Module 12: Holdup Problem

Information Economics (Ec 515)  ·  George Georgiadis

Standard Holdup Problem

- Canonical model by Hart and Moore (1988)
- 2 contracting parties: A buyer and seller can trade a quantity \( q \in [0, 1] \) at a price \( P \).
- Buyer’s valuation \( v \) and seller’s production cost \( c \) are uncertain when contracting takes place and can be influenced by investments

\[
\begin{align*}
\text{Buyer: } & v \in \{v_L, v_H\} \text{ and } \Pr(v_H) = j \text{ at cost } \psi(j) \\
\text{Seller: } & c \in \{c_L, c_H\} \text{ and } \Pr(c_L) = i \text{ at cost } \phi(i)
\end{align*}
\]

- For example:
  - Buyer invests in marketing to increase price that he can sell the good at.
  - Seller invests in modern infrastructure to reduce production cost.

- Ex-post payoff levels are

\[
\begin{align*}
\text{Buyer: } & vq - P - \psi(j) \\
\text{Seller: } & P - cq - \phi(i)
\end{align*}
\]

- **Timing:**

1. The buyer and the seller contract.
   - Contract specifies quantity \( q \) to be traded at price \( P \).
2. Each party simultaneously chooses his investment level \( i \) and \( j \).
3. Both parties learn state of nature \( \theta = (v, c) \).
4. The contract is executed (possibly after renegotiation).
First Best

- Assume that

\[ c_H > v_H > c_L > v_L \]

- Ex-post efficient level of trade is

\[ q = 1 \text{ if } \theta = (v_H, c_L) \]
\[ q = 0 \text{ otherwise} \]

- The total expected surplus is given by

\[ \max_{i,j} \{ ij (v_H - c_L) - \psi (j) - \phi (i) \} \]

so the first best investment levels satisfy

\[ i^{fb} (v_H - c_L) = \psi' (j^{fb}) \]
\[ j^{fb} (v_H - c_L) = \phi' (i^{fb}) \]

Nash Equilibrium

- \( \theta \) is observable to both parties ex-post, but it is not contractable ex-ante, nor are the investment levels \( i \) and \( j \).

- Assume that ex-post bargaining gives each party half of the surplus.

- The buyer solves

\[ \max_j \left\{ \frac{1}{2} i^* j (v_H - c_L) - \psi (j) \right\} \]

while the seller solves

\[ \max_i \left\{ \frac{1}{2} i^* j^* (v_H - c_L) - \phi (i) \right\} \]

- So in equilibrium, they choose

\[ \frac{1}{2} i^* (v_H - c_L) = \psi' (j^*) \quad \text{and} \quad \frac{1}{2} j^* (v_H - c_L) = \phi' (j^*) \]

- Clearly, \( i^* < i^{fb} \) and \( j^* < j^{fb} \), due to “moral-hazard-in-teams”.

- Solutions?

  - Can we formulate an optimal long-term contract independent of \( \theta \) that mitigates underinvestment?
Default Options

○ Define level of trade $\tilde{q}$ such that

$$\tilde{q} (c_H - c_L) = \phi' (i^{fb})$$

○ Consider the following contractual mechanism (after the state of nature $\theta$ is revealed):

1. Buyer makes a take-it-or-leave-it offer $(P, q)$.
2. Seller accepts $(P, q)$, or rejects it, in which case $\tilde{q}$ is traded at price $\tilde{P}$.
   - $\tilde{P}$ chosen to share the ex-ante surplus according to bargaining weights.

○ We will show that this mechanism implements first best!

○ Buyer will offer $(P, q)$ such that seller is indifferent between accepting the offer and rejecting it.

○ Seller always expects to obtain the default option payoff so he solves

$$\max_i \left\{ \tilde{P} - ic_L\tilde{q} - (1 - i) c_H\tilde{q} - \phi (i) \right\}$$

First order condition: $\tilde{q} (c_H - c_L) = \phi' (i)$, so that $i = i^{fb}$.

○ Buyer maximizes

$$\max_j \left\{ \frac{i^{fb}}{j} (v_H - c_L) - \left[ \tilde{P} - i^{fb} c_L\tilde{q} - (1 - i^{fb}) c_H\tilde{q} \right] - \psi (j) \right\}$$

First order condition: $i^{fb} (v_H - c_L) = \psi' (j)$, so that $j = j^{fb}$.

○ Lesson: By choosing $\tilde{q}$ appropriately, it is possible to induce $(i^{fb}, j^{fb})$.

○ Comments:
– Investment efficiency for the buyer since he’s the residual claimant ...
– ... but why is there investment efficiency for the seller who has no bargaining power at all?
– Incentive to invest comes from availability of default option, which becomes more attractive when cost is $c_L$ and this can be influenced through $i$.

References

