Finance 440 - Turbo Finance

Topic 3
Risk and Return II: The CAPM and Capital Budgeting Under Uncertainty

Goals for Topic 3

Calculate risk-adjusted returns in order to value both financial securities and investment projects.

Derive the Capital Asset Pricing Model (CAPM) -- the most popular model for estimating risk-adjusted required returns.

Understand and appreciate the distinction between market risk and the total risk of a security or project.
Outline of Topics
• Review of Regression Analysis
• Derivation of Asset Pricing Models
  • Systematic vs Idiosyncratic Risk
  • Diversifying idiosyncratic risk
  • Measuring systematic risk
• The Capital Asset Pricing Model (CAPM)
• Extensions
• Discounts rates for Projects

I. Review of Regression Analysis

Consider the relationship between two variables $Y$ and $Z$.
The regression model supposes that they are related as follows:

$$Y = \alpha + \beta Z + \epsilon,$$

where

- $Z$ = the value of the independent variable
- $Y$ = the value of the dependent variable
- $\epsilon$ = a random error for the observation
- $\alpha$ = the point where the line crosses the $Y$ axis
- $\beta$ = the slope of the line (change in $Y$ given a change in $Z$)

and we usually assume that $E[\epsilon] = 0$. 
You may recall that the formula for the best fit is:

\[ Y = \alpha_i + \beta_i Z_t + \epsilon_{it} \]

II. Derivation of Asset-Pricing Models

A. Economy-Wide and Specific Risk

Again, denote the rate of return on security i at time t by \( r_{it} \). Further, let

- \( Z_t \) = the market return
- \( \epsilon_{it} \) = the specific return of firm i at time t

Divide the risk of an individual security into economy-wide risk and specific risk. We do this formally in a regression framework as follows:

\[ r_{it} = \alpha_i + \beta_i Z_t + \epsilon_{it} \]

- The larger is \( \beta_i \), the more subject to market risk is this firm.
- The larger is \( \alpha_i \), the more important is firm-specific risk.
- This firm-specific risk is independent of market risk. Recall from statistics that if two variables are independent, then the variance of the sum equals the sum of the variances. Therefore:

\[ \sigma_i^2 = \beta_i^2 \sigma_z^2 + \sigma_{\epsilon_i}^2 \]
Therefore, the total risk of a security is made up of the risk from market movements and the risk from specific movements.

Two securities can have the same total risk, yet that risk can be made up of very different components.

**Example 3-1. Decomposing the Total Risk of a Stock**

Consider two stocks:

A: An automobile stock with: \( \beta_a = 1.5 \) \( \sigma_{a}^2 = .10 \)

B: An oil exploration company with: \( \beta_b = 0.5 \) \( \sigma_{b}^2 = .18 \)

and the variance of the market risk measure is \( \sigma_z^2 = .04 \).

**What is the total risk of each?**

**Which will have a higher expected rate of return?**

\[ \sigma_A^2 = \]

\[ \sigma_B^2 = \]
B. The Law of Large Numbers

*By the law of large numbers, we can diversify away all specific risk.*

Formally, if we have a number of random variables $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$, each with zero mean and which are not correlated with each other,

then if we take a weighted average of them, with weights $x_1, x_2, \ldots, x_n$,

the average will approach 0 as $n$ gets large, that is:

$$x_1 \varepsilon_1 + x_2 \varepsilon_2 + \ldots + x_n \varepsilon_n = 0$$

when

$$x_1 + x_2 + \ldots + x_n = 1, \ x_i > 0$$

C. Portfolio Risk--A Summary

1. Each security has a rate of return defined by

$$r_{it} = \alpha_i + \beta_i Z_t + \varepsilon_{it}$$

2. A portfolio consists of a set of portfolio weights $x_1, x_2, \ldots, x_n$, where $x_i$ refers to the proportion of the portfolio in security $i$. Note that $x_1 + x_2 + \ldots + x_n = 1$.

