A Theory of Turnover and Wage Dynamics*

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Abstract
We develop a model of turnover and wage dynamics. The main ingredients of the model are insurance, match-specific productivity, and long-term contracting. The model predicts that wages are downward rigid within firms and termination occurs. We apply the model to study the impact of business cycles on subsequent wages and job mobility. Workers hired during a boom have persistent higher future wages if staying with the same firm. However, these boom hires are more likely to be terminated and have shorter employment spells.

Keywords: long-term contracts, business cycles, match-specific productivity

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1 Introduction

Growing evidence has suggested that random economic shocks at the time of employment have large and persistent effects on subsequent labor market outcomes. Baker, Gibbs, and Holmstrom (1994) use personnel records from one firm and identify a cohort effect—a cohort’s average wage at the start of employment affects its wages years later. Subsequent within-firm studies (see Gibbs and Hendricks (2004) for a review) have also documented the cohort effect within the same employment relationship. Beaudry and DiNardo (1991) document the cohort effect in large, representative data sets, and they show that the cohort effect is driven by the lowest unemployment rate since the worker was hired.

Random economic shocks also affect long-term earnings by changing the types of jobs workers choose, and its effects are particularly strong among workers fresh out of school. Oyer (2008) shows that MBAs graduating in recession years are significantly less likely go to Wall Street, and consequently, suffer an estimated lifetime-earnings loss of $1.5 to $5 million. Oreopoulos, von Wachter, and Heisz (Forthcoming) find that a typical recession causes Canadian college graduates to take lower-quality jobs, and as a result, the cumulated earnings in the first ten years of their career are reduced by about five percent. Kahn (2010) shows that a bad economy leads U.S. college graduates to choose less-prestigious occupations, and its negative effects on wages persist even after 15 years.

Despite its negative effects on long-term earnings, a bad economy appears to have a positive effect on job security. Schmieder and von Wachter (2010) use the Displaced Worker Survey and find that the probability of job loss is smaller for workers who were hired in worse labor market conditions. Kahn (2008) exploits a proprietary dataset for large U.S. firms and finds that while jobs created during a recession typically do not last as long, this is mainly because jobs with shorter durations are more likely to be offered in a recession. Once the job heterogeneity is controlled for, workers hired in a recession have longer expected employment spell.

The positive effect on job security is inconsistent with existing theories that explain the negative effect of a recession on long-term earnings. Schmieder and von Wachter (2010) discuss three classes of models on earnings, including human capital accumulation, job matches, and insurance. They note that models based on human capital accumulation and job matches have the opposite prediction on job security. In particular, if workers hired in a bad economy have lower wages because they are less productive, either through worse firm-specific or task-specific human capital
accumulation opportunities or because of worse match qualities, they should have worse rather than better job security.\textsuperscript{1} In models based on insurance, which we will discuss below, worker productivity is assumed to be the same across firms, so there is no turnover.

In this paper, we develop a simple model that integrates insurance, match-specific productivity, and long-term contracts. When match-specific productivity is important, we show that a bad economy lowers long-term earnings, but the workers hired in a bad economy have better employment security. By integrating both insurance and match-specific productivity, our model inherits the properties of insurance models on wage dynamics: wages are downward rigid within an employment relationship. Having match-specific productivity, however, implies that bad matches can be terminated, so turnover does occur. As will be explained in detail in Section 4, the key mechanism of the model is that there is less scope for providing insurance to a worker in a bad economy, so the value of starting an employment relationship is lower. This implies that workers in a bad economy need to be better matches to be hired. Consequently, the average match quality of workers hired in a worse economy is better, resulting in longer employment spells.

Both insurance and match-specific productivity are familiar and important ideas in labor economics. Harris and Holmstrom (1982) show that many features of the labor market can be explained by an insurance model in which firms can commit to long-term contracts for risk-averse workers who are free to move. Beaudry and DiNardo (1991) find that the Harris-Holmstrom model offers a better description of wage dynamics than does a spot market model or a full insurance model. However, these insurance models do not generate turnover because the productivity of the workers is identical across all firms, making turnover meaningless. For turnover patterns, a leading model is Jovanovic (1979), which is based on learning and match-specific productivity. Its celebrated prediction, that turnover probability is inverse-U shaped with job tenure, has been confirmed by empirical findings; see, for example, Farber (1994). In Jovanovic (1979), the initial expected match quality is the same for all hired workers. Here, in contrast, we assume that before hiring a worker, the firm observes a signal about his match-specific productivity. This allows the firm to base its hiring decision on the signal observed and also allows the average match quality of the hired workers to differ with the economic condition at the time of hiring.

The rest of the paper is organized as follows. We set up the model in Section 2. Section 3 describes the main features of the optimal employment contract, which are then used to study the

\textsuperscript{1}See Gibbons and Waldman (2004, 2006) for the theory of task-specific human capital.
effects of random economic shocks on subsequent labor market outcomes in Section 4. Section 5
discusses how the results of the paper change with alternative formulations. Section 6 concludes.

2 Setup

There is one worker and one firm. Both live for two periods and do not discount the future.
There are two possible states of the economy: boom and bust. We denote $\theta_L > 0$ as the general
productivity level in a bust and $\theta_H > \theta_L$ as the general productivity level in a boom. The state of
the economy is i.i.d. across periods. In each period, the probability of a boom state is $p \in (0, 1)$.
Let $\theta_i, i = 1, 2$, be the general productivity in period $i$. The state of the economy is public
information and becomes known at the beginning of each period.

The worker’s productivity in the firm depends both on the state of the economy and on his
match-specific quality. If the worker is a good match, his output in the firm in period $i$ is equal
to $y_i = \theta_i + m > \theta_i$. Otherwise, he is a bad match, and his output is equal to $\theta_i - m$. The match
quality is not known before the worker works for the firm.

