A Theory of Voluntary Disclosure and Cost of Capital

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Abstract

This paper explores the links between firms' voluntary disclosures and their cost of capital. I relate the differences in costs of capital between disclosing and non-disclosing firms to disclosure frictions and equity risk premia. Specifically, I show that firms that voluntary disclose their information have a lower cost of capital than firms that do not disclose. I also examine the extent to which reductions in cost of capital map into improved risk-sharing and/or greater productive efficiency. I prove that high (low) disclosure frictions lead to overinvestment (underinvestment) relative to first-best. Economic efficiency decreases as the disclosure friction increases because of inefficient production in an underinvestment equilibrium. As the disclosure friction continues to increase, the equilibrium switches to overinvestment and further increases in the disclosure friction improve risk-sharing. Importantly the relation between average cost of capital and economic efficiency is ambiguous. A decrease in average cost of capital in the economy only implies an increase in economic efficiency if there is overinvestment.

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1 Introduction

Recently, the relation between corporate disclosure and cost of capital has received considerable attention among academics and regulators. Disclosure can refer either to mandatory or voluntary release of information about firms' financial positions and performance. The cost of capital is the minimum return demanded by investors to invest in a new project. The cost of capital is often viewed as a metric that captures how well financial disclosure achieves its primary function of providing value-relevant information to users of financial statements. For example, the FASB states that "The benefits of financial reporting information include better investment, credit, and similar resource allocation decisions, which in turn result in more efficient functioning of the capital markets and lower costs of capital for the economy as a whole."[1]

In this paper, I study the effect of voluntary disclosure on cost of capital and economic efficiency, where economic efficiency is a combination of productive efficiency and efficient risk sharing. First, I isolate the firm-specific cost of capital effect caused by firms endogenous disclosure decisions from the overall cost of capital effect caused by exogenous changes in economy-wide information factors. Then, I analyze aggregate cost of capital differences across economies, and production and risk sharing efficiency caused by these economy-wide factors. In particular, I address two questions: First, at the individual firm level, do firms that voluntarily disclose more information experience a lower cost of capital? Second, at the macroeconomic level, do endogenous firm disclosures affect average cost of capital in aggregate and what are the consequences of voluntary disclosure on overall economic efficiency? Answering the first question allows us to better understand the economic forces underlying firms' disclosures, their effects on an individual firm's cost of capital, and the cross-sectional differences in costs of capital between disclosing and non-disclosing firms. Providing an answer to the second question could provide rule makers a useful criterion in setting disclosure policy. As noted by Sunder (2002): "cost of capital is an overall social welfare criterion rooted in equilibrium concept."

The first of these two questions refers to the implications of disclosure on cost of capital at the individual firm level within an existing environment. Although the evidence is still relatively recent, a majority of empirical studies document a negative association between disclosure and cost of capital cross-sectionally. When disclosure itself is a choice, the interpretation of empirical results must take into account the issue of endogeneity

[1]FASB Financial Accounting Series, NO.1260-001 July 6 2006, "Conceptual Framework for Financial Reporting: Objective of Financial Reporting and Qualitative Characteristics of Decision-Useful Financial Reporting Information," Section QC53, p.35.

(Skaife, Collins, and LaFond (2004), Nikolaev and Van Lent (2005), and Cohen (2008)). To capture the endogenous disclosure decision, this paper provides a model in which firms choose their disclosure to maximize their market value. In the model, firms with favorable private information are more likely to disclose. Such disclosure of favorable information reveals to the market the firm's lower exposure to systematic risk. Responding to such a disclosure, investors rationally offer a higher price to disclosing firms, leading to a lower cost of capital. This is the first result of the paper. This result delivers the observed cross-sectional association between disclosure and cost of capital within an economy. In other words, different firms endogenously choose different disclosures. Investors in turn rationally value firms at different prices, leading to different costs of capital. Thus disclosure and cost of capital, both endogenous in the model, appear to be negatively associated and are driven by the underlying voluntary disclosure incentive.

The second question refers to the effects of an economy-wide exogenous information factor on the relation between overall cost of capital in the economy and efficiency at the macroeconomic level. Several empirical papers attempt to connect cost of capital to economic efficiency (e.g., Morck, Yeung, and Yu (2000) and Chen, Berger, and Li (2006)). Similar to Dye (1985), this paper introduces a disclosure friction under which investors are unable to fully distinguish between firms that choose not to disclose and firms that cannot disclose. The disclosure friction is a measure of the overall information availability in the economy. I show that average cost of capital captures how well financial markets function at insuring imperfectly diversified investors against disclosure risk. Building on the firm-level result, in contrast to existing literature, I show that greater information availability in the economy increases the average cost of capital because more information increases the dispersion of prices post disclosure; this itself leads, as noted by Hirshleifer (1971), to lower risk-sharing efficiency.[2]

Furthermore, at the macroeconomic level, information availability affects real decisions (investment) as well as risk-sharing (among risk-averse investors facing remaining uncertainty associated with the firms' cash flows). In this paper, I distinguish the efficiency effect due to improved risk-sharing from the efficiency effect due to improved productive decisions. The second main result of this paper is that the average cost of capital is a good proxy for efficiency only if one starts from a disclosure friction that is relatively high. A higher disclosure friction improves risk-sharing. In contrast if the disclosure

[2]In resolving uncertainty, information also erodes risk-sharing opportunities when it is publicly revealed before trading. "Public information ... in advance of trading adds a significant distributive risk" (Hirshleifer 1971, p. 568). However, my result differs from Hirshleifer (1971) in that I show how changes to voluntary disclosure may lead to greater price dispersion and study aggregate cost of capital, while Hirshleifer focuses on efficiency after price dispersion has increased.

friction is relatively low, decreasing the friction increases efficiency as it improves productive inefficiency. Overall economic efficiency is maximal when financial disclosures are either perfectly informative or completely noisy. In other words, I show that regulators will have to balance the productive efficiency problem against the risk-sharing problem in setting public disclosure policy. Empirically the result further suggests that different cross-country characteristics among regulatory environments and accounting practices are driven by their economic primitives.

The model in this paper extends a voluntary disclosure model (Dye (1985)) by incorporating an asset pricing framework (commonly referred as Mossin-Lintner-Sharpe model). In the economy, each of a large number of risk-averse investors owns a firm's new project whose expected cash flows, if financed, contain an idiosyncratic and a common cash flow component. Each firm decides whether or not to disclose private information about its idiosyncratic cash flows. Investors observe public disclosures (if any) and rationally price each firm. Similar to Dye (1985), there exists a disclosure friction: some firms cannot credibly communicate their information. Consequently, firms that disclose but cannot get their message out are pooled with those firms that intentionally did not disclose. This disclosure friction might be interpreted as information asymmetry between firms and investors or as a proxy for the complexity of the economic operations to be disclosed.[3] Alternatively, one might view this friction as a summary measure of the regulatory oversight of corporate disclosure and the quality of accounting standards.[4] This disclosure friction affects the proportion of firms voluntarily disclosing and I show how it can work to reduce cost of capital, both at the firm level (if a particular firm discloses more relative to its peers) and at the aggregate market level (if all firms disclose more overall). The model highlights information asymmetries between firms and investors, rather than among different investors. The results are derived with constant relative risk averse investors and general probability distributions (not necessarily the normal distribution).

Next I elaborate on the economic intuition for my results. First, I find that, if the disclosure friction is sufficiently high, firms making more voluntary disclosures have a lower cost of capital. The rationale for this result is the relation between voluntary disclosures and investors' updated estimate of the firms' systematic risk per dollar of expected cash

[3]For example, it may be easier to disclose information in a well-established industry than in a new venture or a firm engaging in complex financial operations. Investors may also not pick up the information sent by firms, either because they did not pay attention to the release of information or cannot understand the firm's information.

[4]A common measure in mandatory disclosure of accounting quality is the variance on the information disclosed. Although this disclosure friction is related to voluntary disclosure, these two metrics are similar in that it measures the level of information inferred by markets.

flows. Conditional on a voluntary disclosure, investors expect more expected cash flows which dilute the firms' sensitivity to systematic risk, in turn decreasing cost of capital and increasing market value. Indeed, I show that, from the perspective of managers, firms disclose if and only if such a disclosure reduces their cost of capital, consistent with the common use of the statement in the empirical literature (although not with prior analytical work in this area). In summary, the model matches the observed cross-sectional association between disclosure and cost of capital. It shows that firms that disclose do so to increase their market value, which in turn, reduces their cost of capital, while the remaining firms do not disclose because they were unable to due to the disclosure friction, or doing so would have increased their cost of capital.

Second, I consider the overall cost of capital of all firms in the economy, averaging the market returns of all disclosing and non-disclosing firms. I distinguish two sources of economic (in)efficiency: imperfect risk-sharing for the initial owners of the firms and productive inefficiencies tied to the asymmetric information about firms that did not disclose. I compare overall cost of capital and economic efficiency. When the disclosure friction is high, I show that, if more firms voluntarily disclose their information, the dispersion of market prices increases, which implies an increase in average cost of capital and worsens risk-sharing. In this situation, an increase in cost of capital is perfectly aligned with a deterioration in risk-sharing and therefore is a valid metric for the analysis of the efficiency consequences of disclosure. The nature of the productive inefficiency depends on the disclosure friction. If the disclosure friction is relatively low, firms that do not disclose are viewed as having low value and high cost of capital and do not invest. As a result, a low disclosure friction leads to underinvestment by some high-value firms which may not have disclosed because they were unable to due to the disclosure friction. Conversely, if the disclosure friction is high, investors anticipate that more firms that did not disclose are high-value firms that could not disclose; therefore, non-disclosing firms are priced higher and, thus, are able to invest which leads to an overinvestment problem. Finally, I examine the tension between risk-sharing and investment efficiency, and find that economic efficiency depends on the level of the disclosure friction, with risk-sharing concerns less (more) important than productive efficiency with a low (high) disclosure friction and underinvestment (overinvestment).

The formal analysis yields several key observations that I will discuss in more detail later. One, given that asset pricing theory captures the firm's exposure to non-diversifiable risk, it requires a proper understanding of firm's strategic disclosure. Two, the links between the disclosure friction and the cost of capital are very different at the firm and the aggregate level. Three, the disclosure friction affects the disclosure decision of managers and, given that such disclosures are then used by investors, has real productive effects and also affects risk-sharing. Fourth, from an empirical perspective, I show that the relation between disclosure and cost of capital depends on the economic primitives.

Related Literature

My paper is related to three strands of literature: voluntary disclosure, accounting quality and cost of capital, and the real effects of disclosure.

The voluntary disclosure literature studies firms' endogenous disclosure decisions and their consequences on the type of information disclosed (Verrecchia (1983) and Dye (1985)). My paper combines voluntary disclosure with asset pricing in the presence of systematic risk. To my knowledge, the only papers that focus on voluntary disclosures and systematic risk are those of Kirschenheiter and Jorgensen (2003, 2007). They focus on disclosures about risk, more applicable to financial products, such as value-at-risk, new ventures, exposure to interest rates. As is common in the voluntary disclosure literature, I consider disclosures about expected or projected cash flows, such as asset values, earnings' forecasts, sales projections, expense reductions or asset acquisitions. While Kirschenheiter and Jorgensen (2003, 2007) do not compute the average cost of capital (average return by all firms), they find that the equity risk premium (return on the market portfolio) is increasing in information availability.[5] They note that these effects should become (arbitrarily) small in a large economy if the disclosure is about the asset's variance (Jorgensen and Kirschenheiter (2003)) but hold in a large economy if the disclosure is about sensitivity to systematic risk (Jorgensen and Kirschenheiter (2007)). I complement their work by introducing a production decision. I also relate the average cost of capital to economic efficiency.

Bertomeu, Beyer, and Dye (2008) study whether firms' voluntary disclosures can reduce asymmetric information in financial markets and lead to cheaper financing. They find that more disclosure occurs in environments with fewer informational frictions, matching the observed association between disclosure and cost of capital. Arya and Mittendorf (2007) show that more voluntary disclosure by firms can lead to less disclosure by outside information providers; leading to lower overall proprietary information publicly revealed and higher value for shareholders. My paper studies another angle of the effects of voluntary disclosures. I show how voluntary disclosures affect the cost of capital (defined as the expected market return) in conjunction with economic efficiency.

[5] I distinguish here the average cost of capital (defined as the equally-weighted average return by all firms) from the equity premium (defined as the value-weighted average return).

My paper also contributes to the literature on the relation between accounting quality and cost of capital. Heretofore, this literature has explored the effects of exogenous information on risk premia. I focus on the endogenous disclosure caused by the information asymmetry between firms and investors, which is different from the information asymmetry among investors in Easley and O'Hara (2004), Hughes, Liu, and Liu (2007) and Lambert, Leuz, and Verrecchia (2008). As in my paper, these papers introduce multiple firms. Easley and O'Hara (2004) find a higher cost of capital if there is more private information and less precise information in a finite economy. In their model, a proportion of investors receive information and the others do not. They explain that the uninformed investors will demand a higher risk premium for trading securities on which they face information risk. However, Hughes, Liu, and Liu (2007) prove that this result does not hold when the economy becomes large, as more information may only affect the (aggregate) market premium but not a firm's cost of capital directly. They prove that information about the systematic factor is the only information priced by the market. Lambert, Leuz, and Verrecchia (2007) derive whether the presence of additional information in a multi-asset economy would increase or decrease cost of capital. They find that a disclosing firm has a lower cost of capital than this same firm in the economy prior to disclosure; however, after disclosure has occurred, a disclosing firm may have a higher cost of capital than a firm that did not disclose. Armstrong, Banerjee, and Corona (2008) also consider a multi-firm model where information quality affects cost of capital through systematic risk. They show that observed beta and information quality are negatively related for positive beta stocks but positively related for negative beta stocks. One problem, noted by Christensen, De la Rosa, and Feltham (2008), is that, if there are no real decisions, disclosure should only affect the timing of resolution of uncertainty, and thus a commitment to disclosure does not increase welfare of the manager disclosing or, even, that of investors. In comparison to this literature, I show that firms that choose to disclose have an unambiguously lower cost of capital after disclosure has occurred than firms that choose not to disclose. I further introduce a production decision and study the disclosure effects on efficiency.

Yee (2007) and Gao (2008) contribute to the literature on exogenous information and cost of capital by specifically analyzing the consequences of disclosure on risk premia and efficiency. I extend their work on disclosure, cost of capital and efficiency from exogenous to endogenous disclosure. Yee (2007) ties earnings quality and production to efficiency and shows that higher earnings quality leads a firm to invest less but increases investors' expected utilities. Gao (2008) focuses on the relationship between cost of capital and welfare. Adding investment, the accounting quality changes the investment decision and risk allocation. There is a discrepancy between cost of capital and investors' welfare arising from the fact that the cost of capital does not internalize risk allocation. However, both of these papers highlight that the results are derived within a single-firm economy and would become less clear in a large economy. Among other modeling differences, my paper introduces a large number of firms and investors, and studies each investors' optimal portfolio choice.

A third strand of literature analyzes the benefits and costs associated with disclosure and their potential impacts on economic efficiency. These studies focus predominantly on the "real" costs associated with disclosure or non-disclosure (competition, trading costs, manipulation), while I elaborate an asset pricing model where disclosures are associated with the firm's market expected return. This literature documents that coarsening information may be desirable for shareholders (Arya et al (1998), Demski (1998), Arya and Glover (2008) and Einhorn and Ziv (2008)). Another set of papers further consider the real consequences of disclosure, and whether particular disclosures can lead to higher surplus for shareholders. Similar to these papers, I introduce an investment decision. Kanodia (1980) studies changes to accounting information quality and their effects on investment efficiency.[6] Kanodia, Singh, and Spero (2005) also provide insights on the role of disclosure on investment opportunities and identify how more or less disclosure is socially desirable. This last strand of literature addresses issues on the role of disclosure at the macroeconomic level.

I provide several contributions in this study. I unify the three strands of literature in a common theory that speaks to differences between the cost of capital of disclosing and non-disclosing firms but also aggregate risk premia and efficiency. To my knowledge, my study is the first to link together voluntary disclosure on cash flows, cost of capital and economic efficiency. The theoretical motivations for considering these questions as part of a single framework are clear. Regulation of accounting practices should be concerned with the overall improvement in economic efficiency, which requires the use of observable metrics such as the cost of capital. However, such regulation needs to take into account its effect on the strategic decisions of managers and how such decisions will show up in cross-sectional empirical studies. Further, I develop a model in which the predictions hold in a large economy which allows me to separate diversifiable from non-diversifiable risk and derive cross-sectional implications. From a robustness standpoint, my results

[6] Kanodia and Lee (1998) also consider the interaction between investment and disclosure and how disclosure in periodical performance reports influences the managers' real decisions. Liang and Wen (2006) investigate the effects of the accounting measurement basis on the capital market pricing and efficiency of the firm's investment decisions. Other papers show the disclosure effect on measuring intangibles (Kanodia, Sapra, and Venugopalan (2004)) and also accounting for derivatives (Kanodia, Mukherji, Sapra, and Venugopalan (2000)).

do not require the normality assumption but do require constant relative risk-aversion.[7] Finally, I provide several empirical implications: disclosure should be associated with lower cost of capital in cross-sectional studies, and disclosing to reduce cost of capital (as often stated in empirical studies) is equivalent to disclosing to maximize value. The model has also implications for the use of accounting information in asset pricing models. From an asset pricing perspective, accounting has traditionally been viewed as producing information about expected cash flows, i.e. the numerator of a net present value calculation. My framework further suggests measures of voluntary disclosure quality as possible proxies for the firm's exposure to systematic risk, i.e. the denominator or beta in a net present value calculation. In Appendix A, I present different tables to position my paper specifically in the cost of capital literature.[8]

The rest of the paper proceeds as follows. Section 2 presents the model. Sections 3-4 determine the characteristics of the two types of equilibria of the model. Section 5 examines the impacts of an exogenous disclosure friction on cost of capital both at the firm level and the macroeconomic level. Section 6 focuses on the efficiency implications of more information availability. Section 7 concludes. All proofs are in Appendix B.

