Liquidity Flooding, Asset Prices and the Real Economy*

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Abstract

This paper develops a model for understanding the relationship between liquidity injections, asset prices, and growth of the real economy. We study the interaction between two sectors in the economy with different degrees of financial friction in the context of liquidity injections for economic stimulus. We show that if too much liquidity is injected into the economy, overheating can build up in the sector with lower friction (e.g., the financial sector), crowding out the demand for liquidity in the sector with higher friction (e.g., the real sector). The crowding-out occurs in a self-reinforcing spiral because of feedback between liquidity inflows, asset prices and firm asset collateral values. The crowding-out effect originates in the different degrees of asset specificity across the two sectors. The model characterizes conditions under which crowding-out is more likely, with empirical implications. The paper highlights the effect of financial frictions on the allocation and distribution of liquidity in an economy.

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1 Introduction

During and after the severe financial crisis of 2007-2009, central banks around the world adopted traditional as well as unconventional monetary policies to inject liquidity into their banking systems in an attempt to save the real economy from recession. The liquidity injections came in various forms in different countries. However, one common aim of the monetary expansion policies was to ensure the banking systems would have sufficient liquidity to extend loans to the real sector, thus stimulating the growth of the real economy. Economic stimulus with liquidity injections, however, does not always work. Very often, overheating builds up in asset (financial) markets while the real economy makes little recovery. What happened in recent years in China, now the world’s second largest economy, is an illustrative case in point.

Facing a sharp decline in the external demand for exports and the danger of plunges in economic growth, the Chinese government implemented an economic stimulus package unprecedented in history, notably through some very aggressive monetary expansion policies. Figure 1 shows the money supply of M2 in China. The official figure shows that a total amount of RMB 7.37 trillion (around USD 1.08 trillion) of bank credit was injected into China’s economy in the first half of 2009, up 200% on the same period a year earlier (when the global financial crisis was not yet in full swing). One immediate and pronounced phenomenon following the liquidity injections was the surge in house prices, with a 50% increase within one year in many cities. Asset prices were also climbing in other asset classes, like commodities. Ironically, in the name of stimulating the real economy, small and medium-sized businesses in China have been experiencing the most difficult times ever in financing and corporate liquidity after the economic stimulus.

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1 In the U.S., the Federal Reserve adopted an unconventional policy of credit easing through a combination of lending to financial institutions, providing liquidity directly to key credit markets, and purchases of long-term securities (Bernanke (2009)). Central banks in Europe and Japan used similar ‘quantitative easing’ policies, while many emerging economies undertook aggressive credit expansion.

2 In our paper, as shown later, overheating can be defined as the situation in which an increase, rather than a decrease, in the interest rate coincides with the increase in the supply of liquidity because of the stronger force of demand.

3 Data is from the statistics and analysis department of the People’s Bank of China, China’s central bank (http://www.pbc.gov.cn/publish/english/963/index.html).

4 The data are from China Real Estate Index System (CREIS).
The underground real interest rate surged to 30% in 2010 in some regions. Most small and medium-sized firms with little collateral have been virtually left out of the bank credit market. The *People’s Daily*, China’s leading official newspaper, wrote: “Massive funds pulled out the real sector and flowed into the real estate sector, crowding out the real economy.”

The experience of China is not unique. Many emerging economies are experiencing a similar problem. Notably, the pre-crisis asset bubble in the U.S., which was the origin of the 2007-2009 crisis, was also rooted in an environment of massive liquidity injections and credit expansion.

In this paper, we provide one perspective for understanding the relationship between liquidity injections, asset prices and growth of the real economy. Specifically, we study the interaction (competition) between two sectors in the economy with different degrees of financial friction in the context of liquidity injections, and its consequence for economic growth.

We model an economy with two sectors, labeled sector *I* and sector *II*. The two sectors differ

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7. In fact, in response to the bursting of the Internet bubble in early 2000, the Federal Reserve Bank had adopted a policy of unprecedented credit easing, with the Federal funds (effective) rate having been historically low at below 2.5% for a prolonged period between 2001 and 2005 (http://www.federalreserve.gov/releases/h15/data.htm).
in firm asset specificity: firms in sector \( I \) have higher asset specificity than firms in sector \( II \). Sector \( I \) can be interpreted as the real sector in the economy while sector \( II \) is the financial sector (e.g., housing, commodities, equities, etc.). In fact, assets tend to be specific (in operation) across firms for real business (Williamson (1985, 1986)), while assets in the financial sector tend to be more homogeneous. More generally, however, the two sectors could be both real sectors/industries featuring different degrees of asset specificity.

We first show that there is positive feedback between bank lending, asset prices and collateral values in the economy. This result builds on the insights of Shleifer and Vishny (1992) and Benmelech and Bergman (2011b). In the model, at the initial date, the economy is hit by an \textit{unexpected} negative (systemic) liquidity shock; firms with assets in place need to make a liquidity investment to enable their project to deliver cash flow in the future. If commercial banks increase their lending to the corporate sector (i.e., provide liquidity support to firms), firms will realize their cash flow thus increasing the available cash in the industry.\(^8\) However, the friction of unverifiability of cash flow coupled with collateral constraints may limit effective banking lending. At the intermediate date, firms have diverging views about economic prospects; some become optimistic and some pessimistic, which motivates them to trade their assets in the secondary asset market. If there is high cash flow in the industry, the asset price is high due to the “cash in the market” effect.\(^9\) The high asset price in the secondary market would lead to a high collateral value of the firm’s asset, which eases financial frictions and facilitates lending. Therefore, the rational expectation of more cash in the industry would increase banks’ willingness to lend more in the first place. A feedback loop forms between bank lending and corporate cash flows. By injecting liquidity into the commercial banking system, the central bank can influence the economy by shifting this loop.

Then, we show that the strength of feedback, however, is asymmetric across sectors. This is because the response of the asset price to liquidity injections is asymmetric across sectors. In a

\(^8\) The term ‘firm’ is used in a broad sense, encompassing individual investors. It is possible that an individual rather than a corporation invests in the asset (financial) market, e.g., to buy houses.

secondary asset market, optimistic firms cannot only use their own liquidity when buying but can also borrow from the sellers by pledging their assets as collateral (i.e., leverage). Hence, the asset price in a secondary market reflects not only the opinion of optimists but also the total liquidity they can access. For sector \( II \), with lower asset specificity, optimistic firms are able to leverage more when buying, considering that their lower asset specificity raises debt capacity more (Williamson (1988)). Therefore, the asset price in sector \( II \) responds more strongly to liquidity injections than that in sector \( I \) does.

This asymmetry between sectors creates a ‘crowding-out’ effect. If too much liquidity is injected into the economy, the asset price and thus the collateral value in sector \( II \) can increase so fast that it leads to a rise in the real interest rate in the economy. This is because when the collateral value increases, firms’ collateral constraints are relaxed; more firms qualify to borrow and compete for loans, pushing up the interest rate. As the collateral price in sector \( I \) responds only weakly, the higher interest rate reduces (i.e., crowds out) the liquidity entering that sector. The crowding-out manifests in a self-reinforcing spiral as more liquidity flowing into sector \( II \) pushes up the interest rate, and the increased interest rate leads to additional liquidity flowing out of sector \( I \) (into sector \( II \)), pushing up the interest rate further, and so on. In short, too much liquidity injected actually reduces the liquidity entering sector \( I \). If, on the other hand, too little liquidity is injected, sector \( I \) of course cannot obtain much liquidity. We show that, under certain conditions, there exists a unique optimal amount of liquidity injection for maximizing the liquidity in sector \( I \) (e.g., the real sector).

One would expect that more (less) liquidity leads to a decrease (increase) in interest rates. The Japanese experience during the 1980s, however, is a stark example of monetary contraction accompanying a (slight) decrease in real interest rates. In fact, the monetary tightening in Japan in the latter 1980s triggered the fall in asset prices and thus reduced the collateral values of firm assets; the reduction in the creditworthiness of Japanese corporations at least in part contributed to the less effective demand on credit, decreasing interest rates (see, e.g., Bernanke and Gertler (1995); Benmelech and Bergman (2011b)).\(^{10}\) What happened recently in China can be regarded

\(^{10}\)Bernanke and Gertler (1995) write: “the crash of Japanese land and equity values in the latter 1980s was the
as the same sort of problem the Japanese faced, but in the completely opposite direction. That is, the massive liquidity injections and credit expansion in China created overheating in some sectors (e.g., the real estate sector) which led to the effective demand for credit shooting up, in turn causing the real interest rates to rise. This potentially generates a downward ‘crowding-out’ spiral between sectors, as China’s official media reported.

