Abstract. This paper shows how standard arguments supporting the imposition of price caps break down in the presence of demand uncertainty. In particular, though in the deterministic case the introduction or lowering of a price cap (above marginal cost) results in increased production, increased total welfare, decreased prices, and increased consumer welfare, we show that all of the above comparative statics predictions fail for generic uncertain demand functions. For example, for price caps sufficiently close to marginal cost, a decrease in the price cap always leads to a decrease in production and total welfare under certain mild conditions. Under stronger regularity assumptions, all of the monotone comparative statics predictions from the deterministic case also do not hold for a generic uncertain demand if we restrict attention to price caps in an arbitrary fixed interval (as long as the price caps are binding for some values in that interval).

Keywords: Price caps, demand uncertainty, monotone comparative statics.
1. Introduction

There is a common economic perception that, in the context of a monopoly or oligopoly, price ceilings or caps can be effective in combatting the exercise of market power. In words of a Federal Energy Regulatory Commissioner (Walsh (2001)): “If you cap those prices, you eliminate any incentive to withhold ... you may as well sell into the market at a capped price as long as you’re covering your running cost and making a reasonable profit.”

The classical rationale for the use of price caps is well known. Consider the case of a monopolist and a standard downward-sloping demand curve. If competition was perfect, the resulting price $p_{pc}$ would be equal marginal cost, and the produced quantity $q_{pc}$ would maximize efficiency and welfare. However, an unconstrained monopolist will maximize profits by equalizing marginal revenue and marginal cost. The resulting quantity $q_m$ is less than the socially optimal quantity $q_{pc}$ and leads to a price of $p_m > p_{pc}$. The imposition of a price cap at a level between $p_{pc}$ and $p_m$ results in an increased quantity produced by the monopolist because its marginal revenue is constant up to a production level where the price cap is no longer binding. As a result, the imposition of a price cap at a level between $p_{pc}$ and $p_m$ increases quantities produced, increases welfare, and decreases prices. Moreover, the "optimal" price cap is exactly $p_{pc}$, the perfectly competitive price. Thus, price caps and, in particular, price caps close to marginal costs, seem like an attractive tool to increase consumer welfare and total efficiency.

A typical criticism of the theory is on practical grounds. For example, it is difficult in practice to know what $p_{pc}$ is (See Clarkson and Miller (1982), p. 461 ff.). The purpose of this paper is to show that the predictions of the deterministic theory change drastically if demand is uncertain.

In order to make this point, the paper examines the impact of price caps in the context of a one-stage Cournot model. We first consider a simple, deterministic, Cournot game in which $n \in \mathbb{N}$ firms choose production quantities. Profit of each firm depends on the quantity produced by the firm and the price of the good which is a function of the total quantity produced by the industry. In addition, each

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1The focus here is not on macroeconomic situations in which price controls might be imposed to control inflation, but on situations in which price caps are or have been imposed on particular products such as apartment rents, electricity, gasoline, interest rates on credit card balances, or college tuition.

2Cournot models are widely used in applications, for example, in the analysis of electricity markets. See Daughety (1988) for an anthology of work on Cournot models and general applications. Also see Carlton and Perloff (1994).

3All our results thus apply to the monopoly case $n = 1$. 
firm has a constant marginal cost \( c \). In the context of this model, we examine the introduction of a price cap \( \bar{p} > c \). Theorem 1 shows that, if we respectively restrict attention to the equilibrium with the lowest or highest production quantity, the following monotone comparative static results hold in the deterministic case: (i) production is nonincreasing in the price cap; (ii) total welfare is nonincreasing in the price cap; (iii) as the price cap \( \bar{p} \) approaches marginal cost, total welfare converges to the efficient level; (iv) consumer welfare is nonincreasing in the price cap; (v) average prices are well defined (in the sense that firms produce a positive quantity) and nondecreasing in the price cap. These results seem to confirm the intuition supporting the use of price caps, in particular price caps which are close to marginal cost.

We show that these results do not extend to environments with stochastic revenues. For this purpose, we consider an extension of the above model where demand is uncertain and risk neutral firms choose the production quantities before the demand realization (or, equivalently, revenue realization) is known. In this environment, none of the above five predictions is true in general. The technical reason why the results of the deterministic case do not immediately generalize is that with stochastic demand, the single-crossing property that underpins the deterministic monotone comparative statics results does not have to hold. As a matter of fact, we show that, under weak assumptions, all of the conditions (i)–(v) fail for a generic uncertain demand schedule. Theorem 2 shows that if demand is stochastic and some additional regularity assumptions are satisfied, (i)–(ii) actually will be reversed for price caps close to marginal cost. In general, the quantitative effect of price caps close to marginal cost will depend on the exact form of the uncertainty. Theorem 3 provides a characterization and shows that if low realizations of demand are possible, although perhaps very improbable, the breakdown of production (and thus welfare) associated with a low price cap (above but close to marginal cost) will be severe. However, perverse effects are not limited to price caps close to marginal cost. Theorems 4 and 5 show that, under stronger regularity conditions, for a generic demand schedule, the above monotonicity properties (points (i)–(ii) and (iv)–(v)) fail in any interval in which the price cap is binding.

The paper is organized as follows. The next section introduces the model. Section 3 contains the results. Section 4 concludes. All proofs can be found in the Appendix.
2. Model

We consider the case of \( n \) symmetric firms, where \( n = 1, 2, \ldots \) is an arbitrary natural number. Each firm produces the same homogeneous good at a constant marginal cost \( c \) which is the same for all firms.\(^4\)

Demand is given by a continuous price function \( P(Q, \theta) \) which depends on \( Q \in \mathbb{R}^+ \), the total quantity produced in the industry, and some random variable \( \theta \in \mathbb{R} \) distributed according to a distribution \( F \) with bounded support.\(^5\) We assume that a high realization of \( \theta \) leads to higher prices than a low realization, i.e. that \( P(Q, \theta) \) is increasing in \( \theta \) for any fixed value of \( Q \). We also assume that for each fixed \( \theta \) the price function \( P(Q, \theta) \) is decreasing in \( Q \). While we do not require \( P \) to be differentiable, we assume there exists \( L_1, L_2 > 0 \) such that

\[
|P(Q, \theta) - P(Q', \theta)| \leq L_1 \cdot |Q - Q'| \quad \text{and} \quad |P(Q, \theta) - P(Q, \theta')| \geq L_2 \cdot |	heta - \theta'| \quad \text{for all } \theta, \theta' \in \mathbb{R} \text{ and } Q, Q' \in \mathbb{R}^+.\(^6\)