At the beginning of period 1, the firm observes a signal $\alpha \in [0, 1]$, which denotes the probability
that the worker is a good match. We assume that $\alpha$ has a non-zero density function $f$ with
support in $[0, 1]$. Given $\alpha$, the worker’s expected match quality with the firm in period 1 is equal to
$m_1 = (2\alpha - 1)m$. Upon observing the signal, the firm decides whether to make an offer to the worker.
If no offer is made, both parties receive their outside options and the game ends. The firm’s outside
option is normalized to 0, and the worker’s outside option is given by $u(\theta_1) + pu(\theta_H) + (1 - p)u(\theta_L)$,
where $u$ is a twice-differentiable, increasing, and concave function. One interpretation of the
worker’s outside option is that the worker can engage in home production that produces $\theta_i$ each
period.

If the firm makes an offer, the employment contract specifies a triple $(w_1, h(m_2, \theta_2), w_2(m_2, \theta_2))$.
Here, $w_1$ is the period-1 wage, $h$ is the probability that the firm retains the worker in period 2, $w_2$
is the worker’s period-2 wage if retained, and $m_2$ is the worker’s expected match quality in period 2.
If the worker rejects the offer, the parties again receive their outside options. If the worker
accepts the offer, he receives his period-1 wage $w_1$ and the period-1 output is realized. We have
suppressed the dependence of $w_1$, $h$, and $w_2$ on $\theta_1$ to simplify notation. The firm is able to commit
to this contract.
Since the worker’s match quality is equal to his output minus the general productivity level in period 1, the exact match quality is fully revealed at the end of period 1, i.e., \( m_2 \in \{-m, m\} \). The one-to-one relationship between the period-1 output and the match quality implies that an equivalent setup is to have the contract depend on the period-1 output instead. We choose the current setup to simplify notation.

At the beginning of period 2, \( \theta_2 \) is revealed. With probability \( h(m_2, \theta_2) \), the worker is retained, and with probability \( 1 - h(m_2, \theta_2) \) the worker is laid off. If the worker is laid off, he receives his outside option \( u(\theta_2) \), and the firm receives 0 in period 2. If the worker is retained and decides to stay, he receives \( w_2(m_2, \theta_2) \), and the firm receives \( \theta_2 + m_2 - w_2(m_2, \theta_2) \). The worker can also decide to take his outside option, in which case he receives his outside option \( u(\theta_2) \) and the firm receives 0 in period 2. To simplify exposition, we assume that \( w_2(m_2, \theta_2) \geq \theta_2 \) when \( h(m_2, \theta_2) > 0 \), so that if the worker is not laid off, he will not leave.

In the setup above, we made a few simplifying assumptions to illustrate better the model’s main mechanism. First, both the match quality and the general level of productivity are binary variables. The predictions of the model are robust to more general distributions of match quality and productivity levels, as shown at the end of the paper. Second, the firm’s outside option is independent of the general productivity level. This assumption simplifies notation. The same patterns of turnover and wage dynamics also arise as long as the value of the employment relationship relative to the sum of the outside options is larger in a boom. Moreover, when this condition holds, the model’s predictions remain the same when it is extended to allow for multiple firms and workers.

### 3 Properties of Turnover and Wage Dynamics

In this section, we discuss the general properties of turnover and wage dynamics by characterizing the optimal contract that maximizes the firm’s profit. Our first result shows that if the worker is retained by the firm, his wage is downward rigid.

**LEMMA 1.** For a retained worker, his period-2 wage is given by

\[
 w_2(m_2, \theta_2) = \begin{cases} 
 w_1 & \text{if } \theta_2 < w_1 \\
 \theta_2 & \text{otherwise} 
\end{cases}
\]

Lemma 1 states that the period-1 wage \( w_1 \) serves as a wage guarantee such that the wage of a retained worker is at least \( w_1 \) even if his outside wage is lower. The reason for the downward
rigidity is the same as in Harris and Holmstrom (1982). In particular, consider a worker retained by the firm in period 2. Suppose he experiences a wage loss; the firm can then smooth the worker’s pay by raising the period-2 wage and lowering the period-1 wage. This change generates a larger value for the relationship since it offers better insurance to the worker. Consequently, if the wage falls, the firm can increase its profit through such changes. This implies that the wage must be downward rigid in the optimal contract. While being downward rigid, the wage is not upward rigid. Whenever the retained worker’s outside option exceeds the period-1 wage, the period-2 wage increases to match it. This lack of full insurance arises because the worker cannot commit to the contract. The worker’s opportunity to take the outside option therefore limits the scope of insurance.

While the optimal contract offers insurance by giving a wage guarantee to the retained worker, it does not eliminate employment risk. When the worker’s outside option exceeds the period-1 wage, the retention rule is efficient—the worker is retained if and only if he is a good match. When the worker’s outside option falls below the period-1 wage, the retention rule specifies that the bad match is terminated if match quality is sufficiently important. The next lemma gives the exact retention rule.

**Lemma 2.** Let $w_1$ be the period-1 wage in the optimal contract. The retention rule satisfies the following.

(a) If $\theta_2 \geq w_1$, the retention rule is efficient:

$$h(m_2, \theta_2) = \begin{cases} 1 & \text{if } m_2 = m \\ 0 & \text{if } m_2 = -m. \end{cases}$$

(b) If $\theta_2 < w_1$, the good match is retained: $h(m, \theta_2) = 1$. The bad match is terminated if and only if match quality is sufficiently important:

$$h(-m, \theta_2) = \begin{cases} 1 & m < \frac{u(w_1) - u(\theta_2)}{u'(w_1)} - (w_1 - \theta_2) \\ 0 & m > \frac{u(w_1) - u(\theta_2)}{u'(w_1)} - (w_1 - \theta_2), \end{cases}$$

and

$$h(-m, \theta_2) \in [0, 1] \text{ if } m = \frac{u(w_1) - u(\theta_2)}{u'(w_1)} - (w_1 - \theta_2).$$

Termination occurs in this model because the worker’s productivity depends on his match quality. Terminating a bad match therefore has the benefit of increasing the value of the relationship through higher productive efficiency. This contrasts with Harris and Holmstrom (1982) and
Beaudry and DiNardo (1991) in which no termination occurs because the worker’s productivity is the same in all firms. In particular, when the outside option exceeds the period-1 wage, the retention rule is efficient from the production standpoint—the worker is retained if he is a good match and is terminated otherwise. The efficiency follows because by Lemma 1, termination does not affect the worker’s payoff in this case. Therefore, the optimal retention rule simply maximizes the joint output.

