Analyst Forecasts : The Roles of Reputational Ranking and Trading Commissions

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Abstract

This paper examines how reputational ranking and trading commission incentives influence the forecasting behavior of sell-side analysts. I develop a model in which two analysts simultaneously forecast a company's earnings. Each analyst attempts to maximize his compensation, which is a linear combination of his trading commissions and his expected reputational value in the labor market. My main predictions from this model are as follows: (i) Reputational concerns alone do not provide analysts with sufficient incentive for honest reporting. Indeed, a reputational payoff structure in which the reward for being the only good analyst is sufficiently larger than the penalty for being the only bad analyst makes the incentive problem even worse. (ii) Trading commissions alone also provide perverse incentive. While, with single analyst, there is no honest reporting, with multiple analysts, the information content of analyst forecasts go up because of the analysts' desire to coordinate. (iii) Trading commissions, together with reputational concerns, provide better incentive for honest reporting than reputational concerns or trading commissions alone.

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1 Introduction

Financial analysts play a crucial intermediary role between the companies traded in the capital market and investors. Analysts collect information about a company from multiple sources, analyze the information and make forecasts about various financial indicators of the company. Investors use these forecasts when making decisions to buy, sell or hold stocks of the company. Sell-side analysts, in particular, are employed by investment advisory firms and provide forecasts to institutional and retail investors. In finance and accounting empirical studies, earnings forecasts of sell-side analysts are typically used as proxies for investors' earnings expectations. An implicit assumption underlying such research is that an analyst seeks to minimize the mean squared forecast error, and thus, truthfully reveals his private information to the investors. While mean squared error is a useful statistical concept and its minimization is probably the appropriate objective function in a Robinson Crusoe economy, it is unclear that a strategic analyst adopts such an objective function. Indeed, the forecasting strategy of a strategic analyst is an open question that begs investigation.

Two of the most important metrics that determine a sell-side analyst's compensation are his commissions from trade generation, and his Institutional Investor ranking (Groysberg, Healy and Maber 2008),[1] an annual ranking, by major institutional investors, of an analyst's reputation relative to his peer group in the market. While the trading commissions component has always been part of an analyst's compensation, it has become even more important after the recent regulatory changes (e.g., Sarbanes-Oxley Act 2002 (Section 501), Global Settlement 2003) that prohibit directly linking an analyst's compensation to investment banking activities (Jacobson, Stephanescue and Yu 2010).

Given these two metrics, casual empiricism suggests that a sell-side analyst trades off two main incentives while deciding upon an earnings forecast: trading commissions and his relative

[1] The authors used eighteen years of compensation (proprietary) data for all senior and junior sell-side analysts of a large, top-tier investment bank. They found that analysts with top three or runner-up rating in the Institutional Investor (II) ranking received 61% (100%) higher total (bonus) compensation than an unranked peer. Furthermore, analysts who covered stocks at the third trading volume quartile earned 48% (64%) higher total (bonus) compensation than analysts at the first quartile. These findings are also consistent with the information the authors gathered while interviewing research directors of eleven leading investment banks, including the sample bank.

reputation for forecast accuracy. An analyst is paid a certain percentage of the trading commissions he generates for his brokerage-firm employer; the greater the price movements caused by an analyst's forecasts, the higher the trade generated for the brokerage firm. Therefore, an analyst has a strong incentive to move the price of the company's stock to the maximum extent possible, perhaps by misrepresenting his information. However, an analyst's desire to generate trade today is disciplined by his incentive to build a reputation for forecast accuracy. Reputation is important because highly reputed analysts have a greater impact on future price movements (Stickel 1992; Park and Stice 2000) and, therefore, generate more trade in the future (Jackson 2005) and higher brokerage commissions for their firms (Irvine 2004). More reputed analysts also receive better compensation and have greater job mobility than their less reputed counterparts (Mikhail, Walther and Willis 1999; Leone and Wu 2002; Hong and Kubik 2003). Consequently, an analyst faces the classic short- versus long-term tradeoff - providing accurate forecasts that enhances his reputational ranking for future benefits versus potentially misleading investors with forecasts that generate high current-period trading commissions.

In this paper, I develop a simple analytical model to answer two questions: first, how this tradeoff affects analysts' forecasting behavior; second, to what extent analysts' private information gets impounded into the security prices.

Preview of the Model. I consider a single-period model, which adequately proxies for multi-period effects, with three players: two analysts and one investor who represents the capital market. At $t = 0$, each analyst receives a private signal about the earnings of a company that both are covering. The company's earnings are assumed to be either high or low. Analysts' signals and forecasts are also represented using binary values. An analyst can have either good or bad talent. A good analyst receives a more precise signal than a bad one. Neither the analysts nor the market knows the talent of each analyst; they only know the prior distribution of an analyst's talent. The signals of good analysts are perfectly correlated, conditional on the state (earnings). If either or both of the analysts are bad, their signals are conditionally independent.

At $t = 1$, each analyst makes a forecast about the earnings of the company, and the market prices the company's stock based on the analyst forecasts. At $t = 2$, the company's earnings are publicly reported. The market now compares the forecasts and the actual realization of earnings and updates the reputation of each analyst. An analyst's reputation is defined as the market's belief about his talent. The objective of each analyst is to maximize his compensation, which is a linear combination of the trading commissions he generates for his brokerage-firm employer and his expected reputational value in the labor market. The reputational value component - derived from each analyst's reputational ranking payoff - captures the idea that part of an analyst's compensation depends on not only his own reputation but also his reputation relative to his peer group in the market.

Preview of the Results. In the results section, I begin by characterizing the equilibrium features of my model with only one analyst. This analysis helps highlight the differences in the equilibrium behavior when there is a second analyst, which introduces the strategic interaction (with the first analyst) component in the model. I start with two benchmark cases in which an analyst is maximizing either his trading commissions or his expected reputational value in the labor market. Next, I characterize the equilibrium forecasting behavior of an analyst when he is concerned about both his trading commissions and his reputational value in the market.

When an analyst's sole objective is to maximize his trading commissions, his earnings forecast can only partially reveal his private signal. The intuition of this reporting behavior is that the analyst, with his only objective being to maximize his trading commissions, will tend to report in such a way that moves the price to the maximum. In order to move the price, an analyst will tend to report against the prior expectations of the market; I call this the analyst's "against-the-prior" incentive. Thus, when the analyst's signal opposes the prior, his forecast will be consistent with both his private signal and his against-the-prior incentive. However, when the analyst's signal matches the prior, following his against-the-prior incentive contradicts his private signal, leading to the loss of information in equilibrium.

When an analyst's sole objective is to maximize his expected reputational value in the market, he can credibly communicate his private information only if the prior of earnings is at the intermediate range (not very high or very low). At extreme priors, an analyst with an unlikely signal (one that contradicts the prior) will infer that he is probably a bad type, and, therefore, expects that the communication of his private signal will lead to a downward revision of his reputation in the market. Recall that neither the analyst nor the market knows the analyst's talent. Thus, to appear to be a good type, the analyst's forecast will always match the prior, regardless of his signal. The reputational concerns of an analyst thus creates a "conformist" bias in an analyst's forecasts, which leads to no information transmission at extreme priors.

The focus of this paper, then, is to explore how trading commissions and reputational ranking incentives, together, influence the forecasting behavior of an analyst in the presence of another analyst, as well as allowing for the possibility that the private signals of the analysts may be conditionally correlated on state. The main results are as follows :

(i) *Endogenous discipline by multiple analysts.* With only a trading commission incentive, while there is no fully revealing equilibrium with only one analyst, there is a fully revealing equilibrium under a certain range of parameters with more than one analyst.

(ii) *Asymmetry in reputational ranking payoffs.* If the reputational reward for being the only good analyst is sufficiently higher than the reputational penalty for being the only bad analyst, then there is less information revelation by analysts in equilibrium.

(iii) *Positive role of trading commissions.* Trading commissions, together with reputational concerns, provide better incentive for honest reporting than reputational concerns or trading commissions alone.

The first result highlights the role of the second analyst in information revelation in equilibrium when each analyst is concerned with maximizing his trading commissions. In contrast to the benchmark case with only one analyst, in my model, each analyst can coordinate with the other analyst to move the price in equilibrium, generating positive trading commissions. Each analyst's trading commissions now depend on his private signal, and, thus, he can credibly communicate his private information to the market. The intuition is that each analyst faces a tradeoff: on one hand, there is the against-the-prior incentive, as was the case with a single analyst; on the other hand, there is now an additional incentive to "coordinate" with the other analyst to issue the same forecast, because dissimilar forecasts will not move the price. Moreover, the coordination incentive increases with the conditional correlation between the analysts' signals. Thus, the analysts' incentives to coordinate induce them to condition their forecasts on their private signals, leading to information revelation in equilibrium.

The intuition of the second result is the following: if the objective of each analyst is to maximize his expected reputational value in the labor market, each will tend to maximize the likelihood of being the only good analyst and minimize the likelihood of being the only bad analyst. In order to be perceived as the only good analyst, each analyst will want to differentiate himself from the other. However, note that the analyst knows neither the private signal of the other analyst nor his or the other analyst's talents. The only information each analyst has is his own signal and the possibility that his and the other analyst's signals are conditionally correlated. Given this information, and assuming that the other analyst will report his own signal (in a symmetric Nash equilibrium), the best response for each analyst will be to use mixed strategies, randomizing his reports and thus differentiating himself. However, randomization will lead to less information transmission in equilibrium compared to full revelation. In contrast, to avoid being perceived as the only bad analyst, each analyst will want to increase the likelihood of moving in conjunction with - rather than differentiating himself from - the other analyst. Again assuming that the other analyst will report his own signal, the best response for each analyst will be to report his own signal as well.

On balance, if the reputational reward for being the only good analyst is sufficiently higher than the reputational penalty for being the only bad analyst, then each analyst will have a greater incentive to differentiate himself from the other analyst by randomizing his reports, which leads to less information transmission in equilibrium. For example, on Wall Street, "All-Star" analysts are paid substantially higher than their average counterparts, yet analysts are not penalized as much if they rank lower. My result suggests that such asymmetry in a reputational payoff structure can influence analysts to reveal less information in equilibrium.

The third result of this paper is the potentially positive impact of the trading commission incentive in the sense that trading commissions, along with reputational concerns, provide better incentive to analysts than reputational concerns or trading commissions alone. This result is robust even in the case of a single analyst.

To develop the intuition of this result, first consider a setup that addresses only reputational concerns. Assume that each analyst receives signal that contradicts the prior of earnings, when the prior is sufficiently precise. Similar to the case with a single analyst, each analyst will tend to follow the prior to appear to be a good type, leading to no information transmission in equilibrium. Now, imagine that the analysts also care about their commissions from trade generation. The trading commission incentive will influence each analyst to report against the prior to increase price movement. Thus, with the additional incentive of trading commissions, the analysts will be able to credibly communicate their signals, which was not possible with the reputational incentive alone. This combination of incentives leads to more information revelation in equilibrium.

Contribution. My paper contributes to the literature on the relationship between analyst forecasts and expert's reputation in primarily two ways. First, to the best of my knowledge, it is the first paper that develops a model of analysts' forecasting behavior using a simple tradeoff: maximizing current-period trading commissions versus generating future relative reputational payoffs. This tradeoff is important because anecdotal evidence and empirical studies show that these two motives are the main components of an analyst's compensation (Groysberg, Healy and Maber 2008). In addition, adding a current-period profit motive to reputational concerns affects the features of equilibrium. Second, the consideration of multiple analysts introduces an element of strategic interaction. On one hand, each analyst competes against the other to receive a higher reputational ranking; on the other hand, the analysts (implicitly) coordinate forecasts so that each receives maximum trading commissions.

To elaborate further on the first contribution, as mentioned above, there have been several papers, such as Ottaviani and Sorensen (2001, 2006a, 2006b) and Trueman (1994), that develop models in which an expert is maximizing his reputational value; however, none of them address trading commissions or any other profit motives. The inclusion of a current-period trading commission motive changes some features of equilibrium, including the increase in informativeness of equilibrium.

There are also a few papers that model an expert's short- and long-term tradeoffs, as discussed in the literature review above. In contrast to Prendergast and Stole's (1996) paper, which relies on the difference between actual and expected investment to make inferences about the manager's ability, in my model, the market makes inferences about an expert's talent by updating its belief based on the expert's forecast and the actual realization of the state variable for which the forecast has been made. Similarly, Dasgupta and Prat (2008) focus on how career concerns reduce information revelation, while in my paper, the focus is on how the addition of the trading commission motive improves information revelation. Also, in their paper, when the traders are maximizing only trading profits, there is always a fully revealing equilibrium, which is not true in my model.

My paper is closest in setup to Jackson's (2005), although there are two crucial differences. First, the equilibrium in my model is a function of the prior of the state variable (company's earnings), which is assumed to be half in Jackson's model. It can be easily shown in my model that if the prior is half, there is always a fully revealing equilibrium, as in Jackson's model. However, in my model, the equilibrium forecasting behavior of the analysts is interesting when the prior is not half. Second, one major focus of Jackson's paper is to show that on average an analyst's forecast is optimistic in equilibrium, a result that depends primarily on the author's assumption that investors face short-sales constraints. In contrast, the focus of my paper is to show how adding a trading commission incentive alleviates - but does not fully mitigate - the conformist bias due to analysts' reputational concerns. In my model, there are no short-sales constraints.

Finally, to elaborate on the second contribution, by considering the strategic interaction between analysts, my paper contributes to the expert's reputation literature by integrating relative reputational payoff considerations into a model in which experts take simultaneous actions. The main difference between my paper and Effinger and Polborn's is that in my model, the analysts move simultaneously, unaware of each other's actions. Moreover, some of the assumptions in Effinger and Polborn can be very restrictive in the context of analysts' earnings forecasts, the focus of my model. Effinger and Polborn (2001) imply that if the reputational reward for being the only smart agent is sufficiently large, then there is anti-herding: the second mover always reports against the first mover's action regardless of his own signal. In contrast, if the reward is not that high, the second mover may herd with the first mover by reporting in the direction of the first mover's action, ignoring his own signal. In sequential moves, the consideration of relative reputation typically leads to information loss (anti-herding or herding). In my model, while a payoff structure in which reputational reward is sufficiently higher than the reputational penalty leads to information loss, a payoff structure with sufficiently high reputational penalty improves information revelation in equilibrium.

1.1 Related Literature

This paper brings together two important strands of literature: sell-side analysts' forecasting behavior and expert's reputation. The first strand focuses on the forecasting strategies of a sell-side analyst under different incentives. For example, Beyer and Guttman (2008) consider a case in which a sell-side analyst's payoff depends on his commission from trade generation as well as his loss from forecast errors. They find a fully separating equilibrium in which the analyst biases his forecast upward (downward) if his private signal reveals relatively good (bad) news. Morgan and Stocken (2003) consider a financial market setting in which the investors are uncertain about the incentives of the security analyst, who makes stock recommendations that the investors use for their investing decisions. The analyst is not concerned about generating trade for his brokerage firm. The authors show that the investors' uncertainty about the analyst's incentives makes full information revelation impossible. There are two classes of equilibria: "partition equilibria", a la Crawford and Sobel (1981), and "semiresponsive" equilibria, in which analysts with aligned incentives can effectively communicate only unfavorable information about a company.

The second strand focuses on how reputational concerns influence an expert's professional advice to a decision maker. The expert has an informative signal about the state of the nature. he takes an action, possibly by providing advice or making a forecast about the state, which will be used by an uninformed decision maker. The expert is only concerned with having a reputation of being well-informed. Ottaviani and Sorensen (2001) show that when an expert does not know his talent and is maximizing his expected reputational payoff in the market, then he can credibly communicate his private information to the decision maker only if the prior of the state is in the intermediate range. At an extreme prior, no information is communicated in equilibrium. Trueman (1994) shows that when an analyst knows his talent, a good type always reveals his private signal to the market, while the bad type can do so only in the intermediate range of prior. At an extreme prior, the bad type can credibly communicate only part of his information. None of the papers consider trading commissions or any other profit motives of an expert in addition to reputational concerns.

There are at least three papers that do consider profit motives. Prendergast and Stole (1996), Dasgupta and Prat (2008) and Jackson (2005) model an expert's short- and long-term tradeoff: current-period profit motives versus future reputation benefits. In Prendergast and Stole (1996), a manager's objective is to maximize both current profits from his investment decisions and end-of-period market perception of his ability. However, in their model, the market never sees the "realized" effects of his decisions. Inferences about the manager's ability - his reputation - are drawn from the difference between actual and expected investment. Dasgupta and Prat (2008) consider a multi-period setting in which traders care about their trading profits as well as their future reputation. However, in their model, they focus on showing that the career concerns of traders reduce information revelation in equilibrium. Furthermore, when the traders are maximizing only trading profits, there is always a fully revealing equilibrium.

Jackson (2005) considers a single-period model in which a sell-side analyst is maximizing a linear combination of trading commissions and his reputation in the market while making an earnings forecast. He shows, both theoretically and empirically, that on average, an analyst's forecast is optimistic in equilibrium. Optimistic analysts generate more trade, and highly reputed analysts generate higher future trading volume. That the forecast is optimistic in equilibrium results from the combination of two scenarios: if the analyst's reputational concerns are sufficiently high (a "good" analyst), then he can always credibly communicate his private signal to the market; however, if his concerns for future reputation are not that high ("evil" analyst), then he always issues a high forecast in equilibrium, regardless of his signal. It's worth noting that the full-revelation result in the first case crucially depends on the author's assumption that the prior of the state variable (earnings) is half. Furthermore, the result that the analyst's optimism occurs in equilibrium depends primarily on Jackson's assumption that investors face short-sales constraints.

Finally, there are two papers that discuss the role of relative reputation and relative performance in an expert's behavior when experts move sequentially. Effinger and Polborn (2001) consider a model in which two experts, moving one after the other, are making business decisions about their respective firms. Unlike the reputational herding literature (e.g., Scharfstein and Stein 1990; Graham 1998), they assume that an expert's payoff will depend not only on his reputation, but also on his relative reputation vis-a-vis the other expert. They find that if the value of being the only smart expert is sufficiently large, the second mover always opposes his predecessor's move, regardless of his own signal ("anti-herding"); otherwise, herding may occur. Bernhardt, Campello and Kutsoati (2004) consider a case in which analysts make sequential forecasts, and their compensations are based both on their absolute forecast accuracy and their accuracy relative to other analysts following the same firm. The authors show that if the relative performance compensation is a convex function, then the last analyst strategically biases his forecast in the direction of his private information. However, a concave function induces the last analyst to bias his report toward the consensus.

The paper is organized as follows. Section 2 sets up the model. Section 3 discusses equilibrium features of the model with a single analyst. Section 4 defines and characterizes the most informative equilibrium of the model with two analysts. Section 5 discusses possible empirical implications of the predictions of my model. Section 6 draws conclusions from my analysis. Section 7 and 8 are the appendices, which provide derivations of some expressions and all the proofs.

2 The Model

There are three players: two analysts, $i \in \{1, 2\}$, and one investor, who represents the market. There are three dates. At $t = 0$, each analyst receives a private signal $s_i \in S_i \equiv \{h, l\}$ of the earnings $x \in X \equiv \{H, L\}$ of a company. $H(h)$ and $L(l)$ can be interpreted as high and low respectively, where $L(l) < H(h)$. Upon observing his signal, at $t = 1$, each analyst makes a forecast (or message), $m_i \in M_i \equiv \{h, l\}$. The market observes the forecasts (m_i, m_{-i}), and prices the company's stock as $P(m_i, m_{-i})$. An analyst's talent is $\theta \in \Theta \equiv \{g, b\}$, good or bad. We can think of the analyst's talent as his type. A good analyst receives a more precise private signal about the earnings of the company than a bad analyst. Neither the market nor the analysts know θ (this captures the idea that the analysts don't know their talents sufficiently more than the market); they only know the prior distribution, $\Pr(\theta = g) \equiv \lambda \in (0, 1)$. Also, everyone knows the prior distribution of earnings, $\Pr(x = H) \equiv q \in (0, 1)$. Finally, at $t = 2$, when earnings x is reported by the company, the market compares the realized x with the analyst forecasts (m_i, m_{-i}), and updates its belief about each analyst's talent, which I define as the analyst's reputation, $\Pr(\theta_i = g|m_i, m_{-i}, x)$. The time line of the game is shown in Figure 1.

Information structure. The precision of each analyst's private signal is

$$\gamma_\theta = \Pr(s = x|x, \theta) \tag{1}$$

I assume $1 > \gamma_g > \gamma_b = \frac{1}{2}$. The probability that a good analyst receives a matched signal (i.e.,

$s = x$), conditional on earnings and his talent, is γ_g, which is higher than that (i.e., γ_b) of a bad analyst.The assumption of $\gamma_b = \frac{1}{2}$ implies that the bad analyst receives a completely noisy signal of the earnings. Since analysts don't know their talents, they only know the unconditional probability, which is defined as $\gamma \equiv \Pr(s = x|x) = \lambda\gamma_g + (1-\lambda)\frac{1}{2} > \frac{1}{2}$. We can also interpret γ as the average signal quality (precision) of an analyst.

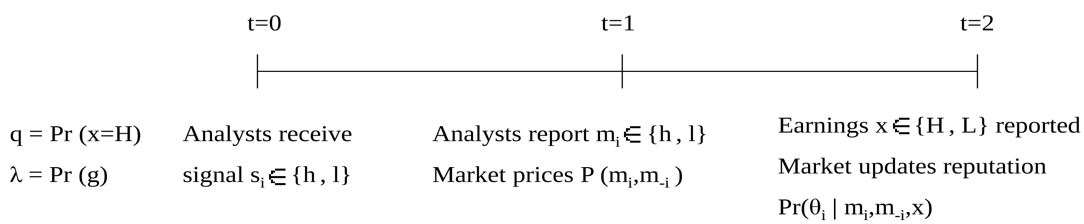

Figure 1 : Sequence of Events

Correlation between signals. Private signals of good analysts are perfectly correlated conditional on state x and talent θ. However, if one of the analysts is bad, then their signals are conditionally independent.[2] Thus, the probability that two good analysts observe matched signals is $\Pr(s_i = x, s_{-i} = x \mid x, g_i, g_{-i}) = \gamma_g$, and not γ_g^2. This assumption[3] is similar to that of

[2] The qualitative aspects of my results will not change as long as the correlation between the good analysts is more than that between the bad analysts.

[3] Instead of perfect correlation, the level of correlation could have been more general, $\rho \in (0, 1]$ as in Graham (1999). In that case, the probability that two good analysts observe ex-post correct signals is $\rho\gamma_g + (1-\rho)\gamma_g^2$, a convex combination of two extreme cases when analysts receive perfectly correlated signals (γ_g) and conditionally independent signals (γ_g^2). Similarly, the likelihood that two good analysts receive different signals is $(1-\rho)\gamma_g(1-\gamma_g)$. It turns out that assuming a more general correlation level doesn't change the qualitative aspects of my results.

Scharfstein and Stein (1990).

