

Social Media Marketing: How Much Are Influentials Worth? *

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Abstract

This paper studies the competition between firms for social media influencers. Firms spend effort to convince influencers in a social network to recommend their products. The results show that the value of an influencer depends only on the structure of the influence network and not the level of competition in the product market. Influencers who exclusively cover a high number of consumers are more valuable to firms than those who mostly cover consumers also covered by other influencers. The total effort expended by firms is completely determined by the in-degree distribution of the network. Firm profits are highest when there are many consumers with a very low in-degree or with very high in-degree. Consumers with an intermediate level of in-degree contribute negatively to profits and high in-degree consumers increase profits when market competition is not intense. Prices are generally lower when consumers are covered by many influencers, however, firms are not always worse off with lower prices. The nature of consumer response to recommendations makes an important difference. When first impressions dominate, firm profits for dense networks are higher, but when recommendations have a cumulative influence profits are reduced as the network becomes dense. An extension models the incentives firms provide to influencers and how these affect influencers' network formation behavior. Direct monetary incentives lead to dense networks and consumers covered by many influencers. Such networks are only beneficial to firms when market competition is not intense. When the market is more competitive, firms are better off providing smaller direct incentives just enough to ensure a sparse coverage of consumers by influencers.

1 Introduction

The emergence of social media is transforming the way firms approach consumers. Nielsen reports¹ that 92% of consumers trust product recommendations from people they know, vastly exceeding any form of advertising or branded communication. Recognizing the power of word-of-mouth, marketers began to reach out to influential consumers, hoping that customers can convince their peers more effectively than traditional advertising would.

In order to utilize the value of the vast consumer-to-consumer communication networks facilitated by social media, companies need to: i) identify influencers, and ii) convince influencers to recommend their products. There are a number of upcoming service providers offering assistance with the first task. The most notable example is Klout.com, a site that collects data about consumers from social networks to estimate their influential power. While the task of assigning individual influence scores is not easy and frequent adjustments to the methodology are necessary,² various startups have made good progress in this direction, some of them being able identify influencers at a very granular level.³

Once marketers have identified influencers the next step is to get them to recommend the marketed product. There are different approaches to convincing influencers, but all of them involve considerable effort from firms. The activities firms conduct can range from simple communications that demonstrate the value of the product through offering extra perks to directly paying influencers. For example, Cathay Pacific offers lounge access to customers (of any airline) that have a high Klout score.⁴ Other companies such as online fashion retailers Bonobos and Gilt offer discounts, whereas Capital One provides increased credit card rewards for customers with high influence scores. Some of these offers go to the length of giving away products for free.⁵

It is apparent that companies spend considerable effort trying to identify and win over influencers, but it is not clear what the value of each influencer is and how this value depends on the influence network between consumers. This problem is reflected in the widespread and intense discussions

¹ “Global Trust in Advertising and Brand Messages”, Nielsen, April 2012

² “Klout, Controversial Influence-Quantifier, Revamps Its Scores”, Businessweek, August 14, 2012

³ “Finding Social Media’s Most Influential Influencers”, Businessweek, October 18, 2012

⁴ “Free Cathay Pacific Lounge Access at SFO via Klout.. If You Are Cool Enough,” available at <http://thepointsguy.com/2012/05/klout-offers-some-free-cathay-pacific-lounge-access-at-sfo/>

⁵ “Gilt Gives Discounts to Match Klout Scores”, available at <http://allthingsd.com/20120305/gilt-gives-discounts-to-match-klout-scores/>

among practitioners about the return on investment in social media marketing efforts.⁶

The conventional wisdom suggests that consumers who have many peers listening to them are valuable, but this simple prescription that only considers the reach of each influencer does not take into account the potential overlap between consumers covered by different influencers. In a competitive environment, it is crucial to understand how firms should value each influencer depending on which of their peers these influencers can have an impact on. Another commonly held view is that the more links and communication there is, the better off marketers who rely on influencers are. Again, this train of thought ignores the potential for competition and the possibility that influencers may recommend the competitor's product.

In order to rigorously study the problem of how much effort to spend on influential consumers in a competitive environment, we develop an analytical model addressing the following questions:

- How much effort should competing firms spend on each influencer depending on the intensity of competition?
- What is the role of the network structure? Are highly connected influentials always the most valuable?
- What is the effect of the influence network on prices, firm profits, and consumer surplus?
- How should firms incentivize influentials? Should they offer direct (monetary) benefits or should they convince them in other ways?
- How will influentials change their network in response to different incentives?

The main model includes two competing firms who try to win over influencers in a network. Firms exert effort on each influencer and when they succeed the influencer recommends their product. Consumers who receive recommendations about only a single product provide a unit margin to the firm selling that product, whereas those who receive recommendations about both products provide a lower margin to both firms. The exact margin depends on the intensity of competition and can be as low as zero in a very competitive environment.

We find that the equilibrium is symmetric where the two firms follow the same strategies, but the effort levels for each influencer differ based on his or her position in the network. We identify

⁶ "Driving Business Results With Social Media", - Businessweek, January 21, 2011,

the equilibrium effort in the most general fashion: for each influencer in any network. Surprisingly, the effort levels are independent of the intensity of product market competition between firms. This follows from the phenomenon that firms value influencers for both offensive and defensive purposes. On one hand, winning over an influencer makes it possible to convey a message to consumers who do not otherwise receive recommendations about a product. On the other hand, winning over an influencer prevents the competing firm from having its product recommended to consumers. The combination of the two effects makes it equally important to spend effort on influencers in both a competitive and a non-competitive environment (offense pays off more in the latter, defense in the former).

The equilibrium effort level for a given influencer is a function of the network structure. In particular, the value of an influencer to firms highly depends on whether consumers covered by this influencer are covered by other influencers and if yes, by how many of them. Adding up the effort levels of one firm for all influencers reveals that the total effort exerted is determined by a very simple network statistic: the in-degree distribution of the influence network. Consumers with a small in-degree, those who are influenced by only a few influencers, contribute to effort levels less than those with a high in-degree. As a consequence, highly connected influencers are valuable, but only if they cover consumers who are not covered by many other influencers. Firm profits depend on the network structure in an interesting way as the profit is a U-shaped function of the in-degrees. Networks where each consumers covered by very few, and those where each consumer is covered by a large number of influencers are the most profitable, whereas networks where each consumer is covered by an intermediate number of influencers are the least profitable.

To study how recommendations influence purchase decisions in more detail, we extend the model to include a more elaborate influence process that allows us to examine consumer consideration and choice together firms' pricing decisions. In equilibrium, we find that firms employ mixed pricing strategies while using effort levels similar to those of the basic model to convince influencers. We find that prices depend on the network structure in an interesting fashion. Firms charge lower prices to consumers covered by more influencers. As a result, in sparse networks, where consumers are typically covered by a few influencers, prices are generally high as firms are able to extract almost all surplus from consumers who only consider one product. At the other extreme, when consumers are under the influence of many influencers, prices go down as price competition increases. Surprisingly,

these lower prices do not always hurt firms. When first impressions about a product determine consumers' product considerations, highly covered consumers can increase firm profits as firms cut back on their efforts trying to convince influencers. When recommendations have a cumulative effect, firms spend relatively heavily on winning over influencers even in dense networks, resulting in lower profits.

Finally, we study how firms should incentivize influencers to form their networks and how consumers react to different incentives. In principal, firms have two different ways of making influencers recommend their product. We find that when firms use a direct, possibly monetary incentive, influencers establish more links, expanding their network. This can benefit or hurt firms depending on the level of competition. The main result is counterintuitive: when the product market is competitive firms should not offer a very high monetary benefit to influencers so as to avoid too much coverage. When the market is not that competitive, firms should encourage influencers to build their network with substantial monetary benefits.

The rest of the paper is organized as follows. In the next section, we review the relevant literature, then, in Section 3 we introduce the model. In Section 4, we derive the equilibrium of the basic model. Next, in Section 5, we examine the pricing decisions and in Section 6, we study the incentives provided by firms to influencers and the resulting endogenous network formation behavior. Finally, in Section 7, we conclude and discuss the practical implications and limitations of our results.

2 Relevant Literature

The importance of word-of-mouth communications in marketing has been long recognized by the literature, starting with the widely employed product diffusion model of Bass (1969). Most of the early models concerned aggregate effects, but with the new developments in network analysis and the availability of network data, academic research recognized the need to understand the role of the underlying network. For example, Goldenberg et al. (2001) studied the diffusion process on a grid as a special type of network. First on simple, then on more complex networks the marketing literature has uncovered several important network properties and their contribution to the diffusion process (Goldenberg et al. 2009, Katona et al. 2011, Yoganarasimhan 2012), helping to identify influencers (Tucker 2008, Trusov et al. 2010). At the same time, computer scientist have also addressed the question of how to seed a network optimally to maximize viral spread (Kempe et al. 2003, Stonedahl

et al. 2010).