When the outside option falls below the period-1 wage, the retention rule is no longer efficient. While the good match is again retained, the bad match is not always terminated. For the bad match, the retention rule compares the benefit of production efficiency with the cost of loss of insurance. On the one hand, terminating a bad match is beneficial from a joint-production standpoint since the worker is more productive with his outside option. On the other hand, terminating the worker is costly because it exposes the worker with the risk of a lower wage. Note that reducing the risk of the worker benefits the firm by allowing it to lower the period-1 wage.

Lemma 2 shows that a bad match is terminated if the magnitude of the match quality, \( m \), is higher than \((u(w_1) - u(\theta_2))/u'(w_1) - (w_1 - \theta_2)\). The concavity of \( u \) implies that this threshold is positive, so a bad match is retained when \( m \) is below the threshold. By keeping a bad match, the retention rule reflects the value created by providing insurance to the risk-averse worker. When the worker is more risk averse, he values insurance more and the threshold is higher. As the worker becomes less risk averse, this threshold decreases and reaches zero as the worker becomes risk neutral. In this case, the worker does not value insurance, and the retention rule is efficient.

Lemma 2 also shows that the optimal retention rule is stochastic when the magnitude of the match quality is equal to the threshold \((u(w_1) - u(\theta_2))/u'(w_1) - (w_1 - \theta_2)\). In this case, the marginal benefit of terminating a bad match is equal to the marginal cost of reducing the insurance provided. Note that the retention threshold depends on period-1 wage \( w_1 \), which is determined endogenously and is affected by the exogenous condition of the economy. In other words, the state of the economy affects jointly the wage offered and the retention rule. Proposition 1 below describes the optimal contract as a function of the state of the economy.

**PROPOSITION 1.** The optimal contract depends on the state of the economy.

\( (a.) \) In a bust state, the retention rule is efficient, i.e.,

\[
h(m_2, \theta_2) = \begin{cases} 
1 & \text{if } m_2 = m \\
0 & \text{if } m_2 = -m 
\end{cases}
\]
and
\[ w_1 = \theta_L. \]

(b.) In a boom state, the good match is always retained, i.e., \( h(m, \theta_L) = 1 \). For the bad match, there exist two cutoffs \( \underline{M} < \overline{M} \) such that

(i) if \( m \leq \underline{M} \), the bad match is retained, i.e.,
\[ h(-m, \theta_L) = 1, \]
and \( w_1 \) satisfies
\[ u(w_1) = \frac{u(\theta_H) + (1 - p)u(\theta_L)}{2 - p}; \]

(ii) if \( m \in (\underline{M}, \overline{M}) \), the bad match is retained with probability
\[ h(-m, \theta_L) = 1 - \frac{u(w_1)(2 - p) - u(\theta_H) - (1 - p)u(\theta_L)}{(1 - p)(1 - \alpha)(u(w_1) - u(\theta_L))}, \]
and \( w_1 \) satisfies
\[ -w_1 + \theta_L + \frac{u(w_1) - u(\theta_L)}{u'(w_1)} = m; \]

(iii) if \( m \geq \overline{M} \), the bad match is terminated, i.e.,
\[ h(-m, \theta_L) = 0, \]
and \( w_1 \) satisfies
\[ u(w_1) = \frac{u(\theta_H) + (1 - p)(1 - p)\alpha u(\theta_L)}{1 + (1 - p)\alpha}. \]

Proposition 1 shows that in a bust state, the retention rule is efficient. Moreover, the worker’s wages in both periods are equal to his outside productivities, so the employment contract offers no insurance to the worker hired in a bust. The reason is that in a bust state, the worker’s outside productivity is at its lowest possible level, so the worker’s period-2 outside productivity cannot decrease further, i.e., the worker’s outside options are downward rigid. Since the main value of the employment contract is to protect the wage from falling downwards, no further insurance will be offered when the outside options already have this property.

In a boom state, in contrast, some amount of insurance will be offered, and consequently, the retention rule is no longer efficient. In particular, the worker’s period-1 wage is strictly below his outside productivity, and in return, he receives partial insurance in period 2. The amount of insurance provided in the optimal contract again reflects the trade-off between avoiding the bad
match and providing insurance discussed at the end of Lemma 2. When match quality is less important \((m \leq M)\), the marginal benefit of offering more insurance—in the form of lowering the probability that the bad match is terminated in a bust—exceeds the marginal cost of keeping it. The optimal contract therefore does not terminate the bad match in a bust. When match quality is more important \((m \geq M)\), the gain from insurance falls short of the loss of keeping the bad match, and the bad match is terminated in a bust. When match quality falls in the intermediate range, the marginal benefit of insurance is equal to the marginal cost of keeping the bad match, so a bad match is retained with some probability.

The key property of the optimal employment contract is that employment relationships starting in the boom state provide more insurance. The reason is simple. The worker’s outside option is higher in a boom state, so there is more scope for insurance. In particular, it is more attractive in a boom state for the worker to accept a lower starting wage (relative to his outside productivity) for future insurance. This lower cost of providing insurance implies that the value of the employment relationship is higher in a boom state, making the parties more likely to start a relationship. The next section uses this observation to explore the effect of business cycles on employment dynamics.