3. The overall rate of return on the portfolio is just a weighted average of the individual returns.

$$r_P = x_1 r_1 + x_2 r_2 + \ldots + x_n r_n =$$

$$x_1 [\alpha_1 + \beta_1 Z + \varepsilon_1] + x_2 [\alpha_2 + \beta_2 Z + \varepsilon_2] +$$

$$\ldots + x_n [\alpha_n + \beta_n Z + \varepsilon_n]$$
We can do some rearranging to get:

\[ r_P = \alpha^* + \beta^* Z + \epsilon^* \]

where

\[ \alpha^* = x_1 \alpha_1 + x_2 \alpha_2 + \ldots + x_n \alpha_n \]
\[ \beta^* = x_1 \beta_1 + x_2 \beta_2 + \ldots + x_n \beta_n \]
\[ \epsilon^* = x_1 \epsilon_1 + x_2 \epsilon_2 + \ldots + x_n \epsilon_n \]

4. The major points are:

a) Our portfolio has only market risk, all of the specific risk has been diversified away.

b) The amount of market risk is given by \( \beta^* \). The total risk of a well-diversified portfolio equals its economy-wide risk:

\[ \sigma_p^2 = [\beta^*]^2 \sigma_z^2 \]

\[ \sigma_p^2 = [\beta^*]^2 \sigma_z^2 \]

(4)

\[\text{The risk of a well-diversified portfolio depends only upon the market risk of the securities included in the portfolio. This is measured by the average } \beta.\]
D. Measuring Market Risk

1. Construct the following portfolio:

   a) It includes every security in the market, and
   b) The weight of each security in the portfolio is proportional to its relative size in the economy.

Example 3-2. Constructing the Market Portfolio

Suppose that there are only two stocks in the economy, IBM and Xerox. You have the following information:

<table>
<thead>
<tr>
<th>Stock</th>
<th>#Shares</th>
<th>Price (per share)</th>
<th>Market Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBM</td>
<td>10,000</td>
<td>$300</td>
<td>$3,000,000</td>
</tr>
<tr>
<td>Xerox</td>
<td>20,000</td>
<td>$50</td>
<td>$1,000,000</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>$4,000,000</td>
</tr>
</tbody>
</table>

x(IBM) = 3/4  x(Xerox) = 1/4

What happens to shares if the price of IBM increases to $400 and the price of Xerox increases to $100?

Notice that if prices change, our portfolio remains the correct one without our buying or selling. Thus, it is called a passive portfolio.
Then:

\[
\text{value IBM} = 400(10,000) = 4,000,000 \\
\text{value Xerox} = 100(20,000) = 2,000,000
\]

The realized return on the market portfolio =

\[
\frac{3}{4}(4m - 3m)/3m + \frac{1}{4}(2m - 1m)/1m = 50%
\]

\[
x_{\text{new}}(\text{IBM}) = 4/6 \quad x_{\text{new}}(\text{Xerox}) = 2/6
\]

Holding the market now means holding 4/6 of your money in IBM and 2/6 of your money in Xerox.

Before you held 3/4 of your money in IBM and 1/4 of your money in Xerox.

The portfolio shares changed due to the change in value, not due to trading.

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**E. The Market Model**

Conceptually, the measure of market returns should include all traded assets in the economy. In practice, some broad market index, with return $r_m$, is typically used. We can express the return on security $i$ relative to the "market model" we have developed as:

\[
 r_i = \alpha_i + \beta_i r_m + \epsilon_i
\]

(This is the familiar notion of dividing of risk into market risk and specific risk.) However, we can now be more precise:

\[
 \beta_i = \rho_{im}\sigma_i/\sigma_m = \rho_{im}\sigma/\sigma_m
\]

where $\rho_m$ = correlation between the return on security $i$ and the market portfolio. $\sigma_m^2$ = variance of the rate of return of the market portfolio.
III. The Capital Asset Pricing Model

The CAPM is a special case of the market model described above. It specifies a particular value for the intercept term, \( \alpha \), in the market model.

Instead of the general market model:

\[
  r_i = \alpha_i + \beta_i r_m + \epsilon_i
\]

The CAPM specifies:

\[
  r_i = (1-\beta_i)r_f + \beta_i r_m + \epsilon_i
\]

Does this restricted case make sense?
- What does it imply for the return on a risk-free asset?
- What does it imply about the return on an asset that tracks the market?

The CAPM equation can be rewritten as:

\[
  r_i - r_f = \beta_i(r_m - r_f) + \epsilon_i
\]

Thus, the CAPM posits a linear relationship between the risk premium of a security and the risk premium of the market.

The CAPM can also be written as a linear relationship between
the \( \beta \) of a security and its expected rate of return.

\[
  E[r_i] - r_f = \beta_i E[r_m - r_f]
\]

where

- \( E[r_i] \) = the expected rate of return on the security
- \( E[r_m] \) = the expected rate of return on the market portfolio
- \( r_f \) = the riskless rate
- \( \beta_i = \rho_{im} \sigma_i / \sigma_m \)

The relationship is called the security market line (SML).
A. The Security Market Line

The CAPM implies that differences in expected returns can be attributed entirely to differences in betas.