The model has two empirical implications. The first is a cross-sectional implication. The model implies that for a country with a poorer contracting institution (see, e.g., LaPorta, Lopez-de Silanes, Shleifer, and Vishny (1997, 1998); Djankov, Hart, McLiesh, and Shleifer (2008)), overheating and crowding-out between sectors are more likely. The second is a time-series implication. At times of greater uncertainty about economic prospects, crowding-out is more likely to occur in response to liquidity injections, which relates to “flight to safety” phenomena.

Our paper highlights the effect of financial frictions on the allocation and distribution of liquidity in an economy. The financial friction in our model is the unverifiability of cash flows combined with collateral constraints (see, e.g., Hart and Moore (1994, 1998)). In the two-sector economy setting, we show that liquidity not only tends to move to the sector with lower friction (i.e., the allocation effect) but also that the sector with lower friction can crowd out the other sector (i.e., the crowding-out effect). That is, in the presence of the allocation effect, liquidity in both sectors increases with liquidity injections but the increase in one sector is greater than in the other. However, when the crowding-out effect happens, more liquidity injected increases the liquidity in one sector but reduces that in the other. In the literature on fiscal policy, the crowding-out effect means that an increase in government spending can lead to a reduction in private investment, because more spending can increase interest rates due to increased borrowing (see, e.g., Blanchard (2008)). Our paper demonstrates the crowding-out effect across two (private) sectors in the context of liquidity injections. The mechanism of the crowding-out effect is different, through the feedback between liquidity injections (credit expansion), collateral values, and interest rates.

result (at least in part) of monetary tightening; ... [T]his collapse in asset values reduced the creditworthiness of many Japanese corporations and banks...".

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Benmelech and Bergman (2011b) first built a framework for studying the interplay between financing frictions, liquidity, and collateral values. The authors show that the credit channel monetary policy transmission mechanism sometimes does not work because firm asset collateral values do not increase with liquidity injections (i.e., credit traps). We build on Benmelech and Bergman’s framework to examine an economy with two sectors, providing a new dimension of limitations of monetary policy in economic stimulus. We show that as long as the collateral value of one sector increases faster than that of the other, there is the possibility of crowding-out. This way we explain why overheating in one sector can hurt the other sector. We also provide a new micro-foundations for Benmelech and Bergman’s insight on the effect of liquidity injections on asset prices. We use an industry-equilibrium framework with heterogeneous beliefs to endogenize the collateral value of bank loans. This framework enables us to endogenize why the two sectors are different in the bank credit market, and thus to explain why crowding-out can occur between the two sectors.

The work by Shleifer and Vishny (1992) and Geanakoplos (2010) helps explain why asset prices in financial markets do not only depend on asset fundamentals but also on the aggregate liquidity in the economy. Shleifer and Vishny (1992) adopt an industry equilibrium approach with asset specificity. Geanakoplos (2010) uses a general-equilibrium framework with heterogeneous beliefs. In this literature, aggregate liquidity in the economy is exogenously given. In our paper, we show the effect of policy (liquidity injections) on the aggregate liquidity, and thus demonstrate the implications of monetary policies on asset prices.

Acharya and Naqvi (2012) show that abundant liquidity can cause moral hazard problems of loan officers inside banks, inducing excessive credit volume and having asset price implications. Allen and Gale (2000) study the consequences of credit expansion and highlight the problem of risk shifting of borrowers and its asset price implications. Kiyotaki and Moore (1997) incorporate the financial friction of contracting imperfections and collateral constraints into a macroeconomic model and build a framework to explain why relatively small shocks might suffice to cause

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11 A strand of finance literature uses a general equilibrium framework to study the interplay between liquidity, leverage, and asset prices (e.g., Holmstrom and Tirole (1997); Acharya, Shin, and Yorulmazer (2010); Acharya and Viswanathan (2011)).
large business cycle fluctuations. Compared with the above work, our paper focuses on the interplay between two sectors in the economy that have different degrees of financial friction. The mechanism generating the asset price change is heterogeneous beliefs with asset specificity, instead of moral hazard problems of risk shifting.

Our paper relates to the literature on the credit channel view of monetary policy transmission (Bernanke and Blinder (1988); Gertler and Hubbard (1989); Gertler and Gilchrist (1994); Kashyap and Stein (1994); Kashyap, Lamont, and Stein (1994); Kashyap and Stein (1995); Bernanke and Gertler (1995); Stein (1998); Kashyap and Stein (2000), etc.). This literature argues shocks to monetary policy affect the economy through their impact on financial frictions and the availability of credit. Indeed, as in Benmelech and Bergman (2011b), one role that liquidity injections play in our model is to push up the asset price (collateral values), easing firms’ collateral constraints and thus mitigating the financial friction.\(^{12}\)

The paper is organized as follows. In Section 2, we present the model and solve the two equilibria: the one-sector economy equilibrium and the two-sector economy equilibrium. In Section 3, we discuss empirical implications of the model. Section 4 concludes.

2 Model

In this section, we present the model and the equilibria.

2.1 Model setup

We use a model setup similar to that in Benmelech and Bergman (2011b), but introduce two sectors into the economy and consider heterogeneous beliefs among firms. Specifically, we consider an economy with two sectors, labeled as sector I and sector II. The two sectors differ in firm

\(^{12}\)There is a growing theoretical literature on the recent financial crisis of 2007-2009. Some papers are related to ours, including Kashyap, Rajan, and Stein (2008); Stein (2009); Mendoza (2010); Shleifer and Vishny (2010b,a, 2011); Bebchuk and Goldstein (2011); Diamond and Rajan (2011); Stein (2012).
asset specificity, which we will elaborate on. Each sector consists of a continuum of self-employed firm-households with measure one.\textsuperscript{13} For ease of exposition, we do not distinguish between the two sectors at this stage; so for now we can regard that there is only one sector. Later, we will model the interplay between the two sectors in detail. In the economy, there is also a set of commercial banks that supply capital to firms, and a central bank. The model has three dates: $T_0$, $T_1$ and $T_2$. There is no discount factor between $T_1$ and $T_2$ or the time between these two dates is very short.

\subsection*{2.1.1 Firms}

Each firm has an asset \textit{in place} at $T_0$ - an identical investment project across all firms. Firms undertook their project before $T_0$, which is expected to generate a constant cash flow $C$ at $T_1$ and a random cash flow $\tilde{x}$ at $T_2$, where $\tilde{x}$ has one of two realizations, $\tilde{x} \in \{u, d\}$, and $u > d > 0$.

Only a part of the cash flow of the project is contractible. More specifically, the cash flow $C$ is uncontractible while a part of the cash flow $\tilde{x}$ is contractible. The contractible part is a constant amount $X$, where $0 \leq X \leq d$; the remaining part $\tilde{x} - X$ is uncontractible. As is standard in the incomplete contracting literature (e.g., Hart and Moore (1998)), the interpretation is the following.

The project’s cash flow is unverifiable. If the owner of a project defaults at $T_2$, outside investors (i.e., debt-holders) obtain the control right of the asset; outside investors can only realize a cash flow $X$ when they seize and operate the asset at $T_2$ due to asset specificity. That is, the term $X$ measures asset specificity; the lower $X$, the higher the asset specificity. Alternatively (and intuitively), we can think that the payoff of the project at $T_2$ has two components: the cash flow $\tilde{x} - X$ and the liquidation or salvage value of the project’s (fixed) asset, $X$; while the cash flow is unverifiable, the project’s (fixed) asset can be contracted as collateral, and outside investors can realize its liquidation (salvage) value. In this case, the term $X$ equivalently measures firm

\textsuperscript{13}See Mendoza (2010) for a setup of self-employed firm-households.
Williamson (1988) stresses the link between asset specificity, the liquidation value of assets, and debt capacity. He argues that assets with low specificity have high liquidation values, which raise debt capacity.\textsuperscript{15}

### 2.2 Liquidity shock

The economy (firms) suffers an \textit{unexpected} (systemic) liquidity shock at $T_0$ (as in the business cycle literature, e.g., Kiyotaki and Moore (1997)). That is, a firm has to invest an additional amount $I$ at $T_0$ to enable its project to deliver the cash flow $C$, where $I < C$; otherwise its project delivers zero cash flow at $T_1$.