4 Implications of Business Cycles on Turnover and Wages

In this section, we apply the properties of the employment contract derived in the previous section to study its implications on hiring, wages, and subsequent turnover over the business cycle. We assume that match quality is sufficiently important so that the following condition holds:

\[
m > \frac{u(\theta_H) - u(\theta_L)}{u'(\theta_H)} - (\theta_H - \theta_L). \tag{Large Match Quality}
\]

This condition ensures that the optimal contract terminates the bad match in period 2. We make this assumption mostly for analytical simplicity so that the ex post threshold of termination does not depend on the ex ante timing of hiring. The plausibility of the assumption clearly depends on the conditions of the labor market and the types of jobs considered. The assumption is more likely to be satisfied when the skill requirements for the job are complex and the market for such jobs is thin. Match quality is likely to be more important, for example, when the job requires a large combination of different skills.

To study the impact of the business cycle on the hiring threshold, recall that \(\alpha\) is the firm’s signal about the probability of the worker being a good match. It is clear the expected value of the
relationship is increasing in $\alpha$, so the firm makes an offer to the worker if and only if the signal is above a threshold. Denote $\alpha_H$ and $\alpha_L$ as the thresholds for the boom and bust states, respectively. An immediate consequence of Proposition 1 is that the hiring threshold is lower in a boom state.

**Lemma 3.** The marginal worker hired in a boom state has a lower expected match quality than the marginal worker hired in a bust state:

$$\alpha_H < \alpha_L = \frac{1}{3}.$$  

The value of $1/3$ of the hiring threshold in a bust state is specific to the model setup. It results from that the optimal contract in a bust state offers zero insurance. Since no insurance is offered, the hiring decision is entirely based on the value of experimenting with the worker’s match quality. When the worker has a $1/3$ probability of being a good match, his expected output from match-specific productivity—taking into account that he will be terminated if discovered to be a bad match—is exactly zero. In other words, the bust state’s threshold $\alpha_L$ solves $(2\alpha_L - 1)m + \alpha_L m = 0$, where $(2\alpha_L - 1)m$ is the expected match-specific productivity in period 1.

While the value of $1/3$ is specific to the setup, it is a more general property that the hiring threshold in a boom state is lower. The reason for this was alluded to at the end of the previous section—the value of starting an employment relationship is larger in a boom state than in a bust state. The extra value results from the additional insurance the firm can provide to the worker hired in a boom. The value of insurance implies that for a worker whose expected match productivity from the employment relationship (taking into account that the worker will be terminated in period 2 for being a bad match) is zero, there is still gain from the employment relationship. Therefore, the value from the employment relationship can be positive even if the worker’s expected match-specific productivity is negative. This implies that the marginal worker in a boom state has a negative expected match-specific productivity.

Lemma 3 provides one reason for the variations of job-finding rates during the business cycle. Hall (2005) reports that the increase in the unemployment rate during the recession is driven largely by changes in job-finding rates instead of job-separation rates. This model shows that job-finding rates can drop in recessions because firms become "pickier" by raising the bar in terms of match quality. Evidence from the popular press suggests that this mechanism may be at work. A 2011 Wall Street Journal article quotes Paul Marchand, head of talent acquisition at a New York food-and-beverage company, as saying, "People think that with all the available talent, time-to-fill would
go down, but it’s just the opposite. When you’re still trying to find quality candidates, it’s actually taken longer.” Such emphasis on "quality candidates" suggests that companies are looking for better matches during the recession.

The difference in the hiring thresholds implies that the initial labor market condition affects the composition of the workers hired, which directly affects the subsequent patterns of wages and turnover. The next proposition reports a few empirically testable predictions on the impact of business cycles. The empirical literature has distinguished between voluntary and involuntary turnover. To connect our results to the empirical findings, we define turnover as voluntary if the worker’s new wage is strictly higher than his previous wage. Otherwise, turnover is involuntary. There are other ways to distinguish voluntary and involuntary turnover in this model. For example, we can define that a worker leaves involuntarily if \( h = 0 \), and it yields the same qualitative result. The current definition is chosen because it depends on variables that are empirically observable.

**PROPOSITION 2.** The initial state of the economy has persistent effects on wages and turnover.

(a) A worker hired in a boom has a higher period-2 wage if he stays with the same firm. But the initial state has no effect if the worker leaves.

\[
E_{\alpha,\theta_{2}}[w_{2}(m, \theta_{2})|\alpha \geq \alpha_{H}, \theta_{1} = \theta_{H}] > E_{\alpha,\theta_{2}}[w_{2}(m, \theta_{2})|\alpha \geq \alpha_{L}, \theta_{1} = \theta_{L}];
\]

\[
E_{\alpha,\theta_{2}}[w_{2}(-m, \theta_{2})|\alpha \geq \alpha_{H}, \theta_{1} = \theta_{H}] = E_{\alpha,\theta_{2}}[w_{2}(-m, \theta_{2})|\alpha \geq \alpha_{L}, \theta_{1} = \theta_{L}].
\]

(b) A worker hired in a boom is more likely to leave both voluntarily and involuntarily.

\[
E_{\alpha,\theta_{2}}[w_{2}(-m, \theta_{2})] > w_{1}|\alpha \geq \alpha_{H}, \theta_{1} = \theta_{H}] > E_{\alpha,\theta_{2}}[w_{2}(-m, \theta_{2})] > w_{1}|\alpha \geq \alpha_{L}, \theta_{1} = \theta_{L}];
\]

\[
E_{\alpha,\theta_{2}}[w_{2}(-m, \theta_{2})] \leq w_{1}|\alpha \geq \alpha_{H}, \theta_{1} = \theta_{H}] > E_{\alpha,\theta_{2}}[w_{2}(-m, \theta_{2})] \leq w_{1}|\alpha \geq \alpha_{L}, \theta_{1} = \theta_{L}].
\]

Consequently, a worker hired in a bust has a longer average job tenure.

Proposition 2(a) shows that the average period-2 wage of boom hires is higher than that of bust hires. Since the average period-1 wage of boom hires is also higher, this is the cohort effect—a cohort’s future wages are related to the initial wage upon entering the firm. As mentioned in the introduction, many empirical studies have documented the cohort effect and several theories have been put forward to explain it. The mechanism here is most related to Beaudry and DiNardo (1991). The optimal contract determines the initial wage and the future wages jointly, so both are affected by the labor market condition at the signing of the contract.

