Fair Value Accounting and Debt Contracting Efficiency[*]

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Abstract

We examine the impact of alternative fair value concepts on the effectiveness of debt covenants in mitigating investment policy distortions caused by debt. We model non-contractible project selection and an interim choice of whether to continue or abandon the project. A debt covenant violation cedes the interim choice to creditors. We consider covenants based on the value 'in-use' (continuation-value), the value 'in-exchange' (abandonment-value), and the maximum or minimum of the two. We find that continuation-value and minimum-value dominate the other two fair value measures. Continuation-value covenants effectively reduce continuation inefficiencies for a given project, while minimum-value covenants better mitigate asset substitution problems. An implication of these results relevant to standard setters is that flexibility in fair value measures may contribute to contracting efficiency.

JEL Classification: G32, M40, M48

1 Introduction

The development of International Financial Reporting Standards (IFRS) and the convergence between IFRS and U.S. Generally Accepted Accounting Principles (GAAP) have increased the prospect that fair value will serve as a basis for valuing firms' assets. Interest in the use of fair value in accounting has been further elevated by recent events associated with the meltdown of financial markets.[1] The appropriate measure of fair value for accounting purposes remains an open issue. Within the GAAP framework, Statement of Financial Accounting Standards (SFAS) No. 157 defines alternative measures of fair value and allows some discretion in selecting among the alternatives (FASB, 2008). IFRS contain similar provisions as exhibited by International Accounting Standard (IAS) No. 32 which also allows for discretion in determining the fair value of financial instruments (IASB, 2003). In this study, we shed light on policy implications of fair value by examining how alternative fair value measures impact the efficacy of accounting-based debt covenants in reducing inefficiencies that stem from debt-induced distortions in shareholder preferences regarding new investment.

In our model, the firm obtains project financing from competitive, risk-neutral creditors in the form of pure discount debt that matures at a future date when project payoffs are realized.[2] The model adds a debt covenant and project selection to Décamps and Faure-Grimaud's (2002) depiction of the effect of debt on continuation decisions. We assume that the shareholders' choice of project is non-contractible reflecting their discretion over post-contracting investment policies.[3] In practice, the degree of post-contracting discretion can range from small, as for debt-financed real estate investments that involve discretion in property management, to large, as for

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[1] In particular, considerable controversy has surrounded the application of fair value in mark-to-market treatment of asset-backed securities given the illiquidity of markets for such assets.
[2] All of our results continue to hold in a qualitative sense for coupon debt.
[3] By non-contractible we are assuming that either post-contracting investment choices are not observable to creditors, or that it is prohibitively costly to write contracts contingent on those choices.

general obligation debt. At an interim date, the firm produces a financial report and must decide whether to continue or abandon the project. The debt contract includes a technical default provision (covenant) that, if violated, transfers the continuation decision to the creditor. The assumption of competitive, risk-neutral creditors implies that debt proceeds equal the expected value of debt conditional on the creditor's rational conjectures of the firm's investment policy.

We focus on how alternative measures of fair value impact on the effectiveness of debt covenants in resolving conflicts of interest between shareholders and creditors at project selection and continuation stages. SFAS No. 157 refers to an asset's value "in-use", the value of the asset as a part of a larger whole such as a business unit, and the asset's value "in-exchange", the price that the asset commands individually if offered for sale (FASB, 2008). Similarly, IAS No. 36, defines the "recoverable amount" in asset impairment as the greater of the value in-exchange or the value-in-use (IASB, 2004).[4] We therefore model the project as consisting of a continuation-value process representing the value "in-use" and an abandonment-value process representing the value "in-exchange". The alternative fair value measures we consider are continuation-value, abandonment-value, the minimum of continuation- and abandonment-value (hereafter 'minimum-value'), and the maximum of continuation- and abandonment-value (hereafter 'maximum-value').[5] The use of maximum-value corresponds to the SFAS No. 157 statement that "the highest and best use of the asset establishes the valuation premise used to measure the fair value of the asset." Financial reports based on the minimum-value, which do not appear in existing accounting standards, can be viewed as a form of accounting conservatism.

[4] While this definition is in the context of asset impairment, it reflects a concept of fair value that may be used more broadly as IFRS move toward fair value as the guiding principle in financial statement presentations.

[5] As a benchmark, we also consider covenants that depend on both continuation and abandonment values for a fixed project and are "first-best" with respect to implementing efficient continuation decisions. As we will show, such covenants may fail to mitigate inefficiencies in the initial project selection. We rule out consideration of more complex functions of continuation and abandonment values. This seems reasonable given the typical threshold-type contracts observed in practice.

Our analysis suggests that no single concept of fair value provides a superior basis for accounting based contracts. The nature of a firm's investment opportunities determine which fair value measure yields the highest firm value when used in debt covenants. This suggests to policy makers that allowing for some discretion in the basis of fair value measures may provide for greater efficiency in resolving the time inconsistency problems that arise in levered firms.

In order to assess the effects of fair value measures on debt covenants, we first examine how the measures of fair value impact the effectiveness of debt covenants absent any discretion over the initial project choice so that only the interim continuation choice remains. The presence of debt creates an incentive for shareholders to continue projects that, from a firm value perspective, should be abandoned. For example, shareholders strictly prefer continuation when abandonment proceeds are insufficient to cover the debt obligation, regardless of how unpromising continuation may be. At worst, continuation offers shareholders some chance of receiving a payoff. In contrast, creditors strictly prefer abandonment when the proceeds cover the debt obligation and therefore guarantee that they receive their maximum payment. A debt covenant serves to reduce continuation inefficiencies ultimately borne by shareholders by allocating control rights to shareholders when continuation is efficient and to creditors when abandonment is efficient. Neither shareholders nor creditors always select the highest-and-best use represented by the maximum-value measure of fair value.

This simpler setting allows us to derive analytic results and to illustrate the forces that determine the relative efficiency of the fair value measures. Consistent with intuition, the debt's maturity value and the likelihood of creditor control act as partial substitutes for one another in satisfying the creditor's breakeven constraint. Covenants based on any of the fair value measures reduce continuation inefficiencies relative to granting unconditional control to shareholders. We find that, for a fixed project, the minimum-value covenant Pareto dominates the abandonment-value covenant and that the continuation-value covenant Pareto dominates the maximum-value covenant. In both cases, the Pareto dominating contract transfers control to creditors in states where shareholders would otherwise inefficiently continue, but grant no new control to shareholders in states where creditors would otherwise inefficiently abandon.

We next analyze the effects of covenants on initial project selection decision. After the debt contract is in place, shareholders have an incentive to choose projects that either involve unprofitable risks or enhance the prospects of continuation in the event that creditors control the abandonment decision. The combination of a knockout option in the form of abandonment at an interim date and an option to repay at maturity make closed-form solutions infeasible. Accordingly, results in this case require the use of numerical analysis as is common in the finance literature on capital structure.[6]

Specifically, we consider the role of covenants in mitigating the asset substitution problem whereby shareholders may have incentive to choose low-NPV projects with high volatility (Smith and Warner, 1979), which we model as discretion over the drift and volatility parameters of the continuation-value process. The debt contract parameters are set based on a conjectured project choice that the managers are free to deviate from; however, creditors are rational and anticipate any such deviations so that, in equilibrium, the manager chooses the conjectured project despite the opportunity to do otherwise.

The ability to abandon the project at the interim date makes volatility valuable, by itself. Thus, high volatility projects are inferior only if they are accompanied by a sufficiently large decrease in drift to offset the value created by volatility. The tradeoff between drift and volatility

[6] See Goldstein, Ju and Leland (2001), Hennessy and Whited (2005, 2007), Leland (1998), and Tsyplakov (2008). A precedent in the accounting literature is Verrecchia (1986).

determines the degree of value destruction and the attractiveness of high-volatility projects to shareholders. Shareholders have no incentive to increase volatility when doing so requires too large a sacrifice in drift. In such cases, concerns of continuation inefficiencies dominate concerns over project selection and we find that reports based on continuation-value are the most efficient in terms of overall firm value. The minimum-value, abandonment-value and maximum-value covenants also induce efficient project selection in this case; however, they entail greater inefficiencies at the interim continuation decision than the continuation-value covenant.

If alternative projects involve a moderate sacrifice of drift in exchange for volatility, the minimum-value covenant performs best. The additional continuation inefficiencies of the minimum-value covenant relative to the continuation-value covenant are small and fall more severely on riskier projects. The value of improving project selection more than offsets the minimum-value covenant's incremental inefficiencies at the interim continuation decision. While abandonment-value covenants can also discourage inefficient project selection, such contracts entail substantially more continuation inefficiencies than the minimum-value covenant.

In addition to the asset substitution problem, we consider the use of fair value measures when shareholders face project choices that alter the abandonment-value process by diverting resources or making idiosyncratic investments. Shareholders have an incentive to diminish abandonment-value in order to make continuation more likely at the interim stage, which they can accomplish by lowering the drift of the abandonment-value process. The key tradeoff is how much volatility must be given up for a reduction in drift. As with the asset substitution case, if the alternative projects are sufficiently unattractive, concerns over continuation decisions dominate the choice of covenant and the continuation-value covenant yields higher overall firm value than the other fair value measures. The continuation-value covenant also dominates the others when there is only a moderate reduction or increase in volatility for a low drift process.[7] When the low drift project entails a sufficiently large increase in volatility, none of the covenants can be structured to discourage inferior projects. Shareholders are then better off structuring a continuation-value debt covenant that implements the inferior project.

Our analysis relates to several studies on project selection and debt contracting. Leland (1998) models project selection as the ability to switch volatility between low and high values whereas we allow for one-time project choices that involve an interim continuation decision and accounting-based debt covenants. Gigler, Kanodia, Sapra and Venugopalan (2009) develop a model with a non-stochastic abandonment value and show that a continuation-value covenant can implement a first-best continuation choice. We find that a similar result in our richer setting with a stochastic abandonment-value if the firm has no discretion over the initial project selection and a covenant based on both continuation- and abandonment-values is feasible. However, the first-best result no longer holds when we introduce project selection. Also, while Gigler, et al. find that conservative accounting measures are inefficient in their setting, our findings regarding the efficiency of the minimum-value covenant suggest that conservative accounting in the sense of a fair value measure can be value-enhancing when abandonment values are stochastic and shareholders have post-contracting discretion in project selection.[8]

Chava and Roberts (2007) and Nini, Smith and Sufi (2009) lend empirical support to our focus on the use of covenants to transfer control rights to creditors, with specific reference to shareholders' tendency toward asset substitution.[9] Both papers find that covenant violations

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[7] In these cases, the abandonment-, minimum- and maximum-value covenants can also be structured to discourage inefficient project selection; however, they do so at a greater cost in terms of inefficiencies at the interim continuation decision and are therefore inferior.

[8] In this vein, Göx and Wagenhofer (2008) examine the use of impairment accounting in mitigating post-contracting moral hazard, whereas we focus on post-contracting project-selection.

[9] Also see the survey paper by Roberts and Sufi (2009).

grant creditors significant influence over the firms' subsequent investments and lead to reductions in capital expenditures, consistent with the tendency of creditors to abandon projects in our model. Chava and Roberts find that transfers of control are more frequent when firms have more exposure to agency and informational problems. Bradley and Roberts (2004) find that such covenants are more common for smaller, high growth firms that are more prone to asset substitution problems similar to those that we consider in our numerical simulations. Finally, Zhang (2008) finds evidence that conservative accounting benefits creditors *ex post* by signaling default risk and allowing for corrective actions, which benefits shareholders *ex ante* via lower lending costs. This result is consistent with our model's tradeoff between control rights and maturity value, in addition to the effectiveness of the conservative minimum-value covenant in discouraging inefficient project choices.[10]

The remainder of this paper proceeds as follows Section 2 lays out the model. Section 3 characterizes covenants and their effect on efficiencies at the interim continuation decision, without regard to project selection. Section 4 develops numerical analysis of the setting where the firm has discretion over the initial project choice. Section 5 provides concluding remarks.

2 Model

2.1 Basic Structure

This section defines the project's continuation-value and abandonment-value processes. It also identifies the shareholders' and creditors' decision criteria for abandoning the project at the interim date. The model consists of three dates illustrated in Figure 1. At Time 0, the firm borrows funds from creditors using zero coupon debt to invest in a project that will be chosen after signing the debt contract. At Time 1, the project may be abandoned, in which case the

[10] Other studies on the relation between lending and accounting conservatism include Bushman and Piotroski (2006), Ball, Bushman and Vasvari (2008) and Beatty, Weber and Yu (2008).

shareholders receive any residual of the abandonment-value over the value of the debt. If the project is instead continued, then it yields a terminal value at Time 2 and shareholders receive any residual of the terminal value over the face value of the debt.

