A THEORY OF
‘PROMINENT’ DISCLOSURE

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ABSTRACT: ‘Prominence’ plays an important role in financial reporting – an entity might assign an item to the footnotes, report it below the line, or comment on it in a press release versus the MD&A. We propose a model where quantitative disclosures are classified as more or less prominent, based on technically vague information. In our model, an entity must give sufficiently informative quantitative disclosures high prominence, sufficiently uninformative quantitative disclosures low prominence, while for a range of disclosures discretion is permitted. We show that a market of rational actors will react more strongly to negative (than positive) univariate quantitative disclosures when conditioned on either classification (low or high prominence). This effect strengthens as vagueness increases. We then examine how the reporting and bundling of multivariate items might occur in response to vagueness. We find that for unambiguously good or bad quantitative news, bundling increases with vagueness, and the market puts more weight on bad quantitative news disclosures than good quantitative news disclosures when conditioned on disclosure type. However, when multivariate quantitative news is mixed (good and bad items occur together), the probability of bundling is independent of vagueness, the market reaction to a bundle’s classification is muted, and the market’s reaction to unbundled items is amplified.

KEY WORDS: Prominence; First-order vagueness; Soft information; Bundling; Asymmetric market response.
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I. Introduction

It has long been recognized that emphasis or salience plays an important role in conveying meaning to a communication, whether it be verbal or textual. To the extent that a communication contains items of varying significance, making an item prominent conveys to the receiver of the message that the item is important. Accounting disclosures apply the convention in a number of ways: when an item has been assigned to main statements (versus the footnotes); when an item is reported above the line (versus below the line) on an income statement; when an item is classified as an unusual item (versus an ordinary item); or when the firm uses different public outlets (of varying prominence) to disclose its information. In principle, highlighting important information serves to make markets more efficient by providing more transparency to investors, and we would therefore would expect regulators to express an interest in prominence. To roughly illustrate this claim, a word search for the term prominent/prominence of the SEC’s website\(^1\) indentified 513, 369, and 1321 documents respectively for Regulatory Actions, Staff Interpretations, and Public Comments. Despite being widely referred to as a desired objective, the term is vague, and the exact role of prominence in disclosures has not been well articulated. To that end, we present a model that describes how prominence might function in reporting financial information to the public, and provides a framework for interpreting a number of findings in the accounting literature.

A number of accounting studies in the academic literature have documented the use of prominence in public communications of financial data. They include McVay (2006), who produces evidence consistent with managers opportunistically shifting expenses from core expenses (cost of goods sold and selling, general, and administrative expenses) to special items. While this activity does not change bottom-line earnings, it overstates (more prominent) core earnings. Files at al. (2009) have examined three degrees of prominence: the headline of a press

release, the body of a press release, or a footnote. They find stock market reactions consistent with the expected convention – the strongest reaction for headlines, and the weakest reactions for footnotes. Davis and Tama-Sweet (2012) and Mayhew (2012) introduce the idea of disclosure outlets (e.g., MD&A versus press releases) and describe how an outlet choice may influence the stock market reaction to the information. Under the assumption that a press release is a more prominent outlet than MD&A, Davis and Tama Sweet hypothesize and find evidence that managers will shift more pessimistic language to the MD&A. Earlier work dealing with prominence includes Maines and McDaniel (2000), who also report findings consistent with a prominence convention with nonprofessional subjects (MBA students).

A general explanation provided for the results above has been that users have cognitive limitations and that prominence guides users’ attention to the most relevant information. Indeed, certain adaptive conventions have evolved over long (e.g. prehistoric) periods of time and have become part of our cognitive structures. However, given that these conventions are already in their behavioral repertoire, they may currently be applied by sophisticated (i.e., fully rational) users who do not have memory or attention constraints with respect to the data that they process, yet recognize the possibility that prominence represents a convention that may add meaning that cannot be otherwise communicated.

We model how prominence provides useful information to fully rational financial statement users in the presence of a special type of qualitative information, which is technically vague. We examine a setting where, when permitted some classification discretion (e.g., wiggle room), the firm will chose that classification which maximizes its market value, and demonstrate that there are useful ways to exploit vague information. In what follows, we set up the problem using a model of first-order vagueness. We then go on to show how vague information constrains the reporting process and provides non-trivial information to outside stock market participants who then make sense of it.

The first part of the paper examines a univariate quantitative disclosure, which is (hard) information. The production of this information is paired with information that is informative of the that number’s impact on firm value, but is observable only to firm insiders. Because a
publicly accepted continuous accounting measure for the inside information does not exist, the firm is limited to a (binary) qualitative disclosure indicating whether the quantitative information should be given prominence (or not). This might be accomplished by choosing whether or not to provide a special highlight for the number in some way, and captures the binary nature of many forms of qualitative classification (e.g., reasonable/not reasonable, ordinary/extraordinary, transitory/core). The range for this information has three regions – two end regions where results can be definitely classified as prominent/not prominent (and where the firm has no discretion), and an interior region which is indeterminate and permits wiggle room to the firm in making its classification (i.e., the area where actors may agree to disagree).  

We get the result that negative quantitative disclosures are more likely to be classified as less (versus more) prominent than positive quantitative disclosures, which is consistent with Davis and Tama-Sweet and McVay’s findings. In addition, our model suggests that that the (fully rational) market will react more strongly to negative quantitative information conditioned on its classification. We derive a measure of vagueness which corresponds to the size of the indeterminate region for the inside information and show that as the size of the indeterminate region grows, the effect indicated above increases, suggesting that vagueness has an important role to play in understanding disclosures. We also show that when the level of vagueness exceeds a certain amount, there exist alternative equilibria with uninformative disclosures. We suggest however, that the informative disclosure equilibria are more robust than the uninformative equilibria.

The asymmetric reactions to bad and good news that we find are relevant to a number of ongoing discussions in academic accounting. They include Kothari, Shu and Wysocki (2009) who find evidence that the market reaction to bad news is stronger than the reaction to good news. To explain this, they suggest that for a range of incentives (e.g., career concerns), firms will withhold bad information until the cumulative effect on market value reaches a threshold, at which point it is released. In contrast, they suggest that firms “leak” good news on a more frequent basis. On the other hand, Skinner (1994) suggests that litigation risk may encourage managers to reveal bad news more quickly thereby lessening the impact of any particular bad

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2 This information is like a bread roll – hard on the outside and soft in the middle.
news disclosure. Overall, the documented empirical reactions to good and bad news disclosures varies, with an apparent tendency to find stronger market reactions to good news earnings announcements than bad news earnings announcements, with the reverse tendency for non-mandated disclosures.\(^3\)

An important point of departure for our paper from the above discussion is that we do not posit an acceleration or delay, per se, to create asymmetric market reactions to good and bad quantitative news – all of our information comes out at once. As such, we present and explore a variant of discretion (other than timing) due to the inherent vagueness of classifications, and our simple model does not require us to delve into many of contextual complications which otherwise cloud the incentives to make disclosures.\(^4\)

The second part of the paper extends the model to two simultaneous quantitative (hard information) items, each of which must be classified as either i) prominent or ii) not prominent. We assume that if the quantitative items are given the same classification, they are then pooled and reported as a single sum in a single classification.\(^5\) Conversely, if the quantitative items are given different classifications, then the items are reported separately (i.e., with different classifications). The first case is referred to as “bundling” and the second is referred to as “unbundling.”\(^6\) We show that for unambiguously positive or unambiguously negative quantitative components, the likelihood of bundling increases as vagueness increases. We again get the result that market reaction is stronger when the disclosure has negative quantitative

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\(^3\) Earnings are disclosed in a complex environment. As examples, Soffer, Thiagarajan and Walther (2000), who, in the spirit of Skinner, find evidence suggesting that preannouncements dilute the market reaction to negative earnings, but the effect is restored when a longer time period including the preannouncement is studied. Collins, Pincus and Xie, (1999) suggest a ‘liquidation effect’ where negative earnings signal a movement toward liquidation values being more relevant. Aboody and Kasznik (2000) investigate the role of stock options. Our approach is perhaps most consistent with Freeman and Tse (1992) who argue that earnings have various components – that may be more or less transitory.