Reputational payoff. Each analyst's reputational payoff in the market depends not only on his own reputation but also on the reputation of the other analyst. This assumption can be interpreted as an abstraction of the practice by institutional investors of annually ranking analysts' reputations, and the analysts' compensations being linked to their reputational rankings. As illustrated in Table 1,

Table 1: Reputational Ranking Payoffs

	good	bad
good	y, y	$\bar{z}, \underline{z}$
bad	$\underline{z}, \bar{z}$	$0, 0$

$$\overline{z} = u_i(g_i, b_{-i}) > y = u_i(g_i, g_{-i}) > 0 = u_i(b_i, b_{-i}) > \underline{z} = u_i(b_i, g_{-i}) \tag{2}$$

I call $u_i(\theta_i, \theta_{-i})$ the reputation ranking payoff of analyst i, when his and the other analyst's types are θ_i and θ_{-i}, respectively. Note that the reputational ranking payoff of an analyst is higher when he is good and the other is bad, as opposed to when both of them are good. Similarly, the reputational payoff of an analyst is lower when he is bad and the other is good, as opposed to when both of them are bad. Furthermore, the analyst's reputational value – given the forecasts and the realized state – is defined as

$$U_i(m_i, m_{-i}, x) = \sum_{\theta_i} \sum_{\theta_{-i}} u_i(\theta_i, \theta_{-i}) \Pr(\theta_i, \theta_{-i} \mid m_i, m_{-i}, x) \tag{3}$$

For example, analyst i's reputational value in the market, given his forecast $m_i = h$, the other analyst's forecast $m_{-i} = l$ and the actual realization of earnings $x = H$, is $U_i(m_i^h, m_{-i}^l, x^H) = y \Pr(g_i, g_{-i} \mid m_i^h, m_{-i}^l, x^H) + \overline{z} \Pr(g_i, b_{-i} \mid m_i^h, m_{-i}^l, x^H) + \underline{z} \Pr(b_i, g_{-i} \mid m_i^h, m_{-i}^l, x^H)$.

Objective function. Each analyst is employed by a brokerage firm that compensates him for the trading commissions he generates in a given period and for his reputational value in the labor market. The analyst's reputation is important to a brokerage firm because an analyst with a higher reputational value generates more trading volume in the future, given that the capital market uses the analyst's reputation as a prior for his talent in the subsequent period and that a highly reputed analyst generates greater price movements. Accordingly, each analyst's objective is to maximize a linear combination of the trading commissions he generates for the brokerage firm and his expected reputational value in the market. Specifically, analyst i's objective function is

$$V_i(m_i \mid s_i) \equiv \alpha \pi_i(m_i \mid s_i) + (1-\alpha) R_i(m_i \mid s_i),\ \alpha \in [0,1] \tag{4}$$

where $\pi_i(m_i \mid s_i)$ and $R_i(m_i \mid s_i)$ are the trading commissions and the expected reputational value components of the analyst's compensation, respectively. The parameter α denotes the relative weight the brokerage firm places on the trading commissions versus the analyst's reputational value when setting the analyst's compensation. σ_{-i} is the other analyst's (i.e., -i) strategy. An analyst's strategy is defined as $\sigma_i : S_i \to \triangle(M_i)$, a mapping from the analyst's signal space, S_i, to a probability distribution over his message space, M_i. Note that the possible strategies include mixed strategy options. Trading commissions are defined as

$$\pi_i(m_i \mid s_i) \equiv \mathbb{E}_{m_{-i}}[|P(m_i, m_{-i}) - P_0| \mid s_i, \sigma_{-i}] \tag{5}$$

where $P_0 = E[x]$, the price at $t = 0$. The more the price moves subsequent to the analyst forecasts, regardless of the direction of the movement, the greater the trading volume and the higher the trading commissions for the analysts. Expected reputational value of each analyst is defined as

$$R_i(m_i \mid s_i) \equiv \mathbb{E}_{x, m_{-i}}[U_i(m_i, m_{-i}, x) \mid s_i, \sigma_{-i}] \tag{6}$$

where $U_i(m_i, m_{-i}, x)$ has been defined earlier in (3).

3 Single Analyst Benchmarks

In this section, I discuss the equilibrium features of the model with only *one* analyst. This analysis will help underscore the differences in equilibrium behavior when there is a second analyst, which introduces an element of strategic interaction (with the first analyst) in the model. I start with two benchmark cases in which an analyst is concerned with maximizing *either* his trading commissions *or* his reputation in the labor market. Finally, I characterize the analyst's equilibrium forecasting behavior when he is maximizing *both* his trading commissions *and* his reputational value in the market.

Benchmark 1 (Trading commission motive): When an analyst's objective is to maximize his trading commissions, he solves

$$\max_{\tilde{m}} |P(\tilde{m}) - P_0| \tag{7}$$

his best strategy will then be to move the price $P(m)$ from the price at $t = 0$ as much as possible to generate maximum trade. Suppose his strategy is defined as $\sigma_h \equiv \Pr(m^h \mid s^h)$ and $\sigma_l \equiv \Pr(m^h \mid s^l)$. The prices subsequent to high and low reports can be expressed as

$$\begin{aligned} P(m^h) &= \frac{x^H[\sigma_h\gamma + \sigma_l(1-\gamma)]q + x^L[\sigma_h(1-\gamma) + \sigma_l\gamma](1-q)}{\sigma_h[\gamma q + (1-\gamma)(1-q)] + \sigma_l[(1-\gamma)q + \gamma(1-q)]} \\ P(m^l) &= \frac{x^H[(1-\sigma_h)\gamma + (1-\sigma_l)(1-\gamma)]q + x^L[(1-\sigma_h)(1-\gamma) + (1-\sigma_l)\gamma](1-q)}{(1-\sigma_h)[\gamma q + (1-\gamma)(1-q)] + (1-\sigma_l)[(1-\gamma)q + \gamma(1-q)]} \end{aligned} \tag{8}$$

The details of these calculations are shown in Appendix A. Also, $P_0 = E[x] = x^H q + x^L(1-q)$. Furthermore, the price subsequent to the high report is higher than that subsequent to the low report, as we can expect:[4]

$$P(m^h) \geqslant P_0 \geqslant P(m^l) \tag{9}$$

The analyst's trading commissions from high and low forecasts are, respectively,

$$\begin{aligned} \pi(m^h) = |P(m^h) - P_0| &= \left|\frac{(x^H - x^L)q(1-q)(2\gamma-1)(\sigma_h - \sigma_l)}{\sigma_h[\gamma q + (1-\gamma)(1-q)] + \sigma_l[(1-\gamma)q + \gamma(1-q)]}\right| \\ \pi(m^l) = |P(m^l) - P_0| &= \left|\frac{(x^H - x^L)q(1-q)(2\gamma-1)(\sigma_l - \sigma_h)}{(1-\sigma_h)[\gamma q + (1-\gamma)(1-q)] + (1-\sigma_l)[(1-\gamma)q + \gamma(1-q)]}\right| \end{aligned} \tag{10}$$

Equations (10) indicate that an analyst's trading commissions primarily depend on his forecasting strategies (σ), the prior probability of earnings (q), and his average signal precision (γ), which is a function of his prior reputation and the signal precision of the good analyst.

[4] Technically, the ordering in (9) is true when the condition $\sigma_h \geqslant \sigma_l$ is valid. This condition implies that there is a higher likelihood of reporting high when the analyst receives a high signal than when he receives a low signal. This is a more natural assumption than $\sigma_h \leqslant \sigma_l$, in which case the price after a high forecast drops from the original price (i.e., P_0), and the price after a low forecast rises. Accordingly, I call an equilibrium a *natural equilibrium* if the strategy pair (σ_h, σ_l) always satisfies the condition $\sigma_h \geqslant \sigma_l$. An equilibrium in which the opposite, i.e., $\sigma_h < \sigma_l$, is true is called a *perverse equilibrium*. In this paper, I focus only on natural equilibria.

Property 1 shows that an analyst's trading commissions increase with his prior reputation and signal precision. This property of an analyst's trading commissions is consistent with the empirical regularities that analysts with higher prior reputation have a greater impact on price movements (Stickel 1992; Park, and Stice 2000), and therefore, generate more trading commissions for their brokerage-firm employers (Irvine 2004).

Property 1. *An analyst's trading commissions (i.e., $\pi(m)$) increase with his prior reputation (i.e., λ) and his signal quality (i.e., γ)*

Proof. All proofs are in Appendix B. □

Note also in (10) that the trading commissions do not depend on the analyst's private signal. In fact, if the market (naïvely) believes that the analyst is truthfully revealing his private signal (i.e., $m = s$), then the analyst has an incentive to deviate by simply reporting against the prior of the earnings. More specifically, suppose the prior is optimistic, i.e., $q > \frac{1}{2}$. Then given the market's naïve conjecture, the analyst will always issue a low earnings forecast, regardless of his private signal, since a low forecast will generate the maximum trading volume given the optimistic prior. Similarly, with the same conjecture, an analyst will always report a high forecast when the prior is pessimistic. The following lemma formalizes this intuition.

Lemma 1. *(Impossibility of full revelation) When an analyst is concerned with maximizing* only *trading commissions, there is no fully revealing equilibrium*[5] *in any interval of $q \in (0, 1)$. Specifically, if the market conjectures that the analyst is revealing his private signal, then an analyst will always report high if the market's prior is pessimistic, and report low if the market's prior is optimistic, regardless of his private signal.*

So, what is an equilibrium when an analyst's only objective is to maximize his trading commissions? In the following proposition, I show that in equilibrium, an analyst can only partially reveal his private signal. Specifically, if the prior of earnings is pessimistic, he will report high if he receives a high signal; however, he will strictly randomize between a high and a low report if he receives a low signal. In effect, a low forecast reveals unambiguously that

[5] A fully revealing equilibrium is defined as an equilibrium in which the message m is a one-to-one map of signal s. In this paper, I call an equilibrium fully revealing if $m = s$.

the analyst received a low signal; in contrast, a high forecast can be issued for either a high or a low signal, implying that a high forecast has less information content than a low forecast at pessimistic priors. Similarly, if the prior is optimistic, then the analyst will report low when he receives a low signal, but will randomize between high and low reports if he receives a high signal. The equilibrium regions are illustrated in Figure 2.

Proposition 1. *(Characterization of Equilibrium)*

If an analyst is concerned with maximizing only *trading commissions, then there exists an equilibrium, which can be expressed as follows:*

(i) if $q \in (0, \frac{1}{2})$, then an analyst with high signal will report high; however, an analyst with low signal will strictly randomize between high and low reports; the farther q is from $\frac{1}{2}$, the less information is revealed

(ii) if $q \in (\frac{1}{2}, 1)$, then an analyst with low signal will report low; however, an analyst with high signal will strictly randomize between high and low reports; the farther q is from $\frac{1}{2}$, the less information is revealed

(iii) if $q = \frac{1}{2}$, the analyst fully reveals his private signals.

The intuition of this result is that the analyst, with his only objective being to maximize his trading commissions, will tend to report so as to move the price to the maximum. If the prior is pessimistic, then a high forecast will move the price more than a low forecast, and thus, will generate maximum trading volume and higher trading commissions for the analyst. Similarly, a low forecast in the case of an optimistic prior will generate the maximum trading commissions. I call this – the incentive to report against the prior – an analyst's "against-the-prior" incentive.

Suppose the prior of earnings is pessimistic (i.e., $q < \frac{1}{2}$). The analyst's against-the-prior incentive will induce him to report high, regardless of his signal. Now, if the analyst receives a high signal, then a high earnings forecast is consistent with both his private signal and his against-the-prior incentive. Therefore, the analyst will report high with a high signal. However, if the analyst receives a low signal, then a high forecast, although consistent with his against-the-prior incentive, is not consistent with his private signal, and thus, he will strictly randomize between high and low reports. Therefore, for a pessimistic prior, the low report has more information content than a high report. While a low report reveals, unambiguously, that the analyst has received a low signal, a high report can be issued by the analyst for both high and low signals.

Note that only at $q = \frac{1}{2}$ can the analyst fully reveal his private signals. The intuition is that at $q = \frac{1}{2}$, the prior is neither optimistic nor pessimistic; the price moves the same amount regardless of the analyst's forecast, making him indifferent between reporting high and low. In fact, at $q = \frac{1}{2}$, the prior is the most diffused; there is maximum uncertainty about the future values of earnings, x; the uncertainty decreases as the prior becomes more precise. It is easy to see that $Var(x)$ is maximum at $q = \frac{1}{2}$ and decreases as q moves farther away from $\frac{1}{2}$.

We will expect that the analyst will be most likely move the price to the maximum extent possible at diffused priors, and thus, to generate maximum trading commissions when the prior is around the point $q = \frac{1}{2}$. An analyst's ability to move the price and to generate more trade diminishes as the prior becomes more precise. This intuition is formalized in the next lemma.

Lemma 2. *An analyst's equilibrium trading commissions are maximum at $q = \frac{1}{2}$, decrease with q as q moves away from $\frac{1}{2}$, and approach zero as either $q \to 0$ or $q \to 1$.*

Benchmark 2 (Reputation Motive): When an analyst's only concern is to maximize his reputation in the labor market, he solves the following problem,

$$\max_{\tilde{m}} \mathbb{E}_x[Pr(\theta = g \mid \tilde{m}, x) \mid s] \tag{11}$$

After the analyst issues an earnings forecast, and the earnings have been reported, the market compares the analyst's forecast with the earnings and updates the analyst's reputation, $Pr(\theta = g \mid m, x)$, the market's belief that the analyst is of good type. Since the market does not have access to the analyst's private signal, the best way to assess the analyst's type is to check whether his forecast and the reported earnings match. If the forecast and the earnings match, the analyst's reputation is favorably updated; if they do not, then his reputation is downgraded. The following lemma formalizes this intuition.

Lemma 3. *(Updating Reputation)*

(i) $\Pr(g \mid m^h, x^H) \geq \lambda \geq \Pr(g \mid m^l, x^H)$

(ii) $\Pr(g \mid m^l, x^L) \geq \lambda \geq \Pr(g \mid m^h, x^L)$

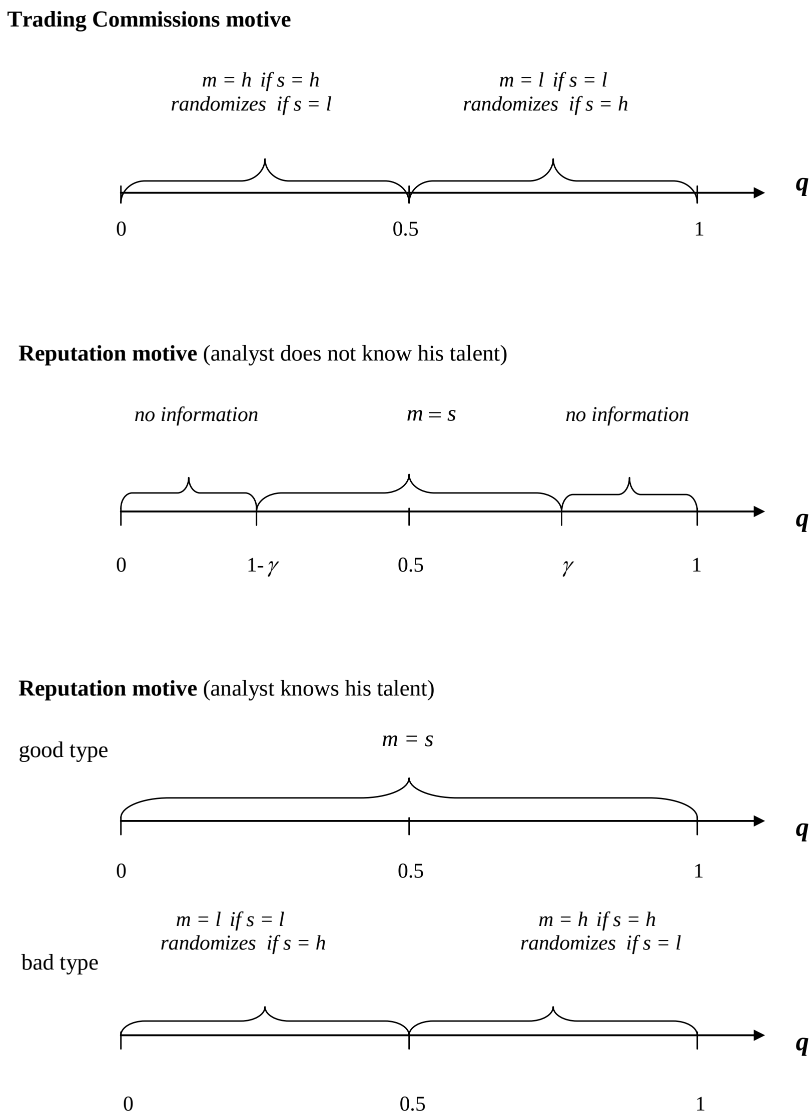

Figure 2 : Benchmark Equilibrium Regions

The intuition of this result is that a forecast consistent with the reported earnings implies that the analyst must have received a very precise private signal, the hallmark of a "good" analyst. Thus, the market updates its belief favorably that the analyst is good. On the other hand, if the analyst's forecast does not match the reported earnings, the market downgrades the analyst's reputation, thinking that his private signal was not precise. Lemma 3 is also consistent with the empirical regularities (e.g., Stickel 1992; Hong, Kubik and Solomon 2000) that lower forecast errors or a higher forecast accuracy results in higher assessments of an analyst's reputation. In my stylized model, a forecast that matches the reported earnings implies higher forecast accuracy.

Characterizing the analyst's equilibrium forecasting behavior, the next lemma shows that an analyst can credibly communicate his private information only if the prior is in the intermediate range, i.e., $q \in [1-\gamma, \gamma]$. he cannot reveal any information credibly if the prior is extreme (Ottaviani and Sorensen 2001). The equilibrium regions are shown in Figure 2. The intuition is that while maximizing his expected reputation in the market, the analyst's best strategy is to forecast $m = h$ if $\Pr(x = H \mid s) \geq \Pr(x = L \mid s)$ and $m = l$ if $\Pr(x = L \mid s) \geq \Pr(x = H \mid s)$. This strategy implies $m = h$ if $q \geq 1-\gamma$, and $m = l$ if $q \leq \gamma$. Thus, a fully revealing equilibrium occurs in the intermediate range of prior, $q \in [1-\gamma, \gamma]$. However, if the prior is extreme, either $q < 1-\gamma$ or $q > \gamma$, then the analyst will not be able to credibly communicate any information to the market, leading to a "babbling" equilibrium. The following lemma summarizes this result and has been proved in Ottaviani and Sorensen (2001). I do not prove the lemma here.

Lemma 4. *(Characterization of Equilibrium)*
When an analyst is concerned with maximizing only *his reputational value in the market, there exists an equilibrium, which can be expressed as follows:*
(i) if $q \in [1-\gamma, \gamma]$, then there is a fully revealing equilibrium,
(ii)if $q \notin [1-\gamma, \gamma]$, then no information is communicated.

The intuition for the noninformative region, i.e., $q \notin [1-\gamma, \gamma]$, is as follows. Consider $q > \gamma$, i.e., $\Pr(x = H \mid s) > \Pr(x = L \mid s)$ or $x = H$ is more likely than $x = L$. Now, if the analyst receives a low signal, he infers that he has a higher likelihood of being a bad type since at $q > \gamma$, $\Pr(g \mid s = l) < \lambda = \Pr(g)$. Thus, to appear to be a good type, and to secure a favorable reputation, the analyst will tend to follow the prior by reporting $m = h$ regardless of his private signal. Knowing this, the market will completely ignore whatever the analyst forecasts if $q > \gamma$, and thus, no information is transmitted in equilibrium. Similarly, if $q < 1 - \gamma$, in order to show that he has received a consistent signal, which is $s = l$, and to appear to be a good type, the analyst will again follow the prior by reporting $m = l$ regardless of his private signal and the market will ignore the forecast. The analyst, thus, has a "conformist" bias – conforming to the prior by ignoring his own signal – at either very high or very low priors.

The result that no information can be communicated at extreme priors changes drastically when the talent of an analyst is known to the analyst but not to the market (see Figure 2, last panel). When the analyst knows his talent, the good type always reveals his private signal, even at extreme priors. The bad type, on the other hand, reveals his private signal only if $q \in [1 - \gamma_b, \gamma_b]$; however, if $q < 1 - \gamma_b$, he forecasts low if he receives a low signal but strictly randomizes between high and low forecasts if he receives a high signal. Note that in the model, $\gamma_b = \frac{1}{2}$. Also, if $q > \gamma_b$, then the bad type forecasts high if he receives a high signal but strictly randomizes between high and low forecasts if he receives a low signal (Trueman 1994).

The intuition is that when an analyst receives an inconsistent signal at extreme priors, say, a low signal at high prior, then his reporting strategy can depend on another piece of information, his talent, which was missing when he didn't know his type. A good analyst, although having a low signal at extremely high prior, will risk reporting low – his own signal – expecting a huge gain in his reputation if the realization of state is actually low. A bad analyst, on the other hand, cannot risk as much as a good type since the precision of his signal is lower than that of the good analyst. In fact, it can be shown that the expected reputational gain for revealing his own signal is higher for a good type than a bad type. Thus, at extreme priors, whereas a good analyst can credibly communicate his private signals, a bad analyst can only partially do so.

3.1 Both Trading Commissions and Reputation

In this section, I define and characterize the equilibrium when a single analyst is concerned with both the trading commissions and the reputation motives. The term equilibrium refers to a perfect Bayesian Nash equilibrium.

Definition 1. *An equilibrium consists of an analyst's forecasting strategy σ and the market's pricing rule $P(m)$ such that* *(i) for each $s \in \{h, l\}$, the analyst solves*

$$\max_{\tilde{m}} \{\alpha|P(\tilde{m}) - P_0| + (1-\alpha)\mathbb{E}_x[Pr(\theta = g \mid \tilde{m}, x) \mid s]\} \tag{12}$$

(ii) for each $m \in \{h, l\}$, the market follows the pricing rule $P(m) = E[x|m]$

(iii) given m and the realization of x, the market's belief about the analyst type, $Pr(\theta = g \mid \tilde{m}, x)$, is consistent with Bayes' rule.

Condition (i) states that the analyst maximizes his net compensation of trading commissions and expected reputational value for each of his signal types, taking the market's pricing rule as given. Condition (ii) says that the market prices the company's stock as an expected value of x conditional on the analyst's forecast. Condition (iii) states that the market's belief about the analyst's type is consistent with Bayes' rule.

