Rare Disasters, Financial Development, and Sovereign Debt*

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Abstract

We study the implications of the interaction between rare disasters and financial development for sovereign debt markets. In our model, countries vary in their financial development, by which we mean the extent to which shocks can be hedged in international capital markets. The model predicts that low levels of financial development generate key empirical features of sovereign debt in emerging economies: high credit spreads associated with lower debt-to-GDP ratios than those of developed countries.

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1 Introduction

Rare-disaster models have proved useful in advancing our understanding of many asset pricing phenomena.[1] In this paper, we study the implications of the interaction between rare disasters and financial development for sovereign debt markets.

One intriguing fact about these markets is that emerging economies pay high credit spreads on their sovereign debt, despite generally having much lower debt-output ratios than developed countries. Reinhart, Rogoff and Savastano (2003) call this phenomenon "debt intolerance."

The debt intolerance of emerging markets is at odds with the predictions of the classic Eaton and Gersovitz (1981) model of sovereign debt.[2] A key cost of defaulting in this model is the loss of access to capital markets. Since real output growth is generally more volatile in emerging markets than in developed countries, the loss of market access is more costly for emerging markets. So, other things equal, emerging markets should be less likely to default, pay lower credit spreads on their sovereign debt, and have higher debt capacity.

The idea that the high output volatility characteristic of emerging markets makes their default cost high and their probability of default low contradicts another finding stressed by Reinhart et al (2003): emerging markets tend to be serial defaulters.

In this paper, we propose a model of sovereign debt where countries vary in their level of financial development. By financial development, we mean the extent to which countries can hedge shocks to their economy in international capital markets.[3] We show that low levels of financial development generate debt intolerance.

As in Tourre (2016) and Bornstein (2017), we write our sovereign-debt model in continuous time. A significant technical advantage of this approach is that the model can be solved in closed form up to an ordinary differential equation (ODE) with intuitive boundary conditions.

[1] Examples include the equity premium (Rietz (1988), Barro (2006), Barro and Jin (2011), and Gabaix (2012)), the predictability of excess returns (Wachter (2013)), the corporate bond spread (Bhamra and Strebulaev (2011)), and the returns to the carry trade (Burnside, Eichenbaum and Rebelo (2011) and Farhi and Gabaix (2015)).

[2] See Aguiar and Amador (2014) and Aguiar, Chatterjee, Cole, and Stangebye (2016) for recent surveys of the sovereign debt literature.

[3] Another aspect of financial development might reflect the country's access to commitment mechanisms such as posting collateral or depositing money in escrow accounts that can be seized by creditors. We do not consider these mechanisms because sovereign debt is in practice generally unsecured. Mendoza, Quadrini, and Rios-Rull (2009) also emphasize the importance of financial development, which they interpret as a country's ability to enforce domestic financial contracts to hedge idiosyncratic risks.

The representative agent has the continuous-time version of Epstein and Zin (1989) preferences proposed by Duffie and Epstein (1992). This formulation allows us to separate the impact of risk aversion and intertemporal substitution on credit spreads and debt capacity. This separation plays an important role in our quantitative analysis.

Output follows the jump-diffusion process considered by Barro and Jin (2011), in which the size distribution of jumps is governed by a power law. The presence of rare disasters is key to generating default in our model.

As in Aguiar and Gopinath (2006) and Arellano (2008), we assume that upon default the country suffers a decline in output and loses access to international capital markets. It then regains access to these markets with constant probability.

Outside of the default state, the country can invest in a risk-free international bond, hedge diffusion shocks, partially hedge rare-disaster risk, and issue non-contingent debt that can be defaulted upon.

Our model includes two frictions that make markets incomplete: limited commitment and limited spanning.[4] To isolate the impact of limited commitment, suppose there is full spanning so that all shocks can be hedged. As in Kehoe and Levine (1993) and Kocherlakota (1996), the country's debt capacity is reduced to a level such that in equilibrium the country weakly prefers repaying its outstanding debt over defaulting on it.[5] Since hedging is more cost effective than defaulting in terms of managing the country's risk, the country never defaults and the credit spread on sovereign debt is zero.

The second form of market incompleteness is limited spanning. In practice, manifestations of limited spanning include a country's inability to issue debt with long maturities or debt denominated in local currency. To simplify the analysis, we model limited spanning as the country's limited access to financial securities that can be used to hedge rare disasters. In the tradition of Eaton and Gersovitz (1981), we assume that these limits are exogenous. This exogeneity assumption is consistent with the key finding of the literature on the original-sin hypothesis: the degree of market incompleteness is more closely related to the size of the economy than to the soundness of fiscal and monetary policy or other

[4]Bai and Zhang (2010) combine these two forms of market incompleteness to explain the Feldstein-Horioka puzzle. They consider a limited-enforcement model in the spirit of Kehoe and Levine (1993), so in their model default does not occur in equilibrium.

[5]For other early important contributions on the implications of limited commitment, see Alvarez and Jermann (2000, 2001), Kehoe and Perri (2002), Albuquerque and Hopenhayn (2004), and Cooley, Marimon, and Quadrini (2004).

fundamentals (Hausmann and Panizza (2003) and Bordo, Meissner, and Redish (2004)).

Our model includes two special cases. The first is exogenously incomplete markets sovereign debt models, as in Eaton and Gersovitz (1981). The second is endogenously incomplete markets due to limited commitment, as in Kehoe and Levine (1993) and Kocherlakota (1996). In the exogenously incomplete markets case, the country has limited access to financial markets (financial development is low) and hence relies on default to manage rare disasters. In the limited-commitment endogenously incomplete markets case, the country has access to the full set of Arrow securities (financial development is high) and hence avoids using default to manage rare disasters.[6]

One key result is that the more limited is the spanning of assets at a country's disposal, the more severe is its debt intolerance. When spanning is limited, it is not optimal of fully hedge risks that can be hedged.[7] The country uses the available hedging instruments to increase its debt capacity by ensuring that default is not triggered by shocks that can be hedged. So, countries with more limited spanning hedge less and endure more volatility in consumption. These countries are also more likely to default, so lenders charge them a higher credit spread to cover the expected default losses. Reducing the span of assets available to a country reduces debt capacity, increases credit spreads, and limits the ability to smooth consumption. In other words, it produces debt intolerance.

Our model suggests that Shiller's (1993) proposed creation of a market for perpetual claims on countries' Gross Domestic Produce (GDP) could significantly improve welfare in emerging markets. By increasing a country's ability to hedge its risks, GDP-linked bonds would lower credit spreads, increase debt capacity and reduce consumption volatility.

The paper is organized as follows. Section 2 presents our model. Section 3 discusses the model's solution. Section 4 characterizes the first-best solution that obtains under full spanning and full commitment. Section 5 calibrates our model using Argentinean data and explores its quantitative properties. Section 7 discusses the sensitivity of the model's implications to parameter values. Section 8 contains results for a version of the model with expected utility. Section 6 concludes.

[6]In Corporate Finance, Bolton, Wang, and Yang (2018) analyze optimal corporate liquidity and risk management, executive compensation, and debt capacity in a setting where the entrepreneur has inalienable human capital and limited commitment in continuous time. Default does not occur in their model as it features full spanning with limited commitment by the entrepreneur.

[7]This result is related to work by Caballero and Krishnamurthy (2003). These authors use a different class of models that builds on the work of Holmstrom and Tirole (1998) to show that limited financial development exacerbates underinsurance in emerging markets.

2 Model Setup

We consider a continuous-time model where the country's infinitely-lived representative agent receives a perpetual, exogenous, stochastic stream of output. As we show in Section 3, default occurs in equilibrium. Upon default, the country endures distress costs that take the form of a fall in output. Below, we introduce the law of motion that governs output in the absence of default.

2.1 Output Process

We model output, Y_t, as a jump-diffusion process. This process is consistent with the evidence presented in Aguiar and Gopinath (2007) which suggests that permanent shocks are the primary source of fluctuations in emerging markets. Both diffusion and jump shocks are important in generating our model's main predictions.

The law of motion for output, Y_t, is given by:

$$\frac{dY_t}{Y_{t-}} = \mu dt + \sigma d\mathcal{B}_t - (1-Z)d\mathcal{J}_t \,, \quad Y_0 > 0 \,, \tag{1}$$

where μ is the drift parameter, σ is the diffusion-volatility parameter, $\mathcal{B}$ is a standard Brownian motion process, and $\mathcal{J}$ is a pure jump process. If a jump does not occur at t, $d\mathcal{J}_t = 0$. If a jump occurs at t, $d\mathcal{J}_t = 1$ and output falls from Y_{t-} to $Y_t = ZY_{t-}$. We call $Z \in (0,1)$, the fraction of output retained by the country after the jump, the recovery fraction.

We assume that Z follows a well-behaved cumulative distribution function, $G(Z)$ and that jumps are governed by a Poisson process with a constant arrival rate, λ. There is no limit to the number of jumps that can occur over a fixed time interval and the occurrence of a jump does not affect the likelihood of future jumps.

Since the expected percentage output loss upon the arrival of a jump is $(1 - \mathbb{E}(Z))$, the expected growth rate of output in levels is given by:

$$g = \mu - \lambda(1 - \mathbb{E}(Z)) \ . \tag{2}$$

Here, the term $\lambda(1 - \mathbb{E}(Z))$ represents the reduction in the expected growth rate associated with jumps.

We can write the dynamics for logarithmic output, $\ln Y_t$, in discrete time as follows:

$$\ln Y_{t+\Delta} - \ln Y_t = \left(\mu - \frac{\sigma^2}{2}\right)\Delta + \sigma\sqrt{\Delta}\,\epsilon_{t+\Delta} - (1-Z)\nu_{t+\Delta} \,, \tag{3}$$

where the time-t conditional distribution of $\epsilon_{t+\Delta}$ is a standard normal random variable and $\nu_{t+\Delta} = 1$ with probability $\lambda\Delta$ and zero with probability $(1-\lambda\Delta)$. Equation (3) implies that the expected change of $\ln Y$ over a time increment Δ is $(\mu - \sigma^2/2)\,\Delta - \lambda(1 - \mathbb{E}(Z))\Delta$. The term $\sigma^2/2$ is the Jensen-inequality correction associated with the diffusion shock.

2.2 Preferences

We assume that the lifetime utility of the representative agent, V_t, has the recursive form proposed by Kreps and Porteous (1978), Epstein and Zin (1989), and Weil (1990). We use the continuous-time version of these preferences developed by Duffie and Epstein (1992a):

$$V_t = \mathbb{E}_t \left[\int_t^\infty f(C_u, V_u) du \right], \tag{4}$$

where $f(C, V)$ is the normalized aggregator for consumption C and utility V. This aggregator is given by:

$$f(C, V) = \frac{\rho}{1 - \psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma)V)^{\chi}}{((1-\gamma)V)^{\chi - 1}}. \tag{5}$$

Here, ρ is the subjective discount rate and

$$\chi = \frac{1 - \psi^{-1}}{1 - \gamma}. \tag{6}$$

This recursive, non-expected utility formulation allows us to separate the coefficient of relative risk aversion (γ) from the elasticity of intertemporal substitution (ψ). This separation plays an important role in our quantitative analysis. The time-additive separable CRRA utility is a special case of recursive utility where the coefficient of relative risk aversion, γ, equals the inverse of the elasticity of intertemporal substitution (EIS), $\gamma = \psi^{-1}$, implying $\chi = 1$. In this case, $f(C, V) = U(C) - \rho V$, which is additively separable in C and V, with $U(C) = \rho C^{1-\gamma}/(1-\gamma)$.