Firms decide on the quantity they wish to produce before the realization of \( \theta \) is known. Because firms are assumed to be risk neutral, they will maximize their expected profits. This means that in the case of no price caps, firm \( i \) chooses its production \( q \) to maximize its expected profits

\[
\pi(q, Q_{-i}) = E(q \cdot (P(q + Q_{-i}, \theta) - c)),
\]

where \( Q_{-i} \) is the quantity produced by all other firms. Similarly, in the more general case of a price cap \( \overline{p} \), firm \( i \) chooses its production \( q \) to maximize its expected profits given by

\[
\pi(q, Q_{-i}, \overline{p}) = E(\pi_\theta(q, Q_{-i}, \overline{p})),
\]

where

\[
\pi_\theta(q, Q_{-i}, \overline{p}) = q \cdot (\min(P(q + Q_{-i}, \theta), \overline{p}) - c)
\]

are the profits for a given realization of \( \theta \). To guarantee that there exists a profit-maximizing quantity, we assume that \( \lim_{Q \to \infty} P(Q, \theta) < c \) for all \( \theta \in \mathbb{R} \).

In the following, we say that production is gainful if \( E(P(0, \theta)) > c \). Clearly if \( E(P(0, \theta)) < c \), no firm will ever produce a positive quantity no matter what the price cap is. As the analysis of the case \( E(P(0, \theta)) = c \) is straightforward and without practical interest, we will in the following assume that production is gainful.

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\(^4\)We consider constant marginal costs because in this case the comparative statics results are very clear cut and simple to state. It is straightforward to generalize Theorems 2, 4 and 5 to the case of a convex cost function.

\(^5\)While \( F \) has bounded support, it will be convenient to assume that \( P(Q, \theta) \) is defined for all \( \theta \in \mathbb{R} \) and \( Q \in \mathbb{R}^+ \).

\(^6\)If \( P \) were continuously differentiable, we could replace the above assumption with the requirement that \( \frac{d}{dQ} P < 0 \) and \( \frac{d}{d\theta} P > 0 \).
We are interested in pure strategy symmetric Nash Equilibria, i.e. quantities \( q \) that solve 
\[
q \in \arg \max q' \pi(q', (n-1) \cdot q, \bar{p}).
\]
For any fixed price cap \( \bar{p} \), denote the set of all such equilibria by \( q^*(\bar{p}) \). Since a price cap below \( c \) leads to no production, we restrict ourselves in the following to price caps \( \bar{p} \) which are strictly larger than \( c \).

### 3. Results

#### 3.1. Existence

We start by establishing the existence of the studied equilibria.

**Proposition 1.** For any price cap \( \bar{p} \) the set \( q^*(\bar{p}) \) is nonempty.

Existence of symmetric equilibria for similar models in which marginal costs are equal to zero was proven by Roberts and Sonnenschein (1976). The argument of Roberts and Sonnenschein relies on the nonnegativity of the inverse demand function and therefore does not directly apply in the presence of a positive \( E(\min(P(Q, \theta), \bar{p}) - c) < 0 \) for some \( \bar{p} \) and \( Q \). Instead, we use Lemma 1 below to establish a regularity property of the best response correspondence \( b(\cdot, \bar{p}) \).

**Lemma 1.** Let \( b(x, \bar{p}) = \arg \max q \pi(q, x, \bar{p}) \) be the best response of a player if his opponents together produce \( x \in \mathbb{R}_+ \) and the price cap is \( \bar{p} \).

Fix a price cap \( \bar{p} > c \) and quantity \( x_o \in \mathbb{R}_+ \). Then \( \lim_{x \uparrow x_o} b(x, \bar{p}) \) and \( \lim_{x \downarrow x_o} b(x, \bar{p}) \) exist\(^8\) and

\[
\lim_{x \uparrow x_o} b(x, \bar{p}) = \min_{x \uparrow x_o} b(x_o, \bar{p}) \leq \max_{x \uparrow x_o} b(x_o, \bar{p}) = \lim_{x \downarrow x_o} b(x, \bar{p}),
\]

where \( \min_{x \uparrow x_o} b(x, \bar{p}) \) and \( \max_{x \uparrow x_o} b(x, \bar{p}) \) denote respectively the smallest and largest element in \( b(x, \bar{p}) \).\(^9\)

The existence of equilibria now follows immediately from Lemma 1 along the same lines as in Roberts and Sonnenschein (1976) or Milgrom and Roberts (1994).

By Proposition 1, the set \( q^*(\bar{p}) \) is nonempty. Note that it is also compact as the best response correspondence has a closed graph. We will denote the smallest and largest element in \( q^*(\bar{p}) \) by \( q^*_L(\bar{p}) \) and \( q^*_H(\bar{p}) \), respectively.

\(^7\)See also Example 1 in Milgrom and Roberts (1994).

\(^8\)When we say that \( \lim_{x \uparrow x_o} b(x, \bar{p}) \) exists, we mean that there exists a real number denoted by \( \lim_{x \uparrow x_o} b(x, \bar{p}) \) such that for any sequence \( x_m \to x_o \) and for any sequence \( q^m \in b(x_m, \bar{p}) \) it is the case that \( q^m \to \lim_{x \uparrow x_o} b(x, \bar{p}) \). The statement for \( \lim_{x \downarrow x_o} b(x, \bar{p}) \) has an analogous meaning.

\(^9\)The existence of \( \min_{x \uparrow x_o} b(Q-1, \bar{p}) \) and \( \max_{x \uparrow x_o} b(Q-1, \bar{p}) \) is guaranteed; the best response correspondence has a closed graph with values that are uniformly bounded. (The existence of a uniform upper bound follows from \( \lim_{Q \to \infty} P(Q, \theta) < c \) for all \( \theta \in \mathbb{R} \).)
3.2. Deterministic Case. If we refer to total and consumer welfare in the context of deterministic demand, we mean respectively \( \int_0^Q (P(x, \theta) - c) \cdot dx \) and \( \int_0^Q (P(x, \theta) - \min(P(Q, \theta), \bar{p})) \cdot dx \) respectively. If demand is not deterministic by total and consumer welfare, we mean respectively the expected values of the above two expressions.

**Theorem 1.** Assume deterministic demand and restrict attention to the equilibrium with the highest or lowest production quantity. Then,

(i) production is nonincreasing in the price cap;
(ii) total welfare is nonincreasing in the price cap;
(iii) as price cap approaches marginal cost, total welfare converges to the efficient level;
(iv) consumer welfare is nonincreasing in the price cap;
(v) average prices are well-defined\(^{11}\) (i.e. firms engage in production) and non-decreasing in the price cap.

