Matching Markets: Theory and Practice

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In 1962 David Gale and Lloyd Shapley published one of the most influential papers in game theory starting the literature in matching theory.

Until early 2000s the main practical application of matching theory was entry level labor markets such as the U.S. hospital-intern market.

This trend has recently changed as matching theory found new applications in (often large scale) resource allocation problems of social importance.

Research on two of these new applications has especially been influential in practice:

- School Choice
- Kidney Exchange
Kidney Exchange

- Shortly after the first few matching papers on kidney exchange are published in 2004-2005, several regions in the U.S. and a few countries launched centralized kidney exchange programs.
- In his June 2011 Congress testimony, Dr. Myron Gutmann, Assistant Director at NSF emphasized, research on kidney exchange has resulted in measurable gains for the U.S. taxpayer.
- Similarly, in a recent NSF - Science Nation story, Nancy Lutz, program director at NSF remarked:

  “In addition, it’s especially rewarding to see such a clear and immediate benefit to the public. This research moved from abstract, academic theory to real world, direct impact very quickly.”
School Choice

- Shortly after the first matching paper on school choice is published in 2003, several school districts adopted mechanisms advocated in this paper. These school districts include:
  - New York City
  - Boston
  - Chicago
  - Denver
  - New Orleans

- Perhaps more strikingly, these mechanisms are adopted throughout England by all local authorities (more than 150 of them) by 2007.
In this series of lectures, we will see that the catalyst for the recent success of matching theory has been the strong theory tradition pioneered by Gale & Shapley coupled with careful modeling bringing the theory and the practice closer.

Prior to 1990s, research on matching was mostly focused on two-sided matching markets. The influential monograph of Roth & Sotomayor (1990) gives a very clear picture of the focus of matching literature in this era.

While Shapley & Scarf (1974) and Hylland & Zeckhauser (1977) introduced two early models of “one-sided matching,” these models received much less attention until the late 1990s.
The Rise of One-Sided Matching

- This trend has changed considerably starting with late 1990s. Not only the focus on one-sided matching models increased, matching theorists developed new models at the interface of one-sided matching and two-sided matching.

- It is this recent trend which eventually lead to a number of new applications, including school choice and kidney exchange.

- The last few years have also seen significant advances in two-sided matching theory with the introduction of Hatfield & Milgrom (2005) matching with contracts model.

- Having learned from past experience, market designers have immediately explored the potential links of this exciting model with one-sided matching. This approach has already resulted in a brand new application of matching theory: Cadet-branch matching.
Matching Markets: The Path Between Theory & Practice

Timeline

1960

1970

1980

1990

2000

2010

Key Contributions

Contributions

Two-Sided Matching

Gale & Shapley AMM 62

Kelso & Crawford Econometrica 82

Dubins & Freedman AMM 81 Roth MOR 82

Roth Econometrica 82

Roth JPE 84

Roth & Sotomayor 1990

1960

1970

1980

1990

2000

2010

Allocation via Priorities

Balinski & Sönmez JET 99

Hatfield & Milgrom AER 05

Hatfield & Kojima JET 10

Sönmez & Switzer 2011

Shapley & Scarf JME 74

Abdulkadiroğlu & Sönmez JET 99

Abdulkadiroğlu & Sönmez AER 03

Hyland & Zeckhauser JPE 77

School Choice Reforms in:

New York City
Boston
Chicago
Denver
England
New Orleans

Kidney Exchange Clearinghouses:

New England Program for Kidney Exchange
Alliance for Paired Donation
National Matching Scheme (England)
for Paired Donation
National KPD Pilot Program (USA)

Combinatorial Optimization

Gallai MTAMKIK 63 MTAMKIK 64
Edmonds CJM 65

One-Sided Matching

(Unit-Demand Indivisible Goods Allocation)

NRMP & Various other labor markets summarized in Roth & Peranson AER 99

Hatfield & Milgrom AER 05

Echenique AER 12

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Thanks to Roth & Sotomayor (1990), the two-sided matching literature prior to 1990s is well-known.

In these lectures our focus is the subsequent literature, with a special emphasis on the path between the theory and the practice.

The lectures are organized as follows:

- **Lecture 1**: Overview and One-Sided Matching
- **Lecture 2**: Kidney Exchange
- **Lecture 3**: School Matching
- **Lecture 4**: Cadet-Branch Matching