Firms differ in their level of internal capital at $T_0$. Suppose the amount of internal capital of a firm at $T_0$ is $A$, which means that the firm needs an amount of external capital, $B = I - A$, to be able to make its liquidity investment. We assume that $B$ has a probability distribution (pdf), $f(B)$, across firms, within the support $[0, I]$. $F(\cdot)$ denotes the cumulative distribution function (cdf) of $f(\cdot)$. Clearly, giving the distribution of $B$ is equivalent to giving the distribution of $A$.\textsuperscript{16}

Faced with limited internal capital, firms seek to raise external capital by borrowing from commercial banks.\textsuperscript{17} The borrowing (debt) is short-term, that is, a firm needs to repay its debt at $T_1$. We will show that long-term debt with maturity $T_2$ is not optimal or infeasible. Firms that do not make the liquidity investment can deposit their spare internal capital with commercial banks at $T_0$.

It is common knowledge that firms will have diverging (heterogeneous) beliefs at $T_1$. For simplicity, we assume that there are two types of beliefs at $T_1$: high (optimistic) beliefs and low (pessimistic) beliefs. For the optimistic (respectively pessimistic) beliefs, the probability of realizing $u$ of $\bar{x}$ is $\theta_H$ (respectively $\theta_L$), where $\theta_H > \theta_L$. That is, optimism corresponds to

\textsuperscript{14}Firm asset collateralizability at $T_1$ is different, which is measured by $P$, as shown later.

\textsuperscript{15}See Benmelech (2009) and Benmelech and Bergman (2009, 2011a) for evidence.

\textsuperscript{16}For simplicity and without loss of generality, we do not explicitly model firms’ investment and financing decisions before $T_0$, which is not the focus of the paper.

\textsuperscript{17}Rajan (1992) provides justifications for debt financing.
Pr [ \tilde{x} = u ] = \theta_H and pessimism to Pr [ \bar{x} = u ] = \theta_L. Ex ante, before T_1, the probability of being optimistic is \pi.

There is a secondary asset market at T_1, where firms with heterogeneous beliefs trade their assets. As in Miller (1977); Harrison and Kreps (1978); Geanakoplos (2010), short-selling is not allowed for the secondary market.

2.2.1 Commercial banks

There is a large number of commercial banks that make loans to firms. Each individual commercial bank is price-taking. That is, the lending by any one bank does not affect the market-wide interest rate, at which firms can borrow. Denote the net interest rate of bank loans by r. As the cash flow C of a firm’s project is not contractible, the only means to force a firm to repay is to contract the firm’s asset (project) as collateral. If the firm does not repay, the bank can threaten to liquidate the firm’s project to sell in the secondary market at T_1. We denote by P the market price of the asset (project) in the secondary market. If a firm has the full bargaining power in renegotiating with its bank, then a firm will never be able to commit to repay more than P at T_1 (e.g., Hart and Moore (1994)). Therefore, the collateral value of a firm’s asset at T_1 is P.\(^{18}\)

Both P and r will be endogenized.

2.2.2 Central bank

After the economy suffers the systemic liquidity shock, the central bank chooses an amount of liquidity, Q, to inject into the commercial banking system at T_0, where Q \in [0, \overline{Q}]; \overline{Q} is the maximum amount of liquidity the central bank can inject, which reflects the government’s constraint in economic stimulus. We interpret liquidity as loanable funds. Essentially, as in

\(^{18}\)We will prove that P > X, so the long-term debt with maturity at T_2 is not optimal or feasible for some firms since they can raise less external financing by using long-term debt than by using short-term debt. This is in the spirit of Hart and Moore (1994) on the optimal debt maturity choice. Also, if the support of B is assumed to be [X, I], long-term debt becomes infeasible for all firms.
Benmelech and Bergman (2011b), we have abstracted away the institution and assumed that the central bank can directly determine the amount of loanable funds in the commercial banking system. This simplification is to capture the fact the central bank can use various policy tools to influence bank credit available to the economy, for example, the policies of direct lending to financial institutions, equity injections to increase bank capital, and so on.\footnote{It is also worth noting that all quantities in our model are in ‘real’ and not ‘nominal’ terms. Essentially, we abstract away the nominal side of the economy (e.g., inflation) and focus on the real side.}

The liquidity injection $Q$ is essentially ‘outside liquidity’ in the spirit of Kiyotaki and Moore (2002) while the liquidity ultimately extended by the private sector (i.e., the bank deposits by non-investing firms) is ‘inside liquidity’. As shown later, in our model the liquidity injection $Q$ gets fully repaid by the private sector (i.e., investing firms) at $T_1$.

The central bank’s objective is to maximize the number of firms in sector $I$ (e.g., the real sector) that can make the liquidity investment. For simplicity and without loss of generality, we assume that each commercial bank obtains a fixed amount of liquidity, aggregating to $Q$.

Figure 2 summarizes the main setup of the model.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2}
\caption{Main setup of the model}
\end{figure}

\footnote{An unexpected liquidity shock}
\footnote{Liquidity injections $Q$}
2.3 One-sector economy equilibrium

In this subsection, we solve for the equilibrium of the one-sector economy. We first state the equilibrium concept.

**One-sector economy equilibrium** An equilibrium of the one-sector economy consists of the following four elements:

i) Firms optimize their investment and borrowing choices at $T_0$ given the interest rate $r$; 

ii) Commercial banks optimize their lending decisions at $T_0$ given the collateral value of firm asset, $P$, and the market interest rate $r$; 

iii) The bank credit market clears at $T_0$. That is, the aggregate amount of bank credit that all firms obtain is equal to the total supply of liquidity; 

iv) The secondary asset market clears at $T_1$. That is, there is an asset market equilibrium at $T_1$ where firms trade based on their beliefs. The equilibrium asset price is $P$.

2.3.1 Solving for the equilibrium

We examine equilibrium elements ii), iii) and iv) in order, and finally check element i).

First, we consider the decisions of commercial banks at $T_0$. If commercial banks rationally anticipate that the collateral value of a firm’s asset is $P$ at $T_1$, and given the interest rate $r$, they would grant a loan to a firm with a maximum amount $\frac{P}{1+r}$. Hence, the marginal firm that can undertake the liquidity investment, denoted $B^*$, is

$$B^* = \frac{P}{1+r}. \quad (1)$$

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20 Considering that each commercial bank has a fixed amount of funding to lend, and given the market interest rate $r$, an individual commercial bank has no incentive to use an interest rate different from $r$. In fact, if it charges an interest rate lower than $r$, its profits become less. On the other hand, if it charges an interest rate higher than $r$, it loses all its customers. So it is optimal for the individual commercial bank to use the market interest rate $r$ as well. Essentially, the commercial banks behave competitively and are price-takers.
We will verify, by considering their participation conditions, that firms \( B \in [0, B^*] \) undertake the liquidity investment while firms \( B \in (B^*, I] \) do not.

Second, the credit market must clear such that the total supply of funds should equal the total demand for funds at \( T_0 \). The supply of funds is from commercial banks, which have two sources of funding: the liquidity injection \( Q \) and the deposits from the non-investing firms \( \int_{B^*}^{I} (I - B) f(B) dB \). The total demand of funds (by the investing firms) is \( \int_{0}^{B^*} B f(B) dB \). Thus, we have

\[
\int_{B^*}^{I} (I - B) f(B) dB + Q = \int_{0}^{B^*} B f(B) dB. \tag{2}
\]

Adding \( \int_{0}^{B^*} (I - B) f(B) dB \) to both sides of this equation, equation (2) can be equivalently rewritten as

\[
\int_{0}^{I} (I - B) f(B) dB + Q = I \cdot F(B^*). \tag{2'}
\]

Equation (2’) has an intuitive interpretation. Each project requires the amount \( I \) of investment and thus the total amount of investment is \( I \cdot F(B^*) \), which is financed by the liquidity injection \( Q \) and the aggregate inside liquidity, the internal capital of all firms.

Third, solving for the equilibrium of the secondary asset market at \( T_1 \), we obtain the equilibrium asset price \( P \). At \( T_1 \), there are two types of firms that participate in the market: optimists and pessimists. The asset valuation for optimists, denoted \( E^H(\tilde{x}) \), is the expected value of cash flow \( \tilde{x} \) given their beliefs, that is \( E^H(\tilde{x}) = u \cdot \theta_H + d \cdot (1 - \theta_H) \). Likewise, the asset valuation of pessimists, denoted \( E^L(\tilde{x}) \), is \( E^L(\tilde{x}) = u \cdot \theta_L + d \cdot (1 - \theta_L) \). Clearly, \( E^H(\tilde{x}) > E^L(\tilde{x}) \). The difference in valuations between optimists and pessimists motivates them to trade. Also, as in Shleifer and Vishny (1992), only industry participants, who have previous periods of experience in managing assets, can operate the assets to generate cash flows at \( T_2 \). Thus, buyers in the secondary market are optimists. Nevertheless, optimists may face liquidity constraints when buying (as in Shleifer and Vishny (1992); Geanakoplos (2010)) and may not have sufficient
liquidity to buy the quantity of assets they desire. Therefore, the asset price reflects both the valuation of optimists and the liquidity they can access.