2.3 Financial Assets and Market Structure

It is well known that market incompleteness is critical to generating default in sovereign-debt models. If the country could trade in a complete set of contingent assets and perfectly commit to all promised contingent repayment paths, default would not occur in equilibrium. As discussed in the introduction, our model includes two sources of market incompleteness. The first is limited commitment: the country cannot commit to repaying its debt. The second is limited spanning: markets for certain shocks are incomplete. To capture the notion that some shocks are harder to hedge than others, we assume that large jump shocks might not be insurable.

We denote the country's financial wealth by W_t. Under normal circumstances, the country has four investment and financing opportunities: (1) it can insure its diffusion risk through hedging contracts; (2) it can buy insurance against certain jumps; (3) it can borrow in the sovereign debt market at an interest rate that is the sum of the risk-free rate, r, and an endogenous credit spread, π_t; and (4) it can save at the risk-free rate, r. Upon default on its sovereign debt, the country enters autarky and loses access to all four investment and financing opportunities. It regains this access with a constant probability.

Diffusion Risk Hedging Contracts. We assume that diffusive shocks are idiosyncratic and that markets for contracts that hedge these shocks are perfectly competitive. An investor who holds one unit of the hedging contract at time t receives no upfront payment, since there is no risk premium for bearing idiosyncratic risk, and receives a gain or loss equal to $\sigma d\mathcal{B}_t = \sigma \left(\mathcal{B}_{t+dt} - \mathcal{B}_t\right)$ at time $t + dt$. We normalize the volatility of this hedging contract to be equal to the output volatility parameter, σ. This hedging contract is analogous to a futures contract. We denote the country's holdings of diffusion risk contracts at time $t-$ by Θ_{t-}.

Jump Insurance Contracts and Premia. We assume that jump shocks are idiosyncratic and that markets for contracts that hedge these shocks are perfectly competitive. Consider an insurance contract initiated at time $t-$ that covers the following jump event: the *first* stochastic arrival of a downward jump in output with a recovery fraction in the interval $(Z, Z + dZ)$ at jump time $\tau^{\mathcal{J}} > t-$ for $Z \geq Z^*$. Here, Z^* is a parameter that describes the level of financial development. The higher the value of Z^*, the less developed are financial markets and the fewer jump insurance opportunities the country has.

Because jumps are recurrent, insurance contracts for all intervals $(Z, Z + dZ)$ for $Z \geq Z^*$ are available for trading. The buyer of a unit of this insurance contract makes continuous insurance premium payments. This insurance premium is equal to $\lambda dG(Z)$, the product of the jump intensity, λ, and the probability $dG(Z)$ that the recovery fraction, Z, falls in the interval $(Z, Z + dZ)$ for $Z \geq Z^*$. Once the jump event occurs at time $\tau^{\mathcal{J}}$, the buyer stops making payments and receives a one-time *unit* lump-sum payoff. Conceptually, this insurance contract is analogous to one-step-ahead Arrow securities in discrete-time models. In practice, this insurance contract is similar to a credit default swap.[8]

We denote the country's holdings of jump-risk insurance contracts at time $t-$ contingent on a recovery fraction $Z \geq Z^*$ by $X_{t-}(Z)$. The country pays an insurance premium to hedge jump risk at a rate $X_{t-}(Z)\lambda\, dG(Z)$ before the *first* jump with $Z \geq Z^*$ arrives at time $\tau^{\mathcal{J}}$. At this time, the country receives a lump-sum payment $X_{t-}(Z)$ if the recovery fraction is in the interval $(Z, Z + dZ)$. Since the country can purchase insurance for all possible values of $Z \geq Z^*$, the total jump insurance premium per period is given by:

$$\Phi_{t-} = \lambda \int_{Z^*}^{1} X_{t-}(Z) dG(Z) \equiv \lambda\, \mathbb{E}\left[X_{t-}(Z)\, \mathcal{I}_{Z \geq Z^*}\right] , \tag{7}$$

where the expectation, $\mathbb{E}[\,\cdot\,]$, is calculated with respect to the cumulative distribution function, $G(Z)$. The indicator function $\mathcal{I}_{Z\geq Z^*}$ equals one if $Z \geq Z^*$ and zero otherwise. This indicator function imposes the restriction that jump insurance is available only for $Z > Z^*$.

Sovereign Debt. We use D_t to denote the level of sovereign debt: $D_t = -W_t > 0$ when $W_t < 0$. As in discrete-time settings, sovereign debt is borrower-specific, non-contingent, uncollateralized, and short term.[9] Sovereign debt is continuously repaid and reissued. The borrowing process continues until the country defaults and resumes once the borrower reenters the sovereign-debt market. Sovereign debt is held and priced in competitive markets by well-diversified foreign investors. The maximal amount of debt that the country can issue is stochastic and endogenously determined in equilibrium by the creditors' break-even condition and the borrower's optimal default decisions.

Sovereign Default. The country has the option to default at any time on its sovereign debt. As emphasized by Zame (1993) and Dubey, Geanakoplos and Shubik (2005), the possibility of default provides a partial hedge against risks that cannot be insured because of limited financial spanning.

Let $\tau^{\mathcal{D}}$ denote the endogenous time of default. There are two costs of defaulting. The first is an output loss that proxies for the disruptions of economic activity associated with default. As in Aguiar and Gopinath (2006), we assume that output drops permanently from

[8]Pindyck and Wang (2013) discuss a similar insurance contract in a general equilibrium setting with economic catastrophes.

[9]Auclert and Rognlie (2016) show that sovereign debt models with short-term debt have a unique Markov perfect equilibrium.

$Y_{\tau^{\mathcal{D}}}$ to $\alpha Y_{\tau^{\mathcal{D}}}$ upon default, where $\alpha \in (0,1)$ is a constant. The second default cost is the loss of access to financial markets. Upon default, the country enters a state of autarky in which it cannot issue sovereign debt, save, or hedge its exposure to both diffusive and jump shocks.

While in autarky, the country re-gains its normal access to financial markets with probability ξ per unit of time. Upon exiting autarky at time $\tau^{\mathcal{E}}$, the country starts afresh with no debt ($W_{\tau^{\mathcal{E}}} = 0$) and its output is continuous, so $Y_{\tau^{\mathcal{E}}} = Y_{\tau^{\mathcal{E}}-}$.

Suppose that the country wants to borrow an amount D_t at time t. Creditors quote a credit spread π_t for the period $(t, t+dt)$. The likelihood of default is positive over a time increment dt because uninsured jumps can occur and the country has the option to default. When it does not default, the country pays principal and interest to creditors, $D_t(1 + (r + \pi_t)dt)$, and issues an amount D_{t+dt} of new debt at time $t + dt$. When an uninsured jump occurs, the borrower may find it optimal to default on its debt and pay nothing to creditors.[10] In Section 3, we solve for the equilibrium credit spread, π_t, and the debtor's optimal default strategies.

Optimality. The country chooses its consumption, diffusion and jump risk hedging demands, sovereign debt issue, and default timing to maximize the utility of the representative agent, defined by equations (4)-(5), given the process specified in equation (1), and equilibrium pricing of sovereign debt and insurance contracts for diffusion and jump shocks.

3 Model Solution

In this section, we solve our model using dynamic programming. We call the state in which the country has access to financial markets the normal regime. We denote by $V(W_t, Y_t)$ the representative agent's value function for the normal regime and by $\widehat{V}(Y_t)$ the value function for the autarky regime. The autarky value function depends only on contemporaneous output because financial wealth is zero in autarky.

The continuous-time formulation allows us to solve the model in closed form up to an ordinary differential equation with economically intuitive boundary conditions. We proceed in three steps: (1) analyze the decision problem in the normal regime; (2) characterize the boundary conditions for endogenous default; and (3) convert the two-dimensional optimiza-

[10] To simplify, we consider only the possibility of complete default. See Yue (2010) and Asonuma, Niepelt, and Ranciere (2017) for models with partial default.

tion problem into a one-dimensional problem using the fact that the value functions and decision rules are homogeneous in wealth and output.

3.1 The Normal Regime

We first caracterize the law of motion for financial wealth, which is the country's flow budget constraint. Then, we write the Hamilton-Jacobi-Bellman (HJB) equation and derive first-order conditions.

Law of Motion for Financial Wealth. Financial wealth, W_t, evolves according to:

$$dW_t = [(r+\pi_{t-})W_{t-} + Y_{t-} - C_{t-} - \Phi_{t-}]dt + \sigma\Theta_{t-}d\mathcal{B}_t + X_{t-}(Z)\mathcal{I}_{Z\geq Z^*}\, d\mathcal{J}_t - W_{t-}\,\mathcal{I}_t^{\mathcal{D}}\, d\mathcal{J}_t\,. \quad (8)$$

The first term on the right side of equation (8) is interest income/expenses, $(r+\pi_{t-})W_{t-}\,dt$ plus output, $Y_{t-}\,dt$, minus consumption, $C_{t-}\,dt$, and minus the jump-insurance premium, $\Phi_{t-}dt$. When $W_{t-} > 0$, the country has no debt outstanding and $\pi_{t-} = 0$. When $W_{t-} < 0$, the country pays interest at a rate $(r + \pi_{t-})$, where π_{t-} is the equilibrium credit spread demanded by creditors.

The second term on the right side of equation (8), $\sigma\Theta_{t-}d\mathcal{B}_t$, is the realized gain or loss from diffusion risk hedging contracts. Since diffusion shocks are idiosyncratic with zero mean, the country incurs no up-front payment. The third term, is the lump-sum payment, $X_{t-}(Z)$, from the jump insurance contract when a jump arrives ($d\mathcal{J}_t = 1$) and the realized Z is hedgeable, i.e., $Z \in (Z^*, 1)$.

The last term in equation (8) ensures that financial wealth drops from W_{t-} immediately to zero upon default ($\mathcal{I}_t^{\mathcal{D}} = 1$). By local non-satiation, it is never optimal for the country to purchase jump insurance for values of Z that trigger default, so $X_{t-}(Z) = 0$ whenever $\mathcal{I}_t^{\mathcal{D}}(Z) = 1$.

Hamilton-Jacobi-Bellman Equation. The value function $V(W,Y)$ in the normal regime satisfies the following HJB equation:[11]

$$
\begin{aligned}
0 = \max_{C,\Theta,X} \; & f(C,V(W,Y)) + [(r+\pi)W + Y - C - \Phi]\, V_W(W,Y) \\
& + \frac{\Theta^2\sigma^2}{2} V_{WW}(W,Y) + \mu Y V_Y(W,Y) + \frac{\sigma^2 Y^2}{2} V_{YY}(W,Y) + \Theta\sigma^2 Y V_{WY}(W,Y) \\
& + \lambda \mathbb{E}\left[\left(V(W+X,ZY)\mathcal{I}_{Z\geq Z^*} + V(W,ZY)\mathcal{I}_{\underline{Z}<Z\leq Z^*} + \widehat{V}(\alpha ZY)\mathcal{I}_{Z\leq \underline{Z}}\right) - V(W,Y)\right],
\end{aligned} \tag{9}
$$

where the expectation $\mathbb{E}[\,\cdot\,]$ is evaluated with respect to the cumulative distribution function, $G(Z)$. The HJB equation states that at the optimum, the sum of the country's normalized aggregator, $f(C,V)$, and the expected change in the value function V (the sum of all the other terms on the right side of equation (9)) must equal zero.

The expected change of the value function, $V(W,Y)$, is driven by changes in W and Y, the country's diffusion and jump risk hedging demands, and the country's default decisions. The second and third terms of equation (9), describe the drift and volatility effects of wealth W on the expected change of the value function $V(W,Y)$. The fourth and fifth terms reflect the drift and volatility effects of output Y on the expected change of $V(W,Y)$. The sixth term, $\Theta\sigma^2 Y V_{WY}(W,Y)$, captures the effect of the country's intertemporal diffusive shock hedging demand on the drift of $V(W,Y)$.