In order to maximize both his trading commissions and expected reputational value in the market, an analyst solves (12) : $\max\limits_{\tilde{m}} \{\alpha|P(\tilde{m}) - P_0| + (1-\alpha)\mathbb{E}_x[Pr(\theta = g \mid \tilde{m}, x) \mid s]\}$, or equivalently,

$$\max_{\tilde{m}} \{\alpha\pi(\tilde{m}) + (1-\alpha)R(\tilde{m} \mid s)\}$$

where $\pi(\tilde{m}) = |P(\tilde{m}) - P_0|$ and $R(\tilde{m} \mid s) = \mathbb{E}_x[Pr(\theta = g \mid \tilde{m}, x) \mid s]$ by definitions (4) and (5). To minimize notational clutter, I define $\triangle V_h \equiv V(m^h \mid s^h) - V(m^l \mid s^h)$, where $V(m \mid s) \equiv \alpha\pi(m) + (1-\alpha)R(m \mid s)$ follows from (4). $\triangle V$ is the the difference in the expected payoff of the analyst for forecasting $m = h$ and $m = l$ when he has received $s = h$. Similarly, $\triangle V_l \equiv V(m^h \mid s^l) - V(m^l \mid s^l)$. Also, $\triangle R_h \equiv R(m^h \mid s^h) - R(m^l \mid s^h)$ and $\triangle R_l \equiv R(m^h \mid s^l) - R(m^l \mid s^l)$. However, since the trading commissions of an analyst do not depend on his private signals, $\triangle\pi \equiv \pi(m^h) - \pi(m^l)$. By definition, $\triangle V_j = \alpha\triangle\pi + (1-\alpha)\triangle R_j,$ $j \in \{h, l\}$. To emphasize the roles of the analyst's strategies (σ_h, σ_l), the prior of state (q), and the relative weight of trading commissions versus reputational payoff (α) in the analysts' expected payoffs, I write $\triangle V_h(\sigma_h, \sigma_l, q, \alpha)$ for $\triangle V_h$. Similarly, I use $\triangle\pi(\sigma_h, \sigma_l, q)$ for $\triangle\pi$, and $\triangle R_h(\sigma_h, \sigma_l, q)$ for $\triangle R_h$

There will be a fully revealing equilibrium if the following inequalities are satisfied:

$$\begin{aligned} V(m^h|s^h) &\geq V(m^l|s^h) \\ V(m^h|s^l) &\leq V(m^l|s^l) \end{aligned}$$

or, equivalently,

$$\begin{aligned} \triangle V_h(\sigma_h &= 1, \sigma_l = 0, q, \alpha) \geq 0 \quad (13) \\ \triangle V_l(\sigma_h &= 1, \sigma_l = 0, q, \alpha) \leq 0 \quad (14) \end{aligned}$$

Inequality (13) implies that $\alpha \triangle \pi(\sigma_h = 1, \sigma_l = 0, q) + (1-\alpha) \triangle R_h(\sigma_h = 1, \sigma_l = 0, q) \geq 0$, or

$$\alpha \triangle \pi(\sigma_h = 1, \sigma_l = 0, q) \geq -(1-\alpha) \triangle R_h(\sigma_h = 1, \sigma_l = 0, q) \quad (15)$$

which means that for an analyst with a high signal, the optimal forecast will be high if his expected gain in trading commissions for a high forecast is at least as good as his reputational loss in forecasting high. In other words, each analyst is trading off his short-term gain in trading commissions with his long-term losses in reputational value in the market. Similarly, inequality (14) implies

$$\alpha \triangle \pi(\sigma_h = 1, \sigma_l = 0, q) \leq -(1-\alpha) \triangle R_l(\sigma_h = 1, \sigma_l = 0, q) \quad (16)$$

Proposition 2 characterizes the equilibrium forecasting behavior of an analyst when he is maximizing his trading commissions and his expected reputational value in the labor market. The equilibrium regions are shown in Figure 3.[6]

Proposition 2. *(Characterization of Equilibrium)*
If an analyst's objective is to maximize both *his trading commissions and reputational value in the market, then there exists an* $\alpha_{\max} \in (0,1)$ *such that for any* $\alpha \leq \alpha_{\max}$*, there exists an equilibrium, which can be expressed as follows:*
(i) if $q \in [\underline{q}(\alpha), \overline{q}(\alpha)]$*, there is a fully revealing equilibrium*
(ii) if either $q \in (0, q_{\min}(\alpha))$ *or* $q \in (q_{\max}(\alpha), 1)$*, then no information is communicated in equilibrium*
(iii) if $q \in [q_{\min}(\alpha), \underline{q}(\alpha))$*, then an analyst with a low signal will forecast low; however, an analyst with a high signal will strictly randomize between high and low forecasts,*
(iv) if $q \in (\overline{q}(\alpha), q_{\max}(\alpha)]$*, then an analyst with a high signal will forecast high; however, an analyst with a low signal will strictly randomize between high and low forecasts,*
where,

[6] To avoid notational clutter, α in the parenthesis of q has been dropped in Figure 3.

$\underline{q}(\alpha) = 1 - \overline{q}(\alpha)$, $q_{\min}(\alpha) = 1 - q_{\max}(\alpha)$

Proposition 2 states that if the relative weight of the trading commissions component of a sell-side analyst's compensation is not very high (which implies that the relative weight of reputational concerns is sufficiently large), then there are three types of equilibrium – fully revealing, partially revealing and non-informative – at different intervals of the prior of earnings. At intermediate priors, there is a fully revealing equilibrium. At sufficiently high priors, the analyst will report high if he receives a high signal, and will strictly randomize between high and low reports when he receives a low signal. In contrast, at sufficiently low priors, the analyst will strictly randomize if he receives a high signal, but will report low when he receives a low signal. At extreme priors, no information is communicated in equilibrium.

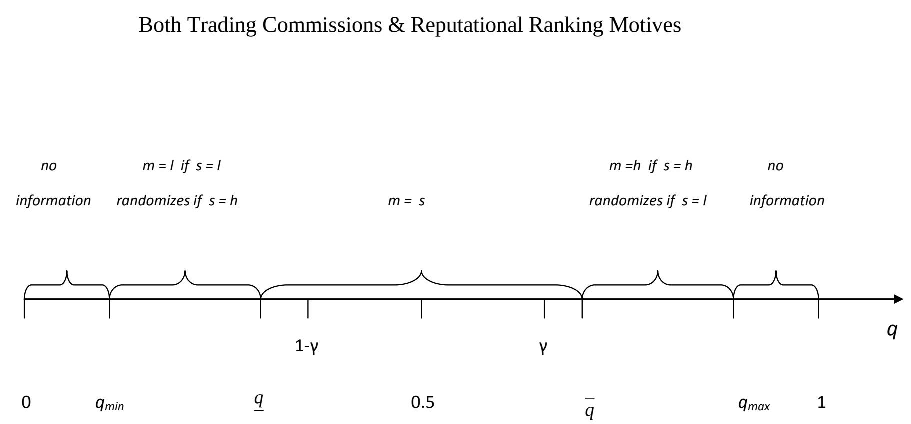

Figure 3: Equilibrium Regions (Single Analyst)

Three aspects of this equilibrium are noteworthy. First, in contrast to benchmark 1, in which the analyst is concerned about maximizing only his trading commissions, in this case, there is a fully revealing equilibrium for some interval of priors. Consistent with casual empiricism, reputational concerns do increase the information content and thus, the credibility of an analyst's forecasts. Second, at extreme priors, no information is communicated in equilibrium. As in benchmark 2, in which the analyst is concerned only about his reputation in the market, at extreme priors – when the prior is either close to one or zero – the public information of earnings is very precise, and thus an analyst with an unlikely signal (for example, a low signal at very high prior or a high signal at very low prior) will tend not to reveal his signal, apprehending that reporting such a signal will negatively impact his reputation in the market. As I have shown in lemma 2, at extreme priors, where the trading commissions are very small due to less likelihood of price movements, the conformist bias due to the reputational concerns of the analyst persists. Third, there is an upper bound of the relative weight of the trading commissions (i.e., α) vis-a-vis the reputational incentive. Above this upper bound, the trading commission incentive will dominate and we will get an equilibrium similar to that in benchmark 1.

Proposition 3. *(Informativeness of Equilibrium)*
Suppose $\alpha \leq \alpha_{\max}$. The region of priors (i.e., q) for which there is a fully revealing equilibrium increases, and the region of priors for which there is a noninformative equilibrium decreases as the relative weight of trading commissions (i.e., α) increases.

Formally, if $\alpha_2 > \alpha_1$, then $\overline{q}(\alpha_2) \geqslant \overline{q}(\alpha_1)$ and $q_{\max}(\alpha_2) \geqslant q_{\max}(\alpha_1)$

Proposition 3 comprises one key result of this paper. This result shows that, as long as an analyst is sufficiently concerned about his reputation (i.e., $\alpha \leq \alpha_{\max}$, which means that the relative weight on reputational concerns is sufficiently large), trading commissions, along with reputational concerns, provide better incentive for honest reporting than reputational concerns alone. Specifically, the relative weight of trading commissions increases the region of fully revealing equilibrium and decreases the region of noninformative equilibrium. This result is striking because while the trading commission incentive, by itself, discourages the truthful revelation of an analyst's private signal, when added to the reputational concerns incentive, it improves the information content of an analyst's forecast.

The intuition of this result is as follows. Recall our discussions in benchmark 2, in which reputational concerns create a conformist bias in an analyst's forecast. At sufficiently high or low priors, an analyst will conform to the prior by ignoring his own private information, and the market, aware of this strategy, will completely ignore the analyst's forecast, leading to no information transmission in equilibrium. To be more specific, suppose the prior of earnings is very pessimistic, i.e., q is sufficiently less than $\frac{1}{2}$. An analyst with only reputational concerns who receives a high signal will have no incentive to report high due to his conformist bias. However, with the addition of a trading commission motive, the analyst now has an against-the-prior incentive, which will motivate him to report high at pessimistic prior. Thus, when the two motives are combined, the analyst can credibly communicate his high signal at very low prior, which was not possible with only a reputational concern.

4 Two Analysts: Equilibrium

In this section I define and characterize the equilibrium of the model with two analysts. Throughout, the term equilibrium refers to a perfect Bayesian Nash equilibrium with symmetric strategies. Given that the analysts don't know their types and are identical in every aspect except in their private signals, the most plausible equilibrium is an equilibrium with symmetric strategies.

4.1 Equilibrium Definition

Definition 2. *An equilibrium consists of analysts' forecasting strategies (σ_i, σ_{-i}), and the market's pricing rule $P(m_i, m_{-i})$ such that*
(i) for each $s_i \in S_i$, analyst i solves

$$\max_{\tilde{m}_i} \{\alpha \pi_i(\tilde{m}_i \mid s_i) + (1-\alpha) R_i(\tilde{m}_i \mid s_i)\} \tag{17}$$

where $\pi_i(\tilde{m}_i \mid s_i)$ and $R_i(\tilde{m}_i \mid s_i)$ follow from definitions (5) and (6)
(ii) for each pair of (m_i, m_{-i}), the market follows the pricing rule $P(m_i, m_{-i}) = E[x|m_i, m_{-i}]$
(iii) given (m_i, m_{-i}) and the realization of x, the market's belief about analyst i's type, $Pr(\theta_i \mid m_i, m_{-i}, x)$, is consistent with Bayes' rule.

Condition (i) states that each analyst maximizes his net compensation of trading commissions and expected reputational value for each of his signal types, taking the market's pricing rule and the other analyst's strategy as given. Condition (ii) says that the market prices the company's stock as an expected value of x conditional on the analyst forecasts, (m_i, m_{-i}), taking into account their reporting strategies. Condition (iii) states that the market's beliefs about the analysts' types are consistent with Bayes' rule.

I will first characterize the equilibria of two extreme cases : $\alpha = 1$, i.e., when the analysts are only concerned with their trading commissions profits, and $\alpha = 0$, i.e., when the analysts only care about their reputational ranking payoffs.

4.2 Trading commission motive

In this section, I characterize equilibrium for the case in which each analyst's only motive is to maximize his trading commissions, without any concern for reputational payoff. Analyst i's objective is

$$\max_{\tilde{m}_i} \mathbb{E}_{m_{-i}}[|P(m_i, m_{-i}) - P_0| \mid s_i, \sigma_{-i}] \tag{18}$$

Given his signal s_i, analyst i's optimal strategy will be to move the price as much as possible from the price at $t = 0$ in order to generate maximum trading commissions. However, the price movement depends not only on his own forecast but also on the forecast of the other analyst. Thus, while choosing which forecast to make, high or low, analyst i will take into account both his own private signal s_i and the strategy of the other analyst $\sigma_{\text{-}i}$, and conditional on this information, he will compare his expected trading commissions under $m_i = h$ and $m_i = l$.

I define $\sigma_{ij} \equiv \Pr(m_i^h | s_i^j)$ for $j \in \{h, l\}$ and $i \in \{1, 2\}$. For example, $\sigma_{ih} = \Pr(m_i^h | s_i^h)$ means that the likelihood that analyst i with signal $s_i = h$ will report $m_i = h$ is σ_{ih}. Furthermore, since I am focusing on equilibria with only symmetric strategies, $\sigma_{ij} = \sigma_{-ij} \equiv \sigma_j$. Suppose that the conjectured strategy is that each analyst is revealing his private signal, i.e., $\sigma_{ih} = 1$ and $\sigma_{il} = 0\ \forall i$. Then, using $P(m_i, m_{-i}) = E[x \mid m_i, m_{-i}]$,

$$\begin{aligned}
\pi_i(m_i^h | s_i^h) &= \mathbb{E}_{m_{-i}}[|P(m_i^h, m_{-i}) - P_0| \mid s_i^h, \sigma_{-i}] \\
&= |E[x|m_i^h, m_{-i}^h] - E[x]| \Pr(m_{-i}^h \mid s_i^h, \sigma_{-i}) + |E[x|m_i^h, m_{-i}^l] - E[x]| \Pr(m_{-i}^l \mid s_i^h, \sigma_{-i}) \\
&= |E[x|s_i^h, s_{-i}^h] - E[x]| \Pr(s_{-i}^h \mid s_i^h) + |E[x|s_i^h, s_{-i}^l] - E[x]| \Pr(s_{-i}^l \mid s_i^h) \qquad (19)
\end{aligned}$$

Note that there are two important probabilities that each analyst needs to consider when choosing his optimal forecast: $\Pr(x^H | s_i, s_{-i})$, the posterior of $x = H$, given his and the other analyst's signals, and $\Pr(s_{-i}|s_i)$, the likelihood of the other analyst's signal given his own signal. For example, $\Pr(x^H \mid s_i^h, s_{-i}^h)$ is calculated as follows:

$$\Pr(x^H \mid s_i^h, s_{-i}^h) = \frac{\Pr(x^H)\Pr(s_i^h, s_{-i}^h|x^H)}{\Pr(x^H)Pr(s_i^h, s_{-i}^h|x^H) + \Pr(x^L)Pr(s_i^h, s_{-i}^h|x^L)} \tag{20}$$

where

$$\Pr(s_i^h, s_{-i}^h \mid x^H) = \sum_{\theta_i}\sum_{\theta_{\text{-}i}} \Pr(\theta_i, \theta_{-i})\Pr(s_i^h, s_{-i}^h \mid x^H, \theta_i, \theta_{-i})$$

which is calculated (details shown in the proof of property 2) as

$$\Pr(s_i^h, s_{-i}^h \mid x^H) = \gamma^2 + \lambda^2\gamma_g(1-\gamma_g) \tag{21}$$

In (20), the joint distribution of the two signals, conditional on the state, depends on two terms: γ^2, which is the joint probability when both the signals are conditionally independent, and an extra term $\lambda^2\gamma_g(1-\gamma_g)$, which captures the effect of the correlation between the signals of two good analysts. This extra term is the difference in the expression $\Pr(g_i, g_{-i})\Pr(s_i^h, s_{-i}^h|x^H, g_i, g_{-i})$ when the signals of good analysts are conditionally correlated $(\lambda^2\gamma_g)$ and when the signals are conditionally independent $(\lambda^2\gamma_g^2\ )$. Henceforth, I will call $\lambda^2\gamma_g(1-\gamma_g) \equiv c$ the "correlation effect"[7] between the analysts' signals. The following property summarizes the results of joint distribution of the analysts' signals. Note that if the analysts have different signals, the correlation effect is negative.

Property 2. *(Joint Distribution of Signals)*
(i) $\Pr(s_i = x, s_{-i} = x \mid x) = \gamma^2 + c$
(ii) $\Pr(s_i = x, s_{-i} \neq x \mid x) = \Pr(s_i \neq x, , s_{-i} = x \mid x) = \gamma(1-\gamma) - c$
(iii) $\Pr(s_i \neq x, s_{-i} \neq x \mid x) = (1-\gamma)^2 + c$

Replacing the relevant probability calculations, $\pi_i(m_i^h|s_i^h)$ in (19), under the conjecture of full revelation of private signals by the analysts, can be simplified as (detailed calculations are

[7] For a more general correlation level $\rho \in (0, 1]$ (see footnote 3), the correlation effect will be $c = \rho\lambda^2\gamma_g(1-\gamma_g)$, strictly increasing in ρ.

shown in the proof of proposition 4):

$$
\begin{aligned}
&\pi_i(m_i^h|s_i^h) \\
&= \; |E[x|s_i^h, s_{-i}^h] - E[x]| \Pr(s_{-i}^h \mid s_i^h) \\
&= \; \left[(x^H - x^L)q(1-q)(2\gamma - 1)\right] \frac{\Pr(s_{-i}^h \mid s_i^h)}{\Pr(s_i^h, s_{-i}^h)} \qquad (22)
\end{aligned}
$$

Note that the second term of $\pi(m_i^h|s_i^h)$ in (19), $|E[x|s_i^h, s_{-i}^l]-E[x]| = 0$, since $E[x|s_i^h, s_{-i}^l] = E[x]$. Two opposite signals, that is, (s_i^h, s_{-i}^l), leave the prior expectation of the state unchanged. A low (high) signal completely offsets the effect of a high (low) signal on the posterior of the earnings. The fraction in (22) implies that there will be positive trading commissions only when both analysts have high signals. The numerator represents the likelihood of the other analyst receiving a high signal when analyst i receives a high signal, and the denominator is the joint probability of both analysts receiving a high signal.

Similarly, trading commissions for a low forecast given a high signal are

$$
\pi_i(m_i^l|s_i^h) = \left[(x^H - x^L)q(1-q)(2\gamma - 1)\right] \frac{\Pr(s_{-i}^l \mid s_i^h)}{\Pr(s_i^l, s_{-i}^l)} \qquad (23)
$$

For a fully revealing equilibrium to exist, the following inequalities need to be satisfied for each i:

$$
\begin{aligned}
\pi_i(m_i^h|s_i^h) &\geq \pi_i(m_i^l|s_i^h) \qquad (24) \\
\pi_i(m_i^h|s_i^l) &\leq \pi_i(m_i^l|s_i^l) \qquad (25)
\end{aligned}
$$

The following proposition characterizes the most informative equilibrium when the analysts are concerned with maximizing only trading commissions.

Proposition 4. *(Characterization of Equilibrium)*
If the analysts' only concerns are to maximize their trading commissions, then there exists an equilibrium, which can be expressed as follows:
(i) if $q \in [1 - q_\pi, q_\pi]$, then there exists a fully revealing equilibrium,
(ii) if $q \in (0, 1 - q_\pi)$, then analysts with a high signal will forecast high; however, analysts with

a low signal will strictly randomize between high and low forecasts,

(iii) if $q \in (q_\pi, 1)$*, then analysts with a low signal will forecast low; however, analysts with a high signal will strictly randomize between high and low forecasts,*

where $\quad q_\pi = \gamma + \frac{2c}{2\gamma - 1}$

This proposition states that there is a fully revealing equilibrium at the intermediate values of prior; however, at sufficiently high or low priors, analysts' private information is revealed only partially. Note that the equilibrium features are very similar to those in benchmark 1 with a single analyst, except that now there is a fully revealing equilibrium under a certain range of parameters, implying that more information is revealed in equilibrium.

The intuition is that each analyst, when deciding between a high and a low forecast, considers a tradeoff. On one hand, trading commissions will be maximized if the analyst's forecast is against the market's prior expectation of the company's future earnings; specifically, if the prior is optimistic (i.e., $q > \frac{1}{2}$), then a low forecast will generate the maximum trading volume by moving the price maximum. As discussed in the case of a single analyst, this is the "against-the-prior" incentive of each analyst. On the other hand, the price will move only if the other analyst forecasts in the same direction as analyst i. I call this the analysts' "coordination" incentive. The coordination incentive is accentuated by the correlation effect. The equilibrium regions are illustrated in Figure 4.

More specifically, consider a case in which the prior is $q > \frac{1}{2}$, and analyst i receives a signal $s = h$. Since the prior is optimistic, the against-the-prior incentive will lead analyst i to make a forecast $m = l$, regardless of his signal. However, given that analyst i received a high signal and the prior is optimistic, there is a higher likelihood that the other analyst will also receive a high signal. Now, a low forecast by analyst i – based only on his against-the-prior incentive – will most likely lead to zero trading commissions, since a low forecast by analyst i and a high forecast by analyst -i (assuming that the other analyst is reporting his private signal in a Nash equilibrium) will not move the price. The importance of the coordination incentive becomes apparent at this point. Analyst i will now consider coordinating with analyst -i to forecast high so that they can move the price, thereby gaining positive trading commissions profits. Given that each analyst is making a forecast without any knowledge about the other analyst's private signal, how do they coordinate?

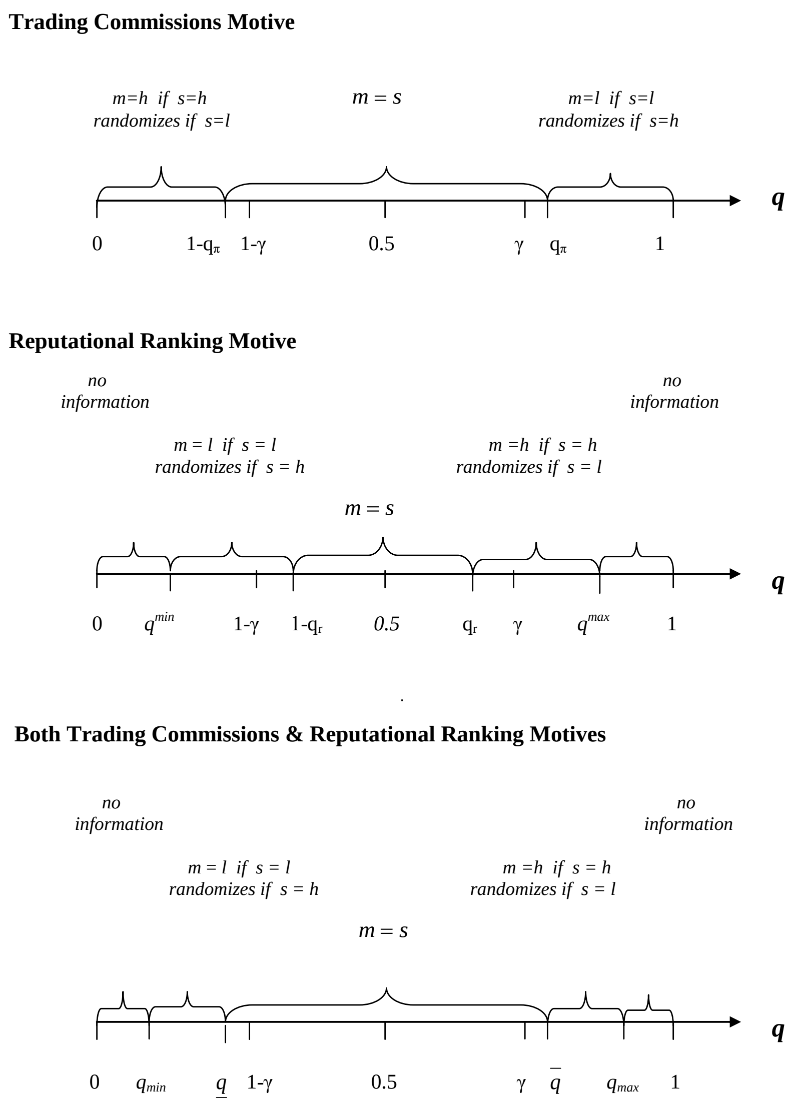

Figure 4: Equilibrium Regions (Two analysts)

Analyst i's signal, along with his knowledge of the prior of state, conveys some probabilistic information about the private signal of the other analyst. The conditional correlation between the analysts' signals makes this probabilistic information even more precise. This knowledge of each other's signals helps the analysts (implicitly) coordinate their forecasts so that they can move the price and generate trading commissions in equilibrium. The coordination incentive, thus, creates an *endogenous discipline* – absent in the case of a single analyst – that makes each analyst's trading commissions dependent on his private signal, and thus, help him credibly communicate his private information in equilibrium.

Thus, the optimal forecasting strategy will depend on the tradeoff between these two incentives. As we can expect, if the prior reputation of the analysts is high, then there is a higher likelihood that the analysts' signals are correlated, and thus, the coordination incentive will dominate the against-the-prior incentive by increasing the region of fully revealing equilibrium. However, at very high or low priors, the against-the-prior incentive will dominate the coordination incentive, leading to a decrease in the region of priors for which a fully revealing equilibrium exists. Furthermore, as the correlation between the analysts' signals increases, the analysts' coordination motive becomes stronger, leading to a higher q_π and a larger region of fully revealing equilibrium. The following lemma summarizes how the equilibrium region changes with respect to the the correlation effect and the analysts' prior reputations.

