$$\mu_X(t) = E[X|T = t] = m_X e^{\frac{1}{2}\kappa(1-\kappa)\sigma_W^2} \, (t/m_X)^{\kappa} \, .$$

Thus the inverse regression function $L(x) = \mu_X^{-1}(x)$ is convex, and is given by

$$L(x) = m_X e^{-\frac{1}{2}(1-\kappa)\sigma_W^2} (x/m_X)^{1/\kappa}.$$

From here one readily deduces the following:

Theorem 5 (Log-normal disclosure intensity). *The disclosure intensity in the noisy log-normal model is given by*

$$\hat{\tau} = \hat{q}(\kappa\sigma_W) \left(1 - \Phi_{\mathrm{LN}}(\widehat{\gamma}^{\mathrm{est}}, \kappa\sigma_W)\right) = 1 - \hat{p}(\kappa\sigma_W)/\hat{q}(\kappa\sigma_W),$$

$$\textit{where } \Phi_{\mathrm{LN}}(\gamma, \sigma) = \Phi_{\mathrm{N}} \left(\frac{\log(\gamma) + \frac{1}{2}\sigma^2}{\sigma} \right),$$

and Φ_{N} *is the standard normal distribution.*

An illustrative example graph of $\hat{\tau}$ against precision as measured by the variable κ is offered in Figure 1.

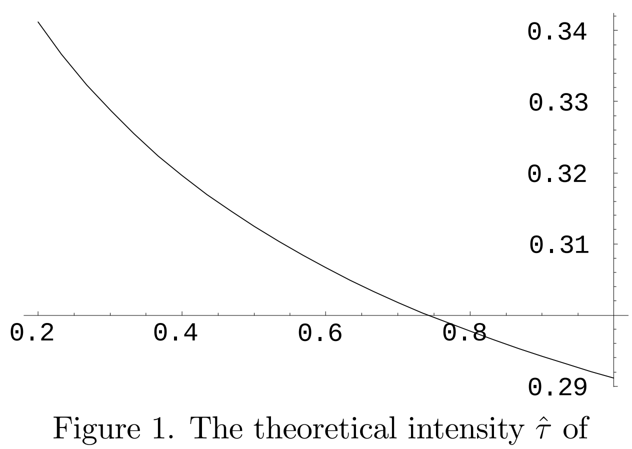

Figure 1. The theoretical intensity $\hat{\tau}$ of voluntary disclosure as a function of aggregate variability.

5 Empirical evidence for disclosure intensity effects

A recent paper in this area is Cousin and de Launois (2006). In their work they consider traditional competing models of conditional volatility: the GARCH specification and a Markov two-state volatility switching model. They argue that changes in the rate of information arrival may cause a switch between high or low (stock return) volatility. In their GARCH framework the specification of conditional variance is given by

$$\sigma_{i,t}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 + \lambda_i N_{i,t}, \tag{14}$$

where the new term $N_{i,t}$ is a proxy[22] for the number of news events specific to company i announced to the stock market per interval t. Their main objective is to compare and contrast the performance of this adjusted GARCH model to a two-state Markov Switching Regression (MSR) model, where now the disclosure intensity determines the probability that a company under consideration is either[23] in a low or high volatility regime.

What is of particular interest for us is that, on the basis of an empirical analysis, they conclude that disclosure intensity is an important explanatory variable for conditional volatility. In the GARCH framework their empirical findings are consistent with our theoretical predictions in that the conditional volatility is increasing in disclosure intensity, and in the MSR framework the probability of being in the high volatility state is increasing in disclosure intensity. Thus their empirical tests appear to be broadly in line with our theoretical predictions. However, before coming to this conclusion we believe it is important to raise a note of caution. What is critical is how Cousin and de Launois measure disclosure intensity. As their Table 1 makes clear, they simply record the frequencies of Factiva disclosures by category. However, if one just records all the raw empirical disclosure intensities for companies, this does not capture the essential features of our generalised Dye model, for the following reason. The theoretical model is of *voluntary* disclosures, that is the Dye model concerns itself only with those news-wires which correspond to

[22] They measure the variable by identifying the frequency of a subset of firm news releases on Factiva.

[23] To be more precise, the disclosure intensity in part determines whether the state regime dummy variable $D_{i,t}$ is above or below a threshold, qualifying whether the firm is in the high volatility regime.

management receiving information about future events that affect their *voluntary* ability to issue the news-wires, and thus may indicate value *above* the Dye cutoff. In addition companies are required under regulatory provisions to make *mandatory* disclosures. Thus the raw data on disclosure intensities is a mix of disclosure 'types', whereas the theory only speaks to voluntary disclosures. Thus, when working with raw disclosure-intensity data, an essential step is to implement an estimation procedure for separating out the voluntary Dye-type disclosures.