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Beaudry and DiNardo (1991) also distinguish between the effects of the initial labor market condition and the tightest labor market condition since the start of the employment relationship. They show that the effect on wages from the initial labor market condition disappears once the tightest labor market condition is controlled for. While our model has only two periods, we can obtain the same result by extending it to multiple periods. This is because, in the optimal contract, wages are downward rigid and adjust upwards only when the worker’s outside option exceeds what is offered by the contract. Therefore, the current wage reflects the best outside option the worker ever had, which corresponds to the tightest labor market condition since the start of the employment relationship.

Proposition 2(a) also shows that the cohort effect, however, applies only to workers who stay with the same firm. If a worker is revealed to be a bad match and is terminated, his wage can drop, and the initial labor market condition no longer matters. There is some support for this prediction. Schmieder and von Wachter (2010) show that most of the wage gains through better labor conditions disappear after a job loss. Brunner and Kuhn (2010) find that about half of the wage gains through better labor conditions are lost after an involuntary turnover. They suggest that long-term contracts are important in generating persistent wage differences, but other factors, such as human capital accumulation, are also relevant.

Proposition 2(b) shows that a worker hired in a boom is more likely to experience both voluntary and involuntary turnover. This is because the hiring threshold is lower in a boom state, so a boom hire is more likely to be a bad match. The empirical evidence on involuntary turnover offers support for our prediction. Schmieder and von Wachter (2010) report that "workers with higher wages due to tight past labor market conditions face a higher risk of layoff." The empirical evidence on voluntary turnover, however, is mixed. Brunner and Kuhn (2010) report that for Austrian workers, "one percentage point increase in the initial employment rate decreases the probability of moving to another employer by 7.5%." In contrast, Oreopoulos, Von Wachter and Heisz (Forthcoming) find that, for Canadian college graduates, bust hires are more likely to switch jobs than boom hires. The mixed findings suggest that turnover is affected by other factors. One possibility is vertical mismatch. In particular, in a recession, fewer high-quality jobs are offered, and the new hires may overqualify for the jobs. Vertical mismatch can lead to future turnover and its importance has been studied by a number of papers (Reder 1955, Okun 1973, McLaughlin and Bils 2001, and Oyer 2006, 2008). Another related reason is the composition of the jobs offered. If high-turnover firms hire more in a recession, this leads to a higher mobility rate for the bust hires. Kahn (2008),
discussed below, shows that the composition of jobs is important for understanding the effect of business cycles on turnover patterns.

Finally, Proposition 2(b) shows that bust hires have longer job tenure, which follows immediately because they are less likely to turnover both voluntarily and involuntarily. This prediction is not supported by earlier evidence. Bowlus (1995) finds a shorter employment duration for recession hires. However, Kahn (2008) points out that this shorter employment duration is driven mostly by the composition of jobs. She finds that while jobs started during a recession do on average end earlier, once firm heterogeneity is controlled for, this pattern reverses. With firm fixed effect controlled for, she shows that a one percentage increase in the national unemployment rate increases the probability that a worker stays for at least one year by five percentage points. Kahn (2008) explicitly points to match-specific productivity as the source for her findings, and this model provides a reason for why a worker hired in a recession is more likely to be a good match.

5 Discussion and Extension

In this section, we discuss how the model’s results change when the firm cannot offer long-term contracts. We also show that the main features of the optimal employment contract remain the same when productivity levels and match qualities are continuous.

5.1 No Commitment

Suppose the firm cannot offer long-term contracts, so the hiring and wage decisions are determined by spot-market transactions. The Result below describes the worker’s wage and turnover patterns in this case.

RESULT. Suppose no long-term contracts can be offered.

(a.) The worker’s wages are equal to his outside option:

\[ w_1 = \theta_1; \]
\[ w_2(m_2, \theta_2) = \theta_2. \]

(b.) The retention rule is efficient:

\[ h(m_2, \theta_2) = \begin{cases} 
1 & \text{if } m_2 = m \\
0 & \text{if } m_2 = -m.
\end{cases} \]
(c.) The hiring rule is efficient in both the boom and bust states:

\[ \alpha_H = \alpha_L = \frac{1}{3}. \]

The above results illustrate the role of long-term contracts (or lack thereof). When long-term contracts are absent, no insurance is offered even if the worker is risk averse. The optimal employment contract therefore maximizes joint output. It follows that both the hiring and retention decisions are efficient from the production standpoint. In terms of hiring, the value of starting the relationship—the difference between the joint payoff in the relationship and the joint outside options—is the same in the boom state as in the bust state. This implies, in particular, that the hiring threshold of match quality is the same in boom and bust states. The threshold level of 1/3 again depends on the specific setup of the model, but the result of efficient hiring is more general. Since both the boom and bust states have the same efficient threshold, the average match quality of the workers hired is also the same. Consequently, initial labor market conditions do not affect future wages and turnover when long-term contracts are absent.

5.2 Continuous Match Quality and Productivity

Next, we relax the assumption that productivity levels and match qualities are both binary. In particular, let the general productivity level \( \theta \in [\underline{\theta}, \overline{\theta}] \), and the match quality \( m \in [\underline{m}, \overline{m}] \). Assume that both have positive density functions. Proposition 3 below shows that the main results of the model are robust to this extension.

