Problems for Review Session: Options

Options and Payoff Diagrams

Consider three call options, each with 3 months to maturity. A call with an exercise price of $35 is selling for $12. A call with an exercise price of $40 is selling for $8. A call with an exercise price of $45 is selling for $4. Consider the following strategy: buy 1 call with an exercise price of $35, buy 1 call with an exercise price of $45, and sell 2 calls with an exercise price of $40.

(a) Graph the gross payoff on this strategy as a function of the stock price at expiration.

(b) Given the call prices, does this strategy represent a profitable arbitrage opportunity? Explain.

Solution:

(a)

(b) It does represent an arbitrage opportunity. From the payoff diagram, if the stock price winds up between $35 and $45 at expiration the position has a positive payoff. But it required a zero initial outlay = 2($8) - $4 - $12.

An Abandonment Option

Your company is considering abandoning a project (call it EuroDisney). You cannot discern whether the currently low revenues are due to bad economic conditions (which will eventually reverse) or to a bad product (which cannot be reversed). Currently, the project is losing 10 million francs per year. Forecasters predict that if now is time 0, then by year 3, economic conditions will improve. At that time, if the product is good, net cashflows should become positive. If the product is bad, then net cashflows will continue to be negative. The actual forecasts are given in the table below:

<table>
<thead>
<tr>
<th>Year</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>CF - good</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>.6</td>
</tr>
<tr>
<td>CF - bad</td>
<td>-10</td>
<td>-10</td>
<td>-10</td>
<td>-10</td>
<td>-10</td>
<td>.4</td>
</tr>
</tbody>
</table>
Since these values are forecasts, they are subject to market risk, as is the current loss of 10 per year. The discount rate of the project is 15%, and the risk-free rate is 3%. The current year's loss (year 0) is already sunk. Only consider cash flows through year 7.

Should you continue the project or abandon it (and if so, when)?

Solution:

Draw the CF time line/tree:

Time:  0   1   2   3   4   5   6   7

      5   10  15  15  15

CF:    -10 -10

Notice that you can collapse the branches of the tree using PV (since once period 3 CFs are realized the firm will either stay or abandon; it would never wait further to decide):

Time:  0   1   2      3      4     5   6   7

      43.48

CF:    -10 -10

Expected PV of cash-flows at t=0: 

\[ \frac{-10}{1.15^2} + \frac{-10}{(1.15)^3} + \frac{.6(43.48) + .4(-38.55)}{(1.15)^3} = -9.24 \]

This suggests abandonment, but value the option first! The option arises in this problem because at t=3, the firm can decide to abandon in the low state, even if it doesn't now. First, find the PV (at t=2, so find \( S_2 \)) of the CF -- the expected PV of payoffs along both branches. This is the value of continuing the project from time 3 forward ignoring the option. Second, find the payoffs along each branch from having the option. In the high state, the option adds nothing, so the payoff is zero. In the low state, the option to wait saves the firm the ensuing loss, or the benefit is 38.55.
Find the risk-neutral probability of the high state: \( \pi_H = \frac{S_2 (1 + r_f) - (-38.55)}{43.48 - (-38.55)} = .586 \), and value the option: \( C_2 = \pi_H \times 0 + (1 - \pi_H) (38.55) = 15.48 \). Discount \( C_2 \) back to time 0 (using \( r = .15 \)), and add the option value to the original NPV calculation to get \( 11.706 + (-9.24) = 2.46 \), the time 0 value of the project, including the option. To verify your answer, take the PV along the preferred branch of the tree using \( r = 15\% \), but using the risk-neutral probability and the risk-free rate to weight the cash-flows in the embedded option that appears at time 3:

\[
\text{Value(t=0)} = \frac{-10}{1.15} + \frac{1}{(1.15)^2} \left[ -10 + \frac{.586(43.48) + .414(0)}{(1.03)} \right] = 2.46.
\]

This is the current value of the project, including the option to wait until the third period to see how uncertainty about product quality is resolved. Thus, the best plan is to wait.