## Discount Rate Review

### Notation:
- \( r_A \): expected return on assets
- \( r_{EU} \): expected return on unlevered equity
- \( r_{EL} \): expected return on levered equity
- \( r_D \): expected return on debt
- WACC: weighted average cost of capital
- \( E \): value of equity
- \( D \): value of debt
- \( V \): firm total value = \( E + D \)
- \( \beta_A \): asset beta
- \( \beta_E \): equity beta
- \( \beta_D \): debt beta

### Perfect Markets (no taxes):

\[
\begin{align*}
  r_A &= r_{EU} = \text{WACC} = \frac{E}{V} r_{EL} + \frac{D}{V} r_D \\
  \beta_A &= \frac{E}{V} \beta_E + \frac{D}{V} \beta_D
\end{align*}
\]

Use these expressions only in perfect markets, zero tax environments.

### Corporate and Personal Taxes:

\[
\begin{align*}
  T^* &= 1 - (1 - \tau_c) \frac{1 - \tau_{pe}}{(1 - \tau_{pd})} \\
  r_A &= r_{EU}
\end{align*}
\]

The tax benefit of debt includes both corporate and personal taxes.

If the firm has an all-equity (unlevered) capital structure, then the tax effects don't drive a wedge between the return on assets and the return on equity.

\[
\begin{align*}
  \text{WACC} &= \frac{E}{V} r_{EL} + \frac{D}{V} (1 - T^*) r_D \\
  r(D/V=1) &= (1 - T^*) r_D
\end{align*}
\]

This can be directly calculated with the levered firm's capital structure and expected returns. With information on betas and market data, you can use debt and equity betas to compute \( r_D \) and \( r_E \). With capital structure data, you can then get WACC.

\[
\begin{align*}
  \text{MM: WACC} &= r_A \left( 1 - T^* \frac{D}{V} \right) \\
  \text{ME: WACC} &= r_A - \frac{D}{V} r_D T^* \frac{1 + r_A}{1 + r_D}
\end{align*}
\]

If you have the data to calculate WACC, but you need to calculate APV, then these formulas allow you to calculate \( r_A \).

*Use MM if \( D \) is fixed in perpetuity.*
*Use ME if \( D/V \) is fixed in perpetuity.*

\[
\begin{align*}
  \beta_A \left( 1 - T^* \frac{D}{V} \right) &= \frac{E}{V} \beta_E + \frac{D}{V} \beta_D \quad \text{if \( D \) fixed} \\
  \beta_A &= \frac{E}{V} \beta_E + \frac{D}{V} \beta_D \quad \text{if \( D/V \) ratio fixed}
\end{align*}
\]

If you have betas, rather than calculating WACC and then getting \( r_A \), use these formulas to get the asset beta and then get the expected return on assets from the CAPM.