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(A, f): Choice with Frames[1]

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We develop a framework for modelling choice in the presence of framing effects. An *extended choice function* assigns a chosen element to every pair (A, f) where A is a set of alternatives, and f is a *frame*. A frame includes observable information that is irrelevant in the rational assessment of the alternatives, but nonetheless affects choice. We relate the new framework to the classical model of choice correspondence. Conditions are identified under which there exists either a transitive or a transitive and complete binary relation R such that an alternative x is chosen in some (A, f) iff x is R-maximal in the set A. We then demonstrate that the framework of choice correspondence misses information, which is essential to economic modelling, and which is incorporated in the extended choice function.

1. INTRODUCTION

The traditional method used by economists to model a choice situation is to describe the set of alternatives from which an economic agent makes his choice. Individual behaviour is specified by assigning to every choice situation a chosen alternative or a collection of possible choices without further specifying how the indeterminacy among the possible choices is resolved.

Mounting evidence from psychology, as well as casual observation and introspection indicates that real-life behaviour often depends on observable information, other than the set of feasible alternatives, which is irrelevant in the rational assessment of the alternatives but nonetheless affects behaviour. For example, a voter may be influenced by the order in which candidates are listed on a ballot; a consumer may condition an online purchase decision on the alternative designated as the default by the retailer; and the choice of a vacation package from a catalogue may depend on whether or not a casino table appears on the front page. We refer to such additional information as a *frame* and to the dependence of choice on the frame as a *framing effect*.

This paper presents a framework for modelling choice in the presence of framing effects which is in the spirit of recent developments in Bounded Rationality and Behavioural Economics. We then relate the new framework to the classical model of choice.

Let X be a finite set of alternatives. According to the standard approach, a choice problem is a non-empty subset of X and a choice function attaches to every choice problem $A \subseteq X$ a single

1. This paper extends and substitutes our previous paper "Two Comments on the Principle of Revealed Preference" posted in June 2006.

element in A. The notion of a choice correspondence, which attaches to every choice problem A a non-empty subset of A, is used to capture indeterminacy in choice.

We enrich the standard model with a set F of frames. For example, F may include various orderings of the set X, a collection of default alternatives, or a set of natural numbers interpreted as the number of elements the decision maker can seriously evaluate. An *extended choice problem* is a pair (A, f) where $A \subseteq X$ is a standard choice problem and $f \in F$ is a frame. An *extended choice function* assigns an element of A to every pair (A, f). An extended choice function induces a choice correspondence by assigning to every set A all the elements chosen from A in some frame.

Bernheim and Rangel (2007) independently develop a framework called choice with ancillary conditions that is similar to the framework of choice with frames. While they focus on the discussion of welfare within the framework, we focus on relating the new framework to the standard model of choice.

Caution should be exercised in applying the model of choice with frames. In many real-life situations the decision maker faces a set of alternatives with additional information that is in fact relevant in the rational assessment of the alternatives and thus should not be regarded as a frame. For example, consider a "matchmaking" situation (A, f) where a male from the set A is to be matched to a female f. In this case, the rational evaluation of a male in A differs as we vary the female f. Matching the male m to the female f_1 is considered by a rational matchmaker to be a different outcome than matching m to f_2. Although this example can formally be included in the framework, it lies outside the scope of this paper, in which we assume that the frame affects choice only as a result of procedural or psychological factors.

The standard model postulates the existence of a transitive and possibly complete binary relation that describes behaviour. In Section 3, we explore the boundaries of this postulate in the context of choice with frames. We identify conditions under which there exists either a transitive or alternatively a transitive and complete binary relation R, such that an element x is chosen in some extended choice problem (A, f) iff x is R-maximal in the set A. The asymmetric component of the relation R must relate x to y iff x is chosen over y in all extended choice problems $(\{x, y\}, f)$. This relation has already appeared in the specific contexts of choice from lists (Rubinstein and Salant, 2006*a*) and choice with a default alternative (Rubinstein and Salant, 2006*b*). Bernheim and Rangel (2007) independently use a similar binary relation.