The last term, which appears in the third line of equation (9), represents the expected change in the value function that occurs as a result of a jump arrival and the concomitant default decision. When a jump arrives at time t ($d\mathcal{J}_t = 1$), the country decides whether to default on its debt after observing the realized recovery fraction, Z. The default decision is characterized by an endogenous, stochastic threshold rule, $\underline{Z}$. For $Z < \underline{Z}$, the country defaults. If $Z \geq Z^*$, the country receives a jump-insurance payment, $X_{t-}(Z)$, and does not default, so its value function at t is $V(W_{t-} + X_{t-}(Z), ZY_{t-})$. If $\underline{Z} < Z \leq Z^*$, the country receives no jump-insurance payment (the jump was not insurable) but does not default, so its value function at t is $V(W_{t-}, ZY_{t-})$. Finally, if $Z \leq \underline{Z}$, the country defaults and pays a distress cost which takes the form of a permanent output loss equal to a fraction $(1-\alpha)$ of the post-jump output $Y_t = ZY_{t-}$. Upon default, the country enters autarky, so its value function at t is $\widehat{V}(\alpha Z Y_{t-})$. We characterize $\widehat{V}(\,\cdot\,)$ in Section 3.2 and $\underline{Z}$ in Section 3.4.

[11] Duffie and Epstein (1992b) generalize the standard HJB equation for the expected-utility case to allow for non-expected recursive utility such as the Epstein-Weil-Zin utility used here.

First-Order Conditions. As in Duffie and Epstein (1992a, 1992b), the first-order condition (FOC) for C is:

$$f_C(C,V) = V_W(W,Y)\,. \tag{10}$$

This condition equates the marginal benefit of consumption, $f_C(C,V)$, to the marginal utility of savings, $V_W(W,Y)$. While under expect utility $f_C(C,V) = U'(C)$, which does not depend on V, here $f_C(C,V)$ is non-separable in C and V.

The FOC for the diffusion-risk hedging demand is:

$$\Theta = -\frac{Y V_{WY}(W,Y)}{V_{WW}(W,Y)}\,. \tag{11}$$

We verify that $V(W,Y)$ is concave in W ($V_{WW} < 0$) so the FOC (11) is sufficient to characterize optimality for Θ.

Equation (11) is similar to the intertemporal hedging demand in Merton (1969) for expected utility and in Duffie and Epstein (1992b) for recursive preferences. Since the country is endowed with a long position in domestic output, we expect its hedging demand to be negative. An interesting property we explore in our quantitative analysis is that it is not optimal to fully hedge diffusion shocks, even though full hedging is feasible. This property reflects the presence of market incompleteness.

The HJB equation (9) implies that the optimal jump risk hedging demand, $X(Z;W,Y)$, solves the following problem:

$$\max_{X} \quad \lambda\,\mathbb{E}\left[\left(V(W+X,ZY) - XV_W(W,Y)\right)\mathcal{I}_{Z\geq Z^*}\right]\,. \tag{12}$$

This problem boils down to maximize $(V(W+X,ZY) - XV_W(W,Y))$ by choosing $X(Z;W,Y)$ for each value of Z that can be insured ($Z \geq Z^*$). The FOC for $X(Z;W,Y)$ is:

$$V_W(W + X(Z;W,Y), ZY) = V_W(W,Y)\,. \tag{13}$$

The intuition for this condition is that it is optimal to choose X to equate the pre- and post-jump marginal utility of wealth. Since output falls upon a jump arrival, without jump insurance, $V_W(W,Y) < V_W(W,ZY)$. The country chooses $X(Z;W,Y) > 0$ to equate the pre- and post-jump marginal utility of wealth.

Value Function. We conjecture and verify that the value function, $V(W,Y)$, is given by:

$$V(W,Y) = \frac{(bP(W,Y))^{1-\gamma}}{1-\gamma}\,, \tag{14}$$

where b is:

$$b = \rho \left[\frac{r + \psi(\rho - r)}{\rho} \right]^{\frac{1}{1-\psi}} . \tag{15}$$

We can interpret $P(W, Y)$ as the certainty equivalent wealth, which is the time-t total wealth that makes the agent indifferent between the status quo (with financial wealth W and output process Y) and having a wealth level $P(W, Y)$ and no output:

$$V(W, Y) = V(P(W, Y), 0) . \tag{16}$$

Next, we turn to the autarky regime.

3.2 Autarky Regime

The autarky value function, $\widehat{V}(Y)$, satisfies the following differential equation:

$$0 = f(Y, \widehat{V}) + \mu Y \widehat{V}'(Y) + \frac{\sigma^2 Y^2}{2} \widehat{V}''(Y) + \lambda \mathbb{E} \left[\widehat{V}(ZY) - \widehat{V}(Y) \right] + \xi \left[V(0, Y) - \widehat{V}(Y) \right] . \tag{17}$$

Wealth is not an argument of the value function because financial wealth is zero in autarky: $W_t = 0$ for $\tau^{\mathcal{D}} < t \leq \tau^{\mathcal{E}}$. The country cannot borrow or lend, so consumption equals output.

The first term on the right side of equation (17) is the net utility flow. The second term, represents the impact of the output drift on marginal utility. The third term, captures the impact of diffusive shocks. The fourth term, describes the possibility of output jumping from Y_t to ZY_{t-} while the country is in autarky. The last term, reflects the possibility of exiting from autarky, which occurs at an exogenous rate, ξ. Upon exit at time $\tau^{\mathcal{E}}$, the country's value function is $V(0, Y_t)$, which corresponds to the normal-regime value function with $W = 0$.

The value function in the autarky regime, $\widehat{V}(Y)$, is:

$$\widehat{V}(Y) = \frac{(b \widehat{p} Y)^{1-\gamma}}{1 - \gamma}, \tag{18}$$

where the coefficient b is given by equation (15) and $\widehat{p}$ is the endogenous (scaled) certainty equivalent wealth in the autarky regime.

3.3 Connecting the Normal Regime with Autarky

The value functions $V(W, Y)$ and $\widehat{V}(Y)$ are connected by recurrent transitions between the normal and autarky regimes (see the two HJB equations, (9) and (17)).

For each level of Y_t, there is an endogenous level of W_t, which we denote by $\underline{W_t} < 0$, that makes the country indifferent between repaying its debt and defaulting:[12]

$$V(\underline{W_t}, Y_t) = \widehat{V}(\alpha Y_t)\,. \tag{19}$$

This value-matching condition implicitly defines the default boundary $\underline{W_t}$:

$$\underline{W_t} = \underline{W}(Y_t)\,. \tag{20}$$

The country faces a borrowing constraint of the form:

$$W_t \geq \underline{W}(Y_t)\,. \tag{21}$$

We refer to $-\underline{W_t}$ as the country's debt capacity, since it is the maximum level of debt that the country can issue.

Equations (9) and (19) imply that whenever $W_t < \underline{W}(Y_t) < 0$, the value of repaying the sovereign debt is lower than the value of defaulting and entering autarky. So, the optimal default strategy is characterized by a threshold rule. Whenever the country's sovereign debt exceeds its endogenous debt capacity, $-\underline{W}(Y_t)$, it is optimal to default.

We need one more condition, discussed in the next subsection, to pin down $\underline{W_t}$, as it is a free boundary. To characterize this condition, it is useful to first simplify the optimization problem by using the homogeneity property to reduce the number of state variables from two to one. We define scaled financial wealth:

$$w_t = \frac{W_t}{Y_t}\,, \tag{22}$$

which is the model's scaled state variable. Similarly, we define $c = C/Y$, scaled diffusion hedging demand, $\theta = \Theta/Y$, scaled jump hedging demand, $x = X/Y$, and scaled jump insurance premium $\phi = \Phi/Y$. The variables x and ϕ are related as follows:

$$\phi(w_{t-}; Z^*) = \lambda \mathbb{E}[x(w_{t-}, Z)\, \mathcal{I}_{Z \geq Z^*}]\,. \tag{23}$$

The scaled certainty-equivalent wealth, $p(w_t)$ is equal to $P(W_t, Y_t)/Y_t$. Euler's theorem implies that $P_W(W_t, Y_t) = p'(w_t)$. The value of $p'(w)$ plays a crucial role in our analysis.

We conjecture and verify that $\underline{W_t}$ has the form $\underline{W_t} = \underline{w} Y_t$, where $-\underline{w} > 0$. The scaled debt capacity, $-\underline{w}$ is constant, because our model has the homogeneity property and our solution is time invariant. The country defaults whenever a jump causes output to drop to a level that raises scaled debt, $-w_t$, above $-\underline{w}$.

[12] Recall that the output loss upon default is permanent. If the country enters autarky at time t its output drops to αY_t and then continues to follow the output process given by equation (1).

3.4 Equilibrium Credit Spread

To calculate the equilibrium credit spread, $\pi(w_{t-})$, it is useful to characterize the default threshold policy in terms of $\underline{Z}$, the stochastic cut-off value for Z. Our model's homogeneity property implies that this cut-off value, $\underline{Z}$, is a function of the pre-jump wealth-output ratio, w_{t-}, which we write as $\underline{Z}(w_{t-})$.

To characterize $\underline{Z}(w_{t-})$, we proceed in two steps. First, consider the case where the country cannot hedge any jump shocks ($Z^* = 1$), so the only way to manage its downside jump risk is by defaulting on its outstanding debt. The country defaults if and only if its post-jump scaled financial wealth lands to the left of its credit limit, which means $w_t \equiv W_t/Y_t = W_{t-}/(ZY_{t-})) = w_{t-}/Z \le \underline{w}$. This inequality implies that the country defaults if and only if $Z \in (0, w_{t-}/\underline{w})$, which implies that the cut-off default threshold is $\underline{Z}(w_{t-}) = w_{t-}/\underline{w}$.

Second, consider the general case where the country can partially hedge jump shocks ($Z^* < 1$). Since hedging is a more efficient way to manage hedgeable jump risk than defaulting, the country always hedges and never defaults for $Z \ge Z^*$. This reasoning implies that the cut-off threshold satisfies: $\underline{Z}(w_{t-}) \le Z^*$. When $Z^* < 1$, the standard local non-satiation argument implies that it is not optimal for the country to purchase jump insurance for Z if it plans to subsequently default in that state.

The general optimal default strategy for all levels of financial development, $0 \le Z^* \le 1$, can therefore be written as:

$$\underline{Z}(w_{t-}) = \min\{w_{t-}/\underline{w}\,, Z^*\}\ . \tag{24}$$

When the country is in debt ($W_{t-} < 0$), the competitive-market zero-profit condition for diversified sovereign debt investors implies that the credit spread, π_{t-}, satisfies:

$$-W_{t-}(1 + rdt) = -W_{t-}(1 + (r + \pi_{t-})dt)\,[1 - \lambda\, G(\underline{Z}(w_{t-}))dt] + \lambda\, G(\underline{Z}(w_{t-}))dt \times 0\,. \tag{25}$$

The first term on the right side of equation (25), is the investors' expected total payment, which is given by the product of the probability of full repayment, $[1 - \lambda\, G(\underline{Z}(w_{t-}))dt]$, and the cum-interest value of debt repayment, $-W_{t-}(1 + (r + \pi_{t-})dt)$, if the country does not default at time t. The second term on the right side of equation (25) is the zero payment that occurs upon default. The left side of equation (25) is the expected total repayment to diversified investors.

Simplifying equation (25), we can write the equilibrium credit spread π_{t-}, as a function of w_{t-}:[13]

$$\pi(w_{t-}) = \lambda\, G(\underline{Z}(w_{t-}))\,. \tag{26}$$

This equation ties the equilibrium credit spread to the country's default strategy. Because there is zero recovery upon default and investors are risk neutral, the credit spread is equal to the probability of default.[14]

Our analysis so far takes debt capacity, $-\underline{w}$, as given. We now determine $-\underline{w}$.

