The Retail Planning Problem
Under Demand Uncertainty

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Many retail store chains carry private label products.

Examples:
- Macy’s carries Alfani, Club Room, and others.
- GAP and Zara: Exclusively private labels.

In addition to deciding inventory levels, the retailer must
- Choose suppliers / establish production facilities.
- Make production and distribution decisions.
We develop a framework to address the retailer’s supplier choice, as well as her production, distribution and inventory decisions.
Related Literature

- Facility Location under Uncertain Demand:
  - Reviews by Aikens (1985), Snyder (2006), and Shen (2007).

- Integrated Supply Chain Models:
  - Balachandran and Jain (1976).
  - Daskin et al. (2002).
  - Shen et al. (2003).

Model Formulation

\[
Z_P = \min \left\{ \sum_i f_i z_i + \sum_{i,j,k} c_{ijk} x_{ijk} + \sum_i d_{ik} w_{ik} + \sum_{i,j} e_{ij} v_{ij} + \sum_{j,k} S_{jk} (y_{jk}) \right\}
\]

\[
\sum_i x_{ijk} = y_{jk} \quad \text{for all } j \text{ and } k
\]

\[
L_i z_i \leq \sum_j \sum_k \alpha_{ijk} x_{ijk} \leq U_i z_i \quad \text{for all } i
\]

\[
\sum_j \alpha_{ijk} x_{ijk} \leq U_i w_{ik} \quad \text{and} \quad \sum_k \alpha_{ijk} x_{ijk} \leq U_i v_{ij} \quad \text{for all } i, j, \text{ and } k
\]

\[
x_{ijk} \geq 0, \quad y_{jk} \geq 0, \quad w_{ik} \in \{0, 1\}, \quad v_{ij} \in \{0, 1\}, \quad z_i \in \{0, 1\} \quad \text{for all } i, j, \text{ and } k
\]

- \( S_{jk} (y) \): Inventory cost (newsvendor model).
- *Fashion industry*: short product lifecycles relative to lead times.
A Basic Result and a Roadmap

- The RPP is strongly NP-hard.
  - Reduction to CPLP (Cornuejols et. al. (1991)).
  - Large-sized instances unlikely to be solvable to optimality.

How to proceed?

- Construct heuristics to obtain a feasible solution.
- Obtain a lower bound on $Z_P$ using a Lagrangean relaxation.
- Evaluate how close the feasible solution is to the lower bound.
  - CVX Heuristic: Average suboptimality gap = 3.4%.
- Analyze the computational results to draw insights.
Convex Programming Heuristic

1: Solves a convex programming relaxation.
   - \( w_{ik} \)'s, \( v_{ij} \)'s and the unfixed \( z_i \)'s relaxed to lie in \([0,1]\).

2: Permanently fix any \( z_i \in \{0,1\} \).

3: Temporarily fixes largest fractional \( z_i \) to 1.
   - Solves remaining problem and rounds to 1 fract. \( w_{ik} \) and \( v_{ij} \).
   - Temporarily fixes smallest fractional \( z_i \) to 0.
   - Solves remaining problem and rounds to 1 fract. \( w_{ik} \) and \( v_{ij} \).

4: Permanently fix the \( z_i \) that yielded lowest total cost.
   - Return to 1 until all \( z_i \)'s have been fixed.

- **LP-based version**: Uses Lagrangean inventory levels to solve a sequence of linear programs.
Lagrangean Relaxation

- Relax $\sum_i x_{ijk} = y_{jk}$. Decomposes problem into:
  
  - $I$ Facility Location Subproblems:
    
    $$L_i^{milp}(\lambda) = \min \left \{ f_i z_i + \sum_k \left[ d_{ik} w_{ik} + \sum_j (c_{ijk} - \lambda_{jk}) x_{ijk} \right] \right \}$$
  
  - $J \times K$ Inventory Subproblems:
    
    $$L_{jk}^{cvx}(\lambda) = \min \{ \lambda_{jk} y_{jk} + S_{jk} (y_{jk}) \}$$

- $L(\lambda) = \sum_i L_i^{milp}(\lambda) + \sum_j \sum_k L_{jk}^{cvx}(\lambda)$

- For any $\lambda \in \mathbb{R}^{J \times K}$, $L(\lambda)$ is a lower bound for $Z_P$. 
Some Analytical Results

Proposition 1

Lagrangean Relaxation solved in closed form for any given \( \lambda \).

