Persuasion in Relationship Finance*

Ehsan Azarmsa§ Lin William Cong†

August 10, 2018

Abstract

Relationship finance features incumbent financiers' observing interim information after initial investment but before continuation decision. The entrepreneurs' endogenous information production and subsequent security issuance to both the incumbent insider and competitive outsider financiers constitute persuasion games with heterogeneous receivers and contingent transfers. Entrepreneurs' endogenous experimentation reduces insiders' information monopoly, but holds up initial relationship formation. Insiders' information production and interim investor competition mitigate the hold-up, and explain empirical links between competition and relational lending. Optimal contracts restore first-best outcomes using convertible securities for insiders and residuals for outsiders. Our findings are robust under continuum actions and partial commitments.

JEL Classification: D47, D82, D83, G14, G23, G28

Keywords: Information Design, Security Design, Bayesian Persuasion, Relationship Lending, Hold-up, Experimentation, Venture Capital, Bank Competition, Contracting.

---

*The authors are grateful to Doug Diamond and Andy Skrzypacz for detailed feedback and suggestions. They also thank Ben Brooks, Peter DeMarzo, Darrell Duffie, Piotr Dworczak, Alex Frankel, Lorenzo Garlappi, Matthew Gentzkow, Itay Goldstein, Yingni Guo, Michel Habib, Zhiguo He, Emir Kamenica, Anil Kashyap, Arthur Korteweg, Robert Marquez, Raghu Rajan, Kyoungwon Seo, Martin Szydlowski, Anjan Thakor, Lucy White, and conference and seminar participants at Berkeley Haas, Boston University, Chicago Booth, Chicago Booth Micro-Lunch, Peking Guanghua, Princeton, UBC Sauder, USC Marshall, Washington University in St. Louis, EFA Meeting, European Summer Symposium in Financial Markets, MFA Meeting, Wharton WINDS, MIT Sloan Junior Finance Conference, and Oxford Financial Intermediation Theory Conference for helpful comments and discussions. Some results in the paper are circulated under a previous draft titled "Insider Investor and Information".

§University of Chicago, Department of Economics and Booth School of Business.

†University of Chicago Booth School of Business. Authors Contact: Will.Cong@ChicagoBooth.edu.

1 Introduction

What is the benefit of raising capital from intermediaries such as banks or venture funds instead of issuing securities in public markets? A large literature on relationship finance reveals that intermediaries can mitigate informational asymmetry and moral hazard when they form relationships with entrepreneurs to become "insider" financiers (e.g., Diamond (1984, 1991), Fama (1985), Ramakrishnan and Thakor (1984), and Kerr, Nanda, and Rhodes-Kropf (2014)). Existing literature recognizes how insiders may hold up entrepreneurs due to information monopoly, but ignores the endogenous nature of information production and design.[1] Yet in reality entrepreneurs' actions not only shape project cash flows, but also alter the informational environment.[2]

Meanwhile, as a confluence of work on Bayes correlated equilibria and Bayesian persuasion, information design is arguably the most active area of research in information economics in recent years (Kamenica (2017)). It has been quickly adopted to address issues in banking regulation, online advertising, entertainment, etc. Yet its application in corporate finance has been limited because contingent transfers prevalent in security design and contracting are typically absent in extant Bayesian-persuasion models, despite the fact that relationship finance provides a most natural setting for a sender to commit to an information structure.

Several questions naturally arise. Do the source and endogenous production of information matter for sequential fund-raising? Do they affect the link between bank orientation and competition? How would the persuasion game play out in the presence of contingent transfers? What are their implications for designing securities for sequential investors? How do heterogeneous financiers as receivers learn and interact?

Motivated by these questions, we model contract and security design and interim experimentation in relationship financing as a Bayesian persuasion game with contingent transfers and heterogeneously-informed receivers. We show that the entrepreneur's endogenous information production reduces an insider's rent from her interim bargaining power, but

[1] Sharpe (1990) and Rajan (1992) discuss the hold-up issues in the context of relationship banking; Admati and Pfleiderer (1994) and Ewens, Rhodes-Kropf, and Strebulaev (2016) argue that staged financing in venture capital can also produce conflicts of interest and hold-ups and give disproportionate bargaining power to the initial venture capitalist, as shown in Fluck, Garrison, and Myers (2006).

[2] Pharmaceutical firms can affect FDA's and investors' decisions by providing additional information and tests (e.g., "Guidance for Industry and FDA Staff. Post market Surveillance Under Section 522 of the Federal Food, Drug and Cosmetic Act" May 16, 2016); Software startups decide on different markets for beta launches because the media attention generated is different, and thus information communicated to potential users ("Why Is Canada Such A Good Testing Ground For Game Releases?" Forbes Tech. Nov 27, 2012); entrepreneurs choose the specific prototype or trial market to work on which produces disparate forms of information; career concerns and managerial choices of projects exhibit similar features.

inefficiently holds up her initial investment. Relationship investors' proprietary information technology and interim competition mitigate this new form of hold-up, and interact to produce the empirically observed non-monotone patterns between interim competition and relationship formation. We then derive a contractual solution to fully restore efficient information production: the entrepreneur optimally promises early insider investors the option to purchase convertible securities in future at a pre-specified price and quantity, and upon the insider's continued financing, issues residual securities to outsider investors.

Specifically, our baseline model considers a capital-constrained entrepreneur with a project that requires two rounds of financing. The first round requires a fixed investment that enables the entrepreneur to "experiment"—broadly interpreted as conducting early-stage activities such as hiring key personnel, acquiring initial users, and developing product prototypes—to produce interim information to persuade investors for continued financing. An investor becomes an "insider" by providing the initial funding. The entrepreneur lacks ex-ante commitment to any specific production technology of interim information, and hence cannot contract on experimentation. But by monitoring and having relationship access, the insider observes interim signals from the experiment informative of the eventual profitability.

After forming the financing relationship, the entrepreneur raises additional capital in a second round by issuing securities to this insider and potentially outsider investors. The insider's informational advantage relative to outsiders gives her bargaining power, and outsiders may learn from her decisions to continue or terminate the project. Following standard persuasion games that feature divergent objectives of the sender and the receiver, we assume that the entrepreneur enjoys private benefit of continuing the project that is difficult to verify or contract upon. The entrepreneur's limited liability and the security choice also contributes to the divergence of sender-receiver objectives.

We first show that the entrepreneur follows a threshold strategy for experimentation. Through designing the informational environment, the entrepreneur makes the insider investor breaks even and indifferent between termination and continuation. On the one hand, this reduces the insider's bargaining power from her information monopoly. On the other hand, the entrepreneur's interim information production is inefficient, rendering the insider incapable of recovering the initial investment in forming the relationship. Projects may not get initial financing to start with—a phenomenon we call Information Production Hold-up (IPH). These results are in sharp contrast to existing theories on bank monitoring and hold-ups on entrepreneurial effort that ignore endogenous information production.

In industries requiring less entrepreneur-specific knowledge, "sophisticated" relationship investors can use their own information technology to evaluate projects' prospect, thus extracting positive interim rent, partially restoring the feasibility of the initial relationship investment. Relationship financing also becomes viable with moderate interim competition, because selling to competitive outsiders encourages more efficient information production by the entrepreneur. Investor competition (reflected through the insider's interim bargaining power) and sophistication (captured by the informativeness of her independent signal) jointly impact the dependence of relationship financing on competition, which is non-monotone in general. In particular, for intermediate levels of investor sophistication, the ease of relationship formation proxied by the initial funding capacity can therefore depend on interim competition in a U-shaped pattern, consistent with empirical findings that other models cannot explain (Elsas (2005) and Degryse and Ongena (2007)).

To explore contractual solutions to this general problem of IPH, we recognize that a designer may not always know the entrepreneur's private benefit and the investor sophistication ex ante. As such, we derive the *robust* optimal design of securities which entails the entrepreneur optimally giving early insiders warrants to purchase some form of convertible securities at pre-specified price and quantity, and issuing residual securities such as equities to outsiders, consistent with the practice in venture financing.

Intuitively, the entrepreneur is biased towards continuation and gets all the ex-ante surplus. At the time of forming a financing relationship, he wants to but cannot commit to efficient information production in the interim. An optimal security therefore should facilitate such a commitment, balancing the entrepreneur's payoff sensitivity to his information production either indirectly through the insider's continuation decision (continuation channel) or directly through internalizing the cost of inefficient continuation himself (payoff channel). The problem involves infinitely-dimensional nested optimization, prompting us to take a novel constructive-proof approach.

Specifically, we propose a set of contracts that restore social efficiency, and show they are the only optimal contracts. We first note that because the insider cannot fully internalize the entrepreneur's private benefit of continuation, the continuation channel is sensitive to investor sophistication. A robust design thus reserves a large enough proportion of second-round financing for competitive outsiders, so that the payoff channel dominates. By giving the insider debt-like securities in bad states of the world, convertible securities maximize the entrepreneur's exposure to the cost of inefficient continuation. Internalizing this cost leads to more efficient information production. Meanwhile, relationship financing is feasible as long as the contract yields the insider enough interim rent in order to recover the investment in the initial round, leaving the security design indeterminate in good states of the world. Consequently, this optimal design uses warrants that specify both the quantity and terms for the insider to purchase a large class of convertible securities, consistent with real-life observations.

Finally, we discuss how our findings remain robust when we allow partial commitment to information design and scalable investment, among others. We also discuss who should enjoy the right for experimentation design. The information production hold-up problem manifests itself under alternative security forms as well, and the economic mechanism apply even beyond relationship lending and staged venture financing. Our study helps underscore and formalize this practical issue, and develops potential contractual solutions. From a theory perspective, our study also sheds light on Bayesian Persuasion games with contingent transfers and sequential heterogeneous receivers, and deepens our understanding of contracting under endogenous information production.

Literature — Our theory foremost contributes to the large literature on relationship financing. Boot (2000), Gorton and Winton (2003), and Srinivasan et al. (2014) survey relationship lending. Theoretical studies on relationship banking focus on information production and control (e.g., Diamond (1984, 1991) and Fama (1985)): while relationship financing can improve financing efficiency (e.g., Petersen and Rajan (1994)), it naturally induces information monopoly (Berger and Udell (1995), Petersen and Rajan (2002), and Rajan (1992)), holding up the entrepreneur's effort in relationship lending (e.g., Santos and Winton (2008) and Schenone (2010)) and venture capital (Burkart, Gromb, and Panunzi (1997) and Ewens, Rhodes-Kropf, and Strebulaev (2016)). We inform the debate by endogenizing the informational environment and analyzing the information production hold-up problem.[3]

Our paper also sheds light on the role of intermediaries and security design in financing innovation (Da Rin, Hellmann, and Puri (2011) and Kortum and Lerner (2001)). Importantly, we add to earlier studies on the extensive use and optimality of convertible securities (e.g., Gompers (1997), Kaplan and Strömberg (2004), and Hellmann (2006)) by endogenizing information design. Instead of deriving the optimality of convertible securities from ex-ante

[3] In this regard, our paper relates to hold-up problem in general, e.g., in Hart and Moore (1988). The contractual solution offered therein and in Aghion, Dewatripont, and Rey (1994), and Nöldeke and Schmidt (1995) do not induce first-best outcomes in our setting due to limited liability. Von Thadden (1995) and Nöldeke and Schmidt (1998) use option contracts to resolve hold-up problems under exogenous information and achieves the first best only under sufficiently low project uncertainty. We endogenize interim information for general project uncertainties.

asymmetry between the issuer and investors (e.g., Stein (1992) and Brennan and Schwartz (1988)), our model features symmetrically informed entrepreneurs and investors and studies informational issues in staged financing. It therefore closely relates to the seminal paper of Cornelli and Yosha (2003), with the key distinction that instead of examining hidden manipulations of non-verifiable signals under an exogenously given information structure, we emphasize endogenous information production (verifiable to the relationship financier) and sequential securities for heterogeneous investors. To our best knowledge, we are the first to show that issuing convertible securities to early insider investors and equities to later outsider investors is optimal and robust. Moreover, while they show that *one* design—convertible securities—can address the issue of window dressing, we characterize *all* robust optimal designs under general security and contract space, and demonstrate they all entail the use of convertible securities.

Empirically, the effect of competition on bank orientation has received significant attention, and Elsas (2005) and Degryse and Ongena (2007) document a puzzling U-shaped effect of market concentration on relationship lending. Extant theories predict either opposing monotone patterns (e.g., Petersen and Rajan (1995) and Dell'Ariccia and Marquez (2004) versus Boot and Thakor (2000) and Dinc (2000)) or suggest a hump-shaped pattern (e.g., Yafeh and Yosha (2001) and Anand and Galetovic (2006)). Our theory offers a first explanation for the empirical findings.

From a theory perspective, our paper contributes to the field of information design (Bergemann and Morris (2017) and Hörner and Skrzypacz (2016)), especially Bayesian Persuasion (Kamenica and Gentzkow (2011) and Ely (2017)). We take a linear programming approach similar to Bergemann and Morris (2017), but allow infinite payoff-relevant states and privately informed receivers. We relax the assumption that the sender's utility from a message completely depends on the expected state (Kolotilin (2017)), or the sender's payoff over the receiver's actions is independent of the state (Gentzkow and Kamenica (2016)). Our discussion on investor sophistication also expands our knowledge about persuasion games with private receiver types.[4]

Importantly, our paper contributes to the literature by featuring security design that endogenizes the dependence of the sender and receiver' payoffs on the state. A closely

[4]Kolotilin (2017) finds a non-monotone effect of the receiver's signal informativeness on sender and receiver's utilities. We differ in that we do not restrict the insider's and the entrepreneur's information production to be independent. When the receiver's type can be flexibly correlated with the signal of a sender's experiment, we restore the intuitive result that the sender's and receiver's utilities are monotone in the receiver's signal precision.

related paper is Szydlowski (2016), which also allows contingent transfers using securities and derives an irrelevant result of security choice when the entrepreneur jointly designs disclosure and security. Our paper is the first to apply information design (and jointly with security design) to relationship financing and contracting under information production hold-up. We also add to the literature by allowing interaction of multiple asymmetrically informed receivers in Bayesian persuasion games.

By so doing, our paper advances the emerging applications of information design in finance. Other recent studies concern topics on capital structure (Trigilia (2017)), government intervention (Cong, Grenadier, and Hu (2017)), and stress tests (Bouvard, Chaigneau, and Motta (2015), Goldstein and Leitner (2015), and Orlov, Zryumov, and Skrzypacz (2017)). One main challenge of the field has been the commitment issue in designing information. We overcome the challenge both by using dynamic relationship to naturally induce commitment and by demonstrating the robustness of our findings to partial commitment.

Finally, our discussion on contracting and security design contrasts with seminal studies such as Holmstrom (1979) and Innes (1990), which concern agents' actions that alter the distribution of cash flows only.[5] The agent in our setting shapes the informational environment, and contracting on continuation and eventual outcomes restores the first best information production—typically unattainable in conventional settings. Our solution approach using a constructive-proof and robustness argument also differs from earlier studies.

2 Relationship Financing and Information

2.1 Model Set-up

Consider a three-period economy with time index $t = 0, 1, 2$. A risk-neutral entrepreneur has a project that requires a fixed investment $I \in (0, 1)$ at $t = 1$, and produces an uncertain cash-flow $X \in [0, 1]$ at $t = 2$ with a prior distribution denoted by a continuous and atomless pdf $f(X)$. In the baseline model, the entrepreneur issues an exogenously given security $s(X)$ at $t = 1$ to finance the project. We assume double-monotonicity for the security, i.e., both $s(X)$ and $X - s(X)$ are weakly increasing in X.[6] The entrepreneur receives a private benefit

[5] Although extant studies on the agency issues in costly experimentation such as Hörner and Samuelson (2013) and Bergemann and Hege (1998) consider information-acquisition effort, the principal cannot isolate information produced by the agent (hidden effort and hidden information).

[6] See, for example, Nachman and Noe (1994), DeMarzo and Duffie (1999), DeMarzo, Kremer, and Skrzypacz (2005) and more recently, Cong (2017). If such monotonicity is violated, either the entrepreneur or

$\varepsilon \in (0, I)$ if the project is financed at $t = 1$, which we assume to be non-contractible because in practice it is hard to quantify or verify (Aghion and Bolton (1992); Dyck and Zingales (2004)).[7] Most of our results are driven by the entrepreneur's limited liability, but this assumption constitutes a realistic source of agency conflict and enables us to discuss robust security design. Moreover, we assume $\mathbb{E}[s(X) - I | X \geq I - \varepsilon] > 0$ to ensure the security can cover the cost of investment when investment is socially optimal. Finally, there is no time discounting.

To best illustrate our economic mechanism and match reality for early business startups, we assume $\mathbb{E}[X - I + \varepsilon] < 0$—an innocuous assumption that implies the absence of direct financing by arms-length investors ex ante, a case for which relationship financing and informational considerations are the most important. That said, with an initial investment $K > 0$ at $t = 0$ the entrepreneur can design an experiment that generates interim information about the distribution of the cash-flow. Specifically, using K received from an initial financier, the entrepreneur (the sender) chooses a finite set of messages $\mathcal{Z}$ and a mapping between the messages and the outcomes, which can be specified as conditional probabilities $\pi(z|X)$ for $z \in \mathcal{Z}$ and $X \in [0, 1]$.[8]

There are competitive investors who can finance the project at $t = 0$ by paying K and one is randomly chosen and becomes an "insider". One may think of $K + I$ as the total investment needed, but raised in stages whereby early experimentation generates interim information (Kerr, Nanda, and Rhodes-Kropf (2014)). For simplicity, we assume the seed investment K generates negligible initial cash flows compared to the final payoff expected by the investor and the entrepreneur, which is similar to normalizing the liquidation value to

the investor can be better off destroying some surplus for some state X, as pointed out in Hart and Moore (1995).

[7] ε could correspond to his utility from the "non-assignable control rent"(Diamond (1993)), or payoff from assets- or business-in-place (Myers and Majluf (1984)), or perquisites that managers appropriate (Jensen and Meckling (1976), see also Winton and Yerramilli (2008); Szydlowski (2016)). Regarding its non-contractibility, for example, the investor cannot promise to pay ε upon terminating the project because otherwise it would lead to entry of fly-by-night firms. Such an arrangement not only leads to adverse selection when ε is privately known by the entrepreneur, but also leads to equilibrium multiplicity and indeterminacy of the informational environment.

[8] We follow the Bayesian Persuasion literature to assume that $(\mathcal{Z}, \pi)$ can be arbitrarily informative, which is justified by interpreting X as the most informative signal the entrepreneur can generate while conducting its usual entrepreneurial activities (Kamenica and Gentzkow (2011)). Furthermore, even though experimentation costs differ significantly across sectors and industries (Nanda and Rhodes-Kropf (2015)), the dispersion within a product category is much smaller, and the cost of experimentation has declined dramatically, particularly in industries such as software and digital media that have benefited from the development of the Internet, because fixed investments in infrastructure and hardware are no longer necessary (e.g., Kerr, Nanda, and Rhodes-Kropf (2014), Palmer (2012), and Blacharski (2013)).

zero (Diamond (1993)) and can be equivalently interpreted as K representing the investment net of initial cash flows.

We assume that all investors observe $(\mathcal{Z}, \pi)$, but with probability $\mu \in [0,1]$ only the insider observes the outcome of the experiment, and with probability $1-\mu$ all investors publicly observe the outcome. μ essentially indicates the extent of the insider's informational advantage through close monitoring and repeated interactions, and we can interpret $1-\mu$ as a reduced-form measure of the "interim competition" between the insider and the outsider commonly modeled in the relationship lending literature (e.g., Petersen and Rajan (1995)). $\mu = 0$ corresponds to perfect interim competition and $\mu = 1$ corresponds to information monopoly by the insider.

After the realization of z, the insider makes a take-it-or-leave-it offer to purchase a λ fraction of the security $s(\cdot)$ at a total price p^I. The entrepreneur decides whether to accept and then if he still needs financing, he sells the security to outsiders who offer a competitive total price p^O for the remaining securities. The investment takes place if and only if I is successfully raised, otherwise the pledged capital is returned to investors.

The players' interim payoffs after the formation of a financing relationship are as follows:

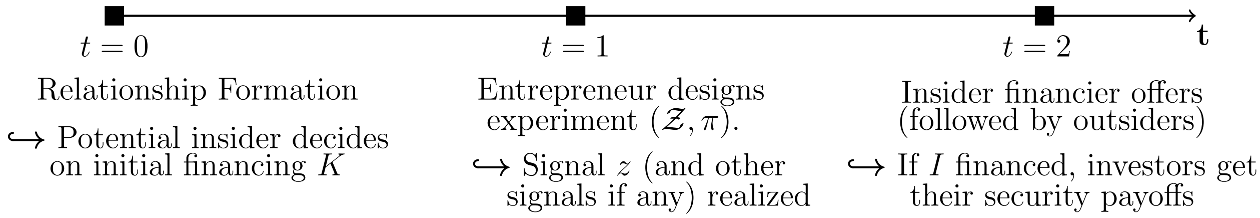
$$
\begin{aligned}
p^O &= \mathbb{E}[(1-\lambda)s(X)|\mathcal{F}^O] && (1)\\
u^E(p^I, p^O; X) &= \left(\varepsilon + X - s(X) + p^I + p^O - I\right)\mathbb{I}_{\{p^I+p^O \geq I\}} && (2)\\
u^I(p^I, p^O; X) &= \left(\lambda s(X) - p^I\right)\mathbb{I}_{\{p^I+p^O \geq I\}}, && (3)
\end{aligned}
$$

where $\mathcal{F}^O$ denotes the outsiders' information set after having observed the insider's continuation or termination action.[9] In addition to the informational advantage, we analyze in Section 3 the case whereby the insider can contract with the entrepreneur at the formation of the financing relationship, potentially using different securities for the insider and outsiders.

To highlight the stark effect of the **information production hold-up (IPH)** problem, we assume that only the entrepreneur has the relevant skill and expertise to design $(\mathcal{Z}, \pi)$ after raising K. This happens when the lender either has no previous experience on the project or it is too costly for him to extract information, e.g. the firm is located in a hardly accessible location, or the investor has no relevant expertise to generate independent signals. Section 4 relaxes the assumption and allows the insider to produce information too.

[9] One can interpret the signal being either public or private as the receiver's private type. Note that the entrepreneur's experimentation affects both the insider and the outsiders' actions, different from Kolotilin, Mylovanov, Zapechelnyuk, and Li (2017) which studies the case of a single receiver with private types.

