We have considered two major topics so far:

I. Present Value
II. Measuring and Adjusting for Risk

The major sub-topics we have covered fit together logically:

I. Present Value

A. Goal of financial manager: maximize the (present) value of the firm's equity. Do this by adopting positive NPV projects.

1. Basic tools for valuation:
   a) basic present value
   b) basic future value
   c) annuities
   d) perpetuities
   e) growing annuities and perpetuities

B. Useful approximation methods for valuation problems:

1. Gordon growth model
2. equivalent annual cost method
C. Be aware but wary of alternatives: pay-back, IRR, accounting profitability, etc.

D. Where things get tricky:

1. Taxes
2. Depreciation
3. different borrowing and lending rates
4. identifying opportunity costs / sunk costs
5. mutually exclusive projects

II. Risk and Return

A. Investors must be compensated for higher risk with a higher expected return.

1. quantify total risk of any asset's returns using standard deviation

B. Distinction between systematic and specific risk; there are gains from diversification!

1. only market risk in well-diversified portfolio
2. extent of reduction in risk of any portfolio is determined by covariances
3. systematic risk remains after diversifying
4. since investors can diversify at a low cost, only systematic component of risk necessitates a higher expected return
C. **CAPM provides**:

1. a way to quantify the systematic risk of any asset: by its beta
2. a way to adjust discount rates to take this risk into account
3. a portfolio strategy to minimize risk for an uninformed investor: invest in a combination of the market portfolio and a risk-free bond.

D. **Graphical tools**:

1. risk/return diagrams and the capital market line
2. the security market line

E. **Acquired Skills**:

1. how to calculate the risk and return of securities and portfolios of securities, and how to interpret these calculations
2. how to calculate beta (given historical data or variance/covariance information)
3. how to adjust discount rates using CAPM
4. how to choose a portfolio with desired risk/return properties
5. how to unlever equity betas to get asset betas to use for project evaluation
Some Practice Problems

1. **Basic present value calculations.** Bill's parents wish to save to put him through college, which he will enter in September, 2005, and from which he will hopefully graduate in June, 2009. They estimate that they will need to pay $32,500 every September at the beginning of each school year. They plan to make annual contributions to a trust fund, with the first contribution coming in September 1999, and the last in September, 2008. If the interest rate is 9%, what equal annual contributions will finance his education, assuming no inflation?

```
  C    C    ...    C        C          C    C

PV of tuition as of 9/2004 = 32.5(P/A, 9%, 4) = $105,290.90
PV of tuition as of 9/98 = 32.5(P/A, 9%, 4)(P/F, 9%, 6) = $62,781.52

10 payments of C:

C = $62,781.52(A/P, 9%, 10) = $9782.62
```

2. **Recognizing incremental costs.** You run a construction business. You own a crane and use 50% of its capacity in your current projects. The remaining capacity is rented out to other firms. These rentals provide net revenues of $30,000 per year. You are bidding on a new contract. The work will require the use of the remaining 50% of the crane’s capacity for the next three years. This use of the crane will also cause maintenance costs to increase by $5,000 per year for the next three years. You are in the process of determining what costs should be charged to the new contract due to its use of the crane, but have received conflicting advice. Mr. Morganfield argues that the new contract should be charged half of the crane’s annual depreciation of $100,000 over the next three years plus the increased maintenance costs, for a charge of $55,000 per year. Ms. Waters argues that you should only look at the incremental costs. The initial purchase of the machine was made on the basis of other projects, and the firm will not get those costs back if the new project is not taken. Therefore, she believes that the new project should only be charged for the increased maintenance, $5,000 per year. What do you think?

Neither are entirely correct. There are two incremental costs, the additional $5,000 in maintenance, and the opportunity cost of losing the revenues of $30,000 per year. Thus the total cost is $35,000 per year. Depreciation is not a cash charge, so it should not be included.
3. **Evaluating alternatives using EAC.** For the indefinite future, your firm needs a machine that costs $75,000. The machine lasts up to 3 years and if used for 3 years, it has a salvage value of 0. If instead, it is replaced after 2 years with a new machine, salvage value is $30,000. If it is replaced after 1 year with a new machine, salvage value is $50,000. The discount rate is 3%. What is the cheapest way to run the machine, continually replacing it after 1, 2, or 3 years? Provide the calculations that support your answer.