(Insert Figure 1 about here)

Given the project choice, the dynamics of continuation and abandonment processes are similar to Décamps and Faure-Grimaud (2002). In their model, there are several dates in which shareholders can choose whether to continue or abandon investment a fixed project whereas we consider just one such date. The continuation- and abandonment-values, v_t and s_t, respectively, evolve as given by the following stochastic differential equations:[11]

$$\frac{\mathrm{d}\,v_t}{v_t} = \mu_v\,\mathrm{d}\,t + \sigma_v\,\mathrm{d}\,z_{vt} \qquad\qquad \frac{\mathrm{d}\,s_t}{s_t} = \mu_s\,\mathrm{d}\,t + \sigma_s\,\mathrm{d}\,z_{st} \tag{1}$$

where z_{vt} and z_{st} are correlated Brownian motions with $\mathrm{d}\,z_{vt}\,\mathrm{d}\,z_{st} = \rho\,\mathrm{d}\,t$. The project requires an investment of s_0 all of which must be financed through pure discount debt with maturity value M due at Time 2.[12] The debt market is competitive and all parties are risk neutral with discount rate r. A debt covenant agreed to at the time of borrowing specifies the technical default conditions under which creditors control the abandonment decision.

If abandoned at Time 1, the project realizes a value s_1 and the shareholders must pay the creditors the present value $\mathrm{e}^{-r}M$ of the Time 2 maturity value or default, giving creditors s_1. If continued, the project realizes a value v_2 at Time 2 and creditors receive the lesser of M or v_2 while shareholders receive any excess of v_2 over M.

[11] Equivalently, $\log(v_t / v_{t-1}) \sim N(\mu_v - \tfrac{1}{2}\sigma_v^2, \sigma_v^2)$ and $\log(s_t / s_{t-1}) \sim N(\mu_s - \tfrac{1}{2}\sigma_s^2, \sigma_s^2)$.

[12] Partial financing of the project yields a reduction in agency costs via a reduction in the face value of debt (Décamps and Faure-Grimaud, 2002). Partial financing of the project directly alters the breakeven condition (4) and *ex ante* payoff to shareholders (5). The remaining expressions differ indirectly via a lower maturity value M, which follows from substituting smaller borrowing proceeds into the breakeven condition (4).

The firm value is the following where I_c is an indicator variable that equals 1 when the project is continued at Time 1 and equals 0 otherwise:

$$e^{-2r}\,E[I_c v_2] + e^{-r}\,E[(1-I_c)s_1] \tag{2}$$

The value of debt given maturity value M and continuation indicator I_c must equal the borrowing proceeds s_0. Given a continuation policy, competitive pricing of the debt implies a maturity value that satisfies the following condition:

$$s_0 = E[I_c\, e^{-2r} \min\{v_2, M\} + (1-I_c)\, e^{-r} \min\{s_1, e^{-r} M\}]. \tag{3}$$

The *ex ante* payoff for shareholders is, therefore,

$$\begin{aligned} &E[I_c\, e^{-2r} \max\{0, v_2 - M\} + (1-I_c)\, e^{-r} \max\{0, s_1 - e^{-r} M\}] \\ &\qquad = e^{-2r}\,E[I_c v_2] + e^{-r}\,E[(1-I_c)s_1] - s_0. \end{aligned} \tag{4}$$

Equation (4) shows that the shareholders' *ex ante* equity value is the firm value less the value s_0 of debt. Shareholders bear the costs of inefficient project selection and continuation policies; however, apart from debt covenants, shareholders cannot commit *ex ante* to an efficient policy. Shareholders do not directly choose the continuation policy represented by I_c; rather, continuation depends on the allocation of control rights specified in the debt contract and the Time 1 preferences of the party controlling the continuation decision; either the shareholders or creditors. These preferences, in turn, depend on the project, itself, and the debt's maturity value.

Absent a debt covenant, shareholders retain unconditional rights over the Time 1 continuation decision. Shareholders, acting in their self interest at Time 1, continue projects in some states for which an optimal *ex ante* continuation policy would have chosen to abandon. For example, shareholders never abandon a project when the abandonment proceeds are insufficient to pay creditors. Abandoning in such cases guarantees that the shareholders receive nothing whereas continuing provides, at worst, an out-of-the-money call option on the project that offers some chance, however remote, of receiving a payoff. Accordingly, without an ability to commit to a continuation policy, the value of the firm (project *cum* abandonment option) is diminished and, given a competitive and rational debt market, shareholders bear the loss. On one hand, a debt covenant that transfers continuation rights to creditors in states where abandonment is optimal from a firm-wide standpoint, but suboptimal for shareholders at Time 1, may be valuable as a commitment device. On the other hand, creditors have incentives to abandon projects too often, such as when abandonment proceeds are sufficient to pay the entire amount of debt, so that transferring decision rights to creditors may lead to inefficient abandonment decisions.

In addition to improving continuation decisions, debt covenants can improve the shareholders' initial project choice. We assume that the firm's investment opportunity set is common knowledge but the choice of a project by shareholders from that set is not contractible. While contracts may successfully place some limits on the set of projects that shareholders can pursue, it would generally be prohibitively costly to totally eliminate shareholders' discretion over future investments. Debt covenants impact project choices by specifying the distribution of control rights with the consequence that shareholders may find some projects unattractive if creditors are likely to control their continuation. While transferring control to creditors may result in inefficient abandonment, this prospect may be worthwhile if it yields an improvement in project choice that outweighs the cost of excessive abandonment.

Fair value information available for debt contracting is provided by the firm. Consistent with the periodic nature of financial accounting reports, this information is determined from the values of projects observable at Time 1 in our model. At issue in later sections are the firm's choices of what project to select under a given covenant and related fair value measure, how best to structure a covenant based on a fair value measure supplied by financial statements, and what fair value measure to employ.[13] Below we identify a benchmark first-best continuation policy for and the shareholders' and creditors' continuation preferences. The actual continuation policy depends on the shareholders' and creditors' preferences and on the allocation of control rights.

2.2 First-Best Continuation Policy

The present value of continuing a given project at Time 1 is

$$\mathrm{e}^{-r}\ \mathrm{E}[v_2 \mid v_1, s_1] = \mathrm{e}^{-r}\ \mathrm{E}[v_2 \mid v_1] = \mathrm{e}^{\mu_v - r}\ v_1 \tag{5}$$

implying a first-best policy of continuing the project if and only if $s_1 \le \mathrm{e}^{\mu_v - r}\ v_1$.

Ex ante (at Time 0) the value of the firm under this strategy is the expected present value of the project if continued plus the present value of an option to exchange the project for its abandonment-value at Time 1 where $I_{s_1 \le \mathrm{e}^{\mu_v - r} v_1}$ denotes an indicator function:[14]

$$\begin{aligned}\mathrm{e}^{-2r}\ \mathrm{E}[I_{s_1 \le \mathrm{e}^{\mu_v - r} v_1} v_2] + \mathrm{e}^{-r}\ \mathrm{E}[I_{s_1 > \mathrm{e}^{\mu_v - r} v_1} s_1] &= \mathrm{e}^{-2r}\ \mathrm{E}[v_2] + \mathrm{e}^{-r}\ \mathrm{E}[\max\{0, s_1 - \mathrm{e}^{\mu_v - r}\ v_1\}] \\ &= \mathrm{e}^{2(\mu_v - r)}\ v_0 + \mathrm{e}^{-r}\ \mathrm{E}[\max\{0, s_1 - \mathrm{e}^{\mu_v - r}\ v_1\}].\end{aligned} \tag{6}$$

Because we later consider an asset substitution problem in which shareholders can pursues a riskier project than was assumed at the time of contracting, it is important to note that increasing the variance of the project's continuation-value process, σ_v^2, alone is insufficient to produce a project substitution problem. The first term in equation (6) shows that the continuation-value variance does not affect the continuation-value, itself; however, if the variance of the continuation-value process is greater than the variance of the abandonment-value

[13] Although firms may supplement a value measure contained within financial statements by disclosure of further value information, the former may enjoy precedence for contracting purposes given its implicit status as a preferred measure for accounting recognition. At this stage, fair value has yet to be adopted as an accounting principle applied across the board in asset valuation. However, as mentioned earlier, there is a trend toward such a principle.

[14] See Margarbe (1978) for the explicit expression of the exchange option in terms of the risk-free rate r and the diffusion parameters from (1).

process ($\sigma_v^2 > \sigma_s^2$), then increasing the variance σ_v^2 increases the value of the abandonment option. The abandonment option allows the firm to exit poorly performing high risk projects while continuing those that perform well. Hence, *ex ante* commitment by shareholders to choose the project with the highest variance holding the drift constant is both desirable and sustainable *ex post*. In order to create a conflict of interest between creditors and shareholders, shareholders must have further discretion with respect to the continuation policy or other aspects of the continuation- or abandonment-value processes (i.e., project selection).

2.3 Shareholders' Continuation Preferences

If shareholders retain the continuation rights at Time 1, then a project will be continued if and only if the present value of their expected payoff from continuing is not exceeded by their expected payoff from abandoning at that time. The former (latter) can be expressed as the value of a long position in a European call option on the project for which the exercise price equals the maturity value (discounted maturity value) of the debt and the exercise date is Time 2 (immediate). Hence, shareholders prefer to continue the project if and only if:

$$e^{-r}\,\mathrm{E}[\max\{0, v_2 - M\} \mid v_1] \geq \max\{0, s_1 - e^{-r} M\}\,. \tag{7}$$

Shareholders will always continue if $s_1 < e^{-r} M$ since abandonment in this case implies they receive nothing and while continuation yields a strictly positive expected value. If $s_1 > e^{-r} M$, then the continuation policy can be expressed as the Time 1 maturity value of the debt plus the aforementioned call option on the project exercisable at Time 2 where $\Phi(\bullet)$ denotes the standard normal distribution:

$$
\begin{aligned}
s_1 &\le e^{-r} M + e^{-r}\, E[\max\{0, v_2 - M\} \mid v_1] \\
&= e^{\mu_v - r} v_1 \Phi\left(\frac{\log(v_1/M) + \mu_v + \sigma_v^2/2}{\sigma_v}\right) + e^{-r} M \Phi\left(-\frac{\log(v_1/M) + \mu_v - \sigma_v^2/2}{\sigma_v}\right) \\
&\equiv \underline{s}_e(v_1).
\end{aligned} \tag{8}
$$

Recall that values of s_1 less than $e^{-r} M$ also imply continuation. Hence, (8) is both necessary and sufficient for continuation by shareholders.

The function $\underline{s}_e$ demarks the threshold for continuation by shareholders and ranges from a minimum of $e^{-r} M$ at $v_1 = 0$, and increases monotonically without bound toward an asymptote $e^{\mu_v - r} v_1$, the first-best threshold. The bold line in Figure 2 provides an illustration of the boundary of set of (v_1, s_1) values for which shareholders prefer to continue the project. Shareholders prefer to continue projects for any (v_1, s_1) values that lie below and to the right of the bold line. This is a strict superset of the values for which continuation is efficient, where the efficient set of continuation-values lies below and to the right of the line $s_1 = e^{\mu_v - r} v_1$.

(Insert Figure 2 about here)

2.4 Creditor's Continuation Preferences

The analogous condition to (7) for creditors is as follows:

$$
e^{-r}\, E[\min\{v_2, M\} \mid v_1] \ge \min\{s_1, e^{-r} M\}. \tag{9}
$$

Similar to the case for shareholders, the necessary and sufficient condition for continuation by creditors can be expressed as the Time 1 present value of the debt's maturity value less a put option on the project exercisable at Time 2 where $\Phi(\bullet)$ denotes the standard normal distribution:

$$
\begin{aligned}
s_1 &\leq e^{-r} M - e^{-r} \mathrm{E}[\max\{0, M - v_2\} \mid v_1] \\
&= e^{\mu_v - r} v_1 \Phi\left(-\frac{\log(v_1/M) + \mu_v + \sigma_v^2/2}{\sigma_v} \right) + e^{-r} M \Phi\left(\frac{\log(v_1/M) + \mu_v - \sigma_v^2/2}{\sigma_v} \right) \qquad (10) \\
&\equiv \underline{s}_d(v_1).
\end{aligned}
$$

The function $\underline{s}_d$ demarks the threshold for continuation by creditors with asymptotes $e^{\mu_v - r} v_1$, the first-best threshold, as v_1 approaches 0, and $e^{-r} M$ as v_1 goes to infinity. The thin line in Figure 2 provides an illustration. Creditors prefer to continue projects for which (v_1, s_1) lies below and to the right of the thin line. This is a strict subset of the efficient continuation region which, in turn, is a strict subset of the values of (v_1, s_1) for which shareholders prefer to continue. The boundaries of the optimal continuation policies of shareholders and creditors never coincide. The difference in thresholds equals the sum of values of a put option and a call option on the project exercisable at Time 2. This difference is strictly positive reflecting the shareholders' greater incentive to continue the project. Direct computations give the following where $\phi(\bullet)$ denotes the standard normal density:

$$
\frac{\partial \underline{s}_d}{\partial \sigma_v} = -e^{\mu_v - r} v_1 \phi\left(\frac{\log(v_1 / M) + \mu_v + \sigma_v^2 / 2}{\sigma_v} \right) < 0 . \qquad (11)
$$

Creditors' willingness to continue projects is strictly declining in the risk of the continuation value, which comes into play when we consider project selection.