Lemma 5. *(Informativeness of Equilibrium)*
(i) q_π *is increasing in correlation effect* c*,*
(ii) q_π *is increasing in prior reputation* λ*.*

4.3 Reputational Ranking Motive

In this section, I characterize the equilibrium for the case in which each analyst's only objective is to maximize his expected reputational value in the labor market, without any concern for generating trading commissions. Analyst i's objective is

$$\max_{\tilde{m}_i} \mathbb{E}_{x,m_{-i}}[U_i(\tilde{m}_i, m_{-i}, x) \mid s_i, \sigma_{-i}] \tag{26}$$

where, as defined previously in (3), $U_i(\tilde{m}_i, m_{-i}, x) = \sum_{\theta_i} \sum_{\theta_{-i}} u_i(\theta_i, \theta_{-i}) \Pr(\theta_i, \theta_{-i} \mid \tilde{m}_i, m_{-i}, x)$.

Analyst i's reputational value depends not only on his own forecast and reputation, but also on the forecast and reputation of the other analyst. Specifically, analyst -i influences analyst i's reputational value in two ways : $m_{\text{-}i}$ affects analyst i's reputation through $\Pr(\theta_i, \theta_{-i} \mid \tilde{m}_i, m_{-i}, x)$, and θ_{-i} affects his reputational ranking payoff through $u_i(\theta_i, \theta_{-i})$.

Suppose analyst i receives signal $s = h$. If he forecasts $m = h$, his expected reputational value will be

$$\begin{aligned}
&R_i(m_i^h \mid s_i^h) \\
&= U_i(m_i^h, m_{-i}^h, x^H) \Pr(m_{-i}^h, x^H \mid s_i^h, \sigma_{-i}) + U_i(m_i^h, m_{-i}^l, x^H) \Pr(m_{-i}^l, x^H \mid s_i^h, \sigma_{-i}) + \\
&\quad U_i(m_i^h, m_{-i}^h, x^L) \Pr(m_{-i}^h, x^L \mid s_i^h, \sigma_{-i}) + U_i(m_i^h, m_{-i}^l, x^L) \Pr(m_{-i}^l, x^L \mid s_i^h, \sigma_{-i}) \qquad (27)
\end{aligned}$$

Furthermore, from (3), using Bayes' Rule,

$$U_i(m_i^h, m_{-i}^l, x^H) = \frac{\sum_{\theta_i} \sum_{\theta_{\text{-}i}} u_i(\theta_i, \theta_{-i}) \Pr(\theta_i, \theta_{-i}) \Pr(m_i^h, m_{-i}^l \mid x^H, \theta_i, \theta_{-i})}{\sum_{\theta_i} \sum_{\theta_{\text{-}i}} \Pr(\theta_i, \theta_{-i}) \Pr(m_i^h, m_{-i}^l \mid x^H, \theta_i, \theta_{-i})} \qquad (28)$$

where for any $j, k \in \{h, l\}$,

$$\begin{aligned}
\Pr(m_i^h, m_{-i}^l \mid x^H, \theta_i, \theta_{-i}) &= \sum_j \sum_k \Pr(s_i^j, s_{-i}^k \mid x^H, \theta_i, \theta_{-i}) \Pr(m_i^h \mid s_i^j) \Pr(m_{-i}^l \mid s_{-i}^k) \\
&= \sum_j \sum_k \Pr(s_i^j, s_{-i}^k \mid x^H, \theta_i, \theta_{-i}) \sigma_{ij} (1 - \sigma_{-ik}) \qquad (29)
\end{aligned}$$

Specifically, if the market believes that the analysts are fully revealing their private signals, i.e., $m_i = s_i$ (or $\sigma_{ih} = 1$ and $\sigma_{il} = 0$) for all i, then (27) can be simplified as

$$\begin{aligned}
&R_i(m_i^h \mid s_i^h) \\
&= \Pr(x^H | s_i^h) \left[U_i(s_i^h, s_{-i}^h, x^H) \Pr(s_{-i}^h \mid x^H, s_i^h) + U_i(s_i^h, s_{-i}^l, x^H) \Pr(s_{-i}^l \mid x^H, s_i^h) \right] + \\
&\quad \Pr(x^L | s_i^h) \left[U_i(s_i^h, s_{-i}^h, x^L) \Pr(s_{-i}^h \mid x^L, s_i^h) + U_i(s_i^h, s_{-i}^l, x^L) \Pr(s_{-i}^l \mid x^L, s_i^h) \right] \\
&= \Pr(x^H | s_i^h) \left[U_i(s_i^h, s_{-i}^h, x^H)(\gamma + \frac{c}{\gamma}) + U_i(s_i^h, s_{-i}^l, x^H)(1 - \gamma - \frac{c}{\gamma}) \right] + \\
&\quad \Pr(x^L | s_i^h) \left[U_i(s_i^h, s_{-i}^h, x^L)(1 - \gamma + \frac{c}{1-\gamma}) + U_i(s_i^h, s_{-i}^l, x^L)(\gamma - \frac{c}{1-\gamma}) \right] \qquad (30)
\end{aligned}$$

The values of $\Pr(s_{-i} \mid s_i \ , x) = \frac{\Pr(s_i, s_{-i} | x)}{\Pr(s_i | x)}$ are evaluated using property 2. The values of $U_i(s_i, s_{-i}, x)$ are derived in Appendix A. Similarly, the reputational value for an analyst when he receives a high signal and forecasts a low report can be expressed as

$$\begin{aligned}
&R_i(m_i^l \mid s_i^h) \\
&= \Pr(x^H|s_i^h)\left[U_i(s_i^l, s_{-i}^h, x^H)(\gamma + \frac{c}{\gamma}) + U_i(s_i^l, s_{-i}^l, x^H)(1-\gamma-\frac{c}{\gamma})\right] + \\
&\quad \Pr(x^L|s_i^h)\left[U_i(s_i^l, s_{-i}^h, x^L)(1-\gamma+\frac{c}{1-\gamma}) + U_i(s_i^l, s_{-i}^l, x^L)(\gamma-\frac{c}{1-\gamma})\right] \qquad (31)
\end{aligned}$$

For a fully revealing equilibrium to exist, the following inequalities are to be satisfied for each i :

$$R_i(m_i^h|s_i^h) \geq R_i(m_i^l|s_i^h) \qquad (32)$$

$$R_i(m_i^h|s_i^l) \leq R_i(m_i^l|s_i^l) \qquad (33)$$

The following proposition characterizes the equilibrium in which each analyst is concerned with maximizing only his reputational value in the market, given that his reputational payoff depends not only on his own reputation but also on the reputation of the other analyst.

Proposition 5. *(Characterization of Equilibrium)*
If the analysts' only incentives are to maximize their expected reputational values in the labor market, then there exists an equilibrium, which can be expressed as follows:
(i) if $q \in [1 - q_r(y, \overline{z}, \underline{z}), q_r(y, \overline{z}, \underline{z})]$*, then there exists a fully revealing equilibrium,*
(ii) if either $q < q^{\min}(y, \overline{z}, \underline{z})$ *or* $q > q^{\max}(y, \overline{z}, \underline{z})$*, then no information is communicated in equilibrium,*
(iii) if $q \in [q^{\min}(y, \overline{z}, \underline{z}), 1 - q_r(y, \overline{z}, \underline{z}))$*, then analysts with a low signal will forecast low; however, analysts with a high signal will strictly randomize between high and low forecasts,*
(iv) if $q \in (q_r(y, \overline{z}, \underline{z}),\ q^{\max}(y, \overline{z}, \underline{z})]$*, then analysts with a high signal will forecast high; however, analysts with a low signal will strictly randomize between high and low forecasts,*
where
$q^{\min}(y, \overline{z}, \underline{z}) = 1 - q^{\max}(y, \overline{z}, \underline{z})$, $q_r(y, \overline{z}, \underline{z}) = \gamma - \eta(y, \overline{z}, \underline{z})$ *and*

$$\eta(y,\overline{z},\underline{z}) = c\left[\frac{\{U_i(s_i = x, s_{-i} \neq x, x) + U_i(s_i \neq x, s_{-i} = x, x)\} - \{U_i(s_i = x, s_{-i} = x, x) + U_i(s_i \neq x, s_{-i} \neq x, x)\}}{\{\gamma U_i(s_i = x, s_{-i} = x, x) + (1-\gamma)U_i(s_i = x, s_{-i} \neq x, x)\} - \{\gamma U_i(s_i \neq x, s_{-i} = x, x) + (1-\gamma)U_i(s_i \neq x, s_{-i} \neq x, x)\}}\right] \qquad (34)$$

Proposition 5 states that the analysts would be able to reveal their private signals only within the intermediate range of priors; however, at extreme priors, they cannot credibly communicate any information to the market. At sufficiently high or low – but not extreme – priors, only partial revelation of their private information is possible. (The equilibrium regions are illustrated in Figure 4 on page 30. To avoid notational clutter, $y, \overline{z}$ and $\underline{z}$ in the parenthesis have been dropped.)

Although the characterization of this equilibrium is similar to that in benchmark 2, in which there is a single analyst who is concerned with maximizing his own reputation, the characterizations differ with respect to two main aspects. First, the informativeness of the equilibrium depends now on additional parameters, reputational payoffs, i.e.,$(y, \overline{z}, \underline{z})$. Second, unlike in benchmark 2, there is now a partially revealing, mixed-strategy equilibrium at sufficiently high or low priors.

In proposition 5, the regions of fully revealing and noninformative equilibrium – which, in turn, define the region of partially revealing equilibrium – depend on the sign and value of η, a term that captures the joint effect of the conditional correlation between analysts' signals and reputational payoffs. The correlation effect is captured by c. The numerator of η is the difference between the sum of reputational value when analysts' received different signals (i.e., $U_i(s_i = x, s_{-i} \neq x, x) + U_i(s_i \neq x, s_{-i} = x, x)$) and the sum of reputational values when analysts received the same signal (i.e., $U_i(s_i = x, s_{-i} = x, x) + U_i(s_i \neq x, s_{-i} \neq x, x)$). The denominator of η is the difference between the expected reputational values of an analyst when he receives a matched signal (i.e., $\gamma U_i(s_i = x, s_{-i} = x, x) + (1-\gamma)U_i(s_i = x, s_{-i} \neq x, x)$) and when he receives an unmatched signal (i.e., $\gamma U_i(s_i \neq x, s_{-i} = x, x) + (1-\gamma)U_i(s_i \neq x, s_{-i} \neq x, x)$). Note that the expected reputational value of an analyst when he receives a matched signal is $\Pr(s_{-i} = x|x)U_i(s_i = x, s_{-i} = x, x) + \Pr(s_{-i} \neq x|x)U_i(s_i = x, s_{-i} \neq x, x)$.

The informativeness of the equilibrium crucially depends on the sign of η. For positive values of η, the informativeness of the equilibrium decreases with η – the higher the η, the lower is the region of fully revealing equilibrium, and the higher is the region of partially revealing equilibrium. In the next proposition, I show how the sign of η – and the informativeness of the equilibrium – changes with the values of reputational payoffs.

Proposition 6. *(Informativeness of Equilibrium)*

The region of priors (i.e., q) for which there is a fully revealing equilibrium decreases *if and only if the reputational reward for being the only good analyst is sufficiently higher than the reputational penalty for being the only bad analyst. Otherwise, the region of priors* increases. *Formally,*

$$\eta(y, \overline{z}, \underline{z}) > (\leq)\ 0 \qquad \textit{if and only if} \qquad (\overline{z} - y) > (\leq)\ (-\underline{z}) + h(\lambda, \gamma_g).y \tag{35}$$

where

$h(\lambda, \gamma_g) = \frac{2\lambda(1+\lambda)}{\frac{1}{f(\lambda,\gamma_g)} - (1-\lambda^2)} - 1 > 0$

$f(\lambda, \gamma_g) = \frac{\gamma_g}{4\lambda\gamma_g + (1-\lambda)^2} + \frac{1-\gamma_g}{4\lambda(1-\gamma_g) + (1-\lambda)^2}$

Proposition 6 states that the information content of an analyst's forecast decreases if the reputational reward for being the only good analyst, (i.e., $(\overline{z} - y)$), is sufficiently larger than the reputational penalty of being the only bad analyst, (i.e., $(-\underline{z})$). The coefficient $h(\lambda, \gamma_g)$ determines to what extent the absolute reputational payoff of both being good analysts (i.e., y) affects the informativeness of the equilibrium.

The intuition is that each analyst – in order to maximize his expected reputational value in the market – would like to increase the likelihood of being the only good analyst (so that he receives $\overline{z}$ more often than y), and decrease the likelihood of being the only bad analyst (so that he receives $\underline{z}$ less often than 0). Now, to increase the likelihood of being the only good analyst, an analyst has to report such that he differentiates himself from the other analyst. Note that the analyst knows neither the private signal of the other analyst nor the talent of himself or the other analyst. The only information he has is his own private signal and the fact that his and the other analyst's signals are possibly correlated. Since we are considering here a symmetric Nash equilibrium, given this information and assuming that the other analyst reports his own signal, the best way for the analyst to differentiate himself is to randomize his reports. Thus, an analyst's incentive to differentiate himself from the other analyst leads to the randomization of reports, which in turn, implies less information transmission in equilibrium.

In contrast, in order to decrease the likelihood of being the only bad analyst – and to receive the reputational penalty less often – an analyst will be less likely to differentiate, and more likely to report in the direction of the other analyst. Again, given an analyst's information set, and assuming that the other analyst is reporting his own signal, the best response for an analyst – to move with the other analyst – will be to report his own signal.

On balance, each analyst's optimal strategy depends on the values of reputational reward (i.e., $(\overline{z} - y)$) relative to reputational penalty (i.e., $-\underline{z}$). The higher the value of reputational reward compared to that of reputational penalty, the greater is the incentive for each analyst to differentiate himself from the other, which leads to more randomization of reports, and less information revelation in equilibrium.

In addition to the reputational reward and penalty payoffs, the absolute reputational payoff of both being good analysts (i.e., y) affects the informativeness of the equilibrium. Each analyst wants to maximize his likelihood of being perceived not only as the only good analyst (and not as the only bad analyst), but also as a good analyst *per se*, which is captured by the reputational payoff y. Now, higher values of y implie greater incentives to be perceived as a good analyst , which, in turn, means a higher likelihood of revealing a private signal, at least in the intermediate range of priors. Recall that in benchmark 2, with a single analyst, the range of priors for which there is a fully revealing equilibrium is $q \in [1-\gamma, \gamma]$. Proposition 6 implies that with higher values of y, it is less likely that the inequality (35) will be satisfied, which in turn, implies a larger fully revealing equilibrium region and less randomization.

The extent to which y influences the informativeness of the equilibrium depends on $h(\lambda, \gamma_g)$. The lower the value of $h(\lambda, \gamma_g)$,the higher the likelihood that inequality (35) will be satisfied, and the less information will be transmitted in equilibrium. Note that $h(\lambda, \gamma_g)$ is decreasing in γ_g. The intuition is that a high γ_g implies a better signal quality for good analysts, and a higher level of conditional correlation between analysts. This further implies that the effect of relative reputational incentives will be more pronounced, and there will be more randomization and less information revelation in equilibrium.

Finally, we can think of scenarios in which there is an asymmetry in relative reputational payoffs. On Wall Street, typically "All-Star" analysts are paid substantially higher than their average counterparts (implying very high $\overline{z}$), and yet analysts are often not penalized equivalently for being ranked lower (implying a low $(-\underline{z})$). In fact, during the period 2000-02, American sell-side analysts in the top ten percentile earned well over $3.5 million, when the median salary was \$1 million (Groysberg, Healy and Maber 2008). Proposition 6 implies that this kind of reputational payoff structure leads to less information content of analyst forecasts.

In contrast, a high penalty (i.e, a high value of $(-\underline{z})$) may arise when there is a very high likelihood that an analyst with a sufficiently poor reputation will be fired from his job. There is empirical evidence that analysts whose forecasts have been less accurate than their peers in the industry are more likely to lose their jobs (Mikhail, Walther and Willis 1999). Proposition 6 implies that this kind of reputational payoff structure will lead to more information content of analyst forecasts. Indeed, Groysberg, Healy and Maber (2008) report that although forecast accuracy is not directly related to an analyst's compensation, it is related to the turnover of analysts, which in our context, may imply that the fear of being fired from the job improves an analyst's forecast accuracy, and thus, increases the information content of analyst forecasts.

4.4 Reputational Ranking and Trading Commissions

In this section, I characterize an equilibrium of the full model. Two analysts, possibly having their private signals correlated, are making simultaneous forecasts of a company's earnings aimed at maximizing their compensations – a linear combination of trading commissions that they generate for their brokerage-firm employers and their expected reputational values in the labor market. Analyst i solves,

$$\max_{\tilde{m}_i} \ \alpha \pi_i(\tilde{m}_i \mid s_i) + (1-\alpha) R_i(\tilde{m}_i \mid s_i) \tag{36}$$

Very similar to the notations I used for a single analyst, let $\triangle V_h \equiv V_i(m_i^h \mid s_i^h) - V_i(m_i^l \mid s_i^h)$, where $V_i(m_i \mid s_i) \equiv \alpha \pi_i(m_i \mid s_i) + (1-\alpha) R_i(m_i \mid s_i)$, as defined in (4). $\triangle V$ is the difference in the expected payoff of analyst i for forecasting $m = h$ and $m = l$ when he has received $s = h$. Similarly, $\triangle V_l \equiv V_i(m_i^h \mid s_i^l) - V_i(m_i^l \mid s_i^l)$. Also, $\triangle \pi_j \equiv \pi_i(m_i^h \mid s_i^j) - \pi_i(m_i^l \mid s_i^j)$ and $\triangle R_j \equiv R_i(m_i^h \mid s_i^j) - R_i(m_i^l \mid s_i^j)$ for $j \in \{h, l\}$. By definition, $\triangle V_j = \alpha \triangle \pi_j + (1-\alpha) \triangle R_j$. To emphasize the roles of the analyst's strategies (σ_h, σ_l), the prior of state (q), reputational ranking payoffs $(y, \overline{z}, \underline{z})$, and the relative weight of trading commissions versus reputational payoff (α) in an analyst's expected payoffs, I write $\triangle V_h(\sigma_h, \sigma_l, q, y, \overline{z}, \underline{z}, \alpha)$ for $\triangle V_h$. Similarly, I use $\triangle \pi_h(\sigma_h, \sigma_l, q)$ for $\triangle \pi_h$, and $\triangle R_h(\sigma_h, \sigma_l, q, y, \overline{z}, \underline{z})$ for $\triangle R_h$.

There will be a fully revealing equilibrium if the following inequalities are satisfied:

$$\triangle V_h(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) \geq 0 \tag{37}$$

$$\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) \leq 0 \tag{38}$$

The following proposition characterizes an equilibrium when both analysts are maximizing their trading commissions and expected reputational value simultaneously.

Proposition 7. *(Characterization of Equilibrium)*
When the analysts' objectives are to maximize both *trading commissions and their reputational values in the market, there exists an* $\alpha_{\max}(y, \overline{z}, \underline{z}) \in (0, 1]$ *such that for any* $\alpha \leq \alpha_{\max}(y, \overline{z}, \underline{z})$*, there exists an equilibrium, which can be expressed as follows:*
(i) if $q \in [\underline{q}(y, \overline{z}, \underline{z}, \alpha), \overline{q}(y, \overline{z}, \underline{z}, \alpha)]$*, there is a fully revealing equilibrium*
(ii) if either $q \in (0, q_{\min}(y, \overline{z}, \underline{z}, \alpha))$ *or* $q \in (q_{\max}(y, \overline{z}, \underline{z}, \alpha), 1)$*, then no information is communicated in equilibrium*
(iii) if $q \in [q_{\min}(y, \overline{z}, \underline{z}, \alpha), \underline{q}(y, \overline{z}, \underline{z}, \alpha))$*, then analysts with a low signal will forecast low; however, analysts with a high signal will strictly randomize between high and low forecasts,*
(iv) if $q \in (\overline{q}(y, \overline{z}, \underline{z}, \alpha), q_{\max}(y, \overline{z}, \underline{z}, \alpha)]$*, then analysts with a high signal will forecast high; however, analysts with a low signal will strictly randomize between high and low forecasts,*
where,

$$\underline{q}(y, \overline{z}, \underline{z}, \alpha) = 1 - \overline{q}(y, \overline{z}, \underline{z}, \alpha), \quad q_{\min}(y, \overline{z}, \underline{z}, \alpha) = 1 - q_{\max}(y, \overline{z}, \underline{z}, \alpha)$$

Proposition 7 states that if the trading commissions component of an analyst's compensation is not very high, then the equilibrium has three regions: first, a fully revealing region, in which each analyst reveals his signal fully, when the prior of earnings is not very far from $\frac{1}{2}$; second, a noninformative region when the prior is extreme, close to either zero or one; and third, a partially informative region in which each analyst randomizes between high and low signals depending on his signal, when the prior diverges from $\frac{1}{2}$ but is not extreme. (The equilibrium regions are illustrated in the last panel of Figure 4 on page 31. To avoid notational clutter, $y, \overline{z}, \underline{z}$ and α in the parenthesis have been dropped.)

While the equilibrium features in proposition 7 are very similar to those in proposition 2 (with only one analyst), the main difference between the two propositions lies in the joint impact of the conditional correlation between analysts' signals and the reputational ranking payoffs on the informativeness of the equilibrium. Adding the second analyst introduces, on one hand, an incentive to implicitly "coordinate" among themselves to issue similar forecasts to generate more trading commissions and, on the other hand, an incentive to compete among themselves, especially with asymmetric reputational reward versus penalty values, to generate maximum reputational values in the labor market. Thus, the equilibrium regions now depend on the reputational ranking payoffs – $y, \overline{z}, \underline{z}$ – in addition to the relative weight of the trading commission motive (α).

In the next proposition, I show that the result shown in proposition 4 for a single analyst that trading commissions, along with reputational concerns, provide better incentive for honest reporting than reputational concerns alone, continues to hold true in the case with two analysts. The intuition here is exactly the same as in the case of a single analyst.

Proposition 8. *(Informativeness of Equilibrium)*
Suppose $\alpha \leq \alpha_{\max}(y, \overline{z}, \underline{z})$. *The region of priors (i.e., q) for which there is a fully revealing equilibrium increases, and the region of priors for which there is a noninformative equilibrium decreases as the relative weight of trading commissions (i.e., α) increases.*

Formally, if $\alpha_2 > \alpha_1$, *then* $\overline{q}(y, \overline{z}, \underline{z}, \alpha_2) \geqslant \overline{q}(y, \overline{z}, \underline{z}, \alpha_1)$ *and* $q_{\max}(y, \overline{z}, \underline{z}, \alpha_2) \geqslant q_{\max}(y, \overline{z}, \underline{z}, \alpha_1)$

5 Empirical Implications

In this section, I relate the predictions of my model to extant empirical regularities. In some cases, the intuitive explanations for those regularities are inconsistent with the forces driving the predictions of my model. Finally, I suggest some additional empirical implications of my model that have not yet been tested.

Implication 1: *Analysts' trading commissions are positively associated with their prior reputation* (property 1).

This implication is obvious and consistent with the findings that highly reputed analysts have a greater impact on stock price movements (Stickel 1992; Park and Stice 2000), generate more trade (Jackson 2005), and generate higher brokerage commissions (Irvine 2004). Indeed, precisely because of this characteristic, a personal reputation for forecast accuracy is important for sell-side analysts, and reputational ranking is one of the most important components of an analyst's compensation (Groysberg, Healy and Maber 2008).

Implication 2: *If an analyst is concerned solely with maximizing trading commissions, his trading commissions and the informativeness of his forecasts increase as the market's prior expectation of a company's future earnings becomes more diffuse (i.e., uncertain)* (proposition 1 and lemma 2).

This implication is consistent with the empirical evidence in Frankel, Kothari and Weber (2006), in which the authors found that the informativeness of an analyst's forecast increases when the uncertainty of a company's returns rises or the trading volume increases.