Determining Scaled Debt Capacity. Using Ito's lemma, we obtain the following law of motion for w_t in the normal regime:

$$dw_t = \mu_w(w_{t-})\, dt + \sigma_w(w_{t-})\, d\mathcal{B}_t + (w_t - w_{t-})\ d\mathcal{J}_t\,, \tag{27}$$

where post-jump scaled financial wealth w_t is given by:

$$w_t = \frac{w_{t-} + x_{t-}}{Z}\mathcal{I}_{Z\geq Z^*} + \frac{w_{t-}}{Z}\,\mathcal{I}_{\underline{Z}(w_{t-})\leq Z\leq Z^*} + 0\times\,\mathcal{I}_{Z<\underline{Z}(w_{t-})}\,. \tag{28}$$

The first term in equation (27) is the drift function, $\mu_w(w_{t-})$, given by:

$$\mu_w(w_{t-}) \;\; = \;\; \left(r + \pi(w_{t-}) - \mu + \sigma^2\right) w_{t-} - \sigma^2\theta(w_{t-}) + 1 - \phi(w_{t-}) - c(w_{t-})\,, \tag{29}$$

where $\pi(w_{t-})$ is the equilibrium credit spread, given by equation (26), $c(w_{t-})$ is the scaled consumption rule, and $\phi(w_{t-})$ is the scaled jump insurance premium given by equation (23).

The second term in equation (27) is the volatility function, $\sigma_w(w_{t-})$, given by:

$$\sigma_w(w_{t-}) = (\theta(w_{t-}) - w_{t-})\sigma\,, \tag{30}$$

where $\theta(w_{t-})$ is the scaled idiosyncratic risk hedging demand.

The last term in equation (27) is the change of scaled financial wealth from w_{t-} to w_t that occurs in response to a jump arrival ($d\mathcal{J}_t = 1$). Equation (28) shows that are three possible scenarios for w_t: (1) if $Z \geq Z^*$, the country receives a lump-sum jump-insurance payment and does not default ($\mathcal{I}_t^{\mathcal{D}} = 0$), so its post-jump w_t is equal to $(w_{t-} + x_{t-})/Z$; (2) if $\underline{Z}(w_{t-}) < Z \leq Z^*$, the country receives no jump-insurance payment (the jump was not insurable) and does not default ($\mathcal{I}_t^{\mathcal{D}} = 0$), so its post-jump w_t is equal to w_{t-}/Z; (3) if $Z \leq \underline{Z}(w_{t-})$, the country defaults ($\mathcal{I}_t^{\mathcal{D}} = 1$), so its post-jump w_t is equal to zero.

[13] When scaled wealth is positive ($w_{t-} > 0$), there is no debt outstanding and the probability of default is zero.

[14] We can generalize our model by incorporating a stochastic discount factor with jump risk premium to price sovereign debt. This generalization produces higher and more volatile credit spreads.

To determine $\underline{w}$, we use the fact that it is not optimal to default in response to a diffusion shock. The intuition for this property is as follows. Since hedging contracts are actuarially fair, it is more efficient to use these contracts to manage diffusion risk than to default. The country accomplishes this goal by setting the volatility of w to zero at $\underline{w}$:

$$\sigma_w(\underline{w}) = 0\,. \tag{31}$$

This condition implies $\theta(\underline{w}) = \underline{w}$. In addition, the country sets $\mu_w(\underline{w}) \geq 0$ so that w moves weakly away from $\underline{w}$, ensuring that $w_t \geq \underline{w}$. As a result, diffusion shocks never cause default. Default occurs only in response to jumps in output that are sufficiently large. It is optimal to repay the debt in response to small jumps to preserve the option of defaulting in the future in response to larger jumps.

Endogenous Relative Risk Aversion $\widetilde{\gamma}$. We introduce a measure of endogenous relative risk aversion, denoted by $\widetilde{\gamma}$, that is useful in interpreting our results.

We define $\widetilde{\gamma}$ as follows:

$$\widetilde{\gamma}(w) \equiv -\frac{V_{WW}}{V_W} \times P(W,Y) = \gamma p'(w) - \frac{p(w)p''(w)}{p'(w)}\,. \tag{32}$$

The first part of equation (32) defines $\widetilde{\gamma}(w)$. The second part follows from the homogeneity property.

The economic interpretation of $\widetilde{\gamma}$ is as follows. Because limited commitment results in endogenous market incompleteness, the country's *endogenous* risk aversion is given by the curvature of the value function $V(W,Y)$ rather than by γ. We use the value function to characterize the coefficient of endogenous absolute risk aversion: $-V_{WW}(W,Y)/V_W(W,Y)$.

We can build a measure of relative risk aversion by multiplying $-V_{WW}(W,Y)/V_W(W,Y)$, with "total wealth." There is no well-defined market measure of the total wealth under incomplete markets. However, the certainty equivalent wealth $P(W,Y)$ is a natural measure, so we use it in our definition of $\widetilde{\gamma}$ in equation (32).[15]

Limited commitment causes $p'(w) \geq 1$ and $p''(w) < 0$, which implies that $\widetilde{\gamma}(w) > \gamma$ (see equation (32)). That is, limited commitment causes the representative agent to be endogenously more risk averse. In contrast, in the first-best solution we describe below, the country fully hedges against diffusion and jump shocks and $\widetilde{\gamma}(w) = \gamma$.

[15] See Wang, Wang, and Yang (2012) for a similar definition in a different setting where markets are exogenously incomplete.

4 First-Best Solution: Full Commitment and Spanning

Before discussing our results under limited commitment and limited spanning, we summarize the first-best (FB) solution that obtains when there is full commitment and full spanning. Full commitment means that the country has to honor all its contractual agreements. Full spanning means that $Z^* = 0$, which represent the highest level of financial development. We use the superscript FB to denote the variables that pertain to the FB solution.

As in Friedman (1957) and Hall (1978), we define non-financial wealth, H_t, for the case where $Z^* = 0$, as the present value of output, discounted at the constant risk-free rate, r:

$$H_t = \mathbb{E}_t \left(\int_t^{\infty} e^{-r(u-t)} Y_u du \right) . \tag{33}$$

Because Y is a geometric jump-diffusion process, we have $H_t = hY_t$, where h is scaled non-financial wealth given by

$$h = \frac{1}{r-g} , \tag{34}$$

and g is given by equation (2). To ensure that non-financial wealth is finite, we require that $r > g$. To ensure that utility is finite, we require the condition:

$$\rho > (1 - \psi^{-1}) \, r . \tag{35}$$

Proposition 1 *Scaled total wealth,* $p^{FB}(w) = P^{FB}(W, Y)/Y = (W + H)/Y$, *is*

$$p^{FB}(w) = w + h , \tag{36}$$

where h *is given by equation (34). The scaled endogenous debt capacity is* $-\underline{w}^{FB} = h$ *and* $w_t \geq \underline{w}^{FB} = -h$. *The optimal consumption-output ratio,* $c_t = c^{FB}(w)$, *is given by:*

$$c^{FB}(w) = m \, p^{FB}(w) = m(w + h \,) . \tag{37}$$

where m *is given by:*

$$m = r + \psi \, (\rho - r) \ . \tag{38}$$

The optimal hedging demand for diffusive shocks for all values of w, $\theta^{FB}(w)$, *is given by:*

$$\theta^{FB}(w) = -h \, . \tag{39}$$

The optimal hedging demand for jump risk, $x^{FB}(w, Z)$, *is given by:*

$$x^{FB}(w, Z) = (1 - Z)h \, . \tag{40}$$

The implied scaled jump insurance premium is constant: $\phi^{FB}(w) = \lambda(1-\mathbb{E}(Z))h$. *There is no default, meaning* $\underline{Z}(w) = Z^* = 0$.

Equation (36) states that the scaled certainty equivalent wealth is given by the sum of w and scaled non-financial wealth h, $p^{FB}(w) = w + h$. Therefore, the country's endogenous relative risk aversion defined in equation (32), $\widetilde{\gamma}(w)$, is constant and equal to γ.

Equation (37) shows that $c^{FB}(w)$ is proportional to scaled total wealth, $p^{FB}(w) = w+h$. Equation (39) shows that the country fully hedges its diffusive risk by taking a short position of h units in the diffusion hedging contract so that the net exposure of its total wealth, $P^{FB}(W,Y)$, to diffusive shocks is zero. Similarly, equation (40) shows that the country fully hedges the jump risk by buying $(1-Z)h$ units of the jump insurance contract for each possible value of Z, so that the net exposure of $P^{FB}(W,Y)$ to jump shocks is zero. As a result, total wealth, $P^{FB}(W,Y)$, is fully hedged and remains constant.

Next, we obtain the closed-form expression for the dynamics of w_t in the FB. By substituting the optimal policies into the drift function, given in equation (29), the volatility function, given in equation (30), and equation (28) for the post-jump w_t, we obtain the law of motion for w_t, given by equation (27), where the drift function is given by:

$$\mu_w^{FB}(w) = 1 + \left(r - m - \mu + \sigma^2\right) w + \left[(\sigma^2 - m) - \lambda(1-\mathbb{E}(Z))\right] h \,. \tag{41}$$

The volatility function is given by

$$\sigma_w^{FB}(w) = -\sigma(w+h)\,, \tag{42}$$

and

$$w_t^{FB} = \frac{w_{t-} + (1-Z)h}{Z}\,. \tag{43}$$

Because the country never defaults, $\underline{Z} = Z^* = 0$, Z is always greater than $Z^* = 0$, hence only the first term in equation (28) is relevant. Substituting $x^{FB}(w,Z) = (1-Z)h$ given in equation (40), we obtain equation (43) for the post-jump w_t.

The law of motion for w_t discussed above ensures that the country's total wealth grows deterministically at a rate $(r-m)$:

$$P^{FB}(W_t, Y_t) = e^{(r-m)t} P^{FB}(W_0, Y_0) = e^{-\psi(\rho - r)t}(W_0 + hY_0)\,. \tag{44}$$

For the case where $\rho = r$, consumption is constant, $C_t = C_0 = r(W_0 + hY_0)$, for all t.[16]

[16]An interesting property is that when the EIS, ψ, is zero, consumption is constant for all admissible values of ρ and r. This result does not hold for expected utility, since this utility function does not allow risk aversion to be finite when the EIS equals zero.

5 Limited-Commitment Solution

In this section, we discuss the solution of our model when there is limited commitment. In this case, default can occur in equilibrium. This possibility constrains the country's debt capacity. The following proposition summarizes the main properties of the solution.