The proof of the theorem is based on Lemma 1 and the following observation.

**Lemma 2.** In the case of deterministic demand (the support of \( \theta \) consists of a single point), \( \pi(q, Q - i, \bar{p}) \) satisfies the single crossing property as a function of \( (q, -\bar{p}) \) for any fixed \( Q - i \).

It is noteworthy that the above result involves all Nash Equilibria. Milgrom and Roberts (1994, Example 1) illustrate their methods considering the Cournot model of Roberts and Sonnenschein (1976). They conclude that an increase in costs leads to a decrease in equilibrium quantities if they restrict themselves to the set of equilibria in which each firm chooses the highest quantity consistent with profit maximization. We are able to generate results involving the set of all equilibria because of the regularity conditions developed in Lemma 1.

3.3. Uncertain Demand. Note that in the case of stochastic demand, the function \( \pi \) will be still a convex combination of functions which (by Lemma 2) have the single crossing property. However, unlike the increasing differences property, the single crossing property is not always preserved under convex combinations.

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\(^{10}\)Thus, in the case where the price cap is binding, consumer welfare is calculated under efficient rationing.

\(^{11}\)In the case of deterministic demand there actually will be just a unique price. We use the term average prices so that it also makes sense to talk about (v) in the context of uncertain demand.
Therefore, in the case of stochastic demand, it is not immediately clear whether \( \pi \) will satisfy the single crossing property as in Lemma 2. Of course, even if \( \pi \) does not satisfy the single crossing property, this does not automatically imply that \( q^*_L(\bar{p}) \) and \( q^*_H(\bar{p}) \) cannot be monotonically decreasing.

We start our investigation by considering price caps close to marginal costs.\(^{12}\)

**Theorem 2.** Assume \( F : \mathbb{R} \to [0, 1] \) is continuously differentiable. Then there exists \( \gamma > 0 \) such that \( q^*_L \) and \( q^*_H \) are monotonically non-decreasing in \((c, c + \gamma)\).

Assume, in addition, \( P \) is continuously differentiable. Then there exists \( \gamma > 0 \) such that either both \( q^*_L \) and \( q^*_H \) are monotonically increasing in \((c, c + \gamma)\) or both are equal to zero on \((c, c + \gamma)\).

Thus, if price caps are sufficiently close to \( c \), then a lowering of the price cap will actually reduce the production level. It might be surprising that even though a price cap close to \( c \) is welfare-maximizing for any \( \theta \) – if \( \theta \) were known, it is not optimal if the exact realization of \( \theta \) is ex ante uncertain.

To see the most basic intuition behind this result, consider the case of a monopolist \((n = 1)\). Imagine the current price cap is \( \bar{p}_1 \) and the monopolist charges some quantity \( q \). The payoff of the monopolist is then a weighted average of the payoff \( q \cdot \bar{p}_1 \) if the price cap is binding, and \( q \cdot P(q, \theta) \) if it is not. Note that if the monopolist knew that the price cap would be binding he would like to increase his production. Only the possibility that the price cap is not binding prevents him from doing so. Now, consider what happens if the price cap goes slightly up to a level \( \bar{p}_2 > \bar{p}_1 \). In this case, if the price cap is binding profits become \( q \cdot (\bar{p}_2 - c) \) while profits remain \( q \cdot (P(q, \theta) - c) \) if the price cap is not binding. There are two resulting effects. The first effect of an increase in the price cap is an increase in the incentive of the monopolist to choose a higher quantity as the benefits of producing a higher quantity increase when the price cap is binding but the disadvantages of producing a higher quantity when the price cap is not binding do not change. But there is a second effect that decreases the monopolist’s incentive to increase the quantity. The probability that the price cap is binding will decrease as the price cap increases. Theorem 2 establishes that under the given assumptions, the first effect dominates the second.

\(^{12}\)Price caps close to marginal costs have received special attention. (See, for example, the comment of the Federal Energy Regulatory Commissioner cited in the Introduction.)
Corollary 1. Assume $P$ is continuously differentiable. Denote the set of all distributions with bounded support for which production is gainful by $\mathcal{F}$. Restrict attention to the equilibrium with the highest or lowest production quantity.

Then the set of $F \in \mathcal{F}$, for which at least one of the conditions (i), (ii) or (iii) of Theorem 1 holds, is nowhere dense in $\mathcal{F}$.

While the above results imply that, unlike in the deterministic case, price caps close to marginal costs are typically not welfare-improving, they leave open the question whether the effect of such price caps are "really bad" or whether, perhaps, they still lead to welfare that, for example, is above what would be achieved without a price cap. The answer to this question, of course, depends on the distribution $F$.

We denote by $\underline{\theta}$ and $\overline{\theta}$ the minimal and maximal point in the support of $F$.

Theorem 3. Let $q^{**}$ be equal to the unique root\(^{13}\) of the equation

$$P(n \cdot q^{**}, \overline{\theta}) = c.$$  

if there is a nonnegative root or equal to zero if the equation does not have a nonnegative root. Then $q^*(\overline{p})$ converges to $\{q^{**}\}$ as $\overline{p}$ converges to $c$, $\overline{p} \searrow c$.

Theorem 3 states that as the price cap gets closer and closer to marginal cost, the firms produce the equilibrium quantity from a competitive market with the lowest demand. Building on the intuitive discussion after Theorem 2, we can provide a basic intuition for this result. A firm deciding about its production quantity must consider the implications of its choice on profits for the different demand realizations, in particular the profits for a binding price cap and also for a price below the price cap, $P(Q, \theta) < \overline{p}$. Then it chooses a quantity that constitutes a good compromise between the different quantities which would be produced if demand were known. As the price cap converges to $c$, the benefits of high production in anticipation of a high demand state will decrease. Therefore, a producer will attach relatively more weight to low demand states while making its decision. As a result, lower production quantities are chosen. In the limit the producer will focus only on the state in which demand is smallest and produce the corresponding quantity.

The theorem immediately allows us to conclude that in certain situations, the introduction of a price cap close to marginal cost will result in a decreased production quantity. In particular, if there is a slight possibility that production will be inefficient, the introduction of a price cap sufficiently close to marginal cost will decrease production to zero.

\(^{13}\)The equation cannot have multiple roots because $P$ is decreasing in quantities.
Corollary 2. Assume that the ex post socially optimal production is equal to zero with some positive probability. Then a price cap sufficiently close to marginal cost $c$ will result in a complete stoppage of production.