Although my predictions are consistent with the evidence in Frankel *et al.*, the explanation in their paper is somewhat different from mine. In Frankel *et al.* (2006), the authors argue that higher return uncertainties and increased trading volumes proxy for a greater demand for analysts' services, which, in turn, leads to greater informativeness in the analysts' reports. In my model, an analyst's report is more informative at higher uncertainties because it is in the best interest of the analyst to reveal his information at diffused priors. An analyst can generate more trading commissions at imprecise priors; however, the price will move and trade will occur only if the analyst's report is informative. My prediction that, at imprecise priors, an analyst generates more informative reports due to his incentive to maximize trading commissions complements Frankel *et al.*'s argument that, at a high uncertainty, there is a heightened demand for informative forecasts.

Next, I discuss how one key result of this paper relates to existing empirical regularities concerning the impact of trading commissions on the accuracy and informativeness of analyst forecasts.

Implication 3: *Trading commissions, together with reputational concerns, lead to higher information content of analyst forecasts than trading commissions or reputational concerns alone.* (propositions 3 and 8).

This prediction contradicts the belief in the popular press and empirical studies that the incentive to generate trading commissions is one of the major sources of conflicts of interest for sell-side analysts. The unsavory role potentially played by trading commissions has recently given rise to regulatory concerns, which have led to considering some forms of brokerage services as *not* "independent"[8] under the Global Settlement (2003).

It is typically argued that linking trading commissions to sell-side analysts' compensation encourages analysts to provide optimistic forecasts, as optimistic forecasts are more effective in generating trading volume. Any investor can act on an optimistic forecast at a relatively low cost by buying a stock, whereas a pessimistic forecast can only be acted upon by someone who owns the stock or who can afford the additional costs of short selling. Although my model does not include short-selling constraints, the forces driving my predictions will continue to hold true even in the presence of such constraints.

In my model, reputational concerns induce analysts to slant their forecasts towards the market's prior, which leads to a "conformist" bias. In contrast, a trading commission incentive motivates analysts to forecast against the market's prior. Thus, trading commissions, together with reputational concerns, provide a stronger incentive to report honestly than either incentive in isolation, by reversing the conformist bias.

Empirical evidence suggests that analysts' forecast biases, which are usually optimistic, are at least partially driven by trading commission incentive (e.g., Agarwal and Chen 2004; Francis, Cowen, Groysberg and Healy 2006; Ljungqvist et al 2005; Gu, Li and Yang 2009). Furthermore, Chen and Jiang (2005) have found that analysts often strategically overweigh their private information when they cover stocks for which the potential trading commissions are high. Assuming a rational market, the results of these studies, taken in conjunction, imply that the incentive to generate trading commissions will reduce the information content of an analyst's forecasts.

In contrast, Clement and Tse (2005) have found that analysts employed by large brokerage firms typically make "bold" forecasts, and yet those bold forecasts are more accurate and informative than herding forecasts. That bolder forecasts have stronger return responses is also consistent with studies by Clement and Tse (2003) and Gleason and Lee (2003). Furthermore, as mentioned previously, Frankel *et al.* (2006) have found that analyst forecasts are more informative when the potential brokerage profits are high.

[8] According to Attorney General of the State of New York (Addendum A, 2003, p.25), "Independent Research Providers may not perform investment banking business of any kind and may not provide *brokerage services in direct and significant competition with the firm*" (emphasis added).

It is worth noting that the empirical studies that emphasize the negative role of trading commissions in creating biases in analyst forecasts do not consider[9] the personal reputational concerns of the analysts. On the other hand, although Frankel et al. suggest trading commissions play a positive role in improving the information content of analyst forecasts, which is consistent with my prediction, their explanation is based primarily on the demand for the analysts' services, not on the incentives of analysts, the focus of my paper and other empirical studies.

On balance, this result of my model underscores the positive role played by trading commissions in sell-side analysts' forecasting behavior, which I believe, contributes to our understanding of how the incentives of trade generation and reputational concerns affect the informativeness of analyst forecasts. Finally, my model has two additional implications, neither of which have yet been tested empirically.

Implication 4: *When analysts are concerned with both trading commissions and their reputational values in the labor market, they will slant their forecasts towards the market's prior expectation, ignoring their private information, if the uncertainty of future earnings is very low* (propositions 2 and 7).

Implication 5: *A reputational ranking payoff structure in which the reward for being ranked higher is sufficiently higher (lower) than the penalty for being ranked lower will lead to less (more) information content of analyst forecasts* (proposition 6).

6 Conclusion

In the empirical accounting and finance literature, analyst forecasts are commonly used as proxies for investors' earnings expectations. The implicit assumption is that analyst forecasts truthfully reveal analysts' private information. In this paper, I demonstrated that given an analyst's incentives to maximize his trading commissions and his reputational value in the labor market, his forecasts often fall short of the truthful revelation of his private information. Indeed, a fully revealing equilibrium exists only at the intermediate values of the market's prior expectation of a company's earnings. At extreme values of the prior, reputational concerns create a "conformist" bias - analysts will tend to bias their forecasts towards the market's prior - leading

[9] except Ljungqvist *et al.* 2005, which considers reputational concerns of analysts indirectly through the institutional ownership of stocks the analysts cover.

to no information revelation in equilibrium. This perverse effect of analysts' reputational concerns may worsen in an asymmetrical reputational payoff structure. For example, a reputational ranking structure in which the reward for being the only good analyst is sufficiently larger than the penalty for being the only bad analyst can potentially impair information transmission.

Trading commissions alone can also create a perverse incentive. For a single analyst solely concerned with maximizing his trading commissions, honest reporting is impossible. However, for multiple analysts, honest forecasting is possible, albeit under a restrictive set of conditions, because of the analysts' desire to coordinate.

More interestingly, the consideration of trading commissions and reputational concerns in conjunction provides a stronger inducement to report honestly than either incentive in isolation. This result underscores the role of the trading commissions incentive in improving the information content of analyst forecasts.

There are several directions in which the stylized model in this paper can be extended. One useful extension would be to analyze a continuous signal and message space for the analysts. When an expert's only objective is to maximize his reputational value in the labor market, it has been shown, for a continuous signal space, that a fully revealing equilibrium cannot exist (Ottaviani and Sorensen 2006a). However, what the equilibrium would be remains an open question, except for a very restrictive case with a multiplicative linear signal structure (Ottaviani and Sorensen 2006b).

Another useful extension would be to analyze an endogenous reputation in a dynamic setting in which the market's posterior belief of an analyst's talent would be used as the prior reputation in the next period. In his seminal paper on managerial reputation, Holmstrom (1999) considers a case in which a manager's productive ability – initially unknown to the manager and the market – is revealed over time through the manager's choice of effort in equilibrium. In cases closer to my model, in which there is no effort choice by an expert, typically one type of expert is assumed to be "honest", i.e., a 'crazy' type who always reveals his private information honestly (e.g., Kreps and Wilson 1982; Benabou and Laroque 1992). In my model, the honesty of an analyst is endogenous and, hence, cannot be assumed. Moreover, integrating the comparison of one expert's reputation relative to others adds further complexity to my model in a dynamic setting.

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7 Appendix A

Calculation of prices. $P(m^h) = E[x|m^h] = x^H \Pr(x^H|m^h) + x^L \Pr(x^L|m^h)$. Also, by Bayes' rule, $\Pr(x^H|m^h) = \frac{\Pr(x^H)\Pr(m^h|x^H)}{\Pr(x^H)\Pr(m^h|x^H)+\Pr(x^L)\Pr(m^h|x^L)}$. Further, $\Pr(m^h|x^H) = \Pr(m^h|s^h)\Pr(s^h|x^H) + \Pr(m^h|s^l)\Pr(s^l|x^H) = \sigma_h\gamma + \sigma_l(1-\gamma)$. Similarly, $\Pr(m^h|x^L) = \sigma_h(1-\gamma)+\sigma_l\gamma$. Taken together, $\Pr(x^H|m^h) = \frac{[\sigma_h\gamma+\sigma_l(1-\gamma)]q}{[\sigma_h\gamma+\sigma_l(1-\gamma)]q+[\sigma_h(1-\gamma)+\sigma_l\gamma](1-q)}$. Similarly, $\Pr(x^L|m^h) = \frac{[\sigma_h(1-\gamma)+\sigma_l\gamma](1-q)}{[\sigma_h\gamma+\sigma_l(1-\gamma)]q+[\sigma_h(1-\gamma)+\sigma_l\gamma](h-q)}$. Finally, $P(m^h) = \frac{x^H[\sigma_h\gamma+\sigma_l(1-\gamma)]q+x^L[\sigma_h(1-\gamma)+\sigma_l\gamma](1-q)}{[\sigma_h\gamma+\sigma_l(1-\gamma)]q+[\sigma_h(1-\gamma)+\sigma_l\gamma](1-q)}$. The price for low report, $P(m^l)$, has been calculated in a similar way.

Analyst's Reputational Value

Using (28), analysts' reputational values are calculated as follows:

$$\begin{aligned}
U_i(s_i = x, s_{-i} = x, x) &= \frac{y\lambda^2\gamma_g + (\overline{z}+\underline{z})\lambda(1-\lambda)\gamma_g\gamma_b}{\lambda^2\gamma_g + 2\lambda(1-\lambda)\gamma_g\gamma_b + (1-\lambda)^2\gamma_b^2} \\
U_i(s_i = x, s_{-i} \neq x, x) &= \frac{\overline{z}\lambda(1-\lambda)\gamma_g(1-\gamma_b) + \underline{z}\lambda(1-\lambda)\gamma_b(1-\gamma_g)}{\lambda(1-\lambda)\gamma_g(1-\gamma_b) + \lambda(1-\lambda)\gamma_b(1-\gamma_g) + (1-\lambda)^2\gamma_b(1-\gamma_b)} \\
U_i(s_i \neq x, s_{-i} \neq x, x) &= \frac{y\lambda^2(1-\gamma_g) + (\overline{z}+\underline{z})\lambda(1-\lambda)(1-\gamma_g)(1-\gamma_b)}{\lambda^2(1-\gamma_g) + 2\lambda(1-\lambda)(1-\gamma_g)(1-\gamma_b) + (1-\lambda)^2(1-\gamma)_b^2} \\
U_i(s_i \neq x, s_{-i} = x, x) &= \frac{\overline{z}\lambda(1-\lambda)(1-\gamma_g)\gamma_b + \underline{z}\lambda(1-\lambda)(1-\gamma_b)\gamma_g}{\lambda(1-\lambda)(1-\gamma_g)\gamma_b + \lambda(1-\lambda)(1-\gamma_b)\gamma_g + (1-\lambda)^2(1-\gamma_b)\gamma_b}
\end{aligned}$$

Replacing γ_b with $\frac{1}{2}$, and after some algebra, reputational values are simplified as follows:

Property 3.

$$\begin{aligned}
U_i(s_i = x, s_{-i} = x, x) &= \frac{2\lambda\gamma_g\left[2y\lambda + (\overline{z}+\underline{z})(1-\lambda)\right]}{4\lambda\gamma_g + (1-\lambda)^2} \\
U_i(s_i = x, s_{-i} \neq x, x) &= \frac{2\lambda\left[\overline{z}\gamma_g + \underline{z}(1-\gamma_g)\right]}{1+\lambda} \\
U_i(s_i \neq x, s_{-i} \neq x, x) &= \frac{2\lambda(1-\gamma_g)\left[2y\lambda + (\overline{z}+\underline{z})(1-\lambda)\right]}{4\lambda(1-\gamma_g) + (1-\lambda)^2} \\
U_i(s_i \neq x, s_{-i} = x, x) &= \frac{2\lambda\left[\overline{z}(1-\gamma_g) + \underline{z}\gamma_g\right]}{1+\lambda}
\end{aligned}$$

8 Appendix B

Proof of Property 1. Since, by definition, $\gamma = \lambda\gamma_g + (1-\lambda)\gamma_b = \frac{1+\lambda(2\gamma_g-1)}{2}$, which is increasing in λ, it is enough to show that $\frac{\partial\pi(m)}{\partial\gamma} \geqslant 0$. I will show here the case for $\frac{\partial\pi(m^h)}{\partial\gamma} \geqslant 0$. The case for $\pi(m^l)$ can be shown analogously.

To show that $\frac{\partial\pi(m^h)}{\partial\gamma} \geqslant 0$, it is enough to show that $\frac{\partial}{\partial\gamma}|\frac{(2\gamma-1)(\sigma_h-\sigma_l)}{\sigma_h[\gamma q+(1-\gamma)(1-q)]+\sigma_l[(1-\gamma)q+\gamma(1-q)]}| \geqslant 0$. As explained earlier in footnote 4 (on page 15), I will consider cases only with $\sigma_h \geqslant \sigma_l$ because such a strategy pair is a more natural assumption than $\sigma_l \geqslant \sigma_h$, in which case the price falls after a high report and rises after a low report.

For any, $\sigma_h \geqslant \sigma_l$, $|\frac{(2\gamma-1)(\sigma_h-\sigma_l)}{\sigma_h[\gamma q+(1-\gamma)(1-q)]+\sigma_l[(1-\gamma)q+\gamma(1-q)]}| = \frac{(2\gamma-1)(\sigma_h-\sigma_l)}{\sigma_h[\gamma q+(1-\gamma)(1-q)]+\sigma_l[(1-\gamma)q+\gamma(1-q)]}$. Accordingly, it is enough to show that $\frac{\partial}{\partial\gamma}[\frac{(2\gamma-1)}{\sigma_h[\gamma q+(1-\gamma)(1-q)]+\sigma_l[(1-\gamma)q+\gamma(1-q)]}] \geqslant 0$. Note that, $\frac{\partial}{\partial\gamma}[\frac{(2\gamma-1)}{\sigma_h[\gamma q+(1-\gamma)(1-q)]+\sigma_l[(1-\gamma)q+\gamma(1-q)]}] = \frac{N}{\{\sigma_h[\gamma q+(1-\gamma)(1-q)]+\sigma_l[(1-\gamma)q+\gamma(1-q)]\}^2}$, where the numerator, $N \equiv 2\sigma_h[\gamma q+(1-\gamma)(1-q)]+2\sigma_l[(1-\gamma)q+\gamma(1-q)]-(2\gamma-1)[\sigma_h(q-(1-q))+\sigma_l(-q+(1-q))]$. Thus, $sgn(\frac{\partial}{\partial\gamma}[\frac{(2\gamma-1)}{\sigma_h[\gamma q+(1-\gamma)(1-q)]+\sigma_l[(1-\gamma)q+\gamma(1-q)]}]) = sgn(N)$. After some algebraic manipulations, $N = \sigma_h+\sigma_l \geqslant 0$. Thus, $\frac{\partial}{\partial\gamma}[\frac{(2\gamma-1)}{\sigma_h[\gamma q+(1-\gamma)(1-q)]+\sigma_l[(1-\gamma)q+\gamma(1-q)]}] \geqslant 0$, and so, $\frac{\partial\pi(m^h)}{\partial\gamma} \geqslant 0$. □

Proof of Lemma 1. I will prove this lemma by contradiction. Suppose that there exists an interval $q \in (0,1)$ such that $\sigma_h = 1$, $\sigma_l = 0$ is an equilibrium. For the putative equilibrium to exist, the following inequalities have to be satisfied. Given $s = h$, $\pi(m^h) \geqslant \pi(m^l)$, and $s = l$, $\pi(m^h) \leqslant \pi(m^l)$. Both the inequalities can hold simultaneously only if $\pi(m^h) = \pi(m^l)$.

Now, given $\sigma_h = 1$, $\sigma_l = 0$, equations (10) can be rewritten as

$$\pi(m^h) = \frac{(x^H - x^L)q(1-q)(2\gamma-1)}{[\gamma q + (1-\gamma)(1-q)]}$$
$$\pi(m^l) = \frac{(x^H - x^L)q(1-q)(2\gamma-1)}{[(1-\gamma)q + \gamma(1-q)]}$$

Thus, for $\pi(m^h) = \pi(m^l)$ to hold, it has to be true that $q = \frac{1}{2}$, which is not an interval, as assumed. This leads to a contradiction. Therefore, there does not exist any fully revealing equilibrium at any interval of prior of earnings.

Furthermore, $\pi(m^h) \geqslant \pi(m^l)$ if $q \leqslant \frac{1}{2}$, which means that given the market's naïve conjecture of full revelation, the analyst has an incentive to issue a high forecast if the prior is pessimistic.

Similarly, $\pi(m^l) > \pi(m^h)$ if $q > \frac{1}{2}$. This completes the proof. $\square$

Proof of Proposition 1. I will first prove part (i) – the case for $0 < q < \frac{1}{2}$ – of the proposition.

Suppose $\sigma_h = 1$, $\sigma_l \in (0,1)$. I will show that if $q \leqslant \frac{1}{2}$, the conjectured pair (σ_h, σ_l) is consistent in equilibrium. Two inequalities that need to hold in equilibrium are: $\pi(m^h) \geqslant \pi(m^l)$ and $\pi(m^h) = \pi(m^l)$. For both of these inequalities to be satisfied, $\pi(m^h) = \pi(m^l)$ $\Leftrightarrow |\frac{(x^H - x^L)q(1-q)(2\gamma-1)(1-\sigma_l)}{[\gamma q+(1-\gamma)(1-q)]+\sigma_l[(1-\gamma)q+\gamma(1-q)]}| - |\frac{(x^H - x^L)q(1-q)(2\gamma-1)(1-\sigma_l)}{(1-\sigma_l)[(1-\gamma)q+\gamma(1-q)]}| = 0$, which leads to, $[\gamma q + (1-\gamma)(1-q)] + \sigma_l[(1-\gamma)q + \gamma(1-q)] = (1-\sigma_l)[(1-\gamma)q + \gamma(1-q)]$. After some algebraic manipulations, $\pi(m^h) = \pi(m^l) \Leftrightarrow q[\sigma_h \gamma + \sigma_l(1-\gamma)] + (1-q)[\sigma_h(1-\gamma) + \sigma_l \gamma] = 1/2$, which simplifies to $\sigma_l = \frac{(2\gamma-1)(1-2q)}{2\{q(1-\gamma)+(1-q)\gamma\}}$.

Note that since $0 < q < \frac{1}{2}$, $\sigma_l > 0$. Also, $q(1-\gamma) + (1-q)\gamma > \frac{1}{2}$ for $q < \frac{1}{2}$. So, $\sigma_l < (2\gamma-1)(1-2q)$. Furthermore, $(2\gamma - 1) < 1$ because $\gamma < 1$, and $(1-2q) < 1$ because $q > 0$. Thus, $\sigma_l < (2\gamma-1)(1-2q) < 1$. Taken together, $\sigma_l \in (0,1)$. Thus, if $0 < q < \frac{1}{2}$, then $\pi(m^h) = \pi(m^l)$, which is consistent with $\pi(m^h) \geqslant \pi(m^l)$ and $\pi(m^h) = \pi(m^l)$, which further implies that $\sigma_h = 1$, $\sigma_l \in (0,1)$ is consistent in equilibrium.

Furthermore, it is easy to show that $\frac{\partial \sigma_l}{\partial q} < 0$ if $0 < q < \frac{1}{2}$, which implies that σ_l decreases – or informativeness of the equilibrium increases – as q gets closer to $\frac{1}{2}$. This proves the last part of (i) that the farther q is from $\frac{1}{2}$, the less information is revealed.

Part (ii) – the case for $\frac{1}{2} < q < 1$ – can be proved in an analogous manner.

I have already shown in the proof of lemma 1 that the analyst fully reveals his private signals in equilibrium, i.e., $\sigma_h = 1$, $\sigma_l = 0$ is an equilibrium only if $q = \frac{1}{2}$. This proves part (iii) of the proposition. $\square$

Proof of Lemma 2. Consider case $0 < q < \frac{1}{2}$. Take trading commissions, $\pi(m^h) = \pi(m^l) = \frac{(x^H - x^L)q(1-q)(2\gamma-1)(1-\sigma_l)}{[\gamma q+(1-\gamma)(1-q)]+\sigma_l[(1-\gamma)q+\gamma(1-q)]}$. By replacing the equilibrium $\sigma_l = \frac{(2\gamma-1)(1-2q)}{2\{q(1-\gamma)+(1-q)\gamma\}}$ from the proof of proposition 1, I get an analyst's equilibrium trading commissions, $\pi(m) = |\frac{(x^H - x^L)q(1-q)(2\gamma-1)(1-\sigma_l)}{(1-\sigma_l)[(1-\gamma)q+\gamma(1-q)]}| = \frac{(x^H - x^L)q(1-q)(2\gamma-1)}{[(1-\gamma)q+\gamma(1-q)]}$. Thus, $\frac{\partial \pi(m)}{\partial q} = (x^H - x^L)(2\gamma-1)\frac{\partial}{\partial q}[\frac{q(1-q)}{(1-\gamma)q+\gamma(1-q)}]$ $= (x^H - x^L)(2\gamma-1)\frac{[(1-2q)\{\gamma - q(2\gamma-1)\} + q(1-q)(2\gamma-1)]}{\{(1-\gamma)q+\gamma(1-q)\}^2}$. Note also that for $0 < q < \frac{1}{2}$, $\gamma - q(2\gamma-1) > 0$. Since $(x^H - x^L)(2\gamma - 1) > 0$, $(1-2q) > 0$ for $0 < q < \frac{1}{2}$ and $q(1-q)(2\gamma-1) > 0$, we can conclude that $\frac{\partial \pi(m)}{\partial q} > 0$. This means that equilibrium trading commissions is increasing in q in

$q \in (0, \frac{1}{2})$. Similarly, I can show that $\frac{\partial \pi(m)}{\partial q} < 0$ in $q \in (\frac{1}{2}, 1)$. Taken together, it is easy to see that $\pi(m)$ decreases as q moves farther from $q = \frac{1}{2}$.We can also see that $\pi(m) \to 0$ as either $q \to 0$ or $q \to 1$.