In Section 4, we discuss the limitations of the model of choice correspondence, and the maximization of a binary relation in particular, in describing behaviour when the decision maker is affected by framing. We present examples of extended choice functions for which the induced choice correspondence misses essential information about choice. In fact, the choice correspondence may carry no information about decision making. In addition, we discuss examples of extended choice functions where the induced correspondence cannot be described by maximizing a preference relation, or alternatively it can but the relation is very far from what we would naturally consider to be the underlying preferences of the decision maker.

Thus, the existence of a binary relation, the maximization of which can describe behaviour, does not imply that exploring the details of a frame-sensitive choice procedure is superfluous. On the contrary, in order to construct rich economic models one often needs a model of choice with frames. In Section 5, we conclude by exploring one such example.

2. EXAMPLES OF EXTENDED CHOICE FUNCTIONS

In this section, we discuss several extended choice models and provide an example of an extended choice function for each.

2.1. *Default alternative*

In this model, one of the alternatives is designated as the default. The collection of frames F is taken to be X. An extended choice problem is a pair (A, x), where $x \in X$ denotes the default alternative that may or may not be available for choice. Zhou (1997), Masatlioglu and Ok (2005, 2006), and Sagi (2006) study axiomatizations within this model.

Example 1. *A decision maker has in mind two functions u and β from X to the reals. Given an extended choice problem (A, x), he chooses x if $x \in A$ and $u(x) + \beta(x) \geq u(a)$ for every other element $a \in A$; otherwise, he chooses the u-maximal element in A. Status-quo bias is modelled by taking $\beta(x)$ to be positive.*

2.2. *List*

The set of alternatives is presented to the decision maker in the form of a list. An extended choice problem is a pair $(A, >)$ where $>$ is an ordering of X. Rubinstein and Salant (2006*a*) study axiomatizations within this model.

Example 2 (Satisficing (Simon, 1955)). *A decision maker has in mind a value function $v : X \to R$ and an aspiration threshold v^*. Given a pair $(A, >)$, he chooses the $>$-first element in A with a value above v^*. If there are none, the $>$-last element in A is chosen.*

2.3. *Limited Attention*

There is a limit on the number of alternatives that the decision maker can actually consider. This number reflects the amount of attention the decision maker devotes to the choice problem. An extended choice problem is a pair (A, n) where n is the number of alternatives the decision maker can actually consider.

Example 3. *A decision maker has in mind two orderings: an "attention" ordering O of X, which determines the alternatives he focuses on, and a "preference" ordering P summarizing his preferences. Given (A, n), the decision maker chooses the P-best element among the first $\min\{n, |A|\}$ elements in A according to the ordering O.*

2.4. *Advertisement*

The intensity of advertising the alternatives within the set X may vary. A frame assigns to every element in X a natural number interpreted as the number of advertisements for this element. However, not all elements in X are available for choice. Thus, an extended choice problem is a pair (A, i) where A is the set of elements available for choice and i is a function that assigns a natural number to every $x \in X$.

Example 4. *A decision maker has in mind a "weight" function u. Given a pair (A, i), he chooses the element $a \in A$, which maximizes $i(x)u(x)$ over all $x \in A$.*

2.5. *Gradual accessibility*

The alternatives are revealed to the decision maker in two stages. In the first stage, the decision maker is unable to assess the alternatives that will appear in the second stage. An extended choice problem is a pair (A, R), where A is the set of elements available for choice in the two stages, $R \subseteq X$, $A \cap R$ is the set of elements appearing in the first stage and $A - R$ is the set of elements appearing in the second stage.

Example 5. *A decision maker has in mind two functions u and υ and a value u^*. Given a problem (A, R), he chooses the u-best element in $A \cap R$ if that element passes the threshold u^* or if $R \supseteq A$. Otherwise, he chooses the υ-best element in $A - R$.*

2.6. *Deadline*

The amount of time that the decision maker can invest in the choice problem is limited. An extended choice problem is a pair (A, t) where t is interpreted as the deadline for making a choice.