\(^4\) In contrast, Fischer and Verrecchia (2000) deal with the complexity issue by assuming that the stock market is uncertain about the manager's reporting objective (e.g., stock options, an intention to do an MBO, risk aversion, litigation environment, etc.). The manager's unknown reporting objective therefore adds noise to the report.

\(^5\) That is, if they could otherwise be broken out within the category, the meaning of a ‘category’ is compromised. Our notion of bundling is similar to what Mullainathan et al. (2008) refer to ‘coarse thinking,’ where individuals co-categorize situations and use one model of inference for all situations in the same category. Similar assumptions underlie “style investing” (Barberis and Shleifer, 2003). A later brief section will discuss a multivariate setting where aggregation (disclosing a sum) is not permitted. For the most part, those latter results mirror the univariate setting.

\(^6\) For a related theoretical treatment, see Dye (2010) whose “bunching” equilibria are discussed below.
content, conditioned on disclosure type. When the quantitative components are of a mixed nature (both favorable and unfavorable quantitative items), however, the results change. In particular, we show that the amount of bundling can be independent of the amount of vagueness. Due to the ambiguous nature of the (mixed) quantitative information, the intensity of the market reaction to bundles is less dependent on the bundle’s classification, and more dependent on whether the disclosure is bundled. That is, information about the weights applied to the quantitative items is more important to valuation when the quantitative news is mixed.

II. Relevant Theoretical Background

Much of what we analyze may be referred to as a combination of hard and soft information. Ijiri (1975) provides an early investigation of soft versus hard information. He notes (p. 36) that the “lack of room for disputes over a measure may be expressed as the hardness of the measure. A hard measure is one constructed in such a way that it is difficult for people to disagree. A soft measure is one that can be easily pushed in one direction or the other.” This perspective is consistent with our notion that vague information contains indeterminate regions where information could be jointly observed by parties with different objectives, they may agree upon what they see, yet they agree to disagree about the datum’s implications.

Another relevant work is Dye and Sridhar (2004), who argue that some data may be softer than other data in the sense that some data are more easily manipulable. While our perspective is similar (we permit wiggle room), it differs from Dye and Sridhar in that they assume all realizations of a random variable have an identical degree of softness (the cost of manipulation is independent of the realization), whereas our approach suggest that, ex post, some realizations are indeterminate (i.e., manipulable) while others are determinate (non-manipulable).7

The paper also has some similarities to Dye (2002) who models binary classifications. It differs in several ways, however, from his analysis, including his assumption that one of the classifications is unambiguously perceived more favorably by investors than the other. In

7 Also see Dye (2002).
contrast, our model views the favorableness of the classification as depending on the context (the sign of the companion quantitative disclosure). Finally, our notion of bundling is similar in some respects to Dye’s (2010) later work on “bunching.” The main objective of bundling, however, is to affect classification, whereas a main objective of bunching is to alter the timing of information.

III. A Model of Univariate Quantitative Information

This section introduces our notion of first-order vagueness. Suppose that the true value of the firm is \( \alpha x \), where

\[
x \in (\infty, \infty) \text{ and } \alpha \in [0,1],
\]

\( x \) and \( \alpha \) are the realizations of independently distributed random variables, and \( \tilde{\alpha} \) is uniformly distributed. We assume that the actual value of \( x \) must be reported (making it quantitative information). On the other hand, we assume that \( \alpha \) is a characteristic (like a color) which can be observed, compared and ranked with other values (e.g. shades) of \( \alpha \), but does not have an agreed upon convention for turning it into a quantitative report (e.g., the measure for light frequency has not been invented yet). \(^8\) The (insider) firm, however, must issue a qualitative report for \( \alpha \), which we designate as assigning a label of either \( L \) (low) or \( H \) (high) to the reported value of \( x \) as a classification of its prominence.

We assume that some classifications are clear-cut while others are more difficult to make. If for example \( \alpha \approx 0 \) or \( \alpha \approx 1 \), the classification of \( x \) is definitely \( L \) or \( H \) respectively. Furthermore we assume that in such cases, the firm has no discretion concerning the classification of \( \alpha \). Conversely, when the value of \( \alpha \) is exactly in the middle of the interval, its classification is

\(^8\) Similar models where one piece of information is relevant but cannot be directly disclosed are Penno and Watts (1991) and Sridhar and Magee (1997). Langberg and Sivaramakrishnan (2008) model a piece of information that may be (exactly) disclosed – but through another channel (analysts).
indeterminate. That is, when $\alpha \approx \frac{1}{2}$, then the firm has discretion over the classification which cannot be challenged by unmodeled third parties (e.g., its auditor or the courts).

We maintain the idea of prominence by imagining that $L$ corresponds, say, to an item found in the footnotes and $H$ might correspond to an item found in the main statements (balance sheet, income statement, or statement of cash flows). Similarly, an $H$ item might be reported above the line and an $L$ item below the line when distinguishing continuing operations from discontinued operations, or ordinary items from unusual items, or in a press release versus MD&A.

**First-Order Vagueness**

To develop a tractable model, we assume first-order vagueness, which means that we model the interval on which $\tilde{\alpha}$ is distributed as having an indeterminate region in the middle surrounded by determinate regions on either end. The term *first-order* is used because we assume that there exists a boundary between the indeterminate and determinate regions. To understand this term, note that second-order vagueness would note that the boundary between indeterminate and determinate is itself indeterminate, requiring two more indeterminate regions characterizing the transition from determinate to indeterminate on either side. This line of reasoning can continue to $n$-th dimensional vagueness, with $n \to \infty$. We therefore cut this line of reasoning at $n = 1$. In doing so we believe that we capture, in large part, the notion of an indeterminate region, while maintaining the crisp boundary required for tractability.\(^9\) The approach to vagueness that we take is sometimes called “supervaluationism,” which emphasizes such a (well-defined) gap. See Fine (1975) for a technical discussion, or Graff (2002) for a brief intuitive statement.

Suppose that the firm is *required* to classify $\alpha$ as an $L$ if $\alpha \in \left[0, \frac{1}{2} - \varepsilon \right]$ and as an $H$ if $\alpha \in \left[\frac{1}{2} + \varepsilon, 1 \right]$, and is given discretion to choose either $L$ or $H$ when $\alpha \in \left[\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon \right]$

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\(^9\) Our model has some similarities to Crawford and Sobel (1982) in that we partition an interval into subintervals. It differs however in that each subinterval in the Crawford and Sobel paper has a unique meaning, while our indeterminate interval has multiple meanings.
where \( \varepsilon > 0 \) is a parameter. See Figure 1.

![Figure 1: First-Order Vagueness](image)

Think of the region \( \left[ \frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon \right] \) as the indeterminate zone, or the firm’s wiggle room. In turn, let \( \varepsilon \) designate the degree of vagueness in that the size of the indeterminate zone increases with \( \varepsilon \).