It can be further shown that the maturity value of the debt and the continuation rights afforded creditors act as partial substitutes, suggesting a demand for covenants that convey such rights as a means to reduce the cost of financing projects. That is, the value of debt is increasing in both the maturity value of debt and in the likelihood that creditors control the continuation decision so that the two can be substituted when crafting a debt contract that satisfies the creditors' breakeven condition, equation (3). Accordingly, we state the following proposition that we later use to demonstrate the effects of debt covenants on firm value:

Proposition 1: *The value of debt is weakly increasing in the extent of creditor control over the continuation decision (the set of time 1 outcomes* (v_1, s_1) *that transfer control to creditors).*

The Appendix includes the proofs of all propositions. In the next section we consider the properties of covenants based on alternative fair measures suppressing, for the moment, the post-contracting project choice. The fair value measures considered include continuation-value, abandonment-value, the minimum of those values (minimum-value), and the maximum of those values (maximum-value).

3 Properties of Covenants for a Fixed Project

3.1 First-Best Covenants for a Fixed Project

Given a fixed project, shareholders can implement the first-best continuation policy through a covenant that transfers continuation rights to creditors if and only if the following condition based on both continuation- and abandonment-values is met:

$$s_1 > e^{\mu_v - r} v_1. \tag{12}$$

From the illustration of continuation policies in Figure 2, it is evident that creditors would make first-best continuation decisions in the region where (12) is satisfied and that shareholders would do so in the region where the inequality in (12) is reversed.

Firm value at Time 0 under a debt contract with the above first-best covenant is given by (6), while (3) ensures the value of the debt is s_0. Firm value is invariant to changes in the maturity value of debt under such a covenant. Given an arbitrary maturity value, the value of the debt at time 0 can be expressed as the discounted expected value conditional on continuing plus the discounted expected value of an abandonment option under a first-best covenant:

$$\begin{aligned}&\mathrm{e}^{-2r}\,\mathrm{E}[I_{s_1 \le \mathrm{e}^{\mu_v - r}v_1}\min\{v_2, M\}] + \mathrm{e}^{-r}\,\mathrm{E}[I_{s_1 > \mathrm{e}^{\mu_v - r}v_1}\min\{s_1, e^{-r}M\}]\\ &\quad = \mathrm{e}^{-2r}\,\mathrm{E}[\min\{v_2, M\}] + \mathrm{e}^{-r}\,\mathrm{E}[I_{s_1 > \mathrm{e}^{\mu_v - r}v_1}(\min\{s_1, e^{-r}M\} - \mathrm{e}^{-r}\,\mathrm{E}[\min\{v_2, M\} \mid v_1])].\end{aligned} \tag{13}$$

It is straightforward to verify that the value of debt is monotonically increasing in the maturity value, implying that there exists a unique M that solves the breakeven condition (3).[15]

The ability to replicate the first-best continuation policy through a debt covenant conditional on project choice is similar to Gigler, et al.'s (2009) result that a continuation policy can be set independently of the debt maturity value when the abandonment-value (in their model) is known *ex ante*. Their use of a fixed abandonment-value allows them to derive the first-best continuation policy based on continuation-value, alone, and then determine the debt's maturity value conditional on that policy. We obtain a similar result here because the continuation policy is independent of creditors' or shareholders' preferred policies were they to have the continuation rights. Subsequent cases consider covenants that cause continuation decisions to depend on shareholder and creditor continuation preferences and, therefore, on maturity values. In these cases we use (3) to determine maturity values. In the event of multiple solutions to (3) for a given continuation policy, we will use the solution with the smallest maturity value.

3.2 Covenants Based on Abandonment-Value

In the case where debt contracts use abandonment-value as their fair value measure, let $\bar{s}$ denote the threshold on the Time 1 abandonment-value s_1 such that continuation rights are conveyed to creditors when $s_1 < \bar{s}$. We note that an optimal covenant based on s_1 includes a lower-tailed region because the creditor's continuation preference approaches the first-best rule

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[15] We note that $\min\{s_1, \mathrm{e}^{-r}M\} - \mathrm{e}^{-r}\,\mathrm{E}[\min\{v_2, M\} \mid v_1] > 0$ for values above the creditor's abandonment threshold $\underline{s}_d$ and this threshold is bounded above by $\mathrm{e}^{\mu_v - r}v_1$. This implies that the second term on the second line of (14) is positive, which represents the added value to creditors from abandoning the project when it is efficient to do so.

for low values of s_1 while the inefficient continuation of shareholders remains strictly positive.[16] Similarly, the optimal covenant does not contain an upper-tailed region because the likelihood of shareholders' inefficient continuation approaches zero for high values of s_1 while the likelihood of inefficient abandonment by creditors remains positive.[17]

Figure 3a compares the continuation decision under an abandonment-value covenant to the first-best rule. The shaded regions in the figure illustrate that there are both inefficient continuation under shareholder control for high s_1 realizations and inefficient abandonment under creditor control for low s_1 realizations. Setting the threshold $\bar{s}$ balances these costs. For example, setting a strict threshold with a high $\bar{s}$ risks giving control to creditors in situations where they abandon regardless of how promising continuing may be.

(Insert Figure 3 about here)

3.3 Covenants Based on Minimum-Value

In the case where debt contracts measure fair value as the minimum of the Time 1 abandonment-value s_1 and continuation-value $e^{\mu_v - r} v_1$ (minimum-value), let $\bar{m}$ be the minimum-value threshold such that continuation rights are conveyed to creditors when $\min\{s_1, e^{\mu_v - r} v_1\} < \bar{m}$. Figure 3b illustrates the inefficient continuation and abandonment that result from a minimum-value covenant. Comparing Figure 3a to Figure 3b shows that applying the same threshold $\bar{s}$ from the abandonment-value covenant to the minimum-value covenant

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[16] Analytically, denote by $\underline{v}_d(s_1)$ the inverse of the threshold function $\underline{s}_d(v_1)$ defined by (11). The difference between the threshold $\underline{v}_d(s_1)$ and the first-best rule approaches zero as s_1 approaches zero. In contrast, the inverse function $\underline{v}_e(s_1)$ of the shareholder's threshold function $\underline{s}_e(v_1)$ defined by (9) remains bounded away from the first-best rule as s_1 approaches zero.

[17] A conference discussant suggested consideration of a covenant under which continuation rights are transferred to creditors when abandonment-value *exceeds* a threshold. Such a covenant is inefficient because shareholders' preferences approximate the first-best abandonment rule for high values of s_1.

conveys incremental continuation rights to creditors in a region where creditors make a decision consistent with the first-best rule. Thus, giving them control in this region reduces excessive continuation by shareholders. This yields the following proposition:

Proposition 2: *Given a covenant threshold $\overline{s}$, a debt covenant based on minimum-value $\min\{s_1, e^{\mu_v - r} v_1\}$ yields less inefficient continuation than a covenant based on abandonment-value s_1 and no more inefficient abandonment. Furthermore, covenants based on minimum-value Pareto-dominate covenants based on abandonment-value.*

The Pareto-dominance in Proposition 2 is due to the fact that, relative to the abandonment-value covenant, the minimum-value covenant grants additional control rights to creditors for Time 1 continuation and abandonment values for which creditors make a first-best continuation choice. This relatively efficient allocation of control rights is accomplished without necessitating a compensating increase in the maturity value of debt.

3.4 Covenants Based on Maximum-Value

This case represents the continuation rule implicit in the highest-and-best-use notion of SFAS No. 157 (FASB, 2008) and IAS No. 36 (IASB, 2004). In the case where debt contracts measure fair value as the maximum of the Time 1 abandonment-value s_1 and continuation-value $e^{\mu_v - r} v_1$ (maximum-value), let $\overline{n}$ denote the maximum-value threshold for shareholders retaining control rights. Creditors then have control when $\max\{s_1, e^{\mu_v - r} v_1\} < \overline{n}$. The maximum-value $\max\{s_1, e^{\mu_v - r} v_1\}$ reflects the choice of the shareholders only if they pursue the first-best continuation rule. When shareholders or creditors control the continuation decision, the maximum-value does not reflect firm value because they either fail to abandon in some cases where $s_1 > e^{\mu_v - r} v_1$ or fail to continue in some cases where $s_1 < e^{\mu_v - r} v_1$. Figure 4a compares the continuation decision under a maximum-value covenant to the first-best rule. In this case, creditors only control the continuation decision when both the abandonment- and continuation-values fall below the threshold $\bar{n}$.

3.5 Covenants Based on Continuation-Value

Last, in the case where debt contracts measure fair value as the Time 1 continuation-value $e^{\mu_v - r} v_1$, let $\bar{v}$ be the continuation-value threshold such that creditors control continuation when $v_1 < \bar{v}$. Figure 4b compares the continuation decision under a continuation-value covenant to the first-best rule. A comparison to the maximum-value covenant in Figure 4a shows that, relative to the maximum-value covenant, the continuation value covenant conveys additional control rights for (v_1, s_1) values in which creditors make the first-best decision to abandon. This reduction in inefficient continuation yields the following proposition:

Proposition 3: *Given a covenant threshold* $\bar{n}$ *, a debt covenant based on the continuation-value* $e^{\mu_v - r} v_1$ *yields less inefficient continuation and no more inefficient abandonment than a covenant based on the maximum-value. Furthermore, covenants based on continuation-value Pareto-dominate covenants based on maximum-value.*

The intuition for Proposition 3 mirrors that for Proposition 2. The additional set of (v_1, s_1) values for which creditors control the continuation decision is one in which they make choices consistent with the first-best rule.

Proposition 3 directly contradicts the highest-and-best-use notion specified by SFAS 157 and IAS 36 (FASB, 2008; IASB, 2004). This notion assumes that managers would choose to continue or abandon based on which results in the highest present value of the project; i.e., the most productive use of the asset. However, this assumption ignores conflicts of interest between shareholders and managers with respect to that choice. The continuation and abandonment inefficiencies in our model arise precisely because managers of a levered firm may have an incentive to continue a project even though abandonment maximizes total firm value.

(Insert Figure 5 about here)

Figure 5 compares the continuation-value to the minimum-value covenant. In effect, a covenant based on continuation-value allows shareholders to retain additional continuation rights by comparison to a covenant based on the minimum-value. As illustrated in Figure 5b, this reduces excessive abandonment by creditors. However, what is not apparent from the figure is that a covenant based on continuation-value involves a reduction in creditors' control rights relative to the covenant based on minimum-value. Proposition 1 states that shareholders must compensate creditors for this reduction in control rights either with a higher maturity value or a stricter threshold (higher v_1 cutoff), *ceteris paribus*. The higher maturity value creates inefficiencies by pushing shareholders' and creditors' continuation preferences further from first-best. Which effect dominates is parameter specific and therefore ambiguous.

In the example of Figure 5, a covenant based on continuation-value dominates. Creditors are compensated by a stricter threshold on continuation-value than that implied by the optimal threshold for minimum-value. The adjustment is small and yields a net reduction in continuation inefficiencies so that overall firm value increases. Allowing for changes to both maturity value and covenant threshold would yield a further increase in firm value. However, we have yet to consider project choice.

Summarizing the case regarding the best fair value measure in a setting where the project is fixed *ex ante* and only the efficiency of continuation at the interim date is at issue, a first-best covenant makes use of both continuation and abandonment values. Such a covenant need not be first-best when project selection is not fixed. In this richer case, covenants and maturity values depend on creditor's conjectures about the drift and volatility parameters of the project selected rather than the project *per se*. Furthermore, the covenant triggers rely on values generated by the projects rather than the project selection itself. Once the covenants are in place, shareholders may have incentives to substitute a project that works more to their advantage under those covenants. Of course, in equilibrium, creditors anticipate this behavior and price the debt accordingly, implying that the shareholders bear the consequences of their inability to commit to first-best project selections.

With a fixed project, imposing a restriction to a single fair value measure for debt contracting purposes results in continuation-value covenants minimizing inefficiencies in continuation choices at the interim date. While a minimum-value covenant allows for less efficient continuation choices at the interim date, as we will show, it is precisely this aspect that raises the prospect of such a covenant dominating continuation-value covenants when project choice is not fixed. Neither abandonment-value nor maximum-value does well in either setting.

4 Covenants with Project Choice

4.1 Asset Substitution Problem

This section extends the preceding analysis by allowing for the shareholders' choice of a project after debt contracting. We characterize the asset substitution problem as one in which inferior projects in terms of present value of expected future cash flows have a higher volatility as measured by the diffusion parameter σ_V and lower drift coefficients μ_V for the continuation process, holding the parameters of the abandonment process constant.[18] Bradley and Roberts (2004) suggest that firms mitigate shareholders' incentives to substitute higher risk, low NPV

[18] As explained earlier, increasing the variance while holding the drift constant increases overall firm value and therefore does not represent an inefficient investment choice.

projects by using covenants that may transfer control rights to creditors.