The role of marketing effort in the diffusion process has been further studied by Mayzlin (2002) who considered the tradeoff between traditional advertising and firm generated buzz, while a stream of papers (Van den Bulte and Lilien 2001, Nair et al. 2010, Iyengar and Valente 2011) examined the effects of marketing effort and social contagion in the drug adoption decisions made by physicians also considering firms' marketing activities. Godes and Mayzlin (2009) ran a field experiment to study how a firm's agents can affect word-of-mouth communications between consumer and find that the effectiveness depends on the nature of ties.

Most papers in the area focus on how a monopolist should target a few consumers in a network to optimize the diffusion of a single product or idea, ignoring competition. A notable exception considering competition in a direct marketing setting is a paper by Zubcsek and Sarvary (2011) who study competing firms advertising to a social network. They find that it is important for advertisers to take into account the existence of the underlying network, especially its density. Despite some similarities, our paper is different in several aspects. Our primary focus is not on seeding strategies that seek to find optimal targets that would maximize the spread of a given campaign. Instead, we analyze how firms should invest to win over influencers in a network in order to capture market share, somewhat akin to Chen et al. (2009). We also take into account that firms can exert different amount of effort engaging in a contest for each influencer. This allows us to quantify the value of each influencer which - as our findings show - depend heavily on the distribution of in-degrees.

A notable paper that also considers competition is by Galeotti and Goyal (2009) who present a simple competitive extension to their model on how to influence influencers. In contrast to our model, they do not study how firms target specific influencers, reducing the decision to a single variable on advertising intensity. Finally, He et al. (2012) consider firm incentives in a competitive setting to induce connections between customers.

Our paper is also related to the literature on targeting and advertising. Our model includes the ability of firms to individually target influencers, related to Chen and Iyer (2002), Chen et al. (2001). Papers such as Iyer et al. (2005), Amaldoss and He (2010) study the strategic effects of targeting and how firms can avoid intense price competition. Similarly to a large proportion of the prior literature on informative advertising, our treatment of pricing decisions builds on the widely adopted formulation by Varian (1980), Narasimhan (1988).

3 Model

3.1 The Consumer Influence Network

There are M consumers in the market, $N \leq M$ of which are influencers. We call consumers who can potentially affect their peers' product choices influencers.⁷ We order consumers so that the first N consumers, that is $i = 1, 2, \dots, N$, are influencers. Each influencer can potentially influence any of his or her peers, whom we call influencees.

A key feature of our model is the underlying influence network structure that determines the influencer-influencee relations. This influence network need not be identical to the underlying network of social connections, as not all of them result in effective influence between consumers. Furthermore, we only consider the influence that takes place in the context of the relevant products in our model. Nevertheless, since influence is likely to take place over social links, hence we think of the influence network as a narrow product-specific subnetwork of the social network.⁸

We model the influence network as a random network. Let I_{ij} be the indicator variable showing if a product recommendation takes place ($I_{ij} = 1$ if yes, 0 otherwise) between influencer i and influencee j . When $I_{ij} = 1$, a unidirectional influence link exists between i and j . We say that influencer i covers consumer j in this case. We use several subscripts to denote consumers throughout the analyses, but for clarity we distinguish between influencers (denoted by i, h) and influencees (denoted by j, g) based on their role.

We take a very general approach to model how the random network is generated, where

$$w_{ij} = \Pr(I_{ij} = 1) \tag{1}$$

measures the expected strength of influence of influencer i on consumer j . We do not specifically model the nature of influence between consumers, which can take several forms. For example, a very simple type of influence is when one consumer informs a peer about a product, making him or her consider it.⁹ A more complex form of influence is when consumers share positive information about a product and try to convince their peers about the product benefits. Regardless of the specific process,

⁷In theory, all consumers could be influencers (and our model allows this), but to paint a more realistic picture, we assume that only a proportion of them has enough influential power to significantly effect others' choices.

⁸In Section 6 we consider an extension, where consumers decide on their influence levels over existing social links.

⁹We use this specification in Section 5 when modeling pricing strategies.

we assume that a higher w_{ij} corresponds to a higher likelihood of an effective correspondence taking place.

The influence network matrix \mathbf{W} is defined as the collection of w_{ij} values. We assume that \mathbf{W} is common knowledge, but that the outcome of each random variable is not observed by players. One can think of \mathbf{W} as the adjacency matrix of the weighted network which is obtained as the expected influence network between consumers. For example, if all $w_{ij} \in \{0, 1\}$, then the influence network is deterministic and the expected network is the same as the actual influence network. However, it is more realistic to assume that players do not observe the exact network, especially since this network is not identical to the underlying social network. Product influence rarely takes place with high probabilities on all social network links. That is, \mathbf{W} represents what firms believe the influence network could be. With the emergence of social network analytic technologies and services these beliefs become more and more accurate. Companies like Klout provide measures about consumer influence levels, and although some of these measures are still rudimentary, there is rapid development in this area. Our model is very flexible with regards to how much information firms have about the influence network ranging from one extreme of complete information (represented by a deterministic \mathbf{W}) to the other extreme of a no or little information. Firms in this case would have some prior belief about how the influence links are distributed, resulting in a stochastic \mathbf{W} matrix with large variances.

An important quantity in our analysis is the total number of influencers that cover a particular influencee. We call this influencee j 's in-degree and denote it by d_j . We also need to count this quantity separately for consumers covered by a particular influencer. In summary, we introduce the notations

$$d_j = \sum_{h=1}^N I_{hj}, \quad \varphi_{ki} = \sum_{j=1}^M \Pr(I_{ij} = 1, d_j = k), \quad \text{and} \quad \varphi_k = \frac{1}{k} \sum_{i=1}^N \varphi_{ki}. \quad (2)$$

That is, φ_{ki} counts the expected number of consumers covered by i , who are influenced by exactly k influencers (including i), whereas φ_k measures the expected frequency of consumers covered by k influencers in the entire network. φ_k can also be interpreted as the expected in-degree distribution of the influence network, whereas φ_{ki} is the expected degree distribution among consumers covered by influencer i . Since our setup is very general in terms of the network structure, we derive the solution for any type of degree distribution. However, in order to ensure an equilibrium in pure strategies,

we assume that φ_{1i} is not too small for any influencer.¹⁰

3.2 Firms and Influencers

We assume that there are two firms ($f = 1, 2$) selling their products to the M consumers. Firms take advantage of social media and target influencers to increase the revenue they can expect from consumers. Consumers thus influence their peers to buy the product of one of these (or both) firms. Let R_f denote the set of consumers who receive recommendations only about the product of firm f , whereas R_{12} the set of those who receive recommendations about both products. In our baseline model, we assume a simple revenue structure for firms who sell their products at zero variable cost. Each consumer $j \in R_f$ provides a revenue of 1 to firm f , whereas a consumer $j \in R_{12}$ provides a revenue of $q \geq 0$ to both firms. In essence, q measures the intensity of competition between the two firms. When $q = 0$, the two firms are competing heavily and cannot extract any profits from consumers that strongly consider both products. As q increases to $1/2$ the competition becomes less intense and while the products are still substitutes, firms can extract the same amount of profit from these consumers as from those only considering one product. As q increases further, the two products eventually become complements.

Although we do not directly model the product market competition in this basic model, there are several micromodels that produce results corresponding to our assumption. For example, if we assume that personalized pricing is possible, Bertrand competition leads to $q = 0$ when consumers in R_{12} compare the two products. Higher q values could be obtained using a bargaining or differentiation models (e.g. in a linear city q would roughly correspond to t/v .) In Section 5, we present such an extended version of the model, where firms make pricing decisions and show that it can correspond to our basic model with any $0 \leq q < 1$. In that model, we assume that firms are able to price discriminate across consumers, but we do not require firms to learn more about consumer identities that they already know from the influence network. Furthermore, even if personalized pricing is not possible, and all consumers pay the same price, one can produce similar results.

We assume that the N influencers have a limited bandwidth to talk to their peers and they only recommend one product. This is consistent with survey results¹¹ showing that influencers

¹⁰Formally, we need $\varphi_{1i} \geq \underline{C}(1/4) \sum_{k=1}^{\infty} \varphi_{ki}$, where \underline{C} depends on the model parameters, but a \underline{C} of around $1/4$ is typically sufficient. This is a technical assumption which does not put too much restriction on the network structure, especially if the w_{ij} probabilities are relatively small.