3.4. Price caps that are not close to marginal cost. Corollary 1 was based on properties of the equilibrium set for price caps close to marginal costs. This raises the question whether the monotonicity properties from Theorem 1 remain true for uncertain demand schedules if we restrict attention to price caps which are still binding, but not necessarily close to $c$. To generalize Corollary 1, we make an extra assumption.\(^\text{14}\) In this section we assume that \textit{marginal profits are decreasing in other firms’ output}, in the sense that $\frac{\partial}{\partial Q} \pi_\theta(q, Q-i)$ is decreasing in $Q-i$ (the output of the other firms) for any $\theta \in \mathbb{R}$, $q \in \mathbb{R}^+$ and $Q-i \in \mathbb{R}^+$.\(^\text{15}\) Note that this implies that, for any given price cap $\bar{p}$, best responses are nonincreasing and thus there exists a unique equilibrium for each price cap. Slightly abusing notation, we denote this equilibrium by $q^*(\bar{p})$.

Let $(p', p'')$ be an interval. We will say that the \textit{price cap is binding} for some $\bar{p} \in (p', p'')$ if for some $\bar{p} \in [p', p'']$, $\Pr(P(n \cdot q^*(\bar{p}), \theta) > \bar{p}) > 0$. Clearly, if this is not the case, i.e., if $\Pr(P(n \cdot q^*(\bar{p}), \theta) < \bar{p}) = 1$ for all $\bar{p} \in [p', p'']$, it has to be that $q^*$ is constant on $[p', p'']$.

In the following, we will say that \textit{production is gainful for a price cap $\bar{p}$} if $E(\min(P(0, \theta), \bar{p})) > c$. Again, if $(\min(P(0, \theta), \bar{p})) < c$ then at a given price cap of $\bar{p}$, no firm would ever produce a positive quantity.

The following result shows that, for stochastic demand distributions, neither produced quantities nor total welfare will typically be nonincreasing functions of the price cap almost everywhere. That is, it does not matter what distribution of $\theta$ one chooses, one can always find a perturbation of that distribution in which the desirable properties of price caps (i) or (ii) of Theorem 1 fail.

\textbf{Theorem 4.} Assume $P$ is twice continuously differentiable and marginal profits are decreasing in other firms’ output. Let $(p', p'')$ be an arbitrary nonempty interval.

\textsuperscript{14}The assumption that marginal profits are decreasing in other firms output is convenient in the proof of Theorem 4 as it stops alternative equilibria from appearing but it does not appear to be essential for the result.

\textsuperscript{15}Actually, for any fixed $n \in \mathbb{N}$, the following results also hold under the slightly weaker alternative assumption that $q \cdot \frac{\partial}{\partial Q^2} P(n \cdot q, \theta) + (1 + \frac{1}{n}) \cdot \frac{\partial}{\partial Q} P(n \cdot q, \theta) \leq 0$ for all $q \in \mathbb{R}^+$ and $q \in \mathbb{R}^+$.\ldots
Let $F$ be the set of all distributions with bounded support such that production is
gainful for price caps in $(p', p'')$ and the price cap is binding for some $\bar{p} \in (p', p'')$.

Recall points (i)-(v) in Theorem 1. Then the set of distributions $F \in F$ for which
at least one of the conditions (i) or (ii) holds for price caps in $(p', p'')$ is nowhere
dense in $F$.

As a decrease in the price cap has a direct effect on average prices and consumer
welfare, an additional assumption is needed to guarantee that (iii) and (iv) do not
hold for a generic demand structure. Let

$$CS_\theta(Q) = \int_0^Q (P(x, \theta) - P(Q, \theta)) \cdot dx$$

be consumer surplus for a fixed $\theta$ if industry wide output is $Q$. We will say that $CS$
is twice continuously differentiable and $CS''_\theta < 0$ if, for all $\theta \in \mathbb{R}$, $CS_\theta(\cdot)$ is twice
differentiable, $CS''_\theta(Q)$ is continuous in both parameters, and $CS''_\theta(Q) < 0$ for all
$Q \in \mathbb{R}_+$ and $\theta \in \mathbb{R}$.

**Theorem 5.** Assume $CS$ is twice continuously differentiable with $CS'' < 0$ and
marginal profits are decreasing in other firms output. Let $(p', p'')$ be an arbitrary
nonempty interval. Let $F$ be the set of all distributions with bounded support such
that production is gainful for price caps in $(p', p'')$ and the price cap is binding for
some $\bar{p} \in (p', p'')$.

Recall points (i)-(v) in Theorem 1. Then the set of distributions $F \in F$ for which
at least one of the conditions (i), (ii), (iv), and (v) holds for price caps in $(p', p'')$
is nowhere dense in $F$.

Our result that lowering a price cap can result in higher average prices is some-
what surprising and means that a regulator trying to lower prices with a price cap
can have his effort backfire. There appear to be empirical examples where lower
price caps coincided with higher average prices, see, for example, California Power
Exchange (2000). (However, the question whether in this case the price increase was
related to uncertainty about future demands or driven by other factors is beyond
this paper.)

4. **Conclusion**

This paper has shown that the simple monotone comparative statics results
which are usually used to justify price caps in the context of deterministic demand
cease to hold if we consider firms facing an uncertain demand function. In the
presence of such uncertainty, imposing a price cap or lowering its level can result
in a reduced production and total welfare. In addition, it can lead to an increase
in average prices and a decrease in consumer welfare.\footnote{Of course, even if average prices increase in response to a decrease in price caps, such price caps could still be justified if the objective is to decrease price volatility rather than improve consumer welfare. (See Verleger (1993) for a discussion of this issue.)} In the case of price caps close to marginal costs, we obtained a characterization of equilibrium production thus making it possible to quantitatively assess the effects of imposing such a cap. Unlike in the case where the future realization of demand is known, price caps close to marginal cost are never welfare maximizing. In the case of price caps “far away” from marginal costs, we showed that all of the standard comparative statics predictions fail for a generic uncertain demand schedule suggesting that also in this case, price caps should be used only with great caution.

Mathematical Appendix

In the following, we sometimes want to underline the dependence of functions or variables on the underlying distribution $F$. In this case, we write $\pi_F$, $q^*_F(p)$, etc. Slightly abusing notation, we continue to write $\pi_{\theta'}$ where $\theta' \in \mathbb{R}$ to denote profits if $F$ is such that $\theta = \theta'$ with probability one, i.e.,

$$\pi_{\theta'}(q, Q^-) = q \cdot (P(q + Q^-, \theta') - c).$$

A.1. Proof of Proposition 1 and related results.