Figure 1: Timeline of the game.



If the project is not financed, all players receive outside options normalized to zero. Intuitively, relationship financing is feasible only if the insider can recover in expectation at least the initial investment K, i.e.,

$$\mathbb{E}[u^I(p^I, p^O; X)] \geq K. \tag{4}$$

Finally we assume $\mathbb{E}[(X - I)\mathbb{I}\{X \geq I - \varepsilon\}] \geq K$, i.e., the project would always be financed through relationship financing (positive NPV ex ante to the financier) if the interim information production is socially efficient. Therefore, the failure to form the financing relationship is purely driven by the entrepreneur's endogenous information production, which differs from the social optimal in general. The timeline of the game is summarized in Figure 1.

2.2 Equilibrium

We work backward by first analyzing the interim persuasion game after the formation of the financing relationship. We discuss exogenous experimentation as a benchmark in order to underscore the **information production hold-up (IPH)** problem under endogenous information production and the way it drastically alters our understanding of relationship finance.

Benchmark with Exogenous Information

Consider an exogenously given $(\mathcal{Z}, \pi)$ that earlier studies specialize to. If signal z is privately observed by the insider, then when $\mathbb{E}[s(X)|z] \geq I$, the insider offers $p^I = I$ to finance the project entirely ($\lambda = 1$ endogenously), assuming any indifference (when $\mathbb{E}[s(X)|z] = I$) is broken by financing more ($\lambda = 1$); else when $\mathbb{E}[s(X)|z] < I$, the insider terminates the project, leading to the outsiders negatively updating their priors and not investing either. The insider's information monopoly essentially gives her full bargaining power over the contractible interim surplus generated, $\max\{\mathbb{E}[s(X)-I|z],0\}$, which corresponds to the well-known information hold-up in earlier models such as Rajan (1992).

In the case where z is publicly observable (which happens with probability $1-\mu$), then both the insider and the outsiders would offer the competitive $p^I = p^O = \mathbb{E}[s(X)|z]$ when $\mathbb{E}[s(X)|z] - I \geq 0$. The entrepreneur extracts the whole interim surplus.

The entrepreneur's expected payoff is thus

$$\begin{aligned} U^E(\mathcal{Z},\pi) &= \mathbb{E}[u^E] = \mu \int_0^1 \sum_{z\in\mathcal{Z}^+} (\varepsilon + X - s(X))\pi(z|X)f(X)dX \\ &\quad +(1-\mu)\int_0^1 \sum_{z\in\mathcal{Z}^+} (\varepsilon + X - s(X) + \mathbb{E}[s(X)|z] - I)\pi(z|X)f(X)dX \\ &= \int_0^1 \sum_{z\in\mathcal{Z}^+} (\varepsilon + X - I)\pi(z|X)f(X)dX - \mu\int_0^1 \sum_{z\in\mathcal{Z}^+} (s(X) - I)\pi(z|X)f(X)dX \end{aligned} \tag{5}$$

where $\mathcal{Z}^+ = \{z|\mathbb{E}[s(X)|z] \geq I\}$ corresponds to the set of signals that justify the investment. Equation (5) follows from applying the law of iterative expectation to $\mathbb{E}[\mathbb{E}[s(X)|z]|z \in \mathcal{Z}^+]$. Similarly, the insider's payoff is:

$$U^I(\mathcal{Z},\pi) = \mathbb{E}[u^I] = \mu\int_0^1 \sum_{z\in\mathcal{Z}^+} (s(X) - I)\pi(z|X)dX \tag{6}$$

Equations (5) and (6) reveal that for a given experiment, the entrepreneur's expected payoff is decreasing and the insider's expected payoff is increasing in μ—a measure of the level of the insider's information monopoly, confirming the results in Petersen and Rajan (1995) that less interim competition leads to higher possibility of financing relationship. By juxtaposing equations (4) and (6), we see that financing relationship is feasible only when the interim rent is sufficiently high (e.g., Nanda and Rhodes-Kropf (2013)), i.e., $\mu\mathbb{E}[(s(X) - I)\mathbb{I}_{\{\mathbb{E}[s(X)|z]\geq I\}}] \geq K$.

We next show that these celebrated results have to be modified once we model the endogenous interim information production and the persuasion game played by the entrepreneur.

Endogenous Experimentation and Information

Here we show that the insider's payoff is not monotone in μ with endogenous information production. In particular, when the insider enjoys information monopoly (μ approaches 1), her *initial investment* to form the relationship is held-up. This *reverse* hold-up is in sharp contrast with the traditional hold-up under exogenous information in the previous literature on relationship finance—in the traditional hold-up, the possibility of the relationship financing is the highest when $\mu = 1$, and the entrepreneur's *effort* is held up.

According to (5), the entrepreneur solves the following maximization problem

$$\max_{(\mathcal{Z},\pi)} \mathbb{E}[(\varepsilon + X - \mu s(X) - (1-\mu)I)\mathbb{I}_{\{\mathbb{E}[s(X)|z]\geq I\}}] \tag{7}$$

Proposition 1 (Endogenous Information). *An optimal experiment for the entrepreneur uses two signals, i.e., $|\mathcal{Z}| = 2$. It generates a signal h to induce investment if $X \geq \max\{\bar{X}, \hat{X}(\mu)\}$, and otherwise generates a signal l to induce termination, where $\bar{X}$ and $\hat{X}(\mu)$ solves:*

$$\begin{aligned} \mathbb{E}[s(X)|X \geq \bar{X}] - I &= 0 && (8)\\ \varepsilon + \hat{X}(\mu) - \mu s(\hat{X}(\mu)) - (1-\mu)I &= 0 \quad \text{if } \varepsilon - (1-\mu)I < 0 && (9)\\ \hat{X}(\mu) &= 0 \quad \text{if } \varepsilon - (1-\mu)I \geq 0 && (10) \end{aligned}$$

Moreover, all optimal experiments lead to the same investment and payoffs, rendering the equilibrium essentially unique.

Proposition 1 characterizes the optimal experimentation after relationship formation. (8) indicates that the continuation based on $X \geq \max\{\bar{X}, \hat{X}(\mu)\}$ makes the insider financier at least break even; (9) indicates that if the private benefit is small relative to the competition, the entrepreneur rationally induces the continuation at X if and only if he can break even; (10) just says that if the private benefit is large relative to interim competition, the entrepreneur always benefit from the continuation, and the threshold is again pinned down by $\bar{X}$ in (8). In equilibrium, while all profitable projects receive continued financing, some inefficient ones do as well due to the entrepreneur's persuasion. When the insider privately observes the experiment outcome, she bears the cost of inefficient continuation and the entrepreneur would like to lower the threshold for investment for his private benefit; but as μ decreases, the entrepreneur's chance of getting a competitive price is higher, helping him better internalize the cost of inefficient continuation. These trade-offs determine the optimal threshold. In the extreme case $\mu = 0$, the entrepreneur sends a high signal for $X \geq I - \varepsilon$, which implements the socially efficient outcome; at the other extreme $\mu = 1$, the entrepreneur decreases the threshold to make the insider indifferent between investment and termination

$(\mathbb{E}[s(X)|z] = I)$.[10]

Corollary 1 (**Information Production Hold-up** (IPH))**.** *For a given* K *and security* $s(\cdot)$*, there exist* $0 < \mu^l < \mu^h < 1$ *such that when* $\mu \in [0, \mu^l) \cup (\mu^h, 1]$*, relationship financing is not feasible.*

While it is straightforward that too much interim competition prevents initial investment from potential insiders, surprisingly with full information monopoly the insider financier could still be held-up. This is because no interim rent would accrue to her under endogenous information production by the entrepreneur. As we show in Section 1, IPH is not driven by the fact that we cannot contract on the verifiable cash flow X, but is driven by the endogenous nature of information production. Contrasting the Corollary with (5) and (6), it should also be apparent that taking information as exogenous in relational financing is not an innocuous assumption—whether information production is endogenous determines whether the entrepreneur or the investor has interim bargaining power.

3 Contractual Solution for IPH

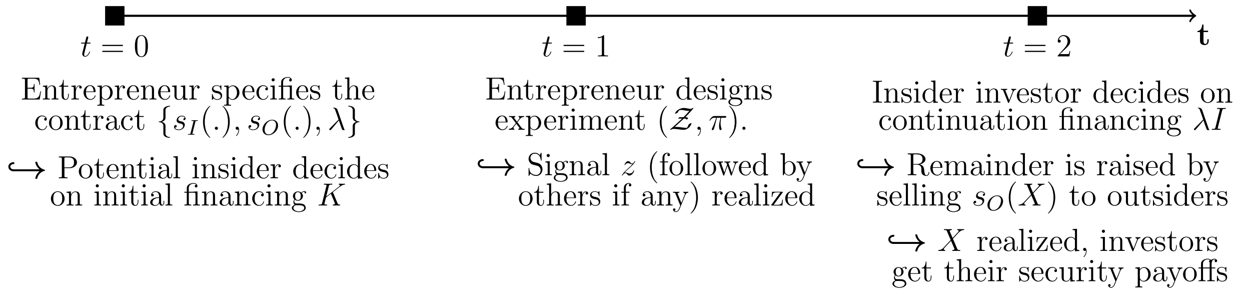
So far we have assumed that the entrepreneur only issues securities at $t = 1$. In this section, we allow long-term contracts at $t = 0$ that specifies the amount of the securities the insider can purchase at $t = 1$ and their payoff.

We assume that the security payoff can depend on X, but not on $(\mathcal{Z}, \pi)$ or the interim signals, which is standard in the Bayesian persuasion literature. It is also realistic to assume that contracts can be contingent on verifiable cash flows but cannot be contingent on the actions that generate information in a particular structure, because the information design space is rich and complex such that the entrepreneur cannot commit to it ex ante due to contract incompleteness. For example, an angel investor does not know what kinds of team members a founder is going to assemble, or what kind of field-specific experiments to conduct.

Moreover, interim signals are often not well-defined and too costly to verify to be useful for security contracts. For example, in Kaplan and Strömberg (2003), the sample of venture capital financing contracts involve contingencies only on an IPO taking place and satisfying certain conditions (e.g. regarding the issue price, proceeds, or market capitalization), but

[10]In Appendix B4, we show that IPH can also reduce the effort distortion introduced by Rajan (1992). Here we focus on information production.

Figure 2: Timeline of the game with optimal security design.



never on other observable and potentially verifiable interim information such as sales revenue, market share, or profitability.

3.1 Long-term Contracts

By long-term contracts, we mean that at time 0, the entrepreneur can contract on λ, the fraction of investment I to be financed from the insider in the second round, and the corresponding payment to the insider $s_I(\cdot)$. Outsiders observe the insider's decision, and finance the remaining $(1-\lambda)I$ by purchasing securities $s_O(\cdot)$ at a competitive price. Therefore, every contract can be summarized by the triplet $\{s_I(\cdot), s_O(\cdot), \lambda\}$. Figure 2 displays the timing of the interactions.

Before we allow endogenous security design, we first restrict our long-term contracts to utilize exogenous security type in the next lemma. For example, if the security type is exogenously restricted to debt contracts, then $s(\cdot)$ is simply pinned down by the face value; if it is restricted to equity contracts, then $s(\cdot)$ is specified by the number of shares. These restricted contracts resolve the traditional information hold-up problem (e.g., Von Thadden (1995)), but not IPH. Therefore, it is really the endogenous security design that helps restore the first-best outcome.

Lemma 1 (Contracting with Exogenous Security Design)**.**

(a) For any given security $s(\cdot)$, without outsider investors ($\lambda = 1$), the equilibrium is unique and no project is financed at $t = 0$ unless $K = 0$.

(b) For any given security $s(\cdot)$, $\exists \lambda \in (0, 1)$ such that the insider receives positive interim rent under the contract $\{\lambda s(\cdot), (1-\lambda)s(\cdot), \lambda\}$.

(c) No contract with a single form of security (including debt, equity, or call options) for both the insider and outsiders implements the first best social outcome.

Note that contract $\{\lambda s(\cdot), (1-\lambda)s(\cdot), \lambda\}$ allows the insider financier to purchase λ fraction of the securities issued at $t = 1$, and sells the remainder to competitive arms-length investors. Part (a) extends Proposition 1 and Corollary 1 to the case where the entrepreneur commits to a long-term contract and consequently the insider does not have monopoly power during the interim about the terms of the contract. This means our results do not rely on the insider investor's having information monopoly.

Part (b) reveals that the insider's interim rent becomes positive for some $\lambda < 1$. This again reflects the importance of "second-sourcing" the investment I (e.g., Farrell and Gallini (1988)), i.e., committing to $\lambda < 1$, because the interim rent is decreasing in λ when λ is big. Even the insider investor has the incentive to decrease his shares of the surplus somewhat to enable the entrepreneur to internalize the cost of inefficient continuation, and thus creating a larger social surplus. Note that although λ here has a similar effect as μ, it is a contract design parameter rather than an exogenous friction.

Finally, Part (c) reveals that long-term contracting with a single form of security cannot restore social efficiency. As we show next, only a set of carefully designed securities which differ for the insider and outsiders can resolve the problem associated with information-production hold-up.

3.2 Robust Optimal Security Design and Contracting

Now we fully allow endogenous design $s_I(X)$ for the insider and $s_O(X)$ for the outsiders.

First, consider the designs that result in the outsiders investing only if the insider continues. We later show that designs outside this set cannot be optimal. Note that the entrepreneur's expected payoff from the contract (s_I, s_O, λ) is then given by:

$$U^E = \int_0^1 (\varepsilon + X - s_I(X) - s_O(X) + p - (1-\lambda)I)\mathcal{I}(X)f(X)dX, \tag{11}$$

where p is the amount raised from the outsiders by selling the security $s_O(X)$, and $\mathcal{I}$ is the probability of investment at state $X \in [0, 1]$. The outsiders' information set contains the public signal z (if any) they receive with probability $1 - \mu$, and the inference from the insider's action of continuation or termination.

Outsiders, being competitive, pay a "fair price" p, given by

$$p = \frac{1}{\mathbb{E}[\mathcal{I}(X)]} \int_0^1 s_O(X)\mathcal{I}(X)f(X)dX \tag{12}$$

Combining (11) and (12) we rewrite

$$U^E = \int_0^1 M(X; s_I, s_O, \lambda)\mathcal{I}(X)f(X)dX,$$

where

$$M(X; s_I, s_O, \lambda) = \varepsilon + X - s_I(X) - (1-\lambda)I. \tag{13}$$

The entrepreneur's optimal design then corresponds to the following maximization problem:

$$\begin{aligned} &\max_{s_I(.), s_O(.), \lambda} \mathbb{E}[M(X; s_I, s_O, \lambda)\mathcal{I}^*(X)] \\ &s.t. \quad \mathbb{E}[(s_I(X) - \lambda I)\mathcal{I}^*(X)] \geq K \quad and \quad s_I(X) + s_O(X) \leq X \quad \forall X \in [0,1], \end{aligned} \tag{14}$$

where the optimization is over the set of designs and the option to walk away from the financing relationship, and the constraints are the IC of the insider to form relationship and the entrepreneur's limited liability. $\mathcal{I}^*(\cdot)$ is the equilibrium investment function under the optimal experiment $(\mathcal{Z}^*, \pi^*)$, and is given by

$$\mathcal{I}^*(X) = \sum_{z \in \mathcal{Z}} \pi^*(z|X)\mathbb{I}_{\{\mathbb{E}[s_I(X) - \lambda I|z] \geq 0\} \cap \{\mathbb{E}[s_O(X) - (1-\lambda)I|\mathcal{F}^O] \geq 0\}}$$

and the optimal experiment given the contract $\{s_I(\cdot), s_O(\cdot), \lambda\}$ solves the following:

$$\begin{aligned} &\max_{(\mathcal{Z}, \pi)} \int_0^1 M(X; s_I, s_O, \lambda)\mathcal{I}(X)f(X)dX \\ &where \quad \mathcal{I}(X) = \sum_{z \in \mathcal{Z}} \pi(z|X)\mathbb{I}_{\{\mathbb{E}[s_I(X) - \lambda I|z] \geq 0\} \cap \{\mathbb{E}[s_O(X) - (1-\lambda)I|\mathcal{F}^O] \geq 0\}\}} \end{aligned} \tag{15}$$

The problem involves infinitely-dimensional nested optimization, and the solution methodologies in conventional contracting and security design problems do not easily apply. We instead take a constructive-proof approach by conjecturing the optimal designs and show this set of designs uniquely achieve the first-best outcome and are indeed optimal in the sense that they maximize the entrepreneur's ex-ante payoff for all $\varepsilon < I$.

Proposition 2. ***(Robust Optimal Design)*** *Optimal design exists implements the first-best social outcome. All optimal designs entail experiments that generates continuation if and only if $X \geq I - \varepsilon$. Moreover, the set of optimal securities that are independent of the entrepreneur's private benefit for continuation all involve the use of convertible securities,* *and are described by* $\{s_I(\cdot), s_O(\cdot), \lambda\}$ *satisfying the following conditions*

$$\lambda \in \left[0, \frac{I - \bar{\varepsilon}}{I}\right], \quad \textit{where} \quad \mathbb{E}\left[(X - I)\mathbb{I}_{\{X \geq I - \bar{\varepsilon}\}}\right] = K, \tag{16}$$

$$s_I(X) = \min\{\lambda I, X\}, \quad \forall\, X < I \tag{17}$$

$$\mathbb{E}[(s_I(X) - \lambda I)\mathbb{I}_{\{X \geq I\}}] = K \tag{18}$$

$$s_O(X) = X - s_I(X), \quad \forall\, X \in [0, 1] \tag{19}$$

Equation (16) reflects the partial indeterminancy of the optimal design because λ can take on a range of values. Equation (17) requires that in the bad states of the world, the security is debt-like. In fact, the shape of the security in the region $X < I - \bar{\varepsilon}$ is indeterminate, but in terms of payoffs, they are equivalent, thus the word "essentially" in how we characterize the securities. (18) ensures that the insider breaks even ex ante, but leaves the security shape indeterminate. Notice when we achieve the first-best social outcome, the insider gets paid only when the project is continued $X \geq I - \varepsilon$. But payoff is zero in $[I - \varepsilon, I)$ anyway as ε cannot be contracted upon, therefore the security payment to the insider is essentially independent of ε. Finally, (19) simply indicates limited liability for outsiders.

Figure 3 provides a concrete illustration using convertible notes for the insider and equities for the outsiders. The dotted line indicates the threshold for continuation vs termination signals.[11]

Because the inefficiency lies in the continuation of bad projects, what matters is how the security allocates the downside exposure (when $X + \varepsilon < I$) between the insider and the entrepreneur (partially through outsider investors if they are present). Giving more exposure to the entrepreneur helps him to internalize the cost of inefficient continuation through reducing his payoff upon continuation (*payoff channel*). Moreover, we require that the design is robust to the entrepreneur's private benefit from continuation, ε.

Such an optimal design should maximize the entrepreneur's downside risk relative to the insider. Therefore the entrepreneur commits to a large enough $1-\lambda$ which exposes his payoff to inefficient continuation. To make his payoff most sensitive to his information production, the entrepreneur uses debt-like contracts in the flat region in Figure 3 to give all the cash flow to the insider in the bad states of the world, effectively committing himself to bear the cost

[11] In Appendix B1, we show equity or debt could be optimal when the entrepreneur is constrained to issue the same security for the insider and the outsiders ($s_I(X) = \lambda s(X)$, $s_O(X) = (1 - \lambda)s(X)$, for some $s(X)$) due to some regularity reasons.

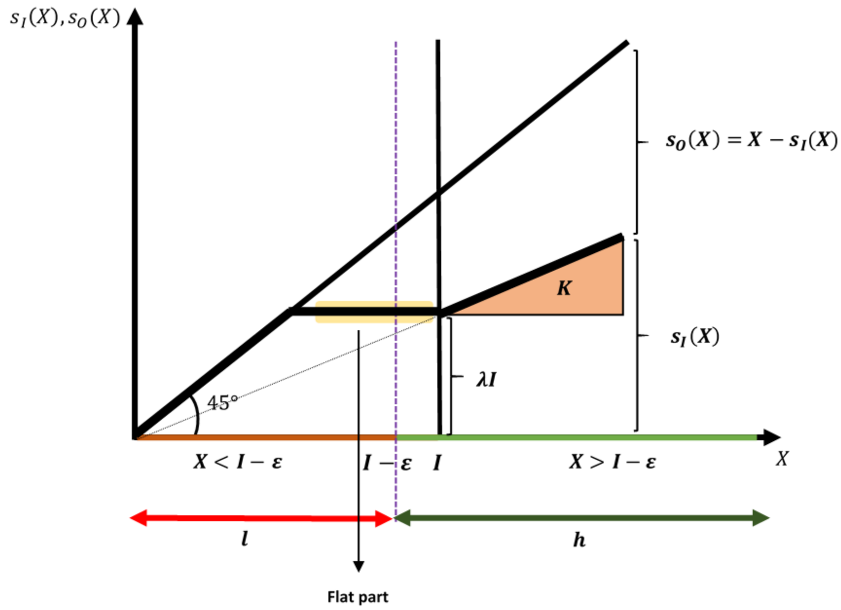


Figure 3: The optimal security under flexible security design.

of inefficient continuation as much as possible, which leads to the first-best outcome.[12] That said, the allocation of the upside exposure only needs to ensure the insider earns enough from the second round to cover the initial investment K. This means the optimal security is indeterminate, despite that the endogenous experimentation yields a unique informational environment that is also socially optimal. The optimal security is partially determinate is a strength rather than a weakness of our model, because it reflects a concept akin to the Modigliani-Miller Theorem: whatever security design achieves can also be achieved with information design. At the same time, it clarifies that optimal designs have to entail some form of convertible securities, consistent with real practice.

Optimal Security and Contract in Practice

Our goal here is not to introduce an alternative mechanism or competing theory for the use of convertible securities or to claim the information design channel is the most dominant. Beyond deriving the optimal security under our setting, Proposition 2 indicates that earlier studies' conclusions are robust to introducing endogenous and flexible information production. Moreover, we characterize the joint optimal securities for both initial insider investors and arms-length outsiders, which extant theories do not discuss, yet is prevalent in

[12]In Appendix B3, we discuss how our contractual setting relates to the classical literature on contracting and moral hazard.

practice.

