In this homework set you will practice using the basic tools of present and future value by solving some practical problems of personal finance. The questions are also intended to focus your attention on how to take into account the effects of taxes and inflation.

1. Suppose that you borrow $25,000 on September 1, 2003 and another $25,000 on September 1, 2004 to finance the portion of tuition and living expenses for graduate school for which your savings are insufficient. Repayment on each of these loans is deferred until graduation. However, interest of 6.5% (quoted annual percentage rate) begins accruing immediately (compounded monthly). The first payment is due September 1, 2005. The loan agreement specifies that the loan be repaid in 120 equal monthly installments.

   a) What will be the total value of your indebtedness on August 31, 2005 after the accrual of interest during your time in school?

   b) What is the monthly payment?

2. Upon graduation, you land a job with a $120,000 salary. The firm also happens to know that your favorite car is the BMW 745i. So to sweeten the deal, the firm also offers to give you $34,000 in cash towards the purchase of the car, or to reimburse you for five years of monthly payments if you lease the car. (Ignore tax implications.)

   If you buy the car: The price of the car is $66,000. You're still short on cash, so if you want to purchase the car, you have to borrow at 10% (the quoted annual percentage rate) by taking out a five-year annuity loan with monthly payments. The loan payments are at the end of each month.

   If you lease the car: The monthly lease payments are $700 for five years. The contract stipulates that at end of the lease you can either purchase the car for $36,000 or walk away. As is the usual practice, the lease payments are at the beginning of each month.

Assume that you know with certainty that you will work for the firm for at least five years, and that you will be able to sell the car for $38,000 at the end of the lease term. You can borrow or lend at an annual rate of 10% (monthly compounding).

   a) Ignoring the firm’s offer, would it be cheaper for you to lease or to purchase the car with a loan if you plan to keep the car for at least five years?

   b) Now consider the firm’s offer. Should you take the $34,000 cash or the $700 monthly reimbursements?
c) Suppose that next year you are in charge of recruiting. You recognize that from the firm’s perspective, both the lease payments and the signing bonus are tax deductible expenses, and that the firm has sufficient profits to use all tax deductions. If the firm’s tax rate is 35%, and it can borrow or lend money at 8% (after corporate taxes, so use this rate for discounting), what size bonus would make the firm indifferent between offering the bonus and the lease deal?

d) Optional–Extra Credit. Now imagine that, as is often the case, your borrowing rate is higher than your lending rate; e.g., your borrowing rate is 12% but your lending rate is 8%. This raises a subtle and tricky issue: Which is the correct discount rate to use in the calculations for parts (a) and (b)? Justify your choice by explicitly identifying the opportunity cost in this situation. How does this affect the answers to parts (a) and (b)? Can you draw any general conclusions about when to discount using the borrowing rate and when to discount using the lending rate?

3. (SPREADSHEET PROBLEM) One day, as you drive around in your BMW, you discover a cottage in the country that’s for sale for $380,000 (it’s a nice cottage…). You want to buy it; and you’ve managed to save more than enough to cover the $80,000 for a down payment. You shop around for mortgage loans and narrow your search down to two alternatives to finance the $300,000 difference; both are 15-year loans.

<table>
<thead>
<tr>
<th>points</th>
<th>quoted annual rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5%</td>
<td>8%</td>
</tr>
<tr>
<td>0%</td>
<td>10%</td>
</tr>
</tbody>
</table>

Points are paid up-front out of savings as a percentage of the mortgage principal amount ($300,000 here). Note that points and mortgage interest payments are tax deductible. Assume that payments are end-of-period, interest is compounded monthly, taxes are also paid monthly (for simplicity), and that your tax rate is 30%, both for ordinary income and capital gains.

a) What are the monthly payments associated with each of these loans?

b) Use a spreadsheet to break down the first three years of payments of each loan into their principal and interest components. Also break out the dollar amount of the tax savings each month due to the deductibility of interest and/or points.
c) Assume that you plan to sell the cottage at the end of three years, and will use part of the proceeds to pay off the remaining balance of the mortgage. The anticipated sales price is $440,000. You pay capital gains tax on the difference between the sales and purchase prices. Do a present value calculation to determine which mortgage to take, discounting after-tax cashflows by an after-tax discount rate. Assume that the appropriate after-tax discount rate to use is 7% (quoted annual percentage rate, with monthly compounding). Intuitively, how would you expect your answer to change if you only intended to own the cottage for one year?

d) Sensing your indecision, the current owners offer to rent you the cottage for $2400 month (fixed over the three years) instead of selling it to you; rental payments are made at the beginning of each month. If you rent you avoid the mortgage payments and also save $300 per month in maintenance costs and property taxes. The rental payments, however, are not tax deductible and you forego the expected capital gain. Should you rent or buy (assuming all else is the same as in parts (a) to (c))? Explain.

e) With all else as in part (d), at what rental rate are you indifferent between renting and buying?

4. On your 35th birthday, it occurs to you that the firm’s retirement account may be insufficient to maintain for you the lifestyle to which you’ve become accustomed. The value of your retirement account on your 35th birthday is $100,000. You plan to retire on your 65th birthday and to live until your 82nd birthday. Your retirement account earns 10% per year, and the inflation rate is 2% forever. Ignore the value of your other assets (e.g., your cottage) and perform calculations as if deposits, withdrawals and compounding of interest occur only once a year.

a) What is the stream of nominal payments on your 66th through 82nd birthdays that provides you with $150,000 of consumption annually in terms of purchasing power today?

b) What nominal balance is needed in your retirement account on your 65th birthday to provide the cash flow stream in (a)?

c) What constant nominal amount would you have to deposit on each of your 36th through 65th birthdays to ensure that the balance is the amount in (b) on your 65th birthday?