Proposition 2 *The optimal consumption-output ratio, $c(w)$, is:*

$$c(w) = mp(w)(p'(w))^{-\psi}\,, \tag{45}$$

where m is given by equation (38). The scaled diffusion risk hedging demand, $\theta(w)$, is:

$$\theta(w) = w - \frac{\gamma p(w)p'(w)}{\gamma(p'(w))^2 - p(w)p''(w)} = w - \frac{\gamma p(w)}{\widetilde{\gamma}(w)}\,, \tag{46}$$

where $\widetilde{\gamma}(w)$ is the endogenous relative risk aversion given by equation (32). For $Z^* \leq Z < 1$, the optimal scaled hedging demand for jump risk, $x(w, Z)$, solves:

$$p'(w) = \left(\frac{Zp((w + x(w,Z))/Z)}{p(w)}\right)^{-\gamma} p'((w + x(w,Z))/Z)\,. \tag{47}$$

The scaled certainty equivalent wealth $p(w)$ in the normal regime, $\widehat{p}$ in the autarky regime, and the scaled financial wealth, $\underline{w} > 0$, satisfy the following two interconnected ODEs:

$$\begin{aligned} 0 = &\left(\frac{m(p'(w))^{1-\psi} - \psi\rho}{\psi - 1} + \mu - \frac{\gamma\sigma^2}{2}\right) p(w) + \left[\, (r + \pi(w) - \mu)\, w + 1 - \phi(w)\,\right] p'(w) \\ &+ \frac{\gamma^2\sigma^2 p(w)p'(w)}{2\,\widetilde{\gamma}(w)} + \frac{\lambda}{1-\gamma}\mathbb{E}\left[\left(\frac{Zp(\breve{w})}{p(w)}\right)^{1-\gamma} - 1\right] p(w)\,, \end{aligned} \tag{48}$$

$$0 = \frac{\rho\left[(b\,\widehat{p}\,)^{-(1-\psi^{-1})} - 1\right]}{1-\psi^{-1}} + \mu + \frac{\lambda(\mathbb{E}(Z^{1-\gamma}) - 1)}{1-\gamma} - \frac{\gamma\sigma^2}{2} + \frac{\xi}{1-\gamma}\left[\left(\frac{p(0)}{\widehat{p}}\right)^{1-\gamma} - 1\right], \tag{49}$$

where $\breve{w}$ is given by

$$\breve{w} = [(w + x(w,Z))/Z]\,\mathcal{I}_{Z\geq Z^*} + (w/Z)\,\mathcal{I}_{\underline{Z}(w)<Z<Z^*} + \underline{w}\,\mathcal{I}_{Z<\underline{Z}(w)}\,. \tag{50}$$

In addition, we have the following boundary conditions:

$$p(\underline{w}) = \alpha\,\widehat{p}\,, \tag{51}$$

$$p''(\underline{w}) = -\infty\,, \tag{52}$$

$$\lim_{w\to\infty} p(w) = w + h\,, \tag{53}$$

where h is given by equation (34).

The equilibrium credit spread is $\pi(w_{t-}) = \lambda G(\underline{Z}(w_{t-})$. The scaled jump insurance premium, $\phi(w_{t-})$, is given by equation (23). The country defaults when $\tau^{\mathcal{D}} = \inf\{t : w_t < \underline{w}\}$.

Equation (45) shows that with limited spanning and/or limited commitment, consumption is a nonlinear function of w, depending on both the certainty equivalent wealth, $p(w)$, and its derivative, $p'(w)$.

Equation (46) determines the country's hedging demand with respect to diffusive shocks. As discussed above, the country hedges to avoid default triggered by diffusive shocks and preserve the option to default in response to rare disasters. Without hedging diffusive shocks, a country at debt capacity would default with probability 50 percent, with default potentially triggered by very small shocks. Substituting equation (46) into equation (30), we obtain:

$$\sigma_w(w) = (\theta(w) - w)\sigma = -\sigma\frac{\gamma p(w)}{\widetilde{\gamma}(w)} < 0\,. \tag{54}$$

In absolute value, the volatility for w is proportional to the ratio between $p(w)$ and endogenous risk aversion, $\widetilde{\gamma}(w)$. We discuss some key properties of $\sigma_w(w)$ below.

Equation (47) determines the country's scaled hedging demand with respect to jump shocks, $x(w, Z)$. As discussed above, for insurable jump shocks ($Z \geq Z^*$) the country hedges its jump risk exposures to equate its pre- and post-jump marginal utility of wealth. The homogeneity property allows us to express this condition in terms of the certainty equivalent wealth, $p(w)$, and the marginal certainty equivalent value of financial wealth, $p'(w)$.

Equations (48) and (49) are the interconnected ODEs for the scaled certainty equivalent wealth in the normal regime ($p(w)$) and the autarky regime ($\widehat{p}$).

Equation (50) includes three mutually exclusive regions for Z: (1) the region where jump risk is insurable, $(Z^*, 1)$. In this region, the country receives a lump-sum jump insurance payment, $x(w, Z)$, and Z in the denominator reflects the output drop associated with the jump, so $\breve{w} = (w + x(w, Z))/Z$; (2) the intermediate region, ($\underline{Z}(w) < Z < Z^*$), where jump insurance is unavailable but the country does not default, so $\breve{w} = w/Z$; (3) the default region ($Z^* \leq Z < 1$). In this region, the country defaults and enters autarky, so its post-jump w_t is zero and its scaled certainty equivalent wealth is $\alpha\widehat{p}$. By using the definition of $\breve{w}$ given in equation (50), which implies $p(\breve{w}) = \alpha\widehat{p}$, and the value-matching condition $p(\underline{w}) = \alpha\widehat{p}$, given by equation (51), we obtain $\breve{w} = \underline{w}$.

Equation (51) follows from the value-matching condition, (19). Equation (52) follows from the zero volatility condition, (31) for w at $\underline{w}$, and $p(\underline{w}) > 0$. This condition together with equation (54) implies that the country's endogenous relative risk aversion, $\widetilde{\gamma}(w)$, approaches infinity, as $w \to \underline{w}$. Equations (51) and (52) jointly characterize the endogenous debt capacity $-\underline{w}$.

Equation (53) states that, as $w \to \infty$, the effect of limited commitment disappears and $p(w)$ converges to $w + h$, which is the FB solution, discussed in Section 4.

Next, we turn to the special case where there is full spanning and hence all jump risks can be hedged ($Z^* = 0$).

Full Spanning and Limited Commitment. As in Kehoe and Levine (1993), when all shocks are insurable at actuarially fair premia, the country never defaults in equilibrium. The country is better off honoring its debt and preserving its debt capacity. Doing so allows the country to borrow at the risk-free rate ($\pi = 0$). The country's temptation to default is an off-equilibrium threat that determines debt capacity by making the country indifferent about whether or not to default.

For this full-spanning and limited-commitment case, equations (51) and (52) are the continuous-time equivalent of the limited-enforcement conditions in Kehoe and Levine (1993), Kocherlakota (1996), and Alvarez and Jermann (2000).

6 Calibration and Quantitative Results

To explore the quantitative properties of our model, we calibrate it with the eleven parameter values summarized in Table 1. We divide these parameters into two groups. The seven parameters in the first group are set to values that are standard in the literature. The four parameters in the second group are calibrated to match key features of data for Argentina.

6.1 Baseline Calibration

We first describe the parameters drawn from the literature. Following Aguiar and Gopinath (2006), we set the risk aversion coefficient (γ) to 2, the annual risk-free rate (r) to 4 percent, and the rate at which the country exits autarky (ξ) to 0.4 per annum. This choice of ξ is consistent with the estimates in Gelos, Sahay, and Sandleris (2011). Following Barro (2009), we set the annual subjective discount rate (ρ) to 5.2 percent.

As in the rare-disasters literature, we assume that the cumulative distribution function

Table 1: PARAMETER VALUES

Parameters	Symbol	Value
risk aversion	γ	2
elasticity of intertemporal substitution	ψ	0.047
subjective discount rate	ρ	5.2%
risk-free rate	r	4%
financial development parameter	Z^*	0.9
output drift (in the absence of jumps)	μ	2.7%
output diffusion volatility	σ	4.5%
jump arrival rate	λ	0.073
power law parameter	β	6.3
default distress cost	α	97.5%
autarky exit rate	ξ	0.25
Targeted observables		
average output growth rate	g	1.7%
output growth volatility		6.66%
average debt-output ratio		15%
unconditional default probability		3%

All parameter values, whenever applicable, are continuously compounded and annualized.

of the recovery fraction, $G(Z)$, is governed by a power law:

$$G(Z) = Z^{\beta}. \tag{55}$$

Following Barro and Jin (2011), we call disasters jump shocks that create realized output losses greater than 10 percent ($Z < 1 - 0.1 = 0.9$). We choose $\beta = 6.3$ and the annual jump arrival rate, $\lambda = 0.073$, so that the annual disaster probability is $\lambda G(0.9) = 0.073 \times G(0.9) =$ 3.8 percent, which is the value estimated by Barro and Jin (2011).

In our baseline calibration, we assume that rare disasters cannot be hedged but non-disaster jumps can be hedged. In other words, the level of financial development in our baseline case corresponds to $Z^* = 0.9$.

We choose the parameter that controls the drift in the absence of jumps (μ), the diffusion volatility (σ), the default distress cost (α), and the intertemporal elasticity of substitution (ψ), to target the following four moments estimated using Argentinean data: an average growth rate of output of 1.7 percent per annum, a standard deviation of the growth rate of output of 6.7 percent, an average debt-to-GDP ratio of 15 percent, and unconditional default probability of 3 percent. Our empirical estimates of the average and standard deviation of the annual growth rate of real GDP for Argentina were obtained using Barro and Ursua's (2008) data for the period 1876-2009.

Our model consolidates the expenditure and borrowing decisions of the private sector and the government. For this reason, we calibrate it to match the ratio of net debt to GDP. In Argentina, as in most countries, a significant fraction of government debt is owned by the domestic private sector. We compute our target for the debt-to-output ratio by calculating the difference between Argentina's debt liabilities and debt assets using the data compiled by Lane and Milesi-Ferretti (2007) for the period from 1970 to 2011. The average net debt-to-GDP ratio during this period is 15 percent.[17] Argentina defaulted 6 times in roughly 200 years, so we target an annual default probability of 3 percent.[18]

We obtain the following parameter values: $\psi = 0.047$, $\mu = 2.7$ percent per annum, $\sigma = 4.5$ percent per annum, and $\alpha = 0.975$. The calibrated value of α implies that the direct costs of defaulting on sovereign debt are equal to 2.5 percent of output. This cost of

[17] The model can be easily calibrated to generate higher average debt-output ratios by increasing $(1 - \alpha)$, the distress cost associated with default, see Table 4.

[18] Argentina defaulted in 1830, 1890, 1915, 1930, 1982, and 2001. See Sturzenegger and Zettlemeyer (2006) for a discussion.

Table 2: Partial Spanning and Debt Intolerance

Z^*	Average debt-output ratio	Default probability	Debt capacity $\lvert \underline{w} \rvert$
1 (No jump hedging)	14.7%	4.0%	20.5%
0.9	**15%**	**3.1%**	**20.7%**
0.5	20.7%	0.1%	24%

All parameter values other than Z^* are summarized in Table 1.

default is conservative relative to the estimates reported by Hebert and Schreger (2016) for Argentina.

In this calibration, the value of the EIS ($\psi = 0.047$) is low so the representative agent has a strong preference for smooth consumption paths.[19] This preference makes the utility cost of default high. Since default is costly, debt capacity is high. At the same time, the preference for smooth consumption means that the country does not save away quickly from the debt region. These properties generate plausible debt-output ratios. We can interpret the low value of the EIS as resulting from rigidities in spending patterns and expenditure commitments that are difficult to change.

6.2 Debt Intolerance

Table 2 shows the impact of different levels of financial development ($Z^* = 0.5, 0.9, 1$) on debt intolerance. We set all other parameters to the values used in our benchmark calibration and summarized in Table 1. To better understand the intuition, we proceed in three steps.

First, we briefly review the properties of the FB case. In the FB, debt capacity is the present discounted value of output, $h = 1/(r - g)$, which represents 4,348 percent of current output. The default probability is always zero since the country fully hedges diffusion and jump risk and never defaults.

Second, we isolate the impact of limited commitment by comparing the full-spanning

[19] There is currently no consensus on what are empirically plausible values for the EIS (see Attanasio and Weber (2010) for a discussion). Our choice is consistent with Hall (1988) who argues that the elasticity of intertemporal substitution is close to zero. It is also consistent with the recent estimates by Best, Cloyne, Ilzetzki, and Kleven (2017) which are based on mortgage data.

limited-commitment case ($Z^* = 0$) to the FB case. When $Z^* = 0$, the country never defaults in equilibrium. In both the FB and the $Z^* = 0$ cases, there is full spanning, so it is cheaper for the country to manage risk by hedging than by defaulting on its sovereign debt. The only difference between the FB and $Z^* = 0$ cases is the presence of limited commitment in the latter case. This limited commitment severely constrains the country's debt capacity: debt capacity is 147 times larger in the FB ($|\underline{w}| = 4,348\%$) than in the $Z^* = 0$ case ($|\underline{w}| = 25\%$). As a result, the country's average debt-to-output ratio is only 19% under limited commitment despite full spanning.