We can interpret the initial-round optimal contract as warrants to the insider to purchase convertible securities in the later round. Conditional on the insider's follow-on investment, the entrepreneur raises the remaining $(1-\lambda)I$ from outsiders by issuing residual securities. Alternatively, we can view the initial round as issuing $\frac{K}{K+\lambda I}$ fraction of convertible securities to the insider, together with warrants allowing the insider to purchase the remaining fraction at a price λI in a later round. One more interpretation is that $K+\lambda I$ is the total amount of financing at the initial round, but paid out in stages, and $(1-\lambda)I$ is a separate issuance to the outsiders. In all these scenarios, although round financing and stage financing in general differ (Cuny and Talmor (2005)), the effect of the optimal design remains the same.

In addition, setting λ can be viewed as designing a later-stage syndication. While the lead investor finances λI, the remainder is financed by syndicate members that observe the leader's action and are competitive.[13]

No matter which interpretation we take, as the cost of initial experimentation goes up, the entrepreneur needs to commit to finance a higher fraction of the follow-on investment from the insider. Take for example a commonly observed security — convertible notes with conversion rate 1:I, i.e., a bond with face value λI can be converted to λ share of equity. One can show that λ is strictly increasing in K.

Finally, we note that the optimal design is partially indeterminate and include most convertible securities used in real-life practice, which other models often cannot account for.

4 Investor Sophistication

So far the financiers rely solely on the entrepreneur's experimentation for learning. In reality, a relationship financier may dictate entrepreneur's information-production activities, or receive additional information beside what the entrepreneur produces. For example, the insider may experiment herself, or use her proprietary business experience and expertise to predict the market demand or project valuation in future financing rounds. Importantly, the insider can set milestones in the initial contract, in which the insider conditions the next round of funding on pre-specified achievements and accomplishments. In fact, we can

[13] Indeed, with a few exceptions all later-stage venture capital investments are syndicated (Lerner (1994)), and partnership agreements often pre-commit venture capitalists to syndicate later-stages of investments (Sahlman (1990)). In this sense, syndication not only can protect the entrepreneur from ex post hold-up by investors and thereby encourage effort (Fluck, Garrison, and Myers (2006)), it also encourages more efficient information production.

interpret the milestones as some "interim signals" that give information about the final cash-flow. We collectively refer to the ability of a relationship financier to utilize such information production technology exogenous to the entrepreneur's design as "*investor sophistication*".

In this section, we show investor sophistication mitigates IPH and discuss how its interaction with interim competition helps us rationalize puzzling empirical patterns in the formation of lending relationships. Then, we show the contractual solution in Proposition 2 is robust to the level of the insider's sophistication. By doing so, we also develop the solution for a Bayesian persuasion game involving multiple heterogeneous receivers with private types.

To incorporate the insider's information production, we allow her to use an exogenous technology to produce an interim signal about the final project cash-flow X through the financing relationship (not observable to the outsiders). In particular, the insider uses experiment $(\mathcal{Y}, \omega_q)$, where $\mathcal{Y} = \{y_1, y_2, \ldots, y_m\}$ is a finite set with m signals. $\omega_q(y_i|X)$ represents conditional distributions. $q \in [0, 1]$ is an index that ranks the informativeness of different experiments, with $q = 0$ for an uninformative experiment. Specifically, for $q > q' \in [0, 1]$, the experiment $(\mathcal{Y}, \omega_q)$ is more informative than $(\mathcal{Y}, \omega_{q'})$ in the sense of Blackwell (1951).[14]

Moreover, for every q, we assume the signals in $\mathcal{Y}$ can be ranked: for every $m \geq i > i' \geq 1$, distribution $f(X|y_i)$ dominates distribution $f(X|y_{i'})$ in the sense of the first order stochastic dominance, i.e. for every $X \in (0, 1)$, we have $F(X|y_i) < F(X|y_{i'})$. This assumption directly implies that $\mathbb{E}_q[s(X)|y_i, z] > \mathbb{E}_q[s(X)|y_{i'}, z]$ for every security $s(\cdot)$ and signal z with a non-degenerate distribution. The following instance is an illustration of the information structure.

Example 1. *Suppose the insider's experiment generates a binary signal, i.e.* $\mathcal{Y} = \{\tilde{h}, \tilde{l}\}$. *Consider the following information structure for* $(\{\tilde{h}, \tilde{l}\}, \omega_q)$*:*

$$\omega_q(\tilde{h}|X) = \begin{cases} \frac{1+q}{2} & I \leq X \leq 1 \\ \frac{1-q}{2} & 0 \leq X < I. \end{cases}$$

The investor receives a signal $y = \tilde{h}$ *with probability* $\frac{1+q}{2} \geq \frac{1}{2}$ *if the project is profitable, and* $y = \tilde{l}$ *with probability* $\frac{1-q}{2} \leq \frac{1}{2}$ *otherwise. Clearly* $\mathbb{E}_q[s(X)|\tilde{h}] \geq \mathbb{E}_q[s(X)|\tilde{l}]$ *for every* $q > 0$

[14]Recall the Blackwell Informativeness Criterion: An experiment provides the decision maker with an updated set of probabilities over the states of nature. One signal is at least as informative as another if, for all utility functions, the decision maker's expected utility from implementing the optimal action (which may vary with his preferences and with the signal he observes) is at least as high with that signal.

and security $s(\cdot)$. Moreover, if $1 > q > q' > \frac{1}{2}$, then the following inequality also holds:

$$\mathbb{E}_q[s(X)|\tilde{h}] > \mathbb{E}_{q'}[s(X)|\tilde{h}] > \mathbb{E}_{q'}[s(X)|\tilde{l}] > \mathbb{E}_q[s(X)|\tilde{l}]$$

It means that experiments with higher values of q generate relatively more extreme signals.

Considering the insider's information production, the entrepreneur now designs an experiment $(\mathcal{Z} \times \mathcal{Y}, \pi)$, where $\pi(.,.|X) : \mathcal{Z} \times \mathcal{Y} \to [0,1]$ is the joint conditional probability of observing the signals. Note that the marginal distributions for $y \in \mathcal{Y}$ must be consistent with the insider's experiment, i.e. $\sum_{z \in \mathcal{Z}} \pi(z, y|X) = \omega_q(y|X)$ for every $X \in [0,1]$ and $y \in \mathcal{Y}$. Moreover, we allow the signals in $\mathcal{Z}$ and $\mathcal{Y}$ to be correlated conditional on the true state of the world.[15]

4.1 Information Production with Investor Sophistication

In this part, we solve for the optimal experiment and the equilibrium payoffs for the entrepreneur and the insider for a given $q \in [0,1]$, when long-term contracting is not feasible. We first derive a simplifying lemma:

Lemma 2. *The optimal experiment (Z, π) is at least as informative as (q, ω_q), in the Blackwell sense. In other words, the outsiders can perfectly infer $y \in \mathcal{Y}$ from observing the realization $z \in \mathcal{Z}$.*

Lemma 2 essentially says that the entrepreneur optimally discloses the insider's signal and when z becomes public, the insider has no information privileges over the outsiders. Intuitively, the entrepreneur benefits from eliminating the insider's informational advantage. A priori, it is not obviously the optimal information structure policy as it could hurt the entrepreneur's payoff through decreasing the probability of continuation. However, obfuscating the signal is not helpful here because the insider's action conveys information to the outsiders anyway and it is harder to hide bad realizations of y.

Denote the signal that $X \geq \max\{\hat{X}(\mu), \bar{X}(y_i)\}$ by x_H, where $\bar{X}(y_i)$ is the solution to $\mathbb{E}_q[s(X) - I|y_i, X \geq \bar{X}(y_i)] = 0$ if a solution exists and is zero otherwise. Denote the

[15] To think about this correlation, note that the entrepreneur can simply include the results from the insider's experiment or a noisy version of them in his experiment report to the investors. Even when the entrepreneur does not observe y, he can still enable the correlation as long as he knows $(\mathcal{Y}, \omega_q)$. Guo and Shmaya (2017) and Kolotilin (2017) derive general results for the case that the sender's and the receiver's experiments must generate *independent* signals conditional on X. In Appendix B2, we illustrate how results are robust to such a requirement.

signal for the opposite $X < \max\{\hat{X}(\mu), \bar{X}(y_i)\}$ by x_L. Now we can characterize the optimal experiment in Proposition 3.

Proposition 3 (**Endogenous Information under Investor Sophistication**). *An optimal experiment exists and requires at most $2|\mathcal{Y}| = 2m$ signals. For every signal $y_i \in \mathcal{Y}$, the entrepreneur sends either $z_i^h = \{y_i, x_H\}$ Otherwise, it sends $z_i^l = \{y_i, x_L\}$.*

Proposition 3 extends Proposition 1 to allow insider sophistication. Lemma 2 implies that the entrepreneur can split the information design problem into m separate information design problem, each following Proposition 1. Since the optimal experiment in Proposition 1 has at most two signals, the optimal experiment in presence of a sophisticated investor has at most $2m$ signals.

Before we investigate the impact of investor sophistication on relationship formation, the following lemma reveals that the first-best outcomes cannot be achieved under investor sophistication alone.

Corollary 2. *For any given $(\mathcal{Y}, \omega_q)$, the equilibrium investment decision is not ex-post socially optimal with a positive probability.*

Therefore, initial long-term contracts with the right security design is integral to achieving the socially optimal investment. To that end, we next discuss whether setting milestones resolves IPH, before discussing optimal contracting and security design under investor sophistication in Section 4.3.

4.2 Setting Milestones

Contracting on interim events is related to "milestones" commonly used in venture financing. For example, the entrepreneur can commit to reaching a pre-specified scale of customer base before seeking continued finance. Would that solve the IPH problem? Perhaps surprisingly, we show the insider cannot increase her expected payoff by contracting on the interim events.

First note that contracting on milestones is different from contracting on the experiment $(\mathcal{Z}, \pi)$. Instead, it is about how continuation is contingent on other information the insider observes. To proceed, we use $\mathcal{Y}^b \subset \mathcal{Y}$ to denote binding signals following which the insider commits to either continue or terminate financing. We denote the set of non-binding signals by $\mathcal{Y}^{nb} = \mathcal{Y} \setminus \mathcal{Y}^b$.

Corollary 3 (**Milestone Futility**). *The insider cannot gain from setting milestones. In particular, the insider's expected payoff is maximized when* $\mathcal{Y}^b = \emptyset$.

Corollary 3 shows that the insider cannot resolve IPH by setting milestones. The reason is that the entrepreneur can flexibly control how the insider updates her prior for every $y \in \mathcal{Y}^{nb}$, regardless of the choice of $\mathcal{Y}^b$. Therefore, the insider's information set following signals in $\mathcal{Y}^{nb}$ does not change, while she potentially makes suboptimal decisions following signals in $\mathcal{Y}^b$.

This also explains why milestones are seldom binding in practice. Besides relying on projections in early stages that are hard to make or enforce, binding milestones render endogenous information production irrelevant, and thus do not help the insider in extracting more interim rent.

4.3 Optimal Contracting and Security Design

The next proposition shows that Proposition 2 is robust to insider sophistication and still achieves the first-best outcome.

Proposition 4 (**Optimal Contracts with Investor Sophistication**). *Regardless of* $(\mathcal{Y}, \omega_q)$*, all robust optimal securities are characterized by (17)-(19).*

The intuition behind Proposition 4 is the following: Because of the flat part of the security, the insider's security entails no risk when the entrepreneur reveals that $X > I - \varepsilon$. Therefore, the insider always continues the project following $z = h$, regardless of her own signal. Hence the insider's action only reveals whether $X > I - \varepsilon$ or not. Hence, by the design of the security, the outsiders invest if and only if the insider invests. Note that the proposition does not concern the optimal experimentation (which Proposition 3 describes) or require any particular structure of $(\mathcal{Y}, \omega_q)$. The proposition holds as long as the experiment results in a first-best continuation action.

In contrast to the case without long-term contracts, the entrepreneur's experiment under the optimal long-term contract does not depend on the insider's experiment. In other words, the entrepreneur might find it optimal to hide some of the insider's information from the outsiders. As a result, the model predicts there is indeed some information asymmetry between the insiders and the outsiders but it does not prevent implementing the socially optimal outcome, because the contractual solution resolves all inefficiencies.

4.4 Relationship Formation, Sophistication, and Competition

How does investor sophistication impact relationship formation? Intuitively, the insider always benefits from having a more informative experiment, as the next corollary shows.

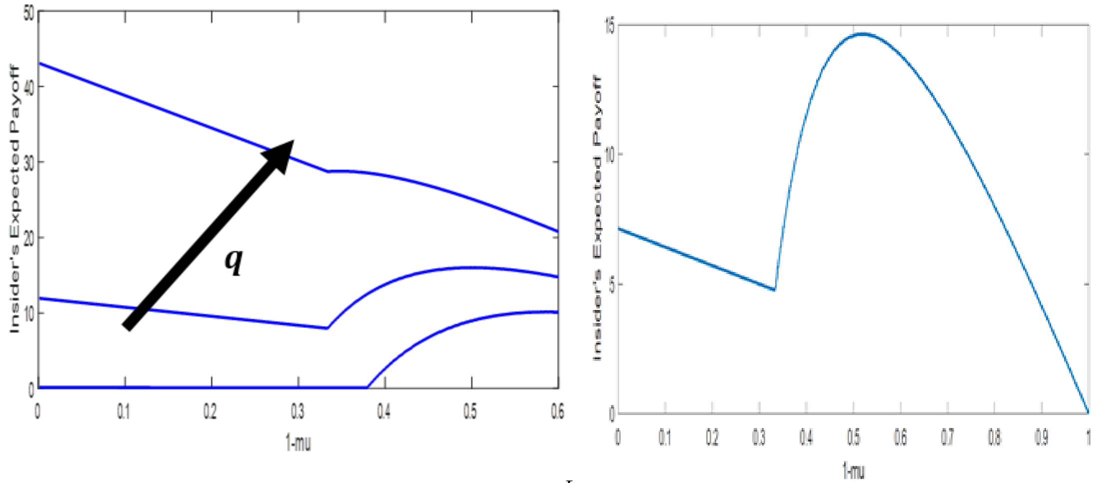
Corollary 4 (**Relationpship and Sophistication**). *For every $\mu \in [0,1]$, the insider's expected payoff $U^I(\mu; q)$ is weakly increasing in q.*

This result is in contrast to Kolotilin (2017) that shows that the receiver's expected payoff is non-monotone in the informativeness of her private signal. Because the insider's expected interim payoff $U^I(\mu; q)$ is higher with higher q, insider sophistication directly facilitates relationship formation.

Furthermore, investor sophistication interacts with interim competition, which can help us rationalize an empirical observation in relationship banking. Prior theoretical predictions on the effect of competition on bank orientation so far has been ambiguous. The investment theory (e.g., Petersen and Rajan (1995) and Dell'Ariccia and Marquez (2004)) argues that as the credit market concentration decreases, the firms' borrowing options expand, rendering banks less capable to recoup in the course of the lending relationship the initial investments in building relationship, which hinders relationship banking; the strategic theory (e.g., Boot and Thakor (2000) and Dinc (2000)) says fiercer interbank competition drives local lenders to take advantage of their competitive edge and reorient lending activities towards relational-based lending to small, local firms, which strengthens relationship banking. Others (e.g., Yafeh and Yosha (2001) and Anand and Galetovic (2006)) suggest that competition can have ambiguous effects on lending relationships, but typically predict an inverted U-shape pattern.

Yet empirically Elsas (2005) and Degryse and Ongena (2007) document a U-shape relationship between likelihood of the lending relationship and the level of competition in the credit market. These two studies stand out because they measure relationship banking directly in terms of duration and scope of interactions, thus improve upon and complement indirect measures such as loan rate (Petersen and Rajan (1995)) or credit availability over firms' life time (Black and Strahan (2002)), for which the impact of competition could be ambiguous in equilibrium (Boot and Thakor (2000)). Our theory offers an explanation.

Proposition 5 (**Relationship and Competition**). $\exists\ \underline{\mu}(q) \in (0,1)$ *such that for* $\mu \in [\underline{\mu}(q), 1]$, *the insider's payoff from the relationship financing,* $U^I(\mu; q)$, *is increasing in the level of interim competition* $(1-\mu)$ *for unsophisticated investors* $(q = 0)$, *decreasing for* *sophisticated investors (sufficiently large q), and U-shaped for investors with intermediate sophistication.*



(a) The "U-shaped" relationship between U^I and $1-\mu$

(b) Non-monotone relationship between $U^I(\mu;q)$ and $1-\mu$

Figure 4: Illustration of the equilibrium capacity of the financing in the initial round as a function of the level of ex-post competition

Figure 4(a) illustrates the relationship between $U^I(\mu;q)$ and μ, the *inverse* measure of competition in our model. In particular, when q takes intermediate values and μ is exogenous, our model thus helps rationalize the findings of Elsas (2005) and Degryse and Ongena (2007).

On the one hand, for a fixed level of private benefit of continuation, lower levels of competition increases the insider's share of the surplus, and is preferred by more sophisticated investors who can produce her own information. On the other hand, higher levels of competition can encourage more efficient information production from the entrepreneur which increases total surplus, thus is preferred by the less sophisticated investors who have no other means to obtain information rent. For intermediate values of sophistication, competition hurts the insider's profit until it replaces the investor's independent information as her main source of interim rent, leading to the local U-shape.

To the extent that small banks are able to collect and act on soft information whereas large banks rely on hard information produced by credit bureaus and alternative sources(e.g., Berger, Miller, Petersen, Rajan, and Stein (2005))—higher q in our model, our theory predicts that competition would reduce relationship formation in regions where large and out-of-market banks are pre-dominant, and the opposite holds for regions with mostly small banks and mutual banks, consistent with empirical evidence in Presbitero and Zazzaro (2011).

Finally, our model reveals that while low levels of interim competition means an insider can recoup the initial cost of forming the relationship, the ease of forming the relationship involves a more complex non-monotonicity: in addition to the U-shape, when the market becomes extremely competitive (μ gets much closer to 0), relationship formation eventually decreases in competition, as seen in Figure 4(b).

5 Discussion and Extension

5.1 Scalable Investment and Continuum Range of Actions

So far, we have assumed a binary investment decision for the project. In this section, we enlarge the set of investment opportunities by allowing scalable investment at $t = 1$ after observing the experimentation outcome.

Specifically, we assume that investing αI generates a stochastic cash flow $r(\alpha)X$, where $r(\cdot) : [0,1] \to [0,1]$ is weakly increasing. For consistency with the baseline model, we assume $r(0) = 0$ and $r(1) = 1$; our baseline model corresponds to $r(\alpha) = 0$ for $\alpha < 1$ and $r(1) = 1$. Without any interim competition ($\mu = 1$), the final payoffs following investment level α and realization of cashflow $r(\alpha)X$ are $u^E = r(\alpha)X - s(\alpha, r(\alpha)X) + r(\alpha)\varepsilon$ and $u^I = s(\alpha, r(\alpha)X) - \alpha I$ respectively, where $s(\cdot,\cdot)$ is a security payment that is generally contingent both on the project scale (or equivalently, level of investment) and the final cash flow, and private benefit of continuation depends on project scale.

In this section, we allow the entrepreneur to choose experiments with infinitely many signals. In particular, we show there are situations that the entrepreneur perfectly discloses X to the investors, since their action set is not finite anymore.

First note that if $\frac{r(\alpha)}{\alpha}$ is weakly increasing in $\alpha \in (0,1]$, namely if the project has the full benefit of scale, the investment decision reduces to a binary decision and apparently all previous results apply. As such, we focus on diminishing return to scale (DRS). Proposition 6 extends our main results to this case.

Proposition 6 **(Scalable Investment).**

(a) The social welfare of investment, $\mathbb{E}[r(\alpha)(\varepsilon+X)-\alpha I]$, *is increasing in interim competition* $1-\mu$. *In particular, when the entrepreneur issues equities, i.e.,* $s(\alpha, r(\alpha)X) = \beta r(\alpha)X$ *for some* $\beta \in (0,1)$ *and* $r(\alpha) = \alpha^\gamma$ *for some* $\gamma \in (0,1)$, *the informativeness of the entrepreneur's experiment weakly decreases in the Blackwell sense, as* μ *increases.*

(b) Suppose $\alpha^*(X;\varepsilon)$ is the optimal level of investment for X and ε, i.e. it is the solution of $\max_\alpha r(\alpha)(X+\varepsilon)-\alpha I$. If $r(\alpha^*(X;\varepsilon))X-\alpha^*(X;\varepsilon)I \geq 0$ with probability one for all values of ε, then a convertible security for insiders and the residual claim for outsiders implement the first best outcome, the entrepreneur chooses the security after the realization of signal z to achieve the desirable level of investment, and fully discloses information.

Proposition 6(a) extends the main result in Section 2.2 that the insider's interim rent is not increasing in her information monopoly, to continuous (non-binary) investment decisions. More information monopoly (higher μ) means less alignment of the entrepreneur's payoff with the social planner's, which leads to inefficient information production. However, the scalable investment helps the insider to recover some interim rent, even if she fully holds up the entrepreneur ($\mu = 1$). When the insider chooses the scale of investment from a continuous set, she should get zero expected rent from the marginal level of investment. The DRS assumption implies she gets positive interim rent overall, if the investment takes place.

Proposition 6(b) extends Proposition 2. To implement the first best outcome under DRS, the security should not only encourage the entrepreneur to experiment efficiently but also incentivize the insider to efficiently scale the project. We thus choose the insider's security to perfectly align the insider's and the social planner's utilities over different scales of investment. We describe the security in greater detail in the appendix.

The key takeaway is that with DRS and a continuum of investment levels, investment decisions are no longer binary (investing I or terminating), and the insider derives partial rent from her informational monopoly. Endogenous information production still leads to reduced rent which potentially renders relationship financing infeasible, consistent with Lemma 1. Moreover, similar to the case of binary investments levels, a contractual solution entailing giving convertible securities to initial investors still achieves the first best outcome, and is robust to investor sophistication.