Because it is not possible to derive closed-form solutions for optimal covenant thresholds given project selection, we resort to numerical simulation.[19] We calibrate the simulation as follows: For our baseline Project *A*, the risk free rate is 0.05 as in Décamps and Faure-Grimaud (2002) which is between 0.045 used by Goldstein, Ju, and Leland (2001) and 0.06 used by Leland (1998). The drift parameters for the project's continuation-value μ_v and abandonment-value μ_s are double and half the risk free rate, respectively. The continuation-value's diffusion parameter σ_v is 0.40 as in Décamps and Faure-Grimaud (2002) while the abandonment-value's diffusion parameter σ_s is half at 0.20, consistent with less volatile values in abandonment. The correlation is set at 0.40 based on the correlation between log asset and log market value returns of *Compustat* firms. The initial continuation- and abandonment-values are set at 100. These parameters imply that the *ex ante* value of the project, inclusive of the abandonment option, is 120.10.

There are four alternatives to the baseline project, Projects $S_1, S_2, S_3,$ and S_4. The volatility for all four projects is set at 0.70, which is higher than Project *A*'s volatility of 0.4. The drift parameters are -0.05, -0.03, -0.003, and 0.005, respectively. These parameters imply that the first-best *ex ante* values of the projects, inclusive of the abandonment option, are 112.40, 115.02, 117.65, and 118.53, respectively. Hence, in the absence of inefficiencies in continuation and project choice, Project *A* would be preferred to all substitutes and Project S_4 would be the best of the substitutes. Table 1 summarizes the simulation parameters.

(Insert Table 1 about here)

[19] This issue arises due to the complexity of the decision problem rather than our distributional assumptions. For example, the use of uniform distributions also requires numerical analysis because solving the equilibrium requires finding roots of high-order polynomials.

Initially, we set covenant thresholds based on different measures of fair value to minimize inefficiencies at the continuation stage assuming that Project *A* would be chosen. *Ex ante* values of equity are 17.89, 19.71, 19.47, and 19.30 under covenants based on abandonment-value, continuation-value, minimum-value, and maximum-value, respectively. As expected, equity values are all below the equity value of 20.10 that would obtain under a first-best covenant for a fixed project. The question now becomes whether these covenants would be sufficient to deter shareholders from defecting to one of the substitute projects, $S_1 - S_4$.

Figure 6 graphically represents the problem by showing shareholders' indifference curves in the two-dimensional space defined by the drift parameter μ_v on the x-axis and volatility parameter σ_v on the y-axis. Each indifference curve represents the locus of combinations of drift and volatility parameters that yield the same *ex ante* equity value as Project *A* given the debt contract that assumes that the firm will choose Project *A*. Shareholders prefer any alternative project located to the northeast of a given indifference curve to projects on that curve. In addition to indifference curves corresponding to fair value based covenants, we also depict indifference curves when shareholders and creditors have unconditional continuation rights.

Figure 6 shows that unconditional shareholder control yields the lowest present value of equity. This is because creditors demand a high face value to compensate for the lack of control rights, which, in turn, virtually guarantees continuation of the project regardless of how low the continuation-value may be.[20] The high face value also tempts shareholders to take on risky projects, which is reflected in the low slope of the shareholders' indifference curve when they have unconditional control. The shareholders' indifference curve with a first-best covenant lies below the indifference curves for the fair value measures, demonstrating that the first-best

[20] The probability of continuation under unconditional equity control for the parameters in the graph is 0.999.

covenant is relatively ineffective in deterring inefficient project selection.

(Insert Figure 6 about here)

Below we examine how covenants based on the different measures of fair value affect project choice. As mentioned, the fair value based covenants exhibited in Figure 6 are optimal in the sense of reducing inefficiencies in continuation decisions in the absence of alternative projects for a given measure of fair value. When a covenant optimized for Project *A* fails to deter one of the alternative projects, we also indicate how the comparisons are affected by tightening the covenant threshold sufficiently to deter defection to an alternative project. We summarize the results of comparisons in Table 2, which also indicates probabilities of default and continuation.

(Insert Table 2 about here)

Choice Between Projects A and S_1: It is evident from Figure 6 that conditionally optimal covenants based on any of the four measures of fair value assuming a choice of Project *A* would deter defection at the project selection stage to S_1. In effect, the loss in value due to the reduction in drift exhibited by S_1 more than offsets the gain from the increase in volatility no matter which covenant is applied. Of the fair value based covenants, the one based on continuation-value delivers the highest equity value.[21]

Choice Between Projects A and S_2*:* Covenants based on abandonment-value and minimum-value assuming a choice of Project *A* would deter defection to Project S_2, while covenants based on continuation-value and maximum-value would not. In Figure 7, we illustrate the effects of tightening the continuation-value covenant to deter defection to Project S_2. Figure 7a depicts the shareholders' indifference curves over projects under the minimum-value

[21] A first-best covenant that assumes Project *A* would also work in this case and would dominate any of the fair value based covenants in maximizing equity value. However, such a covenant would not eliminate defection to any of the other alternative projects.

covenant and the adjusted continuation-value covenant. As we note in Table 2 and Figure 7a, tightening the threshold of the continuation-value covenant to deter defection to projects on the *A* to S_2 line would imply greater inefficiencies at the continuation stage thereby reducing equity value from 19.71 to 18.19 which is less than equity value of 19.47 under a covenant based on minimum-value.

The minimum-value covenant is relatively effective at deterring Project S_2 because it grants additional control rights to creditors in circumstances when they will inefficiently abandon the riskier project. Figure 7b illustrates the covenant thresholds and shareholders' and creditors' Time 1 continuation preferences for Projects *A* and S_1 when the contract is based on a minimum-value covenant.[22] The dashed line depicting the creditors' preferences is shifted downward for the riskier Project S_2. The minimum-value covenant gives control rights to creditors in the region below the horizontal dashed line. The riskier Project S_2 shifts creditors' continuation preferences downward as given by (11). The gap between the creditors' continuation preferences for Project *A* and Project S_2 shows a substantially larger region of abandonment of Project S_2 under the minimum-value covenant relative to the continuation-value covenant, which is denoted by the shaded area on Figure 7b. This increase in the likelihood of abandonment acts as an effective deterrent to choosing Project S_2.

Table 2 shows that the maximum-value covenant that deters defection in this case is nearly identical to the continuation-value covenant that deters defection. This occurs because,

[22] The Project *A* face values given in Table 2 show that the face value of 135.02 for the minimum-value covenant is close to the face value of 133.58 for the continuation-value covenant. The face value is the only parameter of the debt contract that enters the shareholders' and creditors' continuation preference functions $\underline{s}_e$ and $\underline{s}_d$ so that the continuation preferences plotted in Figure 7b are similar under continuation-value and minimum-value covenants optimized for Project *A*.

given the high covenant threshold, the incremental region of the (v_1, s_1) plane for which creditors gain control under the continuation-value covenant *vis-à-vis* the maximum-value covenant lies almost entirely above the shareholder's abandonment threshold $\underline{s}_e$; hence, both shareholders and creditors prefer to abandon in this region. Hence, a covenant based on minimum-value is the best among the fair value covenants for deterring Project S_2. The benefit of the minimum-value covenant stems from the incremental region of creditor control (Compares Figures 4a and 4b). The creditors' abandonment threshold $\underline{s}_d$ shifts downward with project risk. As a result, the region of incremental abandonment with the minimum-value contract is larger with the riskier Project S_2 than with the benchmark Project *A*.

Choice Between Projects A and S_3*:* When covenants are optimized for Project *A*, only the abandonment-value covenant would deter defection to projects that lay on an investment frontier on the line connecting Project *A* to Project S_3. However, the extreme case of complete creditor control illustrated in Figure 6 shows that the other covenants can be tightened to deter Project S_3. Tightening the threshold of a minimum-value covenant to deter defection to S_3 would result in an equity value of 19.00 which is greater than the 17.89 under an abandonment-value covenant. Table 2 shows that the continuation- and maximum-value covenants require extreme tightening to preclude Project S_3, implying that they cede nearly certain control to creditors and each yields an equity value of 14.64, which is well below the 19.00 obtainable with a minimum-value covenant. This suggests that a covenant based on minimum-value is the best among the fair value covenants in this situation.

Choice Between Projects A and S_4*:* None of the covenants based on any of the fair value measures, assuming a choice of Project *A*, deters defection to projects that lay on the line connecting Project A to Project S_4. The covenant thresholds can all be tightened to deter defection to projects on the A to S_4 line. The resulting abandonment-value covenant yields an equity value of 17.68 and the minimum-value covenant yields 18.48. The continuation- and maximum-value covenants both yield 14.36 and, once more, require giving nearly complete control to creditors. If the firm pursues Project S_4 rather than A and sets covenants accordingly, ,the best fair value covenant is based on continuation-value and yields an equity value of 18.07. Hence, a minimum-value covenant remains the best among fair value covenants.[23]

As thresholds under fair value based covenants are tightened, debt maturity values are reduced reflecting the benefits of greater control rights then held by the creditors. The probabilities of default and continuation provide a sense of the relative efficiencies of fair valued based covenants under various thresholds. Assuming a choice of Project A, before thresholds are tightened to deter defections, continuation-value covenants imply the least inefficiencies. This is consistent with our earlier characterization of covenants for a fixed project. However, continuation- and maximum-value covenants require substantial tightening in order to deter defection to Project S_2, resulting in a large increase in the probability of default. This increase in creditor control is partially offset by a lower maturity value, but not enough to produce an equity value higher than under a minimum-value covenant. It becomes difficult to deter defection to an alternative project if an increase in volatility entails only a small reduction in drift. Thus, in our comparisons, continuation-value covenants are ineffective for the higher drift alternatives.

4.2 Wasting Assets Problem

We characterize the wasting assets problem as one in which the alternatives to Project A

[23] In this case, though not in any of the other cases above, a first-best covenant assuming Project S_4 would implement S_4 and yield an equity value of 118.53, which is slightly better than the best fair value based covenant.

have a lower abandonment-value drift μ_s and vary in abandonment-value volatility σ_s. This context represents firms with projects that involve idiosyncratic investments with diminishing value and possibly moral hazard involving diversion of assets. We hold the parameters of the continuation process constant. Similar to before, we consider four alternatives to the baseline project, Projects $W_1, W_2, W_3,$ and W_4. The drift μ_s for all four projects is set at -0.02 as compared to the higher 0.025 drift for Project *A*. Volatility parameters are 0.15, 0.19, 0.2, and 0.22, respectively. These parameters imply that the *ex ante* values of the projects, inclusive of the abandonment option, are 118.24, 118.28, 118.32, and 118.41, respectively. Hence, in the absence of inefficiencies in continuation and project choice, Project *A* is preferred to all substitutes and Project W_4 is the best of the substitutes. Table 3 summarizes the project parameters.

(Insert Table 3 about here)

Initially, we again set covenant thresholds based on different measures of fair value to minimize inefficiencies at the continuation stage assuming that Project *A* would be chosen. As in the asset substitution setting, *ex ante* values of equity are 17.89, 19.71, 19.47, and 19.30 under covenants based on abandonment-value, continuation-value, minimum-value, and maximum-value, respectively. The graph axes are now the drift and volatility parameters of the abandonment-value process. Figure 8 depicts indifference curves for each of the fair value based covenants and for the extreme cases where shareholders or creditors have full control over project continuation. Equity values are increasing upward toward the northeast.

(Insert Figure 8 about here)

The indifference curves for the extreme cases of unconditional shareholder and creditor control suggest the forces at work are different from the asset substitution problem. Whereas the continuation-value parameters considered in the asset substitution problem affect creditors' Time

1 continuation preferences, the parameters of the abandonment-value process do not affect creditors' Time 1 continuation preferences given the realizations of abandonment-value s_1 and continuation-value v_1. This creates the potential for shareholders to take actions that diminish the Time 1 realization of the abandonment-value in order to increase the likelihood that creditors' prefer continuation. The more likely it is that creditors control the continuation decision, the more incentive that shareholders have to reduce the value of abandonment through project selection. Accordingly, Figure 8 shows that full creditor control results in the greatest loss of efficiency rather than full shareholder control as exhibited in Figure 6. Table 4 summarizes the results of project comparisons under the fair value based covenants.

(Insert Table 5 about here)

Choice Between Projects A and W_1*:* Covenants based on any of the four measures of fair value assuming a choice of Project *A* would deter defection at the project selection stage if the frontier of the firm's investment opportunities lay on the line between W_1 and Project *A*. Accordingly, the covenant based on continuation-value is best among the fair value covenants.