¹¹“*Three Surprising Findings About Brand Advocates*”, Zuberance Whitepaper, February 2012, available at

recommend a limited number of products, most of them less than ten per year. Since this already low number of recommendation is split over different product categories, it is plausible to assume that most influencers recommend only one of two competing products. To become the one recommended brand, firms invest resources to convince influencers to recommend their product. For each influencer, let e_{fi} denote the effort spent by firm f on influencer i . We assume that the probability that an influencer recommends a firm is proportional to the effort spent by firms. Formally,

$$\Pr(i \text{ recommends firm } f) = P_{fi}(e_{1i}, e_{2i}) = \frac{e_{fi}^{1/r}}{e_{1i}^{1/r} + e_{2i}^{1/r}}. \quad (3)$$

This is called a Tullock success-function (Tullock 1980), which is often used to model contests and all-pay auction in the literature. The parameter r measures the softness of competition for winning over an influencer. The lower the r , the more firm efforts affect the probability that the firm is picked by an influencer.¹² We assume $r > 1/2$ to ensure a pure-strategy equilibrium. As a result of firm efforts, influencers recommend the appropriate products and consumers receive recommendations about either one or both products. Thus, the payoff of firm f becomes

$$\pi_f = (|R_f| + q|R_{12}|) - \sum_{i=1}^N e_{fi}. \quad (4)$$

Note that we do not explicitly model how firms convince influencers to recommend their products, instead we take a general approach and only focus on how much firms spend. In Section 6, we explore an interesting extension, where firms can provide different types of incentives to influencers.

4 Equilibrium Analysis

Our first goal is to determine the equilibrium effort that firms invest in each influencer. Since the game is symmetric, we will focus on the symmetric equilibrium which, as we will show, is generally the unique pure-strategy equilibrium.

Let us examine the decision of firm 1 for influencer i , where $N_i = \{j : I_{ij} = 1\}$ denotes the set of consumers that i covers. Assume that all of firm 2's effort levels are fixed and that firm 1's effort

<http://zuberance.com/brandadvocateresearch/>

¹²Another way to interpret r is as the exponent of the cost of effort. If we assume a strictly proportional success function $\frac{e_{fi}}{e_{1i} + e_{2i}}$, then the above formulation is equivalent to assuming that the cost of effort is $c(e) = e^r$.

levels for influencers other than i are also fixed. That is, P_{1h} is also fixed for all $h \neq i$ influencers. Since we are focusing on a symmetric equilibrium, we can assume that $P_{1h} = 1/2$ for all $h \neq i$. Then the payoff of firm 1 can be written as

$$\mathbf{E} \pi_1 = \sum_{g=1}^M [\Pr(g \in R_1) + q \Pr(g \in R_{12})] - e_{1i} - \sum_{h \neq i} e_{1h}. \quad (5)$$

Thus, we write the revenue part of the expected profit as a sum where, for each consumer g , we calculate the expected revenue from that consumer. The expected revenue is the probability that the consumer only gets recommendations for product 1 plus q times the probability that s/he gets recommendations for both products. Note that these probabilities do not change for consumers that are not covered by N_i , therefore we restrict our attention to the part of $\mathbf{E} \pi_1$ that varies with e_{1i} :

$$\sum_{g \in N_i} [\Pr(g \in R_1) + q \Pr(g \in R_{12})] - e_{1i} \quad (6)$$

These probabilities further depend on how many other influencers cover g . Let k denote the total number of influencers (including i) that have g in their influence set. For example, if $k = 1$, the only way to access consumer $g \in N_i$ is through influencer i . Thus,

$$\Pr(g \in R_1) + q \Pr(g \in R_{12}) \underset{k=1}{=} \Pr(g \in R_1) = P_{1i}(e_{1i}, e_{2i}) = \frac{e_{1i}^{1/r}}{e_{1i}^{1/r} + e_{2i}^{1/r}}, \quad (7)$$

and this consumer will yield a revenue of 1 iff influencer i recommends firm 1 (the probability of which is P_{1i}). When $k > 1$,

$$\Pr(g \in R_1) + q \Pr(g \in R_{12}) = \frac{1}{2^{k-1}} P_{1i} + q \left(\frac{2^{k-1} - 1}{2^{k-1}} P_{1i} + \frac{2^{k-1} - 1}{2^{k-1}} (1 - P_{1i}) \right). \quad (8)$$

The first term refers to $\Pr(g \in R_1)$, the probability that g only receives recommendations about product 1. This happens if *all* influencers covering g are captured by firm 1. The probability that influencer i is covered is P_{1i} , whereas the other influencers recommend firm 1 with probability $1/2$, hence the $\frac{1}{2^{k-1}}$ multiplier. The probability $\Pr(g \in R_{12})$ is calculated by examining two cases. When influencer i recommends firm 1 (with probability P_{1i}) then the probability that at least one other influencer covering g recommends i is $1 - \frac{1}{2^{k-1}} = \frac{2^{k-1} - 1}{2^{k-1}}$. When When influencer i recommends firm 1 (with probability $1 - P_{1i}$) then in order for g to receive recommendations about both product,

we need to make sure that at least one other influencer recommends product 1. That is, we need to calculate the probability that not all influencers covering g recommend product 2. Since all the probabilities are $1/2$, this is the same values as in the previous case: $1 - \frac{1}{2^{k-1}} = \frac{2^{k-1}-1}{2^{k-1}}$.

After rearranging and plugging in the proportional success function from (3), we obtain

$$\Pr(g \in R_1) + q \Pr(g \in R_{12}) = \frac{1}{2^{k-1}} \cdot \frac{e_{1i}^{1/r}}{e_{1i}^{1/r} + e_{2i}^{1/r}} + q \frac{2^{k-1} - 1}{2^{k-1}}. \quad (9)$$

We can now sum the above according to (6) for all consumers, using the notation φ_{ki} that counts the number of consumers covered by i and exactly $k-1$ other influencers:

$$\sum_{g \in N_i} [\Pr(g \in R_1) + q \Pr(g \in R_{12})] = \frac{e_{1i}^{1/r}}{e_{1i}^{1/r} + e_{2i}^{1/r}} \sum_{k=1}^{\infty} \frac{\varphi_{ki}}{2^{k-1}} + q \sum_{k=1}^{\infty} \varphi_{ki} \frac{2^{k-1} - 1}{2^{k-1}}, \quad (10)$$

Dropping the second term that does not depend on e_{1i} , we can write the profit as

$$\mathbf{E} \pi_1 = \frac{e_{1i}^{1/r}}{e_{1i}^{1/r} + e_{2i}^{1/r}} \sum_{k=1}^{\infty} \frac{\varphi_{ki}}{2^{k-1}} - e_{1i} + C, \quad (11)$$

where C is a constant independent of e_{1i} . The above formula tells us that the value of influencer i for firm 1 depends only on how many other influencers cover i 's influencees. For a particular g influencee of i that is covered by exactly k influencers the value is exactly $\frac{1}{2^{k-1}}$. To derive the equilibrium effort levels, we simply take the first order condition and obtain the symmetric equilibrium. The complete proof can be found in the Appendix, where we check the second order conditions, and examine the uniqueness of the symmetric equilibrium. The following proposition summarizes the main results.

Proposition 1

1. *The symmetric equilibrium effort level for firm f and influencer i is*

$$e_{fi} = \frac{1}{4r} \sum_{k=1}^{\infty} \frac{\varphi_{ki}}{2^{k-1}}.$$

2. *The total effort levels and the profits are*

$$\sum_{i=1}^N e_{fi} = \frac{1}{4r} \sum_{k=1}^{\infty} \varphi_k \frac{k}{2^{k-1}}, \quad \pi_f = \sum_{k=1}^{\infty} \varphi_k \left(\frac{2 - k/r}{2^{k+1}} + q \frac{2^{k-1} - 1}{2^{k-1}} \right).$$

3. *If q is not too large, the above equilibrium is unique.*

The results highlight various interesting features of the equilibrium. The first part provides a very important quantity: the amount a firm invests to convince an individual influencer to recommend its product. The formula shows a nice pattern as the equilibrium effort level is a sum with components corresponding to each consumer that the influencer has potential influence on. Each individual that is only influenced by i contributes $\frac{1}{4r}$ to the equilibrium effort level for i . This is consistent with the literature on contests, as this is the exact effort level that one of two competitors incurs for a prize worth 1 unit. That is, each influencee that is only covered by influencer i contributes 1 unit to i 's worth. However, each influencee that is covered by exactly k influencers (i plus $k - 1$ others) contributes exactly $\frac{1}{2^{k-1}}$ to i 's value to a firm. Surprisingly, this value and the entire effort level does not depend on q . The intuition for this unexpected result is based on the two different purposes of capturing an influencer. One is the offensive reason, to reach consumers who only receive recommendations about the other product. The offensive benefit from such a consumer is q . The other purpose is to prevent competitors from reaching consumers who only receive recommendations about one firm's product. The defensive value is $1 - q$. In the symmetric equilibrium the two purposes are equally likely to be at play, thus, q is canceled out.