Therefore, if we rank a group of assets by their betas, their expected returns should fall on the security market line:

\[ E[R_i] = E[R_f] + \beta_i (E[R_m] - E[R_f]) \]

Questions to think about:

a) What does it mean for an asset to plot above or below the SML?

b) What forces would tend to make it move toward the line if it is actively traded?

c) What if you are the only one who knows that it plots above/below the line?
**Example 3-3. Using the Security Market Line (SML)**

The $\beta$ of General Motors is about 0.54. Using data from past years, assume that $r_f = 0.078$ and $E[r_m] - r_f = 0.083$. Then the expected rate of return on the stock should be:

$$E[r_i] =$$

**Example 3-4. Using the SML to Price Assets**

Suppose that we have done our homework, and have looked at the earnings capacity of a firm. We decide that the probability distribution over the stock price next year is:

<table>
<thead>
<tr>
<th>$P_t$</th>
<th>$95$</th>
<th>$110$</th>
<th>$130$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob.</td>
<td>0.5</td>
<td>0.3</td>
<td>0.2</td>
</tr>
</tbody>
</table>

and suppose that we have the following data:

$\beta_i = 0.75; \quad E[r_m] = 0.20; \quad r_f = 0.05$

No dividend is expected in the next year. Then, what should the stock sell for today, according to the CAPM?
Example 3-5, Zero $\beta$ Company

Imagine a company that sends satellites into the sky.

Suppose that if the space shuttle is able to release the satellites, the price of the stock next year will be $200 per share.

If the space shuttle is unable to release the satellites then the price will be $100 per share.

If there is a 50% chance of successful release and the current riskless rate is 10%, **what should the stock be selling for today?**

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Negative $\beta$ Company

Can a stock have a negative $\beta$? If so, can the expected return be negative?
IV. Variants and Applications of the General Model

The CAPM has come under attack for its inability to sufficiently explain the empirically observed differences in returns across different securities.

Researchers have found that factors other than the return on the market help predict returns.

This has led to the development of a multifactor linear model called the …

A. “Arbitrage Pricing Theory” (APT):

\[ r_i = \alpha_i + \beta_{i1}Z_1 + \beta_{i2}Z_2 + \beta_{i3}Z_3 + \ldots + \varepsilon_i \]

For instance:

- \( Z_1 \) might be the return on the market (as in the CAPM)
- \( Z_2 \) might be the ratio of market value to book value
- \( Z_3 \) might be the interest rate on long-term bonds

Estimates of expected returns based on the APT are also available commercially.

Why might using a broad market index as a proxy for the market portfolio lead to a failure of the CAPM and the need for additional risk factors?
B. Estimating Beta Using Regression Analysis

Estimates of betas for various securities and industries can be purchased from commercial services.

You can also estimate betas yourself...

You need returns data for the asset whose beta you wish to estimate

\( (e.g., \text{if the asset is a stock you need dividends and changes in prices}) \)

and returns on a portfolio that should closely approximate the market portfolio

\( (e.g., \text{the S&P 500 index or the value-weighted index of all NYSE/AMEX stocks}) \).

With this data, construct a time series of returns to the asset and call it \( \{r_{a1}, r_{a2}, \ldots, r_{aT}\} \);

also construct the time series of returns to the market portfolio and call it \( \{r_{m1}, r_{m2}, \ldots, r_{mT}\} \).

Estimate the beta of asset “a” by simply estimating the slope parameter in the regression equation:

\[
  r_{at} = b_0 + b_1 r_{mt} + e_t
\]

using the method of ordinary least squares.
Note that in the estimation procedure just described, we did not take into account the risk-free rate.

Implicitly the risk-free rate was assumed to be constant in the estimation of CAPM betas.

An alternative that does not assume a constant risk-free rate is to regress $r_{i,t} - r_{f,t}$ on $r_{m,t} - r_{f,t}$.

Generally the estimates of $\beta$ are similar using either procedure.

C. Resolution of Uncertainty and Discount Rates

If a project's risk changes significantly during its life, then discount rates must reflect this.

For instance, an event might occur during the project's life that determines its success or failure.

After the event occurs, the project's cash flows are no longer uncertain; they should, therefore, be discounted at the riskless rate.
Example 3-6: Resolution of Uncertainty

The Lyric Opera (located down the street from the Chicago Merc) has an interesting idea for a door prize to appeal to traders:

At the end of three years they will compute the return on the market index (+ dividends):

\[ \frac{\text{S&P 500 (end yr 3)}}{\text{S&P 500 (end yr 2)}} \]

and give a cash award equal to this ratio times $1000.

e.g., if the S&P goes from 150 to 180 the payoff is \((180/150) \times 1000 = 1200\)

Suppose \(E(r_m) = 15\%\) and \(r = 5\%\).