5.2 Commitment to Information Design and Partial Observation

Ex-ante Commitment to Experimentation

Commitment issues are common in applications of information design and Bayesian persuasion. Our setup relaxes this strong assumption that the entrepreneur can commit in the initial round. To see this, note that the insider has limited ability in dictating the entrepreneur's experimentation at the time of initial financing, but can observe it after becoming an insider. In other words, the entrepreneur cannot commit to an experimentation when raising K. Otherwise, the insider can extract rent (as seen in Section 4), and the problem in Section 3 becomes a joint optimization problem on security design and information disclosure for a total issuance of $K + I$, which Szydlowski (2016) addresses. He shows that both equity and debt can implement the optimal design, and the optimal security is indeterminate. In essence, the lack of commitment and contractibility of the experimentation at $t = 0$ in our setting (partially) breaks the indeterminacy in Szydlowski (2016) which has only one round of financing by fully competitive investors.

For illustration, suppose we use debt security to finance the project without investor sophistication, the insider investor receives less reward from high realizations. Therefore, the entrepreneur promises a larger λ to the insider in the second round to compensate for her initial investment K. The entrepreneur, less exposed to the downside (smaller $1 - \lambda$), then chooses less informative signals ex post, which decreases the financing capacity in the first round. Clearly, the choice of security ex ante affects the choice of information design ex post.

Partial Observation of Experimentation

Another assumption we have had thus far is that the insider perfectly observes the entrepreneur's experiment and its signal realization, before making the continuation decision. In other words, we have assumed that not only the investor's monitoring technology rules out misreporting, but the investor also commits to perfectly monitor the entrepreneur's experiment, while she might be better off randomizing between monitoring and not monitoring. We now show that while the first best outcome becomes infeasible, our results on IPH and optimal contracting and security design still hold.

To incorporate the partial observation, we here allow the possibility of the entrepreneur's misreporting and the investor's partial commitment to monitoring:

1. With probability $\alpha \in [0, 1]$, the entrepreneur can misreport the signal z without getting caught even when monitored by the investor.

2. At time $t = 0$, the investor commits to verifying the experiment and its signal realization with probability $\beta \in [0, 1]$.[16]

[16]Note that if the investor monitors the experiment, she makes the continuation decision based on both the reported and the monitored signal. In this case, we allow the investor to commit to punishing the entrepreneur by not investing in the case of misreporting.

Note that our baseline model corresponds to $\alpha = 0$ and $\beta = 1$. In general, the following variant of Corollary 1 and Proposition 2 holds.

Proposition 7 (Partial Observation and Commitment).

(a) *The insider receives no interim rent for extreme values of* μ *and for any values of* $\alpha, \beta \in [0, 1]$.

(b) *For* $\alpha > 0$ *and* $\beta < 1$*, there exists no contractual solution that implements the first best outcome. For small enough values of* α *and large enough values of* β*, the convertible securities are still optimal and robust to* ε*. Relationship financing is infeasible for large enough values of* α *and low enough values of* β*.*

Proposition 7 validates our prior knowledge that the investor's monitoring is essential to relationship financing. But part (a) shows it is insufficient. In fact, a contractual solution involving the outsiders is also required to make optimal investment decisions. Furthermore, note that in contrast to costly state verification models that the investor optimally randomizes between monitoring and not monitoring, Proposition 7(b) shows that the full observation of the investor is required to implement the socially optimal outcome. The main difference is that the investor has no way to punish the entrepreneur for misreporting, as he is protected by his limited liability. Therefore, the entrepreneur always benefit from ex-post misreporting.

5.3 Security Design Right

Section 3 mainly focuses on the entrepreneur's optimal security design problem. In what follows, we show that the insider investor designs optimal security differently from what the entrepreneur does, and in a less socially efficient manner.

Proposition 8 (Security Design Right). *Under both investor sophistication and unsophistication, it is more socially efficient that the entrepreneur designs the security.*

The intuition for Proposition 8 is the following: In order to raise the initial K, the entrepreneur understands he has to provide at least a minimum amount of expected cash flow to the insider investor. Therefore among all designs that generate this amount, he chooses the one that makes him commit to the most informative disclosure policy, which maximizes the total surplus. In other words, ex ante he is the residual claimant and his incentives are more aligned with a social planner. Moreover, if the insider has the security design right, she would choose a less socially optimal security, as she does not consider the private benefit of the entrepreneur from the investment.

6 Conclusion

We model the dynamic financing of innovative projects by relationship and arms-length investors as a mechanism design problem with an embedded Bayesian persuasion game whereby the entrepreneur endogenously produces interim information to seek continued financing. We show that the entrepreneur's (sender's) endogenous experimentation typically reduces the insider's (receiver) informational-monopoly rent, holding up the insider's initial investment to form the relationship in the first place. Investor sophistication (receiver's type) and interim competition can mitigate the problem, and they interact to produce non-monotone patterns of relationship formation and interim competition. Importantly, we derive optimal sequential securities to resolve the information production hold-up: the entrepreneur contracts with investors in the initial round to allow them to purchase convertible securities in a later round, and issue residual claims to competitive outsiders later. The optimal security choice and contracting terms are robust to the entrepreneur's continuation bias, investor sophistication, and scalable investment, and can be applied to similar hold-up problems in persuasion games with contingent transfers and heterogeneous receivers.

Our theory immediately applies to at least two major areas of finance. First, it underscores the impact of endogenous information production and clarifies its interactions with investor sophistication and competition, rationalizing the U-shaped link between relationship lending and competition documented in *relationship lending* that extant theories could not account for. Second, it demonstrates the robustness of convertible securities in the *venture capital*, and derives the optimal contract that is consistent with real-life practice. Given that the solutions to many of the world's biggest problems such as Alzheimer's disease, global warming, and fossil-fuel depletion require large initial funding, reliable financing relationships, and long-term experimentation (Nanda and Rhodes-Kropf (2013) and Hull, Lo, and Stein (2017)), the cost of inefficient information production could be tremendous. Our study helps underscore and formalize this practical issue, and develops potential contractual solutions.

References

Admati, Anat R, and Paul Pfleiderer, 1994, Robust financial contracting and the role of venture capitalists, *The Journal of Finance* 49, 371–402.

Aghion, Philippe, and Patrick Bolton, 1992, An incomplete contracts approach to financial contracting, *The review of economic Studies* 59, 473–494.

Aghion, Philippe, Mathias Dewatripont, and Patrick Rey, 1994, Renegotiation design with unverifiable information, *Econometrica: Journal of the Econometric Society* pp. 257–282.

Anand, Bharat N, and Alexander Galetovic, 2006, Relationships, competition and the structure of investment banking markets, *The Journal of Industrial Economics* 54, 151–199.

Bergemann, Dirk, and Ulrich Hege, 1998, Venture capital financing, moral hazard, and learning, *Journal of Banking & Finance* 22, 703–735.

Bergemann, Dirk, and Stephen Morris, 2017, Information design: A unified perspective, *Working Paper*.

Berger, Allen N, Nathan H Miller, Mitchell A Petersen, Raghuram G Rajan, and Jeremy C Stein, 2005, Does function follow organizational form? evidence from the lending practices of large and small banks, *Journal of Financial Economics* 76, 237–269.

Berger, Allen N, and Gregory F Udell, 1995, Relationship lending and lines of credit in small firm finance, *Journal of Business* pp. 351–381.

Blacharski, Dan, 2013, Top ten ways the cloud has changed how startups launch, *Techie.com November, 26*.

Black, Sandra E, and Philip E Strahan, 2002, Entrepreneurship and bank credit availability, *The Journal of Finance* 57, 2807–2833.

Blackwell, David, 1951, Comparison of experiments, Discussion paper, HOWARD UNIVERSITY Washington United States.

Boot, Arnoud WA, 2000, Relationship banking: What do we know?, *Journal of Financial Intermediation* 9, 7–25.

——— , and Anjan V Thakor, 2000, Can relationship banking survive competition?, *The Journal of Finance* 55, 679–713.

Bouvard, Matthieu, Pierre Chaigneau, and Adolfo de Motta, 2015, Transparency in the financial system: Rollover risk and crises, *The Journal of Finance* 70, 1805–1837.

Brennan, Michael J, and Eduardo S Schwartz, 1988, The case for convertibles, *Journal of Applied Corporate Finance* 1, 55–64.

Burkart, Mike, Denis Gromb, and Fausto Panunzi, 1997, Large shareholders, monitoring, and the value of the firm, *The Quarterly Journal of Economics* 112, 693–728.

Cong, Lin William, 2017, Auctions of real options, *Chicago Booth Working Paper*.

——— , Steven R Grenadier, and Yunzhi Hu, 2017, Dynamic interventions and informational linkages, *Chicago Booth Working Paper Series*.

Cornelli, Francesca, and Oved Yosha, 2003, Stage financing and the role of convertible securities, *The Review of Economic Studies* 70, 1–32.

Cuny, Charles J, and Eli Talmor, 2005, The staging of venture capital financing: Milestone vs. rounds, *Texas A&M University, USA and London Business School, UK. EFA*.

Da Rin, Marco, Thomas F Hellmann, and Manju Puri, 2011, A survey of venture capital research, Discussion paper, National Bureau of Economic Research.

Degryse, Hans, and Steven Ongena, 2007, The impact of competition on bank orientation, *Journal of Financial Intermediation* 16, 399–424.

Dell'Ariccia, Giovanni, and Robert Marquez, 2004, Information and bank credit allocation, *Journal of Financial Economics* 72, 185–214.

DeMarzo, Peter, and Darrell Duffie, 1999, A liquidity-based model of security design, *Econometrica* 67, 65–99.

DeMarzo, Peter M, Ilan Kremer, and Andrzej Skrzypacz, 2005, Bidding with securities: Auctions and security design, *The American Economic Review* 95, 936–959.

Diamond, Douglas W, 1984, Financial intermediation and delegated monitoring, *The Review of Economic Studies* 51, 393–414.

———, 1991, Monitoring and reputation: The choice between bank loans and directly placed debt, *Journal of Political Economy* 99, 689–721.

———, 1993, Seniority and maturity of debt contracts, *Journal of Financial Economics* 33, 341–368.

Dinc, I Serdar, 2000, Bank reputation, bank commitment, and the effects of competition in credit markets, *The Review of Financial Studies* 13, 781–812.

Dworczack, Piotr, and Giorgio Martini, 2017, The simple economics of optimal persuasion, *Working Paper*.

Dyck, Alexander, and Luigi Zingales, 2004, Private benefits of control: An international comparison, *The Journal of Finance* 59, 537–600.

Elsas, Ralf, 2005, Empirical determinants of relationship lending, *Journal of Financial Intermediation* 14, 32–57.

Ely, Jeffrey C, 2017, Beeps, *The American Economic Review* 107, 31–53.

Ewens, Michael, Matthew Rhodes-Kropf, and Ilya A Strebulaev, 2016, Insider financing and venture capital returns, .

Fama, Eugene F, 1985, What's different about banks?, *Journal of Monetary Economics* 15, 29–39.

Farrell, Joseph, and Nancy T Gallini, 1988, Second-sourcing as a commitment: Monopoly incentives to attract competition, *The Quarterly Journal of Economics* 103, 673–694.

Fluck, Zsuzsanna, Kedran Garrison, and Stewart C Myers, 2006, Venture capital contracting: Staged financing and syndication of later-stage investments, *NBER Working Paper*.

Gentzkow, Matthew, and Emir Kamenica, 2016, A rothschild-stiglitz approach to bayesian persuasion., *The American Economic Review* 106, 597–601.

Goldstein, Itay, and Yaron Leitner, 2015, Stress tests and information disclosure, .

Gompers, Paul A, 1997, Ownership and control in entrepreneurial firms: an examination of convertible securities in venture capital investments, *Unpublished working paper, Harvard Business School*.

Gorton, Gary, and Andrew Winton, 2003, Financial intermediation, *Handbook of the Economics of Finance* 1, 431–552.

Guo, Yingni, and Eran Shmaya, 2017, The interval structure of optimal disclosure, *Working Paper*.

Hart, Oliver, and John Moore, 1988, Incomplete contracts and renegotiation, *Econometrica: Journal of the Econometric Society* pp. 755–785.

——— , 1995, Debt and seniority: An analysis of the role of hard claims in constraining management, *American Economic Review* 85, 567–85.

Hellmann, Thomas, 2006, Ipos, acquisitions, and the use of convertible securities in venture capital, *Journal of Financial Economics* 81, 649–679.

Holmstrom, Bengt, 1979, Moral hazard and observability, *The Bell Journal of Economics* pp. 74–91.

Hörner, Johannes, and Larry Samuelson, 2013, Incentives for experimenting agents, *The RAND Journal of Economics* 44, 632–663.

Hörner, Johannes, and Andrzej Skrzypacz, 2016, Learning, experimentation and information design, Discussion paper, Working Paper.

Hull, John C, Andrew W Lo, and Roger M Stein, 2017, Funding long shots, .

Innes, Robert D, 1990, Limited liability and incentive contracting with ex-ante action choices, *Journal of Economic Theory* 52, 45–67.

Ito, Kazufumi, 2016, Functional analysis and optimization, *Lecture Monograph*.

Jensen, Michael C, and William H Meckling, 1976, Theory of the firm: Managerial behavior, agency costs and ownership structure, *Journal of financial economics* 3, 305–360.

Kamenica, Emir, 2017, Information economics, *Journal of Political Economy* 125, 1885–1890.

——— , and Matthew Gentzkow, 2011, Bayesian persuasion, *American Economic Review* 101, 2590–2615.

Kaplan, Steven N., and Per Strömberg, 2003, Financial contracting theory meets the real world: An empirical analysis of venture capital contracts, *The Review of Economic Studies* 70, 281–315.

Kaplan, Steven N, and Per ER Strömberg, 2004, Characteristics, contracts, and actions: Evidence from venture capitalist analyses, *The Journal of Finance* 59, 2177–2210.

Kerr, William R, Ramana Nanda, and Matthew Rhodes-Kropf, 2014, Entrepreneurship as experimentation, *The Journal of Economic Perspectives* 28, 25–48.

Kolotilin, Anton, 2017, Optimal information disclosure: A linear programming approach, *Working Paper*.

——— , Tymofiy Mylovanov, Andriy Zapechelnyuk, and Ming Li, 2017, Persuasion of a privately informed receiver, *Econometrica* 85, 1949–1964.

Kortum, Samuel, and Josh Lerner, 2001, Does venture capital spur innovation?, in *Entrepreneurial inputs and outcomes: New studies of entrepreneurship in the United States* . pp. 1–44 (Emerald Group Publishing Limited).

Lerner, Joshua, 1994, The syndication of venture capital investments, *Financial management* pp. 16–27.

Myers, Stewart C, and Nicholas S Majluf, 1984, Corporate financing and investment decisions when firms have information that investors do not have, *Journal of Financial Economics* 13, 187–221.

Nachman, David C, and Thomas H Noe, 1994, Optimal design of securities under asymmetric information, *The Review of Financial Studies* 7, 1–44.

Nanda, Ramana, and Matthew Rhodes-Kropf, 2013, Innovation and the financial guillotine, Discussion paper, National Bureau of Economic Research.

——— , 2015, Studying the start-up system: Financing experiments, *Science* 348, 1200–1200.

Nöldeke, Georg, and Klaus M Schmidt, 1995, Option contracts and renegotiation: a solution to the hold-up problem, *The RAND Journal of Economics* pp. 163–179.

——— , 1998, Sequential investments and options to own, *The Rand journal of economics* pp. 633–653.

Orlov, Dmitry, Pavel Zryumov, and Andrzej Skrzypacz, 2017, Design of macro-prudential stress tests, *Working Paper*.

Palmer, Maija, 2012, Cloud computing cuts start-up costs, *Financial Times*.

Petersen, Mitchell A, and Raghuram G Rajan, 1994, The benefits of lending relationships: Evidence from small business data, *The Journal of Finance* 49, 3–37.

——— , 1995, The effect of credit market competition on lending relationships, *The Quarterly Journal of Economics,* 110, 407–443.

——— , 2002, Does distance still matter? the information revolution in small business lending, *The Journal of Finance* 57, 2533–2570.

Presbitero, Andrea F., and Alberto Zazzaro, 2011, Competition and relationship lending: Friends or foes?, *Journal of Financial Intermediation* 20, 387–413.

Rajan, Raghuram G, 1992, Insiders and outsiders: The choice between informed and arm's-length debt, *The Journal of Finance* 47, 1367–1400.

Ramakrishnan, Ram TS, and Anjan V Thakor, 1984, Information reliability and a theory of financial intermediation, *The Review of Economic Studies* 51, 415–432.

Sahlman, William A, 1990, The structure and governance of venture-capital organizations, *Journal of Financial Economics* 27, 473–521.

Santos, Joao AC, and Andrew Winton, 2008, Bank loans, bonds, and information monopolies across the business cycle, *The Journal of Finance* 63, 1315–1359.

Schenone, Carola, 2010, Lending relationships and information rents: Do banks exploit their information advantages?, *The Review of Financial Studies* 23, 1149–1199.

Sharpe, Steven A, 1990, Asymmetric information, bank lending, and implicit contracts: A stylized model of customer relationships, *The journal of finance* 45, 1069–1087.

Srinivasan, Anand, et al., 2014, Long run relationships in banking, *Foundations and Trends® in Finance* 8, 55–143.

Stein, Jeremy C, 1992, Convertible bonds as backdoor equity financing, *Journal of Financial Economics* 32, 3–21.

Szydlowski, Martin, 2016, Optimal financing and disclosure, *Working Paper*.

Trigilia, Giulio, 2017, Optimal leverage and strategic disclosure, *Working Paper*.

Von Thadden, Ernst-Ludwig, 1995, Long-term contracts, short-term investment and monitoring, *The Review of Economic Studies* 62, 557–575.

Winton, Andrew, and Vijay Yerramilli, 2008, Entrepreneurial finance: Banks versus venture capital, *Journal of Financial Economics* 88, 51–79.

Yafeh, Yishay, and Oved Yosha, 2001, Industrial organization of financial systems and strategic use of relationship banking, *Review of finance* 5, 63–78.

Appendix A: Proofs of Lemmas and Propositions

A1. A Technical Lemma

The entrepreneur endogenously designs the experiment to maximize his payoff, subject to the insider investors' second-round participation constraint. With finite state space, the signal space as the range of a deterministic mapping from the state space is necessarily finite. Consequently, we can apply the method of Lagrange multipliers directly. But alas, we are dealing with infinite dimensional state space, and in unrestricted signal generation space, we cannot always apply the method of Lagrange multipliers.

That said, the optimal experimentation function is given by the characteristic function of a countable sup-level set of payoff densities, for some cutoff value "multiplier". In other words, the experimentation we look at are conditional probabilities and therefore their space is a Banach space. With this insight, we first prove the following mathematical lemma that allows us to use the method of Lagrange multipliers in the proofs of our lemmas and propositions (see also Ito (2016) for an abstract generalization).

Lemma (A1). *Suppose $w_i(x), m_i(x) : [0,1] \to \Re$ ($1 \le i \le N$) are continuous and bounded functions. Suppose the following maximization problem has a solution:*

$$\begin{aligned} &\max_{\alpha_i(.)\in\mathcal{A}} \int_0^1 \sum_{i=1}^N w_i(x)\alpha_i(x)dx \\ &s.t. \quad \int_0^1 m_i(x)\alpha_i(x)dx \ge 0 \quad \forall\ 1 \le i \le N, \quad and \quad \sum_{i=1}^N \alpha_i(x) \le 1 \quad \forall x \in [0,1], \end{aligned} \tag{20}$$

where $\mathcal{A}$ is the set of all measurable functions over $[0,1]$ that take value from $[0,1]$. Then, there exist non-negative real numbers $\{\mu_i\}_{i=1}^N$, such that the solution to (20) is a solution to the following maximization problem:

$$\max_{\alpha_i(.)\in\mathcal{A}} \int_0^1 \sum_{i=1}^N (w_i(x) + \mu_i m_i(x))\alpha_i(x)dx \quad s.t. \quad \sum_{i=1}^N \alpha_i(x) \le 1 \quad \forall x \in [0,1] \tag{21}$$

Proof. Let $\tilde{\mathcal{A}}^N$ be the set of all $N-$tuples of functions $(\alpha_1(.), \ldots, \alpha_N(.))$ in $\mathcal{A}$ that satisfy $\sum_{i=1}^N \alpha_i(x) \le 1$.

Since all functions are bounded and measurable, then it is easy to check that $\tilde{\mathcal{A}}^N$ constitutes a closed set in ${\mathcal{L}^1}^N$. Therefore, the following maximization problem is well-defined.

$$\max_{(\alpha_1(.),\ldots,\alpha_N(.))\in\tilde{\mathcal{A}}^N} \int_0^1 \sum_{i=1}^N w_i(x)\alpha_i(x)dx \quad s.t. \quad \int_0^1 m_i(x)\alpha_i(x)dx \ge 0 \quad \forall\ 1 \le i \le N \tag{22}$$

Suppose $a^* \in \tilde{A}^N$ is the solution to the problems (20) and (22). It is easy to see that Slater condition, and correspondingly, strong duality holds. Therefore, there exists a vector of non-negative real numbers $\{\mu_i\}_{i=1}^N$ such that a^* solves the following maximization problem as well:

$$\max_{(\alpha_1(.),\ldots,\alpha_N(.))\in\tilde{\mathcal{A}}^N} \int_0^1 \sum_{i=1}^N w_i(x)\alpha_i(x)dx + \sum_{i=1}^N \mu_i \int_0^1 m_i(x)\alpha_i(x)dx \tag{23}$$

Note that (21) is equivalent to (23). □

In fact, for most Bayesian persuasion settings studied in the literature, the signal generation function corresponds to mapping to probability space that is bounded, and therefore lies in a Banach space. This allows us to apply the approach in Bergemann and Morris (2017) beyond finite-state-space settings.

A2. Proof of Proposition 1

First, we show that two signals are enough to implement the optimal information design. In particular, there is an optimal experiment that induces investment when the outcome exceeds some threshold. Then, we show all optimal experiments induce the same investment decisions. Hence, the equilibrium is unique in terms of equilibrium payoffs and investment decisions.

Optimality of a Binary Experiment. According to (5), the entrepreneur is indifferent between experiment $(\mathcal{Z}, \tilde{\pi})$ and a binary experiment $(\{h, l\}, \pi)$ that pools all signals in $\mathcal{Z}^+$ in h and all signals in $\mathcal{Z}^- = \mathcal{Z} \setminus \mathcal{Z}^+$ in l. In other words, we can always let $\pi(h|X) = \sum_{z \in \mathcal{Z}^+} \pi(z|X)$ and $\pi(l|X) = \sum_{z \in \mathcal{Z}^-} \pi(z|X)$. It is easy to see that the insider's expected payoff and the conditional probability of investment is not affected by changing the experiment from $(\mathcal{Z}, \tilde{\pi})$ to $(\{h, l\}, \pi)$.