**Replace every year:**

\[
\begin{align*}
-75,000 & \quad +50,000-75,000 & \quad +50,000-75,000 \\
\end{align*}
\]

\[
\text{Cost} = 75,000 + 25,000/0.03 = $908,333.33
\]

**Replace every two years:**

\[
\begin{align*}
-75,000 & \quad 0 & \quad +30,000-75,000 & \quad 0 & \quad +30,000-75,000 \\
-75,000 & \quad 0 & \quad 0 & \quad 23,517 & \quad 23,517 \\
\end{align*}
\]

\[
\text{Cost} = 75,000 + (23,517/0.03)/(1.03)^2 = $813,916
\]

EAC for payments starting in year 2 = $45,000(A/PV,3%,2) = 23,517.48

**Replace every three years:**

\[
\begin{align*}
-75,000 & \quad 0 & \quad 0 & \quad -75,000 & \quad 0 & \quad 0 \\
\end{align*}
\]

\[
\text{EAC} = 75,000(A/PV,3%,3) = 26,514.78
\]

\[
\begin{align*}
26,515 & \quad 26,515 & \quad 26,515 \\
\end{align*}
\]

\[
\text{Cost} = 26,515/0.03 = 883,825.9
\]
4. **Evaluating alternatives using EAC.** You plan to manufacture a new product. The product can be produced in one of two ways: using old or new equipment. Otherwise the inputs are identical. New equipment costs $18 million, lasts 8 years, and will be replaced by identical equipment at the same cost every 8 years forever. Alternatively, existing equipment which is used to manufacture other products but has excess capacity can be used. The existing equipment (which was just purchased) costs $60 million and normally lasts for 15 years. Using this equipment to produce the new product shortens its life to 10 years. Regardless of your choice, the existing equipment will be replaced by identical equipment at the same cost forever. The cost of capital is 10%.

What is the cheaper way to manufacture the new product? What is the present value of the difference between the cost of the two methods?

\[
\text{EAC(alternative)} = 18m(A/PV,10\%,8) = 3.374m
\]

\[
\text{EAC(current machine under current use)} = 60m(A/PV,10\%,15) = 7.888m
\]

\[
\text{EAC(current machine with additional use)} = 60m(A/PV,10\%,10) = 9.765m
\]

Difference is cost of additional use: 9.765 - 7.888 = 1.876m

Better to shorten life of existing machine.

\[
\text{PV of savings is } (3.374m - 1.876m) / .1 = 14.97m
\]

5. **Gordon Growth Model for Stocks.** Starship Enterprises is currently selling for $120/share. Their dividends are growing at a steady 8% per year under their policy of plowing back 20% of earnings into the company. Earnings at the end of this year are expected to be $30/share.

(a) Find the return on equity (ROE) and the market capitalization rate for Starship Enterprises.

\[\text{ROE} \times \text{plowback} = g, \text{ so } \text{ROE} = 0.08 / 0.2 = 40\%\]

\[\text{price} = \text{DIV}/(r-g)\]

\[\text{DIV} = (1-\text{plowback})\text{EPS} = 30(0.8) = 24\]

(b) Due to their high ROE relative to their market capitalization rate, Starship Enterprises is viewed as a prime takeover target. The hostile takeover genius Darth Raider vows to take over the company and raise the plowback ratio to 50%. If the takeover is expected to occur immediately, what is the effect on the price of Starship Enterprises? (Assume the higher reinvestment rate does not affect the market capitalization rate.)

