**Problems for Review Session: Yield Curve and Equity Issue**

**The Yield Curve**

Gega-ware is an all-equity software firm. It is considering issuing debt for the first time. Assume all cash-flows accrue at the end of the period. The fixed-income analyst observes the following current prices for bonds with similar risk to those to be issued by Gega-ware:

<table>
<thead>
<tr>
<th>Bond-maturity</th>
<th>Coupon rate</th>
<th>Face Value</th>
<th>Current (period 0) Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>10%</td>
<td>$1,000</td>
<td>$1,000</td>
</tr>
<tr>
<td>2 year</td>
<td>5%</td>
<td>$100,000</td>
<td>$90,000</td>
</tr>
<tr>
<td>3 year</td>
<td>8%</td>
<td>$10,000</td>
<td>$9,200</td>
</tr>
<tr>
<td>4 year</td>
<td>12%</td>
<td>$5,000</td>
<td>$5,000</td>
</tr>
</tbody>
</table>

a) Construct the spot yield curve for maturities 1 year through 4 years.

b) Gega-ware plans to issue coupon bonds with face value of $100 million payable in 4 years. The coupon rate is 10%. How much can it expect to raise from this debt issue?

Solution:

a) 1 year: 1000 = 1100/(1+r_0(1)) → r_0(1) = .10 = 10%

2 year: 90000 = 5000/(1.1) + 105000/(1+r_0(2))^2 → r_0(2) = .1085 = 10.85%

3 year: 9200 = 800/(1.1) + 800/(1.1085)^2 + 10800/(1+r_0(3))^3 → r_0(3) = .1135 = 11.35%

4 year: 5000 = 600/(1.1) + 600/(1.1085)^2 + 600/(1.1135)^3 + 5600/(1+r_0(4))^4 → r_0(4) = .1222 = 12.22%

b) P = 10/(1.1) + 10/(1.1085)^2 + 10/(1.1135)^3 + 110/(1.1222)^4 = 93.83 million
Implied Forward Rates and Puttable Bonds

A firm observes the following spot yield curve: \( r_0(1) = 8\% , r_0(2) = 8.25\% , r_0(3) = 8.5\% \). The firm is considering issuing a 3 year bond with face value $10 million and a 10% coupon rate. The bond is puttable at time 2 with an exercise price of $10.5 million. What is the value of the cash flows from the bond at issue ignoring the put option? Assuming that future spot rates equal the implied forward rates, will the put be exercised at time 2? If so, what is its value (at maturity)? If the bondholder separates the coupon payments from the put and sells them separately, what will the bond (without the put) sell for at time 0? What will the put sell for at time 0? How much should the firm expect to raise by issuing the puttable bond?

Solution:
\[
P_0 (\text{ignoring put}) = \frac{1}{1.08} + \frac{1}{(1.0825)^2} + \frac{11}{(1.085)^3} = 10.39
\]

Implied forward rate at time 2 solves:
\[
(1+r_0(3))^3 = (1+r_0(2))^2(1+f_2(1)) \rightarrow f_2(1) = 0.09 = 9\%
\]

\( P_2 = \frac{11}{1.09} = 10.09 < 10.5 \) the exercise price, so the put value = 10.5 - 10.09 = 0.41 million

Bond value without put = 10.39 (from above)

Put value (at t=0 ) = \( \frac{.41}{(1.0825)^2} = .35 \) million

Puttable Bond value = 10.39+.35 = 10.74 million

Issuing Equity in Perfect Markets

Sludge, Inc. has won part of the contract for the Boston Harbor Clean-up. The contract will pay the firm $50 million after 5 years, and another $150 million after the completion of the clean-up in 10 years. The cost to the firm will be an initial $10 million for new equipment, plus $10 million per year in each of the 10 years of the project. (Assume all payments are end-of-year.) The company currently has no debt outstanding. The CFO proposes that the firm raise the initial $10 million investment by a rights offering, so that existing shareholders receive one warrant for every 2 shares they hold. The warrants will mature immediately. Currently, the firm has 2 million shares outstanding with a share price of $27. The new project has the same risk as the firm's existing assets. The beta of the firm's equity is 0.7, the market return is 12%, and the risk-free rate is 5%.

a) Should the firm undertake the Harbor Clean-up project?

b) What does the strike price of the warrants have to be in order to raise the required funds? What is the value of a warrant?

c) What is the effect of the project and the rights issue on the wealth of the existing shareholders? Why?
Solution:

a) required return on equity = .05 + .7(.12 - .05) = .099

\[
NPV = -10 + \left[ \frac{-10}{.099} \left( 1 - \frac{1}{(1.099)^{10}} \right) \right] + \frac{50}{(1.099)^3} + \frac{150}{(1.099)^{10}} = -10 + 27.9 = 17.9
\]

Take the project.

b) Required funds = $10 million, warrants = 1 million, so strike price of warrant must = $10

Asset value before new project = (2 million)$27 = $54 million
Asset value with new project = $54 million + $27.9 million = 81.9 million
Number of shares conditional on exercise of warrants = 2+1 = 3 million
New share price = 81.9/3 = 27.3
Value of warrant = Share price - strike price = 27.3 - 10 = 17.3

c) Original shareholders new share value = $27.3(2 million) = $54.6 million
Original shareholders' warrants value = $17.3(1 million) = $17.3 million
Total wealth of original shareholders = $71.9 million
This equals their original wealth ($54 million) plus the NPV of the project ($17.9 million)