Optimality of Threshold Scheme. Given the experiment generates a binary signal, the entrepreneur solves the following optimization problem.

$$\max_{\pi(h|X)} \int_0^1 [\varepsilon + X - \mu s(X) - (1 - \mu) I] \pi(h|X) f(X) dX \tag{24}$$

$$s.t. \int_0^1 (s(X) - I) \pi(h|X) f(X) dX \geq 0, \quad and \quad \pi(h|X) \in [0, 1]$$

In (24), the entrepreneur maximizes his expected payoff with the participation constraint for the investors. Note that $s(X) \geq I$, with positive probability and the set of measurable functions satisfying the constraint in (24) is a closed and bounded subset of $\mathcal{L}^1$. Therefore, (24) has a solution.

When the constraint in (24) is not binding, the optimal experiment sets $\pi^*(h|X) = 1$ for all values of X for which the value in the bracket is non-negative. It corresponds to the set $\{X \geq \hat{X}(\mu)\}$. When the constraint in (24) is binding, we apply Lemma A1 with $N = 1$. Let $\hat{\lambda}$ be the corresponding multiplier. Then the optimal experiment $\pi^*(h|X)$ solves $\max_{\pi(h|X)} \int_0^1 [\varepsilon + X - I + (\hat{\lambda} - \mu)(s(X) - I)] \pi(h|X) f(X) dX$

Note that the term in the bracket is strictly increasing in X, because

$$\frac{d_+}{d_+ X} [\varepsilon + X - I + (\hat{\lambda} - \mu)(s(X) - I)] = 1 + (\hat{\lambda} - \mu) \frac{d_+}{d + X} s(X) > 1 - \mu \frac{d_+}{d + X} s(X) \geq 1 - \mu \geq 0,$$

where $\frac{d_+}{d_+ X}$ denotes the right derivative. Therefore, the optimal experiment has a threshold scheme, where the threshold $\bar{X}$ satisfies $\int_{\bar{X}}^1 (s(X) - I) f(X) dX = 0$. Since the constraint in this case is binding, $\hat{X}(\mu) \leq \bar{X}$ whereas the opposite holds in the first case. The entrepreneur thus always follows a threshold strategy where the threshold is given by $\max\{\bar{X}, \hat{X}(\mu)\}$.

Uniqueness of Investments and Payoffs. Finally, we show the payoffs and investment decisions in equilibrium is essentially unique. The optimal experiments in general are not unique, as the entrepreneur can split each one of the signals h and l in the optimal experiment $(\{h, l\}, \pi^*)$ into more signals and implement the same outcome. Suppose there is another optimal experiment $(\mathcal{Z}', \pi')$ that induces investment with probability $\mathcal{I}'(X)$ for state $X \in [0,1]$. As discussed earlier, the entrepreneur can replace the experiment with a binary experiment. It implies that the optimization problem in (24) has two different solutions. It is a contradiction. Therefore, $\mathcal{I}(X) = \mathbb{I}_{\{X \geq \max\{\bar{X}, \hat{X}(\mu)\}\}}$. Consequently, the entrepreneur's and the insider's payoffs are the same over equivalent choices of optimal experiment.

Uniqueness of Investments and Payoffs In Mixed Strategies. So far, we have assumed that the insider follows a pure strategy. Even if the insider is allowed to randomize between investment and not investment following a signal, the equilibrium payoffs and investments are unique.

To see this, we show that there exists no mixed-strategy Nash Equilibrium in which the investor terminates the project with positive probability when she is indifferent between continuation and termination. Suppose the contrary that the entrepreneur uses experiment $(\mathcal{Z}', \pi')$ and the investor uses the investment function $i'(.) : \Delta([0,1]) \rightarrow [0,1]$. To induce randomization, there exists a signal that makes the insider indifferent between investment and not investment, i.e. $\exists z \in \mathcal{Z}' : \mathbb{E}[s(X) - I|z] = 0$. If z appears with positive probability, the entrepreneur can increase his payoff by splitting z to z^+ and z^- such that $\mathbb{E}[s(X) - I|z^+] > 0$. This profitable deviation leads to a contradiction.

A3. Proof of Corollary 1

We remind the readers that the corollary does not assert that relationship financing is feasible in $[\mu^l, \mu^h]$. The key message is that there are regions in which relationship financing breaks down.

To show the insider can recover K only for intermediate values of μ, we show that the insider's payoff is single-peaked in μ. The insider's equilibrium interim payoff is as follows:

$$U^I(\{h,l\}, \pi^*(\mu); \mu) = \mu \mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq \max\{\bar{X}, \hat{X}(\mu)\}\}}], \tag{25}$$

where $\pi^*(\mu)$ denotes the optimal experiment for μ. By taking the derivative with respect to μ, we get:

$$\frac{\partial}{\partial \mu} U^I(\{h,l\}, \pi^*(\mu); \mu) = \mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq \max\{\bar{X}, \hat{X}(\mu)\}\}}] + \mu(s(\hat{X}(\mu)) - I) f(\hat{X}(\mu)) \tag{26}$$

Both terms in the right hand side of (26) are weakly decreasing in μ. Moreover, it is easy to see that the insider's payoff is zero for $\mu = 0$ and $\mu = 1$. Given that U^I is continuous and differentiable, the insider's expected payoff is single-peaked with respect to μ. The corollary follows.

A4. Proof of Lemma 1

Proof for Part (a)

When $\lambda = 1$, the outsiders cannot make any offer to the entrepreneur by the design of the contract. Therefore, the insider can fully squeeze the entrepreneur, which is equivalent to the case of $\mu = 1$ in Proposition 1. As is shown in Corollary 1, the relationship financing is infeasible in this case.

Proof for Part (b)

We first show that λ enters into the entrepreneur's payoff in a way similar to that of μ in (5). We then simply apply the result of Corollary 1 to show there are intermediate values of λ for which the insider receives a positive expected payoff after the initial investment.

The entrepreneur's payoff from experiment $(\mathcal{Z}, \pi)$ can be derived as follows: Note that following signal $z \in \mathcal{Z}$, the insider chooses to invest by paying $p^I = \lambda I$ if and only if $\lambda \mathbb{E}[s(X) - I|z] \geq 0$. Following the insider's action, the outsiders realize whether the signal is favorable for investment, $z \in \mathcal{Z}^+$. Consequently, following the insider's investment, the outsiders pay the entrepreneur $p^I = (1-\lambda)\mathbb{E}[s(X)|z \in \mathcal{Z}^+]$. Therefore, the entrepreneur's payoff from experiment $(\mathcal{Z}, \pi)$ is given by:

$$U_\lambda^E(\mathcal{Z}, \pi; \lambda) = \sum_{z \in \mathcal{Z}^+} \int_0^1 (\varepsilon + X - I - s(X) + p^I + p^O)\pi(z|X)f(X)dX$$

$$= \sum_{z \in \mathcal{Z}^+} \int_0^1 (\varepsilon + X - I - s(X) + \lambda I + \mathbb{E}[s(X)|z \in \mathcal{Z}^+])\pi(z|X)f(X)dX$$

$$= \int_0^1 \sum_{z \in \mathcal{Z}^+} (\varepsilon + X - I)\pi(z|X)f(X)dX - \lambda \int_0^1 \sum_{z \in \mathcal{Z}^+} (s(X) - I)\pi(z|X)f(X)dX$$

From this, we see that the entrepreneur's preference over different experiments exactly coincides with (5), if we replace μ with λ. The result follows.

For a given security $s(X)$, suppose the entrepreneur uses the threshold $\bar{X}_s$ for sending high signals when$\lambda = 1$. Then, similar to the argument given in Lemma 4, the insider receives positive interim payoff if:

$$\varepsilon + \bar{X}_s - (s_I(\bar{X}_s) + s_O(\bar{X}_s)) + (1 - \lambda)(s_O(\bar{X}_s) - I) < 0 \tag{27}$$

In short, condition (27) shows that the entrepreneur's gain from the continuation at $X = \bar{X}_s$ does not cover the marginal drop in the price of the security due to inefficient continuation, $(1 - \lambda)(s_O(\bar{X}_s) - I)$. We conclude that the insider can get positive interim rent.

Proof for Part (c)

The investment is socially efficient if and only if $X \geq I - \varepsilon$. However, for contracts of the form $\{\lambda s(\cdot), (1 - \lambda)s(\cdot), \lambda\}$, investment takes place if $X \geq \max\{\bar{X}, \hat{X}(\lambda)\}$. Since both $\bar{X}$ and $\hat{X}(\lambda)$ are less than $I - \varepsilon$ for $\lambda > 0$, investment is inefficient with a positive probability.

A5. Proof of Proposition 2

An optimal design implements the socially optimal outcome when the investment takes place iff $X \geq I - \varepsilon$. We first show that condition (**??**) is necessary for implementing the socially optimal outcome. Then, we introduce a security that maximizes the entrepreneur's expected payoff and implements the socially optimal outcome. Finally, we characterize the set of optimal designs that are robust to ε.

The necessity of (??) for the existence of socially efficient design. Note that both the insider and the outsiders should at least break-even in expectation. Because the insiders and the outsiders cover all the investments and do not gain any private benefit, their total expected payoff cannot exceed $\mathbb{E}[(X-I)\mathbb{I}_{\{X \geq I-\varepsilon\}}] - K$. Therefore, condition (**??**) is necessary.

Social optimality of optimal designs. Note that the social surplus from the relationship financing is bounded by

$$U_{FB}^{E}=\mathbb{E}[(\varepsilon+X-I)\mathbb{I}_{\{\varepsilon+X-I\geq 0\}}]-K \tag{28}$$

We now show that this surplus is achievable for a contract that satisfies the constraints in (14). Note that (17) implies that $M(X;s_I,s_O,\lambda)=\varepsilon+X-I$ for all $X\in[\lambda I,I]$, where $M(\cdot)$ is defined in (13). Since $M(\cdot)$ is increasing in X and $M(I-\varepsilon;s_I,s_O,\lambda)=0$, the entrepreneur sends a high signal for $X\geq I-\varepsilon$, provided the security can cover the investment cost for the insiders and outsiders. Conditions (18) and (19) ensure that is the case. As a result, the entrepreneur receives expected payoff U_{FB}^{ε} and the socially optimal outcome is implemented. It suffices to show that all optimal designs implement the socially optimal outcome.

The set of robust designs. We now argue that the set of contracts specified in (17)-(19) are the only designs that are robust to ε. It is clear that these securities are not specified in terms of ε. To be robust to ε, we need to have $M(I-\varepsilon;s_I,s_O,\lambda)=0$ for all $\varepsilon>0$ that satisfy (**??**). Therefore, we should have $s_I(I-\varepsilon)=\lambda I$ for all ε that satisfy (**??**). We denote the highest value of such ε by $\bar{\varepsilon}$, and show that no investment takes place for $X\leq I-\bar{\varepsilon}$, and thus the specification of contract is irrelevant for these values. Note that according to the solution to (15), the investment function has a threshold scheme for any given contract. Therefore, the participation constraints for the insider and the outsiders imply that the threshold cannot be less than $I-\bar{\varepsilon}$. As such, there would be no investment for $X\in[0,I-\bar{\varepsilon})$. It shows the contingent transfers for these states are irrelevant.

A6. Proof of Lemma 2

We show the entrepreneur designs the experiment in a way that the realized signal z fully reveals the insider's signal y. In other words, for a given signal $z\in\mathcal{Z}$ in the optimal experiment, there exists signal $y\in\mathcal{Y}$ such that $P(y|z)=\frac{\int_0^1\pi(z,y|X)f(X)dX}{\sum_{z\in\mathcal{Z}}\int_0^1\pi(z,y|X)f(X)dX}=1$.

Consider experiment $(\mathcal{Z},\pi)$ and signal $z\in\mathcal{Z}$. Suppose there are $m\geq 2$ distinct signals $\tilde{\mathcal{Y}}(z)=\{y_1,y_2,\ldots,y_l\}\subset\mathcal{Y}$ such that $P(y_i|z)>0$ for all $1\leq i\leq l$. We show the entrepreneur can increase his payoff by splitting signal z into signals $z_1,z_2,\ldots,z_l$, where $\pi(z_i,y_j|X)=\pi(z,y_j|X)\mathbb{I}_{i=j}$.

Let us examine the entrepreneur's expected payoff in these two scenarios. It should be apparent that the insider either chooses $\lambda=1$ or $\lambda=0$ because her expected payoff is linear in her amount of investment. In the first scenario that the signals are pooled, suppose the insider makes the investment for $y\in\tilde{\mathcal{Y}}^{+}(z)\subset\tilde{\mathcal{Y}}(z)$, following signal z. When the signal z is public, the outsiders offer $p^O=\mathbb{E}[s(X)|z,y\in\tilde{\mathcal{Y}}(z)\setminus\tilde{\mathcal{Y}}^{+}(z)]$ if it exceeds the cost of investment I, otherwise they do not make any offer.

Meanwhile, the insider does not pay more than $\max\{p^O,I\}$ even if her valuation given her private signal $y\in\tilde{\mathcal{Y}}(z)^{+}$ exceeds $\max\{p^O,I\}$. As a result, the price of the security increases if the entrepreneur shares the insider's signal with the outsiders. The entrepreneur would indeed be better off by splitting the signal into $z_1,z_2,\ldots,z_l$ in the way discussed.

A7. Proof of Proposition 3

Lemma 2 reveals that the entrepreneur essentially conveys the insider's information to outsiders. T is an information set that nests that of the insider's. As such, the insider's signal alters the problem as if all investors have a different prior belief. Consequently, the entrepreneur essentially faces m different experiment design problems, specified in (7), with the priors $f(X|y_i)$. Proposition 1 then leads us to the optimal experiments under investor sophistication.

The only exception are the cases that either $P(s(X) \geq I|y_i) = 0$ or $\mathbb{E}[X|y_i] > 0$, where $\bar{X}(y_i)$ does not exist. In the first case, there would be no investment by the insider, and consequently the outsiders (according to Lemma 2), regardless of the entrepreneur's choice of signals. For the second case, the entrepreneur optimally induces invest only for the event $X \geq \hat{X}(\mu)$.

A8. Proof of Corollary 2

Suppose the contrary that there exists an insider's experiment $(\mathcal{Y}, \omega_q)$ that leads to the socially optimal investment decisions. To implement the socially optimal outcome, the threshold for all m signals should be $I - \varepsilon$. We thus need to have $\max\{\hat{X}(\mu), \bar{X}(y)\} = I - \varepsilon$ for all $y \in \mathcal{Y}$. Since $\hat{X}(\mu) < I - \varepsilon$ for all $\mu > 0$, then we need to have $\bar{X}(y) = I - \varepsilon$ for all signals in $\mathcal{Y}$. It implies $\mathbb{E}[s(X) - I|z_i^h] = 0$ for $1 \leq i \leq m$. Therefore, the insider receives zero interim expected payoff, failing to recover the initial cost K. Then the insider would not start the relationship financing in the first place, contradicting the outcome being socially optimal.

A9. Proof of Corollary 3

It is easy to show that Lemma 2 still holds: the entrepreneur chooses an experiment strictly more informative than $(\mathcal{Y}, \omega_q)$. It means the entrepreneur still solves m independent information design problem for every signal in $\mathcal{Y}$ to determine the additional information to reveal.

This independence implies that the entrepreneur does not choose a more informative experiment for signals in $\mathcal{Y}^{nb}$, compared to the benchmark case without setting milestone. Moreover, the insider's action following signals in $\mathcal{Y}^b$ is weakly dominated by that without the commitment, as she does not respond to the additional information that the entrepreneur provides for these states. Therefore, the insider does not gain from setting milestones.

A10. Proof of Proposition 4

For every insider's experiment $(\mathcal{Y}, \omega_q)$, (17)-(19) characterize the set of optimal long-term contracts. In particular, we show that under these conditions the entrepreneur optimally designs a binary experiment that sends a high signal if $X \geq I - \varepsilon$, which induces investment. First, suppose the entrepreneur chooses this experiment. (17) implies that the insider always invests if she learns that $X \geq I - \varepsilon$. (18) and (19) together imply that the outsiders also invest if and only if the entrepreneur's experiment sends a high signal. The reason is that the insider's action is binary and only reveals the signal of the entrepreneur's experiment. Therefore, the project is invested if and only if $X \geq I - \varepsilon$. Moreover, by an argument similar to the proof of Proposition 2, it is the optimal experiment for the entrepreneur and these contracts give the entrepreneur the entire social surplus. Now to show that the optimal design has to satisfy (17)-(19), we can use the argument almost verbatim in the proof of Proposition 2.

A11. Proof of Corollary 4

As follows, we show the insider earns higher expected payoff from a more informative experiment. Consider two experiments $(\mathcal{Y}, \omega_q)$ and $(\mathcal{Y}, \omega_{q'})$ with $q' > q$. Therefore, there exists an $m \times m$ Markovian matrix T such that $f_q(X|y_i) = \sum_{j=1}^m T_{ij} f_{q'}(X|y_j)$. Moreover, we can write the insider's expected payoff from experiment $(\mathcal{Y}, \omega_q)$ as:

$$U^I(\mu; q) = \mu \sum_{y \in \mathcal{Y}} P_q(y_i) \mathbb{E}\left[(s(X) - I)\mathbb{I}_{\{X \geq \max\{\hat{X}(\mu), \bar{X}(y)\}\}}\right]. \tag{29}$$

According to the definition of $\bar{X}(y)$ introduced in Proposition 3, if $\bar{X}(y) > 0$ then $\mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq \bar{X}\}}] = 0$. Therefore, we can rewrite (25) as

$$U^I(\mu; q) = \mu \sum_{i=1}^m P_q(y_i) \max\{\int_{\hat{X}(\mu)}^1 (s(X) - I) f_q(X|y_i) dX, 0\} \tag{30}$$

Substituting $f_q(X|y_i)$ by $\sum_{j=1}^m T_{ij} f_{q'}(X|y_j)$, we have

$$\begin{aligned}
U^I(\mu; q) &= \mu \sum_{i=1}^m P_q(y_i) \max\{\int_{\hat{X}(\mu)}^1 (s(X) - I) \sum_{j=1}^m T_{ij} f_{q'}(X|y_j) dX, 0\} \\
&\leq \mu \sum_{i=1}^m P_q(y_i) \sum_{j=1}^m T_{ij} \max\{\int_{\hat{X}(\mu)}^1 (s(X) - I) f_{q'}(X|y_j) dX, 0\} \\
&= \mu \sum_{i=1}^m P_{q'}(y_i) \max\{\int_{\hat{X}(\mu)}^1 (s(X) - I) f_{q'}(X|y_j) dX, 0\} = U^I(\mu; q')
\end{aligned}$$

where the last inequality follows from the identity $P_{q'}(y_j) = \sum_{i=1}^m T_{ij} P_q(y_i)$.

A12. Proof of Proposition 5

The derivative of (29) with respect to μ (when it exists) is

$$\begin{aligned}
\frac{d}{d\mu} U^I(\mu; q) = &\sum_{y \in \mathcal{Y}} P_q(y) \mathbb{E}[(s(X) - I)\mathbb{I}_{\{X \geq \max\{\hat{X}(\mu), \bar{X}_q(y)\}\}}] + \\
&\mu \sum_{y \in \mathcal{Y}} P_q(y)(s(\hat{X}(\mu)) - I) f(\hat{X}(\mu)|y) \mathbb{I}_{\{\hat{X}(\mu) > \bar{X}_q(y)\}}
\end{aligned} \tag{31}$$

To derive the relation between the insider's expected payoff and μ, fix q and consider the following two cases:

1. Suppose $\mu \geq 1 - \frac{\varepsilon}{I}$, which implies $\hat{X}(\mu) = 0$. Then (25) implies that $U^I(\cdot; q)$ is weakly increasing in μ for $\mu \in [1 - \frac{\varepsilon}{I}, 1]$.
2. Suppose $\mu < 1 - \frac{\varepsilon}{I}$, then $\hat{X}(\mu) > 0$. In this range of values of μ, if $\bar{X}_q(y) < \hat{X}(\mu)$ for some $y \in \mathcal{Y}$, then the first term in the right hand side of (31) is positive and the second term is negative. For small enough values of μ the derivative is strictly positive, since the first term dominates the second term. Moreover, the derivative is weakly decreasing, since both of the terms are decreasing in μ. It implies the insider's expected payoff is concave in μ for $\mu \in [0, 1 - \frac{\varepsilon}{I}]$.

Denote $\bar{\mu} \in [0, 1 - \frac{\varepsilon}{I}]$ the maximizer of $U^I(\cdot; q)$. If $\bar{\mu} < 1 - \frac{\varepsilon}{I}$, then the insider's expected payoff is U-shape in μ for $\mu \in [\bar{\mu}, 1]$, which completes the proof.

A13. Proof of Proposition 6

Proof for Part (a)

We first characterize the entrepreneur's information design problem and show how the optimal experiment changes with μ. For a given security $s(\alpha, r(\alpha)X)$ and the entrepreneur's experiment $(\mathcal{Z}, \pi)$, let $F_\pi(dz)$ denote the implied measure of unconditional probabilities over signals $z \in \mathcal{Z}$. We further denote the insider's optimal and the socially optimal action given the signal z by $\alpha^I(z)$ and $\alpha^S(z)$ respectively. In other words,

$$\begin{aligned} \alpha^I(z) &\in argmax_{\alpha\in[0,1]} \quad \mathbb{E}[s(\alpha, r(\alpha)X) - \alpha I|z], \\ \alpha^S(z) &\in argmax_{\alpha\in[0,1]} \quad \mathbb{E}[r(\alpha)(X+\varepsilon) - \alpha I|z] \quad s.t. \quad \mathbb{E}[s(\alpha, r(\alpha)X) - \alpha I|z] \geq 0. \end{aligned} \tag{32}$$

When the signal is privately observed by the insider, she chooses $\alpha^I(z)$. When the signal is publicly observed, all investors invest at the welfare-maximizing level, provided the security covers the investment cost. Thus, the entrepreneur solves the following optimization problem

$$\begin{aligned} \max_{(\mathcal{Z},\pi)} \mu \int_{z\in\mathcal{Z}} & \mathbb{E}[r(\alpha^I(z))(X+\varepsilon) - s(\alpha^I(z), r(\alpha^I(z))X)|z]F_\pi(dz) \\ &+(1-\mu)\int_{z\in\mathcal{Z}} \mathbb{E}[r(\alpha^S(z))(X+\varepsilon) - \alpha I|z]F_\pi(dz) \end{aligned} \tag{33}$$

Equation (33) shows that the entrepreneur faces a trade-off between gaining more rent when the signal is private (the first term) and increasing the efficiency of investment (the second term). As μ increases, the entrepreneur puts smaller weight on the social efficiency of investment, which clearly leads to experiments that implement less socially efficient outcomes.