Example 6. *The decision maker needs time to process the different alternatives. Let υ be a value function and d be a processing time function with the interpretation that an alternative x is available for choice only after $d(x)$ time units. Given a pair (A, t), the decision maker chooses the υ-maximal alternative from among the elements in A with $d(x) \leq t$.*

Note that extended choice functions assign a chosen element to every pair (A, f), where $A \subseteq X$ and $f \in F$. In some cases, it makes sense to restrict the domain of the choice function. For example, in the default model, if the default alternative is always available for choice, the domain of extended choice problems should be restricted to those pairs (A, x) where $x \in A$. In other cases, it seems reasonable to require that the extended function satisfy an invariance property. For example, in the list model, in which the frame is an ordering over the entire set X, it makes sense to require that a choice function assign the same element to any two extended choice problems $(A, >_1)$ and $(A, >_2)$ that order the elements of A identically.

3. CHOICE WITH FRAMES AND THE STANDARD CHOICE MODEL

The interpretation of a choice correspondence C is that the set $C(A)$ contains all the elements that are chosen from the set A under certain circumstances. With this interpretation in mind, an extended choice function c induces a choice correspondence C_c, where $C_c(A)$ is the set of elements chosen from the set A in some frame f. That is,

$$C_c(A) = \{x \mid c(A, f) = x \text{ for some } f \in F\}.$$

The induced choice correspondence reflects the data available to an observer who views the choices of the decision maker, knows that the choices are frame-sensitive but does not have information on the actual frame.

In this section, we analyse the relationship between the model of choice with frames and the standard model of choice correspondence. We discuss three results establishing that a choice correspondence satisfying certain properties is indistinguishable from an extended choice function satisfying analogous properties.

Observation 1. Without imposing any structure, the two models are indistinguishable if the frames are not directly observable. That is, for every choice correspondence C, there exists an extended choice function c such that $C = C_c$.

Indeed, let C be a choice correspondence. Let $F = X$ and define

$$c(A,x) = \begin{cases} x & \text{if } x \in C(A) \\ \text{some } y \in C(A) & \text{if } x \notin C(A) \end{cases}.$$

Clearly, $C = C_c$.

Observation 2. We now investigate circumstances in which an extended choice function can be represented in the correspondence sense as the maximization of a transitive (but not necessarily complete) binary relation. When such a representation is possible, the binary relation is often interpreted as reflecting the decision maker's well-being. In Rubinstein (2006) and Rubinstein and Salant (2008), we express our reservations regarding this approach. We argue that identifying choices with well-being involves a strong assumption and that the two concepts are, in principle, independent of one another.

We focus on extended choice functions in which the frame "triggers" the use of a particular rationale by the individual when making a choice. Formally

Salient Consideration. An extended choice function c is a Salient Consideration function if for every frame $f \in F$, there exists a corresponding ordering $\succ_f$ such that $c(A, f)$ is the $\succ_f$-maximal element in A.

Thus, a Salient Consideration function c induces a correspondence C_c that assigns to every set A all the elements of A that are maximal according to at least one frame-driven consideration. In particular, no element in $C_c(A)$ is Pareto-dominated with respect to the array of rationales $\{\succ_f\}_{f \in F}$ by any other element in A.

Of the examples discussed in Section 2, Examples 1, 2, 4, 5, and 6 are Salient Consideration functions. For example, in Example 5, given any frame R, the rationale $\succ_R$ can be represented by the utility function

$$u_R(x) = \begin{cases} u(x) + M & \text{if } x \in R \text{ and } u(x) \geq u^* \\ v(x) & \text{if } x \notin R \\ u(x) - M & \text{if } x \in R \text{ and } u(x) < u^*. \end{cases}$$

for a large enough M. Example 3 is not a Salient Consideration function. Indeed, if $xOyOz$ but $zPyPx$, then $c(\{x, y, z\}, 2) = y$, while $c(\{y, z\}, 2) = z$.

For any asymmetric and transitive binary relation $\succ$, let $C_\succ$ be the choice correspondence defined by $C_\succ(A) = \{x \in A \mid \text{ there is no } y \in A \text{ such that } y \succ x\}$.

We now show that the following two explanations of a choice correspondence C are indistinguishable in terms of choice observations when the frame is ignored:

(i) $C = C_\succ$ for some asymmetric and transitive (but possibly incomplete) binary relation $\succ$.
(ii) $C = C_c$ for some Salient Consideration function c satisfying property γ-extended.

Property γ-extended. If $c(A, f) = x$ and $c(B, g) = x$, then there exists a frame h such that $c(A \cup B, h) = x$.

