Players with private information can take some action to “signal” their type.
- Taking this action would distinguish them from other types.

Privately informed agents credibly convey information about themselves to another party.

- Studied signaling in the labor market.
- Workers can signal their type by obtaining education.
- In equilibrium: high ability workers get more education, so employers can learn the agent’s type by observing their education level.
- Signal is credible: only high ability workers will be willing to get more years of education.

Setup
- Two types of workers: $\theta_H > \theta_L$ (high ability and low ability)
  - A worker of type $\theta$ produces output which is worth $\theta$ to the employer.

There is a fraction $\lambda \in (0, 1)$ of low type-workers in the market.

- A worker can obtain education level $e$, which is perfectly observable.
  - Assume that $e$ does not affect the worker’s productivity $\theta$.

- Cost of getting education $e \geq 0$ for a type $\theta$ worker is $c(e, \theta)$.

- We assume that:
  - $c(0, \theta) = 0$, $c_e(e, \theta) > 0$, $c_{ee}(e, \theta) \geq 0$ for $\theta \in \{\theta_L, \theta_H\}$. 

\[ c_e(e, \theta_H) < c_e(e, \theta_L) \text{ for all } e. \]

- Second condition implies that getting more education is more costly for low ability workers than for high ability workers.

- Goal: Can high types credibly signal to employers their quality by getting more education than low types.

- If a type \( \theta \) worker gets a wage \( w \) and education \( e \), her utility is

\[ w - c(e, \theta). \]

- Assume that

  1. the outside option of both types of workers is zero; and
  2. there is perfect competition among employers (and so they earn zero profits in expectation).

Benchmark

- Suppose first that workers cannot get education.

  - In this case there is no signaling.

- Equilibrium in this case:

\[ w = \mathbb{E}[\theta] = \lambda \theta_L + (1 - \lambda) \theta_H \]

  - All workers accept this wage (recall that outside option is 0).
  - Firms earn zero profits.
  - High types get a low wage, and low types get a high wage (relative to their productivity).

Education

- Suppose now that workers can get education to signal their type.

- We consider the following game:

  - The worker first learns her type. (Worker is type \( \theta_H \) w.p \( 1 - \lambda \) and type \( \theta_L \) w.p \( \lambda \).)
− After learning her type, worker chooses education level \( e \geq 0 \).
− All firms then observe \( e \) but not \( \theta \). Then they make wage offers to the worker.
− Worker accepts (at most) one offer.

○ In equilibrium, a worker who chooses \( e \) cannot gain by choosing a different \( e' \neq e \).

○ Let \( \mu (e) = \Pr (\theta = \theta_H | e) \).
    − \( \mu (e) \) is the belief of the firms when they observe a worker with education level \( e \).
    − These beliefs are derived from the workers’ strategies.

○ Let \( w(e) \) be the wage that firms offer when they see a worker with education \( e \).
    − Zero profit condition implies that \( w(e) = \mu (e) \theta_H + [1 - \mu (e)] \theta_L \).
    − Wages depend solely on beliefs.
    − If \( \mu (e) \) is not constant in \( e \), then workers with different education level will get different wages.

○ Let \( e(\theta) \) be the education level chosen by a worker with type \( \theta \).

○ Two types of equilibria:
    − Separating equilibria: \( e(\theta_H) \neq e(\theta_L) \).
    − Pooling equilibria: \( e(\theta_H) = e(\theta_L) \).

○ We will focus mainly on separating equilibria (but we will discuss pooling equilibria at the end).

**Separating equilibria**

○ Recall that \( e(\theta_H) \neq e(\theta_L) \) and \( \mu (e) = \Pr (\theta = \theta_H | e) \).

○ In a separating equilibrium, \( \mu (e(\theta_H)) = 1 \) and \( \mu (e(\theta_L)) = 0 \).
    − If firms observe \( e(\theta_i) \), they know that worker has type \( \theta_i \).
    − Thus, workers signal their types through their education level.