With this empirical issue in mind, one procedure could be to exploit the distributional assumptions of the model. The Dye cutoff can be shown to be close to the mean (just below), and one can use this to validate an empirical approach which measures *dimensionless relative intensity,* i.e. excess relative to the mean in proportion to standard deviation. Looking at disclosure intensities above the mean rate ('high rates') abstracts away from mandatory good news disclosures that occur on a regular basis. Restricting attention only to high-intensity disclosure periods, one needs to distinguish between those that approximately correspond to good news (voluntary disclosures) and those that approximately correspond to bad news (mandatory disclosures); the latter are typically driven by regulations put in place to protect investors from delay of bad news disclosure. In order to identify which are good news and which are bad news disclosures, when there is no established standard "message space" for voluntary disclosures, it is suggested here that one could identify good news disclosures as those that give rise to an increase in analysts' consensus forecasts (and so exclude those that give rise to a decline in analysts' consensus forecasts for the company).

In contrast recent research by Rogers, Schrand and Verrecchia (2008) (RSV) use an EGARCH model which allows them to estimate the conditional variance when modelled as being given by one of two functions, the choice depending on the sign of the return shock. The intuition behind this asymmetric modelling assumption is that "bad news" seems to have a more pronounced effect on conditional volatility than has "good news". For many companies there is a strong negative correlation between the current stock returns and future volatility. The tendency for conditional volatility to decline when returns rise (following good news) and rise when returns fall (following bad news) is typically referred to in behavioural finance as the *leverage effect.* RSV propose that, when companies follow a strategy of reporting good news and withholding bad news, this can be described as 'strategic disclosure'. In a setting where good news is taken at face value, bad news below the cutoff threshold has to be inferred by investors; it is this difference (i.e. observed versus inferred) in the formation of expectations that leads to the asymmetric responses in the market. To see this in the limiting case of full disclosure, remove the leverage (asymmetric) effect, whereupon current changes in valuation (impounded in returns) would always be associated with recent news arrival rather than the need for investors to make inferences following non-disclosure. Rather than look at actual disclosures, RSV instead develop two hypotheses about the leverage effect. The first is that the leverage effect is stronger for companies about which there is less private information; that feature is assumed to increase the threshold level of disclosure (implying a lower disclosure intensity). The second is that the leverage effect will be weaker when increased litigation risk affects a company's propensity to adopt a 'strategic disclosure' strategy. RSV report interesting results; however, our research on disclosure intensity suggests an alternative empirical implementation. Specifically, they use the variable PUBINFO as a measure of private information. That measure captures the extent to which information is likely not to be private, because in their analysis, if company returns move together then, ceteris paribus, homogeneity subsists in that sector of industry; so there is less private information when results of company operations are similar. Thus, they do not actually measure disclosure intensities. Accordingly, on the view that our model may have wider empirical applicability than the special two-case scenario investigated by RSV, we suggest that an EGARCH model variant of the standard GARCH model, redesigned so as to refer to disclosure intensities in (14), bears investigation.

6 Conclusion

We have shown that in equilibrium the managers of companies facing higher signal noise will rationally increase their disclosure intensity. That is, working back from observed disclosure intensity, investors rationally infer that, after scaling for size against firm market-capitalization, high intensity disclosing firms are more risky, as management's truthfully disclosed estimates have larger standard deviations. (This can, for instance, be because the managers are subject to greater noise in their operating environments.) The theory developed here, based on generalizations of the established Dye model, suggests both new empirical testing procedures and also critically a different direction in assumed causation. The theory shows why one should not base empirical hypotheses on an a priori assumption that 'better' companies make more voluntary disclosures, since we have shown that it is in fact the companies with the most poorly informed management (facing highest noise) which will – in equilibrium – disclose with the greatest intensity.

The research is subject to a number of caveats. We abide by the assumptions of the Dye model in regard to (truthful) disclosure and the inability of credible disclosure of absence of information. The model is essentially a single-period project model, in which success in one period does not influence successes in later periods. That is, multi-period project dependence (and related disclosure) is not modelled. This is clearly a topic for future research. Furthermore, mangers here make disclosures according to their own optimal cutoff rather than mimicking a different manager type; any other behaviour would require an alternative model.

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