Now, by $\frac{\partial \pi(m)}{\partial q} > 0$ in $q \in (0, \frac{1}{2})$ and $\frac{\partial \pi(m)}{\partial q} < 0$ in $q \in (\frac{1}{2}, 1)$, and the fact that at $q = \frac{1}{2}$, the equilibrium trading commissions are well-defined as $\frac{(x^H - x^L)q(1-q)(2\gamma-1)}{[(1-\gamma)q+\gamma(1-q)]}$ (as I have already shown in the proof of lemma 1), it is straight forward to see that equilibrium trading commissions will be maximum at $q = \frac{1}{2}$. □

Proof of Lemma 3. The following posteriors of an analyst's reputation are calculated using Bayes' rule.

$$
\begin{aligned}
\Pr(g|m^h, x^H) &= \lambda \frac{\gamma_g \sigma_h + (1-\gamma_g)\sigma_l}{\gamma \sigma_h + (1-\gamma)\sigma_l} \\
\Pr(g|m^h, x^L) &= \lambda \frac{(1-\gamma_g)\sigma_h + \gamma_g \sigma_l}{(1-\gamma)\sigma_h + \gamma \sigma_l} \\
\Pr(g|m^l, x^H) &= \lambda \frac{\gamma_g(1-\sigma_h) + (1-\gamma_g)(1-\sigma_l)}{\gamma(1-\sigma_h) + (1-\gamma)(1-\sigma_l)} \\
\Pr(g|m^l, x^L) &= \lambda \frac{(1-\gamma_g)(1-\sigma_h) + \gamma_g(1-\sigma_l)}{(1-\gamma)(1-\sigma_h) + \gamma(1-\sigma_l)}
\end{aligned} \tag{39}
$$

I will prove part (i) of the lemma. Part (ii) can be proved with an analogous argument.

$$
\begin{aligned}
\Pr(g|m^h, x^H) - \Pr(g|m^l, x^H) &= \lambda[\frac{\gamma_g \sigma_h + (1-\gamma_g)\sigma_l}{\gamma \sigma_h + (1-\gamma)\sigma_l} - \frac{\gamma_g(1-\sigma_h) + (1-\gamma_g)(1-\sigma_l)}{\gamma(1-\sigma_h) + (1-\gamma)(1-\sigma_l)}] \\
&= \lambda[\frac{N}{\{\gamma \sigma_h + (1-\gamma)\sigma_l\}\{\gamma(1-\sigma_h) + (1-\gamma)(1-\sigma_l)\}}]
\end{aligned}
$$

where, as defined earlier, analyst's average signal precision, $\gamma = \lambda\gamma_g + (1-\lambda)\gamma_b$ and the numerator, $N \equiv \{\gamma_g \sigma_h + (1-\gamma_g)\sigma_l\}\{\gamma(1-\sigma_h) + (1-\gamma)(1-\sigma_l)\} - \{\gamma_g(1-\sigma_h) + (1-\gamma_g)(1-\sigma_l)\}\{\gamma\sigma_h + (1-\gamma)\sigma_l\}$. After some algebraic manipulations, $N = (1-\lambda)(\gamma_g - \gamma_b)(\sigma_h - \sigma_l) \geqslant 0$ because $\sigma_h \geqslant \sigma_l$. □

Proof of Proposition 2. I will prove this proposition by using lemma 6 below. In part (a) of lemma 6, I first show that if $q \geqslant \frac{1}{2}$, then an analyst with a high signal will always issue a high forecast. However, if he receives a low signal, then to what extent he can credibly communicate his private signal depends on the interval in which the prior (q) lies. Specifically, if the prior is not far from $\frac{1}{2}$, then he creports his low signal; if the prior is sufficiently high, then the analyst randomizes between high and low reports; if the prior is at the extreme, i.e., close to 1, then he will tend to "pool" with the analyst with a high signal by reporting high, leading to no information transmission in equilibrium. Part (b) of lemma 6 describes the analyst's forecasting behavior for $q \leqslant \frac{1}{2}$.

To identify the intervals of priors in which different types of equilibria lie, I define, $\overline{q}(\alpha) \equiv \sup\{q : (\sigma_h = 1,\ \sigma_l = 0)$ is an equilibrium$\}$ and $q_{\max}(\alpha) \equiv \inf\{q : (\sigma_h = 1,\ \sigma_l = 1)$ is the most informative equilibrium$\}$. Similarly, $\underline{q}(\alpha) \equiv \inf\{q : (\sigma_h = 1,\ \sigma_l = 0)$ is an equilibrium$\}$ and $q_{\min}(\alpha) \equiv \sup\{q : (\sigma_h = 0,\ \sigma_l = 0)$ is the most informative equilibrium$\}$. I define an equilibrium to be the *most informative equilibrium* if there exists no other equilibrium, in the feasible set of equilibria, that reveals more information than this equilibrium.

$\underline{q}(\alpha)$ and $\overline{q}(\alpha)$ identify the lower and upper bounds of the prior for which there is a fully revealing equilibrium. Thus, the strategy pair $(\sigma_h = 1,\ \sigma_l = 0)$ is an equilibrium if $q \in [\underline{q}(\alpha), \overline{q}(\alpha)]$. The only equilibrium in the interval of priors $q \in (q_{\max}(\alpha), 1)$ is an equilibrium with strategy pairs $(\sigma_h = 1,\ \sigma_l = 1)$, which implies that no information is transferred in this equilibrium. In the interval, $q \in (\overline{q}(\alpha), q_{\max}(\alpha)]$ an equilibrium with mixed strategies i.e., the pair $(\sigma_h = 1,\ \sigma_l \in (0,1))$ exists.

Finally, I consider only the natural equilibrium in which case $\sigma_h \geqslant \sigma_l$ (see discussions on the reasons for choosing such an equilibrium in footnote 4 on page 15).

Lemma 6. *(a) If $q \geqslant \frac{1}{2}$, then there exists an $\alpha'_{\max}$ such that for all $\alpha \leq \alpha'_{\max} \in (0,1)$, there exits a pair (σ_h, σ_l) satisfying the following equilibrium*

(i) $\sigma_h = 1,\ \sigma_l = 0$ if $q \in [\frac{1}{2}\ , \overline{q}(\alpha)]$

(ii) $\sigma_h = 1,\ \sigma_l \in (0,1)$ if $q \in (\overline{q}, q_{\max}(\alpha)]$

(iii) $\sigma_h = 1 = \sigma_l$ if $q \in (q_{\max}(\alpha), 1)$

(b) If $q \leqslant \frac{1}{2}$, then there exists an $\alpha''_{\max}$ such that for all $\alpha \leq \alpha''_{\max} \in (0,1)$, there exits a pair (σ_h, σ_l) satisfying the following equilibrium

(i) $\sigma_h = 1,\ \sigma_l = 0$ if $q \in [\underline{q}(\alpha), \frac{1}{2}]$

(ii) $\sigma_h \in (0,1),\ \sigma_l = 0$ if $q \in [q_{\min}(\alpha), \underline{q}(\alpha))$

(iii) $\sigma_h = 0 = \sigma_l$ if $q \in (0, q_{\min}(\alpha))$

Proof of Lemma 6. I will prove part (a) of the lemma. Part (b) can be proved by an analogous argument. To prove part (a), I will use the following claims.

Claim 1. *$\triangle R_h \geqslant \triangle R_l$ for any $q \in (0,1)$ for any $\sigma_h \geqslant \sigma_l$*

Proof. By definition, $\triangle R_h \equiv R(m^h|s^h) - R(m^l|s^h) = \Pr(x^H|s^h)[\Pr(g|m^h, x^H) - \Pr(g|m^l, x^H)] - \Pr(x^L|s^h)[\Pr(g|m^h, x^L) - \Pr(g|m^l, x^L)]$, and $\triangle R_l \equiv R(m^h|s^l) - R(m^l|s^l) = \Pr(x^H|s^l)[\Pr(g|m^h, x^H) - \Pr(g|m^l, x^H)] - \Pr(x^L|s^l)[\Pr(g|m^h, x^L) - \Pr(g|m^l, x^L)]$.

Now, $\triangle R_h - \triangle R_l = [\Pr(x^H|s^h) - \Pr(x^H|s^l)][\Pr(g|m^h, x^H) - \Pr(g|m^l, x^H)] + [\Pr(x^L|s^h) - \Pr(x^L|s^l)][\Pr(g|m^h, x^L) - \Pr(g|m^l, x^L)] = [\Pr(x^H|s^h) - \Pr(x^H|s^l)][\Pr(g|m^h, x^H) - \Pr(g|m^l, x^H)] + [\Pr(x^L|s^l) - \Pr(x^L|s^h)][\Pr(g|m^l, x^L) - \Pr(g|m^h, x^L)]$.

However, $\Pr(x^H|s^h) \geqslant \Pr(x^H|s^l)$ and $\Pr(x^L|s^l) \geqslant \Pr(x^L|s^h)$ for any $q \in (0,1)$. Also, by lemma 3, $\Pr(g|m^h, x^H) \geqslant \Pr(g|m^l, x^H)$, and $\Pr(g|m^l, x^L) \geqslant \Pr(g|m^h, x^L)$. Taken together, $\triangle R_h - \triangle R_l \geqslant 0$. □

Claim 2. *$\triangle R_h(\sigma_h = 1, \sigma_l, q) > 0$ for any $q \geqslant \frac{1}{2}$*

Proof. By definition, $\triangle R_h(\sigma_h = 1, \sigma_l, q) = \Pr(x^H|s^h)[\Pr(g|m^h, x^H, \sigma_h = 1) - \Pr(g|m^l, x^H, \sigma_h = 1)] + \Pr(x^L|s^h)[\Pr(g|m^h, x^L, \sigma_h = 1) - \Pr(g|m^l, x^L, \sigma_h = 1)]$. Using the expressions of analyst's reputation from (39) on page 51,

$\Pr(g|m^h, x^H, \sigma_h = 1) - \Pr(g|m^l, x^H, \sigma_h = 1) = \lambda\frac{\gamma_g + (1-\gamma_g)\sigma_l}{\gamma + (1-\gamma)\sigma_l} - \lambda\frac{(1-\gamma_g)}{(1-\gamma)} = \lambda\frac{\gamma_g - \gamma}{(1-\gamma)(\gamma + (1-\gamma)\sigma_l)}$, after some algebraic manipulations. Similarly, $\Pr(g|m^h, x^L, \sigma_h = 1) - \Pr(g|m^l, x^L, \sigma_h = 1) = \lambda\frac{(1-\gamma_g) + \gamma_g\sigma_l}{(1-\gamma) + \gamma\sigma_l} - \lambda\frac{\gamma_g}{\gamma} = -\lambda\frac{\gamma_g - \gamma}{\gamma(1-\gamma+\gamma\sigma_l)}$.

Thus, $\triangle R_h(\sigma_h = 1, \sigma_l, q) = \Pr(x^H|s^h)[\lambda\frac{\gamma_g - \gamma}{(1-\gamma)(\gamma + (1-\gamma)\sigma_l)}] + \Pr(x^L|s^h)[-\lambda\frac{\gamma_g - \gamma}{\gamma(1-\gamma+\gamma\sigma_l)}]$.Also, $\Pr(x^H|s^h) = \frac{q\gamma}{q\gamma + (1-q)(1-q\gamma)}$ and $\Pr(x^L|s^h) = \frac{(1-q)(1-q\gamma)}{q\gamma + (1-q)(1-q\gamma)}$, which implies that $\triangle R_h(\sigma_h = 1, \sigma_l, q)$ is increasing in q. Now, calculating $\triangle R_h$ at $q = \frac{1}{2}$, I derive, $\triangle R_h(\sigma_h = 1, \sigma_l, q = \frac{1}{2}) = \Pr(x^H|s^h; q = \frac{1}{2})[\lambda\frac{\gamma_g - \gamma}{(1-\gamma)(\gamma + (1-\gamma)\sigma_l)}] - \Pr(x^L|s^h; q = \frac{1}{2})[\lambda\frac{\gamma_g - \gamma}{\gamma(1-\gamma+\gamma\sigma_l)}] = \gamma[\lambda\frac{\gamma_g - \gamma}{(1-\gamma)(\gamma + (1-\gamma)\sigma_l)}] - (1-\gamma)[\lambda\frac{\gamma_g - \gamma}{\gamma(1-\gamma+\gamma\sigma_l)}] = \lambda\frac{(\gamma_g - \gamma)(2\gamma - 1)\{\gamma(1-\gamma) + \sigma_l\}}{\{(1-\gamma)(\gamma + (1-\gamma)\sigma_l)\}\{\gamma(1-\gamma+\gamma\sigma_l)\}} > 0$. Thus, $\triangle R_h(\sigma_h = 1, \sigma_l, q) > 0$ for any $q \geqslant \frac{1}{2}$. □

Claim 3. *$\triangle\pi(\sigma_h = 1, \sigma_l, q) < 0$ if $q \geqslant \frac{1}{2}$, and $\triangle\pi(\sigma_h = 1, \sigma_l, q) = 0$ if $q = \frac{1}{2}$ and $\sigma_l = 0$.*

Proof. Using (10) on page 15, and the assumption that $\sigma_h \geqslant \sigma_l$,

$\triangle\pi(\sigma_h, \sigma_l, q) = \pi(m^h) - \pi(m^l)$
$= \frac{(x^H - x^L)q(1-q)(2\gamma-1)(\sigma_h - \sigma_l)}{\sigma_h[\gamma q+(1-\gamma)(1-q)]+\sigma_l[(1-\gamma)q+\gamma(1-q)]} - \frac{(x^H - x^L)q(1-q)(2\gamma-1)(\sigma_h - \sigma_l)}{(1-\sigma_h)[\gamma q+(1-\gamma)(1-q)]+(1-\sigma_l)[(1-\gamma)q+\gamma(1-q)]}$
$= \frac{(x^H - x^L)q(1-q)(2\gamma-1)(\sigma_h - \sigma_l)[(1-2\sigma_h)\{\gamma q+(1-\gamma)(1-q)\}+(1-2\sigma_h)\{(1-\gamma)q+\gamma(1-q)\}]}{\{\sigma_h[\gamma q+(1-\gamma)(1-q)]+\sigma_l[(1-\gamma)q+\gamma(1-q)]\}\{(1-\sigma_h)[\gamma q+(1-\gamma)(1-q)]+(1-\sigma_l)[(1-\gamma)q+\gamma(1-q)]\}}.$

Now, at $\sigma_h = 1$, $\triangle\pi(\sigma_h = 1, \sigma_l, q) =$
$\frac{(x^H - x^L)q(1-q)(2\gamma-1)(1-\sigma_l)[(1-2)\{\gamma q+(1-\gamma)(1-q)\}+(1-2\sigma_l)\{(1-\gamma)q+\gamma(1-q)\}]}{\{[\gamma q+(1-\gamma)(1-q)]+\sigma_l[(1-\gamma)q+\gamma(1-q)]\}\{(1-\sigma_l)[(1-\gamma)q+\gamma(1-q)]\}}$. Thus, $sgn(\triangle\pi(\sigma_h = 1, \sigma_l, q)) = sgn((1-2)\{\gamma q + (1-\gamma)(1-q)\} + (1-2\sigma_l)\{(1-\gamma)q + \gamma(1-q)\}) = sgn([2(1-\sigma_l)\{(1-\gamma)q + \gamma(1-q)\} - 1])$ since $\{\gamma q + (1-\gamma)(1-q)\} = 1 - \{(1-\gamma)q + \gamma(1-q)\}$.

Furthermore, $\{(1-\gamma)q+\gamma(1-q)\}$ is decreasing in q and $\{(1-\gamma)q+\gamma(1-q)\} = \frac{1}{2}$ at $q = \frac{1}{2}$. Thus, if $q \geqslant \frac{1}{2}$, then $\{(1-\gamma)q + \gamma(1-q)\} \leqslant \frac{1}{2}$, and $2(1-\sigma_l)\{(1-\gamma)q + \gamma(1-q)\} \leqslant 1 - \sigma_l$. Finally, $2(1-\sigma_l)\{(1-\gamma)q+\gamma(1-q)\} - 1 \leqslant -\sigma_l < (=)\ 0$ if $\sigma_l \in (0,1](=0)$ and $q \geqslant \frac{1}{2}$. Therefore, $\triangle\pi(\sigma_h = 1, \sigma_l, q) < 0$ if $q \geqslant \frac{1}{2}$, and $\triangle\pi(\sigma_h = 1, \sigma_l, q) = 0$ if $q = \frac{1}{2}$ and $\sigma_l = 0$. □

Claim 4. *If $q \geqslant \frac{1}{2}$, then there exists an $\alpha'_{\max} \in (0,1)$ such that for all $\alpha \leq \alpha'_{\max}$, $\triangle V_h(\sigma_h = 1, \sigma_l, q, \alpha) \geqslant 0$*

Proof. By definition, $\triangle V_h(\sigma_h = 1, \sigma_l, q, \alpha) = \alpha \triangle \pi(\sigma_h = 1, \sigma_l, q) + (1-\alpha) \triangle R_h(\sigma_h = 1, \sigma_l, q)$. If $q \geqslant \frac{1}{2}$, then, by claim 3, $\triangle\pi(\sigma_h = 1, \sigma_l, q) < 0$ ($= 0$ only if $q = \frac{1}{2}$ and $\sigma_l = 0$), and, by claim 2, $\triangle R_h(\sigma_h = 1, \sigma_l, q) > 0$. Now, for any fixed q and σ_l, since $\triangle V_h(\sigma_h = 1, \sigma_l, q, \alpha)$ is continuous in α, there exists an $\overline{\alpha}(\sigma_l, q) \in (0,1]$ (note that $\overline{\alpha}(\sigma_l, q) = 1$ only if $q = \frac{1}{2}$ and $\sigma_l = 0$) such that for all $\alpha \leq \overline{\alpha}(\sigma_l, q)$, $\triangle V_h(\sigma_h = 1, \sigma_l, q, \alpha) \geqslant 0$, where $\overline{\alpha}(\sigma_l, q)$ satisfies the equation, $\triangle V_h(\sigma_h = 1, \sigma_l, q, \overline{\alpha}(\sigma_l, q)) = 0$. Further expanding $\triangle V_h(\sigma_h = 1, \sigma_l, q, \overline{\alpha}(\sigma_l, q)) = 0$, which implies $\overline{\alpha}(\sigma_l, q) \triangle \pi(\sigma_h = 1, \sigma_l, q) + (1 - \overline{\alpha}(\sigma_l, q)) \triangle R_h(\sigma_h = 1, \sigma_l, q) = 0$, which further implies

$$\overline{\alpha}(\sigma_l, q) = \frac{1}{1 + [\frac{-\triangle\pi(\sigma_h=1,\sigma_l,q)}{\triangle R_h(\sigma_h=1,\sigma_l,q)}]}$$

Define $\alpha'_{\max} \equiv \min_{\sigma_l, q} \{\overline{\alpha}(\sigma_l, q)\ \} \in (0,1)$. Thus, for all $\alpha \leq \alpha'_{\max}$, $\triangle V_h(\sigma_h = 1, \sigma_l, q, \alpha) \geqslant 0$ if $q \geqslant \frac{1}{2}$. □

Claim 5. *There exists a $q_{\max}(\alpha) \in (\frac{1}{2}, 1)$ such that for any $q \in (q_{\max}(\alpha), 1)$, $\triangle V_l(\sigma_h = 1, \sigma_l, q, \alpha) > 0$. At $q = q_{\max}(\alpha)$, $\triangle V_l(\sigma_h = 1, \sigma_l^{\min}(q), q, \alpha) = 0$, where $\sigma_l^{\min}(q) = \min\{\sigma_l :$ $(\sigma_h = 1, \sigma_l \in (0,1))$ is an equilibrium for a given $q\}$.*

Proof. By definition, $\triangle V_l(\sigma_h = 1, \sigma_l, q, \alpha) = \alpha \triangle \pi(\sigma_h = 1, \sigma_l, q) + (1-\alpha) \triangle R_l(\sigma_h = 1, \sigma_l, q)$. Now, from equations (10), $\triangle\pi(\sigma_h = 1, \sigma_l, q) = \frac{(x^H - x^L)q(1-q)(2\gamma-1)(1-\sigma_l)}{\gamma q + (1-\gamma)(1-q) + \sigma_l[(1-\gamma)q + \gamma(1-q)]} - \frac{(x^H-x^L)q(1-q)(2\gamma-1)}{(1-\gamma)q+\gamma(1-q)}$. Note that $\frac{(x^H - x^L)q(1-q)(2\gamma-1)(1-\sigma_l)}{\gamma q + (1-\gamma)(1-q) + \sigma_l[(1-\gamma)q + \gamma(1-q)]}$ is increasing in σ_l and $\lim_{\sigma_l \to 1} \frac{(x^H - x^L)q(1-q)(2\gamma-1)(1-\sigma_l)}{\gamma q + (1-\gamma)(1-q) + \sigma_l[(1-\gamma)q + \gamma(1-q)]} = 0$. Thus, $\frac{(x^H - x^L)q(1-q)(2\gamma-1)(1-\sigma_l)}{\gamma q + (1-\gamma)(1-q) + \sigma_l[(1-\gamma)q + \gamma(1-q)]} \geqslant 0$ and $\triangle\pi(\sigma_h = 1, \sigma_l, q) \geqslant -\frac{(x^H-x^L)q(1-q)(2\gamma-1)}{(1-\gamma)q+\gamma(1-q)}$.

By definition, $\triangle R_l(\sigma_h = 1, \sigma_l, q) = \Pr(x^H|s^l)[\Pr(g|m^h, x^H, \sigma_h = 1) - \Pr(g|m^l, x^H, \sigma_h = 1)] + \Pr(x^L|s^l)[\Pr(g|m^h, x^L, \sigma_h = 1) - \Pr(g|m^l, x^L, \sigma_h = 1)]$. From the proof of claim 2, $\Pr(g|m^h, x^H, \sigma_h = 1) - \Pr(g|m^l, x^H, \sigma_h = 1) = \lambda\frac{\gamma_g - \gamma}{(1-\gamma)(\gamma + (1-\gamma)\sigma_l)}$ which is decreasing in σ_l. Thus, $\Pr(g|m^h, x^H, \sigma_h = 1) - \Pr(g|m^l, x^H, \sigma_h = 1) \geqslant \lambda\frac{(\gamma_g - \gamma)}{(1-\gamma)}$ (which is $\lim_{\sigma_l \to 1} \lambda\frac{\gamma_g - \gamma}{(1-\gamma)(\gamma + (1-\gamma)\sigma_l)}$). Again, from the proof of claim 2, $\Pr(g|m^h, x^L, \sigma_h = 1) - \Pr(g|m^l, x^L, \sigma_h = 1) = -\lambda\frac{\gamma_g - \gamma}{\gamma(1-\gamma+\gamma\sigma_l)}$, which is increasing in σ_l. Thus, $\Pr(g|m^h, x^L, \sigma_h = 1) - \Pr(g|m^l, x^L, \sigma_h = 1) \geqslant -\lambda\frac{(\gamma_g - \gamma)}{\gamma(1-\gamma)}$(which is $\lim_{\sigma_l \to 0}[-\lambda\frac{\gamma_g - \gamma}{\gamma(1-\gamma+\gamma\sigma_l)}]$). Therefore, $\triangle R_l(\sigma_h = 1, \sigma_l, q) \geqslant \Pr(x^H|s^l)[\lambda\frac{(\gamma_g - \gamma)}{(1-\gamma)}] + \Pr(x^L|s^l)[-\lambda\frac{(\gamma_g - \gamma)}{\gamma(1-\gamma)}] = \Pr(x^H|s^l)[\lambda\frac{(\gamma_g - \gamma)}{(1-\gamma)}] + \{1 - \Pr(x^H|s^l)\}[-\lambda\frac{(\gamma_g - \gamma)}{\gamma(1-\gamma)}] = [-\lambda\frac{(\gamma_g - \gamma)}{\gamma(1-\gamma)} + \{\lambda\frac{(\gamma_g - \gamma)}{(1-\gamma)}\}(1 + \frac{1}{\gamma})\Pr(x^H|s^l)]$

So far, I have shown that $\triangle\pi(\sigma_h = 1, \sigma_l, q) \geqslant -\frac{(x^H-x^L)q(1-q)(2\gamma-1)}{(1-\gamma)q+\gamma(1-q)}$ and $\triangle R_l(\sigma_h = 1, \sigma_l, q) \geqslant [-\lambda\frac{(\gamma_g - \gamma)}{\gamma(1-\gamma)} + \{\lambda\frac{(\gamma_g - \gamma)}{(1-\gamma)}\}(1 + \frac{1}{\gamma})\Pr(x^H|s^l)]$. Taken together, $\triangle V_l(\sigma_h = 1, \sigma_l, q, \alpha) \geqslant \alpha[-\frac{(x^H-x^L)q(1-q)(2\gamma-1)}{(1-\gamma)q+\gamma(1-q)}] + (1-\alpha)[-\lambda\frac{(\gamma_g - \gamma)}{\gamma(1-\gamma)} + \{\lambda\frac{(\gamma_g - \gamma)}{(1-\gamma)}\}(1 + \frac{1}{\gamma})\Pr(x^H|s^l)]$. However, $\Pr(x^H|s^l) = \frac{q(1-\gamma)}{q(1-\gamma)+(1-q)\gamma} = \frac{q(1-\gamma)}{\gamma - q(2\gamma-1)}$. Thus, $\triangle V_l(\sigma_h = 1, \sigma_l, q, \alpha) \geqslant \alpha[-\frac{(x^H-x^L)q(1-q)(2\gamma-1)}{(1-\gamma)q+\gamma(1-q)}] + (1-\alpha)[-\lambda\frac{(\gamma_g - \gamma)}{\gamma(1-\gamma)} + \{\lambda\frac{(\gamma_g - \gamma)(1+\gamma)}{\gamma(1-\gamma)}\}\{\frac{q(1-\gamma)}{\gamma - q(2\gamma-1)}\}]$.