Choice Between Projects A and W_2*:* Both continuation-value and minimum-value covenants optimized for Project *A* would deter defection to projects that lay on the line from *A* to W_2. The abandonment-value covenant's threshold must be loosened to deter defection from *A* to W_2. Loosening the threshold reduces the attractiveness of W_2 stemming from the shareholders' incentive to reduce the abandonment-value so that creditors are more likely to continue when they have control. But, it also implies an increase in maturity value. In any event, the result is a equity value of 17.68 that is less than 17.89 with no loosening and that is lower than equity value under the other fair value choices. The maximum-value covenant's threshold must also be loosened to discourage a choice of W_2, which yields an equity value of 19.16. The best fair value covenant in this case is the continuation-value covenant.

Choice Between Projects A and W_3*:* Of the covenants optimized for Project *A*, only the minimum-value covenant deters defection to projects on the line between Project *A* and Project W_3. However, loosening the threshold of the continuation-value covenant results in a an equity value of 19.70 that exceeds the equity value if 19.47 with the minimum-value covenant. The looser threshold reduces the incentive of shareholders to diminish the value of the abandonment option by switching from *A* to W_3. The thresholds of both the abandonment- and maximum-value covenants may be loosened to deter W_3; however, they generate more inefficiencies than the continuation covenant and yield equity values of 16.92 and 17.81, respectively.

Choice Between Projects A and W_4*:* It is not possible to deter defection from Project *A* to projects on the line between *A* and Project W_4 under covenants based on any of the fair values when set assuming that *A* would be chosen. Figure 8 shows that W_4 lies above the indifference curve when shareholders have full control over the project. No amount of covenant loosening can deter the selection of W_4. Another choice is to set covenants assuming that W_4 would be chosen. Such a covenant based on continuation-value that deters defection to all other projects results in an equity value of 18.12. This is the best that shareholders can do in this case. [24]

Note that any contract adjustments reduce the likelihood of default and increase the likelihood of continuation, which is needed in order to prevent shareholders from reducing the abandonment-value through project selection as a means of increasing the likelihood of continuing the project. In contrast, contract adjustments to deter asset substitution reduce the

[24] The first-best covenant for W_4 may implement that project, depending on the alternatives available.

likelihood of continuing and therefore realizing the benefit of a riskier continuation value. This suggests that the nature of the shareholders' discretion dictates whether firm value is enhanced by loose or strict covenant thresholds.

5 Conclusion

In this study, we examine how different measures of fair value affect the efficiency of covenants that transfer control decisions to creditors contingent on future investment performance. We characterize projects by stochastic continuation- and abandonment-value processes. Covenants transfer continuation/abandonment rights to creditors if thresholds based on fair value measures are not met. Conflicts of interest between shareholders and creditors arise with respect to project selection and continuation decisions. While firms can eliminate inefficiencies in continuation decisions by appropriate covenants for a fixed project, such covenants may induce sub-optimal project selection with respect to maximizing firm value. Creditors anticipate this conflict so that shareholders bear the costs of discretion to choose projects subsequent to debt contracting.

The choice of fair value measure influences the structure of covenants and their impact on project choices. Fair value measures considered here include continuation-value, abandonment-value, the minimum of continuation- and abandonment-values (minimum-value), and the maximum of continuation- and abandonment-values (maximum-value). The maximum-value corresponds to the FASB and IASB "highest and best use" benchmark for fair value. However, such a choice is sub-optimal from a debt contracting perspective. We find that continuation-value and minimum-value covenants dominate the other fair value measures. The nature of shareholders' investment choices determines which of these two measures yields the highest expected firm value. Minimum-value covenants can be viewed as a manifestation of accounting conservatism and perform well when projects maximizing firm value must compete with riskier lower growth projects. In particular, a riskier project with lower expected value expands the likelihood of inefficient abandonment under minimum-value covenants. This eliminates some of the shareholders' gains from choosing the riskier project and discourages the choice of such projects. Continuation-value covenants dominate in settings where abandonment-values are declining possibly due to agency issues or erosion of assets. In this context, the extra control ceded to creditors by the minimum-value covenant is detrimental because, given a high likelihood of creditor control, shareholders have incentives to diminish the abandonment-value as a means of increasing the likelihood that creditors prefer continuing the project.

Both U.S. GAAP and IFRS specify alternative concepts of fair value that may be applied depending on underlying circumstances. The principal concern of the standard setters seems to be the objectivity of these fair value measures. SFAS No. 157 specifies a pecking order ranging from price quotes in a highly liquid market, prices inferred from recent transactions, or estimates based on some model such as present value of expected future cash flows. However, this concern is distinct from our focus on efficiency in debt contracting. We do not suggest that issues of objectivity are less important; rather that efficiency would still be an important consideration even if those issues could be resolved.

Stepping back to a more fundamental characteristic of accounting practice, it has been suggested that conservatism in accounting contributes to the efficiency of debt contracts. While our study is narrowly addressed to concepts of fair value, we are able to characterize circumstances under which a conservative interpretation of fair value may lead to efficiencies in project selection and continuation decisions when projects are financed through debt.

Broadly speaking our analysis lends support to the view that flexibility in the form of discretion over accounting valuation choices may allow for efficiencies in contracting. The efficiency losses that may unfold from a particular fair value measure suggests a potential welfare loss associated with rigidity of standards when firms seek debt financing of new projects for which it is infeasible to contract all of the specifics of project investments.

Among the limitations of our model is the exclusion of more complex functions of continuation- and abandonment-values than are represented by the minimum or maximum of those values in the design of debt covenants. While an argument can be made for allowing other functions that map continuation- and abandonment-values into an amount to be reported in statements of financial position, these appear to be the only alternatives under discussion by either domestic or international rule making bodies. A related issue is the notion that contracts favor use of amounts reported within financial statements to amounts disclosed by footnotes. We find that firms can implement first-best continuation for a fixed project using contracts based on both continuation- and abandonment-value, which would require the use of two inputs such as from the financial statements and a footnote. However, we also demonstrate that such debt covenants need not be optimal when project selection is non-contractible.

Last, we note that our focus on efficiency properties of covenants that may transfer control rights to creditors ties into recent empirical research by Chava and Roberts (2007) and Nini, Smith and Sufi (2009) on state contingent transfers of control rights from shareholders to creditors upon violation of covenants that implicitly depend on accounting policies in use. It remains for further empirical research to consider the types of value information upon which covenants conveying control rights may be based.

Appendix

Proof of Proposition 1

Define the region in which shareholders control the continuation decision by S and the indicator function for that region by I_s. We can then write the indicator function for continuation as $I_c = I_c(I_s + 1 - I_s)$. The debt value (3) is:

$$\begin{aligned}
&\mathrm{E}[I_c\, \mathrm{e}^{-2r} \min\{v_2, M\} + (1-I_c)\,\mathrm{e}^{-r} \min\{s_1, \mathrm{e}^{-r} M\}] \\
&\quad = \mathrm{E}\Big[(1-I_s)\Big(I_c\, \mathrm{e}^{-2r} \min\{v_2, M\} + (1-I_c)\,\mathrm{e}^{-r} \min\{s_1, \mathrm{e}^{-r} M\}\Big)\Big] \\
&\qquad + \mathrm{E}\Big[I_s \Big(I_c\, \mathrm{e}^{-2r} \min\{v_2, M\} + (1-I_c)\,\mathrm{e}^{-r} \min\{s_1, \mathrm{e}^{-r} M\}\Big)\Big]
\end{aligned} \tag{A1}$$

The shareholder continuation preference $\underline{s}_e(v_1)$ defined by (8) and the creditor continuation preference $\underline{s}_d(v_1)$ defined by (10) allow us to write (A1) as:

$$\begin{aligned}
&\mathrm{E}\Big[(1-I_s)\Big(I_{s_1<\underline{s}_d(v_1)}\, \mathrm{e}^{-2r} \min\{v_2, M\} + I_{s_1>\underline{s}_d(v_1)}\, \mathrm{e}^{-r} \min\{s_1, \mathrm{e}^{-r} M\}\Big)\Big] \\
&\qquad + \mathrm{E}\Big[I_s \Big(I_{s_1<\underline{s}_e(v_1)}\, \mathrm{e}^{-2r} \min\{v_2, M\} + I_{s_1>\underline{s}_e(v_1)}\, \mathrm{e}^{-r} \min\{s_1, \mathrm{e}^{-r} M\}\Big)\Big] \\
&\quad = \mathrm{E}\Big[I_{s_1<\underline{s}_d(v_1)}\, \mathrm{e}^{-2r} \min\{v_2, M\} + I_{s_1>\underline{s}_d(v_1)}\, \mathrm{e}^{-r} \min\{s_1, \mathrm{e}^{-r} M\}\Big] \\
&\qquad + \mathrm{E}\Big[I_s I_{\underline{s}_d(v_1)<s_1<\underline{s}_e(v_1)} \Big(\mathrm{e}^{-2r} \min\{v_2, M\} - \mathrm{e}^{-r} \min\{s_1, \mathrm{e}^{-r} M\}\Big)\Big]
\end{aligned} \tag{A2}$$

By the definition of $\underline{s}_d(v_1)$, $\mathrm{e}^{-r}\, \mathrm{E}[\min\{v_2, M\} \mid v_1] < \min\{s_1, \mathrm{e}^{-r} M\}$ for $s_1 > \underline{s}_d(v_1)$ so that the second term is negative:

$$\begin{aligned}
&\mathrm{E}\Big[I_s I_{\underline{s}_d(v_1)<s_1<\underline{s}_e(v_1)} \Big(\mathrm{e}^{-2r} \min\{v_2, M\} - \mathrm{e}^{-r} \min\{s_1, \mathrm{e}^{-r} M\}\Big)\Big] \\
&\quad = \mathrm{E}\Big[\mathrm{E}\Big[I_s I_{\underline{s}_d(v_1)<s_1<\underline{s}_e(v_1)} \Big(\mathrm{e}^{-2r} \min\{v_2, M\} - \mathrm{e}^{-r} \min\{s_1, \mathrm{e}^{-r} M\}\Big) \mid s_1, v_1\Big]\Big] \\
&\quad = \mathrm{E}\Big[I_s I_{\underline{s}_d(v_1)<s_1<\underline{s}_e(v_1)}\, \mathrm{e}^{-r} \Big(\mathrm{e}^{-r}\, \mathrm{E}\big[\min\{v_2, M\} \mid v_1\big] - \min\{s_1, \mathrm{e}^{-r} M\}\Big)\Big]
\end{aligned} \tag{A3}$$

Thus, any increase in the region of creditor control, which corresponds to a decrease in the region S, weakly increases the value of debt. ■

Proof of Proposition 2

We can write the continuation indicator I_c^s for the abandonment-value covenant as:

$$I_c^s = \underbrace{I_{\mathrm{e}^{\mu_v - r} v_1 \geq s_1}}_{\text{First-best}} + \underbrace{I_{s_1 > \bar{s}} I_{\mathrm{e}^{\mu_v - r} v_1 \leq s_1 \leq \underline{s}_e}}_{\text{Inefficient continuation}} - \underbrace{I_{s_1 < \bar{s}} I_{\mathrm{e}^{\mu_v - r} v_1 \geq s_1 > \underline{s}_d}}_{\text{Inefficient abandonment}} . \tag{A4}$$

Similarly, we can write the continuation indicator I_c^m for the minimum-value covenant as:

$$I_c^m = \underbrace{I_{e^{\mu_v - r} v_1 \geq s_1}}_{\text{First-best}} + \underbrace{I_{b_m \geq \bar{m}} I_{e^{\mu_v - r} v < s_1 \leq \underline{s}_e(v_1)}}_{\text{Inefficient continuation}} - \underbrace{I_{b_m < \bar{m}} I_{\underline{s}_d(v_1) < s_1 \leq e^{\mu_v - r} v_1}}_{\text{Inefficient abandonment}} . \tag{A5}$$

Writing the indicator $I_{b_m < \bar{s}} = 1 - I_{s_1 \geq \bar{s}} I_{e^{\mu_v - r} v_1 \geq \bar{s}} = I_{s_1 < \bar{s}} + I_{s_1 \geq \bar{s}} I_{e^{\mu_v - r} v_1 < \bar{s}}$, first compare the inefficient abandonment regions of the two contracts, given in (A4) and (A5):

$$I_{s_1 < \bar{s}} I_{\mathrm{e}^{\mu_v - r} v_1 \geq s_1 > \underline{s}_d} - (I_{s_1 < \bar{s}} + I_{s_1 \geq \bar{s}} I_{e^{\mu_v - r} v_1 < \bar{s}}) I_{\mathrm{e}^{\mu_v - r} v_1 \geq s_1 > \underline{s}_d} = -I_{s_1 \geq \bar{s}} I_{\underline{s}_d < s_1 \leq \mathrm{e}^{\mu_v - r} v_1 < \bar{s}} = 0 \tag{A6}$$

The last inequality follows from the fact that the sets defined by the two indicator functions in the second equality do not intersect.