If the winners are given the tickets right after the concert, how much should you pay them for their tickets today?

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>-P</td>
<td>1000(1+r_m)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

V. Capital Budgeting Under Uncertainty

- The CAPM tells us the equilibrium expected return associated with each level of beta risk.
- Therefore the CAPM rate is the one we should use to discount cash flows in NPV calculations.
- We will soon be turning to valuing investment projects for capital budgeting purposes. But ... how do we identify the beta for a project?

The beta of a project should reflect the systematic (market) risk associated with the cash flows of that project.

What measures of this risk do we have if the project …

- is in the same line of business as the rest of a publicly-traded firm?
- is in a new line of business for the firm?
- is in a new firm or a private firm?
Example 3-7. A conglomerate operating in several industries.

Let \( r_f = 7\% \), \( E(r_m)-r_f = 8\% \).

<table>
<thead>
<tr>
<th>Industry</th>
<th>( \beta )</th>
<th>( E(r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electronic Components</td>
<td>1.49</td>
<td>18.92%</td>
</tr>
<tr>
<td>Petroleum Refining</td>
<td>0.95</td>
<td>14.60%</td>
</tr>
<tr>
<td>Paper and Allied Products</td>
<td>0.82</td>
<td>13.56%</td>
</tr>
<tr>
<td>Railroads</td>
<td>0.61</td>
<td>11.88%</td>
</tr>
</tbody>
</table>

How would you find the \( \beta \) for the conglomerate?

Asset Betas vs. Equity Betas

The total risk of a firm’s assets is divided unevenly between debt and equity.

To estimate the required return for a project, one must use a weighted average between the required return on debt and the required return on equity.

\[
V = D + E = \text{total value of firm}
\]

\[
\beta_{\text{asset}} = \left( \frac{D}{V} \right) \beta_{\text{debt}} + \left( \frac{E}{V} \right) \beta_{\text{equity}}
\]

This is the formula for unlevering beta.

(What is the \( \beta \) of risk-free debt? What about junk bonds?)
**Applications**

*Estimating the cost of capital for an investment in a new hotel chain.*

Here are some comparables:

<table>
<thead>
<tr>
<th>Hotel Name</th>
<th>Beta of Stock</th>
<th>Debt-Equity Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hilton</td>
<td>0.78</td>
<td>0.16</td>
</tr>
<tr>
<td>Holiday Inn</td>
<td>1.56</td>
<td>2.36</td>
</tr>
<tr>
<td>Ramada</td>
<td>1.05</td>
<td>1.18</td>
</tr>
</tbody>
</table>

Assume that the debt of these hotel chains is risk free. Your hotel chain will have business risk that is similar to these three.

The risk-free rate is 6% and the expected market risk premium is 8%.

*Estimate the cost of capital for the new hotel chain.* (Ignore taxes.)
• Unlevering beta exemplifies a general strategy

• Calculate an overall $\beta$ from the $\beta$ of components
  – $\beta_{\text{assets}}$ from $\beta_{\text{debt}}$ and $\beta_{\text{equity}}$ (or more)
  – $\beta_{\text{conglomerate}}$ from $\beta_{\text{division 1}}$, $\beta_{\text{division 2}}$ …
  – using value weights on the components

• or vice versa … use overall $\beta$ and some components to figure out the $\beta$ of the remaining component

• You can also start from a rate of return and “back out” the $\beta$ that is implied by the CAPM
  – see the review problems for this week!

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Summary

Key Concepts:

*BY THE LAW OF LARGE NUMBERS, WE CAN DIVERSIFY AWAY ALL SPECIFIC RISK.*

The risk of a well-diversified portfolio depends only upon the economy-wide risk of the securities included in the portfolio. This is measured by the average $\beta$.

Definitions:

The security market line (SML) has the following form:

$$ E[R_I] - R_f = \beta_I (E[R_M] - R_f) $$

Notation:

The formula for the best fit is $\beta = \frac{\rho_{\text{sec}, \text{sec}}}{\sigma_{\text{sec}}^2}$.

Divide the risk of an individual security into economy-wide risk and specific risk. In a regression framework:

$$ R_i = \alpha_i + \beta_i \sigma_M + \varepsilon_i $$

The variance of a well-diversified portfolio only depends on portfolio $\beta$ and the market variance:

$$ \sigma_P^2 = [\beta]^2 \sigma_M^2 $$