**PROPOSITION 3.** The optimal contract has the following features.

(a.) For a worker retained by the firm, the wage is downward rigid:

\[
w(m_2, \theta_2) = \begin{cases} w_1 & \text{if } \theta_2 < w_1 \\ \theta_2 & \text{otherwise} \end{cases}.
\]

(b.) Turnover is efficient if the period-2 outside productivity exceeds the period-1 wage. If \( \theta_2 \geq w_1 \),

\[
h(m_2, \theta_2) = \begin{cases} 1 & m_2 \geq 0 \\ 0 & m_2 < 0 \end{cases}.
\]

(c.) If the period-2 outside productivity falls below the period-1 wage, the worker is terminated if and only if the match quality falls below a negative threshold. If \( \theta_2 < w_1 \),

\[
h(m_2, \theta_2) = \begin{cases} 1 & m_2 \geq w_1 - \theta_2 - \frac{u'(w_1) - u(\theta_2)}{u'(w_1)} \\ 0 & m_2 < w_1 - \theta_2 - \frac{u'(w_1) - u(\theta_2)}{u'(w_1)} \end{cases}.
\]
(d.) The period-1 wage increases with the general productivity level and is equal to the outside productivity at the lowest general productivity level.

\[ \frac{\partial w_1(m_1, \theta_1)}{\partial \theta_1} > 0; \]
\[ w_1(m_1, \theta) = \theta. \]

(e.) The hiring threshold decreases with the general productivity level. Let \( m_1(\theta_1) \) be the hiring threshold of match quality. Then

\[ \frac{dm_1(\theta_1)}{d\theta_1} < 0; \]
\[ m_1(\theta) = \frac{1}{3}. \]

Parts (a)-(c) show that the basic features of the optimal employment contract remain unchanged. In particular, wages are downward rigid within the employment relationship, and turnover occurs if the worker’s match quality falls below a threshold. Parts (d) and (e) imply that the average match quality of the hired workers decreases with the general productivity level, and consequently, the initial labor market conditions affect future wages and turnover. Parts (d) and (e) also show that when general productivity is at its lowest level, the optimal contract offers no insurance, and the hiring and retention rules are again efficient, just as in the bust-state contract in the main model.

These results are similar to those in the main section because they follow from the same mechanism. The optimal contract creates value by offering insurance to the worker. When the worker’s outside option is higher, there is greater room for insurance, and as a result, the firm lowers the hiring threshold of match quality. This implies that the initial labor market condition affects the average match qualities of the workers hired, and this results in long-run differences in wages and turnover patterns.

6 Conclusion

This paper develops a model of turnover and wage dynamics. The main ingredients of the model are insurance, match-specific productivity, and long-term contracting. We characterize the optimal employment contract and show that wages are downward rigid within a firm and termination occurs. We apply the model to study the business cycle’s impact on future wages and job mobility. We show that boom hires are on average worse matches and are therefore more likely to leave even if those who remain have higher wages. These predictions shed light on a number of empirical
findings about the persistent effects of initial labor market conditions on a worker’s subsequent labor market outcomes.

In this paper, we choose a simple setup to illustrate the key mechanism—better state of the economy increases the value of the relationship by enhancing the scope for insurance, resulting in firms being more willing to hire worse matches in booms. The simple setup means that many relevant factors that affect wages and turnover are not included. For example, we did not consider match quality at a vertical level. Vertical mismatches can be more severe in a recession when overqualified workers settle for lower-paying jobs, leading to higher future turnover rates. Understanding the interaction between horizontal and vertical matching will be key in assessing the implications of business cycles on labor markets. It will also be useful for policymakers who help facilitate better matching between firms and workers. More research on this topic is needed.
7 Appendix

In this section, we prove the results in the main text. We first set up and analyze the firm’s maximization problem and then use the property of its solution to derive the rest of the results.

The firm chooses the employment contract \((w_1, h(m_2, \theta_2), w(m_2, \theta_2))\) to solve the following constrained maximization problem:

\[
\max_{w_1, h(m_2, \theta_2), w(m_2, \theta_2)} (2\alpha - 1)m + \theta_1 - w_1 + \sum_{\theta_2, m_2} \Pr(\Theta = \theta_2) \Pr(M = m_2) h(m_2, \theta_2) (m_2 + \theta_2 - w(m_2, \theta_2))
\]

subject to

\[
\sum_{\theta_2, m_2} \Pr(\Theta = \theta_2) \Pr(M = m_2) h(m_2, \theta_2) (u(w(m_2, \theta_2)) - u(\theta_2)) \\
\geq u(\theta_1) - u(w_1);
\]

\[
0 \leq h(m_2, \theta_2) \leq 1;
\]

\[
w(m_2, \theta_2) \geq \theta_2 \quad \text{if} \quad h(m_2, \theta_2) > 0.
\]

In the firm’s objective function, \((2\alpha - 1)m + \theta_1 - w_1\) is the firm’s expected profit in period 1. And \(m_2 + \theta_2 - w(m_2, \theta_2)\) is the firm’s profit in period 2 when the state of the economy is \(\theta_2\), the worker’s match quality is \(m_2\), and the worker is kept. In the IR constraint, the left hand side is the worker’s benefit in period 2 from the accepting the firm’s employment contract and the right hand side is his cost in period 1. Note that the IR constraint is binding in the optimal contract because if it were slack, the firm can increase its payoff by lowering the worker’s period-1 wage. The last constraint corresponds to the simplifying assumption that if the worker is not laid off, his period-2 wage is weakly higher than his outside option. Note that since \(h(m_2, \theta_2) \geq 0\), we can rewrite the last constraint as

\[h(m_2, \theta_2)(w(m_2, \theta_2) - \theta_2) \geq 0.\]
To solve the constrained maximization problem, set the Lagrangian as

\[
L = (2\alpha - 1)m + \theta_1 - w_1 + \sum_{\theta_2, m_2} \Pr(\Theta = \theta_2) \Pr(M = m_2)h(m_2, \theta_2)(m_2 + \theta_2 - w(m_2, \theta_2)) + \lambda(u(w_1) + \sum_{\theta_2, m_2} \Pr(\Theta = \theta_2) \Pr(M = m_2)h(m_2, \theta_2)(u(w(m_2, \theta_2)) - u(\theta_2)) - u(\theta_1)) + \sum_{\theta_2, m_2} \Pr(\Theta = \theta_2) \Pr(M = m_2)(\mu_+(m_2, \theta_2)h(m_2, \theta_2) + \mu_-(m_2, \theta_2)(1 - h(m_2, \theta_2))) + \sum_{\theta_2, m_2} \Pr(\Theta = \theta_2) \Pr(M = m_2)\rho(m_2, \theta_2)h(m_2, \theta_2)(w(m_2, \theta_2) - \theta_2).
\]

In the Lagrangian above, \(\lambda\) is the multiplier associated with the IR constraint. It reflects the shadow value to the firm when the worker’s outside option is lowered. We can now prove the main results of the paper by deriving the properties of the Lagrangian.