Following from the above, the second part of the proposition demonstrates that the total effort spent by one firm also does not depend on q . Interestingly, the network structure determines the total effort and profits in a very simple way. If one knows the expected number of individuals covered by exactly k influencers, the total effort and profits are simply a weighted sum of these φ_k coefficients. Figure 1 shows the weights. When $q = 0$, the weight of φ_k is $\frac{2-kr}{2^{k+1}}$ which is a U-shaped function. That is, in a very competitive market only individuals who are only covered by exactly one influencer contribute positively to profit. The number of individuals covered by exactly two influencers does not change profits, whereas those covered by more than two influencers decrease profits. However, those covered by a large number of influencers have only a small negative effect as firms realize that competition is tough and decrease their efforts for highly covered consumers.

It is no surprise that influencers with exclusive coverage (influencing consumers who cannot be reached through other influencers) are the most valuable. The interesting phenomenon is that as the coverage level increases, the profit contribution of highly covered consumers begins to increase at some point. The reason is the strategic effect in the contest for influentials. Firms expect competition

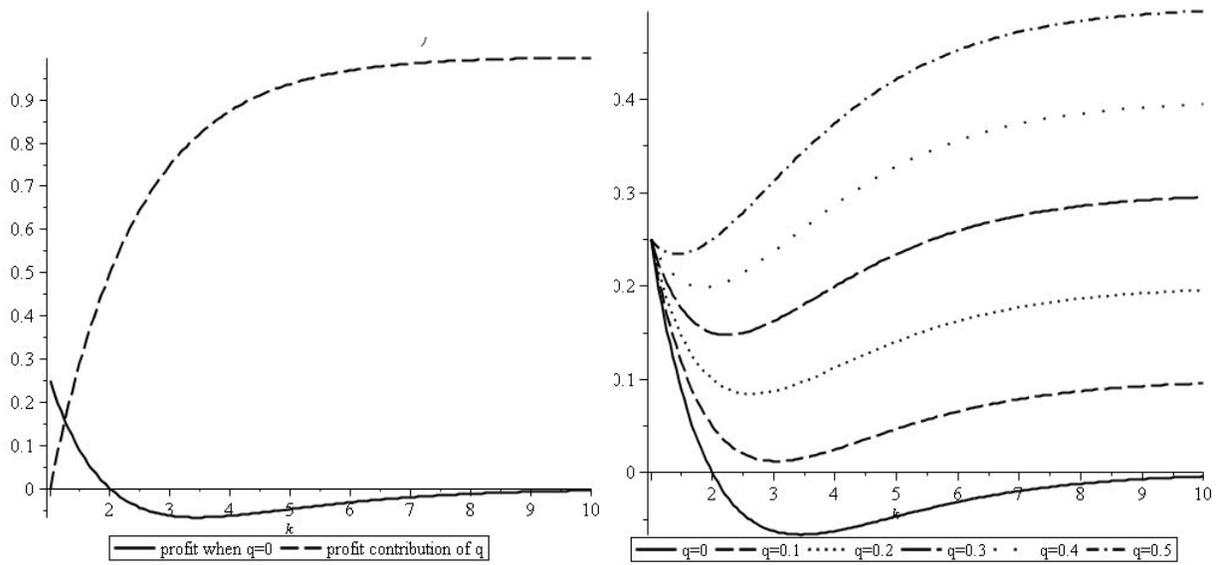


Figure 1: The contribution of different in-degrees to profits. On the left, the solid curve represents $q = 0$ while the dashed curve shows the component multiplied by q . On the right, the profit is depicted for different values of q .

to be tough for influencers covering highly covered consumers and they cut back on their effort to capture them. The reduction in effort exceeds the reduction in revenue for highly covered customers, leading to the U-shaped curve. In particular, when $q > 0$ the profit contribution of highly covered individuals changes in an interesting way. As $k \rightarrow \infty$, the contribution of a consumer covered by k influencers converges to q . Despite the reduced incentives to invest, these consumers contribute to profits positively. As they most likely receive recommendations about both products, they yield a profit of q to each firm.

It is worthwhile to examine how the two parameters measuring competition affect the results. Recall that q measures the softness of the product market competition, whereas r measures the softness of the competition for winning over influencers. As one would expect, profits are increasing in both r and q . As competition becomes softer in either area, firms are better off. However, the value of consumers with different in-degrees changes in an interesting way with these parameters as the following corollary summarizes.

Corollary 1

1. *There exists a \bar{k} such that consumers covered by $k > \bar{k}$ influencers contribute to profits more than those covered by one influencer if and only if $q > \frac{1}{2} - \frac{1}{4r}$.*
2. *Consumers covered by exactly $\lfloor 2(1 + (1 - 2q)r) \rfloor$ influencers contribute the least to firm profits. Their contribution is negative when q and r are low.*

Firstly, since the profit contribution of a consumer covered by k influencers is a U-shaped function of k , it is not clear which consumers are the most valuable to firms. As we show above, the consumers who are worth more are either the ones covered by only one influencer or the ones covered by all of them. If $q > \frac{1}{2} - \frac{1}{4r}$, that is, if competition is generally soft (q and/or r are high), then consumers covered by all influencers are the most valuable in their contribution to profits. When competition either in the product market or for influencers is tough, then we get the opposite results and influencers that have a large exclusive coverage are worth the most.

Secondly, consumers who are covered by an intermediate range of influencers are the least valuable to firms as firms have less incentive to win over influencers covering such consumers. When competition is tough, firms will want to avoid these consumers and the influencers covering them as they decrease profits. This result has the intriguing consequence that firms are better off when

these consumers exit the market. That is, firms not only have the incentive to differentiate between influencers based on their coverage, but different consumers also provide different value to firms based on their position in the network.

Our results also refute the conventional wisdom that denser networks are better for firms that take advantage of word of mouth. Removing influence links can actually increase profits in certain cases. In Section 6 we will investigate further how firms can incentivize influencers to establish the optimal amount of links.

Finally, it is useful to uncover what are the optimal influence networks that firms profit the most from. When q is low ($< \frac{1}{2} - \frac{1}{4r}$), firms would naturally prefer to avoid all competition and, at the same time, cover all consumers. Hence, the optimal influence network is one where each consumer is covered by exactly one influencer. This is a very specific network that is unlikely to be formed in reality. A more realistic setting is when the density of the network is given and we would like to determine the optimal network given the density. Fixing the total number of expected links, as $\sum_{i=1}^N \sum_j = 1^M w_{ij} = \sigma > M$, for a given number of N also fixes the density at σ/N . When q is low, the optimal network will have $\frac{\sigma-M}{N-1}$ consumers covered by all influencers and the remaining consumers covered by exactly one influencer. Figure 2 shows such an optimal network, where a core of consumers are highly influenced and the rest are relatively isolated.

5 Influence, Consideration, and Prices

In our basic model we took a generic, but simplified approach to model the nature of influence and competition. We assumed that firms made a fixed amount of profit per consumer which depended on whether consumers received recommendations about only one or both products. Here, we extend the model to incorporate the pricing decisions. To do so, we model the process of product recommendations between consumers in more detail. If a consumer receives product recommendations from an influencer, the recommended product will enter his/her consideration set with some probability. To account for the impact of multiple recommendations, we use the function $0 \leq \gamma(\ell) \leq 1$ to denote the probability that a consumer receiving exactly ℓ recommendations about a product considers it.

Each consumer has a reservation price normalized to 1 for each product. If there is only one product in the consumer's consideration set, s/he purchases that product as long as the price does not exceed his or her reservation price. When there are multiple products in his or her consideration

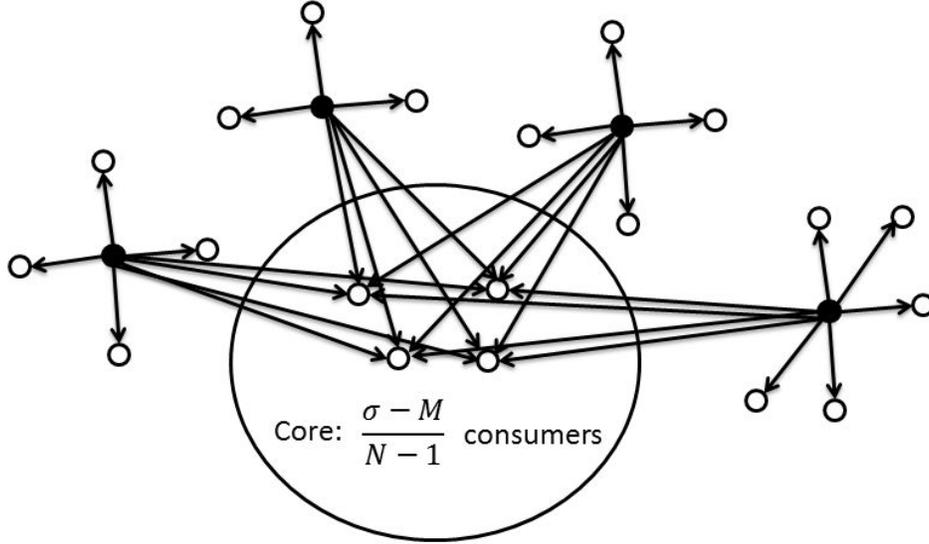


Figure 2: Optimal influence network for firms

set, the consumer chooses the lowest priced product.