Property γ-extended is satisfied by all the examples in Section 2. In Example 1, let $h = x$; in Example 2, let h be an ordering in which x appears first if x is satisfactory or an ordering in which x appears last otherwise; in Example 3, let $h = |\{a \in A \cup B \text{ s.t. } aOx\}| + 1$; in Example 4, let $h(x) = f(x) + g(x)$ and $h(y) = \min\{f(y), g(y)\}$ for every $y \neq x$; in Example 5, let $h = x$ if $u(x) \geq u^*$ and $h = f \cup g \setminus \{x\}$ otherwise; and in Example 6, let h be $t(x)$.

Proposition 1. *A choice correspondence C satisfies $C = C_{\succ}$ for some asymmetric and transitive binary relation $\succ$ iff there is a Salient Consideration function c satisfying property γ-extended, such that $C = C_{\mathrm{c}}$.*

Proof. Assume that $C = C_{\succ}$ where $\succ$ is asymmetric and transitive. For every element $a \in X$, let $\succ_a$ be an extension of $\succ$ to a complete order relation in which only the elements in the set $\{b \mid b \succ a\}$ are ranked above a (the assumption that $\succ$ is transitive and not merely acyclic is necessary for the construction). Let $F = X$. Define a Salient Consideration function c as follows: $c(A,a)$ is the $\succ_a$-maximal element in A.

The function c satisfies property γ-extended. If $c(A,a) = x = c(B,b)$, then x is $\succ$-maximal in both A and B, which means that x is $\succ$-maximal in the set $A \cup B$ and therefore $x = c(A \cup B, x)$.

It remains to verify that $C_{\mathrm{c}} = C_{\succ}$. If $x \in C_{\succ}(A)$, then for no $y \in A$ does $y \succ x$. Therefore, for all $y \in A$, $x \succ_x y$. Hence, $c(A,x) = x$ which implies that $x \in C_{\mathrm{c}}(A)$. Conversely, if $x \in C_{\mathrm{c}}(A)$, then there exists a frame a, such that $c(A,a) = x$. If there were an element $y \in A$ such that $y \succ x$, then by definition $y \succ_a x$ and thus x could not be $\succ_a$-maximal in A. Thus, $x \in C_{\succ}(A)$.

In the other direction, assume that c is a Salient Consideration function satisfying property γ-extended. Since c is a Salient Consideration function, the correspondence C_c satisfies the following two properties:

Property α. If $x \in B \subseteq A$ and $x \in C_c(A)$ then $x \in C_c(B)$.

Property α^+. If $C_c(A)$ is a singleton and $C_c(A) \in B \subseteq A$ then $C_c(B) = C_c(A)$.

Since c satisfies property γ-extended, the correspondence C_c also satisfies:

Property γ. If $x \in C_c(A) \cap C_c(B)$ then $x \in C_c(A \cup B)$.

Lemma 1 relates properties α, α^+, and γ to the maximization of a transitive binary relation and thus concludes the proof.

Lemma 1. *A choice correspondence C satisfies properties α, α^+, and γ iff there exists an asymmetric and transitive binary relation $\succ$, such that $C = C_{\succ}$.*

Proof. It is easy to verify that $C_{\succ}$ satisfies all three properties. In the other direction, Sen (1971) shows that if a choice correspondence C satisfies properties α and γ, then there exists an asymmetric and acyclic binary relation $\succ$ such that $C = C_{\succ}$. To see that $\succ$ is also transitive, assume that $x \succ y$ and $y \succ z$. Then $C(\{x,y\}) = \{x\}$ and $C(\{y,z\}) = \{y\}$. By property α, the alternatives y and z do not belong to $C(\{x,y,z\})$, and thus $C(\{x,y,z\}) = \{x\}$. By property α^+, $C(\{x,z\}) = \{x\}$, which implies that $x \succ z$. ‖

When we start with an extended choice function c, the binary relation $\succ$ constructed in Proposition 1 must be defined by $x \succ y$ if $C_{\mathrm{c}}(\{x,y\}) = \{x\}$ (as in Sen, 1971). Thus, $x \succ y$ iff $c(\{x,y\}, f) = x$ for every frame f. Bernheim and Rangel (2007) define a similar binary relation: xP^*y if for every set A such that $x, y \in A$, $c(A,f) \neq y$ for every frame f. By definition, P^* is acyclic. The relation $\succ$ is finer than P^* and thus property γ-extended is required to assure its acyclicity. Property α^+ assures its transitivity.