Now, we characterize the optimal experiment when the entrepreneur uses equities ($s(\alpha, r(\alpha)X) = \beta r(\alpha)X$) and $r(\alpha) = \alpha^\gamma$. First we find the investors' optimal level of investment given a posterior $f(X|z)$. Following the definition of $\alpha^I(\cdot)$ and $\alpha^S(\cdot)$ provided in (32), we can easily see:

$$\alpha^I(z) = \left(\frac{\gamma\beta\mathbb{E}[X|z]}{I}\right)^{\frac{1}{1-\gamma}}$$
$$\alpha^S(z) = \min\left\{\left(\frac{\gamma\mathbb{E}[X+\varepsilon|z]}{I}\right)^{\frac{1}{1-\gamma}}, \left(\frac{\beta\mathbb{E}[X|z]}{I}\right)^{\frac{1}{1-\gamma}}\right\}$$

Moreover, since the scale of the investment only depends on the expected state $\mathbb{E}[X|z]$, we can WLOG assume that the signal only has information about the expected state. We can then denote the distribution of signals by $G(X)$, where $F(X)$ is a mean-preserving spread of $G(X)$. As a result, we can rewrite the entrepreneur's problem as follows:

$$\max_{G(X)\prec_2 F(X)} \int_0^1 u_\mu(X)dG(X), \tag{34}$$

where

$$u_\mu(X) = \mu a_I(X) + (1-\mu)a_S(X)$$

$$a_I(X) = (\frac{\gamma\beta}{I})^{\frac{\gamma}{1-\gamma}} X^{\frac{\gamma}{1-\gamma}}((1-\beta)X+\varepsilon)$$

$$a_S(X) = \begin{cases} \left(\frac{\beta}{I}\right)^{\frac{\gamma}{1-\gamma}} X^{\frac{\gamma}{1-\gamma}}((1-\beta)X+\varepsilon) & \text{if } \beta X \leq \gamma(X+\varepsilon) \\ I^{-\frac{\gamma}{1-\gamma}}(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}})(X+\varepsilon)^{\frac{1}{1-\gamma}} & \text{if } \beta X > \gamma(X+\varepsilon) \end{cases}$$

The following lemma is useful when we show that $G(\cdot)$ becomes less informative in the Blackwell sense as μ increases.

Lemma (A2). *For every $\mu \in [0,1]$, there exists $X^*(\mu)$ such that the optimal experiment entails pooling all the types below $X^*(\mu)$ and separating all the types above $X^*(\mu)$.*

Proof. First we show that there exists a threshold value $T(\mu)$ such that $u''_\mu(X) > 0$ for $X > T(\mu)$ and $u''_\mu(X) < 0$ for $X < T(\mu)$. Then, the proof follows from Theorem 1 in Dworczack and Martini (2017).

By taking the second order derivative of $a_I(\cdot)$ and $a_S(\cdot)$, we get:

$$a''_I(X) = \left(\frac{\gamma\beta}{I}\right)^{\frac{\gamma}{1-\gamma}} \left(((1-\beta)\left((2X^2 + \frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\right)X + \frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\varepsilon\right) X^{\frac{3\gamma-2}{1-\gamma}}$$

$$a_S(X) = \begin{cases} (\frac{\beta}{I})^{\frac{\gamma}{1-\gamma}}\left(((1-\beta)\left((2X^2 + \frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\right)X + \frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\varepsilon\right) X^{\frac{3\gamma-2}{1-\gamma}} & \text{if } \beta X \leq \gamma(X+\varepsilon) \\ I^{-\frac{\gamma}{1-\gamma}}\left((\frac{\gamma}{(1-\gamma)^2}\right)\left(\gamma^{\frac{\gamma}{1-\gamma}} - \gamma^{\frac{1}{1-\gamma}}\right)(X+\varepsilon)^{\frac{2\gamma-1}{1-\gamma}} & \text{if } \beta X > \gamma(X+\varepsilon) \end{cases} \tag{35}$$

For $\gamma \geq \frac{1}{2}$, $u_\mu(\cdot)$ is convex and full-disclosure is optimal. Moreover, if $\gamma(1+\varepsilon) \geq \beta$, $u_\mu(X)$ just scales with the change of μ, hence the optimal information disclosure is independent of μ. For $\gamma < \min\{\frac{1}{2}, \frac{\beta}{1+\varepsilon}$, there is $T(\mu)$ such that $\left(2X^2 + \frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\right)X + \frac{\gamma(2\gamma-1)}{(1-\gamma)^2}\varepsilon < 0$ if and only if $X < T'$ over the range of $X \in [0,1]$. Therefore, there is a threshold $T(\mu) \in [\frac{\gamma\varepsilon}{\beta-\gamma}, T']$, for which $u''_\mu(X) < 0$ if and only if $X < T(\mu)$. Q.E.D.

□

This characterization of optimal disclosure implies that we only need to show $X^*(\mu)$ is weakly increasing in μ. For $\gamma \geq \frac{1}{2}$, $X^*(\mu) = 0$, hence the statement is straightforward. For $\gamma < \frac{1}{2}$, $X^*(\mu) \in [T(\mu), 1]$, since $u''_\mu(X) < 0$ for $X < T(\mu)$ and they should be pooled. Moreover, note that $X^*(\mu)$ is the solution to $\mathbb{E}[u_\mu(X)|X \leq X^*(\mu)] = u_\mu(\mathbb{E}[X|X \leq X^*(\mu)])$. If such a solution does not exist, then $X^*(\mu) = 1$, i.e. no-disclosure is optimal. Finally, it is easy to check that $\frac{\partial}{\partial\mu\partial X^2}u_\mu(X) < 0$ for $X \in [\frac{\gamma\varepsilon}{\beta-\gamma}, T(\mu)]$. Therefore, $\mathbb{E}[u_{\mu_2}(X)|X \leq X^*(\mu_1)] \leq u_{\mu_2}(\mathbb{E}[X|X \leq X^*(\mu_1)])$ for any $\mu_2 > \mu_1$. Consequently, $X^*(\mu_2) \geq X^*(\mu_1)$. Q.E.D.

Proof for Part (b)

Consider $s^I(\alpha, r(\alpha)X) = \lambda \max\{r(\alpha)X, \alpha I\}$, where λ is defined such that $\mathbb{E}[r(\alpha)X - \alpha I] = \frac{K}{\lambda}$. Moreover, suppose the outsiders receive the residue $s^O(\alpha, r(\alpha)X) = r(\alpha)X - s^I(\alpha, r(\alpha)X)$. We want to show that if the entrepreneur fully discloses X and chooses the optimal level of investment $\alpha^*(X;\varepsilon)$, then both the insider and the outsiders are willing to invest their share of investment, which are $\lambda\alpha^* I$ and $(1-\lambda)\alpha^* I$, respectively.

The insider is always willing to invest α^* as $s^I(\alpha, r(\alpha)X) - \lambda\alpha I \geq 0$ for all values of α. Moreover, the assumption in the proposition implies that $r(\alpha^*)X \geq \alpha^* I$ with probability one. Therefore, the outsiders are also willing to participate because

$$s^O(\alpha^*, r(\alpha^*)X) - (1-\lambda)\alpha^* I = r(\alpha^*)X - \lambda\max\{r(\alpha^*)X, \alpha^* I\} - (1-\lambda)\alpha^* I = (1-\lambda)(r(\alpha^*)X - \alpha^* I) \geq 0$$

It is easy to see the solution is robust to ε and the insider's private experimentation.

A14. Proof of Proposition 7

Proof for Part (a)

Since the insider's payoff is continuous in μ, we only need to prove the insider receives zero expected payoff for $\mu = 0$ and $\mu = 1$. For $\mu = 0$, the insider has no information rent and clearly gets zero expected payoff. We now discuss the case of $\mu = 1$.

Secret Manipulation. First suppose the entrepreneur can secretly change the signal realization with probability $\alpha > 0$. Similar to Proposition 1, the entrepreneur follows a threshold strategy, i.e., there exists $\bar{X}_\alpha \in [0,1]$ such that the experiment generates a high signal for $X \geq \bar{X}_\alpha$. The high signal induces investment if the investor receives non-negative expected payoff from the investment following the high signal, which is equivalent to

$$\alpha \int_0^{\bar{X}_\alpha} (s(X) - I) f(X) dX + \int_{\bar{X}_\alpha}^1 (s(X) - I) f(X) dX \geq 0. \tag{36}$$

The first term in (36) shows the probability that the experiment generates a low signal, but the entrepreneur sends a high signal. Note that for $\alpha = 1$, the inequality does not hold for any threshold value. Therefore, there exists $\bar{\alpha} \in [0,1]$ above which the investment is not feasible. However, for $\alpha \leq \bar{\alpha}$, the entrepreneur chooses $\bar{X}_\alpha$ such that the inequality (36) binds, which implies the investor becomes indifferent between investment and not investment after receiving the high signal. As a result, the investor receives zero interim rent for all values of $\alpha \in [0,1]$.

Random Monitoring. Now consider the case that the insider investor verifies the signal realization with probability $\beta < 1$. This case involves two subcases:

First, the investor cannot commit to punish the entrepreneur for misreporting. In this subcase, there is no signal such as h that always induces investment, because otherwise the entrepreneur would optimally always report h, which leads to negative expected payoff for the investor when she does not monitor. As such, the investor only invests when she monitors, which makes the entrepreneur's reporting strategy irrelevant. Hence the entrepreneur would follow the threshold strategy at $\bar{X}$, in which the investor does not get interim rent.

Now consider the subcase that the investor commits to punish misreporting. in this case, the entrepreneur might use 4 different kinds of signals in his experiment: 1. low signals, such as l_0, that never induce investment. 2. Low signals, such as l_1, that only induce investment when the investor does not monitor. 3. high signals, such as h_0, that only induce investment if they are verified. 4. High signals, such as h_1, that always induce investment. The probability of investment is $(1-\beta)P(l_1) + \beta P(h_0) + P(h_1)$. We next check the IC constraints for truthful reporting for the entrepreneur.

In particular, we show there is no equilibrium that the insider invests without monitoring. Consider the contrary. If $\beta < \frac{1}{2}$, then types l_0 and h_0 prefer to report h_1 instead of truthful reporting. Therefore, we should have $P(l_0) = P(h_0) = 0$, which implies that the investor always invests when she is not monitoring. Thus, she would be better off by not investing at all when she does not monitor. It implies $P(l_1) = P(h_1) = 0$ as well. It is a contradiction. For $\beta \geq \frac{1}{2}$, $P(l_0) = 0$, because he would be strictly better off by reporting h_1. Therefore, the investor without monitoring $h_1 \cup l_1$ realizes. If the investor receives positive payoff from investment following $h_1 \cup l_1$, she should make a loss by investing following h_0, since $P(h_0)+P(l_1)+P(h_1) = 1$. Therefore, the investor would be better off by not investing following h_0, which is a contradiction. Therefore, the investor never invests without monitoring.

Proof for Part (b)

Secret Manipulation. First, we show that the socially optimal outcome cannot be implemented for $\alpha > 0$, then we show the optimal contracts involve using convertible securities, i.e. Condition (17) holds.

Note that once the entrepreneur raises K, he always wants to continue the project, since he receives strictly positive payoff from continuation. Consequently, the entrepreneur misreports the bad signals whenever it is possible, which leads to inefficient continuation. Therefore, the socially optimal outcome is not implementable for $\alpha > 0$.

Now we solve for the optimal contract. If the project is invested with positive probability, then the expected payoff of the entrepreneur from the contract $\{s_I(.), s_O(.), \lambda\}$ is

$$U^E_\alpha(s_I(\cdot), s_O(\cdot), \lambda) = \mathbb{E}[M(X; s_I, s_O, \lambda)(\alpha + (1-\alpha)\mathcal{I}(X))],$$

where $\mathcal{I}(\cdot)$ is the investment function for the case that the entrepreneur cannot secretly manipulate the signal. If $\mathbb{E}[\max\{X - I + \varepsilon, 0\}] > K$, then with an argument similar to the proof of Proposition 2, the following set of convertible securities are optimal and robust to ε, for small enough values of α:

$$\lambda I \le I - \bar{\varepsilon}, \quad s_I(X) = \min\{\lambda I, X\} \ \ \forall\, X < I, \quad \mathbb{E}[(s_I(X) - \lambda I)(\alpha + (1-\alpha)\mathbb{I}_{\{X \ge I-\varepsilon\}})] = K, \quad (37)$$

$$\mathbb{E}[(s_O(X) - (1-\lambda)I)(\alpha + (1-\alpha)\mathbb{I}_{\{X \ge I-\varepsilon\}})] = K, \quad 0 \le s_O(X) \le X - s_I(X) \ \ \forall\, X \in [0,1] \quad (38)$$

Note that the design might not be robust to the insider's experiment $(\mathcal{Y}, \omega_q)$ because the insider's payoff becomes sensitive to the downside realization of the final cash-flow. However, it is still robust to the value of ε, for the insider's ex-post payoff does not depend on ε. Moreover, for big enough values of α, no security can satisfy condition (37). Therefore, relationship financing is infeasible for such big values of α.

Random Monitoring. Consider the case that the investor cannot credibly threaten the entrepreneur to terminate the project when he misreports. The argument for the other case is similar. As it is discussed earlier, in equilibrium, the entrepreneur always reports a high signal and the investor invests if and only if she verifies the signal is truthfully reported. Therefore, the socially optimal outcome cannot be implemented when $\beta < 1$. Moreover, the following convertible securities are optimal and robust to the values of ε.

$$\lambda I \le I - \bar{\varepsilon}, \quad s_I(X) = \min\{\lambda I, X\} \ \ \forall\, X < I, \quad \beta\mathbb{E}[(s_I(X) - \lambda I)\mathbb{I}_{\{X \ge I-\varepsilon\}}] = K,$$
$$\mathbb{E}[(s_O(X) - (1-\lambda)I)(\alpha + (1-\alpha)\mathbb{I}_{\{X \ge I-\varepsilon\}})] = K, \quad 0 \le s_O(X) \le X - s_I(X) \ \ \forall\, X \in [0,1]$$

Clearly, these securities are implementable for large enough values of β. For the case of credible punishments, a robust design might not exist, as the optimal experimentation involves three signals for large values of ε, while it involves two signals for the smaller values. However, for smaller values of ε, the convertible securities specified above are optimal and the equilibrium outcomes are similar.

A15. Proof of Proposition 8

According to Propositions 2 and 4, the entrepreneur's design implements the first best outcome. It is easy to show that the insider receives $\mathbb{E}[(X-I)\mathbb{I}_{\{X\geq I\}}]$ in her optimal design, which implements investment only for $X \geq I$. Therefore, the insider's optimal design does not incorporate the entrepreneur's private benefit of investment, resulting in a lower social welfare.

Appendix B: Extended Discussions

B1. Optimal Design of a Single Security

Due to regulatory concerns, issuance costs, or high cost of communicating with outsiders in the later rounds, the entrepreneur often can only issue one single form of security to both insiders and outsiders. For example, in venture financing the right of first refusal gives its holder the contractual rights but not the obligation to purchase new security issuance before others can purchase that *same security with the same terms*; banks by regulation can only use debt contracts.

As follows, we show that if the entrepreneur can commit to financing λI from the outsider, the unique optimal security is equity. In the absence of such commitment, the optimal security is indeterminate. However, the set of optimal securities include debt contracts, but not necessarily equity contracts.

Specifically, the entrepreneur issues security contract $s(X)$ and determines the fraction to the insiders λ, i.e., $s_I(X) = \lambda s(X)$ and $s_O(X) = (1-\lambda)s(X)$. Proposition 9 characterizes the equilibrium security, optimal level of commitment to outsider competition, and the optimal information production.

Proposition 9 (Optimality of Equity)**.** *Under the single-security constraint and for a given insider's experiment* $(\mathcal{Y}, \omega_q)$*, the entrepreneur optimally issues equities to both the insider and the outsiders. In particular,* $s(X) = X$*, hence* $s_I(X) = \lambda^* X$ *and* $s_O(X) = (1-\lambda^*)X$ *for some* $\lambda^* \in (0,1)$*. The optimal security is unique and it does not implement the first best outcome if* $K > 0$*.*

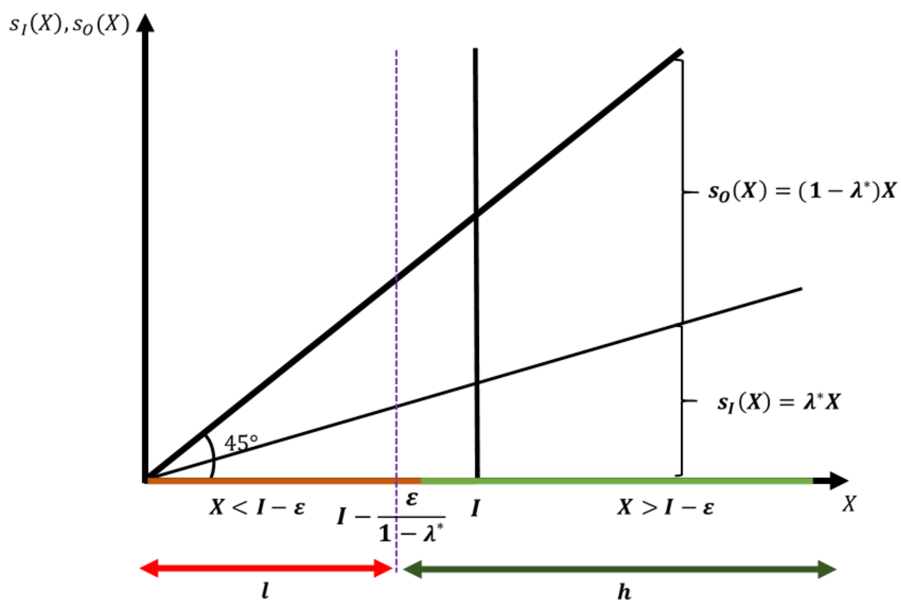


Figure 5: The optimal securities for the entrepreneur under one-security condition; specific example of without investor sophistication.

Proof. We prove the proposition for the case of unsophisticated insider, where she has no proprietary information about the outcome, other than what is provided by the entrepreneur's experiment. The proof for the case of sophisticated investors is similar. The single-security constraint implies that the entrepreneur chooses from contracts in the form of $\{\lambda s(X), (1-\lambda)s(X), \lambda\}$. Therefore, every contract can be represented by the pair of $\{s(X), \lambda\}$.

Denote the entrepreneur's indirect utility from investment at X by $M^s(s,\lambda)$, where:

$$M^s(X;s,\lambda) = \varepsilon + X - \lambda s(X) - (1-\lambda)I = M(X;\lambda s(.),(1-\lambda)s(.),\lambda) \tag{39}$$

The entrepreneur then solves the following maximization problem:

$$\begin{aligned} &\max_{s(.),\lambda,X_c} \mathbb{E}[M^s(X;s,\lambda)\mathbb{I}_{\{M^s(X;s,\lambda)\geq 0\}}] \\ &s.t. \quad \lambda\mathbb{E}[(s(X)-I)\mathbb{I}_{\{M^s(X;s,\lambda)\geq 0\}}] \geq K \end{aligned} \tag{40}$$

Note that the objective function in (40) is bounded by $\mathbb{E}[\max\{\varepsilon+X-I,0\}]$ and the constraint constitutes a closed and bounded subset in a $\mathcal{L}^1 \times [0,1]$ space that contains all combination of regular securities and λ. Therefore, the optimal contract exists. We denote the security $s(X)=X$, by $s_i(\cdot)$.

Definition 1. *Consider the contract $\{s(.),\lambda\}$. If $M^s(0;s,\lambda)<0$, then $\hat{X}(s(.),\lambda)$ is defined to be the solution to $M^s(\hat{X}(s(.),\lambda);s,\lambda)=0$; otherwise, we define $\hat{X}(s(.),\lambda)=0$.*

The next lemma follows immediately from the expression (39) and Definition 1.

Lemma (B1). *Suppose $s_1(.)$ and $s_2(.)$ are two regular securities such that $s_1(X) \geq s_2(X)\ \forall\ X \in [0,1]$.*
a) *$\hat{X}(s_1,\lambda)$ is weakly decreasing in λ.*
b) *$\hat{X}(s_1,\lambda) \geq \hat{X}(s_2,\lambda)$, for every $\lambda \in [0,1]$.*

Now, we are ready to prove the optimality of equity for implementing the payoff channel. Consider a contract $\{s_1,\lambda_1\}$ that satisfies the constraint in (40). Part (b) in Lemma B1 implies:

$$\lambda_1\mathbb{E}[(s_1(X)-I)\mathbb{I}_{\{X\geq\hat{X}(s_1(.),\lambda_1)\}}] \leq \lambda_1\mathbb{E}[(X-I)\mathbb{I}_{\{X\geq\hat{X}(s_i(.),\lambda_1)\}}] \tag{41}$$

Then there exists $\lambda_i \leq \lambda_1$ such that $\lambda_i\mathbb{E}[(X-I)\mathbb{I}_{\{X\geq\hat{X}(s_i(.),\lambda_i)\}}] = K$. Next, we show that the entrepreneur prefers the contract $\{s_i(.),\lambda_i\}$ to $\{s_1,\lambda_1\}$:

$$\begin{aligned} &\mathbb{E}[M(X;s_i,\lambda_i)\mathbb{I}_{\{X\geq\hat{X}(s_i(.),\lambda_i)\}}] = \mathbb{E}[(\varepsilon+(1-\lambda_i)(X-I))\mathbb{I}_{\{X\geq\hat{X}(s_i,\lambda_i)\}}] = \mathbb{E}[(\varepsilon+X-I)\mathbb{I}_{\{X\geq\hat{X}(s_i,\lambda_i)\}}] - K \\ &\geq \mathbb{E}[(\varepsilon+X-I)\mathbb{I}_{\{X\geq\hat{X}(s_1(.),\mu_1)\}}] - \lambda_1\mathbb{E}[(s_1(X)-I)\mathbb{I}_{\{X\geq\hat{X}(s_1(.),\lambda_1)\}}] = \mathbb{E}[M(X;s_1,\lambda_1)\mathbb{I}_{\{X\geq\hat{X}(s_1(.),\lambda_1)\}}] \end{aligned}$$

Therefore, the optimal contract under the single security constraint is equity. For the case with sophisticated investors, λ^* depends on the insider's experiment, although an equity is still uniquely optimal.