We also include firms' pricing decisions in the model. Firms set their prices after the efforts for influencers have been set and committed to.¹³ We assume that firms are able to price discriminate¹⁴ and set a different price p_{fj} for each consumer j . Personalized pricing is more and more common in practice,¹⁵ and the phenomenon has been studied in academia (Chen and Iyer 2002, Zhang 2011). In order to customize prices firms do not have to identify all of the individual customers and exactly determine their willingness to pay.¹⁶ Consistently, we assume that firms only have limited knowledge in the pricing stage. They possess the exact same information about consumers as they have in the first stage, namely the random influence network. When the network is stochastic, this leads to considerable uncertainty with respect to a particular consumer's consideration set, making it

¹³This is consistent with the advertising literature, where advertising decisions typically precede pricing.

¹⁴Formally, we only need that firms are able to discriminate between different types of consumers: those who have different number of influencers covering them. Furthermore, even if firms have to charge the same prices for all consumers the results are not too different. For example, if there are only two influencers, the results are identical.

¹⁵"Shopper Alert: Price May Drop for You Alone", New York Times, August 9, 2012

¹⁶"Web sites change prices based on customers' habits" CNN, June 24, 2005

impossible to extract all the surplus.

As before, we expect a symmetric equilibrium in firm efforts which leads to randomness in which firm the influencer will recommend. Also, the influence network itself is random resulting in uncertainty about what recommendations consumers receive. Consistently with our previous modeling concept, we assume that firms cannot observe individual recommendations.¹⁷ Given all the uncertainty, it is easy to see that there is no pure-strategy equilibrium in the pricing stage: Since there is a positive likelihood that consumers consider both products, but also that they consider only one product, we find an equilibrium with mixed strategies in pricing, similarly to Varian (1980) and Narasimhan (1988). The following proposition describes the prices and the effort levels in the unique symmetric equilibrium that comprises of pure effort strategies. Note that we define a $S_k(t)$ generating function in the proposition to simplify notation, where t takes the value of 0 or 1.

Proposition 2

1. The equilibrium effort level for firm f and influencer i is

$$e_{fi} = \frac{1}{4r} \sum_{k=1}^{\infty} \varphi_{ki} \left(\frac{4}{k} S_k(1) - 2S_k(0) \right), \text{ where } S_k(t) = \sum_{\ell=0}^k \frac{\binom{k}{\ell} \gamma(\ell) (1 - \gamma(k - \ell)) \ell^t}{2^k}.$$

2. The total effort levels and the profits are

$$\sum_{i=1}^M e_{fi} = \frac{1}{4r} \sum_{k=1}^{\infty} \varphi_k (4S_k(1) - 2kS_k(0)), \quad \pi_f = \sum_{k=1}^{\infty} \varphi_k \left(\left(1 + \frac{k}{2r} \right) S_k(0) - \frac{S_k(1)}{r} \right).$$

3. The price p_{fj} is a random variable with support $\left[\frac{\alpha}{\alpha + \beta}, 1 \right]$ and p.d.f. $g(p) = \frac{\alpha}{\beta p^2}$ where

$$\alpha = \sum_{k=1}^{\infty} \Pr(d_j = k) S_k(0), \quad \beta = \sum_{k=1}^{\infty} \Pr(d_j = k) \sum_{\ell=0}^k \frac{\binom{k}{\ell} \gamma(\ell) \gamma(k - \ell)}{2^k}$$

The results demonstrate how the effort levels depend on the degree distribution, similarly to the basic model. Given the more general setup, the coefficients corresponding to the value of a consumer covered by k influencers can be derived from the γ function as given by the $S_k()$ formulas. To better demonstrate the power of our results, we present two examples. The first example shows that our

¹⁷This assumption will not affect the decision on efforts, but will impact pricing. If firms can observe more, they will be able to price discriminate more

basic model is a special case of this general setting, whereas the second example presents a slightly different, but very realistic case.

Example 1 (*First impression*) Suppose $\gamma(0) = 0$ and $\gamma(\ell) = \gamma > 0$ for $\ell > 0$. Then the equilibrium effort levels and profits in Proposition 2 correspond to those in Proposition 1 with $q = 1 - \gamma$.

The influence mechanism defined in this example is a very simple one: the first recommendation a consumer receives about a product makes him/her consider a product with probability γ . Further recommendations about a product do not increase the probability of consideration. This setting captures a situation where first impressions matter overwhelmingly: for example when the recommendation contains a simple piece of information that makes the consumer either consider the product or not. Further recommendations containing the same information do not increase the likelihood of consideration. An extreme example is $\gamma = 1$, when the information contained in the recommendation can be simply thought of as the existence of the product akin to the well known informative advertising models. Substituting into the formulas reveals that $S_k(0) = \gamma(1 - \gamma) + \frac{\gamma^2}{2^{k-1}} - \frac{\gamma}{2^k}$ and $S_k(1) = \frac{k\gamma(1-\gamma)}{2} + \frac{\gamma^2 k}{2^k}$ which, in the case of $\gamma = 1$, exactly reproduces the results of Proposition 1 with $q = 0$. This is not surprising in the sense that when influencers simply make other consumers aware of a product the price competition is intense for those who consider both products. When $\gamma < 1$, the results are also reproduce those in Proposition 1. The corresponding q is $1 - \gamma$, but we also have to multiply all efforts and profits with γ to account for the reduced likelihood that any consumer will consider the product.

It is also interesting to examine the prices. The equilibrium price distribution shows that firms set random prices up to 1 starting at an intermediate value for each consumer. The size of the interval and thus the overall level of prices depends on how many influencers cover the consumer. Given the $\gamma()$ function specified in the example, we obtain $\alpha = \sum_{k=1}^{\infty} \Pr(d_j = k) \left(\gamma(1 - \gamma) + \frac{\gamma^2}{2^{k-1}} - \frac{\gamma}{2^k} \right)$ and $\beta = \sum_{k=1}^{\infty} \Pr(d_j = k) \left(\gamma^2 - \frac{\gamma^2}{2^{k-1}} \right)$ suggesting two effects. First, prices generally increase as γ decreases due to the softened price competition as less and less consumers consider both products, consistently with $q = 1 - \gamma$ decreasing with γ . Secondly, prices also depend on the number of influencers that cover a consumer. As a consumer is covered by more influencers, it is more likely that this consumer considers both products intensifying price competition and reducing prices. Consistently with the results of Proposition 1, firms will thus invest less in influencers with highly covered followers

regardless of the value of q (and regardless of γ here). Even though prices go down as a function of coverage, they never reach zero (as long as $\gamma < 1$), even if a consumer is covered by many influencers.

As a result, lower prices do not always hurt firms. When γ is sufficiently low and consumers are not likely to consider a recommended product, increased coverage by influencers lowers prices, but increases profits at the same time. The intuition follows from a combination of two strategic effects. On one hand, increased coverage results in tougher price competition and lower prices. On the other hand, high coverage leads to careful investments in the effort to convince influentials, leading to less wasteful spending.

Clearly, the influence mechanism in Example 1 has extremely decreasing returns for additional recommendations as $\gamma(\cdot)$ is very concave. Let us consider another example.