Note that it is sometimes possible to represent an extended choice function as maximizing a transitive binary relation even when it is not a Salient Consideration function. We discuss an example in Section 5.

Observation 3. We now investigate conditions under which an extended choice function is "behaviourally equivalent" to the maximization of a complete and transitive binary relation.

For any complete and transitive relation $\succsim$, let $C_{\succsim}(A) = \{x \in A \mid x \succsim y \text{ for every } y \in A\}$. In order to state Observation 3, we need to strengthen property γ-extended:

Property γ^+-extended. If $c(A, f) = x$, $c(B, g) = y$ and $y \in A$, then there exists a frame h such that $c(A \cup B, h) = x$.

Note that restricting property γ^+-extended to the case in which $y = x$ results in property γ-extended.

Examples 2 and 4 satisfy property γ^+-extended, while the remaining examples do not. Let $X = \{x, y, z\}$. In Example 1, let $v(x) = 1$, $v(y) = 2$, $v(z) = 3$ and $\beta \equiv 1{\cdot}5$. Then $c(\{x, y\}, x) = x$ and $c(\{y, z\}, y) = y$, but x is never chosen from $\{x, y, z\}$. A similar situation occurs in Example 3 if $zOxOy$ and $yPzPx$; in Example 5 if $u(z) > u^* > u(x) > u(y)$ and $v(y) > v(z) > v(x)$; and in Example 6 if $v(y) > v(z) > v(x)$ but $d(z) < d(x) < d(y)$.

The next result states that the following two explanations of a choice correspondence C are indistinguishable when ignoring the frame:

(i) $C = C_{\succsim}$ for some complete and transitive binary relation $\succsim$.
(ii) $C = C_c$ for some Salient Consideration function that satisfies property γ^+-extended.

Proposition 2. *A choice correspondence C satisfies $C = C_{\succsim}$ for a complete and transitive binary relation $\succsim$ iff there is a Salient Consideration function c satisfying property γ^+-extended, such that $C = C_c$.*

Proof. Let $C = C_{\succsim}$ where $\succsim$ is complete and transitive and let $\succ$ be its asymmetric component. By definition, $C_{\succ} = C_{\succsim}$. Constructing a Salient Consideration function c as in the proof of Proposition 1, we obtain that $C_c = C_{\succ} = C_{\succsim}$.

To see that c satisfies property γ^+-extended, note that if $c(A, a) = x$ and $c(B, b) = y \in A$, then x is $\succsim$-maximal in A and y is $\succsim$-maximal in B. By $y \in A$ and the fact that $\succsim$ is complete and transitive, x is $\succsim$-maximal in the set $A \cup B$ and thus $x = c(A \cup B, x)$.

In the other direction, let c be a Salient Consideration function satisfying property γ^+-extended. As in Proposition 1, the correspondence C_c satisfies properties α and α^+. Property γ^+-extended implies the following property of C_c:

Property γ^+. If $x \in C_c(A)$, $y \in C_c(B)$ and $y \in A$, then $x \in C_c(A \cup B)$.

Lemma 2 relates properties α, α^+, and γ^+ to the maximization of a complete and transitive binary relation and thus concludes the proof:

Lemma 2. *A choice correspondence C satisfies properties α, α^+, and γ^+ iff there exists a complete and transitive binary relation $\succsim$, such that $C = C_{\succsim}$.*

Proof. The only if part is immediate. Assume C satisfies properties α, α^+, and γ^+. Then C also satisfies property γ, and thus by Lemma 1 there exists an asymmetric and transitive binary relation $\succ$, such that $C = C_{\succ}$. We expand $\succ$ to a complete relation $\succsim$ by defining $x \sim y$ for every two elements x and y, such that $x \not\succ y$ and $y \not\succ x$. Clearly, $C = C_{\succsim}$. To see that $\succsim$ is transitive it is sufficient to show that $\sim$ is transitive. Assume that $x \sim y$ and $y \sim z$. Then, $z \in C(\{y, z\})$, and $y \in C(\{x, y\})$. By property γ^+, $z \in C(\{x, y, z\})$ and thus $z \in C(\{x, z\})$ by property α, implying that $x \not\succ z$. Similarly, $z \not\succ x$ and thus $x \sim z$. ||

4. BETWEEN THE TWO FRAMEWORKS

Propositions 1 and 2 identify conditions under which a Salient Consideration function induces a correspondence that can be represented as the maximization of a transitive (and possibly complete) binary relation. We now discuss the limitations of the model of choice correspondence, and the maximization of a binary relation in particular, in describing behaviour when the decision maker is affected by framing.