Based on the above analysis, we have the asset price, $P$, in the secondary asset market at $T_1$

$$P = \begin{cases} 
E^H(\bar{x}) & \text{if } \Gamma (B^*, r) > E^H(\bar{x}) \\
\Gamma (B^*, r) & \text{if } \Gamma (B^*, r) \in [E^L(\bar{x}), E^H(\bar{x})] \\
E^L(\bar{x}) & \text{if } \Gamma (B^*, r) < E^L(\bar{x})
\end{cases}, \quad (3)$$

where

$$\Gamma (B^*, r) = \frac{\pi \left\{ \int_0^{B^*} [C - B (1 + r)] f (B) dB + \int_{B^*}^I (1 + r) (I - B) f (B) dB \right\} + X}{1 - \pi}.$$ 

The key to understanding the asset price is expression $\Gamma (B^*, r)$, which is in the spirit of Geanakoplos (2010). Optimists as buyers are of total measure $\pi$; among them, some have made the liquidity investment and obtain cash flow $C$ at $T_1$ while the others have no cash flow at $T_1$. Nevertheless, the firms that have made the liquidity investment need to repay the bank loans first and those who have not can withdraw their deposits from banks for asset purchases. Hence, the aggregate internal funds of buyers at $T_1$ are $\pi \left\{ \int_0^{B^*} [C - B (1 + r)] f (B) dB + \int_{B^*}^I (1 + r) (I - B) f (B) dB \right\}$.21 Also, a buyer uses his own asset plus his purchased assets as collateral for borrowing, in which case he can borrow an amount $X$ against each asset. In sum, the aggregate liquidity available to buyers includes the aggregate internal funds of buyers (i.e., the first term of the numerator) and the aggregate liquidity borrowed against the assets in the economy as collateral (i.e., the second term of the numerator). The denominator is the quantity of assets put up for sale.

Furthermore, the asset price is truncated by upper and lower bounds. If the asset price calculated in $\Gamma$ is higher than $E^H(\bar{x})$, this means the total available liquidity is excessive. Thus, in equilibrium, the asset price is $E^H(\bar{x})$, at which the optimists are indifferent between buying and

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21In our model, in equilibrium every participant firm has sufficient cash to repay its debt at $T_1$ even if it is the marginal firm that borrows the highest amount. That is, there is no default.
not, some of whom do not participate in buying, and the total liquidity used to buy is less than
\[
\pi \left\{ \int_0^{B^*} (C - B(1 + r)) f(B) dB + \int_{B^*}^I (I - B) f(B) dB \right\} + X. \]
At the other extreme, if the asset price calculated in \( \Gamma \) is lower than \( E^L(\bar{x}) \), this means that there is too little total available liquidity. Thus, the equilibrium asset price is \( E^L(\bar{x}) \), at which the pessimists are indifferent between selling and not, some of whom do not participate in selling, and the total quantity of assets to sell is less than \( 1 - \pi \).

In what follows, we denote the pricing function of (3) as \( P = p(\bar{x}, B^*, X, r) \).

Finally, we check the firms’ participation condition at \( T_0 \). We find the condition under which a firm is willing to make its liquidity investment. Given the interest rate \( r \), if a firm with internal capital \( A \) borrows an amount \( B = I - A \) to invest in its project, its payoff is \( C - B(1 + r) \). Alternatively, it can deposit its internal capital in commercial banks and realize a payoff of \( A(1 + r) \). Thus, the firm is willing to invest if and only if

\[
C - B(1 + r) \geq A(1 + r) \iff C - I(1 + r) \geq 0. \tag{4}
\]

Note that inequality (4) does not depend on \( A \), which means either all firms or none are willing to make the liquidity investment. For simplicity, we focus on the set of equilibria in which inequality (4) is satisfied, that is where all firms are willing to invest. In this case, whether a firm actually makes the liquidity investment is completely determined by the borrowing constraint, condition (1). We have Theorem 1.

**Theorem 1.** The equilibrium of the one-sector economy is characterized by the triplet \( \{B^*, P, r\} \), which, given \( Q \), solves the system of equations (1) to (3), and satisfies condition (4).

To summarize, the analysis above captures the endogenous feedback loop under liquidity injections between bank lending, the asset price, and the collateral value, illustrated in Figure 3. Bank lending at \( T_0 \) and the aggregate liquidity in industry at \( T_1 \) have a relationship of reciprocal causation.
2.3.2 Characterizing the equilibrium

Now we proceed to characterizing the equilibrium. We are interested in the answers to the following comparative static questions: What is the effect of liquidity injections on the equilibrium asset price and the equilibrium interest rate (i.e., the functions $P(Q)$ and $r(Q)$)? What is the role of asset specificity in the effect?

First, we examine how liquidity injections impact the asset price $P$. Intuitively, liquidity injections enable more firms to make their liquidity investment at $T_0$ and hence increase the liquidity in the industry at $T_1$, which in turn raises the equilibrium asset price.

Formally, we can prove that $\Gamma$ in (3) is an increasing function in $Q$. In fact, from (2), it is easy to obtain that $B^*$ is increasing in $Q$. We can also obtain that the total derivative $\frac{d\Gamma}{dB^*}$ is positive if $C - I(1 + r) > 0$. Therefore, under this condition, $\Gamma$ is certainly increasing in $B^*$ and thus in $Q$.

**Lemma 1.** The equilibrium price $\Gamma$ is increasing in $Q$ if $C - I(1 + r) > 0$ in equilibrium.

*Proof.* See the Appendix. 

\[ \square \]
The intuition for Lemma 1 is the following. In the Appendix, we show that \( \frac{d\Gamma}{dB^*} = \frac{-\pi}{1-\pi} (|C - I(1+r)| f(B^*) - \frac{dr}{dB^*} Q) \). Liquidity injections \( Q \) enable more firms, which are otherwise unable, to make the liquidity investment. Suppose in the economy there is one more firm switching from non-investing to investing at \( T_0 \). Given \( r \), this would increase the liquidity in the industry at \( T_1 \) by an amount \( C - I(1+r) \), which corresponds to the NPV in (4). Further, in the general equilibrium, \( r \) changes, which has the effect of the term \( \frac{dr}{dB^*} Q \). However, in the general equilibrium, the first term always dominates the second term (i.e., \( [C - I(1+r)] f(B^*) - \frac{dr}{dB^*} Q > 0 \)) when \( C - I(1+r) > 0 \). In fact, if \( \frac{dr}{dB^*} \) is positive, we immediately conclude that \( \Gamma \) must increase (even more strongly) in \( Q \) because \( \Gamma = B^*(1+r) \) by equation (1) and \( B^* \) is increasing in \( Q \).\(^{22}\) If \( \frac{dr}{dB^*} \) is negative, clearly \( [C - I(1+r)] f(B^*) - \frac{dr}{dB^*} Q > 0 \), meaning more liquidity in the industry at \( T_1 \). Overall, more liquidity in the industry at \( T_1 \) increases the asset price, in the spirit of Shleifer and Vishny (1992). We show that the condition in Lemma 1, which is also condition (4) in Theorem 1, holds under general parameter values and hence \( \Gamma \) is increasing in \( Q \).

Crucially, we need to consider the lower bound of \( P \) in (3). The asset specificity plays an important role here. If \( X \) is low, the buyers cannot leverage much when buying. Hence, the asset price is low. It is possible that \( X \) is so low that additional liquidity cannot lift the asset price; the asset price is trapped at the lower bound \( E^L(\bar{x}) \) no matter what the size of the liquidity injection \( Q \) (\( \in [0, \bar{Q}] \)) is; that is, \( P \) does not change with \( Q \).

We have Proposition 1.

**Proposition 1.** If \( X \leq \underline{X} \), where \( \underline{X} \) is a (positive) cutoff, the equilibrium asset price \( P(Q) \) is a constant, equal to \( E^L(\bar{x}) \), no matter what the size of the liquidity injection \( Q \) (\( \in [0, \bar{Q}] \)) is. If \( X > \underline{X} \), \( P(Q) \) is (weakly) increasing in \( Q \) under general parameter values.

