Final Review Problems

1. THE YIELD CURVE AND OPTIONS. A large corporation issues a three year "growth bond" promising to pay $5m in one year, $10m in two years, and $15m three years.

   a. Estimate the price of this bond using the following information on a set of three zero coupon bonds of comparable risk:

<table>
<thead>
<tr>
<th>Maturity</th>
<th>Price</th>
<th>Face Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$947.87</td>
<td>$1,000</td>
</tr>
<tr>
<td>2</td>
<td>$886.65</td>
<td>$1,000</td>
</tr>
<tr>
<td>3</td>
<td>$820.89</td>
<td>$1,000</td>
</tr>
</tbody>
</table>

   ANSWER: Implied yields: $Y_1 = 5.5\%$; $Y_2 = 6.2\%$, $Y_3 = 6.8\%$

   $P_0 = \frac{5}{1.055} + \frac{10/(1.062)^2 + 15/(1.068)^3}{1.062^2} = $25.918m

   b. What is the implied 1 period forward rate at time 2?

   ANSWER: $(1.062)^2(1 + r) = (1.068)^3$ $r = 8.01\%$

   c. Now imagine that the bond is callable at time 2 for an exercise price of $13.75m. That is, the issuing corporation has the right to pay the bondholder $13.75m at time 2 to retire the bond. If the one year spot rate at time 2 equals the implied forward rate in part (b), will the bond be called? Explain.

   ANSWER: At time 2, the bond will be worth $15/1.08 = 13.89$. The bond will be called since the corporation pays only 13.75.

   d. Now assume that at time 0 investors anticipate that the realized one year spot rate at time 2 will equal the implied forward rate. What price would you expect to see at time 0 for the callable bond?

   ANSWER: $P_0 = \frac{5}{1.055} + \frac{(10 + 13.75)/(1.062)^2}{1.055} = $25.757m

   e. Use your answer to (a) and (d) to estimate the value of the call option.

   ANSWER: Value of call = $25.918m - $25.757m
2. Assume perfect markets. The Recovery Zone, Inc. (RZI), is a chain of franchises which makes available rest services (cots, pillows, first aid, and alcoholic beverages for parents) for children and parents after leaving Discovery Zone franchises. The firm is all equity financed with 20 million shares of common stock. It has existing assets with market value of $500 million that produce annual earnings of $50 million in perpetuity.

   a. Assuming the market is valuing only the existing assets of RZI, determine the following: the market value of the equity, the required rate of return on the equity, the earnings per share and the stock price per share.

       **Answer:** Market value of equity is given at $500 million, so price per share is $25. Earnings per share = $2.50. The required return solves $25 = $2.50/r so r=10%.

Next suppose that RZI has been conducting research to develop a new product line to introduce at their stores. If the development is successful and the new product line is introduced (probability = 30%) then the new product line will require a $50 million investment and will generate annual earnings of $4.5 million in perpetuity. The new product is expected to have demand insensitive to the business cycle and an appropriate discount rate for the new investment is considered to be 8%. If the development effort is unsuccessful (probability = 70%) then the investment will be abandoned --- NPV = 0.

   b. When the market finds out about the possibility of the new product line, but not whether or not the project will be taken, what is the new value of the stock price?

       **Answer:** The market value should rise by the expected NPV of the new project. If the project works out, it has an NPV of:

       $4.5 million/.08 - $50 million = $6.25 million

       The expected increase in firm value is .3($6.25 mil.) + .7($0) = $1.875 million. This will result in a price increase of $.09375 per share.

Next, suppose that development effort is successful, the new project is undertaken and it is financed with a new equity issue.

   c. How many shares of equity will the firm have to issue and what price will the firm receive for the shares?

       **Answer:** Since the project is known to be a success, the stock price increases further to reflect the full value of the project prior to the equity issue. That is, the total firm value accruing to old shareholders increases by the NPV of $6.25 mil. relative to the initial $500 million value, so the price per share is $506.25 mil./20 mil. = $25.3125. The new shareholders receive a fair price for their $50 million, so the number of shares issued is $50 mil / $25.3125 = 1,975,308.64
Active board member, P.O'Lonius, argues that the investment is a bad idea. He claims that the expected return on the new investment is less than the firm's cost of capital and that the new project financed with equity will be dilutive (it will reduce earnings per share, thus harming the original shareholders).

d. Show that P.O'Lonius's claim that earnings per share will be lower (than in part a) is true. What do you think of his concerns about the advisability of the investment (address each concern)?

Answer: New EPS = $54.5 million / 21,975,308.64 = $24.80, so his math is correct although his logic is wrong. The old shareholders are clearly better off because they benefit from the NPV of the new project, and have claim to a less risky stream of earnings.

