Module 10: Relational Contracts

Information Economics (Ec 515) · George Georgiadis

○ Insofar, output has been assumed to be
  – observable and verifiable; i.e., it can be enforced by a court of law;
  – or not observable at all (by the agent).

○ Both are extreme assumptions!
  – Performance evaluation is often based on “soft information”.

○ Suppose that output is observable to both parties, but not verifiable.

○ Parties can use repeated interaction to ensure that the firm pays for good performance.

A Simple Model of Discretionary Bonuses

○ Time $t \in \mathbb{N}$.

○ Output is binary: $x_t \in \{H, L\}$.

○ In each period $t$, agent chooses effort $a_t = \Pr \{x_t = H\} \in [0, 1]$.
  – Cost of effort $c \frac{a_t^2}{2}$, where $c < H - L$.

○ Compensation consists of base salary $s$ and bonus $b$ that principal “promises” to pay if $x_t = H$.
  – Both parties observe $x_t$, but it cannot be contracted on, so the principal can renege on her promise to pay $b$.

○ Both parties are risk neutral and discount rate is $\delta \leq 1$.
  – The larger $\delta$ is, the more the parties “care” about the future.
Timing: In each period $t$

1. Principal offers a compensation package $\{s, b\}$.
   - $\{s, b\}$ is chosen to maximize her expected discounted profit.

2. Agent accepts or rejects it in favor of alternative employment $\bar{U} = L$.

3. If the agent accepts, he exerts effort $a_t \in [0, 1]$ at cost $c \frac{a_t^2}{2}$.
   - The principal does not observe the agent’s effort choice.

4. Both parties observe $x_t$.

5. If $x_t = H$, the principal chooses whether to pay the agent bonus $b$.

First best outcome:

- Total surplus:
  $$S(a_t) = (1 - a_t) L + a_t H - \underbrace{\frac{c a_t^2}{2}}_{\text{effort cost}}$$

- Principal chooses effort: $a_t \in \max \left\{ L + a_t (H - L) - c \frac{a_t^2}{2} \right\}$
  - First order condition: $(H - L) - ca > 0$ for all $a \in [0, 1] \implies a^{fb} = 1$.
  - Pays salary $s = L$ and $b = 0$.

One-shot Game (no repetition):

- Principal will renege on promise to pay bonus $b$.
- Agent will choose effort $a = 0$ and produce output $x_t = L$.
- Principal will pay salary $s = L$, earning 0 profit.

Repeated Game

- The equilibrium of the one-shot game is also an equilibrium of the repeated game.
  - Can we construct another equilibrium where agent chooses higher effort?
- WLOG we can assume that the agent uses a grim-trigger strategy.
This is the worst possible penalty here (Abreu, Pearce and Stachetti, JET, 1984).

If the principal reneges on promise, then the agent chooses $a = 0$ forever after (giving the principal profit 0).

- Given contract $\{s, b\}$, if agent believes that principal will pay bonus:

$$ a_t \in \arg \max_a \left\{ s + ab - c \frac{a^2}{2} \right\} $$

- First order condition: $a^*(b) = \frac{b}{c}$ (for $b \leq c$).

- Agent will accept contract if $s + a^*(b) b - c \frac{(a^*(b))^2}{2} \geq L$.

- Principal will offer minimum salary that agent will accept: $s + a^*(b) b = L - \frac{b^2}{2c}$

- Then the principal’s expected profit (per period) is

$$ V(b) = \underbrace{L + a^*(b) (H - L)}_{\text{net profit}} - \underbrace{[s + a^*(b) b]}_{\text{payroll cost}} = \underbrace{b (H - L)}_{c} - \frac{b^2}{2c} $$

- Will the principal choose to pay the bonus if $x_t = H$? Yes if

$$ (H - s - b) + \delta V(b) \geq (H - s) + \delta 0 $$

$$ \iff b \leq \delta V(b) $$

$$ \iff b \leq 2 \left( H - L - \frac{c}{\delta} \right) $$

- The cost of paying the bonus is $\$b$.
- The cost of not paying the bonus is $\$ \delta V(b)$.

- Principal solves

$$ \max_b \underbrace{\frac{b (H - L)}{c}}_{\text{net profit}} - \frac{b^2}{2c} $$

s.t. $b \leq \underbrace{2 \left( H - L - \frac{c}{\delta} \right)}_{ \text{payroll cost}}$

- First order condition: $\frac{(H-L)}{c} \cdot b > 0$ for all $b \leq c$.

- Solution: $b^* = \min \left\{ c, 2 \left( H - L - \frac{c}{\delta} \right) \right\}$

- Observe that $b^*$ increases in $\delta$. 
Lessons Learned

- In a long-term relationship, the principal has a reputation to protect.

- By reneging on her promise to pay the bonus, she loses her reputation, and the agent will not exert effort in the future.

- The value of reputation increases in her patience (i.e., in $\delta$).

- Principal would like to promise bonus $b = c$ (to induce first best effort), but this promise is not credible unless she is patient enough.

References


Board S., (2011), Lecture Notes.