**Proof.** See the Appendix. \(\square\)

In Proposition 1, asset specificity plays an important role in determining the asset price.\(^{22}\) In fact, if \( C - I(1+r) \) is positive and close to zero, \( \frac{dr}{dB^*} \) must be negative; if \( \frac{dr}{dB^*} \) is positive, \( C - I(1+r) \) must be far above zero and exceed the effect of \( \frac{dr}{dB^*} \). Overall, \( [C - I(1+r)] f(B^*) - \frac{dr}{dB^*} Q > 0 \) if \( C - I(1+r) > 0 \). See the Appendix.
The reason is that asset specificity determines the leverage level of buyers (i.e., the amount of external funds buyers can access) and thus influences the total liquidity available for purchases and thus the asset price. Proposition 1 gives the cleanest case for the asset price not responding much to liquidity injections. In fact, if the value $X$ is low such that $X \leq \bar{X}$, the asset price $P$ is trapped at the lower bound $E_L(\tilde{x})$ and becomes insensitive to liquidity injections. Only when the financing friction is sufficiently low ($X$ sufficiently high) can liquidity injections influence the asset price.\(^{23}\)

It is worth noting that for the benefit of a clean analysis and for our purpose, we have divided $X$ into two regions in the analysis: $X \leq \bar{X}$ and $X > \bar{X}$. The merit of the cleanness can be further seen later when we discuss Figures 4a and 4b. The results of the paper however hold generally (for some $Q$).

Next, we examine the response of the equilibrium interest rate to the liquidity injection. Intuitively, liquidity injections can raise the asset price and thus the collateral value (for bank loans). The higher collateral value means that more firms can borrow and compete for loans. As the supply of loans also increases with liquidity injections, the equilibrium interest rate can either go up or down.

From the optimal lending condition (1), we have the equilibrium interest rate as $r = \frac{P(Q)}{B^*(Q)} - 1$, where $P(Q)$ has the properties of Proposition 1 and $B^*(Q)$ is uniquely determined by market clearing (2). In fact, we can prove that $\frac{dB^*}{dQ} = \frac{1}{P_J(B^*)} > 0$, which implies that $B^*(Q)$ is increasing in $Q$. Therefore, if $P(Q)$ is constant, then $r$ is certainly decreasing in $Q$. If $P(Q)$ is increasing in $Q$, it is not unambiguous whether $r$ is decreasing or increasing in $Q$. Indeed, we prove that $r$ can go in either direction depending on whether $P(Q)$ increases faster or more slowly than $B^*(Q)$. Finally, when $Q$ is very low (close to 0), $r$ is generally decreasing in $Q$. To see this, we consider the equilibrium interest rate when $Q = 0$; in this case, all bank loans are ‘inside liquidity’ and no cash flow at $T_1$ is used to repay the interest for ‘outside liquidity’ $Q$, and hence the asset price relative to the threshold $B^*$ (that is, $\frac{P(Q)}{B^*(Q)}|_{Q=0}$) is high; when $Q$ increases away from zero, a part

\(^{23}\)In this sense, buyers’ internal funds and external funds are complementary in purchasing assets; more internal funds push up the asset price only when the external funds are above a threshold.
of the cash flow at $T_1$ starts to be repaid as the interest on ‘outside liquidity’, and hence \( \frac{P(Q)}{B^*(Q)} \) decreases.

Proposition 2 summarizes the relationship between $r$ and $Q$.

**Proposition 2.** If the financing friction is large, $X \leq X$, the equilibrium interest rate $r(Q)$ is strictly decreasing in $Q \in [0, Q]$. For lower financing frictions, $X > X$, under some distribution $f(B)$ and parameters, $r(Q)$ decreases first and then increases in $Q$, that is, there exists a minimum $r(Q)$, denoted $r_{\text{min}}$, for an interior $Q \in (0, Q)$.

**Proof.** See the Appendix.

Liquidity injections affect the asset price and thereby the equilibrium interest rate. That is, the interest rate equilibrates the (effective) demand for liquidity, captured by the collateral value (asset price) $P$, and the supply of liquidity, captured by the threshold $B^*$, through $1 + r = \frac{P(Q)}{B^*(Q)}$. Proposition 2 delineates two important cases. First, if the asset price is constant, then the interest rate certainly decreases in liquidity injections because the supply of loans increases with liquidity injections. Second, the asset price may increase very fast, faster than $B^*$. In this case, the interest rate increases with liquidity injections. In fact, the latter case happens when the density $f(B)$ is very thick in some region of $B$. Note that $\frac{dB^*}{dQ} = \frac{1}{f(B^*)}$. If the density of $f(B)$ is very high, $B^*$ increases very slowly in $Q$, not as fast as $P(Q)$ does, hence the interest rate increases.

**Remark** We can explain the above mechanisms in the framework of the demand and supply equilibrium.\(^{24}\) When more liquidity is injected, which is on the supply side, the interest rate typically falls. However, in the model, the supply also influences the demand. More liquidity injected pushes up the asset price and thus the collateral value; hence the effective demand for liquidity also increases. When the force of demand is stronger than that of supply, the interest rate increases rather than decreases.

\(^{24}\) See also Benmelech and Bergman (2011b).
Figures 4a and 4b illustrate the equilibrium interest rate under the demand and supply equilibrium. The effective demand of liquidity is a decreasing function of the interest rate (for a given collateral value), i.e., the higher the interest rate, the lower the demand. Thus, more supply of liquidity typically causes the equilibrium interest rate to fall, which is the case of Figure 4a. However, in general equilibrium, the supply may also change the effective demand, i.e., the effective demand curve shifts upward because the collateral value increases. The demand curve may move upward very fast, and hence the equilibrium interest rate is non-monotonic in liquidity supply, which is the case of Figure 4b. More specifically, in our model, the aggregate supply of liquidity is $Q + \int_{B^*}^{I} (I - B) f(B) dB$ and the aggregate demand is $\int_{0}^{B^*} B f(B) dB$. By (2'), we can equivalently rewrite the supply as $S = Q + \int_{0}^{I} (I - B) f(B) dB$ and the demand as $D(r; Q) = I \cdot F\left(\frac{P(Q)}{1+r}\right)$, where both supply and demand are functions of $Q$. In Figure 4a, $P(Q)$ is a constant while in Figure 4b, $P(Q)$ is increasing in $Q$.

(a) Liquidity market equilibrium with $x \leq \underline{x}$  
(b) Liquidity market equilibrium with $x > \underline{x}$

Figure 4: Liquidity market equilibrium
Figure 5 depicts the response of the interest rate to liquidity injections for the one-sector economy under two cases of Proposition 2.

2.4 Two-sector economy equilibrium

In this subsection, we consider the economy with two sectors: sector I and sector II. In order to highlight the main mechanism of the model, we assume that the two sectors differ only in their asset specificity. That is, we assume that the cash flow of the project, the internal capital distribution of firms, and other aspects are identical across the two sectors. The only difference is that the two sectors have different $X$s, the contractible part of the cash flow at $T_2$. Sector I has a higher friction, a lower $X$ denoted $X^I$, where $X^I \leq X$, while sector II has a lower friction, a higher $X$ denoted $X^{II}$, where $X^{II} > X$.

The central bank’s objective is to maximize the number of firms in sector I that can undertake their liquidity investment by choosing an optimal amount of liquidity injection. Essentially, the central bank can control the aggregate liquidity to inject into the economy but cannot control

\[\text{Figure 5: The response of the interest rate to liquidity injections for the one-sector economy}\]

\[\text{25 For cleanliness, we consider that } X^{II} > X \geq X^I. \text{ When } X^{II} > X^I \geq X, \text{ the result in this subsection (i.e., the unique optimal amount of liquidity injection as will be shown in Proposition 3) still holds. However, a stricter condition is required, that is, the maximum amount of allowed liquidity injection } Q \text{ cannot be too large. The reason is that when } X^{II} > X^I > X, \text{ the interest rates in both sectors are increasing in the liquidity inflow when the liquidity inflow is sufficiently high; hence, the crowding-out effect occurs only when } Q \text{ is not very large.}\]
the allocation of the liquidity across the two sectors, which is determined by commercial banks
based on market forces. Market forces mean that commercial banks make loans only based on
firms’ collateral values (no matter which sector firms are from) and there is a single interest
rate in the economy (for both sectors). Also, the central bank cannot control the (real) interest
rate directly. In fact, central banks generally control only the overnight interest rate but cannot
control long-term real interest rates – the ultimate interest rates that matter for real businesses
(see Blinder (1998, p.70)). Also, in our model, one \( r \) may map into two \( Q \)s (i.e., not a one-to-one
mapping) as \( r(Q) \) is non-monotonic for sector \( II \).