Third, we study the impact of financial development. Reducing spanning from $Z^* = 0$ to the value used in our benchmark calibration, $Z^* = 0.9$, produces a large decline of debt capacity, from 25 to 20.7 percent. Because of limited spanning, the country uses default to manage large jump shocks: when $Z^* = 0.9$, the probability of default is 3.1 percent.

Eliminating entirely the ability to hedge jump risk ($Z^* = 1$) results in a large rise in the probability of default relative to the benchmark case (from 3.1 to 4 percent), even though the decline in debt capacity and average debt-output ratio is small (from 20.7 to 20.5 percent and from 15 to 14.7 percent, respectively). The large rise in the probability of default occurs because when $Z^* = 1$, the only way to manage large jump risk is to default on sovereign debt. In contrast, when $Z^* = 0.9$, roughly half of the jump shocks can be hedged ($1 - G(Z^*) = 49$ percent).

Improving financial development by decreasing Z^* from 0.9 to 0.5 has a dramatic impact on the debt capacity, average debt-output ratio, and default probability: debt capacity rises from 20.7 to 24 percent of output, the average debt-output ratio increases from 15 to 20.7 percent, and the probability of default drops to close to zero, from 3.1 to 0.1 percent.

In sum, Table 2 shows that low financial development causes debt intolerance. This table also shows that improving financial development from a low level has a large positive impact on the country's ability to borrow and the credit spread it pays on its sovereign debt.

6.3 Economic Mechanisms and Quantitative Implications

In this subsection, we use our benchmark calibration to explore the economic mechanisms at work in our model. Figures 1, 2, and 3 show the values of key variables as a function of scaled financial wealth, w, for economies with different levels of financial development. Recall that in our benchmark calibration we set $Z^* = 0.9$, which means that jumps that

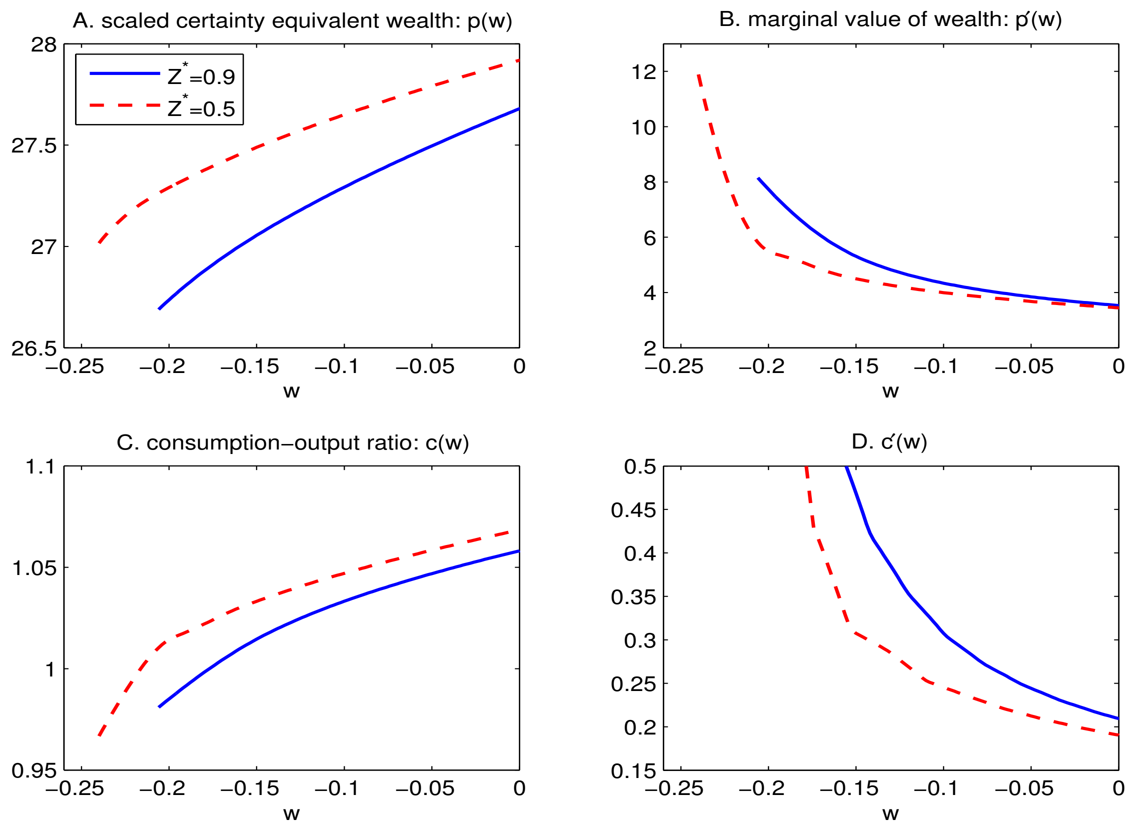

Figure 1: Scaled certainty equivalent wealth $p(w)$, marginal certainty equivalent value of wealth $p'(w)$, scaled consumption $c(w)$, and marginal propensity to consume $c'(w)$ for two levels of financial development: $Z^* = 0.5$ and $Z^* = 0.9$. The support of w is $[\underline{w}, \infty)$, where $\underline{w} = -20.6$ percent and $\underline{w} = -24.0$ percent for $Z^* = 0.9$ and 0.5, respectively.

generate output losses greater than 10 percent cannot be hedged.

It is useful to first briefly review the properties of the FB case. Recall that total wealth is additively separable ($p^{FB}(w) = w + h = w + 43.5$), the marginal value of wealth is one ($p'(w) = 1$), the marginal propensity to consume (MPC) out of wealth is constant and equal to $m = r + \psi(\rho - r) = 4.1$ percent, and the scaled consumption rule is $c(w) = 0.041 \times (w + h)$. Next, we discuss the impact of limited spanning and limited commitment on the quantitative properties of our model.

Certainty equivalent wealth, marginal value of wealth, consumption, and the MPC. Panels A and B of Figure 1 display the scaled certainty-equivalent wealth, $p(w)$, and the marginal certainty-equivalent value of wealth, $P_W(W, Y) = p'(w)$, respectively. The function $p(w)$ is increasing and concave, which implies that $p'(w)$ is decreasing in w. Panels C and D display the consumption-output ratio, $c(w)$, and the MPC out of wealth, $c'(w)$, respectively. The function $c(w)$ is increasing and concave, which implies that $c'(w)$ is decreasing in w. As w goes to infinity, $p(w)$ approaches $p^{FB}(w) = w + h$, $p'(w)$ approaches one, $c(w)$ approaches $c^{FB}(w) = m(w + h)$, and $c'(w)$ approaches the MPC obtained in the FP, $m = 0.041$.

Consider our baseline case where $Z^* = 0.9$ (our proxy for the status quo in emerging markets). The scaled certainty-equivalent wealth, even when the country has no debt, is $p(0) = 27.68$, which is only 64 percentage of h, the FB value. The marginal value of wealth at $w = 0$ is $p'(0) = 3.53$. Consumption is $c(0) = 1.06$, which is only 60 percent of $mh = 0.041 \times 43.48 = 1.76$, the FB value. The MPC is $C_W(0, Y) = c'(0) = 20.9\%$, which is about five times of 4.1%, the FB value. In sum, limited commitment and limited spanning have a large quantitative impact on the properties of the economy.

Next, we discuss the impact of financial development under limited commitment. We compare our baseline case with an economy where $Z^* = 0.5$, which corresponds to a high level of financial development since the country can hedge jumps that generate output losses smaller than 50 percent.

The higher is financial development (lower Z^*), the higher is $p(w)$ because more risks are hedged and the representative agent faces less uncertainty. As a result, the marginal value of wealth, $p'(w)$, is lower. Consumption is higher because both a higher $p(w)$ and a lower $p'(w)$ cause $c(w)$ to be higher (see equation (45)).

To quantify the impact of financial development, suppose that $w = -15$ percent, which is the average debt-to-output ratio in the baseline calibration. The value of $p(-0.15)$ is 27.05 in the economy with $Z^* = 0.9$, which is one percent lower than in the economy with $Z^* = 0.5$. The marginal value of wealth, $p'(-0.15)$, is 5.31 in the economy with $Z^* = 0.9$, which is 18 percent higher than in the economy with $Z^* = 0.5$. The consumption-output ratio, $c(-0.15)$, is 1.01 in the economy with $Z^* = 0.9$, which is 2 percent lower than in the economy with $Z^* = 0.5$. The MPC out of wealth, $c'(-0.15)$, is 0.47 in the economy with $Z^* = 0.9$, which is 52 percent higher than in the economy with $Z^* = 0.5$. Finally, as discussed above, the endogenous debt capacity, $-\underline{w}$, increases from 20.6 percent to 24.0 percent as financial development improves.

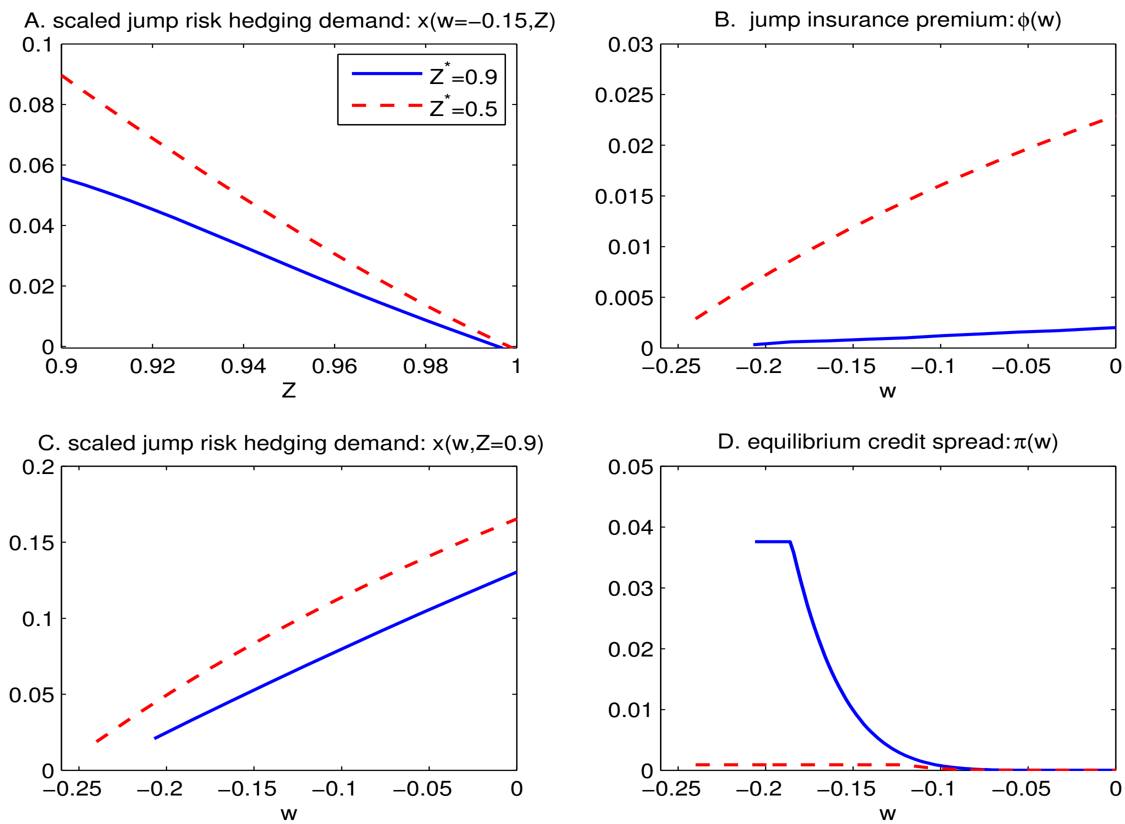

Figure 2: Scaled jump risk hedging demand $x(w, Z = 0.9)$, scaled jump risk hedging demand $x(w = 0, Z)$, jump insurance premium payment $\phi(w)$, and the equilibrium credit spread $\pi(w)$ for two levels of financial development: $Z^* = 0.5$ and $Z^* = 0.9$.