2 Model

2.1 Timeline

The economy is populated by a large number of investors and firms. I briefly describe the main sequence of events.

At date 0, each investor is endowed the ownership of a single project, which entitles the owner to the future cash flows (CF) of the project if the firm is eventually financed. I later refer to this project as "the firm."

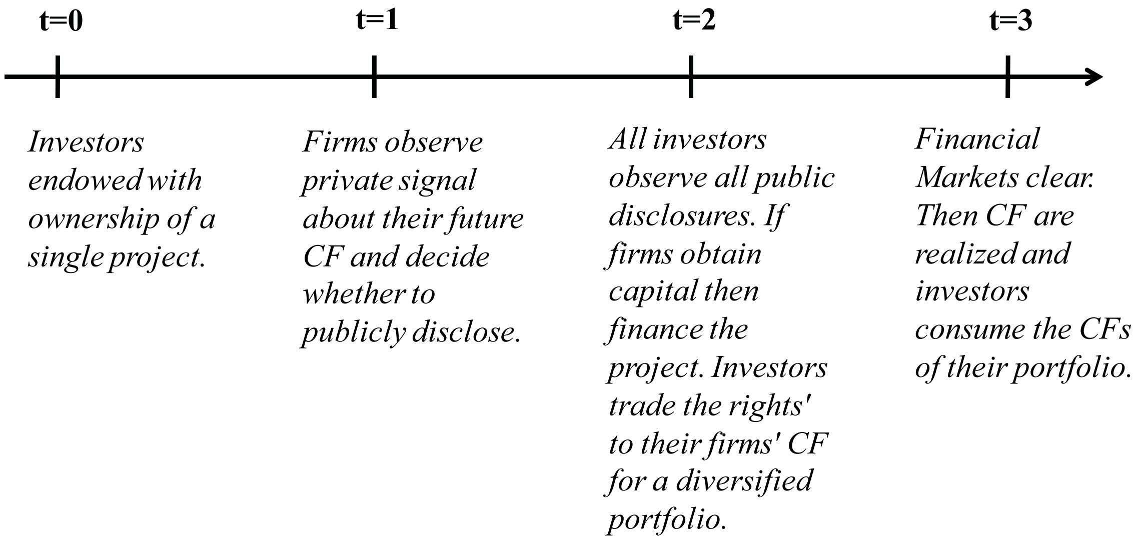
At date 1, each firm receives private information about the future cash flow of the project and may then choose to disclose. The disclosure problem is described in more details in Subsection "Firm Sector."

At date 2, all investors observe all public disclosures (if any). Firms' projects valued at a positive price are financed. Investors trade the rights to their firms' cash flow for a diversified portfolio. Their portfolio choice decision is described in Subsection "Investors' Problem."

[7] The CRRA utility fits better observed risk-taking behavior than constant absolute risk-aversion (Camerer and Ho (1994)).

[8] As shown in table 3, many of the results are new and do not fit in preexisting frameworks.

Figure 1: Timeline



At date 3, financial markets clear; the market-clearing prices are determined in Subsection "Competitive Equilibrium." Then, uncertainty about the firm's cash flows is realized, and investors consume the cash flows received from their portfolio.

2.2 Firm Sector

I discuss here the characteristics of the firms and describe the events occurring at date 1.

Firm's Cash Flows

There is a continuum of firms, and each can generate a (potential) stochastic cash flow π if the firm is financed, net of the required investment, and zero otherwise.[9] To focus on a multi-firm economy, I restrict attention to a setting in which the project is financed or not, and do not consider scales of investment.

I assume that $\pi = \epsilon + y$, where ϵ is a firm-specific i.i.d. random variable (the indexation on each firm is omitted to save space) and y is a systematic risk factor (common for all firms). This factor model approach is similar to Jorgensen and Kirschenheiter (2003, 2007) and Hughes, Liu, and Liu (2007).[10]

[9] The role of firms is identical to the neo-classical view. Firms are only executing the project if financed. Their role is to maximize the value of the current owner.

[10] Note that under the assumptions of the capital asset pricing model (e.g., Mossin (1966)), firms' cash flows can always be decomposed into an idiosyncratic and a (suitably constructed) systematic factor and

The firm-specific shock ϵ captures the firm's idiosyncratic (diversifiable) risk, has a distribution $H(.)$, density $h(.)$ with mean $\mathbb{E}(\epsilon) = \theta$ and full support over $\mathbb{R}$ and is independent of y.

The common shock y is assumed to have a density $f(.)$ and full support over $[\underline{y}, +\infty)$ (where $\underline{y} > -\theta$) and $\mathbb{E}(y) = 0$.[11] Without loss of generality, I normalize the "mass" of all firms in the economy to one; therefore, defining $CF_m(y)$ as the payoff in unit of consumption of all firms (hereafter "market portfolio"), $CF_m(y)$ must be equal to $Prob(Inv)(\mathbb{E}(\epsilon|Inv) + y)$, where Inv represents the event that the firm is financed and $Prob(Inv)$ is the probability of a firm to be financed. For example, if all firms are financed, $CF_m(y) = \theta + y$. Following this observation, I will denote a realization of y as a "state of the world."

Disclosure Decisions

All firms observe a perfect signal s on their idiosyncratic cash flow ϵ.[12] The role of the factor model presented earlier is to focus the analysis on information about the idiosyncratic component of firm's cash flows, and exclude any information about the systematic component that would clearly work to realize some of the systematic risk and thus reduce risk premia (Hughes, Liu, and Liu (2007), Lambert, Leuz, and Verrecchia (2007, 2008)). In practice, one would expect a single firm's disclosures to contain relatively little information about the state of the overall economy, as compared to information about the firm's own business.[13] Thus, for firms that are not too large relative to the economy, the effect of information about systematic risk - if any - is likely to be small for the vast majority of existing firms.[14]

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thus, such a decomposition is without loss of generality.

[11]The restriction to $\mathbb{E}(y) = 0$ is without loss of generality; if $\mathbb{E}(y) \neq 0$, one could relabel $y' = y - \mathbb{E}(y)$, with mean zero, and $\epsilon' = \theta + \mathbb{E}(y)$, with no change to the results or analysis. In other words, a revision of the economy's growth would be captured in this model by the common mean of the firm-specific factor θ.

[12]The results, and proofs, are unchanged if one assumes instead that firms receive a noisy signal, say ρ, on ϵ. Given that the estimation risk on ϵ is purely idiosyncratic, it would not be priced, and thus one could relabel the model by replacing ϵ by $\epsilon' = \mathbb{E}(\epsilon|\rho)$. Using ϵ', all the results will carry over.

[13]The aggregation of information of all small firms should unravel some information about the state of the economy. Seyhun (1992) reports evidence that aggregate insider trading in small firms predicts future stock returns in larger firms. However it is unlikely that a small firm alone would provide significant information about the common shock.

[14]I do not mean that this effect would necessarily vanish in a large economy, since it is always theoretically possible to have a disclosure by a very small firm to be very informative on all other firms (and thus, incidently, on systematic uncertainty); however, this result is economically implausible, as very little would likely be learnt about the economy (e.g., GDP or consumption growth, level of the index) through the incremental disclosure of one firm.

Firms decide whether to release their private information. As is common in the voluntary disclosure literature, I assume disclosure must be truthful. If the firm chooses to voluntarily disclose, then with probability $\eta \in (0, 1)$, the message sent by the firm is not received; thus η is a measure of the disclosure friction. Investors cannot distinguish between firms which chose not to disclose from firms whose disclosures were not received. Conversely, with probability $1 - \eta$, firms choosing to disclose succeed in disclosing and their private signal becomes public information.[15] Like a "lock and key", disclosure and the probability it is received are both needed for a private signal to become public: first, the firm needs to choose to release its information (the lock) and then investors need to receive it correctly (the key).

The disclosure friction η is a modeling device to capture the informational asymmetry between firms and outside investors. One can think of all firms choosing to provide either an informative disclosure or a noisy one. With probability $1 - \eta$, an informative disclosure is understood by the receiver. Of course, none of the noisy disclosures can be understood. One can also view η as a proxy for the complexity of the economic operations to be disclosed or whether a firm lacks sufficient credibility for truthful disclosure (Stocken (2000)). Investors may also not pick up the information sent by firms, either because they did not pay attention to the release of information or cannot understand the firm's information (Hirshleifer and Teoh (2003)). The probability η may also be interpreted as a measure of the accounting quality of accounting standards: a firm will report mandatory disclosures but may then schedule a presentation (Bushee, Jung, and Miller (2008)) and/or additional financial notes/disclosures that may or may not guide the investors in understanding the information released. In the model investors either internalize the firms' disclosures or do not.[16] I will refer to the parameter η as a disclosure friction. But the reader can refer to the different interpretations mentioned before to relate to a more specific context.

Firms take the set of possible prices as given when they disclose. I define P_ϵ (to be endogenously determined) as the market price if the firm's signal $s = \epsilon$ is observed. The price $P_\emptyset$ is offered if no additional signal is revealed, pooling firms that voluntarily retained their information with firms that disclosed but the information was not received. Further, a negative price implies that the firm is not financed, thus producing a certain

[15] This friction does not correspond to the Dye's specification but is mentioned in his paper as one alternative assumption to model non-proprietary information. In my model, using the alternative formulation is more analytically tractable.

[16] There may be costs associated in decreasing η (e.g., often cited implementation costs of the Sarbanes-Oxley Act as noted in Gao, Wu, and Zimmerman (2008)). Such costs would endogenize η in a straightforward manner and thus are not incorporated explicitly in the model.

cash flow of zero.

Firms maximize the value of their current owner and disclose if and only if $P_\epsilon \geq P_\emptyset$. I denote the optimal disclosure threshold ϵ^*, above which all firms decide to voluntarily disclose.

2.3 Investors' Problem

Now I discuss the characteristics of investors, and describe the events occurring at date 2.

Preferences

Investors are each initially endowed with one firm.[17] They have a constant relative risk-aversion (CRRA) utility function $u(x) = x^{1-\alpha}/(1-\alpha)$, where x is final consumption and $\alpha > 0$ is an investor's Arrow-Pratt relative risk-aversion coefficient.[18] Each investor is initially undiversified; thus, it is optimal for him to sell the project and invest in a diversified portfolio.[19]

Portfolio Choice

At date 2, investors observe the information disclosed by all firms and sell their asset in a competitive market; and (optimally) want to diversify away their idiosyncratic risk.

Post disclosure, investors differ in the disclosure of their firm. If the firm did not disclose, the investor can sell his firm for a price $P_\emptyset$. There is also a continuum of investors who can sell their firm for a price P_ϵ where $\epsilon \in \mathbb{R}$. In short-hand, I denote the value of a firm P_δ, where $\delta \in \{\emptyset\} \cup \mathbb{R}$. Because the conditions for two-fund separation hold in this economy (see Cass and Stiglitz (1970)), investors trading in a complete financial market would always choose to hold only a combination of the market portfolio and the risk-free asset. Their portfolio choice problem can thus be written as follows:

[17] All the results of the model carry over for: (i) other types of ownership (certain investors own multiple projects or share ownership), (ii) if some investors do not own a project, (iii) if all investors also have an i.i.d. personal wealth, in addition to their project. The only required assumption is that not all investors are perfectly diversified ex-ante. One of the main results (the difference in returns between disclosing or non-disclosing firms) is robust to any strictly concave utility function but the CRRA assumption is required for the comparative statics and efficiency comparisons.

[18] if $\alpha = 1$, $u(.)$ is set to $u(x) = \ln(x)$.

[19] If the investor was also the manager, issues relating to signalling with security design could arise; for example, entrepreneurs/investors may possibly have used debt finance and signal their information by retaining some of their firm's cash flow(see Bertomeu, Beyer, and Dye (2008) for a model in which the firm is financed with debt). The role of investors as managers is studied within the context of partnerships in Huddart and Liang (2005).

$$(\Gamma_\delta) \qquad \max_{\gamma_{\delta f}, \gamma_{\delta m}} \int f(y) U \left\{ P_\delta(\gamma_{\delta f} + \gamma_{\delta m} CF_m(y)) \right\} dy$$
$$\text{s.t } \; P_\delta \geq P_\delta \gamma_{\delta f} + P_\delta \gamma_{\delta m} P_m \tag{1}$$

Problem (Γ_δ) corresponds to the portfolio choice problem of an investor whose firm disclosed δ. In this problem, P_m is the price of the market portfolio, defined as

$$P_m = \mathbb{E}(CF_m(y))/\mathbb{E}(R_m)$$

The market price P_m (equivalently $\mathbb{E}(R_m)$) is taken as given by investors but is endogenized in the next section. Note that P_m is the *ex-ante* price, before y is realized. Let $\gamma_{\delta f}$ be the proportion of the initial wealth the investor puts in the risk-free asset and $\gamma_{\delta m}$ be the proportion of the wealth he invests in the market portfolio. Equation (1) is the investor's budget constraint. That is, the investor can use his current wealth, as determined by his holdings in the single firm disclosing δ, to purchase any combination of the risk free asset and market portfolio.

2.4 Competitive Equilibrium

I discuss here the sequence of events occurring at date 3; specifically, I state the definition of a competitive equilibrium and derive the equilibrium market prices.

No-Arbitrage

As a result of the factor decomposition, each firm's cash flow contingent on a disclosure can be written in terms of a basket of the market portfolio and the risk-free asset; in the absence of arbitrage, its value can be computed as the value of this basket of securities.

Assume initially that all firms in the economy invest. Consider a firm disclosing ϵ. An investor is willing to value this firm but only knows the value of the market portfolio P_m. The value of this firm disclosing ϵ can be written in terms of P_m, because:

(i) Holding the firm yields the investor a cash flow $\epsilon + y$, where ϵ is known and y is not yet known.

(ii) Holding $\epsilon - \mathbb{E}(\epsilon)$ units of the risk-free asset and one unit of the market portfolio yields a cash flow $\epsilon - \mathbb{E}(\epsilon) + \mathbb{E}(\epsilon) + y = \epsilon + y$.

If the investor is rational, he should value these two investments identically and thus should value this disclosing firm at $P_m + \epsilon - \mathbb{E}(\epsilon)$. Recall that the unit price of the risk-free asset can be normalized to one. This concludes the argument when firms all invest. More generally, if some firms do not invest, the cash flow can be decomposed as follows:

$$\underbrace{\epsilon - \mathbb{E}(\epsilon|Inv)}_{\text{units of risk-free asset}} + \underbrace{\frac{1}{Prob(Inv)}}_{\text{units of market portfolio}} \underbrace{Prob(Inv)\,(\mathbb{E}(\epsilon|Inv) + y)}_{CF_m}$$

In other words, the firm's cash flow is the sum of $\epsilon - \mathbb{E}(\epsilon|Inv)$ units of a risk-free bond and $\frac{1}{Prob(Inv)}$ units of the market portfolio. The unit price of the market portfolio P_m is equal to $\frac{\mathbb{E}(CF_m)}{\mathbb{E}(R_m)}$, where $\mathbb{E}(R_m)$ is the expected market portfolio return. When valuing this basket of assets, the firm's no-arbitrage price must be:

$$P_\epsilon = \epsilon - \mathbb{E}(\epsilon|Inv) + \frac{1}{Prob(Inv)} \frac{\mathbb{E}(CF_m)}{\mathbb{E}(R_m)}$$

Second, consider a firm that does not disclose and is financed. The firm's cash flow is $\epsilon + y$, where ϵ is unknown to investors but can be perfectly diversified by holding a portfolio of all non-disclosing firms. As a result the price $P_\emptyset$ of this firm should be that of a firm paying $\mathbb{E}(\epsilon|ND) + y$, where ND represents the event that the firm did not disclose. By no-arbitrage, it must hold that:

$$P_\emptyset = \mathbb{E}(\epsilon|ND) - \mathbb{E}(\epsilon|Inv) + \frac{1}{Prob(Inv)} \frac{\mathbb{E}(CF_m)}{\mathbb{E}(R_m)}$$

If a non-disclosing firm is not financed, I set $P_\emptyset = 0$.