**Proof of Lemma 1:** From the Lagrangian, the FOC with respect to \(w_1\) gives that

\[
u'(w_1) = \frac{1}{\lambda}.
\]

Since the worker is retained, \(h(m_2, \theta_2) > 0\). Then the FOC with respect to \(w(m_2, \theta_2)\) gives that

\[
-1 + \lambda u'(w(m_2, \theta_2)) + \rho(m_2, \theta_2)u'(w(m_2, \theta_2)) = 0.
\]

This implies

\[
u'(w(m_2, \theta_2)) = \frac{1}{\lambda + \rho(m_2, \theta_2)} \leq \nu'(w_1),
\]

so

\[
w(m_2, \theta_2) \geq w_1.
\]

Now the complementarity slackness condition of the last constraint in the Lagrangian gives that (again noting \(h(m_2, \theta_2) > 0\))

\[
\rho(m_2, \theta_2)(w(m_2, \theta_2) - \theta_2) = 0.
\]

Now consider two cases. First, \(\theta_2 < w_1\). In this case,

\[
w(m_2, \theta_2) - \theta_2 > 0.
\]

since \(w(m_2, \theta_2) \geq w_1\) by above. This implies that \(\rho(m_2, \theta_2) = 0\). The strict concavity of \(u\) then gives that \(w(m_2, \theta_2) = w_1\) since

\[
u'(w_1) = \frac{1}{\lambda} = \frac{1}{\lambda + \rho(m_2, \theta_2)} = u'(w(m_2, \theta_2)).
\]
In the second case, \( \theta_2 \geq w_1 \). Recall that by assumption \( w(m_2, \theta_2) \geq \theta_2 \). Now suppose to the contrary that \( w(m_2, \theta_2) > \theta_2 \). This then implies that \( u'(w(m_2, \theta_2)) < u'(\theta_2) \) so \( \rho(m_2, \theta_2) > 0 \). But the complementarity condition of the last constraint in the Lagrangian then implies \( w(m_2, \theta_2) = \theta_2 \), which is a contradiction. This proves that \( w(m_2, \theta_2) = \theta_2 \) when \( \theta_2 \geq w_1 \).

**Proof of Lemma 2:** From the Lagrangian, the FOC with respect to \( h(m_2, \theta_2) \) gives

\[
m_2 + \theta_2 - w(m_2, \theta_2) + \lambda(u(w(m_2, \theta_2)) - u(\theta_2)) + \mu_+(m_2, \theta_2) - \mu_-(m_2, \theta_2) + \rho(m_2, \theta_2)(u(w(m_2, \theta_2)) - u(\theta_2)) = 0.
\]

Now consider two cases. First, \( \theta_2 \geq w_1 \). In this case, Lemma 1 implies \( w_2(m_2, \theta_2) = \theta_2 \). Substituting this into FOC-h, we get

\[
m_2 + \mu_+(m_2, \theta_2) - \mu_-(m_2, \theta_2) = 0.
\]

This implies that

\[
h(m_2, \theta_2) = 1.
\]

if and only if \( m_2 > 0 \). This proves (a.) part.

In the second case, \( \theta_2 < w_1 \). In this case, Lemma 1 implies \( w(m_2, \theta_2) = w_1 \). Substituting this into FOC-h, we have

\[
m_2 + \theta_2 - w_1 + \lambda(u(w_1) - u(\theta_2)) + \mu_+(m_2, \theta_2) - \mu_-(m_2, \theta_2) + \rho(m_2, \theta_2)(u(w_1) - u(\theta_2)) = 0.
\]

Now if

\[
m_2 + \theta_2 - w_1 + \lambda(u(w_1) - u(\theta_2)) > 0,
\]

then \( \mu_-(m_2, \theta_2) > 0 \) (since \( \rho(m_2, \theta_2)(u(w_1) - u(\theta_2)) \geq 0 \)). This implies that \( h(m_2, \theta_2) = 1 \).

Similarly, if

\[
m_2 + \theta_2 - w_1 + \lambda(u(w_1) - u(\theta_2)) < 0,
\]

then \( h(m_2, \theta_2) = 0 \). To see this, suppose to the contrary that \( h(m_2, \theta_2) > 0 \). Then complementarity slackness condition of the last constraint in the Lagrangian implies that \( \rho(m_2, \theta_2) = 0 \) since \( u(w_1) - u(\theta_2) > 0 \). This allows us to rewrite the FOC as

\[
m_2 + \theta_2 - w_1 + \lambda(u(w_1) - u(\theta_2)) + \mu_+(m_2, \theta_2) - \mu_-(m_2, \theta_2) = 0.
\]

It follows that if \( m_2 - w_1 + \lambda(u(w_1) - u(\theta_2)) < 0 \), we must have \( \mu_+(m_2, \theta_2) > 0 \), contradicting \( h(m_2, \theta_2) > 0 \). It follows that we must have \( h(m_2, \theta_2) = 0 \) in this case.
Finally, by substituting $\frac{1}{u'(w_{1})}$, we obtain the conditions in the (b.) part.