Proof of Lemma 1. The statement of the lemma consists of two parts, one involving $\lim_{x \downarrow x_0} b(x, p)$, the other involving $\lim_{x \uparrow x_0} b(x, p)$. We will prove the former; the proof of the latter is analogous.

Assume the first part of the lemma is not true, thus there exists a sequence $x_k \downarrow x_0$ and a sequence $q_k \in b(x_k, p)$ such that $q_k$ converges to some point $q^A$ which is not equal to the point $\max b(q, p)$ which we will denote by $q^B$.

As the best response correspondence has a closed graph, it must be the case that $q^A$ is itself a best response. The definition of $q^B$ therefore implies that $q^B \geq q^A$.

As we assumed that $q^A$ is not equal to $q^B$, it follows that $0 \leq q^A < q^B$.

For the sake of notation, it will be convenient to denote $x_k - x_0$ by $h_k$. Also, let $\phi(x) = E(\min(P(x, \theta), p) - c)$ for all $x \in \mathbb{R}_+$. The remainder of the proof is organized in several steps.

1. We already saw that both $q^A$ and $q^B$ must be best responses against $x_0$, in particular it must be that

$$q^A \cdot \phi(x_0 + q^A) = q^B \cdot \phi(x_0 + q^B).$$
and thus (as $0 \leq q^A < q^B$ implies $0 < q^B$)

$$\frac{q^A}{q^B} \cdot \phi(x_o + q^A) = \phi(x_o + q^B).$$

Note that $0 < q^B$ and $q^B \cdot \phi(x_o + q^B) \geq 0$ implies that $\phi(x_o + q^B) \geq 0$. But then $q^A < q^B$ implies that $\phi(x_o + q^A) > 0$. Hence,

$$\phi(x_o + q^A) > \phi(x_o + q^B).$$

(2) Since $q^A$ is a best response against $x_o$, it must be that

$$q_k \cdot \phi(x_k + q_k) = (q_k + h_k) \cdot \phi(x_o + q_k + h_k) - h_k \cdot \phi(x_o + q_k + h_k) \leq q^A \cdot \phi(x_o + q^A) - h_k \cdot \phi(x_o + q_k + h_k).$$

(3) Since $q_k$ is a best response against $x_k$, it must be that

$$(q^B - h_k) \cdot \phi(x_k + (q^B - h_k)) \leq q_k \cdot \phi(x_k + q_k).$$

(4) Combining the inequalities of the two previous steps, we get that

$$(q^B - h_k) \cdot \phi(x_k + (q^B - h_k)) \leq q^A \cdot \phi(x_o + q^A) - h_k \cdot \phi(x_o + q_k + h_k).$$

Note that $x_k + (q^B - h_k) = x_o + q^B$ and apply the first equation of Step 1 to conclude that

$$h_k \cdot \phi(x_o + q_k + h_k) \leq h_k \cdot \phi(x_o + q^B).$$

As $h_k > 0$ and $x_o + h_k = x_k$ this means that

$$\phi(x_k + q_k) \leq \phi(x_o + q^B).$$

(5) Our assumptions on $P$ guarantee that $\Phi$ is continuous. Thus, as $x_k \to x_0$ and $q_k \to q^A$, the last step implies that

$$\phi(x_o + q^A) \leq \phi(x_o + q^B).$$

The last inequality in Step 5 contradicts the last inequality of Step 1. The contradiction shows that the statement of the lemma must be correct. □

Proof of Proposition 1. Lemma 1 implies that the function $\max b(\cdot, \overline{p})$ is continuous but for upwards jumps. Thus, by Theorem 1 in Milgrom and Roberts (1994), the equation

$$q = \max b((n - 1) \cdot q, \overline{p})$$

has a solution. The conclusion follows. □

A.2. Proof of Theorem 1 and related results.
Proof of Lemma 2. Consider two quantities \( q' > q'' \) and two price caps \( p' < p'' \). Assume that, for a fixed \( Q - i \), \( \pi_\theta(q', Q - i, p'') > \pi_\theta(q'', Q - i, p'') \). We will show that \( \pi_\theta(q', Q - i, p') > \pi_\theta(q'', Q - i, p') \). We will compare the change in profits as the price cap changes from \( p'' \) to \( p' \). It will be convenient to distinguish three cases:

1. The price cap \( p' \) is binding for both quantities \( q' \) and \( q'' \). As \( p' > c \), this implies that producing the higher quantity is better after the price cap was decreased to \( p' \).

2. The price cap \( p' \) is binding if production has the lower level \( q'' \) but is not binding if production is \( q' \). As \( p' < p'' \), this means that after the decrease of the price cap to \( p' \) production at a level \( q'' \) became less attractive while production at a level of \( q' \) yields the same profits as before. As producing the higher quantity was better before, it also must be better now, after the decrease in the price cap.

3. The price cap \( p' \) is not binding for both quantities \( q' \) and \( q'' \). In this case profits are the same both after and before a decrease in the price cap. The claim follows.

The second part of the proof which involves weak inequalities is analogous. □

Lemma 3. If \( \pi \) satisfies the single crossing property then \( q^*_L(p) \) and \( q^*_H(p) \) are both nonincreasing functions of \( p \).\(^{17}\)

Proof of Lemma 3. Define \( \phi(Q) = E(\min(P(Q, \theta), p) - c) \). Using Lemma 1, we already showed in the proof of Theorem 1 that \( \max b(\cdot, p) \) is continuous but for upward jumps. Similarly, Lemma 1 implies that \( \min b(\cdot, p) \) is also continuous but for upward jumps.

As \( \pi \) satisfies the single crossing property, the Monotonicity Theorem of Milgrom and Shannon (1994, p. 162, Theorem 4) implies that for any fixed \( Q - i \) the functions \( \min b(Q - i, \cdot) \) and \( \max b(Q - i, \cdot) \) are nonincreasing. Note that \( q^*_L(p) \) and \( q^*_H(p) \) are respectively equal to the lowest and highest intersection of \( \min b(Q - i, \cdot) \) and \( \max b(Q - i, \cdot) \) with the diagonal. Theorem 1 in Milgrom and Roberts (1994) implies that \( q^*_L(p) \) and \( q^*_H(p) \) are nonincreasing. □

Proof of Theorem 1. Lemmas 2 and 3 immediately imply that (i) holds. Note that total welfare

\[
\int_0^Q \left( P(x, \theta) - c \right) \cdot dx
\]

\(^{17}\)As always in this paper, we only consider price caps \( p \) which lie above marginal cost \( c \).
is a monotonic function of total production $Q$ as long as $P(Q, \theta) \geq c$. However, independently of the price cap, equilibrium total production will always satisfy $P(Q, \theta) \geq c$ as otherwise firms could decrease production. Point (ii) follows.