Market-Clearing and Risk Premium

I close the model by recovering the expected market return $\mathbb{E}(R_m)$ from equilibrium restrictions. To avoid situations with multiple equivalent equilibria, I assume that a firm that does not expect to be financed conditional on its disclosure will not disclose (e.g., if there is some small cost for disclosure).[20] It follows that only firms that did not disclose may not be financed. Therefore, there are two possible equilibrium candidates: (1) over-investment equilibria, in which all firms invest and receive a positive price even if they do

[20]The results are unchanged when this restriction is lifted; except that there may be many economically equivalent equilibria in which some low-value firms choose to disclose but still do not receive financing.

not disclose, (2) underinvestment equilibria, in which firms that do not disclose - whether voluntarily or involuntarily - are not financed.

I now introduce the overinvestment equilibrium, where the optimal disclosure threshold ϵ^* is replaced by ϵ^{over}.

Definition 1 *An "overinvestment" competitive equilibrium is a set of optimal portfolio choice, expected market portfolio return and disclosure threshold* $(\gamma_{\delta f}^{over}, \gamma_{\delta m}^{over}, \mathbb{E}(R_m^{over}), \epsilon^{over})$ *such that:*

(i) $\gamma_{\delta f}^{over}, \gamma_{\delta m}^{over}$ *solve the maximization problem* (Γ_δ)

(ii) $P_{\epsilon^{over}} = P_\emptyset$

(iii) $P_\emptyset \geq 0$ *and all firms are financed.*

(iv) $\forall y,\ \ 0 = (1-(1-\eta)(1-H(\epsilon^{over})))\gamma_{\emptyset f}^{over} P_\emptyset + (1-\eta)\int_{\epsilon^{over}}^{+\infty} \gamma_{\epsilon f}^{over} P(\epsilon) h(\epsilon) d\epsilon$

Condition (i) is the optimality condition for investors. Condition (ii) is the optimality for firms: the optimal disclosure threshold ϵ^{over} is determined so that a firm receiving signal ϵ^{over} is indifferent between disclosing or retaining its private information. It should be noted from the previous discussions that $P_{\epsilon^{over}}$ and $P_\emptyset$ depend on the expected market return. Condition (iii) ensures that all firms invest in the economy. Finally, condition (iv) represents the market-clearing constraint in the risk-free asset: the supply in the risk-free asset is equal to the demand in the risk-free asset.

$$\underbrace{0}_{\text{net supply}} = \underbrace{(1-(1-\eta)(1-H(\epsilon^{over})))\gamma_{\emptyset f}^{over} P_\emptyset}_{\text{non-disclosing firms' total demand}} + \underbrace{(1-\eta)\int_{\epsilon^{over}}^{+\infty} \gamma_{\epsilon f}^{over} P(\epsilon) h(\epsilon) d\epsilon}_{\text{disclosing firms' total demand}}$$

Investors in non-disclosing firms demand the same quantity of risk-free asset $\gamma_{\emptyset f}^{over} P_\emptyset$, whereas investors in disclosing firms differ in the quantity of the risk-free asset they demand conditional on the disclosure ϵ, $\gamma_{\epsilon f}^{over} P(\epsilon)$. The risk-free asset is in zero net supply; then, by Walras Law, market-clearing of the risk-free asset and the investor's budget constraints imply market-clearing for the market portfolio, and this second market-clearing condition can be omitted.

Conditions (i) and (iii) are standard in the general equilibrium literature and condition (ii) is standard in the disclosure literature; the model nests both general equilibrium concerns and endogenous disclosure in a common framework. Further, the disclosure decision and the expected market return are inter-related. On one hand, both $P_{\epsilon^{over}}$ and $P_\emptyset$

are functions of the expected market return $\mathbb{E}(R_m^{over})$ and thus solving for ϵ^{over} requires some knowledge of the expected market return. On the other hand, the market clearing constraint (iv) depends on the disclosure threshold ϵ^{over}.

I next consider the underinvestment equilibrium with the optimal threshold ϵ^* replaced by ϵ^{under}.

Definition 2 *An "underinvestment" competitive equilibrium is a set of optimal portfolio choice, expected market portfolio return and disclosure threshold* $(\gamma_{\delta f}^{under}, \gamma_{\delta m}^{under}, \mathbb{E}(R_m^{under}), \epsilon^{under})$ *such that:*

(i) $\gamma_{\delta f}^{under}, \gamma_{\delta m}^{under}$ *solve the maximization problem* (Γ_δ)

(ii) A non-disclosing firm receives $P_\emptyset = 0$ *and is not financed*

(iii) $P_{\epsilon^{under}} = 0$

(iv) $\forall y, \;\; 0 = (1-(1-\eta)(1-H(\epsilon^{under})))\gamma_{\emptyset f}^{under} P_\emptyset + (1-\eta)\int_{\epsilon^{under}}^{+\infty} \gamma_{\epsilon f}^{under} P_\epsilon h(\epsilon) d\epsilon$

The definition of the equilibrium is similar to the previous one in terms of conditions (i), (iii), and (iv). The main difference is that firms whose information is not received (either voluntarily withheld or because they were unable to transmit) are not financed. Specifically condition (ii) corresponds to firms that would be priced negatively by investors, were their projects executed and, thus, are not financed (their cash flow is zero). Therefore the market portfolio cash flow in state y is $CF_m(y) = (1-\eta)\int_{\epsilon^{under}}^{+\infty}(\epsilon + y)h(\epsilon)d\epsilon$. This corresponds to the firms successfully disclosing their information and a positive pricing in equilibrium.

At this stage, the term overinvestment and underinvestment may seem an abuse of language, given that no such property has yet been formally shown. However, stepping ahead, I will demonstrate in Sections 3 and 4 that such labels are indeed justified; hopefully, the reader will pardon this logical misstep and benefit from the convenience of using the terminology early in the discussion.

2.5 First-Best Benchmark

The first-best solution to the model is defined as the optimal financing threshold ϵ^{FB} chosen by a fully-informed, efficiency-maximizing planner. Note that, in the first-best, all investors should be given the same well-diversified portfolio ex-ante and thus the first-best problem is equivalent to maximizing the ex-ante CRRA utility of the investor.[21]

$$\max_{\tilde{\epsilon}} \frac{1}{1-\alpha} \int f(y) \left(\int_{\tilde{\epsilon}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right)^{1-\alpha} dy$$

Proposition 1 *Firms are financed if and only if their signal about future cash flows ϵ is above ϵ^{FB} where ϵ^{FB} is uniquely defined as follows:*

$$\epsilon^{FB} = -\frac{\int y f(y) (\int_{\epsilon^{FB}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon)^{-\alpha} dy}{\int f(y) (\int_{\epsilon^{FB}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon)^{-\alpha} dy} \in (0, \theta) \tag{2}$$

First-best prescribes not to finance firms whose expected cash flows are too low. The fundamental tension in the first-best solution is between increasing expected aggregate consumption, and decreasing total aggregate risk by closing down some firms. The first-best threshold lies between 0 and θ. At one extreme, financing a firm with zero idiosyncratic value ($\epsilon = 0$) would increase risk without increasing aggregate consumption. At the other extreme, a firm with the expected unconditional cash flow ($\epsilon = \theta$) yields a positive non-diversifiable cash flow component $\theta + y$, which is always strictly greater than the payoff if the firm is not financed.

3 Overinvestment Equilibrium

I first solve the overinvestment equilibrium. This equilibrium exists if the non-disclosing price $P_\emptyset$ is positive. I decompose the problem in three steps. First, I take the expected market portfolio return $\mathbb{E}(R_m)$ as given and derive the optimal disclosure threshold from condition (ii) in definition (1). Second, I solve for the expected market return $\mathbb{E}(R_m)$ based on constraints (i) and (iv) from definition (1). Third, I collect these results and formally state the competitive equilibrium of the model.

[21]For convenience, I focus on the "anonymous" or symmetric solution, in which the planner does not advantage certain investors over others. The solution ϵ^{FB} is unchanged if one considers the complete set of Pareto-efficient solutions in which the planner may favor certain investors over others.

3.1 Disclosure Threshold Dependent on the Disclosure Friction

Non-Disclosure Price

To find the optimal threshold, it is first helpful to consider the price offered to firms which do not disclose as a function of different possible disclosure thresholds $\hat{\epsilon}$ (the latter being not necessarily the optimum); to stress the dependence on the disclosure threshold, I denote this price $P_{\emptyset}(\hat{\epsilon})$.

Lemma 1 *$P_{\emptyset}(\hat{\epsilon})$ is U-shaped in $\hat{\epsilon}$, attainting a global minimum at a unique point $\hat{\epsilon}^{over}$ given by:*

$$\int_0^{\hat{\epsilon}^{over}} H(\epsilon)d\epsilon = \eta \int_{\hat{\epsilon}^{over}}^{+\infty} (1 - H(\epsilon))d\epsilon \tag{3}$$

The fact that $P_{\emptyset}(\hat{\epsilon})$ is non-monotonic in $\hat{\epsilon}$ is not surprising. Suppose for example that $\hat{\epsilon}$ is small (all firms want to disclose). Then, investors know that non-disclosure is involuntary and thus they will offer a price corresponding to the unconditional cash flow expectation θ. At the other extreme, suppose that $\hat{\epsilon}$ is large (no firm discloses). Then, investors observe only firms not disclosing in the economy and thus also give a price corresponding to the expected cash flow θ. In-between these two extremes, the market infers that firms with high signals are more likely to disclose, self-selecting out of the non-disclosure outcome.

More precisely, the function $P_{\emptyset}$ is U-shaped and driven by the tension between two conflicting effects. The first effect is the *dilution* effect: if the threshold increases, the proportion of firms intentionally (unintentionally) not disclosing increases (decreases). The price is driven down, as high-value firms are diluted. The second effect is the *self-selection* effect: as the threshold increases, firms deliberately withholding their information receive higher signals on their future cash flows and thus the price increases.

For low values of $\hat{\epsilon}$ (most firms disclose), the dilution effect dominates and $P_{\emptyset}$ is decreasing in $\hat{\epsilon}$. In other words, moving to a regime in which fewer firms disclose decreases the price offered to firms that do not disclose. As the threshold increases, more firms *voluntarily* withhold and their average ϵ increases. The self-selection effect prevails for high values of $\hat{\epsilon}$ (few firms disclose) and $P_{\emptyset}$ is increasing in $\hat{\epsilon}$.

Optimal Disclosure Threshold

The optimal disclosure threshold ϵ^{over} is the signal at which the firm is indifferent between disclosure and non-disclosure, i.e. $P_{\epsilon^{over}} = P_{\emptyset}$. Proposition 2 shows that an optimal disclosure threshold exists and is unique.

Proposition 2 *There exists a unique optimal disclosure threshold ϵ^{over} such that*

(i) if $\epsilon < \epsilon^{over}$, the firm retains its private information.

(ii) if $\epsilon \geq \epsilon^{over}$, the firm discloses it.

ϵ^{over} does not depend on the expected market portfolio return $\mathbb{E}(R_m)$, and satisfies $\epsilon^{over} = \hat{\epsilon}^{over}$ (obtained in Lemma 1).

As in standard disclosure models, firms with favorable news disclose and those with less favorable news withhold. While P_δ depends on the expected market return $\mathbb{E}(R_m)$, the disclosure threshold ϵ^{over} does not.

Proposition 2 shows that the equilibrium threshold is located exactly at the (interior) value of ϵ that minimizes $P_\emptyset$. Therefore, the disclosure game points to the outcome that makes not disclosing the least attractive. I interpret this result as a partial unraveling of types: market forces minimize payoffs to firms which do not voluntarily give their information. The disclosure friction $\eta \in (0, 1)$, however, prevents this payoff from becoming small, as low-signal firms know that they can be pooled to other firms who did not disclose involuntarily. While I will focus on endogenous disclosure in the rest of the paper, the result also points to certain practical cases in which actual accounting rules are able to set the threshold, such as materiality rules or conservative accounting practices (Basu (1997), Heitzman, Wasley, and Zimmerman (2008)). In my model, such rules would always decrease the price difference between the disclosing and non-disclosing firms, making both types of firms more alike and qualitatively weakening the effects of the voluntary disclosure regime.

Comparative Statics

The optimal threshold might potentially depend on the exogenous parameters of the model: the disclosure friction η and the risk aversion of the investors α. In corollary 1, I describe several comparative statics tying the disclosure decisions to these fundamental characteristics of the economy.

Corollary 1 *The threshold ϵ^{over}: (i) is increasing in the disclosure friction η, (ii) is positive and less than θ, (iii) does not depend on the risk-aversion of investors α.*

As is common in the disclosure literature, a higher disclosure friction increases the proportion of firms *voluntarily* withholding. This comparative static is well-understood in the disclosure literature and thus I do not pursue it further here. Firms that should not have invested in first-best do not disclose yet are financed; in this respect, the equilibrium is consistent with its terminology of overinvestment. The asymmetric information between firms and outside investors, combined with a high disclosure friction, leads investors to infer that non-disclosing firms, are predominantly firms unable to disclose and thus are likely to have favorable news. Non disclosing firms have a higher valuation when disclosure frictions are higher, which leads to inefficient investments being financed. Effectively, there are three types of firms that do not disclose: (a) firms that should be financed in first best that wanted to disclose but could, (b) firms that should be financed in first-best but voluntarily withheld, (c) firms that should not be financed in first-best and withheld. Only type (c) causes the investment inefficiency; type (b) may not be blamed on the grounds of efficiency since its voluntary non-disclosure only causes a reallocation of wealth from type (a).

The disclosure threshold ϵ^{over} does not depend on the market risk premium and thus on the risk-aversion parameter α. All firms are financed and once they execute their projects, they are identically affected by the common shock y, which is additively separable from the idiosyncratic cash flow ϵ. An empirical implication of this property is that the amount of voluntary disclosure should be insensitive to the business cycle. For example, according to the model, one should not observe much time series variation in aggregate levels of disclosure as compared to, say, cross-country or cross-industry variations. Moreover, the aggregate level of disclosure should not be related to characteristics of the overall economy, such as GDP growth or market return.

3.2 Market Risk Premium

I determine next the expected market portfolio return $\mathbb{E}(R_m^{over})$ and the competitive equilibrium of the economy.

Proposition 3 *For high levels of disclosure frictions ($\eta \geq \eta^{over}$), there exists an overinvestment competitive equilibrium*
$(\gamma_{\delta f}^{over}, \gamma_{\delta m}^{over}, \mathbb{E}(R_m^{over}), \epsilon^{over})$*, where:*

(i) $\gamma_{\delta f}^{over} = 0$ *and* $\gamma_{\delta m}^{over} = \mathbb{E}(R_m^{over})/\theta$

(ii) the expected market return is equal to:

$$\mathbb{E}(R_m^{over}) = \frac{\theta}{\theta + Q^{over}}$$
$$where \;\; Q^{over} = \frac{\int y f(y)(\theta + y)^{-\alpha} dy}{\int f(y)(\theta + y)^{-\alpha} dy} < 0$$

(iii) and the optimal disclosure threshold is ϵ^{over} *defined in equation* (3)

After the disclosure stage, agents have personal wealth P_δ (where δ may vary across agents), the market value of their firm. Each agent, then, makes different portfolio choice decisions, choosing a different quantity of risk-free asset and market portfolio. Under the assumption of CRRA utilities, agents invest a fixed share of their wealth in the market portfolio that does not depend on their wealth (they invest in proportion to their wealth). Aggregating all such consumers yields a simple expression for the equity premium that corresponds to the market premium for a representative agent owning all the firms and having the same CRRA utility function as each individual consumers.[22]

The expected market portfolio return $\mathbb{E}(R_m^{over})$ given in equation (4) can be rewritten as follows:

$$\underbrace{\mathbb{E}(R_m^{over}) - 1}_{\text{risk premium}} = \frac{-Q^{over}}{\theta + Q^{over}} = \frac{-Q^{over}}{P_m^{over}} \tag{4}$$

The expected market return has, as predicted by the asset pricing theory, a return higher than the risk-free rate because the market portfolio is exposed to undiversifiable systematic risk. The term $-Q^{over}/P_m^{over}$ corresponds to the equity premium in the CAPM framework.

One important aspect of the model is that the equity premium does not depend on the informational frictions and/or characteristics of the disclosure environment. By Proposition 3, the equity premium can be fully characterized by the behavior of the representative agent who, by construction, owns all the wealth and thus does not bear the extra risk due to disclosure. The result may be surprising given existing results in the CARA-Normal framework, where it is typically obtained that the risk-premium decreases in the disclosure friction (Jorgensen and Kirschenheither (2003)) or, in a pure exchange economy,

[22] The result also suggests some caution in interpreting single-agent models of disclosure outside of the CRRA framework. If utility functions are CARA, for example, there will exist a representative agent; however, the preference of this representative agent will depend on the wealth of all agents, which in turn will depend on the disclosure threshold ϵ^*; as a result, a comparative static on the disclosure threshold would require adjusting the preferences of the representative agent - which would lead to considerable analytical difficulties.

may decrease in exogenous accounting quality (Lambert, Leuz, and Verrecchia (2007), Gao (2008)). One limitation of CARA is its unappealing risk-taking properties; for example, CARA implies that a wealthy investor would own as much (in total dollar amount) of the market portfolio as a poor investor; the richer investor would invest all of his/her extra wealth in the risk-free asset. The idea that risk premia should decrease with a greater disclosure friction is, to some extent, an artefact of the CARA formulation, which does not occur if investors invest in the market portfolio relative to their wealth which seems realistic.