We assume throughout that the market is risk-neutral and that all participants (the firm and the outside market) understand the probabilities, payoff structure, and rules of the game (common knowledge).\(^{10}\) The market, however, is unable to observe the realization of \( \alpha \). Let \( C \in \{L, H\} \) denote the firm’s public classification of \( \alpha \). Upon observing \( x \) and \( C \), the market will value the firm at

\[
\alpha^*[C, x],
\]

where \( \alpha^*[C, x] \) denotes the market’s expected value of \( \tilde{\alpha} \) given \( C \) and \( x \). The firm chooses \( C \) to maximize this expression, within the bounds permitted by \( \varepsilon \).

\(^{10}\) Without first-order vagueness, the assumption of common knowledge results in agreement, rather than disagreement as we see here. See Aumann (1976) who models the phenomenon of “agreeing to disagree.” His assumptions result in the endogenous disappearance of disagreement, while we assume that disagreement (between good and bad quantitative news outcomes) is exogenously permitted.
Informative Equilibrium for the Univariate Case

Noting that the firm must classify the disclosure as an $L$ if $\alpha < \frac{1}{2} - \varepsilon$, and it must classify the disclosure as an $H$ if $\alpha > \frac{1}{2} - \varepsilon$, consider the following disclosure policy for the remaining region:

if $x \geq 0$ (good news), then classify the disclosure as an $H$ if $\alpha$ is in the indeterminate zone, and if $x < 0$ (bad news), then classify the disclosure as an $L$ if $\alpha$ is in the indeterminate zone.

Consider the implications of this policy: Suppose that $x \geq 0$. Then the market’s incremental valuation for the disclosure will be $\frac{1 - 2\varepsilon}{4} x$, if classified as an $L$ is, and $\frac{3 - 2\varepsilon}{4} x$, if classified as an $H$:

$$\alpha^*[L, x] = \frac{1 - 2\varepsilon}{4}, \quad \text{Threshold} \quad \frac{1}{2} - \varepsilon \quad \frac{1}{2}, \quad \alpha^*[H, x] = \frac{3 - 2\varepsilon}{4}$$

Figure 2: Classification of Positive Quantitative News
When \( x < 0 \), the market’s incremental valuation for the disclosure will be \( \frac{1 + 2\varepsilon}{4} x \), if \( L \) is disclosed and \( \frac{3 + 2\varepsilon}{4} x \), if \( H \) is disclosed:

\[
\alpha^*[L, x] = \frac{1 + 2\varepsilon}{4} \\
\text{Threshold:} \quad \frac{1}{2 + \varepsilon} \\
\alpha^*[H, x] = \frac{3 + 2\varepsilon}{4}
\]

Figure 3: Classification of Negative Quantitative News

Thus to the extent that we restrict our comparison of bad news and good quantitative news disclosures to those given more prominence (or those given less prominence), we see from this model that the market will react more strongly to bad news. That is, the market weight, \textit{contingent on classification}, is stronger by \( \varepsilon \) when the news is negative. The ex ante expected reaction, of course, remains the same.\(^{11}\) A broader range of good information is classified as prominent than bad information,\(^ {12}\) but this results in the narrower range of bad information producing an amplified stock market reaction.

\(^{11}\) Compare this to Langberg and Sivaramakrishnan (2008) who show that disclosures of bad news are more precise and occur less frequently, and Suijs (2008) who finds that an accounting system that reports bad news more precisely than good news results in a higher firm value. Both of these papers derive results under different conditions (analyst production of information and intergenerational risk sharing respectively), and do not address the issue of prominence.

\(^{12}\) Similarly, Bowen, Davis, and Matsumoto (2005) find that managers emphasize in press releases that metric (GAAP or pro forma) portraying the better firm performance.
For example, if \( \varepsilon = \frac{1}{6} \), then the classification (prominent/non-prominent) thresholds for good news and bad quantitative news are \( \frac{1}{3} \) and \( \frac{2}{3} \) respectively, meaning that if good news and bad news were, say, ex ante equally likely, then prominent good (bad) quantitative news will appear \( \frac{1}{3} \left( \frac{1}{6} \right) \) of the time and prominent bad quantitative news will appear \( \frac{1}{6} \left( \frac{1}{3} \right) \) of the time. The market weights attached to bad quantitative news are uniformly higher than the weights attached to good quantitative news, if we condition by classification. That is, the weights assigned to prominent bad and non-prominent bad news quantitative disclosures are \( \frac{5}{6} \) and \( \frac{1}{3} \) respectively, while the weights assigned to prominent good and non-prominent good news quantitative disclosures are \( \frac{2}{3} \) and \( \frac{1}{6} \) respectively.

**Proposition 1**: The univariate equilibrium depicted in Figures 2 and 3 exists for all \( \varepsilon \in \left[ 0, \frac{1}{2} \right] \).

**Uninformative Equilibrium for the Univariate Case**

The equilibrium depicted in Figures 2 and 3 is unique as long as \( \varepsilon < \frac{1}{\sqrt{8}} \). When \( \varepsilon \geq \frac{1}{\sqrt{8}} \), there exists an uninformative equilibrium depicted in Figure 4:
By assumption, firm must report L when $\alpha < \frac{1}{2} - \varepsilon$, and H when $\alpha > \frac{1}{2} + \varepsilon$. It may use its discretion otherwise and reports H when $\frac{1}{2} - \varepsilon \leq \alpha \leq \tau$, and L when $\tau < \alpha \leq \frac{1}{2} + \varepsilon$. Then the expected value of $\tilde{\alpha}$ given a report L is $\frac{1 + 4\varepsilon^2 - 2\tau^2}{4 - 4\tau}$, and given a report H is $\frac{1 + 4\varepsilon^2 - 2\tau^2}{4\tau}$.

As long as $\varepsilon \geq \sqrt{1/8} \approx .353553$, these expressions are equal (and the report uninformative) when

$$\tau = \frac{1}{2} \left(1 - \sqrt{8\varepsilon^2 - 1}\right) \text{ or } \tau = \frac{1}{2} \left(1 + \sqrt{8\varepsilon^2 - 1}\right).$$

The uninformative equilibrium is constructed in such a way that the firm uses its discretion to assign the H report to a subinterval of low $\alpha$ weights and the L report to subinterval of high $\alpha$ weights, so that when taking into consideration the mandatory assignments, the overall weight assigned by the market to the H report exactly counterbalances that assigned to the L report.\textsuperscript{13} Because $\varepsilon \geq .353553$, the possibility of an uninformative equilibrium requires a substantial

\textsuperscript{13} This construction follows an approach used by Wagenhofer (1990) who notes that his pooling equilibrium “balance[d] the beliefs of the uniformed players” with non-adjointing disclosure intervals.
amount of vagueness (the maximum amount is \( \varepsilon = \frac{1}{2} \)). It also appears to be less natural than the informative equilibrium in that all parties must carefully solve for the appropriate parameter, \( \tau \).

That said, one might speculate an uninformative equilibrium as credible in a setting where, for some reason (e.g., risk-aversion), the firm wishes to ex ante commit to an uninformative disclosure. To the extent that the market believes this commitment, it would rationally ignore the classifications \( L \) and \( H \). The informative disclosure equilibria – which also exist at higher levels of vagueness – appear more robust, however, from an intuitive perspective. That is, the construction of the uninformative equilibria appears to be rather knife edge in nature.\(^{14}\) We also question how reasonable it would be for a manager to keep her position if the stock market viewed her disclosures as irrelevant.