We have the central bank’s optimization problem

\[
\max_Q B^{*I} \quad \text{(Program 1)}
\]

\[
\text{s.t. } r = \frac{P^i}{B^{*i}} - 1 \quad \text{(5a)}
\]

\[
\int_{B^{*i}}^{I} (I - B) f(B) dB + Q^i = \int_0^{B^{*i}} f(B) dB \quad \text{(5b)}
\]

\[
P^i = p(\bar{x}, B^{*i}, X^i, r) \quad \text{(5c)}
\]

\[
Q = \sum_i Q^i, \quad \text{(5d)}
\]

where \( i = I \) and \( II \).

In the above optimization problem, the variables with superscript \( i \) denote the variables for
sector \( i \), where \( i = I \) and \( II \). In particular, \( Q^i \) is defined as the net amount of outside-sector
liquidity entering sector \( i \), or the net liquidity inflow into sector \( i \). The central bank chooses an
optimal amount of liquidity injection, \( Q \), to maximize the threshold of firms in sector \( I \) that make
the liquidity investment. Equations (5a), (5b) and (5c) correspond, in order, to the respective
equations (1), (2) and (3) of the one-sector economy. The three equations ((5a) through (5c))
give the equilibrium within each sector. The interaction between the two sectors is given by (5a)
and (5d). First, equation (5a) characterizes that there is a single interest rate, in equilibrium, for
the two sectors. Second, equation (5d) states that the aggregate outside-sector liquidity of the
two sectors must equal \( Q \).
In the two-sector economy, there is an integrated bank credit market at $T_0$, through which capital can flow across sectors. That is, a non-investing firm in one sector saves its spare internal capital in commercial banks and can actually become an ultimate liquidity provider to investing firms in the other sector. In fact, in Program 1, even when $Q = 0$, $Q^i$ might not be equal to zero, in which case, measuring the capital flow across sectors. In contrast, the secondary asset markets for sectors $I$ and $II$ at $T_1$ are segmented. That is, only industrial participants active in a particular sector can buy assets from their peers in the same sector.

Under certain conditions, the central bank’s optimization problem has a unique solution - there is a unique optimal amount of liquidity injection. The conditions are such that the interest rate is U-shaped and monotonically decreasing in liquidity injections for sectors $I$ and $II$, respectively, as shown in Figure 5 (see Proposition 2).

**Proposition 3.** Under some distribution $f(B)$ and parameters, the central bank has a unique optimal amount of liquidity injection, $Q^*$.

To understand the optimal liquidity injection, we can think that there are two steps in solving for the two-sector equilibrium for a given $Q$ in Program 1. First, given $Q^i$ for each sector, solve the equilibrium within each sector, that is, solve for the triplet $\{B^{*i}, P^i, r^i\}$. In particular, we obtain the function $r^i(Q^i)$. Second, by considering the link between the two sectors, (5a) and (5d), we can work out $Q^i$ (for $i = I$ and $II$), for a given $Q$. That is, by considering $r^I(Q^I) = r^{II}(Q^{II}) = r$ and $Q^I + Q^{II} = Q$, we obtain the unique $Q^I$ and $Q^{II}$. This way we determine the equilibrium $Q^I$ (and thus $B^{*I}$) for a given $Q$. Then, we can find the optimal $Q^* (\in [0, Q])$ that maximizes $B^{*I}$.

Figure 6 depicts the optimal liquidity injection in the two-sector equilibrium according to the two steps described above. In the figure, the optimal amount of liquidity injection is $Q^* = Q^I + Q^{II}$. If the liquidity injection exceeds this level, all additional liquidity goes to sector $II$ and further, some liquidity in sector $I$ is actually *squeezed out* due to the increased interest rate. Therefore, overall, the liquidity in sector $II$ increases while that in sector $I$ decreases.
It is important to examine the process by which the new equilibrium is reached when some additional amount of liquidity (beyond $Q^*$) is injected. In Figure 6, in which an additional amount of liquidity $(Q'_{II} + Q'_{I}) - (Q'_I + Q'_{II})$ is injected, we examine how $Q^I$ reaches $Q''$. Suppose initially all additional liquidity enters sector $II$; this would push up the interest rate; the higher interest rate would squeeze some liquidity out of sector $I$, which means that more liquidity would flow into sector $II$, pushing up the interest rate further, and so on in a spiral. Essentially, there is a downward spiral in reaching the new equilibrium. That is, one sector enters a liquidity-asset price ‘inflationary’ cycle while the other enters a ‘deflationary’ cycle, and the two cycles reinforce each other.

In sum, we can divide the amount of liquidity injection, $Q$, into two regions: $Q \leq Q^*$ and $Q > Q^*$. In the region of $Q \leq Q^*$, the liquidity in both sectors increases with liquidity injections. In the region of $Q > Q^*$, more liquidity injected increases the liquidity in one sector but reduces that in the other, that is, the ‘crowding-out’ effect occurs. The ‘crowding-out’ occurs in a spiral.

We have Proposition 4.

**Proposition 4.** When $Q \leq Q^*$, there is an ‘allocation’ effect, i.e., liquidity in both sectors increases with liquidity injections but sector $I$ obtains less liquidity than sector $II$. When $Q > Q^*$,
there is a ‘crowding-out’ effect, i.e., more liquidity injected increases the liquidity in sector II but reduces it in sector I; the crowding-out occurs in a self-reinforcing spiral.

Note that in order to highlight the main mechanism of the model, we have assumed that the two sectors differ only in their asset specificity while keeping other aspects identical in the two sectors. However, if we assume that sector II has a strictly lower project cash flow but a higher $X$, it is easy to show that sector II can still crowd out sector I; this clearly gives welfare implications.

**Proposition 5.** Suppose the project cash flow ($C,u$ and $d$) in sector I is strictly higher than that in sector II while the $X$ is lower in sector I. It is still possible that sector II obtains more liquidity than sector I, and more liquidity injected increases the liquidity in sector II but reduces that in sector I. That is, the sector with lower NPV projects crowds out the sector with higher NPV projects.

Before closing this section, we give a numerical example for the results of the model, illustrating the existence of relevant parameters in the propositions. That is, the numerical example is to highlight the qualitative (rather than quantitative) aspect of the model.

We choose parameter values as simple as possible. We set the parameter values for the project as $I = 1$, $C = 2.2$, $\pi = 0.4$, $E^H(\bar{x}) = 1.3668$, $E^L(\bar{x}) = 1.0334$. The firm distribution is $f(B) = \log \frac{1}{1-B}$ for $B \in [0,1)$. As the maximum total amount of liquidity that any one sector can demand is $Q^{\text{max}} = \int_0^1 B f(B) dB = 0.75$, we set $Q = 2Q^{\text{max}} = 1.5$.

Given that $E^L(\bar{x}) = 1.0334$, we can calculate the threshold $X$ in Proposition 1, which is $X = 0.0501$. We set $X^I = 0.05$ and $X^{\text{II}} = 0.35$, where $X^I < X < X^{\text{II}}$.

For sector I, we can work out that the asset price is $P^I = 1.0334$ for any $Q^I \in [0,0.75]$. The asset price is trapped at $E^L(\bar{x}) = 1.0334$. The interest rate $r^I(Q^I)$ is a strictly decreasing function of liquidity inflow $Q^I$.

For sector II, the asset price $P^{\text{II}}(Q^{\text{II}})$ is (weakly) increasing in $Q^{\text{II}}$. When $Q^{\text{II}}$ is small,
liquidity injections are not sufficient to push the asset price above $E^L(\bar{x})$ and the asset price is $P^{II} = 1.0334$; when $Q^{II}$ is big enough, $P^{II}(Q^{II})$ is strictly increasing in $Q^{II}$, with the maximum asset price being $P^{II}(Q^{II} = 0.75) = 1.3667$ (which is lower than $E^H(\bar{x}) = 1.3668$). As for the equilibrium interest rate $r^{II}(Q^{II})$, it is non-monotonic and ‘U’-shaped, with the minimum interest rate being $r_{\text{min}} = 0.3270$.