First, a choice correspondence induced by an extended choice function may miss essential information about choice. Consider Example 4 in the Advertisement model where $c(A,i) = \arg\max_{x \in A} i(x)u(x)$. In that case, $C_c(A) = A$ for any choice problem A. This is the coarsest possible choice correspondence, and it carries no information about decision making.

Second, reasonable extended choice functions induce choice correspondences that cannot be described as the maximization of a transitive binary relation. Consider, for example, a decision maker who has in mind an ordering $xOyOz$. He is observed making choices in each of two frames, which are interpreted as moods. In a *Good* mood he maximizes O and in a *Bad* mood he minimizes O. In this case, $C_c(\{x,y,z\}) = \{x,z\}$, and thus any binary relation $\succsim$ that explains C_c must yield either $x \succ y$ or $z \succ y$. However, $C_c(\{x,y\}) = \{x,y\}$ and $C_c(\{y,z\}) = \{y,z\}$, which is a contradiction.

Third, even when there exists a transitive relation that explains the extended choice function, its interpretation may not be straightforward. The explanatory relation combines the preferences of the decision maker and the procedure he is using, but does not allow distinguishing between the two. Consider Example 3 in the Limited Attention model. An extended choice problem is a pair (A,n) and the extended choice function c picks the P-best element among the first $\min\{n,|A|\}$ elements in A according to the ordering O. Then, the explanatory binary relation must be defined by $a \succ b$ if aPb and aOb. (If aPb and aOb, then b is never chosen from $\{a,b\}$ and thus it must be that $a \succ b$. If bPa, then $c(\{a,b\},2) = b$ and thus $a \not\succ b$, and if bOa, then $c(\{a,b\},1) = b$ and thus $a \not\succ b$.) Let us verify that $C_\succ = C_c$. If $a \in C_\succ(A)$, then there is no $x \in A$ such that xPa and xOa, and thus $c(A,n) = a$ for $n = |\{x \in A \text{ s.t. } xOa\}| + 1$. If $a \notin C_\succ(A)$, then there is an element b such that bOa and bPa and thus a is never chosen from (A,n) for every n. Note that the relation $\succ$ treats the two relations P and O completely identically even though one of them expresses the preferences of the decision maker while the other is merely the attention ordering.

Similarly, when an extended choice function c induces a choice correspondence C_c, which can be described as the maximization of a preference relation $\succsim$, the interpretation of the symmetric component of $\succsim$ does not necessarily have any indifference meaning. Recall Example 2 in the List model in which the decision maker chooses the first element x in the list, which satisfies $v(x) > v^*$ and the last element otherwise. The binary relation $\succsim$, for which $C_c = C_\succsim$, can be represented by the utility function $u(x) = 1$ if $v(x) > v^*$ and 0 otherwise. (To see this, note that a satisfactory element a is $\succsim$-maximal in A and thus $a \in C_\succsim(A)$. It is also chosen from A when it appears first in the ordering and thus $a \in C_c(A)$. A non-satisfactory element a appears in $C_\succsim(A)$ only when there are no satisfactory elements in A. It appears in $C_c(A)$ only if A does not include satisfactory elements and a appears last among the elements of A.) But the seeming indifference between two elements does not imply that the decision maker is indecisive or indifferent between them. One could argue that resolving this indifference between the satisfactory elements is immaterial since it does not influence choice. This is not quite accurate. For example, if a social planner is able to affect the ordering of the alternatives, he would want v-better elements to appear earlier in the list.