IV. A Model of Multivariate Quantitative Information

In this section we consider what happens when the firm bundles information. To accommodate this idea, we assume that the firm has two numbers to disclose, \( x_1 \) and \( x_2 \). Each number has an associated weight, \( \alpha_i \), such that the contribution of \( x_i \) to firm value is \( \alpha_i x_i \). We assume, as before, that the \( \tilde{\alpha}_i \) are uniformly and independently distributed on the unit interval and independent of the \( \tilde{x}_i \). The \( \tilde{x}_i \), in turn, are independent of each other with \( x_i \in \{-1, 1\} \), and the probability of \( x_i = -1 \) or \( +1 \) equals \( \frac{1}{2} \). As before, the indeterminate zone is \( \alpha_i \in [\alpha_L, \alpha_H] \), where the boundaries \( \alpha_L = \frac{1}{2} - \varepsilon \) and \( \alpha_H = \frac{1}{2} + \varepsilon \) are exogenously set (first-order vagueness).

We interpret bundling as the requirement that the firm report the sum, \( x_1 + x_2 \) (rather than the components \( x_1 \) and \( x_2 \)) when \( x_1 \) and \( x_2 \) are similarly classified (as either an \( L \) or an \( H \) ). That

\(^{14}\) In particular, apply the following robustness test: assume that there are common beliefs that the firm plays the uninformative equilibrium with probability \( 1 - \sigma \), and the informative equilibrium with probability \( \sigma \), where \( \sigma \in (0, 1) \). The uninformative equilibrium does not hold up as \( \sigma \to 0 \), while the informative equilibrium holds up as \( \sigma \to 1 \).
is, when the components \( x_1 \) and \( x_2 \) are similarly classified, they are *pooled together* and not reported separately (or sub-categorized). Conversely, the firm will separately report \( x_1 \) and \( x_2 \) when it classifies one as an \( L \) and the other and an \( H \).

In this scenario, the firm must sometimes bundle involuntarily. For example, if \( \alpha_1 < \alpha_L \) and \( \alpha_2 < \alpha_L \), then the firm must internally classify each component as an \( L \), and publicly reports the sum (bundle), \( x_1 + x_2 \) as an \( L \). Conversely there are times when the firm must involuntarily unbundle, or classify \( x_1 \) and \( x_2 \) separately, such as when \( \alpha_1 < \alpha_L \) and \( \alpha_2 > \alpha_H \). In this case, the firm must classify and publicly report \( x_1 \) as \( L \) and \( x_2 \) as \( H \). Consistency (1) follows directly from the definition of bundling.

**Consistency (1):** If one component is indeterminate and the other is determinate, then if the firm wishes to bundle, it must assign the classification of the determinate component to the bundle.

For the case where both components are indeterminate, we impose an additional consistency requirement. Within the indeterminate zone, a lack of agreement need not imply that the resulting evidence be uninformative when reported by self-interested entities. That is, while ordered data may not have an absolute measure, certain classification decisions may be objectively ruled out – thereby putting some restrictions on manipulation. For example, if one datum \( x_1 \) is more prominent than another, \( x_2 \), then we assume a firm is not allowed to classify \( x_2 \) as prominent and \( x_1 \) as not prominent because the observable order is violated.

**Consistency (2):** If both components are indeterminate, the firm is permitted to classify the bundle \( x_1 + x_2 \) as either an \( L \) or an \( H \), but if the firm decides to unbundle (and call one component an \( L \) and the other an \( H \)), the classification must be consistent with the ordering of \( \alpha_i \) and \( \alpha_j \). For example if \( \alpha_i < \alpha_j \), then the firm is required to classify \( x_i \) as an \( L \) and \( x_j \) as an \( H \) if it unbundles.

When an unbundled disclosure is made, the value of the firm equals
\[ \alpha_i^{U} [C_i, C_2, x_1, x_2] x_1 + \alpha_i^{U} [C_i, C_2, x_1, x_2] x_2, \]

where \( C_i \in \{H, L\} \) is the classification applied to \( x_i \), and \( \alpha_i^{U} \) is the weight applied to \( x_i \) by the outside market. An unbundled disclosure requires that \( C_1 \neq C_2 \), for if \( C_1 = C_2 \), we assume that the observations are pooled, and \( x_1 + x_2 \) reported. Specifically, \( \alpha_i^{U} [H, L, x_1, x_2] x_i \) denotes the weight applied to \( x_i \) when the disclosure is unbundled, \( x_i \) is classified as an \( H \) and \( x_2 \) classified as an \( L \). Similarly, \( \alpha_i^{U} [L, H, x_1, x_2] x_i \) denotes the weight applied to \( x_i \) when the disclosure is unbundled, \( x_i \) is classified as an \( L \) and \( x_2 \) classified as an \( H \). When an bundled disclosure is made, value of the firm equals

\[ \alpha^B [C_B, x_1 + x_2] (x_1 + x_2), \]

where \( C_B \in \{H, L\} \) designates the classification of the (bundled) disclosure, and \( \alpha^B [C_B, x_1 + x_2] \) represents the weight applied by the market to the disclosed sum, \( x_1 + x_2 \).

When one component is indeterminate and the other is determinate, then the classification of the determinate component determines the classification if bundling occurs. For example, suppose that \( \alpha_1 < \alpha_L \) and \( \alpha_L \leq \alpha_2 \leq \alpha_H \), then the firm must classify \( x_1 \) as an \( L \), but has discretion over how it classifies \( x_2 \). If it classifies \( x_2 \) as an \( L \), the values of \( x_1 \) and \( x_2 \) are automatically presented as a sum, or bundled by Consistency (1). This is attractive to the firm when

\[ \alpha^B [L, x_1 + x_2] (x_1 + x_2) \geq \alpha_i^{U} [L, H, x_1, x_2] x_1 + \alpha_2^{U} [L, H, x_1, x_2] x_2. \]

That is, the only alternative to bundling the disclosure as an \( L \) is to unbundle the disclosure and classify \( x_2 \) as an \( H \).

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Consider next the case where both measures are in the indeterminate zone. Given that $C_1 \neq C_2$, an unbundled disclosure implies that (given Consistency (2)), if $\alpha_i < \alpha_j$, then $x_i$ must be classified as an $L$ and $x_j$ classified as an $H$. Thus if $\alpha_i < \alpha_2$, then the firm will prefer to bundle when either

$$\alpha^B[L, x_1 + x_2]x_i + \alpha^U[L, H, x_1, x_2]x_i + \alpha_2^U[L, H, x_1, x_2]x_2,$$

or

$$\alpha^B[H, x_1 + x_2]x_i + \alpha_1^U[L, H, x_1, x_2]x_i + \alpha^U[L, H, x_1, x_2]x_2.$$ 

**Equilibrium for the Multivariate case**

Table 1 presents a Bayesian Nash equilibria (the two shaded cells will be explained below). For this to be an equilibrium, the risk-neutral outside market will take expectations over all conditions giving rise to a particular disclosure and produce weights that in turn lead to the firm to make the disclosures indicated in Table 1 such that those disclosures maximize its market value. We will refer to this equilibrium as (E1).