The optimal amount of liquidity injection for the economy is $Q^* = 0.6113$, at which the distribution of the liquidity injection across the two sectors is $Q^{I*} = 0.4163$ and $Q^{II*} = 0.1950$, respectively.

Figure 7a shows the asset price response in each sector to its (net) liquidity inflow $Q^i$ and Figure 7b depicts the interest rate response in each sector.
(a) Asset price responses to liquidity inflow

(b) Interest rate responses to liquidity inflow

**Figure 7:** A numerical example
3 Empirical implications

We have shown that if too much liquidity is injected into the economy, there is the possibility of crowding-out between the two sectors that have different degrees of friction. In this section, we intend to understand in what situations crowding-out is more likely and what its implication is for the optimal liquidity injection. We derive two implications: a cross-sectional one and one in the time series.

Recall that the term $X$ is the contractible part of the cash flow of the project. It differs across sectors. For a given sector such as the real sector, it also differs across countries. In fact, for a country with poorer contracting institutions, creditors can realize a lower fraction of the project cash flow in the case of default even if collateral is present (e.g., LaPorta, Lopez-de Silanes, Shleifer, and Vishny (1997, 1998); Djankov, Hart, McLiesh, and Shleifer (2008)). This can be due to an inefficiency of contract enforcement or high transaction costs in financial markets in liquidating collateral. Specifically, holding $X^{II}$ (the contractible part of the cash flow for sector $II$) constant for all countries, we expect $X^I$ (the contractible part of the cash flow for sector $I$) to be lower for a country with poorer contacting institutions.

By conducting a comparative statics analysis on two countries with the same $X^{II}$ but different levels of $X^I$, we have the following prediction.

**Implication 1** (Cross-sectional implication) A country with poorer contracting institutions has a lower fraction of injected liquidity entering the higher-friction sector for a given level of liquidity injections, is more likely to suffer crowding-out between sectors, and ex ante should have a lower level of optimal liquidity injections.

*Proof.* See the Appendix.

As for the time-series implication, our main interest is in the uncertainty about economic prospects. Specifically, consider a mean-preserving spread of $\theta_L$ and $\theta_H$, that is, hold the average of $\theta_L$ and $\theta_H$ constant while increasing their distance, $\theta_H - \theta_L$. Then, the distance $\theta_H - \theta_L$
measures the degree of divergence in investors’ opinions. We can characterize the uncertainty about economic prospects by the degree of divergence in opinions. The higher the degree of divergence, the higher the uncertainty about economic prospects. We have Implication 2.

**Implication 2** (Time-series implication) At a time of higher uncertainty about economic prospects, crowding-out is more likely for a given level of liquidity injections.

*Proof.* See the Appendix. □

Implication 2 describes a form of “flight-to-safety” phenomena.\textsuperscript{26} The intuition of Implication 2 is as follows. We consider a pair \((\theta_L', \theta_H')\), which is a mean-preserving spread of the pair \((\theta_L, \theta_H)\), that is, \(\theta_L' < \theta_L < \theta_H < \theta_H'\). Under the mean-preserving spread, the lower bound of the asset price decreases, that is, \(E^{\theta_L'}(\tilde{x}) < E^{\theta_L}(\tilde{x})\). For sector \(I\), if its original asset price binds at the lower bound \(E^{\theta_L}(\tilde{x})\), the divergence of opinion reduces the asset price. In contrast, for sector \(II\), under the divergence of opinion, the asset price either remains unchanged or increases. Therefore, the gap in asset prices between the two sectors widens. This makes crowding-out more likely for a given level of liquidity injections. Figure 8 illustrates the intuition.

![Figure 8: Greater uncertainty widens gap in asset prices in the two sectors](image)

\textsuperscript{26} Among others, Kamara (1994); Longstaff (2004) document evidence on the flight-to-safety phenomenon.
Miller (1977) has argued that an increase in belief disagreements combined with short-selling constraints tends to increase the (over)valuation of the asset.\textsuperscript{27} We highlight that the valuation increases are asymmetric between two sectors when investors in the two sectors have different degrees of liquidity constraints in buying assets, originating in asset specificity. Implication 2 shows the consequence of this asymmetry because of an increase in uncertainty and belief disagreements.

4 Conclusion

The paper studies why economic stimulus by way of liquidity injections may not work. We highlight the friction of imperfect financial contracting. We show that there is positive feedback between liquidity injections and firm asset collateral values in an economy. This feedback, however, is asymmetric across sectors. The sector with lower friction has stronger feedback. This asymmetry creates a crowding-out effect. For the sector with lower friction, the collateral value responds strongly to liquidity injections, which can push up the interest rate; the increased interest rate reduces the liquidity in the sector with higher friction as the collateral value in that sector responds feebly. Therefore, even if the collateral value in one sector increases with liquidity injections, as long as the increase is not as fast as that in the other sector, there is the possibility of crowding out, which holds back economic recovery.

Our paper has two empirical implications. The model implies that overheating and crowding-out are more likely in a country with a poorer contracting institutions and at a time of greater uncertainty about economic prospects. Authorities should be more cautious about the crowding-out effect in these situations.

The paper highlights limitations of monetary policy in stimulating economic growth. The paper implies that monetary policy in conjunction with fiscal policy to target some specific sectors/industries might have better effects in economic stimulus.\textsuperscript{28} Regulation on the leverage

\textsuperscript{27}See also Harrison and Kreps (1978); Scheinkman and Xiong (2003). For an excellent survey of the disagreement approach to the modeling of bubbles, see Hong and Stein (2007).

\textsuperscript{28}See also Bebchuk and Goldstein (2011) for related discussions.
level in more speculative industries might be helpful in reducing the crowding-out effect.

References


A Appendix

Proof of Lemma 1

By using the credit market clearing condition (2), the expression of $\Gamma(B^*, r)$ in (3) can be rewritten as $\Gamma(B^*, r) = \frac{\pi \left[C \cdot F(B^*) - Q (1 + r)\right] + X}{1 - \pi}$.

By substituting (1) into this equation, $r$ can be eliminated. That is, $\Gamma(B^*) = \frac{\pi C \cdot F(B^*) + X}{(1 - \pi) + \pi \frac{Q}{1 - r^*}}$. 

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From (2), $B^*$ is completely and uniquely determined by $Q$. Hence, $\Gamma$ can be expressed in terms of $Q$:

$$\Gamma (Q) = \frac{\pi C \cdot F (B^* (Q)) + X}{(1 - \pi)} + \pi \frac{Q}{B^* (Q)}.$$ 

Now we derive the total derivative $\frac{d\Gamma}{dQ}$.

From (2), we can work out

$$\frac{dB^*}{dQ} = \frac{1}{f(B^*)}. \tag{A1}$$

From the expression $\Gamma(B^*, r)$ in (3), we have the total derivative:

$$\frac{d\Gamma}{dB^*} = \frac{\partial \Gamma}{\partial B^*} + \frac{dr}{dB^*} \frac{\partial \Gamma}{\partial r}$$

$$= \frac{\partial \Gamma}{\partial B^*} + \frac{d}{dB^*} \left( \frac{P}{B^*} - 1 \right) \frac{\partial \Gamma}{\partial r}$$

$$= \frac{\partial \Gamma}{\partial B^*} + \left( \frac{1}{B^*} \frac{dP}{dB^*} - \frac{P}{B^{*2}} \right) \frac{\partial \Gamma}{\partial r}, \tag{A2}$$

where the equality in the second line uses the optimal lending condition (1). Given the interior region of (3) in which $P = \Gamma$, we can rewrite equation (A2) as

$$\frac{d\Gamma}{dB^*} = \frac{\partial \Gamma}{\partial B^*} + \left( \frac{1}{B^*} \frac{d\Gamma}{dB^*} - \frac{\Gamma}{B^{*2}} \right) \frac{\partial \Gamma}{\partial r}. \tag{A3}$$

Noticing that the term $\frac{\partial \Gamma}{\partial B^*}$ appears on both sides of equation (A3), we solve for this term

$$\frac{d\Gamma}{dB^*} = \frac{\frac{\partial \Gamma}{d\Gamma} - \frac{\Gamma}{B^{*2}} \frac{\partial \Gamma}{\partial r}}{1 - \frac{1}{B^*} \frac{\partial \Gamma}{\partial r}}. \tag{A4}$$

Also, from the expression of $\Gamma(B^*, r)$ in (3), we have

$$\frac{\partial \Gamma}{\partial B^*} = \frac{\pi}{1 - \pi} \left[ C - I (1 + r) \right] f (B^*)$$

$$\frac{\partial \Gamma}{\partial r} = \frac{\pi \left\{ \int_0^{B^*} -B f (B) dB + \int_B^{I - B} (I - B) f (B) dB \right\}}{1 - \pi} = -\frac{\pi}{1 - \pi} Q,$$
where the second equality follows from the credit market clearing condition (2).