Example 2 (*Cumulative influence*) Suppose $\gamma(\ell) = 1 - (1 - \gamma)^\ell$ with $\gamma > 0$. Then $S_k(0) = (1 - \gamma/2)^k - (1 - \gamma)^k$ and $S_k(1) = \frac{k}{2} \left((1 - \gamma/2)^{k-1} - (1 - \gamma)^k \right)$

Subsequent recommendations for the same product can have a positive effect on consideration. For example, when influencers convey more complex information, every new piece of information can make influencees consider the product. Even more so, if recommendations involve some sort of social pressure or network effects, where more recommendations lead to an increased likelihood of consideration (as in Katona et al. (2011)). The results in this case are somewhat different. In contrast to the case where first impressions matter the most, price competition gets very intense for consumers covered by many influencers, prices tending to zero. This leads to a pattern similar to that in Proposition 1 with $q = 0$. Deriving the profit reveals that

$$\pi_f = \sum_{k=1}^{\infty} \varphi_k \left((1 - \gamma/2)^{k-1} \left(1 - \gamma/2 \left(1 + \frac{k}{2r} \right) \right) - (1 - \gamma)^k \right),$$

which results in positive profit contribution only for consumers covered by a few influencers. Consumers covered by more will reduce profits, but the decline will be slower as γ decreases.

Comparing Examples 1 and 2 shows an interesting pattern. For $\gamma = 1$ the results are identical, but for $\gamma < 1$ there are important differences. In both cases, consumers covered by few influencers are valuable, but highly covered consumers have opposing impact on profits in the two cases. When the first recommendations matter the most, dense networks are more profitable for firms, whereas in case of cumulative influence, dense network always reduce profits, but less so as density increases.

On the flipside, consumers covered by only a few influencers contribute to profits more in case of cumulative influence. Figure 3 shows a comparison between profits in the two cases.

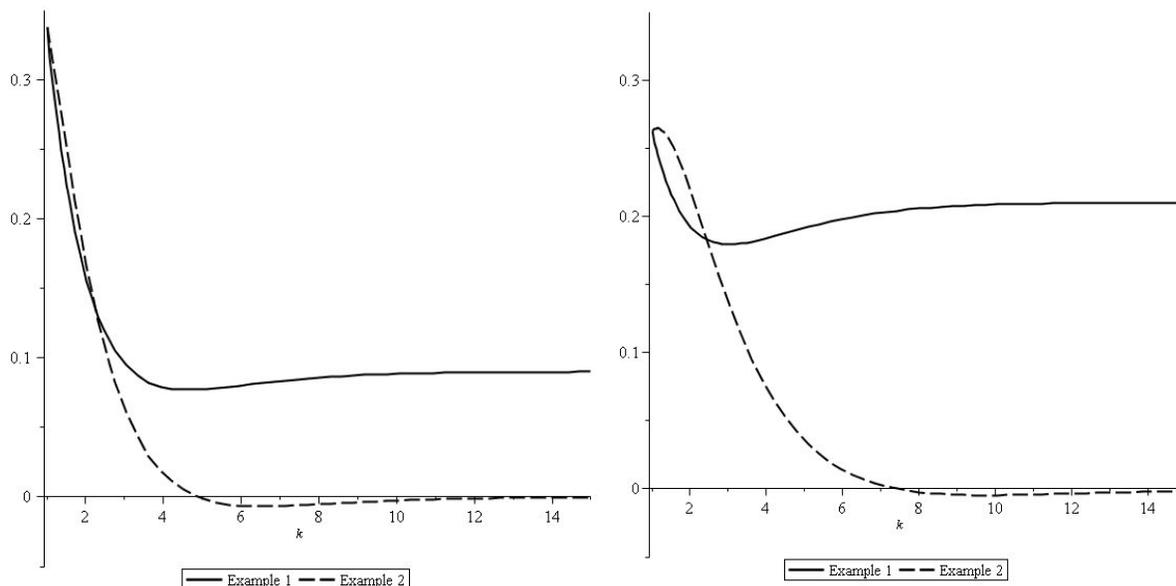


Figure 3: Firm profits under different influence mechanisms. On the left, $r = 2, \gamma = 0.9$. On the right $r = 2, \gamma = 0.7$

6 Endogenous Network Formation and Influencer Incentives

Throughout the paper, we have assumed that the influence network is exogenously given. However, there is increasing pressure on consumers to become influential among their peers in order to get perks and benefits from firms. Klout - the influence score tracking service - even created a Klout Perks API that makes it easier for firms to offer perks to their highly influential customers.¹⁸ In this section, we examine how endogenizing the influence network formation affects our results. We also study how firms should incentivize consumers to build the influence network. As we shown in the main model, firms sometimes do not want consumers to be too much connected to each other, hence we study what the optimal level of these incentives are in different social networks.

¹⁸ “Your Influence (and Klout Score) is Worth Money” - available at <http://www.stateofsearch.com/your-influence-and-klout-score-is-worth-money/>

First, we describe the network formation process. As before, we assume that there are N influencers out of M consumers in the market.¹⁹ We assume that there is a network of social connections between consumers that provide the basis for influence to take place. Let u_{ij} be an indicator of a link, that is, $u_{ij} = 1$ if there is a link between consumers i and j and $u_{ij} = 0$ otherwise, forming the underlying social network matrix \mathbf{U} . Whenever an influencer i wants to establish an influence link of w_{ij} , it can only be done through an underlying social link. In other words $w_{ij} \leq u_{ij}$ for all (i, j) pairs. Exerting influence on another consumer is costly, we assume that the cost of building an influence link of w_{ij} strength costs the influencer $C(w_{ij})$. For the sake of tractability, we assume that the cost function is quadratic and is sufficiently high, that is, $C(w_i) = \frac{c}{2}w_{ij}^2$ with a sufficiently high c parameter.²⁰

In order for influencers to build their networks and attempt to influence their peers, they have to receive some benefit. We assume that this benefit comes directly from firms. When firm f exerts e_{fi} effort to convince influencer i , some of this effort directly benefits influencer i as a monetary transfer. The parameter α_f measures the percentage of the effort e_{fi} that directly goes to the influencer. That is, in our model influencer i receives a payment of $\alpha_f e_{fi}$ from firm f . In reality this payment does not have to be cash, instead it can be in the form of a rebate, points for future purchases, or simply a price reduction. We do not model the exact nature of the payment, our assumption only specifies that $\alpha_f e_{fi}$ directly increases i 's utility, whereas the remaining $(1 - \alpha_f)e_{fi}$ does not change i 's utility, only his or her probability to recommend firm f .

We assume that the above decisions take place in the following sequence. First, firms simultaneously announce and commit to α_f , the percentage of effort that directly benefits influencers. Second, influencers simultaneously build the network by selecting each consumer they want to influence and the strength of the relationship. Third, firms simultaneously determine their effort levels, exactly as in Section 3. As before, we are looking for equilibria that are symmetric with respect to the firm strategies and we also require subgame-perfection. We first determine the possible equilibrium networks for fixed α_f .

Proposition 3 *For fixed α_1 and α_2 values, a W random influence network is an equilibrium outcome*

¹⁹An alternative approach is to assume that becoming an influencer is costly and to also endogenize the decision of each consumer to become an influencer. The results, however, would be similar.

²⁰This ensures that $w_{ij} < 1$ in equilibrium.

if and only if each w_{ij} satisfies.

$$\frac{(1 - w_{ij}/2)(\sum_{g=1}^N u_{gj})^{-1}}{w_{ij}} = \frac{4rc}{\alpha_1 + \alpha_2}.$$

Thus, the equilibrium w_{ij} is decreasing in r , c , $(\sum_{g=1}^N u_{gj}) - 1$, and increasing in α_1, α_2 .

The proposition shows us the equilibrium influence network structure. Influence takes place over existing social connections, but with varying strength. How hard influencers try to influence a consumer depends on his/her position in the social network. What determines the strength of incoming influence links for a consumer is the number of influencers that the consumer is connected to in the social network. Interestingly, consumers with connections to more influencers will be influenced to a lesser extent. The intuition for this results is that the competition between firms trickles down to the level of influencers who get lower benefits from consumers for which firms compete more intensely.

The proposition also sheds light on the relationship between the social network and influence network in our model. While the social network is potentially dense with most nodes having tens or hundreds of connections, the influence network is typically much sparser, especially when influencing someone is costly. The degree distribution of the influence network is thus shifted towards lower degrees compared to the social network, with much more consumers influenced by only a few influencers.

Another important implication of the proposition is that the incentives given by firms change the structure of the influence network. The strength of all influence links increases with α_1 and α_2 . In other words, firms can incentivize influencers to form stronger links by increasing the portion of their effort that is a direct transfer to influencers.

Corollary 2 *In any symmetric equilibrium $\alpha_1 = \alpha_2 = \alpha^* > 0$ holds. $\alpha_1 = \alpha_2 = \alpha^* = 1$ is an equilibrium outcome if and only if q is high or a high proportion of consumers have only one social connection.*

The result shows a surprising pattern. When the market is not very competitive, firms spend all their effort on directly increasing the utility of influencers. The reason is that a high percentage of direct payoff incentivizes influencers to build a dense network that benefits firms given the low

level of competition (high q). On the other hand, when competition is intense, firms do not want to provide such strong incentives as too dense coverage by influencers would hurt their profits. But firms do not want to provide too little incentive as that would leave many consumers untouched by influencers. Therefore, firms set the percentage of direct benefit at an intermediate level, such that resulting influencer coverage is not too low, but not too high either.