If demand is deterministic, then in equilibrium demand is equal to supply, i.e. for all $q \in q^\ast$, $P(n \cdot q, \theta) \leq \overline{p}$. Indeed, $P(n \cdot q, \theta) > \overline{p}$ in connection with the continuity of $P(\cdot, \theta)$ and $\overline{p} > c$ would mean that firms could increase their profits by slightly increasing production. Therefore (i) together with the observation made above and the monotonicity assumptions on the inverse demand function $P$ implies (iii) and (iv). Property (v) now follows directly from (i) and (iv). □

A.3. Proofs for Section 3.3.

Proof of Theorem 2. We will prove the statement of the theorem for $q^\ast_L$, the argument for $q^\ast_H$ is analogous. Consider first the case where $P$ is twice continuously differentiable in both of its parameters. Note that if $q^\ast_L(p') = 0$ for some $p' > c$ then $q^\ast_L(p'') = 0$ for all $p'' \in (c, p')$ and the theorem holds. Assume therefore, without loss of generality, that $q^\ast_L(p') > 0$ for all $p' > c$.

Define $q^{\max}$ to be the solution to $P(q^{\max}, \theta) = c$, where $\theta$ is the maximal point in the closure of the support of $F$. Let $b(x, \overline{p}) = \arg \max_q \pi(q, x, \overline{p})$ be the best response of a player if her opponents together produce $x$ and the price cap is $\overline{p}$.

Note that

$$
\frac{d^2}{dq \cdot dp} \pi(q, Q - i, \overline{p}) = \frac{d}{dq} \int_{\tilde{\theta}}^{\infty} q \cdot dF(\theta)
$$

$$
= (1 - F(\tilde{\theta})) + \frac{\delta}{\delta q} P(q + Q - i, \overline{\theta}) \cdot q \cdot f(\overline{\theta})
$$

$$
\geq (1 - F(\tilde{\theta})) - \frac{L_1}{L_2} \cdot q^{\max} \cdot f(\overline{\theta})
$$

where $\tilde{\theta}$ is equal to $\overline{\theta}$ if $P(q + Q - i, \overline{\theta}) > \overline{p}$, equal to $\overline{\theta}$ if $P(q + Q - i, \overline{\theta}) < \overline{p}$ and given by $P(q + Q - i, \overline{\theta}) = \overline{p}$ otherwise. If, in addition, $q \in b(Q - i, \overline{p})$ then,

$$
0 \leq (1 - F(\tilde{\theta})) \cdot q \cdot (\overline{p} - c) + \int_{-\infty}^{\tilde{\theta}} (P(q + Q - i, \theta) - c) \cdot q \cdot dF(\theta)
$$

$$
\leq (1 - F(\tilde{\theta})) \cdot q \cdot (\overline{p} - c) + \int_{-\infty}^{\tilde{\theta}} (\overline{p} - c - (\overline{\theta} - \theta) \cdot L_2) \cdot q \cdot dF(\theta)
$$

$$
= q \cdot (\overline{p} - c) + \int_{-\infty}^{\tilde{\theta}} (\theta - \overline{\theta}) \cdot L_2 \cdot dF(\theta)
$$
and thus either \( q = 0 \) or \( \int_{-\infty}^{\bar{\theta}} (\bar{\theta} - \theta) \cdot dF(\theta) \leq \frac{(\bar{\theta} - c)}{L_2} \).

Note that \( \int_{-\infty}^{\bar{\theta}} (\bar{\theta} - \theta) \cdot dF(\theta) \) is monotonically increasing in \( \bar{\theta} \). Choose \( \gamma > 0 \) such that if \( \int_{-\infty}^{\bar{\theta}} (\bar{\theta} - \theta) \cdot dF(\theta) \leq \frac{\gamma}{L_2} \) then \( \bar{\theta} \) is small enough so that

\[
(1 - F(\bar{\theta})) - \frac{L_1}{L_2} \cdot q_{\text{max}} \cdot f(\bar{\theta}) \geq \frac{1}{2}.
\]

Putting the above observations together conclude that if \( q, x \in \mathbb{R}_+ \) and \( \bar{\theta} > c \) satisfy \( \bar{\theta} - c < \gamma, q \leq \max b(x, \bar{\theta}) \), and \( q > 0 \) then \( \frac{d^2}{dq \, dq} \pi(q, x, \bar{\theta}) \geq \frac{1}{2} \).

Assume \( q^*_k \) is not monotonically nondecreasing in \((c, c + \gamma)\). Then there exist sequences \( r^n, s^n \in (c, c + \gamma) \) such that \( s^n - r^n \to 0, r^n < s^n \), and \( q^*_k(r^n) > q^*_k(s^n) \).
Without loss of generality, assume there exists \( t \in (c, c + \gamma) \) such that \( s^n, r^n \to t \) as \( n \to \infty \). Note that \( \gamma \) was chosen so that \( \frac{d^2}{dq \, dq} \pi(q, x, \bar{\theta}) > \frac{1}{4} \) for all \((q, x, \bar{\theta})\) in some neighborhood of \( \{ (q', (n - 1) \cdot q, t) : q < q'(t), q' \leq \max b((n - 1) \cdot q, t) \} \).

Then, however, Theorem 1 from Edlin and Shannon (1996) implies that \( b(x, \bar{\theta}) \) is monotonically increasing (as a function of \( \bar{\theta} \)) in some neighborhood of \( \{(n - 1) \cdot q, t) : q < q'(t) \} \) which means that \( q^*_k \) has to be monotonically increasing in some neighborhood of \( t \). Contradiction!

Now consider the case where \( P \) does not satisfy the differentiability assumptions made above. Choose \( \gamma > 0 \) such that if \( \int_{-\infty}^{\bar{\theta}} (\bar{\theta} - \theta) \cdot dF(\theta) \leq \frac{\gamma}{L_2} \) then \( \bar{\theta} \) is small enough so that

\[
(1 - F(\bar{\theta})) - \frac{L_1}{L_2} \cdot 8 \cdot q_{\text{max}} \cdot f(\bar{\theta}) \geq \frac{1}{2}.
\]

If \( q^*_k \) is not monotonic on \((c, c + \gamma)\), then there are points \( r, s \in (c, c + \gamma) \) such that \( r < s \) and \( q^*_k(r) > q^*_k(s) \).