4 Underinvestment Equilibrium

4.1 Risk Premium and Optimal Disclosure Threshold

The underinvestment equilibrium shares with the previous equilibrium the existence of a representative agent, specifically risk premia can be obtained from the solution in a one-person economy. However, one major difference in this economy is that the total consumption available in the economy (the payoff of the market portfolio) depends on how many firms are financed (and thus not financed), which depends on the probability of disclosure and the disclosure friction. Therefore, the disclosure friction may now affect risk premia, through its real effects on aggregate wealth.

Proposition 4 *If the level of disclosure frictions is low ($\eta \leq \eta^{under}$), there exists an underinvestment competitive equilibrium, which is given as follows:*

(i) $\epsilon^{under} = \epsilon^{FB}$

(ii) $\gamma_{\delta f}^{under} = \gamma_{\emptyset m}^{under} = 0$ *and* $\gamma_{\epsilon m}^{under} = \frac{\mathbb{E}(R_m^{under})}{(1-\eta)\int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon}$

(iii) $\mathbb{E}(R_m^{under})$ *is given as follows:*

$$\begin{aligned} \mathbb{E}(R_m^{under}) &= \frac{\int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon}{\int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon + (1 - H(\epsilon^{under}))Q^{under}} \\ \textit{where} \quad Q^{under} &= \frac{\int y f(y) (\int_{\epsilon^{under}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon)^{-\alpha} dy}{\int f(\tilde{y}) (\int_{\epsilon^{under}}^{+\infty} (\epsilon + \tilde{y}) h(\epsilon) d\epsilon)^{-\alpha} d\tilde{y}} < 0 \end{aligned}$$

In the underinvestment equilibrium, all firms that should not have invested in first-best do not disclose and therefore are not financed. Thus, this equilibrium prescribes efficient shut-down of all low-value firms. However, there are also high-value firms that, with probability η, could not disclose and are not financed, leading to underinvestment relative to first-best.

Neither the optimal disclosure threshold nor the risk premium depend on the disclosure friction η. Intuitively, the economy functions in a "constrained" first-best environment, in which a proportion η of efficient firms are simply not financed, but for the remaining proportion $1-\eta$ of efficient firms, investments are made according to the first-best rule.

The expected market portfolio return $\mathbb{E}(R_m^{under})$ can be written as follows:

$$\mathbb{E}(R_m^{under}) - 1 = \frac{-(1-H(\epsilon^{under}))Q^{under}}{\int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon)d\epsilon + (1-H(\epsilon^{under}))Q^{under}} > 0$$

Notice that in the underinvestment equilibrium, some firms do not invest and this leads to a decrease in the exposure of the market portfolio to the systematic risk.

4.2 Uniqueness of Equilibrium

I investigate next the type of equilibrium that can be sustained as a function of the disclosure friction. Let $Q(x)$ be defined as follows:

$$Q(x) = \frac{\int y f(y)(\int_x^{+\infty}(\epsilon+y)h(\epsilon)d\epsilon)^{-\alpha}dy}{\int f(y)(\int_x^{+\infty}(\epsilon+y)h(\epsilon)d\epsilon)^{-\alpha}dy}$$

Expression $Q(x)$ is a measure of the risk premium required by investors depending on the financing threshold. In other words as x increases fewer firms are financed. Note that if x converges to $-\infty$, $Q(x)$ converges to Q^{over} whereas if x is equal to ϵ^{FB} then $Q(x) = Q^{under}$. Ordering Q^{under} and Q^{over} boils down to study the variations of $Q(x)$ for a change in x. An increase in x means fewer firms are financed. Risk-averse investors' wealth diminishes as some firms with positive cash flows do not execute their project. In response to the negative wealth effect investors require a higher risk premium, which translates into a lower $Q(x)$. Simultaneously reducing the number of financed firms also decreases their exposure to systematic risk. Investors require less insurance and thus a lower risk premium or equivalently a higher $Q(x)$.

Lemma 2 *(i) If $Q(x)$ increases in x then $\epsilon^{under} \leq \epsilon^{over}$ and $\eta^{under} \leq \eta^{over}$.*

(ii) If $Q(x)$ decreases in x then $\epsilon^{under} \geq \epsilon^{over}$ and $\eta^{under} \geq \eta^{over}$.

The overinvestment equilibrium exists as long as the non-disclosing firms invest. This condition is satisfied if investors believe that many high-value firms could not disclose, i.e. the disclosure friction η is high. Conversely, when the disclosure friction is low, investors are able to identify non-disclosing firms as low-value firms and therefore they do not invest in a non-disclosing firm. Confronting these two forces, I show that one can be confronted with two scenarios. Case (i) corresponds to the case where there is a unique underinvestment equilibrium for a given η if $\eta \leq \eta^{under}$ whereas there exists a unique overinvestment equilibrium for a given η if $\eta \geq \eta^{over}$. However between η^{under} and η^{over} there is neither an underinvestment nor an overinvestment.[23] Case (ii) implies a multiplicity of equilibria between η^{under} and η^{over}. The overinvestment and underinvesment equilibria overlap.

In both scenarios a large decrease in the disclosure friction worsens the lemon's problem conditional on non-disclosure and thus may cause a reduction in investment even if this means also not financing some high-value firms. The result suggests that sufficiently large decreases in the disclosure friction should reduce the total aggregate level of investment. The analysis points to possibly unwelcome consequences of greater disclosure. For example, during the 2008 financial crisis, a move toward mark-to-market accounting in bank financial statements may have provided more accurate information, but simultaneously may have triggered the shutdown of other (possibly) healthy institutions that were unable to disclose because of the complexity of their net positions and financial instruments.

The results presented in the next sections hold independently of the variation of $Q(x)$ in x.

5 Disclosure and Cost of Capital

5.1 Expected Cash Flows and Market Sensitivity

In this Section, I relate a firm disclosure to its cost of capital. Assume that the economy is such that $\eta \geq \eta^{*}$. Prices of a disclosing and a non-disclosing firm are characterized by the two following components:

[23] For $\eta \in (\eta^{under}, \eta^{over})$ there exist mixed-strategy equilibria where non-disclosing firms are not financed with a positive probability. These equilibria are not studied as they are not very realistic.

$$P(\epsilon) = \underbrace{\epsilon}_{\text{Idiosyncratic CF}} + \underbrace{Q^{over}}_{\text{Systematic pricing}}$$

$$P_{\emptyset} = P(\epsilon^{over}) = \underbrace{\epsilon^{over}}_{\text{Idiosyncratic CF}} + \underbrace{Q^{over}}_{\text{Systematic pricing}}$$

The first component in the above equation corresponds to inferences about the idiosyncratic cash flow. It is increasing in the signal and, given that firms that do not disclose have, on average, low value, it is also greater for firms disclosing than for firms not disclosing. The second component corresponds to the pricing of the firm's systematic risk, and is identical for disclosing and non-disclosing firms. While the total amount of systematic risk borne by the disclosing and non-disclosing firms is identical, the total amount of risk per unit of expected cash flow is not. Because disclosing firms have higher cash flows, the sensitivity to systematic risk is diluted. This rationalizes the empirical positive association between voluntary disclosure and earnings quality documented, for example, by Francis, Nanda, and Olsson (2008).

5.2 Disclosure and Market Beta

To convert these results into statements about firm's expected returns, I define next a firm's cost of capital as investors' expected cash flow over the market price. To set up ideas, the risk premium corresponds to the cost of capital for the market portfolio.

Formally, let $R_\delta \equiv (\mathbb{E}(\epsilon|\delta))/P_\delta$ be defined as the expected return for a firm disclosing δ (i.e., the ratio of its expected cash flow to its price), where $\delta \in \{\emptyset\} \cup \mathbb{R}$. Finally, let $R_D = \mathbb{E}(R_\delta|\delta \neq \emptyset)$, be the expected return conditional on disclosure.

From asset pricing models, one knows that a firm less (more) sensitive to systematic risk has a lower (higher) expected market return. The measure of the firm's sensitivity to systematic risk is the market β measured by the covariance of the firm's return with the market portfolio return over the variance of the market portfolio return. I relate the market β to the cost of capital in this model.

Lemma 3 *Suppose $\eta \geq \eta^{over}$. The firm's cost of capital can be expressed as follows:*

$$R_{\emptyset} = \underbrace{1}_{\textit{Risk-free}} + \beta_{\emptyset} \underbrace{(\mathbb{E}(R_m^{over}) - 1)}_{\textit{Risk Premium}}$$

$$R_D = \underbrace{1}_{\textit{Risk-free}} + \beta_D \underbrace{(\mathbb{E}(R_m^{over}) - 1)}_{\textit{Risk Premium}}$$

where:

$$\beta_\emptyset = \underbrace{\frac{V(y)}{P_\emptyset P_m^{over}}}_{covariance} / \underbrace{\frac{V(y)}{P_m^{over2}}}_{market\ variance}$$

$$\beta_D = \underbrace{\int_{\epsilon^{over}}^{+\infty} \frac{V(y)}{P(\epsilon) P_m^{over}} \frac{h(\epsilon)}{(1 - H(\epsilon^{over}))} d\epsilon}_{covariance} / \underbrace{\frac{V(y)}{P_m^{over2}}}_{market\ \ variance}$$

Disclosing and non-disclosing firms differ by their sensitivity to the risk premium. Note at this point that non-disclosing firms have an additional variance term due to the fact that their signal is imperfectly known; however, this extra variance is diversifiable and therefore it is not priced. In particular, the variance due to the estimation risk on ϵ does not appear in $\beta_\emptyset$.

5.3 Firms' Cost of Capital and Disclosure

Proposition 5 compares average expected returns of disclosing firms R_D (averaged over all firms who disclosed successfully) and non-disclosing firms $R_\emptyset$.

Proposition 5 *In the overinvestment equilibrium region ($\eta \geq \eta^{over}$), $\beta_D < \beta_\emptyset$. That is, disclosing firms have a lower cost of capital than non-disclosing firms, i.e. $R_D < R_\emptyset$.*

The market beta of firms disclosing is lower than the market beta of firms that do not disclose. This effect is due to the fact that disclosing firms have, on average, higher idiosyncratic cash flows than non-disclosing firms. In turn, these future gains dilute some of the sensitivity to the systematic shock, offsetting the systematic risk. In contrast, non disclosure implies low cash flows, and thus more systematic risk per unit of cash flow and a higher beta. Proposition 5 establishes that non-disclosing firms earn a higher return than disclosing firms in environments where the disclosure friction is relatively high.

Discussion

This result sheds light on the mixed empirical findings in the current capital market literature on cost of capital. Welker (1995) and Sengupta (1998) analyze firm disclosure rankings given by financial analysts and find that firms rated as more transparent have a lower cost of capital. Botosan (1997) and Botosan and Plumlee (2002) show that firms disclosing more information in their annual reports have lower cost of capital. Francis et al. (2004, 2005) and Ecker et al. (2006) proxy for accounting quality using residual accruals volatility and find similar results. Chen, Berger, and Li (2006) also provide evidence that more firm-specific information in stock returns is related to a lower cost of equity. Finally, Francis, Nanda, and Olsson (2008) find that more voluntary disclosure is associated with a lower cost of capital. Most of these papers relate to information that is in large part voluntarily given, and thus where strategic considerations about what to disclose play an important role. This paper provides a simple intuitive framework to better identify some of the economic forces at play in these empirical findings. Note that, in comparison, models with exogenous disclosures do not have this implication, because once the signal is disclosed, the cost of capital of a disclosing firm is not necessarily lower than that of its peers.

The result should be separated from other standard models of disclosure (e.g., Verrecchia (1983) or Dye (1985)) that do not incorporate systematic risk. In such models, the primary object of interest is the instantaneous response of the market price to disclosure. Such response would also exist here (the non-disclosing firm's price would decrease) but the notion of cost of capital studied here is measured as the return for the (possibly long) period post disclosure, excluding the disclosure event. The benefit of using this approach is that it predicts long-term effects of disclosure, as observed empirically, versus a short adjustment. Further, the standard model predicts that, when not disclosing, a firm's market price would decrease which would lead to negative returns and/or the counterfactual empirical implication that the cost of capital of non-disclosing firms (as proxied by their market return) would be lower than the cost of capital of disclosing firms.

An additional remark to be made at this point is that, in their decision process, value-maximizing firms that disclose, do so to reduce their cost of capital, as noted in many empirical studies. In the model, if a firm with $\epsilon > \epsilon^{over}$ has not disclosed, it would have received a price $P_\emptyset$ and a higher cost of capital. This does not mean, however, that all firms may disclose to reduce their cost of capital. Firms that voluntarily withhold, also do so to reduce their cost of capital. Voluntary disclosure policy is driven by the problem of minimizing cost of capital in financial markets.

Finally, the discussion focuses on the overinvestment equilibrium; if the disclosure friction is low, non-disclosing firms are not financed and therefore one cannot compare the costs of capital between disclosing and non-disclosing firms. More realistically, in the data there may be noise in observing whether firms disclose. If with some probability, some disclosing firms are classified as non-disclosing by the econometrician, then, in the underinvestment equilibrium, firms that did not disclose due to a measurement error should have in expectation the same return as disclosing firms.[24] Thus, a mild extension of the model would suggest that, when the disclosure friction becomes small so that the underinvestment equilibrium occurs, there should not be any difference in costs of capital between disclosing and non-disclosing firms. Controlling for the level of the disclosure friction, in this respect, would help refine the empirical findings.

5.4 Disclosure Friction and Average Cost of Capital

Price Dispersion

Before the average cost of capital in the economy is formally stated, it is useful to first derive the price dispersion induced by the disclosure friction. I provide in Lemma 4 two additional technical properties of the model.

Lemma 4 *Denote $\Delta(.;\eta)$ the distribution of P_δ. Let $\eta \geq \eta'$:*

(i) In the overinvestment region, $\Delta(.;\eta)$ second-order stochastically dominates $\Delta(.;\eta')$.

(ii) In the underinvestment region, $\Delta(.;\eta')$ first-order stochastically dominates $\Delta(.;\eta)$.

The first part of lemma 4 (i) demonstrates that a lower disclosure friction increases the variability of market prices in the overinvestment region. That is, more disclosure implies a wider range of reported signals while less disclosure implies an "average" price for non-disclosing firms. It follows that a profit-maximizing but risk-averse investor would always prefer a greater disclosure friction in the overinvestment region. This finding echoes to some extent the finding in Levine and Smith (2008) who show that more variance does not mean less usefulness. The second part (ii) shows that the result is reversed when the disclosure friction is sufficiently low and falls in the underinvestment region. In the underinvestment equilibrium region, decreasing the disclosure friction raises the chances of a successful disclosure, which implies, because the disclosure threshold coincides with first-best, that more firms choose the efficient investment.

[24]The model extends readily to such a measurement error, but to save on notations, I did not include it.

Analysis of Average Cost of Capital

I analyze next the average cost of capital in the economy ($\mathcal{R} \equiv \mathbb{E}(R_\delta)$), which is the unconditional expected return averaging all the firm-specific returns in the economy. The average cost of capital captures the regression output in a cross-sectional equally-weighted empirical study. It is typically different from the risk premium, which is computed as the return of the market portfolio. However, a link between the two concepts is that the latter corresponds to the return of each firm, weighted by its size in the market portfolio (or value-weighted).

Proposition 6 *(i) In the overinvestment region, disclosing and non-disclosing firms' average cost of capital decrease in the disclosure friction. Further overall average cost of capital decreases in the disclosure friction.*

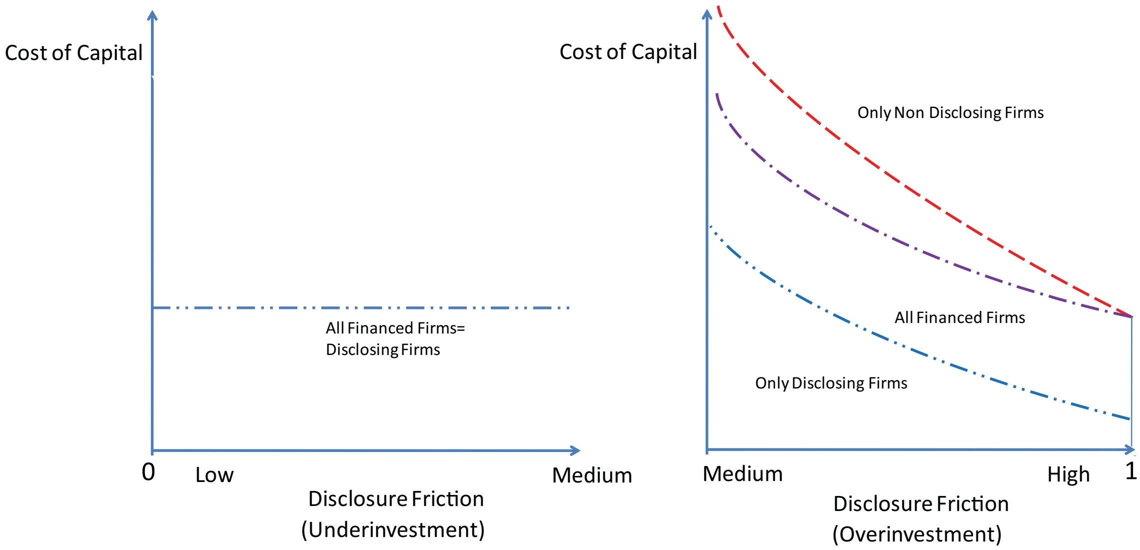
(ii) In the underinvestment region, average cost of capital does not depend on the disclosure friction.