3. OPTIONS. Say that you have written a put on Borland Computer Corp., and the company subsequently becomes entangled in a lawsuit in which they will either win and come out $500 million ahead, or lose and come out $500 million behind. What happens to the value of your position?

Answer: We know that increased variance makes the holder of a put (or call) better off since the option is more likely to be in the money. We also know that options are a zero sum game between the writer (seller) of the option and the buyer of the option. So if the buyer of the put is better off, you must be worse off.

4. OPTIONS. Sketch the payoff diagram for the following portfolios of options:

a. Short one call in Sears at exercise price of $45.
   Own one share of Sears stock.
   Long one put in Sears at Exercise price of $38.

b. Would you be likely to take this position if you felt you were more optimistic about Sear's prospects than the overall market seemed to be?

Answer: Definitely not. Buying the put is insurance against a drop in Sears stock, which you would not want to pay for if you though the stock would do unexpectedly well. Similarly, writing a call is risky if the stock price is expected to rise.
5. **DO-IT-YOURSELF.** In this problem assume that capital markets are perfect and that all debt is riskless. The risk-free rate is 6%. A firm worth $100 million has $60 million debt and 4 million shares of equity outstanding. The required return to the firm's productive assets is 9.6%. An investor takes $3000 of her own money, borrows another $2000, and buys $5000 worth of the firm's stock.

a. What is the expected return to the firm's equity?
b. What is the expected return to the investor's portfolio?

Suppose now the firm issues $10 million worth of new debt, and distributes the entire proceeds to the shareholders via a $2.50/share special dividend.

c. What is the expected return to the firm's equity after the dividend paid?
d. If the investor uses her entire income from this special dividend to buy risk free bonds, what is the expected return to her portfolio?

(a) \( D = \$60 \text{ mil.} \) so \( E = \$40 \text{ mil.} \). Using the WACC formula,

\[
\frac{60}{100} \times 0.06 + \frac{40}{100} r_e = 0.096.
\]

Thus \( r_e = 15\% \).

(b) The return to the portfolio is \( \frac{5000(0.15) - 2000(0.06)}{3000} = 21\% \). Note that the return is relative to the amount of personal funds invested. This corresponds to using the WACC formula with \( E = 5000 \), and \( D = -2000 \). As above, borrowing is denoted by a negative \( D \).

(c) The total corporate debt is increased to 70 mil., and equity falls to 30 mil. after the 10 mil. dividend is paid out; total asset value is unaffected. The equity holders are no better or worse off, since they have 30 mil. in equity + 10 mil. in dividends. Again using the WACC formula,

\[
0.096 = \frac{7}{10} \times 0.06 + \frac{3}{10} r_e
\]

Thus \( r_e \) increases to 18\%, reflecting the increased financial risk.

(d) The return must be the same as before the dividend was paid (21\%). An additional $2.50 per share was borrowed to pay the dividend. By using the dividend to buy debt (lend), leverage is reduced by the investor by an exactly offsetting amount.
6. BEM Corp. has three operating divisions:

<table>
<thead>
<tr>
<th>Share of Company Value</th>
<th>Asset β</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baked Goods</td>
<td>.4</td>
</tr>
<tr>
<td>Electric Appliances</td>
<td>.5</td>
</tr>
<tr>
<td>Medical Supplies</td>
<td>.1</td>
</tr>
</tbody>
</table>

Their debt/equity ratio is 3 to 2, and the expected return on their equity is 14%. Treasury bills yield 6.8%, and the expected return on the market is 13.5%.

a. What is the debt β of BEM?

\[
\text{Asset beta} = .4(.1) + .5(1.2) + .1(.6) = .7 \\
\text{Asset beta} = (2/5)(\text{equity beta}) + (3/5)(\text{debt beta}) \\
.14 = .068 + (\text{equity beta})(.135 - .068) \Rightarrow \text{equity beta} = 1.075 \\
.7 = (2/5)(1.075) + (3/5)(\text{debt beta}) \Rightarrow \text{debt beta} = .45
\]

b. BEM's debt is in the form of 20 year bonds with 10% annual coupon payments. What is the maximum price you should be willing to pay for $10,000 face value of these bonds? (Assume that the yield curve is flat.)

\[
\text{annual coupon} = .1(10,000) = 1,000 \\
\text{discount rate} = .068 + .45(.135 - .068) = 9.815\% \\
\text{price} = 1,000(P/A, 9.815\%, 20) + 10,000(P/F, 9.815\%, 20) = 10,159.51
\]