<table>
<thead>
<tr>
<th>$x_i$</th>
<th>$x_2$</th>
<th>$\alpha_i &lt; \alpha_L$</th>
<th>$\alpha_i &lt; \alpha_L$</th>
<th>$\alpha_i &lt; \alpha_L$</th>
<th>$\alpha_i &lt; \alpha_L$</th>
<th>$\alpha_i &lt; \alpha_L$</th>
<th>$\alpha_i &lt; \alpha_L$</th>
<th>$\alpha_i &lt; \alpha_L$</th>
<th>$\alpha_i &lt; \alpha_L$</th>
<th>$\alpha_i &lt; \alpha_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-1$</td>
<td>$-1$</td>
<td>Bundle as an $L$</td>
<td>Bundle as an $L$</td>
<td>Bundle as an $L$</td>
<td>Bundle as an $L$</td>
<td>Bundle as an $L$</td>
<td>Bundle as an $L$</td>
<td>Bundle as an $L$</td>
<td>Bundle as an $L$</td>
<td>Bundle as an $H$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$+1$</td>
<td>Bundle as an $L$</td>
<td>Bundle as an $L$</td>
<td>Bundle as an $L$</td>
<td>Bundle as an $L$</td>
<td>Unbundle</td>
<td>Unbundle</td>
<td>Unbundle</td>
<td>Unbundle</td>
<td>Bundle as an $H$</td>
</tr>
<tr>
<td>$+1$</td>
<td>$-1$</td>
<td>Bundle as an $L$</td>
<td>Bundle as an $L$</td>
<td>Bundle as an $L$</td>
<td>Bundle as an $L$</td>
<td>Unbundle</td>
<td>Unbundle</td>
<td>Unbundle</td>
<td>Unbundle</td>
<td>Bundle as an $H$</td>
</tr>
<tr>
<td>$+1$</td>
<td>$+1$</td>
<td>Bundle as an $L$</td>
<td>Bundle as an $L$</td>
<td>Bundle as an $L$</td>
<td>Bundle as an $L$</td>
<td>Unbundle</td>
<td>Unbundle</td>
<td>Unbundle</td>
<td>Unbundle</td>
<td>Bundle as an $H$</td>
</tr>
</tbody>
</table>

Table 1: A Bundling equilibrium (E1)
Note first of all, that a bundled disclosure $x_1 + x_2 = -2$ unambiguously means that $x_1 = -1$ and $x_2 = -1$, and thus the (outside) market always knows for sure when the firm has a realization corresponding to Row 1. Similarly, a bundled disclosure of $x_1 + x_2 = 2$ unambiguously means that $x_1 = +1$ and $x_2 = +1$, and thus the market always knows for sure when the firm has a realization corresponding to Row 4. Finally, a bundled disclosure of $x_1 + x_2 = 0$ means either that $x_1 = +1$ and $x_2 = -1$; or $x_1 = -1$ and $x_2 = +1$, meaning that the firm unambiguously has a realization corresponding to either Row 2 or Row 3. Thus we may think of Table 1 as representing 3 different sub-games, i) an unfavorable quantitative news sub-game (Row 1), ii) a favorable quantitative news sub-game (Row 4), and iii) a mixed quantitative news sub-game (Row 2 or Row 3).

**Proposition 2:** Table 1 (E1) constitutes an equilibrium.

Proof: See appendix

Rows 1 and 4

Consider first Rows 1 and 4 of Table 1. When the quantitative news is unambiguously negative ($x_1 = -1$ and $x_2 = -1$), the bundling is predominately $L$, and when the quantitative news is unambiguously positive, ($x_1 = 1$ and $x_2 = 1$) the bundling is predominately $H$. For Rows 1 and 4, the probability of a bundled disclosure is $(.5 + 2\varepsilon)^2$. For Row 1 (4) the probability of a $L$ ($H$) bundled disclosure is $(.5 + \varepsilon)^2$ and the probability of an $H$ ($L$) bundled disclosure is $(.5 - \varepsilon)^2$. This is summarized in Figure 5.
Consider next the market weights applied to a disclosure in Rows 1 and 4. If the disclosure is bundled, the market will simply multiply $-2$ (Row 1) or $+2$ (Row 4) by a number. On the other hand if the firm unbundles, the disclosure for each cell in Row 1 (Row 4) will be identical. That is, the market sees a $-1$ disclosure classified as an $H$ and another $-1$ disclosure classified as an $L$ for Row 1. Similarly the unbundled disclosures for Row 4 are also identical for each cell. Thus we can calculate an expected weight to be applied to a unbundled disclosure for either Row 1 or Row 4. Figure 6 summarizes the results:
It is clear from Figure 6 that if we condition by $H$-bundled, or $L$-bundled, the market reaction to a negative quantitative disclosure is higher than a positive one. A similar result is found with the unbundled disclosures. We also see that these effects are magnified with increasing levels of vagueness, thus extending the results from the univariate case to the multivariate case for bundled disclosures. Figure 6 shows that for all levels of vagueness, and all types of disclosures, an $H$-bundled negative news disclosure has the strongest reaction and a less prominent $L$-bundled positive news disclosure has the weakest reaction, and that these differences increase with vagueness. Notice that several of the curves intersect. The points of intersection can be verified to be at $\varepsilon = .25$, indicating that vagueness also affects the ordering of the weights across the bundling/unbundling distinction as well. In particular, unbundled disclosures (which always have both $L$ and $H$ classifications per disclosure) have intermediate weight for low levels of vagueness. Yet as vagueness increases, the unbundled disclosures are no longer have
unambiguously intermediate weights – the unbundled disclosures have higher weights than the \( H \) – bundled positive news disclosures, and the good news unbundled disclosures have lower weights than the \( L \) -bundled bad news disclosures. An immediate implication of this discussion is that (equilibrium) bundling need not imply opacity (e.g., camouflage), due to the fact that some bundled disclosures have higher (lower) market weights than unbundled ones.

**Proposition 3:** For Rows 1 and 4, bundling increases with vagueness, the market weights more heavily bad news than good news conditioned on disclosure type, and the strongest (weakest) market reaction is to bundled bad (good) news disclosures. For lower levels of vagueness, \( \varepsilon < .25 \), the market reaction to all types of disclosures may be ordered: the strongest reaction to bundled prominent disclosures, the weakest reaction to a less prominent bundled disclosures and intermediate reactions to unbundled disclosures.

**Rows 2 or 3**

Consider next the sub-game represented by Rows 2 or 3. Certain results from Proposition 3 will not hold when we examine this sub-game. Quantitative items \( x_1 \) and \( x_2 \) are of opposite signs if and only if \( x_1 + x_2 = 0 \). Thus when the market observes a bundling with this value, it unambiguously knows that the outcome comes from either Row 2 or Row 3 (but not which row). An interesting feature of this sub-game is that when the market observes an unbundled disclosure with opposite signs, it will for Rows 2 and 3 also recognize whether the positive \( x_i \) has been classified as an \( H \) and the negative \( x_i \) classified as an \( L \) (or vice versa). Consequently, disclosures in this setting convey more information than for Rows 1 and 4.

Consider first the case where the positive \( x_i \) has been classified as an \( H \) and the negative \( x_i \) classified as an \( L \). Label that as a “match”, and the opposite case as a “mismatch.” We use these terms to reflect the sense that the firm would wish to put a higher weight on the positive quantitative component. Consider the equilibrium (E1) which is characterized by \( L \) bundling in the shaded cells of Table 1.
In this case, it appears that voluntary bundling buries negative information (mismatches). The only unbundled mismatch that is disclosed is mandatory. By holding back other than the worst case (that negative outcome which has the highest weight $\alpha_i > \alpha_H$ and the positive outcome has the lowest weight $\alpha_i < \alpha_L$), when the worst possible case does appear as an unbundled disclosure – disclosed as a mismatched $L$ and an $H$ -- the market assigns its lowest value. Conversely, the unbundled matched disclose isolates the very best outcomes and obtains the highest market price. Overall, for mixed quantitative news, the market reaction to a bundle’s classification is muted, and the market’s reaction to unbundled items is magnified relative to the case of unambiguously good or bad quantitative news. This reveals that unbundling reveals more information than bundling when the quantitative news is mixed than when it is homogeneous – the assignment of $L$ or $H$ classifications is more informative when the quantitative items are of opposite sign.
Another interesting feature of Rows 2 and 3 is that the probability of an unbundled disclosure

\[ 2(0.5 - \varepsilon)^2 + 0.5 - \varepsilon (2\varepsilon) + \frac{1}{2} (2\varepsilon)^2 = 0.5 \]

That is, the likelihood of a unbundled disclosure is independent of the level of vagueness. This result contrasts that of Rows 1 and 4 where (see Figure 5) the maximum probability for an unbundled disclosure equals \(0.5\) and then declines to zero as vagueness increases.