Proposition 6 provides the result that maps the disclosure friction to average cost of capital. In the overinvestment if regulators' objective is to lower the average cost of capital for disclosing firms, they would achieve this by increasing the disclosure friction (i.e., making it harder for firms to disclose when they want to). A lower disclosure friction increases the number of disclosures which in turn causes more cross-sectional dispersion in market prices. On a value-weighted basis, this would not affect the market risk premium . On an equally-weighted basis, firms with lower prices and higher costs of capital are over-represented (as compared to the value-weighted portfolio) implying a greater average cost of capital.

The overinvestment equilibrium and interior disclosure friction leads to the highest possible average cost of capital (see Figure 2). For any level of the disclosure friction such that $\eta \leq \eta^{under}$ (underinvestment equilibrium), firms receive an average cost of capital equal to the cost of capital of all disclosing firms.[25] In the underinvestment region non-disclosing firms are no longer financed and aggregate cost of capital falls and remains constant for all $\eta \leq \eta^{under}$. I confront in Figure 2 the average cost of capital in the economy with the average costs of capital of disclosing firms (R_D) and non-disclosing firms (R_{ND}) as a function of the exogenous disclosure friction. Figure 2 illustrates that if the disclosure friction is low (if underinvestment), disclosing firms' average cost of capital

[25]The average cost of capital in the economy is an unconditional expectation whereas the average cost of capital of disclosing and non-disclosing firms is a conditional expectation.

Figure 2: Average Costs of Capital



is flat whereas it decreases with a higher disclosure friction if the disclosure friction is relatively high (if overinvestment). Non-disclosing firms' average cost of capital becomes extremely large when the disclosure friction is equal to η^{over} as investors offer them a price close to zero. As drawn on the figure on the right hand side, the overall average cost of capital is a weighted average of the disclosing and non-disclosing firms' cost of capital.

A comparison between the risk premium and the average cost of capital highlights the importance of the choices made in designing empirical cross-sectional studies. In the overinvestment region, the value-weighted cost of capital does not depend on the disclosure friction while the equally-weighted cost of capital is decreasing in the disclosure friction. Increased voluntary disclosures are one of the often stated objectives of regulators. From a regulatory perspective, the SEC in its 2003 report on "Management's Discussion and Analysis of Financial Condition and Results of Operations" proposes increasing disclosure quality.[26] The MD&A requirements encourage firms to disclose any useful information to investors even if not mandated: "…in identifying, discussing and analyzing known material trends and uncertainties, companies are expected to consider all relevant information, even if that information is not required to be disclosed."[27] My analysis shows that such rules should lead to more voluntary disclosures, but possibly

[26] see the 2003 report "Commission Guidance Regarding Management's Discussion and Analysis of Financial Condition and Results of Operations."

[27] A detailed report "Interpretation: Commission Guidance Regarding Management's Discussion and Analysis of Financial Condition and Results of Operations" SEC 17 CFR Parts 211, 231 and 241, Release Nos. 33-8350; 34-48960 is available at http://www.sec.gov/rules/interp/33-8350.

also higher average cost of capital in the overinvestment region.

Discussion

The model has implications for existing research on cost of capital. Recently, several studies have examined the consequences on firm's average cost of capital of changes to accounting standards (which may or may not improve accounting quality). Barth, Landsman, and Lang (2007) find evidence that firms applying IAS generally have higher value-relevant information than domestic standards. Leuz and Verrecchia (2000) report that a sample of firms voluntarily switching from German to IASB standards decreased their cost of capital. The interpretation of this empirical finding depends on whether this shift was: (i) a voluntary disclosure, (ii) an exogenous change in accounting quality (e.g., if firms needed to use IFRS for other non-strategic reasons). Under (i), the model would be consistent with IFRS having more quality than German standards, since only high-value firms would have chosen to shift (and reduce their cost of capital). Under (ii), the model predicts that the decrease in cost of capital should have been caused by a decrease in accounting quality (interpreted here as an increase in the disclosure friction) in the IAS standard. Comparing (i) and (ii), the interpretation of IFRS having more accounting quality than German standards depends on whether the shift was predominantly a strategic choice or a natural experiment.

6 Efficiency and Average Cost of Capital

Finally I address the relation between the exogenous disclosure friction and economic efficiency; and from a policy evaluation standpoint, whether economic efficiency can be measured by the average cost of capital. I use the term of efficiency to describe how well the accounting system maximizes investors' ex-ante expected utility (another common term is welfare). The term of efficiency is preferred here to that of welfare to stress that I look at the ex-ante utility of identical agents, and not at other important but unmodeled redistributive implications (see Kanodia (1980) or Gao (2008) for discussions on these issues). Analyzing efficiency is useful, because in theory any efficient outcomes could lead to welfare improvements in an economy with non-identical agents if a planner were to make fixed transfers (second-welfare theorem); separating efficiency from welfare considerations, in this respect, allows me to distinguish accounting from reallocative effects.

Economic efficiency in the model can be decomposed in two aspects: productive efficiency, i.e. whether a lower disclosure friction implements more efficient production, and risk-sharing efficiency, i.e. to what extent can financial markets help investors insure against diversifiable risk.

6.1 Productive Efficiency

I look first at the productive distortions which appear in second-best. In short-hand, I define as productive efficiency the extent to which firms' investment decisions correspond to the first-best investment decisions.[28]

Proposition 7 *(i) In the overinvestment region, productive efficiency does not depend on the disclosure friction.*

(ii) In the underinvestment region, productive efficiency decreases in the disclosure friction.

The model predicts that a lower disclosure friction leads to higher productive efficiency if the disclosure friction is initially sufficiently low. Since the disclosure friction is low enough that the equilibrium is one of underinvestment, further lowering the friction will reduce the underinvestment. A lower friction allows fewer high-value firms to be pooled with low-value firms.

In contrast, when the equilibrium has overinvestment, increases or decreases to the disclosure friction do not affect the productive efficiency. All firms are financed regardless of the disclosure friction and, thus, the accounting system fails to function as an informative signal for investment decisions.

6.2 Risk-Sharing Efficiency

Financial markets help ex-ante undiversified investors diversify their idiosyncratic risk ϵ in the financial market, thus leading to improved risk-sharing. However, as noted in Lemma 4, a lower disclosure friction may imply more dispersion in personal wealth, leading to additional disclosure risk. I define here risk-sharing efficiency by considering investors' expected utility for a given investment.[29]

[28] As most notions of efficiency, productive efficiency is, in general, an incomplete order, in that the efficiency of different economies may not be comparable (if two economies have different firms that invest or do not invest). However, in my model, productive efficiency in one type of equilibrium is always comparable: the disclosure friction increases or decreases the mass of firms that underinvest or overinvest.

[29] Again, as in the case of productive efficiency, I define the term only loosely here. A formal definition is available on request.

Proposition 8

(i) In the overinvestment region, risk-sharing efficiency increases in the disclosure friction.

(ii) In the underinvestment region, risk-sharing efficiency decreases in the disclosure friction.

I show that the risk-sharing efficiency depends on whether the disclosure friction is used for investment purposes. If there is overinvestment, the disclosure friction does not affect the firms' investment decision and thus disclosure has no purpose in that respect. However lowering the disclosure friction, because trades are realized after the disclosure decision, increases risk for ex-ante investors.[30] In comparison, I find that, whenever the disclosure friction is used to decide on investment decisions, a lower disclosure friction always leads to less underinvestment but the range of prices is not affected by the disclosure friction. A lower disclosure friction always leads to better risk-sharing, in that it makes the likelihood of the low payoff associated to firm shutdown less likely.

6.3 Economic Efficiency

I derive the investors' expected utility (or economic efficiency) for a change in the level of the disclosure friction. To compute economic efficiency, one needs to contrast both productive efficiency with ex-ante risk-sharing motives.

Proposition 9 *Economic efficiency is maximum at either $\eta = 0$ or $\eta = 1$.*

As the disclosure friction moves away from η^{over} or η^{under}, either productive efficiency or risk-sharing improves, leading to an overall improvement in economic efficiency. Proposition 9 shows that the global efficiency optimum takes the form of a corner (or "bang-bang") social policy with either a complete resolution of the risk-sharing with no disclosure (maximum disclosure friction), or a complete resolution of the production inefficiency with full disclosure (no disclosure friction).

Corollary 2 *An increase in average cost of capital implies a decrease in economic efficiency in the overinvestment region.*

[30]However, it should still be noted that the fact that accounting is not used for investment purposes $\eta \geq \eta^*$ is endogenously derived from the model; and thus the statement requires a proper analysis of investment and disclosure.

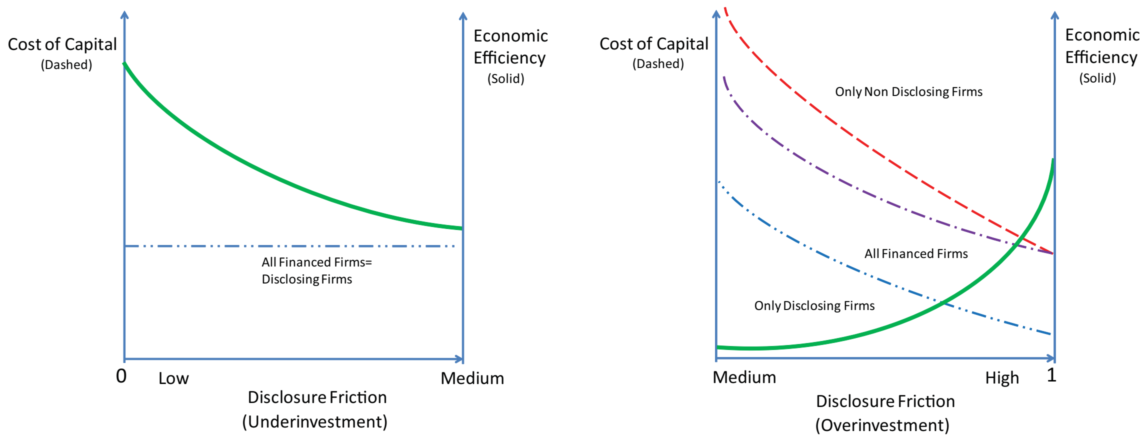
As noted in the introduction, linking cost of capital to efficiency is important, since cost of capital is a metric that can be empirically observed to evaluate a new accounting regulation. I show that the average cost of capital is always aligned with economic efficiency in the overinvestment region. The average cost of capital is a proxy for economic efficiency when the disclosure friction is high; specifically, in my model, the average cost of capital captures how well financial markets function at diversifying idiosyncratic risk. However, when the disclosure friction is relatively low, the average cost of capital may be artificially low because high-beta non-disclosing firms are not financed. I represent the overall costs of capital and investors' expected utility (called on Figure 3 "Economic Efficiency") as a function of the disclosure friction on Figure 3. As illustrated when the disclosure friction is above η^{over} (overinvestment), the investors' expected utility and the average cost of capital (and also disclosing and non-disclosing firms' average costs of capital) are negatively associated. Shyam Sunder's (2002) question "How would the rule maker know which rule will reduce the cost of capital?" can be addressed as follows: if the initial disclosure friction is relatively high and cannot be substantially decreased without large cost to the economy, rule makers should try to increase it to decrease average cost of capital.

Further when one compares the efficiency with no friction versus the maximum disclosure friction, if efficiency is greater without the disclosure friction, then average cost of capital and efficiency are misaligned (as shown on the left hand side of Figure 3). It is easy to verify that this condition will be verified when risk-aversion is low, so that the production efficiency concerns dominates risk-sharing concerns. The condition, however, is not sufficient to maximize efficiency, given that average cost of capital is constant when $\eta < \eta^{under}$; in this case, maximizing efficiency requires also maximizing the aggregate level of investment.

On the other hand, average cost of capital is aligned with efficiency when risk-sharing motives dominate productive efficiency concerns (i.e., when $\eta = 1$ is preferred to $\eta = 0$). In this case, the efficient accounting policy minimizes average cost of capital and, in the overinvestment region, a decrease in average cost of capital implies an increase in efficiency. This condition will be satisfied when risk-aversion is large or few firms have signals below ϵ^{FB}.

In summary, I show that the alignment between average cost of capital and economic efficiency depends on investors' risk-aversion and the distribution of firm's cash flows. From a practical perspective, then, an implication of the result is that policy evaluation using average cost of capital should depend on which economy is being analyzed. In developed economies, for example, one may possibly expect more high-value firms because

Figure 3: Cost of Capital vs Economic Efficiency



such firms are filtered by existing institutions. Further, because the state may be offering a safety welfare net, one may also expect investors to be less risk-averse. One would then expect in such economies regulators to implement a lower disclosure friction and relatively high average cost of capital for disclosing firms. This prediction would be reversed in the case of emerging economies. In such economies, one would expect investors to be more risk-averse due to possible liquidity needs or more severe lemon's problems due to the lack of pre-established institutions.[31] A testable empirical implication is that countries that are well-developed should choose a low disclosure friction, and higher average cost of capital for disclosing firms, moderate to high average cost of capital and higher investment as compared to emerging economies.

6.4 Mandatory Disclosures

Regulators often prone transparency. I discuss next the benefits of exogenously imposing the release of information by those firms that strategically chose not to disclose. Of course, doing so may not be costless, the costs of implementing more transparency have been criticized in the implementation of the Sarbanes-Oxley Act. Given that the downside of these additional costs is logically straightforward, I focus instead on the benefits of such mandatory disclosures and how such benefits will be perceived in firms' cost of capital.

[31] A recent literature discusses whether developing economies may have more severe informational frictions than developed countries and examines implications for growth.

Corollary 3 *In the overinvestment region, mandatory disclosure implies productive efficiency; however, in the underivestment region, mandatory disclosure does not change productive efficiency or risk-sharing.*

The first part of the statement is intuitive. Mandatory disclosure complements the accounting system in detecting low-value firms. The effect of mandatory disclosure, however, is ambiguous, given that it puts additional risk on ex-ante investors. However, it can be easily verified that mandatory disclosure always increases efficiency if $\eta = 0$ is preferred to $\eta = 1$ under the no-mandatory disclosure regime. However, mandatory disclosure fails to produce any real effects in the case of a low disclosure friction. This means that if imposing such regulation were to be costly, it would be a pure deadweight loss to the economy.

These observations can be reframed in the context of current discussions about the success of the regulations surrounding the Sarbanes-Oxley Act. One question is whether accounting quality (measured here by a disclosure friction) was too low, and led to overinvestment on many low-value firms. If full disclosure was the preferred level of information availability, lowering the disclosure friction would have been optimal. However, if the latter was infeasible or impractical, a surrogate policy consists of mandatory disclosures. On the other hand, if the disclosure friction was already low, and the sequence of corporate scandals around the year 2000 was a one-time event; then, mandatory disclosures may not have improved efficiency.

7 Concluding Remarks

This paper provides a theory that ties together voluntary disclosure, cost of capital and economic efficiency. My model captures three main salient components: there are multiple firms and investors, voluntary disclosures are endogenous but affected by an exogenous disclosure friction,and the disclosure friction has real efficiency consequences on risk-sharing and production. I make several main observations.

(i) If the disclosure friction is high, firms that disclose have lower cost of capital than firms that do not disclose.

(ii) An increase from a low to a high disclosure friction implies an increase in aggregate investment.

(iii) Economies with a high disclosure friction feature overinvestment, while those with a low disclosure friction feature underinvestment.

(iv) If the disclosure friction is high (low), a decrease in the disclosure friction implies an increase (no change) in average cost of capital and an decrease (increase) in economic efficiency.

(v) Mandatory disclosures may increase efficiency only if the initial disclosure friction is relatively high.

As a path for future work, the analysis suggests several links between information availability driven by the disclosure friction and asset pricing; if one interprets information availability in our model as a proxy of accounting quality then empirical analysis should offer a more systematic methodology to use accounting quality as an asset pricing factor. Moreover, more work is necessary to unravel how to measure changes to accounting quality that fit well the cross-section of stock returns. Finally, I focused on a one-period economy, in order to use results on aggregation with a disclosure game with multiple firms. However, a dynamic model would complement my paper to understand the time-series properties of disclosures in which investors can smooth their consumption over time.

Appendix A

The objective of this appendix is to position my paper within the literature on cost of capital, defined as the expected market return. As mentioned before, the literature on cost of capital is predominantly divided into the literature using endogenous information (voluntary disclosure) on one hand and the literature on exogenous information on the other hand. I present next in table 1 the different assumptions used in the cost of capital literature and in my paper and in table 2 the questions addressed in the literature. I finally gather in table 3 the main results of the papers in the literature and mine.