Define $f(q) \equiv \alpha[-\frac{(x^H-x^L)q(1-q)(2\gamma-1)}{(1-\gamma)q+\gamma(1-q)}] + (1-\alpha)[-\lambda\frac{(\gamma_g - \gamma)}{\gamma(1-\gamma)} + \{\lambda\frac{(\gamma_g - \gamma)(1+\gamma)}{\gamma(1-\gamma)}\}\{\frac{q(1-\gamma)}{\gamma - q(2\gamma-1)}\}]$ so that $\triangle V_l(\sigma_h = 1, \sigma_l, q, \alpha) \geqslant f(q)$. Note that $f(q)$ is a quadratic function of q. Furthermore, $\lim_{q\to 0} f(q) < 0$, $f(q = \frac{1}{2}) < 0$, and $\lim_{q \to 1} f(q) > 0$. Thus, there exists a unique $q_{\max}(\alpha) \in (\frac{1}{2}, 1)$ such that $f(q = q_{\max}(\alpha)) = 0$ and $f(q) > 0$ if $q \in (q_{\max}(\alpha), 1)$. The other point at which $f(q) = 0$ has to be some $q > 1$, which is not feasible.

Therefore, there exists a unique $q_{\max}(\alpha) \in (\frac{1}{2}, 1)$ such that for $q \in (q_{\max}(\alpha), 1)$, $\triangle V_l(\sigma_h = 1, \sigma_l, q, \alpha) \geqslant f(q) > 0$.

Furthermore, by the definition of $q_{\max}(\alpha)$, at $q = q_{\max}(\alpha)$, $\triangle V_l(\sigma_h = 1, \sigma_l^{\min}(q), q, \alpha) = 0$, where $\sigma_l^{\min}(q) = \min\{\sigma_l : (\sigma_h = 1, \sigma_l \in (0,1))$ is an equilibrium for a given $q\}$. $\square$

Claim 6. *Suppose* $\alpha \leq \alpha'_{\max}$. $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, \alpha) \leqslant 0$ *if* $q \in [\frac{1}{2}\ , \bar{q}(\alpha)]$

Proof. By definition, $\triangle R_l(\sigma_h, \sigma_l, q) = \Pr(x^H|s^l)[\Pr(g|m^h, x^H) - \Pr(g|m^l, x^H)] +$

$\Pr(x^L|s^l)[\Pr(g|m^h,x^L)-\Pr(g|m^l,x^L)]$. Using (39), at $\sigma_h=1,\sigma_l=0$, $\Pr(g|m^h,x^H)-\Pr(g|m^l,x^H)$ $=\lambda\frac{(\gamma_g-\gamma)}{\gamma(1-\gamma)}$ and $\Pr(g|m^h,x^L)-\Pr(g|m^l,x^L)=-\lambda\frac{(\gamma_g-\gamma)}{\gamma(1-\gamma)}$. Therefore, $\triangle R_l(\sigma_h=1,\sigma_l=0,q)$ $=\lambda\frac{(\gamma_g-\gamma)}{\gamma(1-\gamma)}[\Pr(x^H|s^l)-\Pr(x^L|s^l)]=[\lambda\frac{(\gamma_g-\gamma)}{\gamma(1-\gamma)}][\frac{q-\gamma}{q(1-\gamma)+(1-q)\gamma}]\leqslant 0$ if $q\leqslant\gamma$ and >0 if $q>\gamma$.

Also, by claim 3, $\triangle\pi(\sigma_h=1,\sigma_l=0,q)<(=)\ 0$ if $q>(=)\ \frac{1}{2}$. Thus, for any $\alpha\leq\alpha'_{\max}$, $\triangle V_l(\sigma_h=1,\sigma_l=0,q,\alpha)=\alpha\triangle\pi(\sigma_h=1,\sigma_l=0,q)+(1-\alpha)\triangle R_l(\sigma_h=1,\sigma_l=0,q)<0$ if $q\leqslant\gamma$. Since, by definition, $\overline{q}(\alpha)\equiv\max\{q:\sigma_h=1,\ \sigma_l=0\ \}$, there exists a $\overline{q}(\alpha)>\gamma$ such that $\triangle V_l(\sigma_h=1,\sigma_l=0,\overline{q}(\alpha),\alpha)=0$, and $\triangle V_l(\sigma_h=1,\sigma_l=0,q,\alpha)\leqslant 0$ if $\frac{1}{2}\leqslant q\leqslant\overline{q}(\alpha)$. □

Now, I return to proving part (a) of lemma 6. By claims 4 and 6, for any $\alpha\leq\alpha'_{\max}$, $\triangle V_h(\sigma_h=1,\sigma_l,q,\alpha)\geqslant 0$ and $\triangle V_l(\sigma_h=1,\sigma_l=0,q,\alpha)\leqslant 0$ if $q\in[\frac{1}{2},\overline{q}(\alpha)]$. This proves part (i). Also, by claims 4 and 5, for any $\alpha\leq\alpha'_{\max}$, $\triangle V_h(\sigma_h=1,\sigma_l,q,\alpha)>0$ and $\triangle V_l(\sigma_h=1,\sigma_l,q,\alpha)>0$ if $q\in(q_{\max}(\alpha),1)$. This proves part (iii). Part (ii) follows from the fact that $\triangle V_l(\sigma_h=1,\sigma_l,q,\alpha)=0$ if $\overline{q}(\alpha)<q\leqslant q_{\max}(\alpha)$ (since $\triangle V_l(\sigma_h=1,\sigma_l=0,q,\alpha)\leqslant 0$ if $q\leqslant\overline{q}(\alpha)$ and $\triangle V_l(\sigma_h=1,\sigma_l,q,\alpha)>0$ for any $q>q_{\max}(\alpha)$ by claims 6 and 5 respectively) and by claim 4 that for any $\alpha\leq\alpha'_{\max}$, $\triangle V_h(\sigma_h=1,\sigma_l,q,\alpha)\geqslant 0$. This completes the proof of part (a).

Part (b) can be proved in an analogous argument for any $\alpha\leq\alpha''_{\max}$. By symmetry, it can be shown that $\underline{q}(\alpha)=1-\overline{q}(\alpha)$ and $q_{\min}(\alpha)=1-q_{\max}(\alpha)$. □

Finally, I return to completing the proof of proposition 2. Define $\alpha_{\max}\equiv\min\{\alpha'_{\max},\alpha''_{\max}\}\in(0,1)$. Now, using the results of lemma 6, the results of proposition 2 follow directly. □

Proof of Proposition 3. Suppose $\alpha\leq\alpha_{\max}$. I will first show that if $\alpha_2>\alpha_1$, then $\overline{q}(\alpha_2)\geqslant\overline{q}(\alpha_1)$. By definition, $\overline{q}(\alpha)\equiv\max\{q:(\sigma_h=1,\ \sigma_l=0)$ is an equilibrium$\}$, and thus, $\triangle V_l(\sigma_h=1,\sigma_l=0,\overline{q}(\alpha),\alpha)=0$ and $\triangle V_l(\sigma_h=1,\sigma_l=0,q,\alpha)<0$ for any $q<\overline{q}(\alpha)$. For $\alpha=\alpha_1$, $\triangle V_l(\sigma_h=1,\sigma_l=0,\overline{q}(\alpha_1),\alpha_1)=0$. However, by claim 3, $\triangle\pi(\sigma_h=1,\sigma_l=0,\overline{q}(\alpha_1))<0$. Therefore, $\triangle R_l(\sigma_h=1,\sigma_l=0,\overline{q}(\alpha_1))>0$.

Now for $\alpha_2>\alpha_1$, I get $\triangle V_l(\sigma_h=1,\sigma_l=0,\overline{q}(\alpha_1),\alpha_2)=\alpha_2\triangle\pi(\sigma_h=1,\sigma_l=0,\overline{q}(\alpha_1))+(1-\alpha_2)\triangle R_l(\sigma_h=1,\sigma_l=0,\overline{q}(\alpha_1))<0$. Since, by definition, $\triangle V_l(\sigma_h=1,\sigma_l=0,\overline{q}(\alpha),\alpha)=0$, there exists $\overline{q}(\alpha_2)\geqslant\overline{q}(\alpha_1)$ such that $\triangle V_l(\sigma_h=1,\sigma_l=0,\overline{q}(\alpha_2),\alpha_2)=0$. Thus, $\overline{q}(\alpha_2)\geqslant\overline{q}(\alpha_1)$.

Next, I will show that if $\alpha_2>\alpha_1$, then $q_{\max}(\alpha_2)\geqslant q_{\max}(\alpha_1)$. By definition, $q_{\max}(\alpha)\equiv\min\{q:(\sigma_h=1,\ \sigma_l=1)$ is an equilibrium$\}$, and also, from claim 5, $\triangle V_l(\sigma_h=1,\sigma_l^{\min}(q_{\max}(\alpha)),q_{\max}(\alpha),\alpha)=$

0. For $\alpha = \alpha_1, \triangle V_l(\sigma_h = 1, \sigma_l^{\min}(q_{\max}(\alpha_1)), q_{\max}(\alpha_1), \alpha_1) = 0$. However, by claim 3, $\triangle\pi(\sigma_h = 1, \sigma_l^{\min}(q_{\max}(\alpha_1)), q_{\max}(\alpha_1)) < 0$. Thus, $\triangle R_l(\sigma_h = 1, \sigma_l^{\min}(q_{\max}(\alpha_1)), q_{\max}(\alpha_1)) > 0$.

Now for $\alpha_2 > \alpha_1$, I get, $\triangle V_l(\sigma_h = 1, \sigma_l^{\min}(q_{\max}(\alpha_1)), q_{\max}(\alpha_1), \alpha_2) = \alpha_2 \triangle \pi(\sigma_h = 1, \sigma_l^{\min}(q_{\max}(\alpha_1)), q_{\max}(\alpha_1)) + (1 - \alpha_2) \triangle R_l(\sigma_h = 1, \sigma_l^{\min}(q_{\max}(\alpha_1)), q_{\max}(\alpha_1)) < 0$. Again, since from claim 5, $\triangle V_l(\sigma_h = 1, \sigma_l^{\min}(q_{\max}(\alpha)), q_{\max}(\alpha), \alpha) = 0$, there exists $q_{\max}(\alpha_2) \geqslant q_{\max}(\alpha_1)$ such that $\triangle V_l(\sigma_h = 1, \sigma_l^{\min}(q_{\max}(\alpha_2)), q_{\max}(\alpha_2), \alpha_2) = 0$. Thus, $q_{\max}(\alpha_2) \geqslant q_{\max}(\alpha_1)$. $\square$

Proof of Property 2. By definition,

$$\begin{aligned}
&\Pr(s_i, s_{-i} \mid x) \\
&= \sum_{\theta_i}\sum_{\theta_{-i}} \Pr(\theta_i, \theta_{-i}) \Pr(s_i, s_{-i} \mid x, \theta_i, \theta_{-i}) \\
&= \Pr(g_i, g_{-i}) \Pr(s_i, s_{-i} \mid x, g_i, g_{-i}) + \Pr(g_i, b_{-i}) \Pr(s_i, s_{-i} \mid x, g_i, b_{-i}) + \\
&\quad \Pr(b_i, g_{-i}) \Pr(s_i, s_{-i} \mid x, b_i, g_{-i}) + \Pr(b_i, b_{-i}) \Pr(s_i, s_{-i} \mid x, b_i, b_{-i}) \\
&= \lambda^2 \Pr(s_i, s_{-i} \mid x, g_i, g_{-i}) + \lambda(1-\lambda) \Pr(s_i \mid x, g_i) \Pr(s_{-i} \mid x, b_{-i}) + \\
&\quad (1-\lambda)\lambda \Pr(s_i \mid x, b_i) \Pr(s_{-i} \mid x, g_{-i}) + (1-\lambda)^2 \Pr(s_i \mid x, b_i) \Pr(s_{-i} \mid x, b_{-i})
\end{aligned}$$

The last three terms of the third equality follow from the assumption that the signals of analysts are conditionally independent if at least one of them is bad. The first term, $\Pr(s_i, s_{-i} \mid x, g_i, g_{-i})$, involves the correlation of signals between two good analysts for which the values are (i) γ_g if $s_i = s_{-i} = x$, (ii) $1 - \gamma_g$ if $s_i = s_{-i} \neq x$, and (iii) 0 if $s_i \neq s_{-i}$.

I will prove only part (ii) of property 2. The rest can be proved in a similar way. I will first show that $\Pr(s_i = x, s_{-i} \neq x \mid x) = \gamma(1-\gamma) - \lambda^2\gamma_g(1-\gamma_g)$.

By definition, $\Pr(s_i = x, s_{-i} \neq x \mid x) = \lambda^2 \Pr(s_i = x, s_{-i} \neq x \mid x, g_i, g_{-i}) + \lambda(1-\lambda) \Pr(s_i = x \mid x, g_i) \Pr(s_{-i} \neq x \mid x, b_{-i}) + (1-\lambda)\lambda \Pr(s_i = x \mid x, b_i) \Pr(s_{-i} \neq x \mid x, g_{-i}) + (1-\lambda)^2 \Pr(s_i = x \mid x, b_i) \Pr(s_{-i} \neq x \mid x, b_{-i}) = 0 + \lambda(1-\lambda)\gamma_g(1-\gamma_b) + (1-\lambda)\lambda\gamma_b(1-\gamma_g) + (1-\lambda)^2\gamma_b(1-\gamma_b) = -\lambda^2\gamma_g(1-\gamma_g)] + \lambda^2\gamma_g(1-\gamma_g) + \lambda(1-\lambda)\gamma_g(1-\gamma_b) + (1-\lambda)\lambda\gamma_b(1-\gamma_g) + (1-\lambda)^2\gamma_b(1-\gamma_b) = -\lambda^2\gamma_g(1-\gamma_g) + \gamma(1-\gamma)$. The first term in the second equality is zero by (iii) above; in the third equality, an extra term $\lambda^2\gamma_g(1-\gamma_g)$ is first subtracted and then added back; in the fourth equality, the last four terms add up to $\gamma(1-\gamma)$ which is $\Pr(s_i = x, s_{-i} \neq x \mid x)$ with no conditional correlation between the signals. The first term in the fourth equality is negative because of the fact that the signals of the analysts are different.

Using the same argument, it is straightforward to show the other part of (ii), i.e., $\Pr(s_i \neq x, s_{-i} = x \mid x) = \gamma(1-\gamma) - \lambda^2\gamma_g(1-\gamma_g)$. This completes the proof. $\square$

Proof of Proposition 4. (i) For a fully revealing equilibrium to exist, inequalities (24) and (25) are to be satisfied for all i. Before solving the equilibrium, I will first show how I derived the expressions for $\pi_i(m_i^h|s_i^h)$ in (22) and $\pi_i(m_i^l|s_i^h)$ in (23).

By definition, $E[x|s_i^h, s_{-i}^h] = x^H \Pr(x^H \mid s_i^h, s_{-i}^h) + x^L \Pr(x^L \mid s_i^h, s_{-i}^h)$. Also, $\Pr(x^H|s_i^h, s_{-i}^h) = \frac{\Pr(x^H)\Pr(s_i^h, s_{-i}^h|x^H)}{\Pr(x^H)\Pr(s_i^h, s_{-i}^h|x^H)+\Pr(x^L)\Pr(s_i^h, s_{-i}^h|x^L)} = \frac{q\{\gamma^2+c\}}{q\{\gamma^2+c\}+(1-q)\{(1-\gamma)^2+c\}}$, by using results from property 1. Thus, $E[x|s_i^h, s_{-i}^h] = \frac{x^H q\{\gamma^2+c\}+x^L(1-q)\{(1-\gamma)^2+c\}}{q\{\gamma^2+c\}+(1-q)\{(1-\gamma)^2+c\}}$. Further, $E[x|s_i^h, s_{-i}^h] - E[x] = \frac{x^H q\{\gamma^2+c\}+x^L(1-q)\{(1-\gamma)^2+c\}}{q\{\gamma^2+c\}+(1-q)\{(1-\gamma)^2+c\}} - \{x^H q + x^L(1-q)\} = \frac{(x^H-x^L)q(1-q)(2\gamma-1)}{q\{\gamma^2+c\}+(1-q)\{(1-\gamma)^2+c\}}$.

However, $E[x|s_i^h, s_{-i}^l] = E[x]$ because $\Pr(x^H|s_i^h, s_{-i}^l) = \frac{\Pr(x^H)\Pr(s_i^h, s_{-i}^l|x^H)}{\Pr(x^H)\Pr(s_i^h, s_{-i}^l|x^H)+\Pr(x^L)\Pr(s_i^h, s_{-i}^l|x^L)}$ $= \frac{q\{\gamma(1-\gamma)-c\}}{q\{\gamma(1-\gamma)-c\}+(1-q)\{\gamma(1-\gamma)-c\}} = q$, using property 2. Taken all together, I get (22)

$$\pi_i(m_i^h|s_i^h) = \left[(x^H - x^L)q(1-q)(2\gamma-1)\right]\frac{\Pr(s_{-i}^h \mid s_i^h)}{\Pr(s_i^h, s_{-i}^h)}$$

(23) is calculated similarly. Now, (24) implies,

$$\begin{aligned}
&\left[(x^H - x^L)q(1-q)(2\gamma-1)\right]\frac{\Pr(s_{-i}^h \mid s_i^h)}{\Pr(s_i^h, s_{-i}^h)} \geq \left[(x^H - x^L)q(1-q)(2\gamma-1)\right]\frac{\Pr(s_{-i}^l \mid s_i^h)}{\Pr(s_i^l, s_{-i}^l)} \\
\Leftrightarrow\ &\frac{\Pr(s_{-i}^h \mid s_i^h)}{\Pr(s_i^h, s_{-i}^h)} \geq \frac{\Pr(s_{-i}^l \mid s_i^h)}{\Pr(s_i^l, s_{-i}^l)} \\
\Leftrightarrow\ &\frac{\Pr(s_{-i}^h \mid s_i^h)}{\Pr(s_i^h)\Pr(s_{-i}^h \mid s_i^h)} \geq \frac{\Pr(s_{-i}^l \mid s_i^h)}{\Pr(s_i^l, s_{-i}^l)} \\
\Leftrightarrow\ &\Pr(s_i^l, s_{-i}^l) \geq \Pr(s_i^h, s_{-i}^l) \\
\Leftrightarrow\ &q\{(1-\gamma)^2 + c\} + (1-q)\{\gamma^2 + c\} \geq \gamma(1-\gamma) - c \\
\Leftrightarrow\ &q \leq \gamma + \frac{2c}{2\gamma - 1} \equiv q_\pi
\end{aligned} \tag{40}$$

Similarly, (24) leads to $q \geq 1 - \gamma - \frac{2c}{2\gamma-1} = 1 - q_\pi$, which, along with (40), completes the proof of part (i).

The proofs of (ii) and (iii) will follow similar arguments as in proposition 1, and will be included in the future updated version of the paper. $\square$

Proof of Lemma 4. (i) Obvious from the definition of q_π.

(ii) Replacing the value of c, I get $\frac{2c}{2\gamma-1} = \frac{2\lambda^2\gamma_g(1-\gamma_g)}{\lambda(2\gamma_g-1)} = \frac{2\lambda\gamma_g(1-\gamma_g)}{2\gamma_g-1}$. Now, further replacing the value of $\gamma = \frac{1+\lambda(2\gamma_g-1)}{2}$, I get $q_\pi = \frac{1+\lambda(2\gamma_g-1)}{2} + \frac{2\lambda\gamma_g(1-\gamma_g)}{2\gamma_g-1}$, which is increasing in λ. $\square$

Proof of Proposition 5. By (32), and using (30) and (31), when analyst i receives a high signal, he will forecast high if $R_i(m_i^h \mid s_i^h) \geq R_i(m_i^l \mid s_i^h)$, which implies

$$\begin{aligned}
&\Pr(x^H|s_i^h)\left[U_i(s_i^h, s_{-i}^h, x^H)(\gamma + \frac{c}{\gamma}) + U_i(s_i^h, s_{-i}^l, x^H)(1-\gamma-\frac{c}{\gamma})\right] + \\
&\Pr(x^L|s_i^h)\left[U_i(s_i^h, s_{-i}^h, x^L)(1-\gamma+\frac{c}{1-\gamma}) + U_i(s_i^h, s_{-i}^l, x^L)(\gamma-\frac{c}{1-\gamma})\right] \\
\geq\ &\Pr(x^H|s_i^h)\left[U_i(s_i^l, s_{-i}^h, x^H)(\gamma + \frac{c}{\gamma}) + U_i(s_i^l, s_{-i}^l, x^H)(1-\gamma-\frac{c}{\gamma})\right] + \\
&\Pr(x^L|s_i^h)\left[U_i(s_i^l, s_{-i}^h, x^L)(1-\gamma+\frac{c}{1-\gamma}) + U_i(s_i^l, s_{-i}^l, x^L)(\gamma-\frac{c}{1-\gamma})\right] \qquad (41)
\end{aligned}$$

However, by symmetry, $U_i(s_i^h, s_{-i}^h, x^H) = U_i(s_i^l, s_{-i}^l, x^L)$, and $U_i(s_i^h, s_{-i}^l, x^H) = U_i(s_i^l, s_{-i}^h, x^L)$, and after some algebra, (41) implies,

$$D * [q\gamma - (1-q)(1-\gamma)] \geq c\left[\{U_i(s_i^h, s_{-i}^l, x^H) + U_i(s_i^l, s_{-i}^h, x^H)\} - \{U_i(s_i^h, s_{-i}^h, x^H) + U_i(s_i^h, s_{-i}^h, x^L)\}\right] \qquad (42)$$

where $D \equiv \left[\{\gamma U_i(s_i^h, s_{-i}^h, x^H) + (1-\gamma)U_i(s_i^h, s_{-i}^l, x^H)\} - \{\gamma U_i(s_i^l, s_{-i}^h, x^H) + (1-\gamma)U_i(s_i^l, s_{-i}^l, x^H)\}\right]$.

Dividing both sides of (42) by D (by claim 7 below, $D > 0$), and replacing the expression of D back, I derive that $R_i(m_i^h \mid s_i^h) \geq R_i(m_i^l \mid s_i^h) \Leftrightarrow q \geq 1 - (\gamma - \eta) \equiv 1 - q_r$ where

$$\eta = c\left[\frac{\{U_i(s_i^h, s_{-i}^l, x^H) + U_i(s_i^l, s_{-i}^h, x^H)\} - \{U_i(s_i^h, s_{-i}^h, x^H) + U_i(s_i^h, s_{-i}^h, x^L)\}}{\{\gamma U_i(s_i^h, s_{-i}^h, x^H) + (1-\gamma)U_i(s_i^h, s_{-i}^l, x^H)\} - \{\gamma U_i(s_i^l, s_{-i}^h, x^H) + (1-\gamma)U_i(s_i^l, s_{-i}^l, x^H)\}}\right]$$

More generally, $\eta(y, \overline{z}, \underline{z})$ can be expressed as

$$\eta = c\left[\frac{\{U_i(s_i = x, s_{-i} \neq x, x) + U_i(s_i \neq x, s_{-i} = x, x)\} - \{U_i(s_i = x, s_{-i} = x, x) + U_i(s_i \neq x, s_{-i} \neq x, x)\}}{\{\gamma U_i(s_i = x, s_{-i} = x, x) + (1-\gamma)U_i(s_i = x, s_{-i} \neq x, x)\} - \{\gamma U_i(s_i \neq x, s_{-i} = x, x) + (1-\gamma)U_i(s_i \neq x, s_{-i} \neq x, x)\}}\right.$$

Claim 7.

$$\{\gamma U_i(s_i = x, s_{-i} = x, x) + (1-\gamma)U_i(s_i = x, s_{-i} \neq x, x)\} - \{\gamma U_i(s_i \neq x, s_{-i} = x, x) + (1-\gamma)U_i(s_i \neq x, s_{-i} \neq x, x)\} > 0$$

Proof. Using property 3 (appendix A on page 48), and replacing the values of $U_i(s_i, s_{-i}, x)$, I derive $(1-\gamma)U_i(s_i = x, s_{-i} \neq x, x) - \gamma U_i(s_i \neq x, s_{-i} = x, x) = (1-\gamma)[\frac{2\lambda\left[\overline{z}\gamma_g + \underline{z}(1-\gamma_g)\right]}{1+\lambda}] - \gamma[\frac{2\lambda\left[\overline{z}(1-\gamma_g)+\underline{z}\gamma_g\right]}{1+\lambda}] = \frac{2\lambda}{1+\lambda}[\overline{z}(\gamma_g - \gamma) - \underline{z}(\gamma_g + \gamma - 1)]$.