Choose a sequence of demand schedules \( P^k \) such that

- \( q^\max_k : P^k(q^\max_k, \bar{\theta}) = c \) satisfy \( q^\max_k \leq 2 \cdot q_{\text{max}} \) where \( q_{\text{max}} : P^k(q_{\text{max}}, \bar{\theta}) = c \);
- \( |P^k(q, \theta) - P^k(q', \theta')| \leq 2 \cdot L_1 \cdot |q - q'| \) and \( |P^k(q, \theta) - P^k(q', \theta')| \leq 2 \cdot L_2 \cdot |\theta - \theta'| \)
  for all \( \theta, \theta' \in \mathbb{R}, q, q' \in \mathbb{R}_+ \) and \( k \in \mathbb{N} \);
- \( P^k \to P \) uniformly on \( \text{Support}(F) \times [0, 2 \cdot q_{\text{max}}] \)
- \( P^k \) is twice continuously differentiable in both parameters
- the lowest equilibrium for a price cap \( s \) is the same for each demand schedule \( P^k \) as in \( P \).

As in the limit the equilibrium set for price cap \( r \) and demand schedule \( P^k \) has to converge to a subset of \( q^*(r) \), the established result for the differentiable demand schedules \( P^k \) implies the result for the nondifferentiable case. \( \square \)

**Proof of Corollary 1.** Let \( F \in F \) be an arbitrary distribution. Note that we can always find a continuously differentiable distributions \( \tilde{F} \in F \) arbitrarily close to \( F \).
Theorem 2 implies that (i) does not hold for $\hat{F}$.

If (i) does not hold for $\hat{F}$, there is an open neighborhood of $\hat{F}$ for which (i) does not hold. We showed that for any $F \in f$, there exists an open set on which (i) does not hold and which is arbitrary close to $F$. This means that the set of $F \in f$ for which (i) holds is nowhere dense in $f$. Analogous arguments establish that the sets of $F \in f$ for which (ii) and (iii) are also nowhere dense. As the union of three nowhere dense sets has to be itself nowhere dense this completes the proof. □

Proof of Theorem 3. Let $\overline{p}^m > c$ be a sequence of numbers such that $\lim m \to \infty \overline{p}^m = c$ and $q^m$ be a sequence of quantities such that $q^m \in q^*(\overline{p}^m)$. Our aim is to show that $q^m \to q^{**}$. For this purpose we examine the sequence $P(n \cdot q^m, \theta)$.

Assume that

$$\liminf_{m \to \infty} P(n \cdot q^m, \theta) < c$$

Then there exists a subsequence $q^{m_k}$ such that $\lim k \to \infty P(n \cdot q^{m_k}, \theta) < c$. As $\theta$ lies in the support of $\theta$, this means that with positive probability the firms will make a per unit loss which is bounded away from zero. Note that on the other hand, the potential per unit gains converge to zero as $\overline{p} \to c$. Thus it must be that $q^{m_k}$ is equal to zero for large enough $k$. In this case, $\lim k \to \infty P(n \cdot q^{m_k}, \theta) < c$ implies $P(0, \theta) < c$, i.e. the equation from the proposition does not have a solution.

Assume now that

$$\limsup_{m \to \infty} P(n \cdot q^m, \theta) > c.$$ 

Then there exists a subsequence $q^{m_k}$ such that $\lim k \to \infty P(n \cdot q^{m_k}, \theta) > c$. This means that $\lim k \to \infty P(n \cdot q^{m_k}, \theta)$ is uniformly bounded away from $c$ for all $\theta$. Thus, for large enough $k$ the price cap will be always binding. As $P(Q, \theta)$ is assumed to be continuous in $Q$, this means that firms would like to increase their production in equilibrium – contradiction. Hence $\limsup_{m \to \infty} P(n \cdot q^m, \theta) \leq c$. □

A.4. Proof of Theorems 4 and 5.

Proof of Theorem 4. Note that it is sufficient to show that for any continuously differentiable $F \in f$ with convex support, there is a $\tilde{F} \in f$ arbitrary close to $F$ such that $q^*$ is nonmonotonic on $(p', p'')$. (Indeed, if this is the case, there will be a neighborhood of $\tilde{F}$ for which $q^*$ is nonmonotonic.)

\(^{18}\)Indeed, either $q^*_L$ and $q^*_H$ are monotonically increasing on some interval $(c, c + \gamma)$ or equal to zero in that interval. In the former case (i) clearly does not hold. In the latter case, (i) would imply that $q^*_L$ and $q^*_H$ are equal to zero for all price caps. This, however, can not be, the fact that production is gainful implies that producing zero cannot be an equilibrium if the price cap is chosen high enough so that it is not binding if total production is zero. The contradiction shows that (i) also in this case cannot be true.
Assume the price cap is binding at some point in \((p', p'')\). If \(q^*\) is not monotonically nonincreasing in \((p', p'')\) then we can choose \(\tilde{F} = F\) and are done. Assume therefore \(q^*\) is monotonically nonincreasing in \((p', p'')\). This fact combined with the assumption that the price cap is binding at some point in \((p', p'')\) implies that the price cap is binding in some proper subinterval of \((p', p'')\). Without loss of generality assume the price cap is binding on \((p', p'')\).

Note that since marginal profits decrease in other firms output and prices are decreasing, \(\pi_\theta(\cdot, Q_{-1})\) is concave for any \(\theta \in \mathbb{R}, Q_{-1} \in \mathbb{R}_+\). As \(\pi_\theta(\cdot, Q_{-1}, p)\) is equal to \(\min(\pi_\theta(\cdot, Q_{-1}), (p - c) \cdot q)\), it is also concave for any \(\theta \in \mathbb{R}, Q_{-1} \in \mathbb{R}_+,\) and \(p > c\).

The equilibrium \(q^*\) is thus equal to the unique root of the first order condition

\[
\int_{-\infty}^{\infty} D_1 \pi_\theta(q, (n - 1) \cdot q, \bar{p}) \cdot dF = 0.
\]

For a given distribution \(F\), price cap \(\bar{p}\), and quantity \(q \in \mathbb{R}_+\), denote the left hand of the above expression by \(\Gamma_F(q, \bar{p})\). It is straightforward to check that, as marginal profits decrease in other firms output, \(\Gamma_F(\cdot, \bar{p})\) is a decreasing function for any distribution \(F\) and price cap \(\bar{p}\).