Table 1: Assumptions

	Production	Multiple firms	CARA utility	normality distribution	Voluntary disclosure
Easley and O'Hara (2004)		X	X	X	
Hughes, Liu and Liu (2007)		X	X	X	
Lambert, Leuz and Verrecchia (2007)		X		X	
Lambert, Leuz and Verrecchia (2008)		X	X	X	
Gao (2008)	X		X	X	
Jorgensen and Kirschenheiter(2003)(2007)		X	X	X	X
My model	X	X			X

Table 2: Questions Addressed

	Disclosing vs Non Disclosing Firms' CoC	Association Risk Premium/ Disclosure	Association Risk Premium/ Efficiency	Association CoC Efficiency
Easley and O'Hara (2004)		X		
Hughes, Liu and Liu (2007)		X		
Lambert, Leuz and Verrecchia (2007)		X		
Lambert, Leuz and Verrecchia (2008)		X		
Gao (2008)		X	X	
Jorgensen and Kirschenheiter (2003, 2007)	X	X		
My model	X	X	X	X

Table 3: Results

	Disclosing vs non-disclosing firms' CoC	Association Disclosure /Risk Premium (RP)	Association Disclosure / overall CoC	Association overall CoC /Efficiency
accounting conventional wisdom	**lower CoC for disclosing firms**	**negative**	**negative**	**negative**
the model	**lower CoC for disclosing firms if overinvestment**	**RP independent of the disclosure friction in a given equilibrium**	**no impact (positive) if underinvestment (overinvestment)**	**negative (no impact) if overinvestment (underinvestment)**
Jorgensen and Kirschenheiter (2003)	**lower CoC for disclosing firms only in finite economy**	**positive**	**no prediction**	**no prediction**
Jorgensen and Kirschenheiter (2007)	**lower CoC for disclosing firms**	**positive**	**no prediction**	**no prediction**
Easley and O'Hara (2004)	**no prediction**	**negative**	**no prediction**	**no prediction**
Hughes, Liu, and Liu (2007)	**no effect**	**negative**	**no prediction**	**no prediction**
Lambert, Leuz, and Verrecchia (2007)	**no prediction**	**negative**	**no prediction**	**no prediction**
Lambert, Leuz, and Verrecchia (2008)	**no prediction**	**negative**	**no prediction**	**no prediction**
Gao (2008)	**no prediction**	**negative if high investment adjustment cost**	**no prediction**	**ambiguous**

Appendix B

Proof of Proposition 1: The social planner solves the following maximization problem:

$$\max_{\tilde{\epsilon}} \int f(y) U \left(\int_{\tilde{\epsilon}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy \tag{A-1}$$

The FOC of the maximization problem (A-1) yields:

$$-h(\epsilon^{FB}) \int f(y)(\epsilon^{FB} + y) U' \left(\int_{\epsilon^{FB}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy = 0 \tag{A-2}$$

As $h(\epsilon^{FB}) > 0$, one can rewrite the above Equation (A-2) as $\Phi(\epsilon^{FB}) = 0$ where:

$$\begin{aligned} \Phi(\epsilon^{FB}) &= -\int f(y)(\epsilon^{FB} + y) U' \left(\int_{\epsilon^{FB}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy = 0 \\ \epsilon^{FB} &= -\frac{\int y f(y) U' \left(\int_{\epsilon^{FB}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right)}{\int f(y) U' \left(\int_{\epsilon^{FB}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right)} dy \end{aligned} \tag{A-3}$$

Expression (A-3) corresponds to Equation (2) in Proposition 1. To prove that ϵ^{FB} is unique and in $(0, \theta)$, it is sufficient to show that: (i) $\Phi' < 0$ and, (ii) $\Phi(0) > 0$ and $\Phi(\theta) < 0$.

$$\begin{aligned} \Phi'(\tilde{\epsilon}) &= \int f(y)(\tilde{\epsilon} + y)^2 h(\tilde{\epsilon}) U'' \left(\int_{\tilde{\epsilon}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy \\ &\quad - \int f(y) U' \left(\int_{\tilde{\epsilon}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy < 0 \end{aligned} \tag{A-4}$$

Further

$$\begin{aligned} \Phi(0) &= -\int y f(y) U' \left(\int_{0}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy \\ &= -\underbrace{\int_{0}^{+\infty} y f(y) U' \left(\int_{0}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy}_{A} \\ &\quad -\underbrace{\int_{\underline{y}}^{0} y f(y) U' \left(\int_{0}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy}_{B} \end{aligned}$$

The first part A is equal to

$$\int_{0}^{+\infty} y f(y) U' \left(\int_{0}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right) dy < \int_{0}^{+\infty} y f(y) U' \left(\int_{0}^{+\infty} \epsilon h(\epsilon) d\epsilon \right) dy$$

because U is strictly concave. Likewise, the second part B is equal to

$$\int_{\underline{y}}^{0} yf(y)U'\left(\int_{0}^{+\infty}(\epsilon+y)h(\epsilon)d\epsilon\right)dy < \int_{\underline{y}}^{0} yf(y)U'\left(\int_{0}^{+\infty}\epsilon h(\epsilon)d\epsilon\right)dy$$

Therefore:

$$\Phi(0) > -\int_{\underline{y}}^{+\infty} yf(y)U'\left(\int_{0}^{+\infty}\epsilon h(\epsilon)d\epsilon\right)dy = -U'\left(\int_{0}^{+\infty}\epsilon h(\epsilon)d\epsilon\right)\int_{\underline{y}}^{+\infty} yf(y)dy = 0$$

Moreover by assumption $\theta + y > 0$ for all y, it follows

$$\Phi(\theta) = -\int f(y)(\theta+y)U'\left(\int_{\theta}^{+\infty}(\epsilon+y)h(\epsilon)d\epsilon\right)dy < 0$$

Proof of Lemma 1:

$$P_{\emptyset}(\hat{\epsilon}) = \underbrace{\frac{1}{\eta+(1-\eta)H(\hat{\epsilon})}\left(\eta\int_{\hat{\epsilon}}^{+\infty}\epsilon h(\epsilon)d\epsilon + \int_{-\infty}^{\hat{\epsilon}}\epsilon h(\epsilon)d\epsilon\right) - \theta}_{\text{value of investment in risk-free asset}} + \underbrace{\frac{\theta}{\mathbb{E}(R_m)}}_{\text{value of investment in market portfolio}}$$

I differentiate $P_{\emptyset}(\hat{\epsilon})$ with respect to $\hat{\epsilon}$:

$$\frac{\partial P_{\emptyset}(\hat{\epsilon})}{\partial\hat{\epsilon}} = -\frac{(1-\eta)h(\hat{\epsilon})\eta}{(\eta+(1-\eta)H(\hat{\epsilon}))^2}\int_{\hat{\epsilon}}^{+\infty}\epsilon h(\epsilon)d\epsilon - \frac{(1-\eta)h(\hat{\epsilon})}{(\eta+(1-\eta)H(\hat{\epsilon}))^2}\int_{-\infty}^{\hat{\epsilon}}\epsilon h(\epsilon)d\epsilon + \frac{(1-\eta)\hat{\epsilon}h(\hat{\epsilon})}{\eta+(1-\eta)H(\hat{\epsilon})} \quad \text{(A-5)}$$

By integration by part and rearranging expression (A-5), I obtain:

$$\frac{\partial P_{\emptyset}(\hat{\epsilon})}{\partial\hat{\epsilon}} = -\frac{(1-\eta)h(\hat{\epsilon})}{(\eta+(1-\eta)H(\hat{\epsilon}))^2}(\eta+(1-\eta)H(\hat{\epsilon}))\hat{\epsilon} - \frac{(1-\eta)h(\hat{\epsilon})}{(\eta+(1-\eta)H(\hat{\epsilon}))^2}\left(\eta\int_{\hat{\epsilon}}^{+\infty}(1-H(\epsilon))d\epsilon - \int_{-\infty}^{\hat{\epsilon}}H(\epsilon)d\epsilon\right) + \frac{(1-\eta)\hat{\epsilon}h(\hat{\epsilon})}{\eta+(1-\eta)H(\hat{\epsilon})} \quad \text{(A-6)}$$

After simplifying expression (A-6), it yields:

$$\frac{\partial P_{\emptyset}(\hat{\epsilon})}{\partial\hat{\epsilon}} = \frac{(1-\eta)h(\hat{\epsilon})}{(\eta+(1-\eta)H(\hat{\epsilon}))^2}\left(\int_{-\infty}^{\hat{\epsilon}}H(\epsilon)d\epsilon - \eta\int_{\hat{\epsilon}}^{+\infty}(1-H(\epsilon))d\epsilon\right) \quad \text{(A-7)}$$

I determine $\hat{\epsilon}^{over}$ such that $\frac{\partial P_\emptyset(\hat{\epsilon})}{\partial \hat{\epsilon}}|_{\hat{\epsilon}=\hat{\epsilon}^{over}} = 0$:

$$\int_{-\infty}^{\hat{\epsilon}^{over}} H(\epsilon)d\epsilon - \eta \int_{\hat{\epsilon}^{over}}^{+\infty} (1 - H(\epsilon))d\epsilon = 0 \tag{A-8}$$

Let us define

$$\Psi(\hat{\epsilon}) = \int_{-\infty}^{\hat{\epsilon}} H(\epsilon)d\epsilon - \eta \int_{\hat{\epsilon}}^{+\infty} (1 - H(\epsilon))d\epsilon \tag{A-9}$$

The function Ψ is increasing in $\hat{\epsilon}$ as

$$\Psi'(\hat{\epsilon}) = \eta + (1-\eta)H(\hat{\hat{\epsilon}}) \geq 0$$

Further when $\hat{\epsilon}$ converges to $-\infty$ then Ψ converges to $-\eta \int_{-\infty}^{+\infty}(1 - H(\epsilon))d\epsilon < 0$ and when $\hat{\epsilon}$ converges to $+\infty$, Ψ converges to $\int_{-\infty}^{+\infty} H(\epsilon)d\epsilon > 0$. If $\hat{\epsilon} \in (-\infty, \hat{\epsilon}^{over}]$ (resp. $\hat{\epsilon} \in (\hat{\epsilon}^{over}, +\infty)$), expression (A-9) is negative (resp. positive). To summarize $P_\emptyset(\hat{\epsilon})$ is decreasing (resp. increasing) if $\hat{\epsilon} \in (-\infty, \hat{\epsilon}^{over}]$ (resp. if $\hat{\epsilon} \in (\hat{\epsilon}^{over}, +\infty)$) where $\hat{\epsilon}^{over}$ is the minimum of $P_\emptyset(\hat{\epsilon})$.

Proof of Proposition 2: Notice that any disclosure equilibrium is determined by a threshold.
I need to solve $P_\emptyset(\hat{\epsilon}) = P(\hat{\epsilon})$ to find the optimal threshold(s). First I prove the existence of this optimal threshold and second its uniqueness.

Existence of the threshold Let $\hat{\epsilon}$ be a threshold (not necessarily the optimal threshold). Notice that $P_\emptyset$ also depends on the threshold $\hat{\epsilon}$. So $P_\emptyset = P_\emptyset(\hat{\epsilon})$. I compare $P_\emptyset(\hat{\epsilon})$ with $P(\hat{\epsilon})$ at the extreme values of $\hat{\epsilon}$. If $\hat{\epsilon}$ goes to $-\infty$, then the price is equal to $-\infty$ and it is less than $P_\emptyset = \mathbb{E}(P(\epsilon)) = \frac{\theta}{\mathbb{E}(R_m)}$. If $\hat{\epsilon}$ goes to $+\infty$, then $P(\hat{\epsilon})$ goes to $+\infty$ and it is greater than $P_\emptyset = \mathbb{E}(P(\epsilon)) = \frac{\theta}{\mathbb{E}(R_m)}$. Therefore by the theorem of intermediary values, there exists a threshold ϵ^{over} such that $P_\emptyset(\epsilon^{over}) = P(\epsilon^{over})$.

Uniqueness of the threshold

I show next that there is a unique solution ϵ^{over} such that $P_\emptyset(\epsilon^{over}) = P(\epsilon^{over})$.

$$\begin{aligned} P_\emptyset(\hat{\epsilon}) &= \frac{1}{\eta + (1-\eta)H(\hat{\epsilon})}\left(\eta \int_{\hat{\epsilon}}^{+\infty} \epsilon h(\epsilon)d\epsilon + \int_{-\infty}^{\hat{\epsilon}} \epsilon h(\epsilon)d\epsilon\right) - \theta \\ &+ \frac{\theta}{\mathbb{E}(R_m)} \end{aligned}$$

Integrating by parts and simplifying, I can expressed the above price as follows:

$$\begin{aligned} P_\emptyset(\hat{\epsilon}) &= \hat{\epsilon} + \frac{\eta}{\eta + (1-\eta)H(\hat{\epsilon})} \int_{\hat{\epsilon}}^{+\infty} (1 - H(\epsilon))d\epsilon \\ &- \frac{1}{\eta + (1-\eta)H(\hat{\epsilon})} \int_{-\infty}^{\hat{\epsilon}} H(\epsilon)d\epsilon - \theta + \frac{\theta}{\mathbb{E}(R_m)} \end{aligned}$$

Similarly $P(\hat{\epsilon})$ can be expressed as:

$$P(\hat{\epsilon}) = \hat{\epsilon} - \theta + \frac{\theta}{\mathbb{E}(R_m)}$$

I compute the difference $D(\hat{\epsilon})$ between $P_{\emptyset}(\hat{\epsilon})$ and $P(\hat{\epsilon})$:

$$D(\hat{\epsilon}) = \frac{1}{\eta + (1-\eta)H(\hat{\epsilon})} \left(\eta \int_{\hat{\epsilon}}^{+\infty} (1 - H(\epsilon))d\epsilon - \int_{-\infty}^{\hat{\epsilon}} H(\epsilon)d\epsilon \right) \tag{A-10}$$

One important feature of the optimal threshold is its independence on the expected market portfolio return as equation (A-10) does not depend on it.
Further the difference is equal to zero at $\hat{\epsilon}^{over}$, implicity solution to equation:

$$\eta \int_{\hat{\epsilon}^{over}}^{+\infty} (1 - H(\epsilon))d\epsilon - \int_{-\infty}^{\hat{\epsilon}^{over}} H(\epsilon)d\epsilon = 0 \tag{A-11}$$

The LHS is equal to $-\Psi$. Therefore for $\hat{\epsilon} \in (-\infty, \hat{\epsilon}^{over}]$, $P_{\emptyset}(\hat{\epsilon}) \geq P(\hat{\epsilon})$ and for $\hat{\epsilon} \in (\hat{\epsilon}^{over}, +\infty)$, $P_{\emptyset}(\hat{\epsilon}) < P(\hat{\epsilon})$. I proved that the optimal threshold ϵ^{over} is unique and equal to the minimum $\hat{\epsilon}^{over}$.
Proof of Corollary 1 The threshold ϵ^{over} is independent of the expected market portfolio return and thus on the risk aversion α.
The function Ψ also depends on the parameter η, and to stress the dependence I denote $\Psi(\eta, \epsilon^{over}) = 0$. As there exists a unique solution ϵ^{over} to $\Psi(\eta, \epsilon^{over}) = 0$, there exists a unique function $J(\eta)$ such that $\epsilon^{over} = J(\eta)$.

- $J(1)$ verifies $\Psi(1, J(1)) = 0$, which gives $J(1) = \theta$.
- $J(0)$ verifies $\Psi(0, J(0)) = 0$, which gives $J(0)$ converging to $-\infty$.

Applying the implicit function theorem,

$$\begin{aligned} J'(\eta) &= -\frac{\partial \Psi}{\partial \eta}(\eta, \hat{\epsilon}) / \frac{\partial \Psi}{\partial \hat{\epsilon}}(\eta, \hat{\epsilon}) \\ &= \frac{\int_{\hat{\epsilon}}^{+\infty} (1 - H(\epsilon))d\epsilon}{\eta + (1-\eta)H(\hat{\epsilon})} \end{aligned} \tag{A-12}$$

Therefore $J'(\eta)$ is positive and ϵ^{over} is increasing in η.
Proof of Proposition 3:
I solve the maximization problem of an investor:

$$\begin{aligned} (\Gamma_\delta) \qquad & \max_{\gamma_{\delta f}, \gamma_{\delta m}} \int f(y) U \left\{ P_\delta(\gamma_{\delta f} + \gamma_{\delta m}(\theta + y)) \right\} dy \\ & \text{s.t } P_\delta = P_\delta \gamma_{\delta f} + P_\delta \gamma_{\delta m} \theta / \mathbb{E}(R_m) \end{aligned}$$

As the utility function $U(.)$ is a CRRA utility, and simplifying the budget constraint, it yields

$$(\Gamma_\delta) \qquad \max_{\gamma_{\delta f},\gamma_{\delta m}} \frac{1}{1-\alpha}\int f(y)\left\{P_\delta(\gamma_{\delta f}+\gamma_{\delta m}(\theta+y))\right\}^{1-\alpha} dy$$
$$\text{s.t} \;\; 1=\gamma_{\delta f}+\gamma_{\delta m}\theta/\mathbb{E}(R_m)$$

The dependence on δ is due to the price P_δ. But from the program Γ_δ, P_δ is only a constant multiplicative term of the objective function and the maximizers $\gamma_{\delta f}$ and $\gamma_{\delta m}$ do not depend on P_δ and so on δ. Thus $\gamma_{\emptyset f}^{over}=\gamma_{\epsilon f}^{over}=\gamma_f^{over}$ and $\gamma_{\emptyset m}^{over}=\gamma_{\epsilon m}^{over}=\gamma_m^{over}$.