Also, $\gamma U_i(s_i = x, s_{-i} = x, x) - (1-\gamma)U_i(s_i \neq x, s_{-i} \neq x, x) = \gamma[\frac{2\lambda\gamma_g[2y\lambda + (\overline{z}+\underline{z})(1-\lambda)]}{4\lambda\gamma_g + (1-\lambda)^2}] - (1-\gamma)\left[\frac{2\lambda(1-\gamma_g)[2y\lambda + (\overline{z}+\underline{z})(1-\lambda)]}{4\lambda(1-\gamma_g)+(1-\lambda)^2}\right] = 2\lambda[2y\lambda + (\overline{z}+\underline{z})(1-\lambda)]\left[\frac{\gamma\gamma_g}{4\lambda\gamma_g+(1-\lambda)^2} - \frac{(1-\gamma)(1-\gamma_g)}{4\lambda(1-\gamma_g)+(1-\lambda)^2}\right] = 2\lambda[2y\lambda + (\overline{z}+\underline{z})(1-\lambda)]\left[\frac{4\lambda\gamma_g(1-\gamma_g)(2\gamma-1)+(1-\lambda)^2(\gamma_g+\gamma+1)}{\{4\lambda\gamma_g+(1-\lambda)^2\}\{4\lambda(1-\gamma_g)+(1-\lambda)^2\}}\right]$.

Now, $(1-\gamma)U_i(s_i = x, s_{-i} \neq x, x) - \gamma U_i(s_i \neq x, s_{-i} = x, x) + \gamma U_i(s_i = x, s_{-i} = x, x) - (1-\gamma)U_i(s_i \neq x, s_{-i} \neq x, x) = \frac{2\lambda}{1+\lambda}[\overline{z}(\gamma_g - \gamma) - \underline{z}(\gamma_g + \gamma - 1)] + 2\lambda[2y\lambda + (\overline{z}+\underline{z})(1-\lambda)]\left[\frac{4\lambda\gamma_g(1-\gamma_g)(2\gamma-1)+(1-\lambda)^2(\gamma_g+\gamma+1)}{\{4\lambda\gamma_g+(1-\lambda)^2\}\{4\lambda(1-\gamma_g)+(1-\lambda)^2\}}\right]$.

Note that to prove that this expression is greater than zero, it is enough to show that $\frac{2\lambda}{1+\lambda}[-\underline{z}(\gamma_g+\gamma-1)] + 2\lambda[\underline{z}(1-\lambda)]\left[\frac{4\lambda\gamma_g(1-\gamma_g)(2\gamma-1)+(1-\lambda)^2(\gamma_g+\gamma+1)}{\{4\lambda\gamma_g+(1-\lambda)^2\}\{4\lambda(1-\gamma_g)+(1-\lambda)^2\}}\right] > 0$. Since $\underline{z} < 0$,by assumption, this is equivalent to prove that $\frac{(\gamma_g+\gamma-1)}{1+\lambda} - (1-\lambda)\left[\frac{4\lambda\gamma_g(1-\gamma_g)(2\gamma-1)+(1-\lambda)^2(\gamma_g+\gamma+1)}{\{4\lambda\gamma_g+(1-\lambda)^2\}\{4\lambda(1-\gamma_g)+(1-\lambda)^2\}}\right] > 0$ or $(\gamma_g+\gamma-1)\{4\lambda\gamma_g+(1-\lambda)^2\}\{4\lambda(1-\gamma_g)+(1-\lambda)^2\} - (1+\lambda)(1-\lambda)\{4\lambda\gamma_g(1-\gamma_g)(2\gamma-1) + (1-\lambda)^2(\gamma_g+\gamma+1)\} > 0$.

Using the $4\lambda\gamma_g+(1-\lambda)^2 > 4\lambda\frac{1}{2}+(1-\lambda)^2 > 1+\lambda^2$ (since $\gamma_g > \frac{1}{2}$), I get $(\gamma_g+\gamma-1)\{4\lambda\gamma_g+(1-\lambda)^2\}\{4\lambda(1-\gamma_g)+(1-\lambda)^2\} - (1+\lambda)(1-\lambda)\{4\lambda\gamma_g(1-\gamma_g)(2\gamma-1)+(1-\lambda)^2(\gamma_g+\gamma+1)\} > (\gamma_g+\gamma-1)(1+\lambda^2)\{4\lambda(1-\gamma_g)+(1-\lambda)^2\} - (1+\lambda)(1-\lambda)\{4\lambda\gamma_g(1-\gamma_g)(2\gamma-1)+(1-\lambda)^2(\gamma_g+\gamma+1)\} = (\gamma_g+\gamma-1)(1+\lambda^2)(1-\lambda)^2 - (\gamma_g+\gamma-1)(1-\lambda^2)(1-\lambda)^2 + (\gamma_g+\gamma-1)(1+\lambda^2)4\lambda(1-\gamma_g) - (1-\lambda^2)4\lambda\gamma_g(1-\gamma_g)(2\gamma-1) = (\gamma_g+\gamma-1)(2\lambda^2)(1-\lambda)^2 + 4\lambda(1-\gamma_g)\{(\gamma_g+\gamma+1)(1+\lambda^2) - (2\gamma-1)(1-\lambda^2)\gamma_g\} > 0$ because $(\gamma_g+\gamma+1) > (2\gamma-1)$, $(1+\lambda^2) > (1-\lambda^2)$ and $1 > \gamma_g$. This completes the proof. □

Returning to the proof of proposition 5, by (33), $R_i(m_i^l \mid s_i^l) \geq R_i(m_i^h \mid s_i^l)$, which implies $q \leq \gamma - \eta = q_r$. This completes the proof of (i).

The proofs of part (ii)-(iv) will follow similar arguments as in proposition 2, and will be included in the future updated version of the paper. □

Proof of Proposition 6. Since the denominator, $D > 0$ by claim 7 above, and $c > 0$, $\eta(y, \overline{z}, \underline{z}) > (\leqslant)\ 0$ if and only if

$$U_i(s_i = x, s_{-i} \neq x, x) + U_i(s_i \neq x, s_{-i} = x, x) > (\leqslant)\ U_i(s_i = x, s_{-i} = x, x) + U_i(s_i \neq x, s_{-i} \neq x, x) \tag{43}$$

$U_i(s_i = x, s_{-i} \neq x, x) + U_i(s_i \neq x, s_{-i} = x, x) = \frac{2\lambda\left[\overline{z}\gamma_g + \underline{z}(1-\gamma_g)\right]}{1+\lambda} + \frac{2\lambda\left[\overline{z}(1-\gamma_g) + \underline{z}\gamma_g)\right]}{1+\lambda} = \frac{2\lambda}{1+\lambda}[\overline{z} + \underline{z}]$.
Also, $U_i(s_i = x, s_{-i} = x, x) + U_i(s_i \neq x, s_{-i} \neq x, x) = \frac{2\lambda\gamma_g[2y\lambda + (\overline{z}+\underline{z})(1-\lambda)]}{4\lambda\gamma_g + (1-\lambda)^2} + \frac{2\lambda(1-\gamma_g)[2y\lambda + (\overline{z}+\underline{z})(1-\lambda)]}{4\lambda(1-\gamma_g)+(1-\lambda)^2} =$
$2\lambda\left[2y\lambda + (\overline{z}+\underline{z})(1-\lambda)\right]\left[\frac{\gamma_g}{4\lambda\gamma_g+(1-\lambda)^2} - \frac{1-\gamma_g}{4\lambda(1-\gamma_g)+(1-\lambda)^2}\right]$.
$\left[\frac{\gamma_g}{4\lambda\gamma_g+(1-\lambda)^2} + \frac{1-\gamma_g}{4\lambda(1-\gamma_g)+(1-\lambda)^2}\right]$

Define $f(\lambda, \gamma_g) = \frac{\gamma_g}{4\lambda\gamma_g+(1-\lambda)^2} + \frac{1-\gamma_g}{4\lambda(1-\gamma_g)+(1-\lambda)^2} > 0$. Now, (43) implies

$$
\begin{aligned}
& \frac{2\lambda}{1+\lambda}[\overline{z}+\underline{z}] > (\leq) 2\lambda\left[2y\lambda + (\overline{z}+\underline{z})(1-\lambda)\right] f(\lambda,\gamma_g) \\
\Leftrightarrow\quad & \overline{z}+\underline{z} > (\leq)\ (1+\lambda)\left[2y\lambda + (\overline{z}+\underline{z})(1-\lambda)\right] f(\lambda,\gamma_g) \\
\Leftrightarrow\quad & \frac{\overline{z}+\underline{z}}{f(\lambda,\gamma_g)} > (\leq)\ (1+\lambda)\left[2y\lambda + (\overline{z}+\underline{z})(1-\lambda)\right] \\
\Leftrightarrow\quad & (\overline{z}+\underline{z})[\frac{1}{f(\lambda,\gamma_g)} - (1-\lambda^2)] > (\leq)\ 2\lambda(1+\lambda)y \\
\Leftrightarrow\quad & \overline{z}+\underline{z} > (\leq)\ [\frac{2\lambda(1+\lambda)}{\{\frac{1}{f(\lambda,\gamma_g)} - (1-\lambda^2)\}}]y \\
\Leftrightarrow\quad & \overline{z} - y > (\leq)\ (-\underline{z}) + h(\lambda,\gamma_g).y
\end{aligned}
$$

where $h(\lambda,\gamma_g) \equiv \frac{2\lambda(1+\lambda)}{\frac{1}{f(\lambda,\gamma_g)} - (1-\lambda^2)} - 1 > 0$, which is proved in claim 8 below.

The proof for other equilibrium regions– for example, partially-revealing and noninformative – can be proved using an analogous argument, and will be included in the future updated version of the paper. □

Claim 8. $h(\lambda,\gamma_g) \equiv \frac{2\lambda(1+\lambda)}{\frac{1}{f(\lambda,\gamma_g)} - (1-\lambda^2)} - 1 > 0$

Proof. I will first show that $\frac{\partial}{\partial\gamma_g}h(\lambda,\gamma_g) < 0$. It is enough to show that $\frac{\partial}{\partial\gamma_g}f(\lambda,\gamma_g) < 0$ since $h()$ is an increasing function of $f()$, which is the only expression that contains γ_g in h. By definition $\frac{\partial}{\partial\gamma_g}f(\lambda,\gamma_g) = \frac{\partial}{\partial\gamma_g}\left[\frac{\gamma_g}{4\lambda\gamma_g+(1-\lambda)^2} + \frac{1-\gamma_g}{4\lambda(1-\gamma_g)+(1-\lambda)^2}\right]$. After some algebraic manipulation, $sgn\left(\frac{\partial}{\partial\gamma_g}f()\right) = sgn\left(-8\lambda(1-\lambda)^2(1+\lambda^2)(2\gamma_g-1)\right)$ which is less than zero. Thus, $\frac{\partial}{\partial\gamma_g}f(\lambda,\gamma_g) < 0$.

Since $h(\lambda,\gamma_g)$ is decreasing in $\gamma_g \in (\frac{1}{2}, 1)$, $h(\lambda,\gamma_g) > \lim_{\gamma_g\to 1} h(\lambda,\gamma_g) = \frac{2\lambda(1+\lambda)}{\frac{1}{\lim_{\gamma_g\to 1} f(\lambda,\gamma_g)} - (1-\lambda^2)} - 1$. Now, $\lim_{\gamma_g\to 1} f(\lambda,\gamma_g) = \frac{1}{(1+\lambda)^2}$. Replacing this value in $h()$, I get, $h(\lambda,\gamma_g) > \frac{2\lambda(1+\lambda)}{(1+\lambda)^2 - (1-\lambda^2)} - 1 = \frac{2\lambda(1+\lambda)}{2\lambda(1+\lambda)} - 1 = 0$. □

Proof of Proposition 7. The proof of this proposition is very similar to that of proposition 2. Following the notations and definitions used in the proof of proposition 2, I define $\overline{q}(y, \overline{z}, \underline{z}, \alpha) \equiv \sup\{q : (\sigma_h = 1,\ \sigma_l = 0)$ is an equilibrium$\}$ and $q_{\max}(y, \overline{z}, \underline{z}, \alpha) \equiv \inf\{q : (\sigma_h = 1,\ \sigma_l = 1)$ is the most informative equilibrium$\}$. Similarly, $\underline{q}(y, \overline{z}, \underline{z}, \alpha) \equiv \inf\{q : (\sigma_h = 1,\ \sigma_l = 0)$ is an equilibrium$\}$ and $q_{\min}(y, \overline{z}, \underline{z}, \alpha) \equiv \sup\{q : (\sigma_h = 0,\ \sigma_l = 0)$ is the most informaitve equilibrium$\}$.

I will prove (i) in two parts, by showing: first, , there exists an $\alpha^{'}_{\max}(y, \overline{z}, \underline{z}) \in (0,1)$, such that for any $\alpha \leqslant \alpha^{'}_{\max}(y, \overline{z}, \underline{z})$, $\triangle V_h(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) \geqslant 0$ if $q \geqslant \frac{1}{2}$ and $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) \leqslant 0$ if $q \leqslant \overline{q}(y, \overline{z}, \underline{z}, \alpha)$ (where $\overline{q}(y, \overline{z}, \underline{z}, \alpha) \geqslant \frac{1}{2}$); second, there exists an $\alpha^{''}_{\max}(y, \overline{z}, \underline{z}) \in (0,1)$, such that for any $\alpha \leqslant \alpha^{''}_{\max}(y, \overline{z}, \underline{z})$, $\triangle V_h(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) \geqslant 0$ if $q \geqslant \underline{q}(y, \overline{z}, \underline{z}, \alpha)$ (where $\underline{q}(y, \overline{z}, \underline{z}, \alpha) \leqslant \frac{1}{2}$) and $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) \leqslant 0$ if $q \leqslant \frac{1}{2}$. Taken together, I will conclude that there exists an $\alpha_{\max}(\overline{z}) \equiv \min\{\alpha^{'}_{\max}(y, \overline{z}, \underline{z}), \alpha^{''}_{\max}(y, \overline{z}, \underline{z})\} \in (0,1)$ such that for any $\alpha \leqslant \alpha_{\max}(y, \overline{z}, \underline{z})$, $\triangle V_h(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) \geqslant 0$ and $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) \leqslant 0$ if $q \in [\underline{q}(y, \overline{z}, \underline{z}, \alpha), \overline{q}(y, \overline{z}, \underline{z}, \alpha)]$.

By definition, $\triangle V_h(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) = \alpha \triangle \pi_h(\sigma_h = 1, \sigma_l = 0, q) + (1-\alpha) \triangle R_h(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z})$. Recall that from proposition 4, $\triangle \pi_h(\sigma_h = 1, \sigma_l = 0, q) \geqslant 0$ if $q \leqslant q_\pi$ (where $q_\pi \geqslant \frac{1}{2}$) and by proposition 5, $\triangle R_h(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}) \geqslant 0$ if $q \geqslant 1 - q_r(y, \overline{z}, \underline{z})$ (where $1 - q_r(y, \overline{z}, \underline{z}) \leqslant \frac{1}{2}$). Since, $\triangle V_h(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha)$ is continuous in α, there exists an $\alpha^{'}_{\max}(y, \overline{z}, \underline{z}) \in (0,1)$ such that for any $\alpha \leqslant \alpha^{'}_{\max}(y, \overline{z}, \underline{z})$, $\triangle V_h(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) \geqslant 0$ if $q \geqslant \frac{1}{2}$.

Again, by definition, $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) = \alpha \triangle \pi_l(\sigma_h = 1, \sigma_l = 0, q) + (1 - \alpha) \triangle R_l(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z})$. Also, by proposition 4, $\triangle \pi_l(\sigma_h = 1, \sigma_l = 0, q) \leqslant 0$ if $q \geqslant 1 - q_\pi$ (where $\frac{1}{2} \geqslant 1 - q_\pi \geqslant 0$) and by proposition 5, $\triangle R_l(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}) \leqslant 0$ if $q \leqslant q_r(y, \overline{z}, \underline{z})$ (where $q_r(y, \overline{z}, \underline{z}) \geqslant \frac{1}{2}$). Also, $\triangle R_l(\sigma_h = 1, \sigma_l = 0, q_r(y, \overline{z}, \underline{z}), y, \overline{z}, \underline{z}) = 0$ and $\triangle \pi_l(\sigma_h = 1, \sigma_l = 0, q_r(y, \overline{z}, \underline{z})) < 0$. Thus, $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q_r(y, \overline{z}, \underline{z}), y, \overline{z}, \underline{z}, \alpha) < 0$ and $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) < 0$ if $q \leqslant q_r(y, \overline{z}, \underline{z})$. However, by definition, $\overline{q}(y, \overline{z}, \underline{z}, \alpha) \equiv \sup\{q : (\sigma_h = 1,\ \sigma_l = 0)$ is an equilibrium$\}$. Therefore, there exists an $\overline{q}(y, \overline{z}, \underline{z}, \alpha) > q_r(y, \overline{z}, \underline{z})$ such that $\triangle V_l(\sigma_h = 1, \sigma_l = 0, \overline{q}(y, \overline{z}, \underline{z}, \alpha), y, \overline{z}, \underline{z}, \alpha) = 0$ and $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) \leqslant 0$ if $q \leqslant \overline{q}(y, \overline{z}, \underline{z}, \alpha)$.

Taken together, there exists an $\alpha^{'}_{\max}(y, \overline{z}, \underline{z}) \in (0,1)$ such that for any $\alpha \leqslant \alpha^{'}_{\max}(\overline{z})$, $\triangle V_h(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) \geqslant 0$ if $q \geqslant \frac{1}{2}$ and $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) \leqslant 0$ if

$q \leqslant \overline{q}(y, \overline{z}, \underline{z}, \alpha)$,where $\overline{q}(y, \overline{z}, \underline{z}, \alpha) > q_r(y, \overline{z}, \underline{z}) \geqslant \frac{1}{2}$. This proves the first part.

By an analogous argument, it can be shown that there exists an $\alpha^{''}_{\max}(y, \overline{z}, \underline{z}) \in (0, 1)$ such that for any $\alpha \leqslant \alpha^{''}_{\max}(y, \overline{z}, \underline{z})$, $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) \leqslant 0$ if $q \leqslant \frac{1}{2}$ and $\triangle V_h(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) \geqslant 0$ if $q \geqslant \underline{q}(y, \overline{z}, \underline{z}, \alpha)$, where $\underline{q}(y, \overline{z}, \underline{z}, \alpha) < 1 - q_r(y, \overline{z}, \underline{z}) \leqslant \frac{1}{2}$. Therefore, there exists an $\alpha_{\max}(y, \overline{z}, \underline{z}) \equiv \min\{\alpha^{'}_{\max}(y, \overline{z}, \underline{z}), \alpha^{''}_{\max}(y, \overline{z}, \underline{z})\} \in (0, 1)$ such that for any $\alpha \leqslant \alpha_{\max}(y, \overline{z}, \underline{z})$, $\triangle V_h(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) \geqslant 0$ and $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) \leqslant 0$ if $q \in [\underline{q}(y, \overline{z}, \underline{z}, \alpha), \overline{q}(y, \overline{z}, \underline{z}, \alpha)]$.

The proofs of (ii)-(iv) are similar to those of the corresponding parts of proposition 2, and will be included in the future updated version of the paper. □

Proof of Proposition 8. This proof is very similar to that of proposition 3. I will show that given, $\alpha \leq \alpha_{\max}(y, \overline{z}, \underline{z})$, if $\alpha_2 > \alpha_1$, then $\overline{q}(y, \overline{z}, \underline{z}, \alpha_2) \geqslant \overline{q}(y, \overline{z}, \underline{z}, \alpha_1)$. The proof of the other part is very similar to that of part two of proposiiton 3 and will be included in the updated version of the paper.

By definition, $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) = \alpha \triangle \pi_l(\sigma_h = 1, \sigma_l = 0, q) + (1 - \alpha) \triangle R_l(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z})$, and $\overline{q}(\overline{z}, \alpha) \equiv \sup\{q : (\sigma_h = 1,\ \sigma_l = 0) \text{ is an equilibrium}\}$. Thus, $\triangle V_l(\sigma_h = 1, \sigma_l = 0, \overline{q}(y, \overline{z}, \underline{z}, \alpha), y, \overline{z}, \underline{z}, \alpha) < 0$ and $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha) < 0$ if $q \leqslant \overline{q}(y, \overline{z}, \underline{z}, \alpha)$.

Consider $\alpha = \alpha_1$. Thus, $\triangle V_l(\sigma_h = 1, \sigma_l = 0, \overline{q}(y, \overline{z}, \underline{z}, \alpha_1), y, \overline{z}, \underline{z}, \alpha_1) = 0$. However, since by proposition 4, $\triangle\pi_l(\sigma_h = 1, \sigma_l = 0, \overline{q}(y, \overline{z}, \underline{z}, \alpha_1)) < 0$, it must be true that $\triangle R_l(\sigma_h = 1, \sigma_l = 0, \overline{q}(y, \overline{z}, \underline{z}, \alpha_1), \overline{z}) > 0$.

Now consider, $\alpha_2 > \alpha_1$. Now, $\triangle V_l(\sigma_h = 1, \sigma_l = 0, \overline{q}(y, \overline{z}, \underline{z}, \alpha_1), \alpha_2) = \alpha_2 \triangle \pi_l(\sigma_h = 1, \sigma_l = 0, \overline{q}(y, \overline{z}, \underline{z}, \alpha_1)) + (1 - \alpha_2) \triangle R_l(\sigma_h = 1, \sigma_l = 0, \overline{q}(y, \overline{z}, \underline{z}, \alpha_1), y, \overline{z}, \underline{z}) < 0$. However, by definition, $\overline{q}(y, \overline{z}, \underline{z}, \alpha) \equiv \sup\{q : (\sigma_h = 1,\ \sigma_l = 0) \text{ is an equilibrium}\}$. Thus, there exists $\overline{q}(y, \overline{z}, \underline{z}, \alpha_2) \geqslant \overline{q}(y, \overline{z}, \underline{z}, \alpha_1)$ such that $\triangle V_l(\sigma_h = 1, \sigma_l = 0, \overline{q}(y, \overline{z}, \underline{z}, \alpha_2), y, \overline{z}, \underline{z}, \alpha_2) = 0$ and $\triangle V_l(\sigma_h = 1, \sigma_l = 0, q, y, \overline{z}, \underline{z}, \alpha_2) \leqslant 0$ if $q \leqslant \overline{q}(y, \overline{z}, \underline{z}, \alpha_2)$. Therefore, $\overline{q}(y, \overline{z}, \underline{z}, \alpha_2) \geqslant \overline{q}(y, \overline{z}, \underline{z}, \alpha_1)$. □