Now compare the inefficient continuation regions in (A4) and (A5):

$$\begin{aligned} &I_{s_1 > \bar{s}} I_{\mathrm{e}^{\mu_v - r} v_1 \leq s_1 \leq \underline{s}_e} - (1 - I_{s_1 < \bar{s}} - I_{s_1 \geq \bar{s}} I_{e^{\mu_v - r} v_1 < \bar{s}}) I_{\mathrm{e}^{\mu_v - r} v_1 \leq s_1 \leq \underline{s}_e} \\ &= I_{s_1 \geq \bar{s}} I_{e^{\mu_v - r} v_1 < \bar{s}} I_{\mathrm{e}^{\mu_v - r} v_1 \leq s_1 \leq \underline{s}_e} = I_{\mathrm{e}^{\mu_v - r} v_1 < \bar{s} \leq s_1 \leq \underline{s}_e} \end{aligned} \tag{A7}$$

The final equality defines a nonempty region since $\bar{s}_e > \mathrm{e}^{\mu_v - r} v_1$. Thus, there is strictly greater excess continuation with a fair value s_1 than with a fair value b_m. This proves the first part of the proposition. The second part follows from this and Proposition 1 because the minimum-value covenant both increases firm value by reducing inefficient continuation and creates slack in the creditors' breakeven condition (3) by granting additional control rights. ■

Proof of Proposition 3

We can write the continuation indicator I_c^n for the maximum-value covenant as:

$$I_c^n = \underbrace{I_{e^{\mu_v - r} v_1 \geq s_1}}_{\text{First-best}} + \underbrace{I_{b_n \geq \bar{n}} I_{e^{\mu_v - r} v_1 < s_1 \leq \underline{s}_e(v_1)}}_{\text{Inefficient continuation}} - \underbrace{I_{b_n < \bar{n}} I_{\underline{s}_d(v_1) < s_1 \leq e^{\mu_v - r} v_1}}_{\text{Inefficient abandonment}}. \quad \text{(A8)}$$

Similarly, we can write the continuation indicator I_c^v for the continuation-value covenant as:

$$I_c^v = \underbrace{I_{e^{\mu_v - r} v_1 \geq s_1}}_{\text{First-best}} + \underbrace{I_{v_1 \geq \bar{v}} I_{e^{\mu_v - r} v_1 < s_1 \leq \underline{s}_e(v_1)}}_{\text{Inefficient continuation}} - \underbrace{I_{v_1 < \bar{v}} I_{\underline{s}_d(v_1) < s_1 \leq e^{\mu_v - r} v_1}}_{\text{Inefficient abandonment}}. \quad \text{(A9)}$$

We can write the covenant indicator $I_{b_n < \bar{n}}$ for the maximum-value covenant as $I_{b_n < \bar{n}} = I_{s_1 < \bar{n}} I_{\mathrm{e}^{\mu_v - r} v_1 < \bar{n}}$. We can write the indicator for the corresponding continuation value covenant as $I_{\mathrm{e}^{\mu_v - r} v_1 < \bar{n}} = I_{v_1 < \mathrm{e}^{r - \mu_v} \bar{n}}$ and compare (A8) to (A9) to give the incremental inefficient continuation and abandonment regions that result from reporting maximum-value versus continuation-value:

$$\begin{aligned} & I_{\mathrm{e}^{\mu_v - r} v_1 < \bar{n}} I_{s_1 \geq \bar{n}} I_{e^{\mu_v - r} v_1 < s_1 \leq \underline{s}_e(v_1)} + I_{e^{\mu_v - r} v_1 < \bar{n}} I_{s_1 \geq \bar{n}} I_{\underline{s}_d(v_1) < s_1 \leq e^{\mu_v - r} v_1} \\ & = I_{\mathrm{e}^{\mu_v - r} v_1 < \bar{n}} I_{s_1 \geq \bar{n}} I_{e^{\mu_v - r} v_1 < s_1 \leq \underline{s}_e(v_1)} + I_{s_1 \geq \bar{n}} I_{\underline{s}_d(v_1) < s_1 \leq e^{\mu_v - r} v_1 < \bar{n}} \\ & = I_{e^{\mu_v - r} v_1 < \bar{n} \leq s_1 \leq \underline{s}_e(v_1)} + 0 \end{aligned} \quad \text{(A10)}$$

The zero incremental inefficient abandonment region stated in the last equality follows from the fact that the sets identified by the indicator functions $I_{s_1 \geq \bar{n}}$ and $I_{\underline{s}_d(v_1) < s_1 \leq e^{\mu_v - r} v_1 < \bar{n}}$ do not intersect. The indicator function $I_{e^{\mu_v - r} v_1 < \bar{n} \leq s_1 \leq \underline{s}_e(v_1)}$ identifies a nonempty set so that the covenant based on the fair value $b_n = \max\{s_1, \mathrm{e}^{\mu_v - r} v_1\}$ yields a larger inefficient continuation region. This completes the proof of the first part of the Proposition. The second part follows from this and Proposition 1 because the continuation-value covenant both increases firm value by reducing inefficient continuation and creates slack in the creditors' breakeven condition (3) by granting additional control rights. ■

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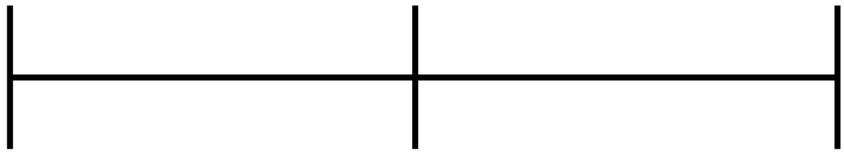


Time 0

- Accounting policy established
- Firm raises s_0 using zero-coupon debt
- Firm invests in project subsequent to issuing debt

Time 1

- Covenant threshold evaluated
- If violated, creditors control continuation
- If project is abandoned, value s_1 is realized; creditors receive $\min\{e^{-r}M, s_1\}$ and shareholders receive $\max\{0, s_1 - e^{-r}M\}$

Time 2

- If project was continued at time 1, value v_2 is realized; creditors receive $\min\{M, v_2\}$ and shareholders receive $\max\{0, v_2 - M\}$

Figure 1: Timeline

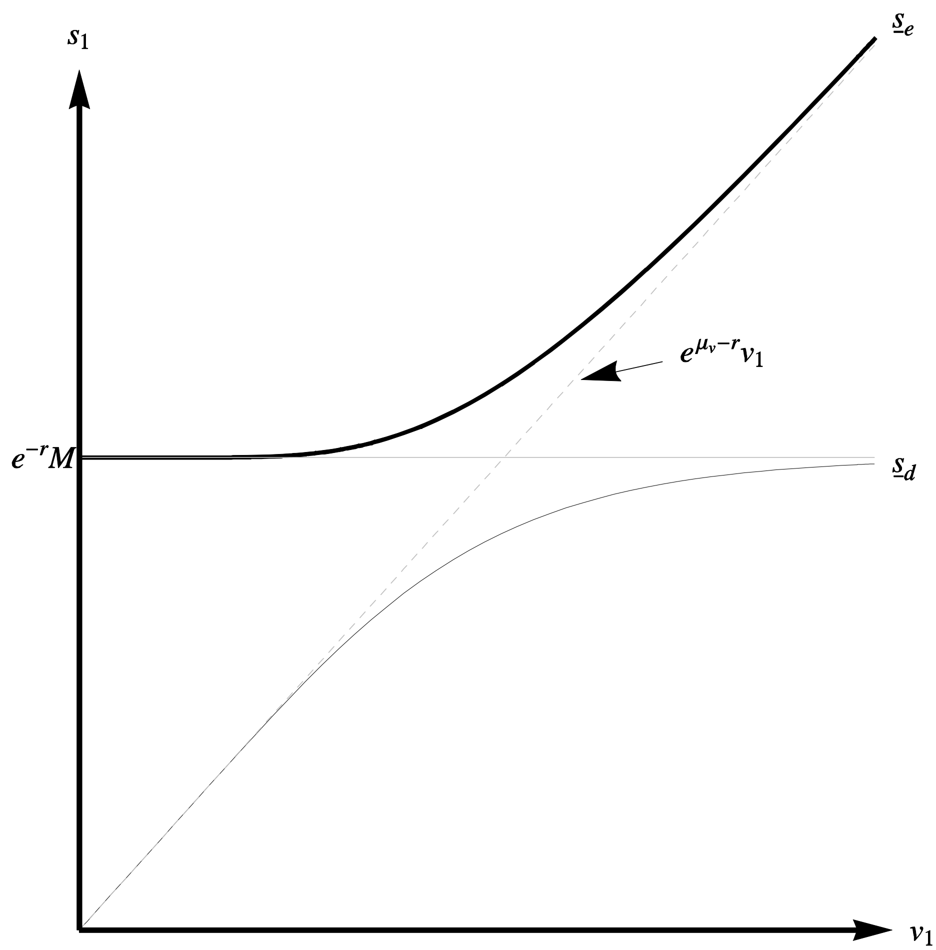
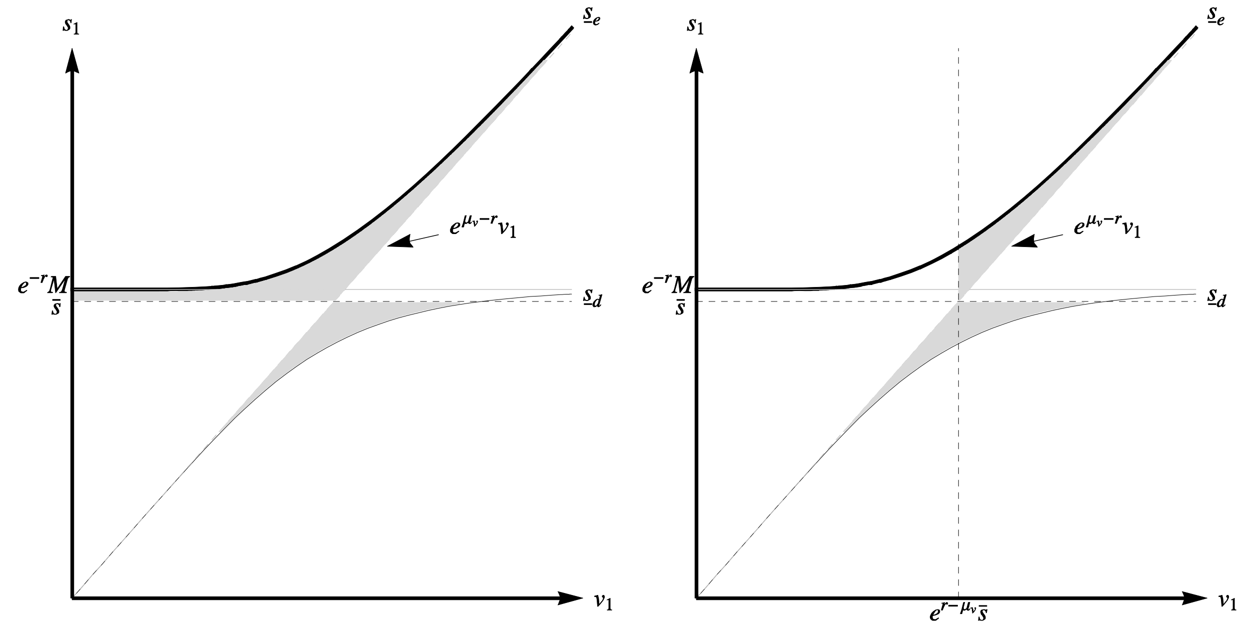


Figure 2: Continuation preferences

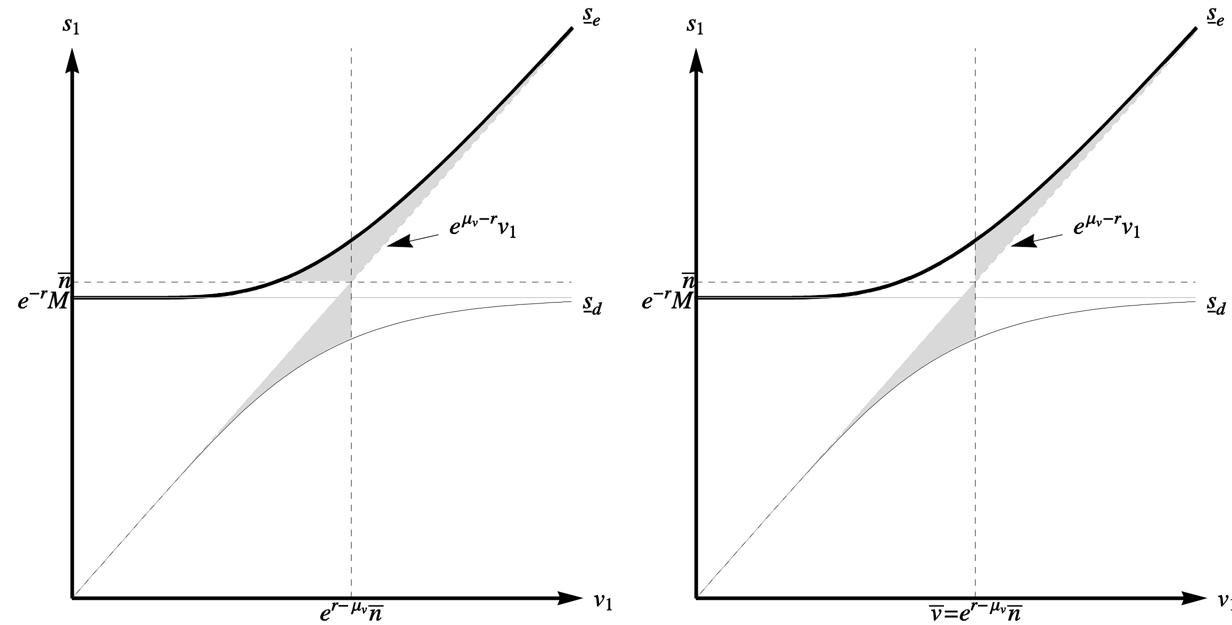
Figure 2 displays the continuation preferences of shareholders and creditors given the time 1 abandonment-value s_1 and continuation-value v_1. Shareholders prefer to continue for values of s_1 that lie below the thick curve $\underline{s}_e$. Creditors prefer continuation for values of s_1 that lie below the thin curve $\underline{s}_d$. The first-best policy continues for values of s_1 that lie below the gray dashed line $e^{\mu_v - r} v_1$. The solid gray line $e^{-r} M$ is the present value of the debt's maturity value at Time 1. This figure uses the project A parameters $r = 0.05$, $\mu_v = 0.1$, $\sigma_v = 0.4$ and the face value $M = 132.8$ that satisfies the creditor's breakeven condition and corresponds to an annual interest rate of about 15.2%.