□

Proposition 9 shows that the optimal single security is equity when commitment in λ is feasible. Recall that the entrepreneur uses the security to best commit himself to efficient interim information production. With a single security, the entrepreneur cannot fully allocate the downside exposure to himself because both insiders and outsiders get the same security, and the outsiders are just a pass-through of interim surplus to the entrepreneur. However, by reducing λ, we naturally reduce the insider's exposure relative to the entrepreneur. Equity is thus optimal when λ is endogenous because it gives the insider the largest upside, allowing her to get enough rent to cover K with the smallest λ. Figure 5 displays the optimal security for the entrepreneur.

Proposition 10 (Optimality of Debt Contracts)**.** *Under the single security-constraint and when committing to raising λI from the insider is not feasible, the optimal security is indeterminate but the set of optimal securities includes debt.*

Proof. When the entrepreneur cannot commit to the way he finances I, the insider gets positive rent only if the signal is private. Therefore, she demands all issues when the signal is good. To give the insider lower rent, the entrepreneur use securities that make the insider break even, i.e.

$$U^I(s(\cdot)) = \mu\mathbb{E}\left[(s(X) - I)\mathbb{I}_{\{X \geq \hat{X}(\mu;s(\cdot))\}}\right] = K \tag{42}$$

The entrepreneur's expected payoff for a given choice of $s(\cdot)$ from this set of securities is

$$U^E(s(\cdot)) = \mathbb{E}\left[(\varepsilon + X - \mu s(X) - (1-\mu)I)\mathbb{I}_{X \geq \hat{X}(\mu;s(\cdot))}\right] = \mathbb{E}\left[(\varepsilon + X - I)\mathbb{I}_{\{X \geq \hat{X}(\mu;s(\cdot))\}}\right] - K. \tag{43}$$

To maximize (43), the entrepreneur chooses a security that maximizes $\hat{X}(\mu; s(\cdot))$. Note that $\hat{X}(\mu; s(\cdot))$ is the solution to the following equality:

$$\hat{X}(\mu; s(\cdot)) : \quad \varepsilon + \hat{X}(\mu) - \mu s(\hat{X}(\mu)) - (1-\mu)I = 0 \Rightarrow \hat{X}(\mu) \leq I - \frac{\varepsilon}{1-\mu}$$

and equality holds if and only if $s(X) = X$ for $X \leq [I - \frac{\varepsilon}{1-\mu}]$. Therefore, all double-monotone securities with $\mathbb{E}\left[(s(X) - I)\mathbb{I}_{\{X \geq I - \frac{\varepsilon}{1-\mu}\}}\right] = K$ and $s(X) = X$ for $X \leq I - \frac{\varepsilon}{1-\mu}$ implement the optimal design for the entrepreneur. Clearly, a debt or a convertible security can implement these two conditions, but equity or call options cannot.

□

In Proposition 10, we study the case that the entrepreneur can only commit to the single security he issues in future. For example, the entrepreneur might commit to the form of security by choosing his institutional investor (Banks, Private Equities, Venture Capitalist, etc). In this case, if the commitment to the future round of investments is not feasible, then debt contracts become optimal. The intuition is the following: The security should be the most sensitive to the losses to discipline the entrepreneur the most. Since a debt contract is the most sensitive security in the down-side, it always can implement the optimal security and it is regardless of μ and the insider's experiment.

B2. Insider's Independent Experimentation

We revisit the entrepreneur's optimal information design, the entrepreneur and the insiders' equilibrium expected payoff and our contractual solution to IPH for the case that the signals of the entrepreneur's experiment are restricted to be independent of the insider's experiment, conditional on the outcome X. In Proposition 11, we show that the U-shape pattern in the insider's payoff as a function of μ still persists. In Corollary 5, consistent with Proposition 5 in Kolotilin (2017), we show the insider's payoff is not increasing in the informativeness of her signal. In Proposition 12, we examine the effect of interim competition on the efficiency of investment decisions and the insider's interim payoff for $q \in [\frac{1}{2}, 1]$.

The results are provided for the specific case that the information structure of the insider's signal follows the one introduced in Example 1, with the only difference that q represents the probability of receiving signal $\tilde{h}_q$ ($\tilde{l}_q$) when $X - I \geq 0$ ($X - I < 0$). As a reminder, we repeat the structure of the insider's private information:

Assumption 1. *The insider's experiment generates a binary signal, i.e.* $\mathcal{Y} = \{\tilde{h}, \tilde{l}\}$, *of the following information structure:*

$$\omega_q(\tilde{h}|X) = \begin{cases} q & I \leq X \leq 1 \\ 1-q & 0 \leq X < I. \end{cases}$$

Proposition 11 (General IPH). *Suppose $\bar{X}(q)$ is the solution to the following equation:*

$$q\int_0^I (s(X)-I)f(X)dX + (1-q)\int_I^1 (s(X)-I)f(X)dX = 0 \tag{44}$$

Moreover, assume $q \geq \bar{q}$, where $\bar{q}$ is the solution to equation $\bar{q}^2 + \bar{q} = 1$. (a) The entrepreneur chooses an experiment that sends a high signal if $X \geq \max\{\bar{X}(q), \hat{X}(\mu)\}$ and sends a low signal, otherwise.

(b) Denote $\bar{K}_C^{\varepsilon}(\mu, q) \equiv \mu\mathbb{E}[(s(X)-I)\mathbb{I}_{X \geq \max\{\bar{X}(q),\hat{X}(\mu)\}}]$ as the capacity of relationship formation for given q and μ. Then, there exists level of sophistication q_1 such that for every $q \in (\frac{1}{2}, q_1)$, $\bar{K}_C^{\varepsilon}(\mu, q)$ is U-shaped in $[\hat{\mu}(q), 1]$ for some $\hat{\mu}(q) \in (0,1)$,. $\bar{K}_C^{\varepsilon}(\mu, q)$ is decreasing in μ for $q \in [q_1, \bar{q}]$.

(c) As $\varepsilon \to 0$, function $\bar{K}_C^{\varepsilon}(\mu; q)$ converges to $\bar{K}_C^{0}(\mu; q)$, which is increasing in μ.

Proof. Similar to the proof of Lemma 1, we prove the lemma for regular securities $s(X)$. First we note that if $\mathbb{E}[s(X) - I|X > I] \leq 0$, then $\bar{X} \geq I$, where $\bar{X}_s$ is the threshold introduced in Proposition 1. In this case, the investor's signal y is not used and the equilibrium experiments, investment functions and payoffs are the same as the ones provided in Proposition 1. As such, the remaining proof is centered on the case $\mathbb{E}[s(X) - I|X > I] > 0$ for simplicity in exposition.

Proof for Part (a)

Similar to the proof of Proposition 1, first we show that for every experiment, there is another experiment with bounded number of signals that gives the entrepreneur the same expected payoff. It helps us to prove the existence of optimal experiments and then characterize them. We refer to the private information (broadly defined) the insider has as *investor type*.

Lemma (B2). *Denote T as the set of investor type and A the set of actions. As long as A and T are finite, for every experiment $(\mathcal{Z}, \pi)$, there exists experiment $(\mathcal{Z}', \pi')$ such that $|\mathcal{Z}'| \leq |A|^{|T|}$ and $U^E(\mathcal{Z}', \pi') = U^E(\mathcal{Z}, \pi)$.*

Proof. For every pure strategy of the investor, such as $\mathfrak{a}(\cdot) : T \to A$, define $\mathcal{Z}(\mathfrak{a})$ as the set of signals in $\mathcal{Z}$, such as z, that the investor chooses $\mathfrak{a}(t)$ when her type is t and she receives z. Note that if $\mathcal{Z}(\mathfrak{a})$ is non-empty, then:

$$\begin{aligned} &\mathbb{E}[u^I(\mathfrak{a}(t), X)|t, z] \geq \mathbb{E}[u^I(a', X)|t, z] \quad \forall t \in T, z \in \mathcal{Z}(\mathfrak{a}) \\ \Rightarrow &\mathbb{E}[u^I(\mathfrak{a}(t), X)|t, \mathcal{Z}(\mathfrak{a})] \geq \mathbb{E}[u^I(a', X)|t, \mathcal{Z}(\mathfrak{a})] \quad \forall t \in T \end{aligned}$$

where $u^I(a, X)$ is the final payoff of the investor from the action a in state X. Now define the experiment $(\mathcal{Z}', \pi')$ as follows: $\mathcal{Z}' = \{z_{\mathfrak{a}}\}_{\mathfrak{a} \in A^T}$, for all $\mathfrak{a}$ that $\mathcal{Z}(\mathfrak{a})$ is non-empty. Moreover, define

$$\pi'(z_{\mathfrak{a}}|X) = \sum_{z \in \mathcal{Z}(\mathfrak{a})} \pi(z|X) \quad \text{if } \mathcal{Z}(\mathfrak{a}, t) \text{ is non-empty}$$

Note that by definition, $z_{\mathfrak{a}}$ is the signal in $\mathcal{Z}'$ that induces the strategy profile $\mathfrak{a}(t)$. We only need to show that $U^E(\mathcal{Z}', \pi') = U^E(\mathcal{Z}, \pi)$. To see this,

$$U^E(\mathcal{Z}', \pi') = \int_0^1 \sum_{t \in T} \sum_{z_{\mathfrak{a}} \in \mathcal{Z}'} u^E(\mathfrak{a}(t), X)\pi'(z_{\mathfrak{a}}|X)g(t|X)f(X)dX$$

$$= \int_0^1 \sum_{t \in T} \left[\int_{z \in \mathcal{Z}} u^E(a(z,t), X)\pi(z|X)dz\right] g(t|X)f(X)dX = U^E(\mathcal{Z}, \pi)$$

where $a(z,t)$ is the investor's action for type t upon receiving signal z. □

Building from the above result, the next lemma proves the existence of an optimal experiment and characterizes it.

Lemma (B3). ***(a)*** *Suppose the investor's action is binary ($A = \{0,1\}$) and investor types are ordered by $T = \{t_1, t_2, \ldots, t_{|T|}\}$ such that posteriors $u^I(1,X) - u^I(0,X)|t_i$ are ranked by first-order stochastic dominance. Then for every experiment, there is an experiment that implements the same expected payoffs and uses at most $|T|+1$ signals.* ***(b)*** *Under these conditions, an optimal experiment exists.*

Proof.

Proof of Lemma B3 Part (a)

Note that under single-property condition, $a(z,t)$ is weakly increasing in t. Therefore, there are at most $|T|+1$ functions of the form $\mathfrak{a} : T \to A$ that $\mathcal{Z}(\mathfrak{a})$ is non-empty. With a similar argument as the one in the proof of the lemma, one can construct an experiment with at most $|T|+1$ signals that implements the same expected payoffs.

Proof of Lemma B3 Part (b)

To show the existence of the optimal experiment, we only need to look at experiments with at most $|T|+1$ signals. In particular, we only need to show that the conditional distributions $\pi(z|X)$, $z \in \mathcal{Z}$, constitute a closed bounded set in $\mathcal{L}^{1^{|T|+1}}$.

For an experiment $(\mathcal{Z}, \pi)$, we can assume it has at most one signal $z_i \in \mathcal{Z}$ such that the entrepreneur chooses $a = 1$ only if her types is t_i or above. $z_{|T|+1}$ is the signal following which no types would invest. Therefore, the part (a) shows that every experiment implements the same expected payoffs with an experiment with following conditions:

$$\begin{aligned} &\int_0^1 (u^I(1,X) - u^I(0,X))\pi(z_i|X)g(t_j|X)f(X) \geq 0 \quad \text{iff } j \geq i, \, \forall 1 \leq i \leq |T|+1 \\ &\sum_{i=1}^{|T|+1} \pi(z_i|X) = 1 \quad \forall X \in [0,1], \quad \textit{and} \quad \pi(z_i|X) \geq 0 \quad \forall X \in [0,1], z_i \in \mathcal{Z} \end{aligned} \tag{45}$$

It is easy to check that the set of experiments satisfying (45) is closed and bounded. Therefore, an optimal experiment exists.

□

As a result of the lemma and the corollary, the optimal experiment exists and has at most three signals. An exhaustive list of candidate signal ranges for such an optimal experiment is $\{m,l\}$, $\{h,l\}$ or $\{m,h,l\}$, where the investor only invests if she receives $(m,\tilde{h})$, $(h,\tilde{l})$ or $(h,\tilde{h})$. We show that for $q \leq \bar{q}$, the optimal experiment is essentially unique and it is of the second form.

Lemma (B4). *No two-signal experiment in the form of $(\{l,m\},\pi)$, where the investor invests iff she receives $(m,\tilde{h})$, is optimal.*

Proof. Suppose the contrary and suppose that there exists an optimal experiment $(\{m,l\},\pi^*_M)$. Therefore, π^*_M solves the following maximization problem:

$$\max_{\pi(m|X)} (1-q)\int_0^I (\varepsilon + X - s(X))\pi(m|X)f(X)dX + q\int_I^1 (\varepsilon + X - s(X))\pi(m|X)f(X)dX$$

$$s.t. \quad (1-q)\int_0^I (s(X)-I)\pi(m|X)f(X)dX + q\int_I^1 (s(X)-I)\pi(m|X)f(X)dX \geq 0$$

$$\pi(m|X) \in [0,1] \qquad \forall X \in [0,1]$$

Let κ be the multiplier corresponding to the constraint. π_M^* then maximizes the following objective function, given the constraint $\pi_M^*(m|.) \in [0,1]$.

$$\max_{\pi(m|X)} (1-q)\int_0^I [\varepsilon + X - s(X) + \kappa(s(X)-I)]\pi(m|X)f(X)dX$$
$$+q\int_I^1 [\varepsilon + X - s(X) + \kappa(s(X)-I)]\pi(m|X)f(X)dX$$

Since both $X - s(X)$ and $s(X) - I$ are weakly increasing functions, there is a threshold value $X_m \in [0,1]$ such that for all $X \geq X_m$, the expression in the bracket is non-negative. According to lemma A1, the optimal experiment among those that only implement signals m and l has a threshold scheme, where the threshold X_m satisfies the following:

$$(1-q)\int_{X_m}^I (s(X)-I)f(X)dX + q\int_I^1 (s(X)-I)f(X)dX = 0$$

In this case, the expected utility of the entrepreneur from $(\{m,l\},\pi_M^*)$ is:

$$U^E(\{m,l\},\pi_M^*) = (1-q)\int_{X_m}^I (\varepsilon + X - s(X))f(X)dX + q\int_I^1 (\varepsilon + X - s(X))f(X)dX \tag{46}$$

By comparing the recent equality with (44), it is easy to see that $X_m < \bar{X}(q)$. Now, we show how the entrepreneur can improve upon π_M^* by introducing signal h (a signal that induces investment regardless of the signal the investor receives). To show this, we consider two cases:

- $s(I) < I$: In this case, we can find a subset $A \subset [I,1]$ such that $\int_A (s(X)-I)f(X)dX = 0$. Then an experiment that sends h for the members of A ($\pi(h|X) = 1$ iff $X \in A$) and sends m for $[X_m, 1] \setminus A$ implements higher payoff for the entrepreneur by $(1-q)\int_A (\varepsilon + X - s(X))f(X)dX$.
- $s(I) = I$: Since $X - s(X)$ is a weakly increasing function, it implies that $s(X) = X$ for all $X \leq I$. Consider small positive values $\eta_1, \eta_2, \eta_3 \geq 0$ that satisfy the following:

$$(1-q)\int_{X_m+\eta_1}^{I-\eta_2} (s(X)-I)f(X)dX + q\int_{I+\eta_3}^1 (s(X)-I)f(X)dX \geq 0$$

$$q\int_{I-\eta_2}^{I} (s(X)-I)f(X)dX + (1-q)\int_{I+\eta_3}^1 (s(X)-I)f(X)dX \geq 0$$

and introduce the following alternative experiment $(\{l,m,h\},\tilde{\pi}_M)$:

$$\tilde{\pi}_M(h|X) = \begin{cases} 1 & X \in [I-\eta_2, I+\eta_3) \\ 0 & X \in [0, I-\eta_2) \cup [I+\eta_3, 1] \end{cases} \qquad \tilde{\pi}_M(m|X) = \begin{cases} 1 & X \in [X_m+\eta_1, I-\eta_2) \cup [I+\eta_3, 1] \\ 0 & X \in [0, X_m+\eta_1) \cup [I-\eta_2, I+\eta_3) \end{cases}$$

It is easy to verify that the experiment $\tilde{\pi}_M$ is designed in a way that the investor invests iff she receives one of $(m,\tilde{l})$, $(h,\tilde{l})$ or $(h,\tilde{h})$. Now, the difference in expected payoffs for the entrepreneur is given by:

$$U^E(\{m,l\},\tilde{\pi}_M) - U^E(\{m,l\},\pi^*_M) = -(1-q)\int_{X_m}^{X_m+\eta_1}(\varepsilon + X - s(X))f(X)dX$$
$$+\,q\int_{I-\eta_2}^{I}(\varepsilon + X - s(X))f(X)dX + (1-q)\int_{I}^{I+\eta_3}(\varepsilon + X - s(X))f(X)dX$$

Now consider the contrary, that the introduced experiment is an optimal experiment. Then $\eta_1^* = \eta_2^* = \eta_3^* = 0$ should satisfy the first order conditions for the following maximization problem:

$$\max_{\eta_1,\eta_2,\eta_3} \quad -(1-q)\int_{X_m}^{X_m+\eta_1}(\varepsilon + X - s(X))f(X)dX + q\int_{I-\eta_2}^{I}(\varepsilon + X - s(X))f(X)dX$$
$$+(1-q)\int_{I}^{I+\eta_3}(\varepsilon + X - s(X))f(X)dX$$

$$s.t. \quad \eta_1,\eta_2 \geq 0, \qquad q\int_{I-\eta_2}^{I}(s(X)-I)f(X)dX + (1-q)\int_{I}^{I+\eta_3}(s(X)-I)f(X)dX \geq 0, \quad and$$
$$(1-q)\left[\int_{X_m}^{X_m+\eta_1}(s(X)-I)f(X)dX + \int_{I-\eta_2}^{I}(s(X)-I)f(X)dX\right] + q\int_{I}^{I+\eta_3}(s(X)-I)f(X)dX \leq 0$$

Let κ_1 and κ_2 be the multipliers for the first constraints, respectively. Since $s(I) = I$, the FOC for η_2 is positive at $\eta_2 = 0$: $[\eta_2]|_{\eta_2=0} : q\varepsilon f(I)$.

Similarly, the FOC for η_3 is positive at $\eta_3 = 0$. Therefore, the optimal values are non-zero. It shows that the experiment $(\{m,l\},\pi^*_M)$ cannot be optimal for any $q \in (\frac{1}{2}, 1)$.

□

Lemma (B5). *No three-signal experiment in the form of $(\{l,m,h\},\pi)$, whereby signals m and h are both sent with positive probability, is optimal.*

Proof. First note we are considering the parameter range $q \in (\frac{1}{2}, \bar{q}]$. Suppose the contrary and there exists a three-signal optimal experiment $(\{l,m,h\},\pi^*_{HM})$, where the investor invests iff she receives one of $(m,\tilde{l})$, $(h,\tilde{l})$ and $(h,\tilde{h})$. Then $\pi^*_{HM}(m|X)$ and $\pi^*_{HM}(h|X)$ solve the following optimization problem.

$$\max_{\pi(h|X),\pi(m|X)} \int_0^I (\varepsilon + X - s(X))(\pi(h|X) + (1-q)\pi(m|X))f(X)dX$$
$$+\int_I^1 (\varepsilon + X - s(X))(\pi(h|X) + q\pi(m|X))f(X)dX \tag{47}$$

$$s.t. \quad q\int_0^I (s(X)-I)\pi(h|X)f(X)dX + (1-q)\int_I^1 (s(X)-I)\pi(h|X)f(X)dX \geq 0$$
$$(1-q)\int_0^I (s(X)-I)\pi(m|X)f(X)dX + q\int_I^1 (s(X)-I)\pi(m|X)f(X)dX \geq 0$$
$$\pi(h|X),\pi(m|X) \in [0,1]$$

Let λ^h and λ^m be the multipliers for the first two restrictions, respectively. Define $c_m(X)$ and $c_h(X)$ as follows:

$$c_h(X) = \begin{cases} \varepsilon + X - s(X) + q\lambda^h(s(X)-I) & 0 \leq X < I \\ \varepsilon + X - s(X) + (1-q)\lambda^h(s(X)-I) & I \leq X \leq 1 \end{cases}$$

$$c_m(X) = \begin{cases} (1-q)(\varepsilon + X - s(X) + \lambda^m(s(X)-I)) & 0 \le X < I \\ q(\varepsilon + X - s(X) + \lambda^m(s(X)-I)) & I \le X \le 1 \end{cases}$$

Then $\pi(h|X)$ and $\pi(m|X)$ solves the following optimization problem subject to $0 \le \pi(h|X), \pi(m|X) \le 1$

$$\max_{\pi(h|X),\pi(m|X)\in[0,1]} \int_0^1 [c_h(X)\pi(h|X) + c_m(X)\pi(m|X)]f(X)dX \tag{48}$$

Now, we appeal to lemma A1. Note that the optimization problem (48) is linear in $\pi(h|X)$ and $\pi(m|X)$. Moreover, it is easy to see that their multipliers are equal at most in a measure-zero subset of $[0,1]$. Therefore, $\pi(h|X), \pi(m|X) \in \{0,1\}$ almost surely. Therefore, there are two subsets $M, H \in [0,1]$ such that the signals m and h are sent for the members in M and H, respectively. Moreover, define $M^1 = M \cap [0,I)$, $M^2 = [I,1] \cap [I,1]$ and define H^1 and H^2, correspondingly.