○ This implies that \( w(e(\theta_H)) = \theta_H \) and \( w(e(\theta_L)) = \theta_L \).
This follows since \( w(e) = \mu(e) \theta_H + [1 - \mu(e)] \theta_L \).

○ We shall show that \( e(\theta_L) = 0 \).
  
  - Towards a contradiction, suppose that \( e(\theta_L) > 0 \).
  
  - If a type \( \theta_L \) worker deviates and chooses \( e = 0 \), she gets wage
    \[
    w(0) = \mu(0) \theta_H + [1 - \mu(e)] \theta_L \geq \theta_L
    \]
    
  - The utility she gets from doing so is
    \[
    w(0) - c(0, \theta_L) \geq \theta_L - c(0, \theta_L) > \theta_L - c(e(\theta_L), \theta_L),
    \]
    so this worker strictly prefers to choose \( e = 0 \) than to choose \( e = e(\theta_L) > 0 \).
  
  ○ Therefore, in a separating equilibrium, it must be that \( e(\theta_L) = 0 \).

○ What about \( e(\theta_H) \)?
  
  ○ This education level has to satisfy two constraints:
    
    \[
    \begin{align*}
    \theta_H - c(e(\theta_H), \theta_L) &\leq \theta_L - c(0, \theta_L) = \theta_L \\
    \theta_H - c(e(\theta_H), \theta_H) &\geq \theta_L - c(0, \theta_H) = \theta_L
    \end{align*}
    \]
    
  - First constraint guarantees that a worker with type \( \theta_H \) prefers to get education level \( e(\theta_H) \) than \( e(\theta_L) \).

  - Second constraint guarantees that a worker with type \( \theta_L \) prefers to get education level \( e(\theta_L) \) than \( e(\theta_H) \).
    
    \[
    \implies c(e(\theta_H), \theta_L) \geq \theta_H - \theta_L \geq c(e(\theta_H), \theta_H)
    \]
    
  - Recall that \( c(e, \theta_L) > c(e, \theta_L) \) for all \( e \).

○ There is a range \([\underline{e}, \bar{e}]\) of values of \( e(\theta_H) \) that satisfy these two inequalities:
  
  - \( \underline{e} \) is such that \( c(\underline{e}, \theta_L) = \theta_H - \theta_L \).
– $\bar{e}$ is such that $c(\bar{e}, \theta_H) = \theta_H - \theta_L$.

○ Any $e(\theta_H) \in [\underline{e}, \bar{e}]$ can be supported as an equilibrium.

○ One thing left: what values does $\mu(e)$ take for $e \neq \{e(\theta_H), e(\theta_L)\}$?
  
  – We cannot derive these beliefs from the workers actions.
  
  – If employers observe $e \neq \{e(\theta_H), e(\theta_L)\}$, what should they believe?

○ Recall that $w(e) = \mu(e) \theta_H + [1 - \mu(e)] \theta_L$.

○ For this to be an equilibrium, both types of workers must prefer to take their equilibrium actions:
  
  – type $\theta_H$ must prefer to get education $e(\theta_H)$ than any $e \neq e(\theta_H)$.
  
  – type $\theta_L$ must prefer to get education $e(\theta_L)$ than any $e \neq e(\theta_L)$.

○ We specify $\mu(e)$ so that workers don’t want to deviate:
  
  – Different ways in which we can do this.
  
  – Idea: “punish” workers with $e \neq \{e(\theta_H), e(\theta_L)\}$ by making $\mu(e)$ small.

### Pooling Equilibria

○ In a pooling equilibrium, $e(\theta_H) = e(\theta_L) = e^*$.
  
  – All workers get the same education level.

○ Zero profits by firms imply that $w(e^*) = (1 - \lambda) \theta_H + \lambda \theta_L$.
  
  – This implies that $\mu(e^*) = 1 - \lambda$.

○ In an equilibrium, no type of worker must benefit from choosing $e \neq e^*$.
  
  – Need to specify $\mu(e)$ so that neither high types nor low types have a profitable deviation.

### References