Jump-risk hedging demand, jump-insurance premium payment, and the credit spread. Panel A of Figure 2 plots $x(w, Z)$ as a function of Z for $w = -15$ percent, the average debt-output ratio targeted in our benchmark calibration. This panel shows that for a given Z^* and w, the hedging demand $x(w, Z)$ is decreasing in Z, which means that the country insures more against bigger losses in order to smooth consumption. Panel B shows that the demand for insurance, $x(w, Z)$, against a 10 percent permanent loss in output ($Z = 0.9$) increases with w, which means that a wealthier country hedges more. That is, hedging and financial wealth are complements. Panel C shows that the scaled jump-insurance premium payment, $\phi(w) = \int_{Z^*}^{1} x(w, Z)dG(Z)$, also increases with w.

Panels A, B, and C together show that hedging demand and hedging costs increase with financial development. Because the jump-insurance premium has to be paid up front, this payment is more costly in utility terms for less financially developed countries. As a result, both the hedging demand and the insurance premium payments are lower for these countries.

As the country's financial development improves (i.e., as Z^* decreases), its risk-sharing opportunities expand, causing its hedging position to increase in absolute value. This increase leads to a rise in debt capacity, $-\underline{w}$.

Panel D plots the equilibrium credit spread, $\pi(w)$, which declines with both the level of financial development and financial wealth w. The credit spread is constant in the region where $w_{t-} \leq -Z^*\underline{w}$, because the threshold default value of Z, $\underline{Z}(w_{t-}) = \min\{w_{t-}/\underline{w}, Z^*\}$, is constant and equal to $Z^* < 1$. This property implies that $\pi(w_{t-}) = \lambda G(Z^*)$ so the credit spread is independent of w.

When financial development is high, the country uses jump insurance contracts to hedge most jump shocks and only uses costly default to manage rare disasters. As a result, the likelihood of default and the credit spread are low. For the case where $Z^* = 0.5$, the equilibrium credit spread is very close to zero for all values of w. In contrast, when financial development is low, the option to default is used to manage most jump shocks and hence default is likely, resulting in a high credit spread. For the case where $Z^* = 0.9$, the equilibrium credit spread is high for debt levels above 15 percent of output.

Panel A of Figure 3 shows that the scaled diffusion hedging demand, $\theta(w)$, increases with w in absolute value. That is, a less indebted country hedges more diffusive risk. As with the case of jump risk, hedging and financial wealth are complements. Even though the country incurs no upfront cost to hedge diffusion shocks, it is not optimal to fully hedge the diffusion risk.

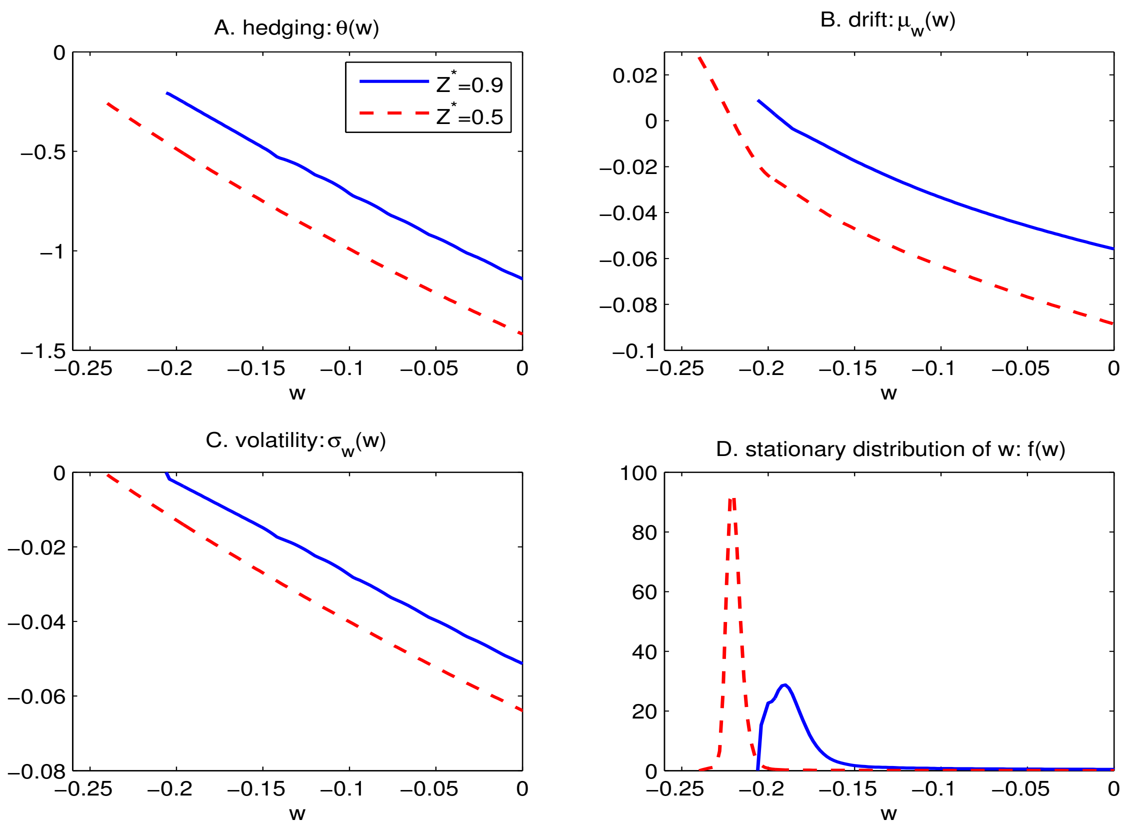

Figure 3: Scaled diffusion risk hedging demand $\theta(w)$, drift $\mu_w(w)$, volatility $\sigma_w(w)$, and the density function for the stationary distribution of w in the normal regime, $f(w)$, for the two levels of financial development: $Z^* = 0.5$, and $Z^* = 0.9$.

Panel B plots the volatility function, $\sigma_w(w)$. Because a less indebted country has a higher $p(w)$ and a lower endogenous relative risk aversion, $\widetilde{\gamma}(w)$, the absolute value of $\sigma_w(w)$ increases with w, as one can see from equation (54). Recall that $\sigma_w(\underline{w}) = 0$. The intuition for this property, which is visible in Panel B, is that it is inefficient for the country to use default to manage continuous diffusive shocks.

Panel C shows the drift function for w, $\mu_w(w)$, which is negative for most values of w. This result follows from the observations that: (a) the country's consumption is often larger than output (see Figure 1); and (b) jump insurance and interest payments drain the country's financial wealth. All these forces move the country further into debt in expectation. However, as the country's debt approaches its maximal capacity, $-\underline{w}$, the country voluntarily adjusts its consumption, insurance demand, and debt level so that $\mu_w(\underline{w}) \geq 0$. This property together with zero volatility condition for w at $\underline{w}$ discussed above are necessary to ensure that the country does not default in response to continuous diffusion shocks.

Next, we turn to the impact of financial development on the idiosyncratic risk hedging demand, $\theta(w)$, and $\sigma_w(w)$. Equation (54) implies that $\sigma_w(w) = -\sigma\gamma p(w)/\widetilde{\gamma}(w) < 0$ and $\theta(w) = w + \sigma_w(w)/\sigma$. The higher the level of financial development, the higher the country's certainty equivalent wealth, $p(w)$, (see Figure 1 and our earlier discussions) and the lower the country's endogenous relative risk aversion, $\widetilde{\gamma}(w)$. Therefore, for a given level of w, a more financially developed country has a more negative $\sigma_w(w)$, and a more negative hedging position, $\theta(w)$.

Panel D displays the probability density function for the stationary distribution of w, $f(w)$, in the normal regime. This panel is consistent with the empirical observation that countries with lower levels of financial development on average have lower debt-to-output ratios. In other words, these countries are debt intolerant.

7 Sensitivity Analysis

We now discuss how a country's average debt-output ratio, average default probability, and debt capacity vary with some key parameters. We change one parameter at a time and fix all other parameters at the values reported in Table 1.

The effect of the EIS, ψ. Table 3 shows the impact of varying the EIS. Recall that risk aversion, γ, is equal to 2. Therefore, we obtain the expected-utility case when ψ is equal

Table 3: THE EFFECT OF THE EIS ψ

ψ	debt-output ratio	default probability	debt capacity $\lvert \underline{w} \rvert$
0	15.3%	3.26%	19.8%
0.047	**15%**	**3.09%**	**20.7%**
0.25	17.9%	1.10%	34.2%
0.5	5.5%	0.65%	42.2%

All parameter values other than ψ are summarized in Table 1.

to 0.5. Increasing ψ from 0.047 to 0.5 substantially reduces the country's debt-output ratio from 15 percent to 5.5 percent and decreases the country's annual default probability from 3.09 percent to 0.65 percent. The intuition for this result is that countries with a higher EIS are more willing to substitute their consumption over time. As a result, these countries have stronger incentives to save away from debt regions where credit spreads are relatively high.

Perhaps surprisingly, raising the EIS from 0.047 to 0.5 more than doubles the country's debt capacity, from 20.7 percent to 42.2 percent. The intuition for this result is that capital markets are more willing to lend to countries with higher intertemporal substitution, since it is less costly (in terms of utility) for these countries to cut consumption in response to adverse shocks to make their debt payments.

The effect of the distress cost, $1-\alpha$. Table 4 illustrates the impact of distress costs and shows that these costs play a key role in allowing the model to generate empirically plausible average debt-output ratios. Increasing the distress cost, $(1-\alpha)$, from 2.5 percent to 5 percent more than doubles the debt capacity from 20.7 percent to 49 percent, significantly raises the debt-output ratio from 15 percent to 37 percent, and decreases the annual default probability from 3.09 percent to 2.96 percent. When default is more costly, debt capacity is higher. At the same time, the country defaults less often despite borrowing more on average.

When the distress cost is zero, the only cost of default is the loss of consumption smoothing opportunities under autarky. Since this utility cost is small, debt capacity is essentially zero (see the last row of Table 4.)

Table 4: The effect of distress cost, $(1-\alpha)$

$(1-\alpha)$	debt-output ratio	default probability	debt capacity $\lvert \underline{w} \rvert$
5%	36.9%	2.96%	49%
2.5%	**15%**	**3.09%**	**20.7%**
1%	5.5%	3.18%	7.2%
0%	0.1%	3.63%	0.2%

All parameter values other than α are summarized in Table 1.

Table 5: The effect of the probability of exiting autarky, ξ

ξ	debt-output ratio	default probability	debt capacity $\lvert \underline{w} \rvert$
0	15.8%	3.06%	21.0%
0.25	**15%**	**3.09%**	**20.7%**
0.5	14.7%	3.13%	18.2%
1	13.2%	3.15%	16.0%

All parameter values other than ξ are summarized in Table 1.

Table 6: THE EFFECT OF RISK AVERSION γ

γ	debt-output ratio	default probability	debt capacity $\lvert \underline{w} \rvert$
1	15.6%	3.25%	19.6%
2	**15%**	**3.09%**	**20.7%**
3	15.4%	2.51%	23%

All parameter values other than γ are summarized in Table 1.

The effect of the probability of exiting autarky, ξ. Table 5 shows the impact of varying ξ. Increasing ξ reduces the expected duration of the autarky regime, $1/\xi$, lowering the cost of defaulting. Since default is less costly, the country defaults more often. In equilibrium, debt capacity falls and the country borrows less.