If M_1 is empty, then signal m is just sent for a subset of $X \in [I,1]$. In this case, the investor invests even if she receives $(m,\tilde{l})$, which is in contrast with the definition of signal m. Therefore, suppose M_1 is non-empty. For $X \in M_1$ we have $c_m(X) \ge \max\{c_h(X), 0\}$. Rearranging the expressions of $c_h(X)$ and $c_m(X)$, we have

$$\frac{q\lambda^h - (1-q)\lambda^m}{q} \ge \frac{\varepsilon + X - s(X)}{I - s(X)} \ge \lambda^m \Rightarrow q\lambda^h \ge \lambda^m \tag{49}$$

Moreover, note that if M_2 is empty, then the investor does not ever invest when she receives m, which is a contradiction with the definition of m. Therefore, M_2 is not empty and there exists $X \in M_2$. For X, we have:

$$\begin{aligned} c_m(X) \ge c_h(X) \quad &\Rightarrow \quad (q\lambda^m - (1-q)\lambda^h)(s(X)-I) \ge (1-q)(\varepsilon + X - s(X)) \\ &\Rightarrow \quad q\lambda^m - (1-q)\lambda^h > 0 \end{aligned} \tag{50}$$

Combining (49) and (50), we get $q\lambda^h > \frac{1-q}{q}\lambda^h \Rightarrow q(1+q) > 1$, which contradicts our assumption about the value of q.

□

Lemmas B4 and B5 imply that the a two-signal experiment with $\{h,l\}$ must be optimal. In this experiment, the investor completely disregards her own signal. It is easy to see that the optimal two-signal experiment has a threshold scheme, where the threshold $\bar{X}(q)$ should satisfy (44). The rest of the results for this part are similar to Lemma 1.

Proof for Part (b) We can rewrite $\bar{K}^{\varepsilon}_C(\mu;q) = \mu\mathbb{E}[(s(X)-I)\mathbb{I}_{\{X \ge \max\{\bar{X}(q),\hat{X}(\mu)\}\}}]$, and define

$$\begin{aligned} \mu^l &= \sup\left\{\mu|\bar{K}^{\varepsilon}_S \text{ is increasing over } [0,\mu]\right\} \\ \mu^h &= \inf\left\{\mu|\bar{K}^{\varepsilon}_S \text{ is decreasing over } \left[\mu, 1 - \frac{\varepsilon}{I - \bar{X}}\right]\right\} \end{aligned} \tag{51}$$

where $\bar{K}^{\varepsilon}_S(\mu) = \mathbb{E}\left[(s(X)-I)\mathbb{I}_{\{X \ge \hat{X}(\mu)\}}\right]$. Since $\bar{X}(q) < I - \varepsilon$, there exists $\hat{\mu}(q)$ such that $\bar{X}(q) = \hat{X}(\hat{mu}(q))$. Because $\bar{X}(q)$ is strictly increasing in q in $[\frac{1}{2}, \bar{q}]$, $\hat{\mu}(q)$ is strictly decreasing in q. If $\hat{\mu}(q) < \mu^l$, then $\bar{K}^{\varepsilon}_C(\mu;q)$ is increasing over $[0,1]$. If that is not the case, then the function is U-shape over $[\mu^h, 1]$.

Proof for Part (c) Note that as ε goes to zero, $\bar{K}^{\varepsilon}_S(\mu)$ becomes strictly increasing in $[0,1]$. Therefore, μ^l converges to one as ε goes to zero.

□

In Proposition 11, the assumption $q \leq \bar{q}$ substantially simplifies the analysis, as the entrepreneur still follows a threshold scheme for disclosure. It is not generally the case, as we show in Lemma B6. In particular, the equilibrium experimentation has a nested-interval structure, consistent with Guo and Shmaya (2017) that finds similar structures under more general settings. For illustration, we take $\mu = 1$ in the next lemma and corollary, the case with interim competition or when $\mu < 1$ are similar.

Lemma (B6). *For $q > \bar{q}$, there exists $q^* > \bar{q}$ such that for $q \geq q^*$, the entrepreneur optimally uses three signals $\{l, m, h\}$. The investor continues the project iff she observes $(m, \tilde{h})$, $(h, \tilde{l})$ or $(h, \tilde{h})$. Specifically, there are $X_m^L(q), X_h^L(q), X_h^H(q) \in (0,1)$ such that the entrepreneur sends h in $[X_h^L(q), X_h^H(q)]$ and m in $[X_m^L(q), X_h^L(q)) \cup (X_h^H(q), 1]$.*

Proof. First, we show that the two-signal experiment introduced in Proposition 11(a) is not optimal for big enough values of q. Then, we show that when a three-signal experiment is optimal, a nested interval structure is used for providing endogenous information.

Non-optimality of two-signal experiments

Suppose the contrary holds that a threshold scheme, with threshold $\bar{X}(q)$ (as introduced in (44)) is optimal. Then Lemma A1 says it is the optimal experiment among all two-signal experiments that the high signal always induces investment. Now consider η_1, η_2 and η_3 that satisfy the following conditions:

$$q \int_{\bar{X}(q)}^{\bar{X}(q)+\eta_2} (s(X) - I) f(X) dX + (1-q) \int_{1-\eta_3}^{1} (s(X) - I) f(X) dX \leq 0$$

$$(1-q) \int_{\bar{X}(q)-\eta_1}^{\bar{X}(q)+\eta_2} (s(X) - I) f(X) dX + q \int_{1-\eta_3}^{1} (s(X) - I) f(X) dX \geq 0$$

If the two-signal experiment is optimal, then the following three-signal experiment should implement a suboptimal investment function for the entrepreneur.

$$\tilde{\pi}(h|X) = \begin{cases} 1 & X \in [\bar{X}(q) + \eta_2, 1 - \eta_3) \\ 0 & X \in [0, \bar{X}(q) + \eta_2) \cup [I - \eta_3, 1] \end{cases} \qquad \tilde{\pi}(m|X) = \begin{cases} 1 & X \in [\bar{X}(q) - \eta_1, \bar{X}(q) + \eta_2) \cup [1 - \eta_3, 1] \\ 0 & X \in [0, \bar{X}(q) - \eta_1) \cup [\bar{X}(q) + \eta_2, 1 - \eta_3) \end{cases}$$

Therefore, $\eta_1 = \eta_2 = \eta_3 = 0$ should be the solution to the following optimization problem:

$$\max_{\eta_1, \eta_2, \eta_3} (1-q) \int_{\bar{X}(q)-\eta_1}^{\bar{X}(q)} (\varepsilon + X - s(X)) f(X) dX - (1-q) \int_{\bar{X}(q)}^{\bar{X}(q)+\eta_2} (\varepsilon + X - s(X)) f(X) dX - q \int_{1-\eta_3}^{1} (\varepsilon + X - s(X)) f(X) dX$$

$$\begin{aligned} s.t. \quad & q \int_{\bar{X}(q)}^{\bar{X}(q)+\eta_2} (s(X) - I) f(X) dX + (1-q) \int_{1-\eta_3}^{1} (s(X) - I) f(X) dX \leq 0 \\ & (1-q) \int_{\bar{X}(q)-\eta_1}^{\bar{X}(q)+\eta_2} (s(X) - I) f(X) dX + q \int_{1-\eta_3}^{1} (s(X) - I) f(X) dX \geq 0 \end{aligned}$$

Suppose κ_1 and κ_2 are the Lagrange multipliers for the above constraints. The FOCs at $\eta_1 = \eta_2 = \eta_3 = 0$ are as follows:

$$\begin{aligned} [\eta_1]|_{\eta_1=0} = 0 \Rightarrow & f(\bar{X}(q))[(1-q)(\varepsilon + \bar{X}(q) - s(\bar{X}(q)) + \kappa_2(s(\bar{X}(q)) - I))] = 0 \\ \Rightarrow & \kappa_2 = \frac{\varepsilon + \bar{X}(q) - s(\bar{X}(q))}{I - s(\bar{X}(q))} \end{aligned} \tag{52}$$

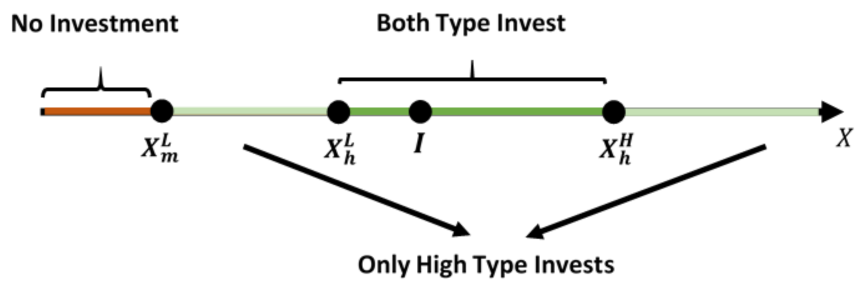


Figure 6: The nested interval structure of the entrepreneur's experimentation

$$\begin{aligned}[\eta_2]|_{\eta_2=0} = 0 \Rightarrow & f(\bar{X}(q))[-q(\varepsilon + \bar{X}(q) - s(\bar{X}(q))) + q\kappa_1(s(\bar{X}(q)) - I) + (1-q)\kappa_2(s(\bar{X}(q)) - I)] = 0 \\ \Rightarrow & -\kappa_1 = \frac{2q-1}{q}\frac{\varepsilon + \bar{X}(q) - s(\bar{X}(q))}{I - s(\bar{X}(q))} = \frac{2q-1}{q}\kappa_2\end{aligned} \tag{53}$$

$$\begin{aligned}[\eta_3]|_{\eta_3=0} = 0 \Rightarrow & f(1)[-(1-q)(\varepsilon + 1 - s(1)) + (\kappa_1(1-q) + \kappa_2 q)(s(1) - I)] = 0 \\ \Rightarrow & \frac{\varepsilon + 1 - s(1)}{s(1) - I} = \frac{\kappa_1(1-q) + \kappa_2 q}{1-q} = \frac{3q^2 - 3q + 1}{q(1-q)}\frac{\varepsilon + \bar{X}(q) - s(\bar{X}(q))}{I - s(\bar{X}(q))}\end{aligned} \tag{54}$$

Note that:

$$\frac{\varepsilon + \bar{X}(q) - s(\bar{X}(q))}{I - s(\bar{X}(q))} \geq \frac{\varepsilon}{I}$$

Therefore, (54) implies that for every $q \in [\frac{1}{2}, 1]$, the following inequality holds:

$$\frac{I}{\varepsilon}\frac{\varepsilon + 1 - s(1)}{s(1) - I} \geq \frac{3q^2 - 3q + 1}{q(1-q)} \tag{55}$$

The RHS in (55) goes to infinity as $q \to 1$, while the LHS is constant. It is the contradiction with the earlier assumption that the two-signal experiment is optimal. Therefore, a three-signal experiment is optimal for large enough values of q.

Nested Interval Structure

Guo and Shmaya (2017) prove the second part of the lemma in their Theorem 3.1 and Discussion 6.3, for securities $s(X)$ such that $\frac{s(X)-I}{\varepsilon+X-s(X)}$ is increasing in X. Note that this condition holds for $s_i(X) = X$.

□

Corollary 5. *For $q \geq q^*$, $\mathcal{I}^q(X) < 1$ for a positive measure of $X \in [I, 1]$, implying a positive probability of inefficient termination. Moreover the investor's interim rent, $\bar{K}_C^{\varepsilon}(q, 1)$, is non-monotone over the region $(\bar{q}, 1]$.*

Corollary 5 provides a counter-intuitive result that the investor's payoff is not globally increasing in investor sophistication. In fact, if the information the investor receives is very accurate, then the entrepreneur adopts a less informative experiment to increase the chance of inefficient continuations with the cost of positive probability of inefficient terminations. Similar to Proposition 5 in Kolotilin (2017), Lemma B6 highlights a "crowding-out effect" of higher levels of the investor's sophistication on the entrepreneur's information production. This effect implies that the investor might be better off by committing to ignoring some part of her independent information, which constitutes an interesting topic for future research.

In the following proposition, we show that while interim competition ($\mu < 1$) can improve the efficiency of investment decisions, it monotonically decreases the insider's interim rent for reasonably sophisticated investors, consistent with Petersen and Rajan (1995).

Proposition 12 (Impact of Competition (extended))**.**

(a) For a given security with interim renegotiation, the equilibrium investment function becomes more socially optimal as interim competition increases (μ decreases).

(b) There exists $q^H < 1$ such that for $q \in (q^H, 1)$, the insider's interim rent is monotone in μ. Therefore, the insider never benefits from interim competition for very high levels of sophistication.

Proof. **Proof for Part (a)**
Suppose $\mu_2 > \mu_1$ and the equilibrium investment functions are respectively $\mathcal{I}_2(X)$ and $\mathcal{I}_1(X)$. We show that:

$$\mathbb{E}[(\varepsilon + X - I)\mathcal{I}_1(X)] \geq \mathbb{E}[(\varepsilon + X - I)\mathcal{I}_2(X)] \tag{56}$$

Suppose the contrary. Note that both $\mathcal{I}_1(X)$ and $\mathcal{I}_2(X)$ are implementable for the entrepreneur tThere are experiments that implement these investment functions), since q is fixed. The optimality of the investment functions implies:

$$\mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_1(X)] \geq \mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_2(X)] \tag{57}$$

Therefore, if (56) does not hold, (57) implies:

$$\mathbb{E}[(X - I)\mathcal{I}_2(X)] > \mathbb{E}[(X - I)\mathcal{I}_1(X)] \tag{58}$$

Furthermore, the fact that the investor receives positive interim payoff for $q > \frac{1}{2}$ and optimality of $\mathcal{I}_2(X)$ for μ_2 implies:

$$\mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_2(X)] > \mathbb{E}[(\varepsilon + (1-\mu_2)(X - I)\mathcal{I}_2(X)] \geq \mathbb{E}[(\varepsilon + (1-\mu_2)(X - I)\mathcal{I}_1(X)] \tag{59}$$

Therefore, (57) and (59) result in:

$$\begin{aligned}
&\mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_1(X)] - \mathbb{E}[(\varepsilon + (1-\mu_2)(X - I)\mathcal{I}_1(X)] \\
\geq &\mathbb{E}[(\varepsilon + (1-\mu_1)(X - I)\mathcal{I}_2(X)] - \mathbb{E}[(\varepsilon + (1-\mu_2)(X - I)\mathcal{I}_2(X)] \\
\Rightarrow &\mathbb{E}[(X - I)\mathcal{I}_1(X)] \geq \mathbb{E}[(X - I)\mathcal{I}_2(X)]
\end{aligned} \tag{60}$$

which contradicts (58). Therefore, (56) holds. It shows the equilibrium investment function become more socially efficient as μ decreases.

Proof for Part (b)
As it is shown in Lemma B6, there exists q^* such that for $q > q^*$, a three-signal experiment is optimal. Fix a level of sophistication in the range q. Therefore, there are functions $M_l(\mu)$, $H_l(\mu)$ and $H_h(\mu)$ such that the entrepreneur sends a high signal for $X \in [H_l(\mu), H_h(\mu)]$ and a medium signal in $[M_l(\mu), H_l(\mu)] \cup [H_h(\mu), 1]$. Moreover, interim competition does not affect the entrepreneur's experimentation if $\varepsilon + (1-\mu)(M_l(1) - I) \geq 0$. Therefore, without loss of generality, we assume μ is small enough that $M_l(\mu) = I - \frac{\varepsilon}{1-\mu}$.

Moreover, as discussed earlier, the insider does not get any interim rent conditional on realization of a medium signal. Therefore, the insider's interim rent is as follows:

$$K_C(\mu; q) = \mu \int_{H_l(\mu)}^{H_h(\mu)} (X - I) f(X) dX = \frac{(2q-1)\mu}{q} \int_{I}^{H_h(\mu)} (X - I) f(X) dX \tag{61}$$

where the last equality comes from the fact that the constraint for sending a high signal is binding. The following lemma characterizes the derivative with respect to μ when $M_l(\mu) = I - \frac{\varepsilon}{1-\mu}$.

Lemma (B7). *If $\varepsilon + (1-\mu)(M_l(1) - I) < 0$, then*

$$\frac{\partial}{\partial \mu} K_C(\mu; q) = \frac{2q-1}{q} \left[\int_I^{H_h} (X-I) f(X) dX - \frac{q(1-q)}{2q-1} \frac{\mu \varepsilon^2}{(1-\mu)^3} f(I - \frac{\varepsilon}{1-\mu}) \right] \tag{62}$$

Proof. By taking derivative from (61) with respect to μ, we get:

$$\frac{\partial}{\partial \mu} K_C(\mu; q) = \frac{(2q-1)}{q} \int_I^{H_h(\mu)} (X-I) f(X) dX + H_h'(\mu) \frac{(2q-1)\mu}{q} (H_h(\mu) - I) f(H_h(\mu)) \tag{63}$$

We know that the conditions for sending high and medium signal binds for all values of μ. Therefore:

$$\begin{aligned} &\frac{\partial}{\partial \mu} \left\{ (1-q) \int_{I - \frac{\varepsilon}{1-\mu}}^{H_l} (X-I) f(X) dX + q \int_{H_h}^{1} (X-I) f(X) dX \right\} = 0 \\ &\Rightarrow -\frac{(1-q)\varepsilon^2}{(1-\mu)^3} f(I - \frac{\varepsilon}{1-\mu}) + (1-q) H_l'(H_l - I) f(H_l) - q H_h'(H_h - I) f(H_h) = 0 \end{aligned} \tag{64}$$

$$\begin{aligned} &\frac{\partial}{\partial \mu} \left\{ q \int_{H_l}^{I} (X-I) f(X) dX + (1-q) \int_I^{H_h} (X-I) f(X) dX \right\} = 0 \\ &\Rightarrow (1-q) H_h'(H_h - I) f(H_h) + q H_l'(H_l - I) f(H_l) = 0 \end{aligned} \tag{65}$$

By combining (64) and (65), we get:

$$H_h'(H_h - I) f(H_h) = -\frac{q(1-q)}{2q-1} \frac{\varepsilon^2}{(1-\mu)^3} f(I - \frac{\varepsilon}{1-\mu})$$

We get (62) by substituting the last equality in (63). □

Note that H_h and H_l are the solution to the following maximization problem, where $M_l = I - \frac{\varepsilon}{1-\mu}$:

$$\begin{aligned} \max_{H_l, H_h} \; & (1-q) \int_{M_l}^{H_l} (\varepsilon + (1-\mu)(X-I)) f(X) dX + \int_{H_l}^{H_h} (\varepsilon + (1-\mu)(X-I)) f(X) dX + q \int_{H_h}^{1} (\varepsilon + (1-\mu)(X-I)) f(X) dX \\ s.t. \quad & (1-q) \int_{M_l}^{H_l} (X-I) f(X) dX + q \int_{H_h}^{1} (X-I) f(X) dX \geq 0 \\ & q \int_{H_l}^{I} (X-I) f(X) dX + (1-q) \int_I^{H_h} (X-I) f(X) dX \geq 0 \end{aligned}$$

According to Lemma B6, we know that both constraints bind. It implies $H_l \to I$ and $H_h \to 1$, as $q \to 1$. Since $f(.)$ and $\frac{\mu}{(1-\mu)^3}$ are bounded ($\mu < 1 - \frac{\varepsilon}{I}$), then the result follows from Lemma B7.

□

B3. Contracting under IPH

When λ is close to 1, payoff channel is not playing a big role, and we can compare IPH to traditional moral hazards of effort provision. The "efficient effort" in our case is to produce continuation only when

$X + \varepsilon > I$, the cost of that effort is the loss of private benefit when we terminate projects. The "action" in our setup is the experimentation, and the outcomes of the experiments correspond to noisy signals of the action.

It is a well-known result that for risk-neutral agents, the optimal security is debt (Innes (1990)). Moreover, even when we can contract on noisy signals of an agent's action, the outcome is generally not first best (e.g., Holmstrom (1979)). Yet the optimal design is partially indeterminate in our model and restores social efficiency. This result only relies on the contractibility of actions the interim information leads to, namely continuation or termination. Our findings point to the key difference between MIP and those in conventional models.

This contrast derives from two subtle differences between our setting and the ones in Holmstrom (1979) and Innes (1990). First, the principal takes an action based on the information produced by the agent, which implies that the agent's effort can affect his final payoff through affecting the principal's continuation decision. In fact, we show in Section A15. that by designing a security that makes the principal's continuation decision, and thus the agent's payoff more sensitive to the agent's effort, we can also align the agent's incentives. Second, which is more important, the agent's effort in our model affects information production, but not the final output X, therefore allowing the principle to know exactly how the entrepreneur's action has affected the final payoff. In other words, upon seeing the cash flow X, everyone knows if the continuation decision is socially optimal or not, hence the entrepreneur ex ante can design the security and contract contingent on the continuation payoff to perfectly incentivize the entrepreneur during the interim to take the right action. In some sense, the noise in the entrepreneur's action — signal to continue or terminate – is orthogonal to the continuation payoff, and making the agent's payoff contingent on both fully solves the agency problem. It is also worth pointing out that unlike the literature on costly experimentation and learning (e.g., Bergemann and Hege (1998) and Hörner and Samuelson (2013)) with hidden effort and hidden information, the principal in our setting observes perfectly the signal the agent produces, which is important for attaining the first best outcome with the optimal contract.

B4. Conventional Effort Distortion

We have emphasized the entrepreneur's role as an information provider, whereas earlier studies concern the entrepreneur's costly effort to improve project cash flows. We now discuss how these two actions interact.

To model effort, we assume the entrepreneur can choose from a set of conditional distributions $f(X|e)$, where $e \in \mathcal{E}$ is the set of available levels of effort. Function $c(\cdot) : \mathcal{E} \to \mathcal{R}$ shows the cost associated with each level of effort. First, suppose $\mathcal{I}(X; e)$ is the equilibrium investment function when effort level e is chosen. Then, $e^* \in \mathcal{E}$ is *constrained first best* if it solves the following maximization problem:

$$e^* \in \quad argmax \ \ \mathbb{E}[(X - I + \varepsilon)\mathcal{I}(X; e)|e] - c(e) \tag{66}$$

It is straight-forward to show that absent investor sophistication and interim competition, there is no effort distortion: given the equilibrium information structure for each level of effort, the entrepreneur chooses the one that is socially preferred. Neither the insider's information monopoly nor IPH distorts entrepreneurial effort. The reason is that when the insider investor gets no rent, the entrepreneur fully internalizes the benefit and the cost of effort. Naturally, in presence of a sophisticated investor and interim competition, the investor may get positive interim rent that distorts effort provision, but still to a lesser extent compared to the case where information production is exogenous and the insider enjoys full monopoly rent.