**Proof of Proposition 1:** For part (a.), note that we can rewrite the worker’s binding IR condition as

$$u(w_1) + \sum_{\theta_2,m_2} \Pr(\Theta = \theta_2) \Pr(M = m_2) h(m_2, \theta_2)(u(m_2, \theta_2)) - u(\theta_2)) = u(\theta_L).$$

Since $u(w(m_2, \theta_2)) - u(\theta_2) \geq 0$ for all $(m_2, \theta_2)$ by Lemma 1, the above then implies

$$w_1 \leq \theta_L.$$ 

Now if $w_1 < \theta_L$, then $\theta_2 > w_1$ for all $\theta_2$, and this implies that $u(w(m_2, \theta_2)) - u(\theta_2) = 0$ for all $(m_2, \theta_2)$. But then this contradicts the IR constraint above. Therefore, we must have $w_1 = \theta_L$. The efficient retention rule then follows from Lemma 2.

For part (b.), $h(m, \theta_2) = 1$ follows directly from Lemma 2. Now consider three cases. First, if the bad match is retained, then the binding IR constraint of the worker gives

$$u(w_1) = \frac{u(\theta_H) + (1-p)u(\theta_L)}{2-p}.$$ 

For the contract to be optimal, Lemma 2 then implies that we must have

$$m \leq \frac{u(w_1) - u(\theta_L)}{u'(w_1)} - (w_1 - \theta_L).$$

Now define

$$M = \frac{u(w_1^\dagger) - u(\theta_L)}{u'(w_1^\dagger)} - (w_1^\dagger - \theta_L),$$

where $w_1^\dagger$ is given by $u(w_1^\dagger) = (u(\theta_H) + (1-p)u(\theta_L))/(2-p)$. In this case, it can be checked that as long as $m \leq M$, the wage and the retention rule in (i) of part (b.) satisfy the Kuhn-Tucker conditions associated with the firm’s maximization problem. Since the firm’s maximization problem is concave (and the domain of the choice variable is essentially bounded), the wage and retention rule is optimal. This proves (i).

Second, if the bad match is terminated, then the binding IR constraint of the worker gives

$$u(w_1) = \frac{u(\theta_H) + (1-p)\alpha u(\theta_L)}{1 + (1-p)\alpha}.$$ 

For the contract to be optimal, Lemma 2 then implies that we must have

$$m \geq \frac{u(w_1) - u(\theta_L)}{u'(w_1)} - (w_1 - \theta_L).$$ 

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Define 
\[ \overline{M} = \frac{u(w_1^U) - u(\theta_L)}{u'(w_1^U)} - (w_1^U - \theta_L), \]
where \( w_1^U \) is given by 
\[ u(w_1^U) = (u(\theta_H) + (1 - p)u(\theta_L))/(1 + (1 - p)\alpha). \]
Just as in the first case, it can be checked that if \( m \geq \overline{M} \), the wage and retention rule in (iii) of part (b.) satisfy the Kuhn-Tucker conditions. This proves (iii). Compared to (i), note that \( \overline{M} \) depends on \( \alpha \) but \( \overline{M} \) is independent of \( \alpha \).

Finally, when \( m \in (\underline{M}, \overline{M}) \), one can check that the choice of \( w_1 \) and \( h(-m, \theta_L) \) in (ii) make the worker’s IR binding, and the Kuhn-Tucker conditions are all satisfied. This proves (ii).

Proof of Lemma 3: \( \alpha_L = 1/3 \) is immediate. To see \( \alpha_H < \alpha_L \), note that the firm’s payoff is continuous and increasing in \( \alpha \). In addition, for a worker with signal \( \alpha_L \), the firm breaks even by paying this worker \( w_1 = \theta_H \) and \( w(m_2, \theta_2) = \theta_2 \) (and by using the same turnover rule). But this contract is not optimal. By reducing \( w_1 \) by \( \varepsilon \) and adjust \( w(m_2, \theta_L) \) accordingly so that the IR again binds, the firm’s profit increases. This implies that the firm’s profit from hiring a worker with signal \( \alpha_L \) is strictly positive, and therefore the cutoff \( \alpha_H \) must be lower than \( \alpha_L \).

Proof of Proposition 2: Straightforward calculations from Proposition 1 and Lemma 3.

Proof of Result: In \( t = 2 \), we clearly have that \( w_2 = \theta_2 \). In this case, no insurance is offered so the retention rule is efficient. In period 1, for a worker with signal \( \alpha \), suppose the firm chooses \( w_1 \). In this case, the firm chooses his first period wage \( w_1 \) to maximize 
\[ (2\alpha - 1)m + \theta_1 - w_1 + \alpha m, \]
such that 
\[ u(w_1) + E[u(\theta_2)] \geq u(\theta_1) + E[u(\theta_2)]. \]

It is clear that the inequality will be binding, and thus, we get 
\[ w_1 = \theta_1, \]
and the firm’s profit becomes 
\[ (3\alpha - 1)m. \]
It follows that the firm will hire the worker if and only if \( \alpha \geq \frac{1}{3} \), and the hiring is independent of the aggregate economic condition.
**Proof of Proposition 3:** Denote $P$ as the CDF of $\theta$ and $Q(m_2|m_1)$ as the CDF of the match quality in period 2 conditional on the match quality being $m_1$ in period 1. Now if the firm hires a worker of expected match quality $m_1$ in period 1, the firm chooses its the employment contract that solves the following program:

$$
\max_{w_1, h(m_2, \theta_2)\in \mathcal{H}} m_1 + \theta_1 - w_1 + \int_{\theta_2, m_2} h(m_2, \theta_2)(m_2 + \theta_2 - w(m_2, \theta_2))dP(\theta_2)dQ(m_2|m_1)
$$

subject to

$$
u(w_1) + \int_{\theta_2, m_2} h(m_2, \theta_2)(u(w(m_2, \theta_2)) - u(\theta_2))dP(\theta_2)dQ(m_2|m_1) \geq u(\theta_1).
$$

$$0 \leq h(m_2, \theta_2) \leq 1.
$$

$$h(m_2, \theta_2)(u(w(m_2, \theta_2)) - u(\theta_2)) \geq 0.
$$

Just as in the main model, we can set up a corresponding Lagrangian, and results in Proposition 3 then follow from exactly the same line as in the main model and we omit the proofs here.■

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References