The aggregate demand of all investors in the risk-free asset is equal to:

$$\begin{aligned}
& (\eta+(1-\eta)H(\epsilon^{over}))P_\emptyset\gamma_{\emptyset f}^{over}+(1-\eta)\int_{\epsilon^{over}}^{+\infty}P(\epsilon)\gamma_{\epsilon f}^{over}h(\epsilon)d\epsilon \\
= & \; \gamma_f^{over}\left((\eta+(1-\eta)H(\epsilon^{over}))(\epsilon^{over}-\theta+\frac{\theta}{\mathbb{E}(R_m)})\right) \\
& +\gamma_f^{over}\left((1-\eta)\int_{\epsilon^{over}}^{+\infty}(\epsilon-\theta+\frac{\theta}{\mathbb{E}(R_m)})h(\epsilon)d\epsilon\right) \\
= & \; \gamma_f^{over}\left(\frac{\theta}{\mathbb{E}(R_m)}-\theta+(\eta+(1-\eta)H(\epsilon^{over}))\epsilon^{over}\right) \\
& +\gamma_f^{over}(1-\eta)\int_{\epsilon^{over}}^{+\infty}\epsilon h(\epsilon)d\epsilon
\end{aligned} \tag{A-13}$$

I rewrite expression $\int_{\epsilon^{over}}^{+\infty}\epsilon h(\epsilon)d\epsilon$ by integrating by parts as follows:

$$\int_{\epsilon^{over}}^{+\infty}\epsilon h(\epsilon)d\epsilon=\int_{\epsilon^{over}}^{+\infty}(1-H(\epsilon))d\epsilon+\epsilon^{over}(1-H(\epsilon^{over})) \tag{A-14}$$

I substitute this expression into equation (A-13) and obtain

$$\begin{aligned}
& \gamma_f^{over}\left(\frac{\theta}{\mathbb{E}(R_m)}-\theta+(\eta+(1-\eta)H(\epsilon^{over}))\epsilon^{over}\right) \\
& +\gamma_f(1-\eta)\left(\int_{\epsilon^{over}}^{+\infty}(1-H(\epsilon))d\epsilon+\epsilon^{over}(1-H(\epsilon^{over}))\right)
\end{aligned} \tag{A-15}$$

Moreover by equation (A-8), I simplify equation (A-15) to:

$$\gamma_f^{over}\left(\frac{\theta}{\mathbb{E}(R_m)}-\theta+\epsilon^{over}+\int_{\epsilon^{over}}^{+\infty}(1-H(\epsilon))d\epsilon-\int_{-\infty}^{\epsilon^{over}}H(\epsilon)d\epsilon\right) \tag{A-16}$$

I also know that the mean θ can be rewritten as:

$$\begin{aligned}
\theta &= \int_{-\infty}^{+\infty}\epsilon h(\epsilon)d\epsilon=\int_{\epsilon^{over}}^{+\infty}\epsilon h(\epsilon)d\epsilon+\int_{-\infty}^{\epsilon^{over}}\epsilon h(\epsilon)d\epsilon \\
&= \int_{\epsilon^{over}}^{+\infty}(1-H(\epsilon))d\epsilon+\epsilon^{over}(1-H(\epsilon^{over}))-\int_{-\infty}^{\epsilon^{over}}H(\epsilon)d\epsilon+\epsilon^{over}H(\epsilon^{over})
\end{aligned}$$

Finally expression (A-16) is equal to $\gamma_f^{over}(\theta + \frac{\theta}{\mathbb{E}(R_m)} - \theta) = \gamma_f^{over} \frac{\theta}{\mathbb{E}(R_m)}$. As the net supply is equal to zero, it yields $\gamma_f^{over} = 0$.
I determine next the expression of the expected market portfolio return in this economy. Injecting the budget constraint (A-13), the maximization problem is equivalent to:

$$(Q_\delta) \qquad \max_{\gamma_m} \frac{P_\delta^{1-\alpha}}{(1-\alpha)\mathbb{E}(R_m)} \int f(y) \left\{\mathbb{E}(R_m) + \gamma_m(\mathbb{E}(R_m)(\theta + y) - \theta)\right\}^{1-\alpha} dy$$

The FOC from the investor's maximization problem with respect to γ_m is equal to:

$$\mathbb{E}\left\{\{\mathbb{E}(R_m)(\theta + y) - \theta\} \{\mathbb{E}(R_m) + \gamma_m(\mathbb{E}(R_m)(\theta + y) - \theta)\}^{-\alpha}\right\} = 0 \qquad \text{(A-17)}$$

As from the market clearing condition, I showed that $\gamma_f^{over} = 0$ then $\gamma_m^{over} = \frac{\mathbb{E}(R_m)}{\theta}$. Thus the FOC is reduced to:

$$\mathbb{E}\left\{\{\mathbb{E}(R_m)(\theta + y) - \theta\} (\theta + y)^{-\alpha}\right\} = 0 \qquad \text{(A-18)}$$

Simplifying,

$$\begin{aligned}
\mathbb{E}(R_m^{over}) &= \frac{\theta \mathbb{E}((\theta + y)^{-\alpha})}{\mathbb{E}((\theta + y)(\theta + y)^{-\alpha})} \\
&= \frac{\theta}{\theta + \mathbb{E}(y(\theta + y)^{-\alpha})/\mathbb{E}((\theta + y)^{-\alpha})}
\end{aligned}$$

I prove next that Q^{over} is negative.

$$Q^{over} = \int \frac{y f(y) U'(\theta + y)}{\int f(y) U'(\theta + y) dy} dy$$

By additivity of the integral,

$$\begin{aligned}
& \int_{\underline{y}}^{+\infty} y f(y) U'(\theta + y) dy \\
= & \int_0^{+\infty} y f(y) U'(\theta + y) dy + \int_{\underline{y}}^0 y f(y) U'(\theta + y) dy
\end{aligned}$$

Moreover $\int_0^{+\infty} y f(y) U'(\theta + y) dy < \int_0^{+\infty} y f(y) U'(\theta) dy$ as U is strictly concave. Likewise $\int_{\underline{y}}^0 y f(y) U'(\theta + y) dy < \int_{\underline{y}}^0 y f(y) U'(\theta) dy$. By assumption $\int_{\underline{y}}^{+\infty} y f(y) dy = E(\tilde{y}) = 0$ and it yields

$$\int_{\underline{y}}^{+\infty} y f(y) U'(\theta + y) dy < U'(\theta) \int_{\underline{y}}^{+\infty} y f(y) dy = 0$$

Under the overinvestment equilibrium, the non-disclosing price $P_{\emptyset}(\epsilon^{over})$ is equal to

$$P(\epsilon^{over}) = \epsilon^{over} + Q^{over}$$

As Q^{over} is independent of η, the derivative of $P(\epsilon^{over})$ w.r.t to η is equal to $J'(\eta) \geq 0$. Thus the non-disclosing price is increasing in the disclosure friction η. Further it implies that there exists a unique η^{over} such that $P_{\emptyset}(\epsilon^{over}(\eta^{over})) = 0$ and ϵ^{over} is always positive.

Proof of Proposition 4:

If the investor has a non-disclosing firm then $\gamma_{\emptyset f}^{under} = \gamma_{\emptyset m}^{under} = 0$ and their utility is equal to zero. However if an investor has a disclosing firm then he solves the maximization problem (Γ_ϵ).

$$(\Gamma_\epsilon) \qquad \max_{\gamma_{\epsilon f}, \gamma_{\epsilon m}} \int f(y) U \left\{ P_\epsilon(\gamma_{\epsilon f} + \gamma_{\epsilon m}(1-\eta) \int_{\epsilon^{under}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon) \right\} dy$$
$$\text{s.t} \;\; P_\epsilon = P_\epsilon \gamma_{\epsilon f} + P_\epsilon \gamma_{\epsilon m}((1-\eta) \int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon))/\mathbb{E}(R_m)$$

Simplifying,

$$(\Gamma_\epsilon) \qquad \max_{\gamma_{\epsilon f}, \gamma_{\epsilon m}} \frac{P_\epsilon^{1-\alpha}}{1-\alpha} \int f(y) \left\{ \gamma_{\epsilon f} + \gamma_{\epsilon m}(1-\eta) \int_{\epsilon^{under}}^{+\infty} (\epsilon + \tilde{y}) h(\epsilon) d\epsilon \right\}^{1-\alpha} dy$$
$$\text{s.t} \;\; 1 = \gamma_{\epsilon f} + \gamma_{\epsilon m} \left((1-\eta) \int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) \right) / \mathbb{E}(R_m)$$

I notice that the maximizers of an individual investor having a disclosing firm do not depend on P_ϵ and thus ϵ. Therefore $\forall \epsilon$, $\gamma_{\epsilon f}^{under} = \gamma_f^{under}$ and $\gamma_{\epsilon m}^{under} = \gamma_m^{under}$.

The aggregate demand of all investors for the risk free asset is equal to:

$$\begin{aligned} & (\eta + (1-\eta) H(\epsilon^{under})) \gamma_{\emptyset f}^{under} P_{\emptyset} + (1-\eta) \int_{\epsilon^{under}}^{+\infty} \gamma_{\epsilon f}^{under} P(\epsilon) d\epsilon \\ = \; & \gamma_f^{under} (1-\eta) \int_{\epsilon^{under}}^{+\infty} P(\epsilon) d\epsilon \end{aligned} \tag{A-19}$$

As the net supply is equal to zero, it yields $\gamma_f^{under} = 0$.

Taking the FOC of the investor problem, it yields

$$\mathbb{E} \left\{ \left(\mathbb{E}(R_m)(1-\eta) \int_{\epsilon^{under}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon - (1-\eta) \int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon \right) \right.$$
$$\left. \left(\mathbb{E}(R_m) + \gamma_m (1-\eta) (\mathbb{E}(R_m) \int_{\epsilon^{under}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon - \int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon) \right)^{-\alpha} \right\} = 0$$

By the market clearing condition, $\gamma_f^{under} = 0$ thus $\gamma_m^{under} = \mathbb{E}(R_m) / \left((1-\eta) \int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon \right)$.

The FOC is then reduced to:

$$\mathbb{E}\left\{\left(\mathbb{E}(R_m)\int_{\epsilon^{under}}^{+\infty}(\epsilon+y)h(\epsilon)d\epsilon-\int_{\epsilon^{under}}^{+\infty}\epsilon h(\epsilon)d\epsilon\right)\right.$$
$$\left.\left(\int_{\epsilon^{under}}^{+\infty}(\epsilon+y)h(\epsilon)d\epsilon\right)^{-\alpha}\right\}=0$$

Simplifying

$$\mathbb{E}(R_m^{under})=\frac{\mathbb{E}\left((\int_{\epsilon^{under}}^{+\infty}\epsilon h(\epsilon)d\epsilon)(\int_{\epsilon^{under}}^{+\infty}(\epsilon+y)h(\epsilon)d\epsilon)^{-\alpha}\right)}{\mathbb{E}\left(\int_{\epsilon^{under}}^{+\infty}(\epsilon+y)h(\epsilon)d\epsilon(\int_{\epsilon^{under}}^{+\infty}(\epsilon+y)h(\epsilon)d\epsilon)^{-\alpha}\right)}$$

Rearranging

$$\mathbb{E}(R_m^{under})=\frac{\int_{\epsilon^{under}}^{+\infty}\epsilon h(\epsilon)d\epsilon}{\int_{\epsilon^{under}}^{+\infty}\epsilon h(\epsilon)d\epsilon+(1-H(\epsilon^{under}))\frac{\mathbb{E}\left(y(\int_{\epsilon^{under}}^{+\infty}(\epsilon+y)h(\epsilon)d\epsilon)^{-\alpha}\right)}{\mathbb{E}\left((\int_{\epsilon^{under}}^{+\infty}(\epsilon+y)h(\epsilon)d\epsilon)^{-\alpha}\right)}} \tag{A-20}$$

I now turn to the determination of the disclosure threshold ϵ^{under}.
The firm observing ϵ^{under} has a cash flow:

$$\epsilon^{under}+y=\underbrace{\epsilon^{under}-\frac{\int_{\epsilon^{under}}^{+\infty}\epsilon h(\epsilon)d\epsilon}{1-H(\epsilon^{under})}}_{\text{units of the risk free asset}}+\underbrace{\frac{1}{(1-\eta)(1-H(\epsilon^{under}))}}_{\text{units of the market portfolio}}CF_m$$

Its price is then $\epsilon^{under}-\frac{\int_{\epsilon^{under}}^{+\infty}\epsilon h(\epsilon)d\epsilon}{1-H(\epsilon^{under})}+\frac{1}{(1-\eta)(1-H(\epsilon^{under}))}P_m^{under}=0$ as by definition this firm has a market price of zero, where

$$\begin{aligned}P_m^{under} &= (1-\eta)\int_{\epsilon^{under}}^{+\infty}\epsilon h(\epsilon)d\epsilon+(1-\eta)(1-H(\epsilon^{under}))Q^{under}\\ \text{with}\ \ Q^{under} &= \int_{\underline{y}}^{+\infty}\frac{yf(y)\left(\int_{\epsilon^{**}}^{+\infty}(\epsilon+y)h(\epsilon)d\epsilon\right)^{-\alpha}}{\int_{\underline{y}}^{+\infty}f(y)\left(\int_{\epsilon^{**}}^{+\infty}(\epsilon+y)h(\epsilon)d\epsilon\right)^{-\alpha}dy}dy\end{aligned}$$

Replacing P_m^{under} by its expression and simplifying it yields:

$$\epsilon^{under}=-\frac{\int yf(y)(\int_{\epsilon^{under}}^{+\infty}(\epsilon+y)h(\epsilon)d\epsilon)^{-\alpha}dy}{\int f(\tilde{y})(\int_{\epsilon^{under}}^{+\infty}(\epsilon+\tilde{y})h(\epsilon)d\epsilon)^{-\alpha}d\tilde{y}}$$

The disclosure threshold ϵ^{under} is independent of the disclosure friction η.

I now prove that Q^{under} is negative. This is similar to the proof of $Q^{over} < 0$.

$$Q^{under} = \int_{\underline{y}}^{+\infty} \frac{y f(y) \left(\int_{\epsilon^{under}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right)^{-\alpha}}{\int_{\underline{y}}^{+\infty} f(y) \left(\int_{\epsilon^{under}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right)^{-\alpha} dy} dy$$

As the utility function is strictly concave,

$$\begin{aligned} &\int_{0}^{+\infty} y f(y) \left(\int_{\epsilon^{under}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right)^{-\alpha} dy \\ &< \int_{0}^{+\infty} y f(y) \left(\int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon \right)^{-\alpha} dy \end{aligned} \tag{A-21}$$

Likewise

$$\begin{aligned} &\int_{\underline{y}}^{0} y f(y) \left(\int_{\epsilon^{under}}^{+\infty} (\epsilon + y) h(\epsilon) d\epsilon \right)^{-\alpha} dy \\ &< \int_{\underline{y}}^{0} y f(y) \left(\int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon \right)^{-\alpha} dy \end{aligned} \tag{A-22}$$

By assumption $\int_{\underline{y}}^{+\infty} y f(y) dy = \mathbb{E}(\tilde{y}) = 0$ which yields

$$Q^{under} < \left(\int_{\epsilon^{under}}^{+\infty} \epsilon h(\epsilon) d\epsilon \right)^{-\alpha} \int_{\underline{y}}^{+\infty} y f(y) dy = 0$$

Further in the underinvestment region the threshold of a firm indifferent between disclosing and not disclosing is still determined by equation (3) as it is independent of the risk premium. The difference here is that the components of the price for non-disclosing firms such that they have exactly a price of zero are different than in the overinvestment region as the expression for Q^{under} is different from Q^{over}. The turning point η^{under} to fall into the underinvestment region is thus determined by $J(\eta^{under}) = -Q^{under}$.

Proof of Lemma 2

If $Q(x)$ is increasing in x then $Q^{under} \geq Q^{over}$ so $\epsilon^{under} \leq \epsilon^{over}$, which implies that $\eta^{under} \leq \eta^{over}$. If $Q(x)$ is decreasing in x then $Q^{under} \leq Q^{over}$, which implies that $\eta^{under} \geq \eta^{over}$.