**Proposition 4:** For Rows 2 and 3, the amount of bundling does not change with vagueness when the quantitative news is mixed (Rows 2 and 3) and the most extreme market reactions are to unbundled disclosures.

**Alternative Pure Strategy Equilibria For Rows 2 and 3**

Three additional pure strategy equilibria exist. To specify them we need focus on only the shaded portion of Table 1. In the first alternative, we replace the two cells where \(L\) bundling occurs, by two cells where \(H\) bundling occurs. We will call this equilibrium (E2). See appendix for a confirmation that this is a Bayesian Nash equilibrium. That said, (E1) and (E2) are symmetric in that E2 switches the payoff to \(L\) bundling with the payoff to \(H\) bundling. Two equilibria are quite similar. To see this, return to Figure 7. The new Figure (corresponding to E2) is almost identical – we switch the labels for \(H\) and \(L\) bundling. In spite of the changes, the main result from (E1) remain. That is, the benefits from matched unbundling and the penalty from mismatched unbundling remain, along with the fact that the probability of unbundling remains constant with vagueness.

Equilibrium (E3) is specified by replacing the northeast cell of the shaded area of Table 1 with an \(H\) bundling, and equilibrium (E4) is specified replacing the southwest cell of the shaded area
with an $H$ bundling. Again, because the payoffs to bundling remain in the same relative position to those for unbundling, not much changes qualitatively from what we see in Figure 7 for (E1).

For proofs of (E2) – (E4), see the appendix.

The existence of an alternative equilibria where the $L$ bundles in the shaded cells are replaced by $H$ bundles suggests that for the mixed quantitative news sub-game, the role of classification is not as important as the role of bundling. That is, the payoffs to either classification are similar (see Figure 7) and what is important is that the firm is permitted to bundle what would otherwise be seen by the market as a mismatch and bury unfavorable disclosures.

V. A Multivariate Setting Without Bundling

As a brief comparison to the previous section, we discuss a multivariate setting without bundling. Suppose that, as before, the firm has two numbers to disclose, $x_1$ and $x_2$, but that the firm makes sub-classifications. That is, the firm discloses each item $x_1$ and $x_2$, and in addition assigns labels to each one. Table 2 contains an equilibrium (offered without proof):

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_1 \rightarrow L$</th>
<th>$x_1 \rightarrow H$</th>
<th>$x_2 \rightarrow L$</th>
<th>$x_2 \rightarrow H$</th>
<th>$x_1 \rightarrow L$</th>
<th>$x_1 \rightarrow H$</th>
<th>$x_2 \rightarrow H$</th>
<th>$x_2 \rightarrow L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-1</td>
<td>$x_1 \rightarrow L$</td>
<td>$x_1 \rightarrow H$</td>
<td>$x_2 \rightarrow L$</td>
<td>$x_2 \rightarrow H$</td>
<td>$x_1 \rightarrow L$</td>
<td>$x_1 \rightarrow H$</td>
<td>$x_2 \rightarrow H$</td>
<td>$x_2 \rightarrow L$</td>
</tr>
<tr>
<td>-1</td>
<td>+1</td>
<td>$x_1 \rightarrow L$</td>
<td>$x_1 \rightarrow H$</td>
<td>$x_2 \rightarrow L$</td>
<td>$x_2 \rightarrow H$</td>
<td>$x_1 \rightarrow L$</td>
<td>$x_1 \rightarrow H$</td>
<td>$x_2 \rightarrow H$</td>
<td>$x_2 \rightarrow L$</td>
</tr>
<tr>
<td>+1</td>
<td>-1</td>
<td>$x_1 \rightarrow L$</td>
<td>$x_1 \rightarrow H$</td>
<td>$x_2 \rightarrow L$</td>
<td>$x_2 \rightarrow H$</td>
<td>$x_1 \rightarrow L$</td>
<td>$x_1 \rightarrow H$</td>
<td>$x_2 \rightarrow H$</td>
<td>$x_2 \rightarrow L$</td>
</tr>
<tr>
<td>+1</td>
<td>+1</td>
<td>$x_1 \rightarrow L$</td>
<td>$x_1 \rightarrow H$</td>
<td>$x_2 \rightarrow L$</td>
<td>$x_2 \rightarrow H$</td>
<td>$x_1 \rightarrow L$</td>
<td>$x_1 \rightarrow H$</td>
<td>$x_2 \rightarrow H$</td>
<td>$x_2 \rightarrow L$</td>
</tr>
</tbody>
</table>

Table 2: A Two-Item Equilibrium Without Bundling
Three alternative equilibria exist. The four (total) equilibria can be specified by changing the labels at a particular cell for the shaded areas to either both $L$ or both $H$. The reasoning is quite similar to that made for Rows 2 and 3 of the bundling scenario.

For the most part, the results of the univariate setting hold, especially when examining Rows 1 and 4, where the quantitative items are unambiguously good or bad.

VI. Conclusions

We examined how prominence arises in financial reporting as a way to communicate technically vague information. We found a number of results that were consistent with the empirical literature and developed new insights concerning how vagueness affects disclosure activity. A point of departure of our paper from most of the previous discussion was that we did not require an acceleration— all of our information comes out at once. Thus, in addition to studying situations where the same outcomes may be reported differently, we also have provided a perspective on why the market reacts as it does to information disclosures.

To facilitate the analyses, we employed the concept of first-order vagueness. The presence of vagueness was represented by an interval with three zones: the two outside zones are determinate and the interior zone indeterminate. By changing the boundaries of these zones, we manipulated the amount of vagueness by changing the size of the gray area. The use of first order-vagueness permitted us to examine a phenomenon that is widely recognized— gray areas. The typical use of the term, gray areas, is a negative one— it is something to be avoided, and difficult to articulate.\footnote{For example, the Record of Proceedings of the SEC Advisory Committee on Improvements to Financial Reporting Open Meeting, March 13, 2008, has panelist Barbara Roper state: “And so maybe they engage in a little gray-area accounting. Maybe they fudge things a little bit, a little bit of minor earnings management.” Later in the transcript, committee member Susan Schmidt Bies notes “I thought it may be helpful to deal with some of these gray areas around judgment, but none of you really focused a lot on it…” http://www.sec.gov/about/offices/oca/acifr/acifrproceedings031308.pdf (Accessed 5/4/2012.)} However, to the extent that accountants rely on language to provide the terms that they use to
justify and understand our classifications, gray areas are an unavoidable aspect of the reporting process.\textsuperscript{16} The use of first-order vagueness may seem paradoxical in that we used it – to make something that is regarded as imprecise more precise – but we argued that it captured the concept of wiggle (fudge) room in a simple and understandable way.