- Lagrangean bound \( = \max_{\lambda} \{ L(\lambda) \} \).
- \( L(\lambda) \) in closed form \( \Rightarrow \) Can solve max. problem directly.

Lemma 1

Lagrangean problem does not possess the integrality property.

- Lagrangean lower bound \( \geq \) convex relaxation lower bound.

Proposition 2

- Conditions so that \( \lambda^*_jk \) can be characterized analytically.
Computational Experiments

- 500 randomly generated problem instances.
  - 5 – 20 candidate facility locations.
  - 10 – 40 stores.
  - 1 – 25 products.

- We evaluate:
  - Objective functions of CVX heuristic, and LP variation;
  - Lagrangean lower bound; and
  - Objective functions of two benchmark heuristics:
    1. Practitioner / Greedy heuristic.
    2. Sequential heuristic.
Suboptimality Gap

% from Lagrangean Lower Bound

- Pract.
- Pract. (NV)
- Seq.
- Seq (Simplified)
- CVX
- LP

- Range
- Mean
- Median
An Insight regarding Inventory Decisions

- Inventory levels in CVX heuristic < newsvendor levels.

**Why?**

CVX solution accounts for effect of inventory to upstream SC costs.

**Intuitively:** a higher inventory level increases

(i) production and distribution costs; and

(ii) costs associated establishing production capacity.

When these costs are accounted for, lower inventory is preferable.

**Take-away**

When managing the entire SC, a lower fill rate may be preferable.
Analyzing the Computational Results

- How does
  - a. the computational time;
  - b. the gap between CVX and the best benchmark heuristic;
  - c. the suboptimality gap; and
  - d. the total expected cost of CVX heuristic

depend on the size and the cost parameters of the problem.

Finding 1: Computational Time.
- Depends primarily on problem size (i.e., $I$, $J$ and $K$).
- Linear regression yields $R^2 = 0.64$.
  - Scales up approximately linearly in problem size.

Finding 2: CVX heuristic is robust to changes in parameters.
- All regressors and their std. errors are close to 0.
Finding 3: Performance Advantage of CVX heuristic is Robust.
- Performance advantage increases in problem size.
- Insensitive to the cost parameters.

Finding 4: Total Expected Cost vs. Problem Parameters.
- Increases in the problem size and the mean demand.
- Key influencing factors:
  1. Inventory underage and overage costs; and
  2. Marginal production and distribution costs.
- Emphasizes value of improved demand forecast.
- Supplier capacity and fixed costs have a secondary effect.
### Summary

- **Integrated SC problem:** Retailer chooses suppliers, and determines production, distribution and inventory planning.
  - Use Lagrangean relaxation to obtain a lower bound.
  - Develop heuristics to obtain feasible solutions.

- **Computational experiments.**
  - Solutions are close to optimal (within 3.4% on average).
  - Suboptimality gap is robust to problem size and parameters.
  - Computational time scales up \( \sim \) linearly in problem size.

- **Insights:**
  1. Lower fill rate may be preferable when managing the entire SC.
  2. Inventory costs are key drivers of total expected SC costs.
    - Fixed costs and supplier capacity have a secondary effect.
Future Research

- Embed this problem in a dynamic environment.
  - Allow for replenishing of inventory.

- Incorporate multiple echelons in the SC.
  - e.g., wholesalers, distribution centers, etc.

- Explicitly model economies of scale.