Choose a point \(\bar{p} \in (p', p'')\). Define \(\tilde{\theta} : P(n \cdot q^*(\bar{p}), \tilde{\theta}) = \bar{p}\). Fix a \(\varepsilon > 0\) and consider the distribution \(\tilde{F}\) given by

\[
\tilde{F}(x) = \begin{cases} 
F(x) & x < \tilde{\theta} - \varepsilon \\
F(\tilde{\theta} - \varepsilon) + \alpha \cdot (F(\tilde{\theta} + \varepsilon) - F(\tilde{\theta} - \varepsilon)) & x \in [\tilde{\theta} - \varepsilon, \tilde{\theta} + \varepsilon) \\
F(x) & x \geq \tilde{\theta} + \varepsilon
\end{cases}
\]

where \(\alpha\) is chosen so that

\[
\int_{\tilde{\theta} - \varepsilon}^{\tilde{\theta} + \varepsilon} D_1 \pi_\theta(q^*(\bar{p}), (n - 1) \cdot q^*(\bar{p}), \bar{p}) \cdot dF
\]

is equal to

\[
\alpha \cdot D_1 \pi_{\theta - \varepsilon}(q^*(\bar{p}), (n - 1) \cdot q^*(\bar{p})) + (1 - \alpha) \cdot \bar{p}.
\]

Note that this definition implies that

\[
\int_{-\infty}^{\infty} D_1 \pi_\theta(q^*(\bar{p}), (n - 1) \cdot q^*(\bar{p}), \bar{p}) \cdot d\tilde{F} = 0
\]

and thus \(q^*(\bar{p})\) is also the unique equilibrium if \(\theta\) is distributed according to \(\tilde{F}\).

Finally observe that in some neighborhood of \(\bar{p}\), the Implicit Function Theorem yields

\[
\frac{dq^*}{d\bar{p}} = -\frac{1}{\int_{-\infty}^{\bar{p}} ((n + 1) \cdot \frac{d\theta}{d\bar{p}} P(n \cdot q, \theta) + n \cdot q \cdot \frac{d^2}{d\theta^2} P(n \cdot q, \theta)) \cdot d\tilde{F}}
\]

\[\text{\textsuperscript{19}}D_1 \pi_\theta, \text{ the derivative with respect to the first parameter exists a.e. As } F \text{ is assumed continuously differentiable and } D_1 P \text{ is bounded it is clear that the integral is well defined.}\]
As \( q \cdot \frac{d}{dq} P(n \cdot q, \theta) + \frac{d}{dq} P(n \cdot q, \theta) \leq 0 \) (by the decreasing marginal profit assumption) and \( \frac{d}{dq} P(n \cdot q, \theta) < 0 \) it follows that \( \frac{dq^*}{dp} > 0 \). □

Proof of Theorem 5. The argument follows the same line as the proof of Theorem 4. After choosing \( \tilde{F} \), observe that in some sufficiently small neighborhood of \( \tilde{p} \), average prices are equal to

\[
\int_{\tilde{\theta}}^{\infty} \tilde{p} \cdot d\tilde{F} + \int_{-\infty}^{\tilde{\theta}} P(n \cdot q^*(\tilde{p}), \theta) \cdot d\tilde{F}
\]

where \( \tilde{\theta} \) is fixed and defined as in the proof of Theorem 4. Using the equality for \( \frac{dq^*}{dp} \) at the very end of the proof of Theorem 4 note that

\[
\frac{d}{dp} \left( \int_{\tilde{\theta}}^{\infty} \tilde{p} \cdot d\tilde{F} + \int_{-\infty}^{\tilde{\theta}} P(n \cdot q^*(\tilde{p}), \theta) \cdot d\tilde{F} \right) = \int_{\tilde{\theta}}^{\infty} 1 \cdot d\tilde{F} + \int_{-\infty}^{\tilde{\theta}} n \cdot \frac{dq^*}{dp} P(n \cdot q^*(\tilde{p}), \theta) \cdot d\tilde{F} = \int_{\tilde{\theta}}^{\infty} 1 \cdot d\tilde{F} \cdot \left( 1 - \frac{\int_{-\infty}^{\tilde{\theta}} n \cdot \frac{d}{dQ} P(n \cdot q, \theta) \cdot d\tilde{F}}{\int_{-\infty}^{\tilde{\theta}} ((n + 1) \cdot \frac{d}{dQ} P(n \cdot q, \theta) + n \cdot q \cdot \frac{d^2}{dQ^2} P(n \cdot q, \theta)) \cdot d\tilde{F}} \right)
\]

It is therefore enough to show that

\[
\frac{\int_{-\infty}^{\tilde{\theta}} n \cdot \frac{d}{dQ} P(n \cdot q, \theta) \cdot d\tilde{F}}{\int_{-\infty}^{\tilde{\theta}} ((n + 1) \cdot \frac{d}{dQ} P(n \cdot q, \theta) + n \cdot q \cdot \frac{d^2}{dQ^2} P(n \cdot q, \theta)) \cdot d\tilde{F}} > 1.
\]

Note that \( CS'' < 0 \) implies that \( \frac{d}{dQ} P(n \cdot q, \theta) + n \cdot q \cdot \frac{d^2}{dQ^2} P(n \cdot q, \theta) > 0 \) for all \( \theta \). But then clearly

\[
\int_{-\infty}^{\tilde{\theta}} ((n + 1) \cdot \frac{d}{dQ} P(n \cdot q, \theta) + n \cdot q \cdot \frac{d^2}{dQ^2} P(n \cdot q, \theta)) \cdot d\tilde{F} > \int_{-\infty}^{\tilde{\theta}} n \cdot \frac{d}{dQ} P(n \cdot q, \theta) \cdot d\tilde{F}.
\]

But then (as the left hand side is negative)

\[
\frac{\int_{-\infty}^{\tilde{\theta}} n \cdot \frac{d}{dQ} P(n \cdot q, \theta) \cdot d\tilde{F}}{\int_{-\infty}^{\tilde{\theta}} ((n + 1) \cdot \frac{d}{dQ} P(n \cdot q, \theta) + n \cdot q \cdot \frac{d^2}{dQ^2} P(n \cdot q, \theta)) \cdot d\tilde{F}} > 1
\]

follows. □

20If average prices are decreasing in the price cap and quantity is increasing in the price cap the implication about consumer and total welfare follow immediately.
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References