(a) \[\text{ROE} \times \text{plowback} = g, \text{ so } \text{ROE} = 0.08 / 0.2 = 40\%\]

\[\text{price} = \text{DIV}/(r-g)\]

\[\text{DIV} = (1-\text{plowback})\text{EPS} = 30(0.8) = 24\]
\[ r = \frac{24}{120} + .08 = 28\% \]

(b) \[ DIV_i^{new} = 30 \times .5 = $15; \quad g^{new} = .4 \times .5 = .20; \quad \text{new price} = \frac{15}{.28 - .2} = $187.50 \]

6. **Portfolio Risk and Return.** Consider two risky securities, X and Y. The expected return on X is 10% and the expected return on Y is 14%. The standard deviation of return on X is 15% and the standard deviation of return on Y is 20%. The correlation between the returns of X and Y is .4. Portfolio A is an equally weighted portfolio of X and Y.

(a) What is the expected return and standard deviation of return of portfolio A?

\[ .5(10\%) + .5(14\%) = 12\% \]

\[ \sigma^2 = .25(15)^2 + .25(20)^2 + 2(.25)(.4)(15)(20) = .0216 \]

\[ \sigma = 14.7\% \]

(b) Suppose you could invest at a riskless rate of 4% and borrow at a rate of 8%. How would you combine Portfolio A with borrowing and/or investing to construct a portfolio that has an expected return of 9%? What is the standard deviation of return on this portfolio?

\[ X(.12) + (1-X)(.04) = .09 \quad \Rightarrow \quad X = .625 \]

\[ \sigma = X(14.7) = 9.1875\% \]

Notice that you combine with 4% because your target return is lower than the expected return on portfolio A, so you will be lending, not borrowing.
7. **Asset vs. Equity Betas.** Partly Peanuts is all equity financed. One-third of their business is in retail nut sales, and two thirds of their business is in brick manufacture. The required return on their equity is 20%. Nuts-2-U is entirely in the nut business. Their D/E ratio is 1/3, and the required return on their equity is 34%. Their debt is risk-free. Assume the risk-free rate is 10% and the expected return on the market is 18%. You want to start a brick company. What is your cost of capital?

\[
\frac{1}{3} \beta_{\text{nuts}} + \frac{2}{3} \beta_{\text{bricks}} = \beta_{\text{equity of PP}}
\]

\[
\beta_{\text{nuts}} = \frac{1}{4} \beta_{\text{debt N2U}} + \frac{3}{4} \beta_{\text{equity N2U}}
\]

\[
.1 + \beta_{\text{equity N2U}}(.08) = .34 \quad \text{so} \quad \beta_{\text{equity N2U}} = 3
\]

This implies \( \beta_{\text{nuts}} = (3/4)3 = 2.25 \)

\[
.1 + \beta_{\text{equity PP}}(.08) = .2 \quad \text{so} \quad \beta_{\text{equity PP}} = 1.25
\]

\[
\frac{1}{3}2.25 + \frac{2}{3} \beta_{\text{brick}} = 1.25 \quad \text{so} \quad \beta_{\text{brick}} = .75
\]

Your cost of capital is estimated to be \( .1 + .75(.08) = .16 \)

8. **CAPM.** Mary has $100,000 invested in a well-diversified portfolio with a \( \beta \) of 0.95. She discovers that the stock of Contraire Intl. has a \( \beta \) of -0.5. The riskfree rate is 5% and the expected return on the market is 12%.

a) How much of her $100,000 investment would she have to put into Contraire Intl. stock to create a zero \( \beta \) portfolio?

\[
(1-X)(.95) + X(-.5) = 0, \quad \text{so} \quad X = .3448, \quad \text{so she should put .3448($100,000) in the stock with a \( \beta \) of .95 and the rest in the negative \( \beta \) stock.}
\]

b) Would the resulting zero \( \beta \) portfolio be riskfree? (Explain)

(a) Solve \( X(.95) + (1-X)(-.5) = 0, \) so \( X = .3448, \) so she should put .3448($100,000) in the stock with a \( \beta \) of .95 and the rest in the negative \( \beta \) stock.

(b) The portfolio has no market risk, but it does have idiosyncratic risk. The idiosyncratic risk is introduced when a large investment is made in the negative \( \beta \) stock.