5. LIMITED ATTENTION

As demonstrated in Section 4, the existence of a binary relation whose maximization describes behaviour does not imply that exploring the details of a frame-sensitive choice procedure is superfluous. These details on how an economic agent's behaviour varies with respect to observable entities are summarized by the extended choice function. Thus, extended choice functions may be the basis for richer economic models that provide an explanation of how seemingly unimportant observables can affect the interaction of economic agents. In preparation for such models, it makes sense to consider families of extended choice functions that have a strong structure. Let us conclude by discussing a simple example.

In the Limited Attention model, an extended choice problem is a pair (A, n), where A is the set of available alternatives and n is the number of elements that the decision maker is capable of evaluating. The number n may be thought of as a measure, which is sometimes observable, of the attention the decision maker devotes to the choice problem. We restrict attention to pairs (A, n) where $n \leq |A|$.

We now characterize the class of extended choice functions described in Example 3. An extended choice function in this class is parameterized by two orderings O and P, such that $c_{O,P}(A, n)$ is the P-best element among the n elements in A that appear first according to O.

The procedure behind this class of extended choice functions has similarities to other two-staged choice procedures suggested in the literature, in which the decision maker constructs an attention set in the first stage and chooses the best element within this set in the second stage. In the procedure discussed here, the attention set consists of the first n elements according to the attention ordering O. In Manzini and Mariotti (2007)'s Rational Shortlist Method, the attention set consists of all elements in A that are not dominated according to a basic binary relation. Another binary relation is then used for maximization within the attention set. In Eliaz and Spiegler (2007), the decision maker is a buyer who faces the menus of two sellers. He constructs a consideration set consisting of the goods offered by the first seller and adds those offered by the second seller if one of them is similar to the best good offered by the first seller.

Three properties characterize the class of all extended choice functions $c_{O,P}$.

The first property states that if alternative a is more accessible to the decision maker than alternative b, then for every set A that contains a and b, the decision maker will not choose b if he devotes little attention to the deliberation process:

Attention 1. If $c(\{a, b\}, 1) = a$, then for every set A that contains a and b, $c(A, 1) \neq b$.

The second property states that if alternative a is chosen over b when the decision maker considers both of them, then for every set A that contains a and b, the decision maker will not choose b when he considers all the elements in the set A:

Attention 2. If $c(\{a, b\}, 2) = a$, then for every set A, $c(A, |A|) \neq b$.

The third property states that adding an element x to a set A, where x is less accessible than all the elements of A, does not alter choice as long as the amount of attention the decision maker devotes to the deliberation process remains the same:

Attention 3. If $c(A, k) = a$ and $c(\{x, y\}, 1) = y$ for every $y \in A$, then $c(A \cup \{x\}, k) = a$.

Clearly, any extended choice function $c_{O,P}$ satisfies Attention 3. To see that $c_{O,P}$ satisfies Attention 1, note that $c_{O,P}(\{a, b\}, 1) = a$ implies that aOb and thus b is not chosen from any set A that contains a and b, when only one element is considered. To see that c satisfies Attention 2, note that $c_{O,P}(\{a, b\}, 2) = a$ implies that aPb and thus b is not chosen from any set A that contains a and b, when all the elements of A are considered.

Any extended choice function c that satisfies the three Attention properties is identical to $c_{O,P}$, where the relations O and P are defined by aOb if $c(\{a, b\}, 1) = a$ and aPb if $c(\{a, b\}, 2) = a$. To see this, note that the relations O and P are complete and asymmetric. The relation O is transitive. Assume to the contrary that xOy, yOz, and zOx, and without loss of generality let $c(\{x,y,z\},1)=x$. By Attention 1, zOx implies that $c(\{x,y,z\},1)\neq x$, which is a contradiction. The relation P is also transitive. Assume to the contrary that xPy, yPz, and zPx and without loss of generality let $c(\{x,y,z\},3)=x$. By Attention 2, zPx implies that $c(\{x,y,z\},3)\neq x$, which is a contradiction.

Finally, $c(A,k)$ is the P-maximal element among the first k elements according to O. To see this, let B be the set of the first k elements in A according to O. Then, by Attention 2, $b=c(B,k)$ is the P-maximal element in B. Otherwise, there is an element $a\in B$ such that aPb and Attention 2 implies that $c(B,k)\neq b$. Since for every element $x\in A-B$ and $a\in B$, we have aOx, we obtain by Attention 3 that $c(A,k)=c(B,k)$.

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