(a) Abandonment-value covenant (b) Minimum-value covenant

Figure 3: Abandonment-value and minimum-value covenants

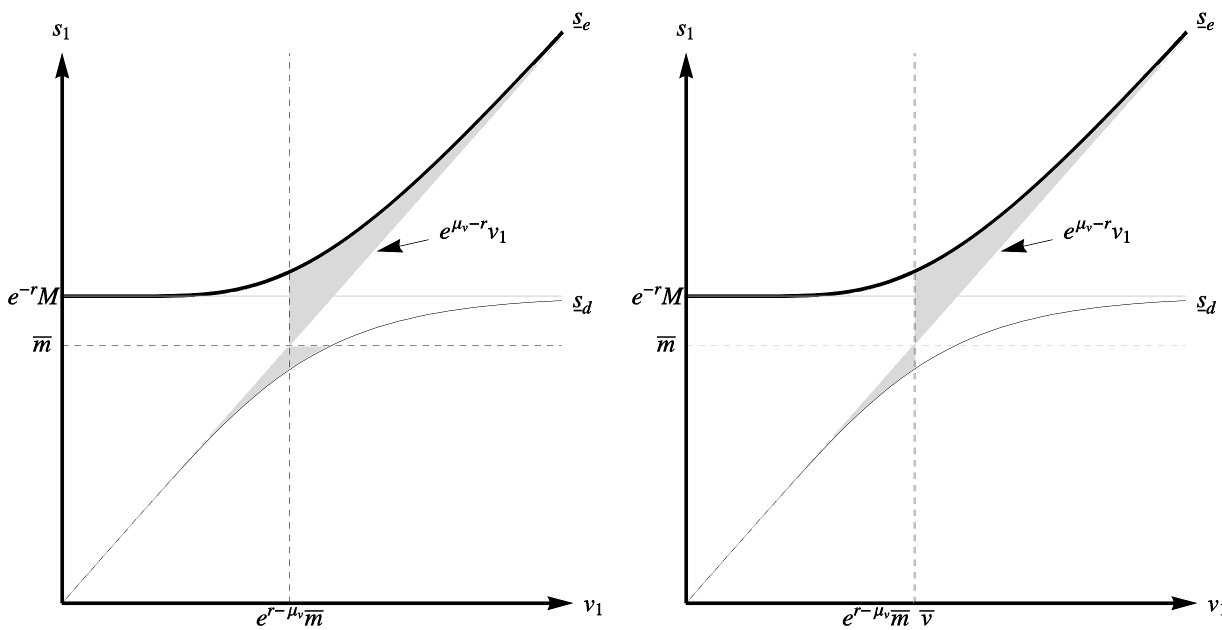
Figure 3 displays the continuation preferences of shareholders and creditors given the time 1 abandonment-value s_1 and continuation-value v_1. It also displays the resulting inefficiencies in continuation decisions. Shareholders prefer continuation for values of s_1 that lie below the thick curve $\underline{s}_e$. Creditors prefer continuation for values of s_1 that lie below the thin curve $\underline{s}_d$. The first-best policy continues for values of s_1 that lie below the gray dashed line $\mathrm{e}^{\mu_v - r} v_1$. The solid gray line $\mathrm{e}^{-r} M$ is the Time 1 present value of the debt's maturity value. The shaded region that lies above the line $\mathrm{e}^{\mu_v - r} v_1$ represents inefficient continuation and the shaded region that lies below the line $\mathrm{e}^{\mu_v - r} v_1$ represents inefficient abandonment. Panel (a) displays inefficiencies that result from a contract in which creditors have decision rights when $s_1 < \bar{s}$ where the black dashed line denotes $\bar{s}$. Panel (b) displays inefficiencies that result from a contract in which creditors have decision rights when $\min\{s_1, \mathrm{e}^{\mu_v - r} v_1\} < \bar{m}$, where the black dashed lines denote the covenant thresholds. The maturity value M and threshold $\bar{s}$ are based on the debt contract that maximizes firm value under the abandonment-value covenant. The minimum-value covenant cutoff $\bar{m} = \bar{s}$ for purposes of illustration and is not the optimal cutoff for the minimum-value covenant contract. This figure uses the Project A parameters $r = 0.05$, $\mu_v = 0.1$, $\sigma_v = 0.4$. The maturity value $M = 135.8$ and the covenant threshold $\bar{s} = 124.1$.



(a) Maximum-value covenant (b) Continuation-value covenant

Figure 4: Maximum-value and continuation-value covenants

Figure 4 displays the continuation preferences of shareholders and creditors given the time 1 abandonment-value s_1 and continuation-value v_1. It also displays the resulting inefficiencies in continuation decisions. Shareholders prefer continuation for values of s_1 that lie below the thick curve $\underline{s}_e$. Creditors prefer continuation for values of s_1 that lie below the thin curve $\underline{s}_d$. The first-best policy continues for values of s_1 that lie below the gray dashed line $\mathrm{e}^{\mu_v - r} v_1$. The solid gray line $\mathrm{e}^{-r} M$ is the Time 1 present value of the debt's maturity value. The shaded region that lies above the line $\mathrm{e}^{\mu_v - r} v_1$ represents inefficient continuation and the shaded region that lies below the line $\mathrm{e}^{\mu_v - r} v_1$ represents inefficient abandonment. Panel (a) displays inefficiencies that result from a contract in which creditors have decision rights when $\max\{s_1, \mathrm{e}^{\mu_v - r} v_1\} < \overline{n}$ where the black dashed lines denote the boundaries of the (v_1, s_1) values for which creditors control the continuation decision. Panel (b) displays inefficiencies that result from a contract in which creditors have decision rights when $v_1 < \overline{v}$, where the black dashed lines denote the covenant thresholds. The maturity value M and threshold $\overline{n}$ are based on the debt contract that maximizes firm value under the maximum-value covenant. The continuation-value covenant cutoff $\overline{v} = \mathrm{e}^{r - \mu_v} \overline{n}$ for purposes of illustration and is not the optimal cutoff for the continuation-value covenant contract. This figure uses the Project A parameters $r = 0.05$, $\mu_v = 0.1$, $\sigma_v = 0.4$. The maturity value $M = 132.10$ and the covenant threshold $\overline{n} = 132.2$.



(a) Minimum-value covenant (b) Continuation-value covenant

Figure 5: Minimum-value and continuation-value covenants

Figure 4 displays the continuation preferences of shareholders and creditors given the time 1 abandonment-value s_1 and continuation-value v_1. It also displays the resulting inefficiencies in continuation decisions. Shareholders prefer continuation for values of s_1 that lie below the thick curve $\underline{s}_e$. Creditors prefer continuation for values of s_1 that lie below the thin curve $\underline{s}_d$. The first-best policy continues for values of s_1 that lie below the gray dashed line $e^{\mu_v - r} v_1$. The solid gray line $e^{-r} M$ is the Time 1 present value of the debt's maturity value. The shaded region that lies above the line $e^{\mu_v - r} v_1$ represents inefficient continuation and the shaded region that lies below the line $e^{\mu_v - r} v_1$ represents inefficient abandonment. Panel (a) displays inefficiencies that result from a contract in which creditors have decision rights when $\min\{s_1, e^{\mu_v - r} v_1\} < \overline{m}$ where the horizontal black dashed line denotes $\overline{m}$ and the vertical black dashed line denotes $e^{r-\mu_v} \overline{m}$. Panel (b) displays inefficiencies that result from a contract in which creditors have decision rights when $v_1 < \overline{v}$, where the black dashed line denotes $\overline{v}$ and the horizontal dashed line denotes the thresholds for the min-covenant. The maturity value M and threshold $\overline{m}$ are based on the debt contract that maximizes firm value under the min-covenant. The min-covenant cutoff $\overline{v}$ solves the creditors' breakeven constraint (3) given the optimal face value for the min-covenant for purposes of illustration and does not represent the optimal combination of maturity value and covenant cutoff for the continuation-value contract. This figure uses the Project A parameters $r = 0.05$, $\mu_v = 0.1$, $\sigma_v = 0.4$. The maturity value $M = 135.0$ and the covenant thresholds $\overline{m} = 107.7$ and $\overline{v} = 103.2$.

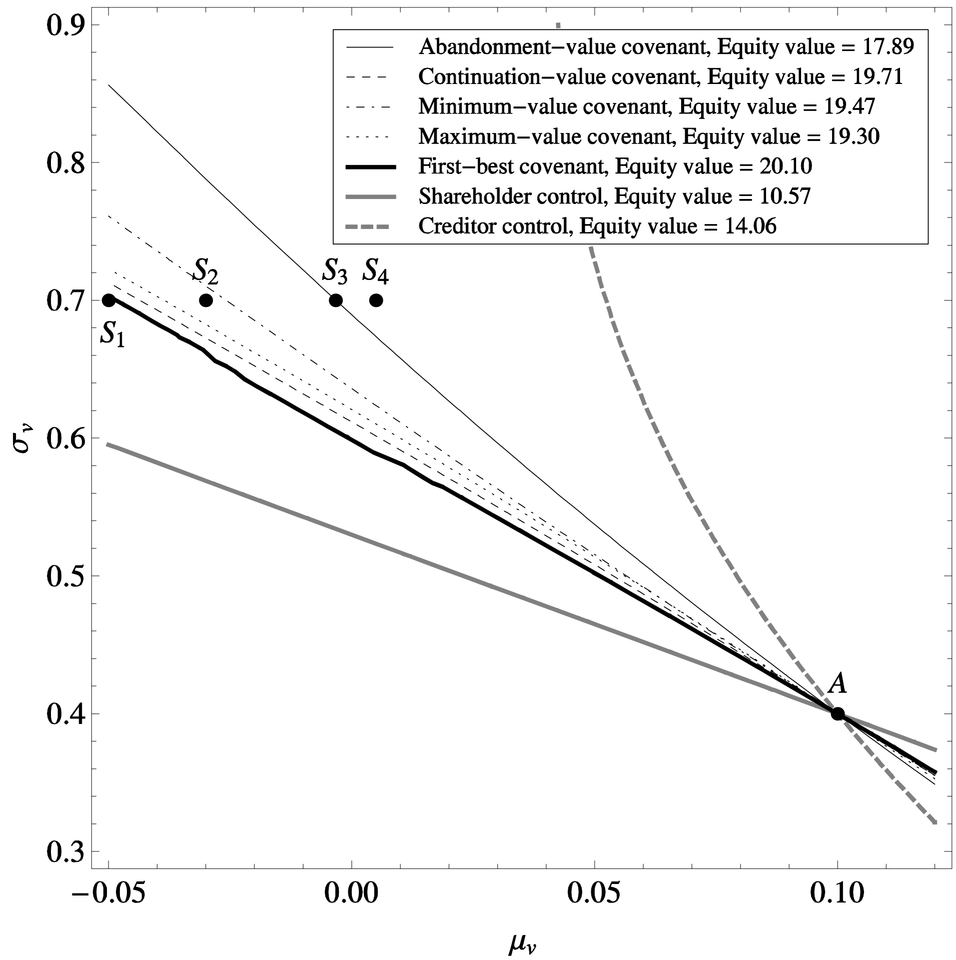
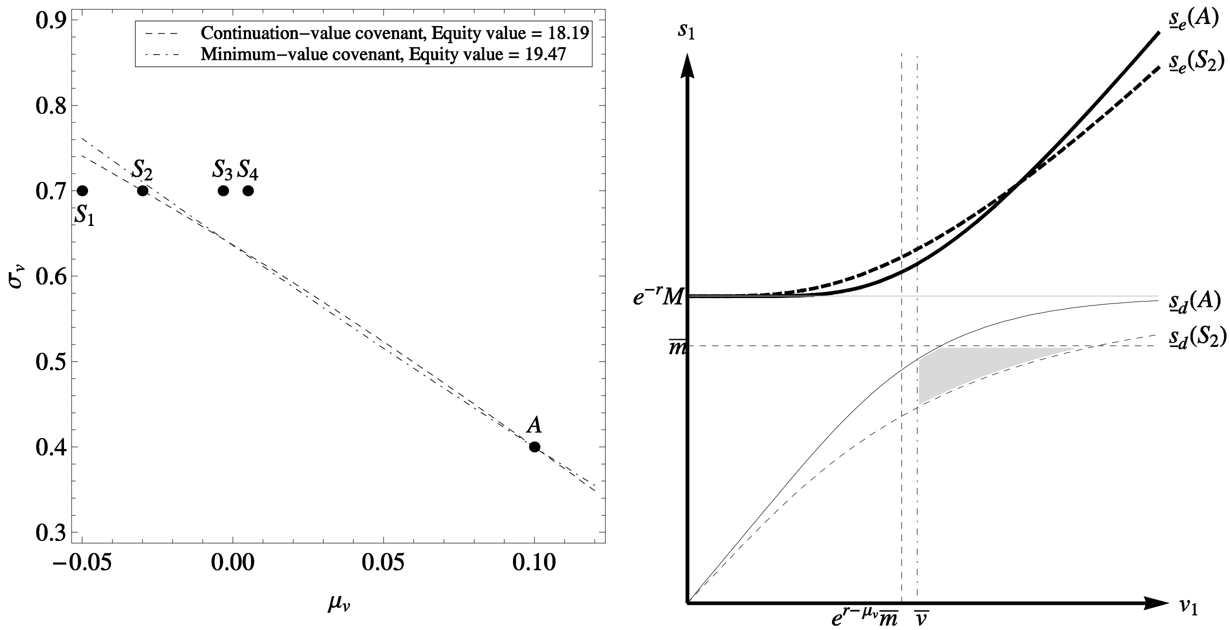