Proof of Lemma 3:

The costs of capital of non-disclosing firms and disclosing firms are respectively:

$$\begin{aligned} R_{\emptyset} &= \frac{\mathbb{E}(\epsilon | ND)}{P_{\emptyset}} = \frac{\epsilon^{over}}{P(\epsilon^{over})} \\ R_D &= \int_{\epsilon^{over}}^{+\infty} \frac{\epsilon}{P(\epsilon)} \frac{h(\epsilon)}{(1 - H(\epsilon^{over}))} d\epsilon \end{aligned}$$

As $P_\emptyset = P(\epsilon^{over}) = \epsilon^{over} + Q^{over}$ and $P(\epsilon) = \epsilon + Q^{over}$, a simple rewriting of the costs of capital yields:

$$\begin{aligned} R_\emptyset &= 1 - \frac{Q^{over}}{P_\emptyset} \\ R_D &= 1 - Q^{over} \int_{\epsilon^{over}}^{+\infty} \frac{h(\epsilon)}{P(\epsilon)(1 - H(\epsilon^{over}))} d\epsilon \end{aligned}$$

Looking closely at the prices, I can express them as a CAPM formulation:

$$\begin{aligned} R_\emptyset &= \underbrace{1}_{\text{Riskfree}} + \underbrace{\frac{P_m^{over}}{P_\emptyset}}_{\beta_\emptyset} \underbrace{(\mathbb{E}(R_m^{over}) - 1)}_{\text{Risk Premium}} \\ R_D &= \underbrace{1}_{\text{Riskfree}} + \underbrace{\int_{\epsilon^{over}}^{+\infty} \frac{P_m^{over}}{P(\epsilon)} \frac{h(\epsilon)}{(1 - H(\epsilon^{over}))} d\epsilon}_{\beta_D} \underbrace{(\mathbb{E}(R_m^{over}) - 1)}_{\text{Risk Premium}} \end{aligned}$$

I derive the beta β as an expression of the covariance between the firm's return and the market portfolio return over the variance of the market portfolio return.

$$\begin{aligned} \beta_\emptyset &= \frac{P_m^{over}}{P_\emptyset} = \underbrace{\frac{V(y)}{P_\emptyset P_m^{over}}}_{\text{covariance}} / \underbrace{\frac{V(y)}{P_m^{over2}}}_{\text{market variance}} \\ \beta_D &= \int_{\epsilon^{over}}^{+\infty} \frac{P_m^{over}}{P(\epsilon)} \frac{h(\epsilon)}{(1 - H(\epsilon^{over}))} d\epsilon = \underbrace{\int_{\epsilon^{over}}^{+\infty} \frac{V(y)}{P(\epsilon) P_m^{over}} \frac{h(\epsilon)}{(1 - H(\epsilon^{over}))} d\epsilon}_{\text{covariance}} / \underbrace{\frac{V(y)}{P_m^{over2}}}_{\text{market variance}} \end{aligned}$$

Proof of Proposition 5:

Further $P_\emptyset(\epsilon^{over}) = P(\epsilon^{over})$. As $\forall \epsilon \geq \epsilon^{over}$, $P(\epsilon^{over}) \leq P(\epsilon)$, $R_\emptyset \geq R_D$.

Proof of Lemma 4:

(i) Let us prove that if $\eta \geq \eta^{over}$, for $\eta < \eta'$, $\Delta(.;\eta')$ second-order stochastically dominates $\Delta(.;\eta)$.

For a price $p < P_\emptyset(\epsilon^{over}(\eta))$, $\Delta(p;\eta) = 0$ otherwise for a price $P_\emptyset(\epsilon^{over}(\eta)) \leq p \leq \overline{p}$, $\Delta(p;\eta) = \{\eta + (1-\eta)H(\epsilon^{over}(\eta))\} + (1-\eta)\{H(p - Q^{over}) - H(\epsilon^{over}(\eta))\}$. Let us define $\eta < \eta'$ and $T(\overline{p})$ as the area between the two curves $\Delta(;\eta)$ and $\Delta(;\eta')$ in $[P_\emptyset(\epsilon^{over}(\eta)), \overline{p}]$, specifically $T(\overline{p}) = \int_{P_\emptyset(\epsilon^{over}(\eta))}^{\overline{p}} (\Delta(p;\eta) - \Delta(p;\eta'))dp$. By additivity of the integral I rewrite $T(\overline{p})$:

$$T(\overline{p}) = \int_{P_\emptyset(\epsilon^{over}(\eta))}^{P_\emptyset(\epsilon^{over}(\eta'))} (\Delta(p;\eta) - 0)dp + \int_{P_\emptyset(\epsilon^{over}(\eta'))}^{\overline{p}} (\Delta(p;\eta) - \Delta(p;\eta'))dp$$

Let us consider $\eta' = \eta + \mu$, then $T(\overline{p})$ becomes:

$$\begin{aligned} T(\overline{p}) &= \int_{P_\emptyset(\epsilon^{over}(\eta))}^{P_\emptyset(\epsilon^{over}(\eta+\mu))} \Delta(p;\eta)dp \\ &\quad + \int_{P_\emptyset(\epsilon^{over}(\eta+\mu))}^{\overline{p}} (\Delta(p;\eta) - \Delta(p;\eta+\mu))dp \end{aligned}$$

Let us further define

$$\begin{aligned} A(\mu) &= \int_{P_\emptyset(\epsilon^{over}(k))}^{P_\emptyset(\epsilon^{over}(\eta+\mu))} \Delta(p;\eta)dp \\ B(\mu) &= \int_{P_\emptyset(\epsilon^{over}(\eta+\mu))}^{\overline{p}} (\Delta(p;\eta) - \Delta(p;\eta+\mu))dp \end{aligned}$$

I differentiate the above expressions w.r.t μ:

$$\begin{aligned} A'(\mu) &= \frac{\partial \epsilon^{over}(\eta+\mu)}{\partial \eta} \Delta(P_\emptyset(\epsilon^{over}(\eta+\mu));\eta) \\ B'(\mu) &= -\frac{\partial \epsilon^{over}(\eta+\mu)}{\partial \eta} \{\Delta(P_\emptyset(\epsilon^{over}(\eta+\mu));\eta) - \Delta(P_\emptyset(\epsilon^{over}(\eta+\mu));\eta+\mu)\} \\ &\quad - \int_{P_\emptyset(\epsilon^{over}(\eta+\mu))}^{\overline{p}} ((1 - H(\epsilon^{over}(\eta+\mu))) \\ &\quad +(1-\eta-\mu)h(\epsilon^{over}(\eta+\mu)) \frac{\partial \epsilon^{over}(\eta+\mu)}{\partial \eta} \\ &\quad -H(p - Q^{over}) + H(\epsilon^{over}(\eta+\mu)) \\ &\quad -(1-\eta-\mu)h(\epsilon^{over}(\eta+\mu)) \frac{\partial \epsilon^{over}(\eta+\mu)}{\partial \eta})dp \\ &= -\frac{\partial \epsilon^{over}(\eta+\mu)}{\partial \eta} \{\Delta(P_\emptyset(\epsilon^{over}(\eta+\mu));\eta) - \Delta(P_\emptyset(\epsilon^{over}(\eta+\mu));\eta+\mu)\} \\ &\quad - \int_{P_\emptyset(\epsilon^{over}(\eta+\mu))}^{\overline{p}} (1 - H(p - Q^{over}))\, dp \end{aligned}$$

When $\mu = 0$, A' can be simplified to:

$$\frac{\partial \epsilon^{over}(\eta)}{\partial \eta} \Delta(P_\emptyset(\epsilon^{over}(\eta));\eta) \geq 0$$

Likewise B' is equal to:

$$-\int_{P_\emptyset(\epsilon^{over}(\eta))}^{\overline{p}} (1 - H(p - Q^{over})) \leq 0$$

Replacing $\frac{\partial \epsilon^{over}(\eta)}{\partial \eta}$ by expression (A-12) and simplifying, I compute $A'(0) + B'(0)$:

$$\int_{\epsilon^{over}(\eta)}^{+\infty} (1 - H(\epsilon))d\epsilon - \int_{P_{\emptyset}(\epsilon^{over}(\eta))}^{\overline{p}} (1 - H(p - Q^{over}))\, dp$$

Consider the change in variable $\epsilon = p - Q^{over}$, I further simplify the above expression by:

$$\int_{\epsilon^{over}(\eta)}^{+\infty} (1 - H(\epsilon))d\epsilon - \int_{\epsilon^{over}(\eta)}^{\overline{\epsilon}} (1 - H(\epsilon))d\epsilon \tag{A-23}$$

If $\overline{\epsilon}$ converges to $+\infty$ then expression (A-23) is equal to zero. Thus $\forall \overline{\epsilon}$, expression (A-23) is positive. It also means that $\forall \overline{p}$, $T(\overline{p}) \geq 0$ i.e. the probability mass of $\Delta(;\eta)$ is more spread out than the probability mass of $\Delta(;\eta')$, which implies that $\Delta(;\eta)$ is "riskier" than $\Delta(;\eta')$.

(ii) Let us prove that if $\eta \leq \eta^{under}$, for $\eta' < \eta$, $\Delta(.;\eta')$ first-order stochastically dominates $\Delta(.;\eta)$.

It has been proven that $P(\epsilon^{under})$ does not depend on η. Thus the range of prices is the same for a given η or η'. Let us take the difference between $\Delta(p;\eta')$ and $\Delta(p;\eta)$ for $p \in [0, \overline{p}]$.

$$\begin{aligned}
& \Delta(p;\eta') - \Delta(p;\eta) \\
= \; & (\eta' + (1-\eta')H(\epsilon^{under})) + (1-\eta')\left(H(p - Q^{under}) - H(\epsilon^{under})\right) \\
& - \left((\eta + (1-\eta)H(\epsilon^{under})) + (1-\eta)\left(H(p - Q^{under}) - H(\epsilon^{under})\right)\right)
\end{aligned}$$

Replacing η by $\eta' + \mu$ it yields

$$\begin{aligned}
& \Delta(p;\eta') - \Delta(p;\eta' + \mu) \\
= \; & (\eta' + (1-\eta')H(\epsilon^{under})) + (1-\eta')\left(H(p - Q^{under}) - H(\epsilon^{under})\right) \\
& - \left((\eta' + \mu + (1-\eta'-\mu)H(\epsilon^{under})) + (1-\eta'-\mu)\left(H(p - Q^{under}) - H(\epsilon^{under})\right)\right) \\
= \; & -\mu(1 - H(\epsilon^{under})) + \mu\left(H(p - Q^{under}) - H(\epsilon^{under})\right) \\
= \; & -\mu\left(1 - H(p - Q^{under})\right) \leq 0
\end{aligned}$$

Therefore $\Delta(;\eta')$ first order stochastically dominates $\Delta(;\eta)$.

Proof of Proposition 6:

If $\eta \geq \eta^{over}$, I will prove that R_D is decreasing in η.

$$R_D = 1 - \int_{\epsilon^{over}(\eta)}^{+\infty} \frac{Q^{over}}{P(\epsilon)} \frac{h(\epsilon)}{(1 - H(\epsilon^{over}(\eta)))} d\epsilon$$

Differentiating R_D w.r.t η, it yields

$$-\frac{1}{(1-H(\epsilon^{over}))^2}h(\epsilon^{over})J'(\eta)\int_{\epsilon^{over}}^{+\infty}\frac{Q^{over}h(\epsilon)}{P(\epsilon)}d\epsilon + J'(\eta)\frac{Q^{over}h(\epsilon^{over})}{(1-H(\epsilon^{over}))P(\epsilon^{over})}$$

Simplifying it yields

$$-\frac{J'(\eta)Q^{over}h(\epsilon^{over})}{(1-H(\epsilon^{over}))}\left(\frac{1}{(1-H(\epsilon^{over}))}\int_{\epsilon^{over}}^{+\infty}\frac{h(\epsilon)}{P(\epsilon)}d\epsilon - \frac{1}{P(\epsilon^{over})}\right) \tag{A-24}$$

As $P(\epsilon^{over}) \leq P(\epsilon)$,

$$\left(\frac{1}{(1-H(\epsilon^{over}))}\int_{\epsilon^{over}}^{+\infty}\frac{h(\epsilon)}{P(\epsilon)}d\epsilon - \frac{1}{P(\epsilon^{over})}\right) \leq 0 \tag{A-25}$$

Also $J'(\eta) \geq 0$ and $Q^{over} < 0$ thus the derivative of R_D with respect to η given by expression (A-24) is negative. Therefore R_D is decreasing in η.
$P_\emptyset(\epsilon^{over}) = P(\epsilon^{over}) = \epsilon^{over} + Q^{over}$. Differentiating $P(\epsilon^{over})$ with respect to η, it yields $J'(\eta) \geq 0$. Thus $R_\emptyset$ is decreasing in η.
If $\eta \leq \eta^{under}$, ϵ^{under} and Q^{under} do not depend on η, so R_D is independent of η.

$$R_D = 1 - Q^{under}\int_{\epsilon^{under}}^{+\infty}\frac{h(\epsilon)}{P(\epsilon)(1-H(\epsilon^{under}))}d\epsilon$$

I turn to the comparative statics on the average cost of capital $\mathcal{R}$.
If $\eta \geq \eta^{over}$, $R_\delta = 1 - \frac{Q^{over}}{P_\delta}$. Thus ordering $\mathcal{R}(\eta)$ and $\mathcal{R}(\eta')$ boils down to ordering $1/P_\delta(\eta)$ and $1/P_\delta(\eta')$. I know that the function $1/P_\delta$ is a convex and decreasing function. I further proved that for $\eta' < \eta$, $\Delta(;\eta)$ second order stochastically dominates $\Delta(;\eta')$. This implies that $\mathbb{E}(1/P_\delta(\eta)) \leq \mathbb{E}(1/P_\delta(\eta'))$.
If $\eta \leq \eta^{under}$ then $\mathcal{R} = R_D$ as non-disclosing firms are not financed and
$R_D = 1 - Q^{under}\int_{\epsilon^{under}}^{+\infty}\frac{h(\epsilon)}{P(\epsilon)(1-H(\epsilon^{under}))}d\epsilon$, independent of η.

Proof of Proposition 7:

For $\eta \geq \eta^{over}$, the non-disclosing price is positive therefore all the firms in the economy invest. Therefore there is always overinvestment.
For $\eta \leq \eta^{under}$ the non-disclosing firms are not financed and there is a proportion of firms $\eta(1 - H(\epsilon^{under}))$ which was unable to disclose their information, although they were efficient firms. Obviously the underinvestment is increasing in η. At $\eta = 0$, all efficient firms invest.

Proof of Proposition 8

By Lemma 4,

(i) If $\eta \geq \eta^{over}$, risk-sharing is improved if the disclosure friction η increases.

(ii) If $\eta \leq \eta^{under}$, the range of prices is not affected by the disclosure friction. However a decreasing disclosure friction affects the distribution of prices. It resolves underinvestment.

All in all lowering the disclosure friction improves risk-sharing (in the sense of second-order stochastic dominance).

Proof of Proposition 9:

For $\eta \geq \eta^{over}$, the investor's indirect utility function with a firm disclosing δ is equal to

$$\frac{1}{1-\alpha} P_{\delta}^{1-\alpha} \int f(y)(\theta + y)^{1-\alpha} dy$$

The indirect utility is concave in P_{δ}. Further I know that for $\eta < \eta'$, $\Delta(.;\eta')$ second order stochastically dominates $\Delta(.;\eta)$, which implies that

$$\begin{aligned} & \frac{1}{1-\alpha} \int P_{\delta}^{1-\alpha} \int f(y)(\theta + y)^{1-\alpha} dy d\Delta(P_{\delta};\eta') \\ \geq & \frac{1}{1-\alpha} \int P_{\delta}^{1-\alpha} \int f(y)(\theta + y)^{1-\alpha} dy d\Delta(P_{\delta};\eta) \end{aligned}$$

For $\eta \leq \eta^{under}$, the investors' aggregate expected utility for a given η is equal to

$$(1-\eta) \int_{\epsilon^{under}}^{+\infty} \frac{1}{1-\alpha} P(\epsilon)^{1-\alpha} \int f(y) \left(\int_{\epsilon^{under}}^{+\infty} (1-\eta)(\epsilon + y) h(\epsilon) d\epsilon \right)^{1-\alpha} dy h(\epsilon) d\epsilon$$

consider $\eta > \eta'$

$$\begin{aligned} & (1-\eta) \int_{\epsilon^{under}}^{+\infty} \frac{1}{1-\alpha} P(\epsilon)^{1-\alpha} \int f(y) \left(\int_{\epsilon^{under}}^{+\infty} (1-\eta)(\epsilon + y) h(\epsilon) d\epsilon \right)^{1-\alpha} dy h(\epsilon) d\epsilon \\ \leq & (1-\eta') \int_{\epsilon^{under}}^{+\infty} \frac{1}{1-\alpha} P(\epsilon)^{1-\alpha} \int f(y) \left(\int_{\epsilon^{under}}^{+\infty} (1-\eta')(\epsilon + y) h(\epsilon) d\epsilon \right)^{1-\alpha} dy h(\epsilon) d\epsilon \end{aligned}$$

Proof of Corollary 2:

By Proposition 6, it has been proven that the average cost of capital in the economy is decreasing in the disclosure friction η, if $\eta \geq \eta^{over}$. Further by Proposition 9, the investors' expected utility increases in the disclosure friction η, if $\eta \geq \eta^{over}$.

Proof of Corollary 3:

For $\eta \geq \eta^{over}$, if mandatory disclosure forces firms which retained voluntarily their information to disclose, all inefficient firms would have to disclose their information and therefore would be closed. For $\eta \leq \eta^{under}$ all firms investing are all efficient, thus mandatory disclosure would not mitigate underinvestment.

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