It also interesting to examine how the α_f levels are affected by r , c and the social network structure. Since α^* is increasing r , a softer competition for the influencers requires a higher percentage of direct benefit to them when the product market competition is intense. The intuition is that firms will spend lower general effort levels on influencers, hence in order to incentivize network building, they have to offer a higher percentage as a direct transfer. Similarly, when it is costlier for influencers to establish their influence, they need higher incentives to create those links. Finally, when the underlying social network becomes denser, α^* decreases as firm want to avoid too much coverage by influencers.

7 Conclusion

In this paper, we have explored the value of influencers in a social network to competing firms. We determined how much each influencer is worth to firms and how much effort firms are willing to spend on influencers in order to win them over. The results show that the value of the influence network is determined by its in-degree distribution. Depending on the level of competition, consumers covered by very few or many influencers contribute most to profits, whereas consumers covered by an intermediate number of influencers often reduce firm profits.

Our results have important implications for social media marketers considering the avenue of influencer marketing. First and foremost, the value of an influencer cannot be described only by the connectivity or reach of the person. One might have numerous connections, but these are often worthless unless they provide exclusive access to some consumers. As a result, seemingly isolated influencers with only a small set of connection can be valuable if their friends cannot be reached otherwise. Any type of influence score that does not take these network properties into account can be misleading. The good news is that a fairly simple statistic, such as the in-degree distribution of consumers covered by an influencer can be very informative.

Second, competition plays a very important role in determining the role of influencers. Even if

a marketer is a monopolist in its own product category, firms have to compete across categories for winning over influencers as most of them recommend only a few brands. An important implication of competition is that one cannot forget about both the offensive and defensive roles of influencer marketing. The reason to win over an influencer is not always to gain additional reach to customers, but it could very well be to prevent a competitor from also reaching one's own customers.

Third, our analysis of the incentives given to influencers sends a clear, but unexpected message for social media marketers, especially if they plan to use these tools on the long term. If a firm provides too much direct, potentially cash, benefits to influencers over time, some of them may alter their behavior in order to take advantage of these incentives. This can lead to increased activity in becoming more and more influential, which is only good for firms up to a certain point. If the influence coverage expands too much, most consumers will be covered by multiple influencers. This, in turn, will reduce firm profits in a competitive market.

Despite the fairly general approach and solutions, our work has a number of limitations. First, we assume that there are only two firms in the market and that they are symmetric. When there are multiple firms competing, one needs to model how consumers react to recommendations about multiple products and how firms compete for these customers. The case corresponding to $q = 0$, when consumers who receive at least two recommendations give zero revenue, is very similar to our basic model and the results are essentially the same. For cases corresponding to $q > 0$, one needs to define several different q values and redo the analysis in a tedious way, leading to similar results. Second, for each influencer we have a simple contest success function that ensures that the influencer will recommend one or the other firm. In reality, there might be an outside option and influencers would not recommend either firm if the efforts are not very high. This would give an incentive for firms to beat not only their competitor, but also the outside option. At the extreme this would lead to a lack of competition for influencers where each firm separately decides whether to capture an influencer or not. Finally, we take a very simplistic approach with respect to what influencers do and whether consumers believe them. The credibility of influencers in an environment where firms heavily invest in trying to convince them is an exciting topic for future research.

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Appendix

PROOF OF PROPOSITION 1: We first determine the individual effort levels that each firm puts out for a given influencer i assuming that the equilibrium is symmetric. Equations (5)-(11) express the profit as a function of e_{fi} , keeping all other effort levels fixed. We can do the same exercise for both firms and see that

$$\mathbf{E} \pi_1 = \frac{e_{1i}^{1/r}}{e_{1i}^{1/r} + e_{2i}^{1/r}} v_i - e_{1i} + C_1, \quad \mathbf{E} \pi_2 = \frac{e_{2i}^{1/r}}{e_{1i}^{1/r} + e_{2i}^{1/r}} v_i - e_{2i} + C_2, \quad (12)$$

which is a symmetric Tullock-contest with value

$$v_i = \sum_{k=1}^{\infty} \frac{\varphi_{ki}}{2^{k-1}}. \quad (13)$$

Differentiating player 1's profit with respect to e_{1i} , we get

$$\frac{\partial \mathbf{E} \pi_1}{\partial e_{1i}} = \frac{e_{1i}^{1/r-1} e_{2i}^{1/r}}{r \left(e_{1i}^{1/r} + e_{2i}^{1/r} \right)^2} v_i - 1 \quad (14)$$

The expected profit is concave in e_{1i} , hence the F.O.C gives the unique maximum for this single variable. Setting $e_{1i} = e_{2i}$, the F.O.C becomes

$$\frac{v_i}{4re_{1i}} = 1 \quad (15)$$

yielding $e_{1i} = \frac{v_i}{4r}$ which is the effort level given in the first part of the proposition. Note that $r > 1/2$ is necessary to ensure that the function in (12) have a positive derivative in 0. When $r < 1/2$, no pure strategy equilibrium exist.

We have determined that the effort levels given above are optimal if the effort levels for all other influentials are fixed. This is sufficient to show that the effort levels in equilibrium must be these, but we need to also make sure that firms do not have an incentive to deviate by changing multiple effort levels. That is, we need to check the second order condition for the maximization problem involving all N variables for a given player. We show that the Hessian is negative definite, yielding that player 1's profit function has a maximum in the above identified equilibrium candidate (corner solutions are ruled out since the derivative is always positive in a given effort variable when its set to

zero as long as $r > 1/2$). First, from (12) and (13) we get that when $e_{1i} = e_{2i}$ the second derivatives forming the diagonal of the Hessian are

$$\frac{\partial^2 \mathbf{E} \pi_1}{\partial e_{1i}^2} = -\frac{1}{4re_{1i}^2} \sum_{k=1}^{\infty} \frac{\varphi_{ki}}{2^{k-1}} < 0.$$

When $e_{1i} = e_{2i}$ and $e_{1i} = e_{2i}$, the (ij) off-diagonal element in the Hessian is

$$\frac{\partial^2 \mathbf{E} \pi_1}{\partial e_{1i} \partial e_{1j}} = \frac{1-2q}{16r^2 e_{1i} e_{1j}} \sum_{k=2}^{\infty} \frac{\varphi_{k(i,j)}}{2^{k-2}} \leq \frac{1-2q}{8r^2 e_{1i} e_{1j}} \left(\sum_{k=2}^{\infty} \frac{\varphi_{ki}}{2^{k-1}} + \sum_{k=2}^{\infty} \frac{\varphi_{kj}}{2^{k-1}} \right),$$

where $\varphi_{k(i,j)}$ denotes the number of consumers covered by exactly k consumers among those who are covered by i and j . This is analogous to the definition of φ_{ki} and it is clear that $\varphi_{k(i,j)} \leq \varphi_{ki}$ and $\varphi_{k(i,j)} \leq \varphi_{kj}$. Furthermore, when φ_{ki} is high enough the diagonal elements dominate the matrix. One can then divide each row by the diagonal element to see that the matrix is negative definite, since all the diagonal elements are -1 in the modified matrix and the off-diagonal elements are smaller than any positive threshold when φ_{ki} is high enough for all i .

For part 2, we need to simply sum the effort levels across all influencers. Recalling that $\sum_{i=1}^N \varphi_{ki} = k\varphi_k$, we easily get the first equation for $\sum_{i=1}^N e_{fi}$. In order to determine profits, we need to calculate the revenues. From a consumer covered by k influencers there are two possibilities firm 1 could earn money. With probability $\frac{1}{2^k}$, the consumer is in R_1 and the revenue for firm 1 is 1. With probability $1 - \frac{1}{2^{k-1}}$ the consumer is in R_{12} yielding a revenue of q . Therefore the total revenue for firm 1 is $\frac{1}{2^k} + q \left(1 - \frac{1}{2^{k-1}}\right)$. Subtracting the investment in effort, we obtain the profit as given in the proposition.