Decreasing the average duration of the autarky regime from four years ($\xi = 0.25$) to one year lowers the debt-output ratio from 15 percent to 13.2 percent, increases the annual default probability from 3.09 percent to 3.15 percent, and reduces the debt capacity from 20.7 percent to 16 percent. When autarky is permanent ($\xi = 0$), as in Eaton and Gersovitz (1981), debt capacity is 21 percent and the average debt-output ratio is 15.8 percent.

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The effect of risk aversion, γ. Table 6 shows that the effect of risk aversion. Increasing γ, raises the cost of default since it is more costly to bear consumption volatility in the autarky regime. With default more costly, the country defaults less often and debt capacity is higher. Increasing γ from one to three increases debt capacity from 19.6 percent to 23 percent and lowers the annual default probability from 3.25 percent to 2.51 percent. The effect of γ on the average debt-output ratio is quite small.

8 An Expected-utility Calibration

In this section, we restrict recursive utility to the expected-utility case generally used in the sovereign-debt literature. We explain why the calibrations traditionally used in this literature rely on a very high discount rate. Then, we show that our key result–low levels of financial development causes debt intolerance–continues to hold for these traditional calibrations.

In the expected-utility case, a low EIS is associated with a high level of risk aversion ($\gamma = \psi^{-1}$). A high γ generates a large debt capacity because the utility cost of defaulting and bearing the consumption volatility associated with autarky is high. This high default cost induces the country to avoid borrowing, so the average debt-output ratio is low or even negative. This tension is absent in our baseline calibration which features a low EIS ($\psi = 0.047$) combined with moderate risk aversion ($\gamma = 2$).

Then how can we match empirical targets within an expected utility framework? As increasing the EIS lowers risk aversion, we next consider an expected-utility-based calibration with a higher EIS: $\psi = 0.5$ and $\gamma = 2$. However, if we use the discount rate proposed by Barro and Jin (2011) and used in our baseline calibration ($\rho = 0.052$), the model does not generate a plausible debt-output ratio. To understand intuition for this result, consider two scenarios. In the first scenario, distress costs, $(1 - \alpha)$, are high, so debt capacity is also high. But a country in debt has a strong incentive to save to avoid the possibility of incurring high default costs. As a consequence, the default likelihood, credit spread, and average debt-to-output ratio are low. In the second scenario, distress costs are low, so the country is willing to borrow and the likelihood of default is high. As a consequence, credit spreads are high and debt capacity is low, resulting in a counterfactually low average debt-to-output ratio.

One solution widely used in the literature is to assume high distress costs so that debt capacity is high and also assume a very high discount rate. This configuration can generate plausible debt-output ratios because high discount rates create an incentive to borrow, even when default costs are high (see Aguiar and Gopinath (2006) and Arellano (2006)).[20] Generating the average debt-output ratio targeted in our calibration (15 percent) requires a value of ρ equal to 20.8%.[21]

Table 7 compares versions of this high-discount-rate calibration with different levels of financial development. We see the same debt intolerance that emerges in our benchmark calibration. Countries with low financial development use default on sovereign debt to manage

[20]Alvarez and Jermann (2001) also find that a low risk aversion and a high discount rate are necessary to match key asset-pricing moments in a general equilibrium asset-pricing model with limited commitment.

[21]Bornstein (2017) compares discrete and continuous-time versions of Arellano's (2008) model and shows that average debt-to-output ratios are lower in the continuous-time version of the model. The reason is as follows. In discrete time, a country can respond to a negative output shock by cutting consumption to reduce its debt for next period, therefore reducing its credit spread. In continuous time, the stock of debt is continuous and an output shock leads to an instantaneous rise in the credit spread. This rise creates an incentive to save away from the debt region.

Table 7: PARTIAL SPANNING AND DEBT INTOLERANCE FOR AN EXPECTED-UTILITY CALIBRATION

Z^*	Debt-output ratio	Default probability	Debt capacity $\lvert\underline{w}\rvert$
1 (No jump hedging)	13.7%	5.69%	18.8%
0.9	**14.5%**	**2.98%**	**19.0%**
0.5	21.0%	0.09%	24.8%

In this table, $\gamma = \psi^{-1} = 2$ and $\rho = 0.208$. Other parameter values excluding Z^* are summarized in Table 1.

their rare-disaster risk. As a result, the probability of default and credit spreads are high and debt capacity is low.

9 Conclusion

In this paper, we present a tractable model of sovereign debt in continuous time. It features the jump-diffusion process for output used in the rare-disasters literature, recursive preferences that separate the role of intertemporal substitution and risk aversion, and partial insurance against jump risk. The model can be solved in closed form up to an ordinary differential equation.

We show that financial development, reflected in limits to the ability to hedge jump risks, generates the debt intolerance that we see in emerging markets: low debt levels are associated with high credit spreads.

In order to focus on the impact of financial development on sovereign debt, we abstracted from two forces that could influence sovereign-credit spreads. The first, is the risk premium demanded by foreign investors to compensate their exposures to the systematic components of sovereign default risk (see, e.g. Pan and Singleton (2008), Longstaff, Pan, Pedersen, and Singleton (2011), Borri and Verdelhan (2015), and Hebert and Schreger (2016).) The second, is the moral hazard problem that is associated with insurance. We plan to address these issues in future research.

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A Appendix: Technical Details

The homogeneity property of the value functions holds for the complete and incomplete market cases, for the normal regime and for autarky.

We conjecture that the value function in the normal regime, $V(W,Y)$, is given by

$$V(W,Y) = \frac{(bP(W,Y))^{1-\gamma}}{1-\gamma} = \frac{(bp(w)Y)^{1-\gamma}}{1-\gamma}\,, \tag{A.1}$$

and the value function in the autarky regime, $\widehat{V}(Y)$, is:

$$\widehat{V}(Y) = \frac{(b\,\widehat{p}\,Y)^{1-\gamma}}{1-\gamma}, \tag{A.2}$$

where b is a constant that will be determined later. We then have

$$V_W = b^{1-\gamma}(p(w)Y)^{-\gamma}p'(w), \tag{A.3}$$

$$V_Y = b^{1-\gamma}(p(w)Y)^{-\gamma}(p(w) - wp'(w)), \tag{A.4}$$

$$V_{WW} = b^{1-\gamma}(p(w)Y)^{-1-\gamma}\left(p(w)p''(w) - \gamma(p'(w))^2\right), \tag{A.5}$$

$$V_{WY} = b^{1-\gamma}(p(w)Y)^{-1-\gamma}\left(-wp(w)p''(w) - \gamma p'(w)(p(w) - wp'(w))\right), \tag{A.6}$$

$$V_{YY} = b^{1-\gamma}(p(w)Y)^{-1-\gamma}\left(w^2p(w)p''(w) - \gamma(p(w) - wp'(w))^2\right). \tag{A.7}$$

Substituting equations (A.1) and (A.3)-(A.7) into the HJB condition (9) and simplifying, we obtain:

$$\begin{aligned}
0 = \max_{c(w),\,\theta(w),\,x(w)} \; & \frac{\left(\frac{c(w)}{bp(w)}\right)^{1-\psi} - 1}{1-\psi^{-1}}\rho p(w) + \left[\,(r + \pi(w) - \mu)\,w + 1 - c(w) - \phi(w)\,\right]p'(w) \\
& + \frac{(\theta(w)\sigma)^2}{2}\left(p''(w) - \frac{\gamma(p'(w))^2}{p(w)}\right) + \frac{\sigma^2}{2}\left(w^2p''(w) - \frac{\gamma(p(w) - wp'(w))^2}{p(w)}\right) \\
& + \theta(w)\sigma^2\left(-wp''(w) - \frac{\gamma p'(w)(p(w) - wp'(w))}{p(w)}\right) + \frac{\lambda}{1-\gamma}\mathbb{E}\left[\left(\frac{Zp(\breve{w})}{p(w)}\right)^{1-\gamma} - 1\right]p(w)\,,
\end{aligned} \tag{A.8}$$

where $\breve{w}$ is given by

$$\breve{w} = \left[(w + x(w,Z))/Z\right]\mathcal{I}_{Z\geq Z^*} + (w/Z)\,\mathcal{I}_{\underline{Z}(w)<Z<Z^*} + \underline{w}\,\mathcal{I}_{Z<\underline{Z}(w)}\,, \tag{A.9}$$

and $\phi(w) = \lambda\mathbb{E}[x(w)\,\mathcal{I}_{Z\geq Z^*}]$.

We can rewrite the first-order conditions for consumption (equation (10)) and diffusion-risk hedging demand (equation (11)) as:

$$c(w) = mp(w)(p'(w))^{-\psi}, \tag{A.10}$$

$$\theta(w) = w - \frac{\gamma p(w)p'(w)}{\gamma(p'(w))^2 - p(w)p''(w)}, \tag{A.11}$$

where $m = \rho^{\psi} b^{1-\psi}$.

The FOC for the jump risk hedging demand, $X(Z;W,Y)$, is given by equation (13). The optimal scaled hedging demand for jump risk, $x(w,Z)$, solves:

$$p'(w) = \left(\frac{Zp((w + x(w,Z))/Z)}{p(w)}\right)^{-\gamma} p'((w + x(w,Z))/Z). \tag{A.12}$$

Replacing equations (A.10) and (A.11) into equation (A.8), we obtain ODE (48) for $p(w)$. Similarly, substituting the conjectured value functions (14) and (18) into the HJB equation (17), we obtain ODE (49) for $\widehat{p}$. The value-matching condition that equates the cost of repaying debt and defaulting, given by equation (19), implies the boundary condition (51). Substituting equation (A.11) into (31), we obtain the boundary condition (52).

Intuitively, self insurance against income shocks becomes as effective as the insurance available in the FB case as $w \to \infty$.

Next, we provides some technical details for the FB case. The conjectured certainty equivalent wealth is given by $p(w) = w + h$. Replacing this value into equations (A.10), (A.11), and (A.12), respectively, we have the optimal consumption, diffusion-risk hedging demand and jump risk hedging demand:

$$c^{FB}(w) = m(w + h), \tag{A.13}$$

$$\theta^{FB}(w) = -h, \tag{A.14}$$

$$x^{FB}(w,Z) = (1 - Z)h. \tag{A.15}$$

Using $p(w) = w + h$ and equation (A.15) into the ODE (48), and using the fact that

$Z^* = 0$ in the FB, we obtain:

$$0 = \left(\frac{m-\psi\rho}{\psi-1} + \mu - \frac{\gamma\sigma^2}{2}\right)(w+h) + [(r-\mu)w+1] + \frac{\gamma\sigma^2}{2}(w+h) + \lambda(\mathbb{E}(Z)-1)h \tag{A.16}$$

$$= \left(\frac{m-\psi\rho}{\psi-1} + \mu\right)(w+h) + [(r-\mu)w+1] + \lambda(\mathbb{E}(Z)-1)h \tag{A.17}$$

$$= \left(\frac{m-\psi\rho}{\psi-1} + r\right)w + \left(\frac{m-\psi\rho}{\psi-1} + \mu - \lambda(1-\mathbb{E}(Z))\right)h + 1\,. \tag{A.18}$$

As equation (A.18) must hold for all $p(w) = w+h$, we must have $\frac{m-\psi\rho}{\psi-1} + r = 0$ which implies that $m = r + \psi(\rho - r)$ as stated in equation (38). Using the fact that $m = \rho^{\psi} b^{1-\psi}$, we obtain the formula (15) for the coefficient b. Finally, substituting $m = r + \psi(\rho - r)$ into equation (A.18), we obtain the value of h:

$$h = \frac{1}{r - [\mu - \lambda(1-\mathbb{E}(Z))]} = \frac{1}{r-g}\,. \tag{A.19}$$