Figure 6: Shareholder indifference curves for asset substitution problem

Figure 6 illustrates shareholders' post-contracting indifference curves over the drift μ_v and volatility σ_v of the continuation-value for the different forms of debt covenants. The point A refers to the reference project and the points S_1 to S_4 refer to alternative projects. The debt contract in each case is priced assuming that the firm pursues project A and sets the debt covenant threshold optimally conditional on that choice. The figure legend lists the equity value associated with choosing project A under the different covenant types. The lines represent the (μ_v, σ_v) combinations that yield the same equity value. Values of (μ_v, σ_v) above and to the right of the indifference curves yield a higher expected value than project A. The thick gray lines represent the project indifference curves when the contract unconditional control to either the shareholders or the creditors. Shareholders never prefer (μ_v, σ_v) combinations that lie below both of these curves and they always prefer (μ_v, σ_v) combinations that lie above both of these curves.



(a) Shareholder indifference curves

(b) Time 1 continuation preferences

Figure 7: Contract adjustments to prevent selection of inferior project S_2

Figure 7a displays the shareholders' post-contracting indifference curves over the drift μ_v and volatility σ_v of the continuation-value for continuation- and minimum-value covenants. The minimum-value covenant threshold is set to its optimal level assuming that the shareholders pursue Project A and the continuation-value threshold is set the lowest level that makes shareholders indifferent between Projects A and S_2. Figure 7b displays shareholders' and creditors' continuation preferences given the minimum-value contract with covenant threshold and face value set to their optimal levels assuming that the shareholders pursue Project A. The solid (dashed) curves denote the preferences when the shareholders have pursued Project A (S_2). Shareholders prefer continuation for values of s_1 that lie below the thick $\underline{s}_e$ curves. Creditors prefer continuation for values of s_1 that lie below the thin $\underline{s}_d$ curves. The straight dashed lines $\overline{m}$ and $\mathrm{e}^{\mu_v - r}\,\overline{m}$ identify the minimum-value covenant thresholds optimized for Project A, where shareholders have control over the continuation decision for (v_1, s_1) values above and to the right of those lines. The vertical dot-dashed line $\overline{v}$ denotes the continuation-value covenant threshold optimized for Project A, where shareholders have control over the continuation decision for (v_1, s_1) values to the right of that line. The shared region highlights the incremental abandonment of Project S_2 under the minimum-value covenant versus the continuation-value covenant.

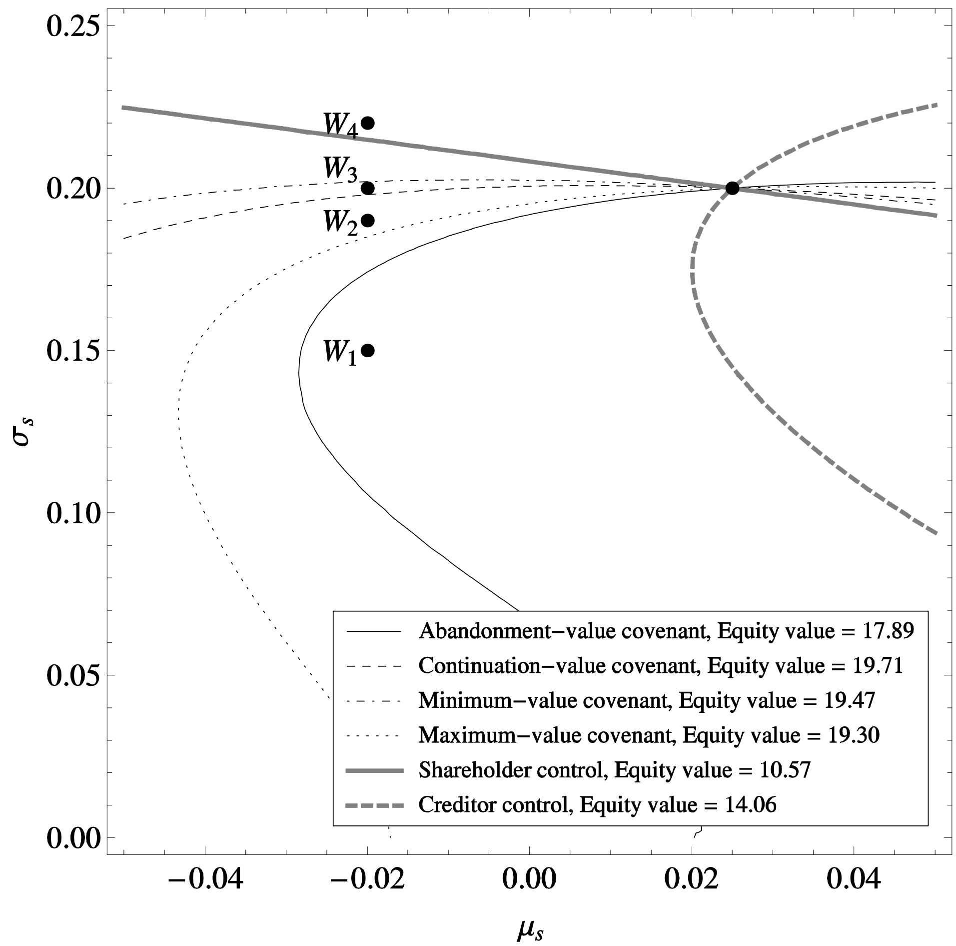


Figure 8: Shareholder indifference curves for wasting assets problem

Figure 8 illustrates the shareholders' post-contracting indifference curves over the drift μ_s and volatility σ_s of the abandonment-value for the different forms of debt covenants. The debt contract in each case is priced assuming that the firm pursues project A and sets the debt covenant threshold optimally conditional on that choice. The points list the (μ_s, σ_s) combinations representing project choices. The figure legend lists the equity value associated with choosing project A under the different covenant types. The lines represent the (μ_s, σ_s) combinations that yield the same equity value. Values of (μ_s, σ_s) above and to the left of the indifference curves yield a higher expected value than project A.

Table 1: Project parameters for asset-substitution problem

Table 1 displays the parameters of the project choices in the asset substitution set.

	Project				
	A	S_1	S_2	S_3	S_4
Continuation-value					
Initial value v_0	100.00	100.00	100.00	100.00	100.00
Drift μ_v	0.100	-0.050	-0.030	-0.003	0.005
Volatility σ_v	0.40	0.70	0.70	0.70	0.70
Abandonment-value					
Initial value s_0	100.00	100.00	100.00	100.00	100.00
Drift μ_s	0.025	0.025	0.025	0.025	0.025
Volatility σ_s	0.20	0.20	0.20	0.20	0.20
Correlation ρ_{vs}	0.40	0.40	0.40	0.40	0.40
Risk-free rate	0.05	0.05	0.05	0.05	0.05
Value with first-best implementation	120.01	112.40	115.02	117.65	118.53

Table 2: Covenant choices and equity values for asset-substitution problem

	Contract implements A vs. alternative S_i			
	S_1	S_2	S_3	S_4
Abandonment-value s_1 covenant				
Equity value	17.89	17.89	17.89	17.68
Debt face value	135.76	135.76	135.76	133.50
Covenant threshold	124.1	124.1	124.1	126.3
P(Creditor control)	0.85	0.85	0.85	0.87
P(Project continues)	0.51	0.51	0.51	0.48
Continuation-value v_1 covenant				
Equity value	19.71	18.19	14.64	14.36
Debt face value	133.58	129.62	129.00	129.00
Covenant threshold	109.8	156.1	283.2	319.6
P(Creditor control)	0.57	0.86	0.99	1.00
P(Project continues)	0.54	0.41	0.35	0.35
Minimum-value $\min\{s_1, e^{\mu v - r} v_1\}$ covenant				
Equity value	19.47	19.47	19.00	18.48
Debt face value	135.02	135.02	132.44	131.57
Covenant threshold	107.7	107.7	118.5	122.5
P(Creditor control)	0.76	0.76	0.87	0.90
P(Project continues)	0.56	0.56	0.49	0.46
Maximum-value $\max\{s_1, e^{\mu v - r} v_1\}$ covenant				
Equity value	19.30	18.19	14.64	14.36
Debt face value	132.10	129.62	129.00	129.00
Covenant threshold	132.2	164.2	297.7	336.1
P(Creditor control)	0.66	0.85	0.99	1.00
P(Project continues)	0.49	0.41	0.35	0.35

Table 3: Project parameters for wasting-assets problem

Table 3 displays the parameters of the project choices in the wasting assets set.

	Project				
	A	W_1	W_2	W_3	W_4
Continuation-value					
Initial value v_0	100.00	100.00	100.00	100.00	100.00
Drift μ_v	0.100	0.100	0.100	0.100	0.100
Volatility σ_v	0.40	0.40	0.40	0.40	0.40
Abandonment-value					
Initial value s_0	100.00	100.00	100.00	100.00	100.00
Drift μ_s	0.025	-0.020	-0.020	-0.020	-0.020
Volatility σ_s	0.20	0.15	0.19	0.20	0.22
Correlation ρ_{vs}	0.40	0.40	0.40	0.40	0.40
Risk-free rate	0.05	0.05	0.05	0.05	0.05
Value with first-best implementation	120.01	118.24	118.28	118.32	118.41

Table 4: Covenant choices and equity values for asset-substitution problem

	Contract implements A vs. alternative W_i			
	W_1	W_2	W_3	W_4
Abandonment-value s_1 covenant				
Equity value	17.89	17.68	16.92	N/A
Debt face value	135.76	139.98	145.45	N/A
Covenant threshold	124.1	119.9	124.1	N/A
P(Creditor control)	0.85	0.81	0.74	N/A
P(Project continues)	0.51	0.57	0.63	N/A
Continuation-value v_1 covenant				
Equity value	19.71	19.71	19.70	N/A
Debt face value	133.58	133.58	134.10	N/A
Covenant threshold	109.8	109.8	107.2	N/A
P(Creditor control)	0.57	0.57	0.55	N/A
P(Project continues)	0.54	0.54	0.55	N/A
Minimum-value $\min\{s_1, e^{\mu v - r} v_1\}$ covenant				
Equity value	19.47	19.47	19.47	N/A
Debt face value	135.02	135.02	135.02	N/A
Covenant threshold	107.7	107.7	107.7	N/A
P(Creditor control)	0.76	0.76	0.76	N/A
P(Project continues)	0.56	0.56	0.56	N/A
Maximum-value $\max\{s_1, e^{\mu v - r} v_1\}$ covenant				
Equity value	19.30	19.16	17.81	N/A
Debt face value	132.10	133.79	141.62	N/A
Covenant threshold	132.2	127.8	118.7	N/A
P(Creditor control)	0.66	0.62	0.52	N/A
P(Project continues)	0.49	0.52	0.60	N/A