Therefore, we can use these expressions to simplify expression (A4):

\[ \frac{d\Gamma}{dB^*} = \left[ C - I (1 + r) \right] f (B^*) + (1 + r) \frac{Q}{B^*}. \]  (A5)

Overall, we have

\[ \frac{d\Gamma}{dQ} \frac{dB^*}{dQ} \frac{d\Gamma}{dB^*} = \left[ C - I (1 + r) \right] f (B^*) + (1 + r) \frac{Q}{B^*}. \]  (A6)

By (A6), we conclude that a sufficient condition for \( \frac{d\Gamma}{dQ} > 0 \) is \( C - I (1 + r) > 0 \).

**Proof of Proposition 1**

For ease of exposition, we reiterate the equilibrium asset price here. That is

\[ P = \begin{cases} 
E^H (\tilde{x}) & \text{if } \Gamma (B^*, r) > E^H (\tilde{x}) \\
\Gamma (B^*, r) & \text{if } \Gamma (B^*, r) \in [E^L (\tilde{x}), E^H (\tilde{x})], \\
E^L (\tilde{x}) & \text{if } \Gamma (B^*, r) < E^L (\tilde{x})
\end{cases} \]

where

\[ \Gamma (B^*, r) = \frac{\pi \left[ \int_0^{B^*} [C - B (1 + r)] f (B) dB + \int_{B^*}^I (1 + r) (I - B) f (B) dB \right] + X}{1 - \pi}. \]

We have proved that \( \Gamma (Q) \) is an increasing function of \( Q \) if \( C - I (1 + r) > 0 \). We show that the condition \( C - I (1 + r) > 0 \) holds under general parameter values. In fact, by \( 1 + r = \frac{\Gamma (B^*, r)}{B^*} \), we have \( C - I (1 + r) = C - I \frac{\Gamma (B^*, r)}{B^*} \), where \( B^* \) is completely determined by \( Q \) and \( r \) is endogenous. *Ceteris paribus*, if \( \pi \) is sufficiently low, \( \Gamma \) is low and thus \( C - I (1 + r) > 0 \). The numerical example in the text is one case.

We consider the lower bound of the asset price, \( E^L (\tilde{x}) \). For our purpose, we choose the
parameters to make sure that the upper bound of the asset price is not binding (i.e., the asset price calculated in $\Gamma(Q)$ is always lower than $E^{H}(\tilde{x})$). We define two cutoffs, $\underline{X}$ and $\overline{X}$, and divide $X$ into three ranges: $X < \underline{X}$, $X > \overline{X}$, and $\underline{X} \leq X \leq \overline{X}$.

The first range of $X$ is the case in which the asset price is trapped at the lower bound no matter what $Q (\in [0, \overline{Q}])$ is. That is, even if $Q = \overline{Q}$, the asset price calculated in $\Gamma$ is still (weakly) below $E^{L}(\tilde{x})$. Therefore, $\underline{X}$ satisfies $E^{L}(\tilde{x}) = \Gamma|_{Q=\overline{Q}, X=\underline{X}}$.

The third range of $X$ is the case in which the asset price is above the lower bound for the whole region of $Q \in [0, \overline{Q}]$. That is, even if $Q = 0$, the asset price calculated in $\Gamma$ is still (weakly) above $E^{L}(\tilde{x})$. Therefore, $\overline{X}$ satisfies $E^{L}(\tilde{x}) = \Gamma|_{Q=0, X=\overline{X}}$.

In the second range of $\underline{X} \leq X \leq \overline{X}$, the asset price is the constant $E^{L}(\tilde{x})$ when $Q$ is low and then increases in $Q$ when $Q$ is higher.

**Proof of Proposition 2**

When $X \leq \underline{X}$, the asset price $P$ is constant in $Q$. Also considering $\frac{dH^*}{dQ} > 0$, we have that when $X \leq \underline{X}$, the equilibrium interest rate $r(Q)$ is strictly decreasing in $Q (\in [0, \overline{Q}])$.

We consider the equilibrium interest rate when $X > \underline{X}$. By $r = \frac{\Gamma}{B^*} - 1$, we have

$$\frac{dr}{dB^*} = \frac{1}{B^*} \frac{d\Gamma}{dB^*} - \frac{\Gamma}{B^{*2}}.$$  \hspace{1cm} (A7)

Plugging (A5) into (A7), we have

$$\frac{dr}{dB^*} = \frac{[C - I(1 + r)] f(B^*) B^* - \frac{1-\pi}{\pi} \Gamma}{(\frac{1-\pi}{\pi} B^* + Q) B^*}.$$  \hspace{1cm} (A8)
By $1 + r = \frac{P(Q)}{B^*(Q)}$, considering $P = \Gamma$, we have

$$
\frac{dr}{dQ} = \frac{dB^*}{dB^*} \frac{dr}{dQ} = \frac{1}{If(B^*)} \left[\frac{C - I (1 + r)}{\lambda_B} f(B^*) B^* - \frac{1 - \lambda_B}{\pi} \Gamma \right] \left(\frac{\lambda_B B^* + Q}{\lambda_B} \right) \frac{1 - \lambda_B}{\pi} \Gamma. 
$$

(A9)

Hence, $\frac{dr}{dQ} > 0$ if and only if $f(B^*) > \frac{1 - \lambda_B}{\pi} \left[\frac{C - I (1 + r)}{\lambda_B} - I \right]^{-1}$ by noting that $\frac{\Gamma}{\lambda_B} = 1 + r$. In equilibrium, we guarantee that $C > I(1 + r)$, so the right-hand side of this condition is bounded. When $f(B^*)$ is sufficiently high (respectively low), $\frac{dr}{dQ}$ is positive (respectively negative). The numerical example in the text illustrates these.

We also consider the special case of $\frac{P(Q)}{B^*(Q)}|_{Q=0}$. In this case, $B^*$ solves $\int_{B^*}^I (I - B) f(B) dB = \int_0^{B^*} B f(B) dB$, and $\Gamma = \frac{\pi C - f(B^*) + X}{1 - \pi}$. Hence, $r$ is determined.

To summarize, when $X > X^*$, we can choose some function $f(B)$ and some $\overline{Q}$ such that $r$ decreases first and then increases in $Q$ within the interval $Q \in [0, \overline{Q}]$.

**Proof of Implication 1**

We consider two countries with the same $X^{II}$ but different levels of $X^I$. The lower $X^I$ is lower than $X$ while the higher $X^I$ is possibly higher than $X$. Then, for the country with a poorer contracting institution, the asset price in sector $I$ is always the constant $E^L(\bar{x})$. In contrast, for the country with a better contracting institution, it is possible that the asset price in sector $I$ is the constant $E^L(\bar{x})$ when $Q$ is low; when $Q$ is high, the asset price increases. In any case, for a given $Q$, the asset price in sector $I$ for the country with a poorer contracting institution is (weakly) lower than that for the country with a better contracting institution, and so is the interest rate for a given $Q$. Therefore, for a given interest rate, the corresponding liquidity injection for the country with a poorer contracting institution is (weakly) lower than that for the country with a better contracting institution. Ex ante, the optimal liquidity injection for the country with a poorer contracting institution should be (weakly) lower than that for the country with a better
Proof of Implication 2

We consider a pair \((\theta'_L, \theta'_H)\), which is a mean-preserving spread of the pair \((\theta_L, \theta_H)\), that is, \(\theta'_L < \theta_L < \theta_H < \theta'_H\). Under the mean-preserving spread, the lower bound of the asset price decreases, that is \(E^{\theta'_L}(\tilde{x}) < E^{\theta_L}(\tilde{x})\). For sector I, if its original asset price binds to the lower bound \(E^{\theta_L}(\tilde{x})\), the divergence in opinions lowers the asset price. In contrast, for sector II, under the divergence of opinions, the asset price either remains unchanged or increases. Therefore, the gap in asset prices between the two sectors increases for a given \(Q\). As does the gap in the interest rates. Therefore, for a given interest rate, the gap in the liquidity allocation between the two sectors widens. That is, for a given level of liquidity injection, overheating and crowding-out become more likely.