We showed that for a market of fully rational actors, price reactions to negative quantitative information exceeded the reactions to positive quantitative information when conditioned on either classification. As vagueness increased, this effect strengthened. We also examined how bundling of multivariate items might occur in response to vagueness. We first considered unambiguously good or bad quantitative news. In this case, we found that bundling increased with vagueness, and that the market weights more heavily bad news bundles than good news. In addition, we showed that for lower amounts of vagueness the market reaction to all types of disclosures may be ordered: the strongest reaction to bundled prominent disclosures, the weakest reaction to a less prominent bundled disclosures and intermediate reactions to unbundled disclosures. When the quantitative news is mixed (good and bad items), however, the likelihood of bundling was independent of level of vagueness, and the market exhibited its most extreme reactions to unbundled disclosures due to the fact that unbundled disclosures are more informative when the quantitative news is heterogeneous.

\textsuperscript{16} See Penno(2008).
References


Appendix

Proof of Proposition 1:

Suppose that firm reports according the conjectured equilibrium. Assume negative quantitative news. In that case the market will assign a weight of \( \frac{1 + 2\varepsilon}{4} \) to an \( L \) classification and \( \frac{3 + 2\varepsilon}{4} \) to an \( H \) classification. The difference between the two weights is \( \frac{1}{2} \). Suppose that \( \alpha \in \left[ \frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon \right] \). Clearly the firm has a strict incentive to report \( L \) according to the conjectured equilibrium. If a firm with discretion reports \( L \), then the weights applied by the market above are correct.

Assume next, positive quantitative news. In that case the market will assign a weight of \( \frac{1 - 2\varepsilon}{4} \) to an \( L \) classification and \( \frac{3 - 2\varepsilon}{4} \) to an \( H \) classification. The difference between the two weights is \( \frac{1}{2} \). Suppose that \( \alpha \in \left[ \frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon \right] \). Clearly the firm has a strict incentive to report \( H \) according to the conjectured equilibrium. If a firm with discretion reports \( H \), then the weights applied by the market above are correct. This completes the proof.

Proof of Proposition 2:

To verify the equilibria we create Tables 3 and 4, indicating the weights and firm value tat the market would assign were it to know (contrary to the model) that the firm’s outcome is in a particular cell.
<table>
<thead>
<tr>
<th>( x_i = -1 )</th>
<th>( x_i = +1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_1 &lt; \alpha_L )</td>
<td>( \frac{5 - \varepsilon}{2} )</td>
</tr>
<tr>
<td>( \alpha_2 &lt; \alpha_L )</td>
<td>( \frac{5 - \varepsilon}{2} )</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( \frac{1.5 + \varepsilon}{2} )</td>
<td>( \frac{5 - \varepsilon}{3} )</td>
</tr>
<tr>
<td>( \frac{5 + \varepsilon}{3} )</td>
<td>( \frac{5 + \varepsilon}{3} )</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( \frac{1.5 + \varepsilon}{2} )</td>
<td>( \frac{5 - \varepsilon}{3} )</td>
</tr>
<tr>
<td>( \frac{5 + \varepsilon}{3} )</td>
<td>( \frac{5 + \varepsilon}{3} )</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( \frac{1.5 + \varepsilon}{2} )</td>
<td>( \frac{5 - \varepsilon}{3} )</td>
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<td>( \frac{5 + \varepsilon}{3} )</td>
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<tr>
<td>( \frac{1.5 + \varepsilon}{2} )</td>
<td>( \frac{5 - \varepsilon}{3} )</td>
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<td>( \frac{5 + \varepsilon}{3} )</td>
<td>( \frac{5 + \varepsilon}{3} )</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( \frac{1.5 + \varepsilon}{2} )</td>
<td>( \frac{5 - \varepsilon}{3} )</td>
</tr>
<tr>
<td>( \frac{5 + \varepsilon}{3} )</td>
<td>( \frac{5 + \varepsilon}{3} )</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
</tbody>
</table>

Table 3: The first entry indicates the expected value of \( \tilde{\alpha}_1 \) and the second entry indicates the expected value of \( \tilde{\alpha}_2 \) given that we are in that cell.
\[ \frac{1}{2} \left( \frac{(5 - \varepsilon)^2}{(5 - \varepsilon)^2 + 4\varepsilon(5 - \varepsilon) + (2\varepsilon)^2} \right) + \frac{4\varepsilon(5 - \varepsilon)}{(5 - \varepsilon)^2 + 4\varepsilon(5 - \varepsilon) + (2\varepsilon)^2} \left( \frac{1 - 5}{2} \right) + \frac{(2\varepsilon)^2}{(5 - \varepsilon)^2 + 4\varepsilon(5 - \varepsilon) + (2\varepsilon)^2} \right) = \frac{(-\varepsilon - 5)^3}{(\varepsilon + 5)^2} \]

Instead, the market will value a firm with an \( H \) bundling \( x_i + x_2 = -2 \) at \( -3 - 2\varepsilon \).

When \( x_i = -1 \) and \( x_j = -1 \), and the firm unbundles, the market will see an assignment of an \( L \) to one \( x_i \) and an \( H \) to the other, and will value the firm at
\[
\left( \frac{2(5 - \varepsilon)^2}{2(5 - \varepsilon)^2 + 2(2\varepsilon)(5 - \varepsilon)} \right)(-1) + \left( \frac{2(2\varepsilon)(.5 - \varepsilon)}{2(5 - \varepsilon)^2 + 2(2\varepsilon)(.5 - \varepsilon)} \right)\left( \frac{-2.5 - \varepsilon}{2} \right)
\]

\[
= -0.5 - 0.5\varepsilon + 2\varepsilon^2 + 2\varepsilon^3
\]

\[
= \frac{-0.5 - 0.5\varepsilon + 2\varepsilon^2 + 2\varepsilon^3}{5 - 2\varepsilon^2}
\]

Graphing these expected payoffs we get:

![Graph showing payoffs for different disclosures](image)

Figure 8: Equilibrium Payoffs to different disclosures in Row 1.

Consider the equilibrium. Figure 8 indicates that a firm who bundles as an L has no incentive to defect from the assumed equilibrium. The single H bundler is unable to do anything else. Unbundling occurs when the firm is unable to classify the bundle as an L. Thus the only alternative unbundling is to bundle as an H. But this results in a lower value.

**Equilibrium (E1) -- Rows 2 and 3.** Quantitative items \( x_1 \) and \( x_2 \) are of opposite signs if and only if \( x_1 + x_2 = 0 \). Thus market observing a bundling knows that the outcome corresponds to a cell form either Row 2 or Row 3. Consider first the equilibrium where both bundlings in the
shaded area of Table 1 are L (equilibrium (E1)). The market observing a disclosure bundled as an L will (weighting the probability of either row at \( \frac{1}{2} \)) will value the firm at

\[
\frac{(0.5 - \varepsilon)^2}{(0.5 - \varepsilon)^2 + (0.5 - \varepsilon)(2\varepsilon) + \frac{1}{2}(2\varepsilon)^2} 0 + \frac{(0.5 - \varepsilon)(2\varepsilon)}{(0.5 - \varepsilon)^2 + (0.5 - \varepsilon)(2\varepsilon) + \frac{1}{2}(2\varepsilon)^2} \left( \frac{-0.5 - \varepsilon}{2} \right)
\]

\[
+ \frac{\frac{1}{2}(2\varepsilon)^3}{(0.5 - \varepsilon)^2 + (0.5 - \varepsilon)(2\varepsilon) + \frac{1}{2}(2\varepsilon)^2} \left( -\frac{2\varepsilon}{3} \right) = \frac{-0.5\varepsilon - \frac{2}{3}\varepsilon^3}{0.5 + 2\varepsilon^2}
\]