For part 3, let \mathcal{G} denote the game we are analyzing in the proposition. Let \mathcal{G}' denote a modified game with $2N$ players, with two players for each influencer. The games \mathcal{G} and \mathcal{G}' are identical, except that in \mathcal{G}' we split each of the two players in \mathcal{G} into N different players that set the N effort levels separately. Each of the N players in \mathcal{G}' that correspond to player 1 in \mathcal{G} gets the entire payoff player gets in \mathcal{G} . It is clear that the N players in \mathcal{G}' face the same maximization problem that player 1 in \mathcal{G} for each variable. Therefore, any equilibrium of \mathcal{G} also has to be an equilibrium of \mathcal{G}' , but not vice versa. However, if we show that \mathcal{G}' has a unique equilibrium in pure strategies \mathcal{G} either has no equilibrium or a unique equilibrium. In order to show that \mathcal{G}' has a unique equilibrium in pure strategies, let us write the profit function of player 1 as

$$\mathbf{E} \pi_1 = \sum_{S \subseteq \{1, \dots, N\}} A_S \prod_{i \in S} P_{1i} - \sum_{h=1}^N e_{1h}. \quad (16)$$

This formulation simply says that the revenue is a multinomial of P_{11}, \dots, P_{1N} with coefficients A_S , where $P_{1i} = \frac{e^{1/r}}{e^{1/r} + e^{2i/r}}$. Simple calculation shows that when q is small enough all coefficients are positive. All the players' profit functions are concave in this case, yielding a unique equilibrium for \mathcal{G}' . Since we have already shown that \mathcal{G} has at least one equilibrium, it must be unique. \square

PROOF OF COROLLARY 1: For part 1, Proposition 1 shows that the profit contribution of a consumer covered by k influencers is

$$a(k) = \frac{2 - k/r}{2^{k+1}} + q \frac{2^{k-1} - 1}{2^{k-1}}. \quad (17)$$

This takes the value of $\frac{1}{2} - \frac{1}{4r}$ for $k = 1$ and increasingly converges to, but never reaches q as $k \rightarrow \infty$. If $q \leq \frac{1}{2} - \frac{1}{4r}$, the maximum contribution is always for $k = 1$. If $q > \frac{1}{2} - \frac{1}{4r}$, let \bar{k} denote the largest solution of

$$\frac{2 - k/r}{2^{k+1}} = q \frac{2^{k-1} - 1}{2^{k-1}}. \quad (18)$$

For $k > \bar{k}$ we get that the RHS exceeds the LHS, completing the proof of part 1.

For part 2, further examining the $a(k)$ function shows that it first decreases, has a unique minimum then increases. To determine the minimum, we solve $a(k) = a(k+1)$ yielding $k = 1 + 2r(1 - 2q)$. This is not necessarily an integer, but we know that the integer that minimizes $a(k)$ is between k and $k + 1$, yielding the stated formula. \square

PROOF OF PROPOSITION 2: Since we are looking for a symmetric equilibrium, we first determine the pricing strategies given symmetric firm effort levels. In a symmetric equilibrium, all influencers recommend both firms with the same probability, which allows us to determine the likelihood that consumer j considers both products (let us denote by β) or only one product (α). Given α and β we can use the mixed strategy equilibrium derived by Varian (1980), Narasimhan (1988) as stated in the proposition. First, to calculate α , let us assume that consumer j is covered by k influencers (this happens with probability $\Pr(d_j = k)$). Let us also assume that ℓ of these k influencers recommend product 1. The likelihood that consumer j considers only this product is then $\gamma(\ell)(1 - \gamma(k - \ell))$. Combining these probabilities for different values of ℓ yields the formula stated in the proposition since the probability that ℓ out of k influencers recommend product 1 is $\binom{k}{\ell}/2^k$. Similar calculations give the value of β .

To derive the equilibrium effort levels, we follow the same lines as in the proof of Proposition 1. The value of winning over influencer i can be written as a sum over the consumers covered by i .

For a particular j consumer covered by exactly k influencers, the value is $\gamma(\ell)(1 - \gamma(k - \ell)) - \gamma(\ell - 1)(1 - \gamma(k - \ell + 1))$ if the consumer receives firm 1's recommendations from $\ell - 1$ out of the the $k - 1$ other influencers and and receivers firm 2's recommendations from $k - \ell$ of them. The probability that $\ell - 1$ of the other $k - 1$ influencers recommend product 1 is $\binom{k-1}{\ell-1}/2^{k-1}$. We can then write the value of winning over influencer i as

$$v_i = \sum_{k=1}^{\infty} \frac{\varphi_{ki}}{2^{k-1}} \sum_{\ell=1}^k \binom{k-1}{\ell-1} [\gamma(\ell)(1 - \gamma(k - \ell)) - \gamma(\ell - 1)(1 - \gamma(k - \ell + 1))]. \quad (19)$$

Using the equality $\binom{k-1}{j-1} - \binom{k}{j-1} = \frac{2j-k}{k} \binom{k}{j}$, we obtain

$$v_i = \sum_{k=1}^{\infty} \frac{\varphi_{ki}}{2^{k-1}} \sum_{\ell=0}^k \binom{k}{\ell} \frac{2j-k}{k} \gamma(\ell)(1 - \gamma(k - \ell)). \quad (20)$$

As in the proof of Proposition 1, the equilibrium effort levels will be $\frac{v_i}{4r}$ resulting in the formula provided in the proposition. \square

PROOF OF PROPOSITION 3: Influencers receive a fixed proportion of firms' efforts. The payoff of influencer i is

$$\pi_i^{infl} = \frac{\alpha_1 + \alpha_2}{4r} \sum_{k=1}^{\infty} \frac{\varphi_{ki}}{2^{k-1}} - \frac{c}{2} \sum_{j=1}^M w_{ij}^2$$

Since the above is additively separable for the different consumers covered by the influencer, let us separate the part for an individual consumer j who is connected to i (that is, $u_{ij} = 1$) as

$$w_{ij} \frac{\alpha_1 + \alpha_2}{4r} \sum_{k=1}^{\infty} \frac{\Pr(j \text{ is covered by } k - 1 \text{ other influencers})}{2^{k-1}} - \frac{c}{2} w_{ij}^2$$

Note that all influencers trying to cover consumer j maximize a similar payoff function and due to the concavity, it is easy to see that the equilibrium is unique, and - as we show - symmetric. If $w_{gj} = w(d)$ for all g that have a social connection to j (such that $u_{gj} = 1$) then $\Pr(j \text{ is covered by } k - 1 \text{ other influencers}) = \binom{d-1}{k-1} w(d)^{k-1} (1 - w(d))^{d-k}$, where $d = \sum_{g=1}^N u_{ij}$. Summing these, we get $(1 - w/2)^{d-1}$, thus the first order condition for influencer i and consumer j becomes

$$\frac{\alpha_1 + \alpha_2}{4r} (1 - w/2)^{d-1} - cw_{ij} = 0$$

This completes the proof if we substitute $w_{ij} = w(d)$, as there is a symmetric equilibrium that must be unique.

□

PROOF OF COROLLARY 2: The profit firms can be calculated as a sum of profits made on each consumer. For a consumer that has d connections in the social network, we can calculate the probability that s/he will be covered by k influencers from the proof of Proposition 3 as $\binom{d}{k}w(d)^k(1-w(d))^{d-k}$. Using these probabilities, we can write the profit contribution of a consumer with d social ties as

$$\pi(d) = \sum_{k=1}^d \binom{d}{k} w(d)^k (1-w(d))^{d-k} \left(\frac{2-k/r}{2^{k+1}} + q \frac{2^k-1}{2^k} \right).$$

The profit function maximized by firms is then $\sum_{d=1}^{\infty} \varphi'_d \pi(d)$, where φ'_d denotes the number of consumers with exactly d social ties, that is, the degree distribution of the underlying social network.

When $q = 0$, simple calculations show that

$$\pi(d) = \left(1 - \frac{w(d)^{d-1}}{2} \right) \left(1 - \frac{w(d)}{2} - \frac{dw(d)}{4r} \right) - (1-w(d))^d.$$

With the exception of $d = 1$, one can show that $\pi(d)$ is increasing in w for small positive values of w , then decreasing until it becomes negative at $\frac{4r}{d+2r}$. A linear combination of different $\pi(d)$'s exhibits a similar pattern as long as the weight of $\pi(1)$ is not too high (when there are not too many consumers covered by only one influencer). Therefore, as long as $q = 0$, $\alpha_1 + \alpha_2 = 0$ cannot be optimal for firms, as that would result in $w(d) = 0$ and both firms would want to deviate by increasing their α_f .

To see when $\alpha_1 = \alpha_2 = 1$ is an equilibrium, we need to examine the $\pi(d)$ functions again. When $q = 1/2$, all $\pi(d)$ functions are increasing in w , therefore firms will want to achieve the highest possible $w(d)$ values by setting $\alpha_1 = \alpha_2 = 1$. The same happens when $\pi(1)$ has a very high weight in the overall profit function. If q is not high and a low proportion of consumers are connected to one influencer, the profit function will be decreasing at interior (less than 1 α values), since we assumed the cost parameter c to be high enough. □