The market observing a disclosure bundled as an H will value the firm at

\[
+ \frac{(0.5 - \varepsilon)(2\varepsilon)}{(0.5 - \varepsilon)(2\varepsilon) + (0.5 - \varepsilon)^2} \left( \frac{-0.5 - \varepsilon}{2} \right) + \frac{(0.5 - \varepsilon)^2}{(0.5 - \varepsilon)(2\varepsilon) + (0.5 - \varepsilon)^2} 0 = \frac{-0.5\varepsilon + 2\varepsilon^3}{0.5 - 2\varepsilon^2}
\]

Finally, the market observing an unbundled disclosure with opposite signs will for Rows 2 and 3 will also recognize if the positive \( x_i \) has been classified as an \( H \) and the negative \( x_i \) classified as an \( L \) (or vice versa). Consider first the case where the positive \( x_i \) has been classified as an \( H \) and the negative \( x_i \) classified as an \( L \). Label that as a match. The market observing a match will price the firm at
\[
\frac{(5-\varepsilon)^2}{(5-\varepsilon)^2 + 2(5-\varepsilon)(2\varepsilon) + \frac{1}{2}(2\varepsilon)^2} \left(1 + \frac{2\varepsilon}{2}\right) + \frac{2(5-\varepsilon)(2\varepsilon)}{(5-\varepsilon)^2 + 2(5-\varepsilon)(2\varepsilon) + \frac{1}{2}(2\varepsilon)^2} \left(\frac{5+\varepsilon}{2}\right)
\]

\[+ \frac{1}{2}(2\varepsilon)^2 \left(\frac{2\varepsilon}{3}\right)\]

\[= \frac{.375 + .25\varepsilon + 1.5\varepsilon^2 - \frac{5}{3}\varepsilon^3}{.25 + \varepsilon - \varepsilon^2}.\]

Consider next the case where the negative \( x_i \) has been classified as an \( H \) and the positive \( x_i \) classified as an \( L \). Label that as a mismatch. The market observing a mismatch will price the firm at \( \frac{-1-2\varepsilon}{2} \). Figure 7 (in text) summarizes the above.

Suppose that a firm assigned by the equilibrium to bundle decides to unbundle. Consistency (1) implies that the unbundled disclosure must classify one component as an \( H \) and a positive the other as an \( L \). This is not permitted if the realization in is in first or last column, thereby eliminating the defection there. Permissible disclosures from a defection must follow Consistency (2), which (by inspecting Rows 2 and 3 of Table 1, means for all of the remaining \( L \) bundles and single \( H \) bundle, that the negative \( x_i \) must be classified as an \( H \) and the positive \( x_i \) classified as an \( L \). Figure 7 shows that such a defection will lower market value, and hence is ruled out. Note that Consistency (1) prevents the \( H \) bundler from posing as an \( L \) bundler, and Figure 7 shows us that an \( L \) bundler from would never be better off by posing an \( H \) bundler (one curve is strictly above the other). Finally we ask whether an unbundled discloser would prefer to bundle. Figure 7 show this will only occur for a mismatch. For Row 2 (Row 3) an inspection of Table 1 reveals that the only time this will occur is when \( \alpha_1 > \alpha_H \) and \( \alpha_2 \leq \alpha_L \) (\( \alpha_2 > \alpha_H \) and \( \alpha_1 \leq \alpha_L \)). In this case the firm is required to unbundle. This demonstrates the equilibrium for (E1) – Rows 2 and 3.
**Equilibrium (E1) -- Row 4:** The equilibrium analysis of Row 4 is symmetrical to the analysis for Row 1.

**Equilibrium (E2) -- Rows 2 and 3.** Consider next the equilibrium where both bundlings in the shaded area of Table 1 are \( H \). Note that the payoffs to bundling as an \( L \) and bundling as an \( H \) have reversed. Consequently bundling as an \( H \) becomes dominant in the shaded area.

**Equilibria (E3) and E(4) -- Rows 2 and 3.** Consider next the equilibrium where one of the bundlings in the shaded area of Table 1 is an \( H \). Note that the payoffs to bundling as an \( L \) and bundling as an \( H \) have reversed. Consequently bundling an \( H \) bundling becomes dominant for that row and an \( L \) bundling remains dominant for the other row.

**Calculation of weights (Rows 1 and 4) for Figure 5:** The Row 1 weight applied to \( x_1 + x_2 \) for an \( L \) bundle is

\[
\left( \frac{(0.5 - \varepsilon)^2}{(0.5 - \varepsilon)^2 + 4\varepsilon(0.5 - \varepsilon) + (2\varepsilon)^2} \right) \left( \frac{1 - 2\varepsilon}{4} \right) + \left( \frac{4\varepsilon(0.5 - \varepsilon)}{(0.5 - \varepsilon)^2 + 4\varepsilon(0.5 - \varepsilon) + (2\varepsilon)^2} \right) \left( \frac{1.5 - \varepsilon}{4} \right)
\]

\[+ \left( \frac{(2\varepsilon)^2}{(0.5 - \varepsilon)^2 + 4\varepsilon(0.5 - \varepsilon) + (2\varepsilon)^2} \right) \left( \frac{1}{2} \right) = \frac{(0.39685 + 0.793701\varepsilon)^3}{2(\varepsilon + 0.5)^2} \]

The Row 1 weight applied to \( x_1 + x_2 \) for an \( H \) bundle is simply \( \frac{2\varepsilon + 3}{4} \)

The Row 1 weight applied to an unbundled disclosure is
\[
\frac{2(5 - \varepsilon)^2}{2(5 - \varepsilon)^2 + 4\varepsilon(5 - \varepsilon)} \left( \frac{1}{2} \right) + \frac{4\varepsilon(5 - \varepsilon)}{4} \left( \frac{2.5 + \varepsilon}{4} \right) = \frac{.25 + .25\varepsilon - \varepsilon^2 - \varepsilon^3}{.5 - 2\varepsilon^2}
\]

The Row 4 weight applied to \( x_1 + x_2 \) for an \( H \) bundle is

\[
\frac{(5 - \varepsilon)^2}{(5 - \varepsilon)^2 + 4\varepsilon(5 - \varepsilon) + (2\varepsilon)^2} \left( \frac{3 + 2\varepsilon}{4} \right) + \frac{4\varepsilon(5 - \varepsilon)}{(5 - \varepsilon)^2 + 4\varepsilon(5 - \varepsilon) + (2\varepsilon)^2} \left( \frac{2.5 + \varepsilon}{4} \right) + \frac{(2\varepsilon)^2}{(5 - \varepsilon)^2 + 4\varepsilon(5 - \varepsilon) + (2\varepsilon)^2} \left( \frac{1}{2} \right) = \frac{.1875 + .625\varepsilon + .25\varepsilon^2 - .5\varepsilon^3}{(5 + \varepsilon)^2}
\]

The Row 4 weight applied to \( x_1 + x_2 \) for an \( L \) bundle is simply \( \frac{1 - 2\varepsilon}{4} \)

The Row 4 weight applied to an unbundled disclosure is

\[
\frac{2(5 - \varepsilon)^2}{2(5 - \varepsilon)^2 + 4\varepsilon(5 - \varepsilon)} \left( \frac{1}{2} \right) + \frac{4\varepsilon(5 - \varepsilon)}{4} \left( \frac{1.5 - \varepsilon}{4} \right) = \frac{.25 - .25\varepsilon - \varepsilon^2 + \varepsilon^3}{.5 - 2\varepsilon^2}
\]